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Interval State Estimation of Distribution Network With Power Flow Constraint

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ABSTRACT Currently, distribution network is faced with many problems, e.g., low automation coverage and less data acquisition. There are also lots of challenges in state estimation, such as imprecise approximation of network parameters and measurement devices as well as integration of distributed generations. In order to deal with these problems of uncertainties in distribution network, an interval state estimation with power flow constraint is proposed in this paper, which is based on the quantitative description of the uncertain parameters, distributed generations, and system measurements with interval numbers. Given the hybrid measurement data, an interval linear state estimation model is established. In order to estimate state values precisely, an iterative Krawczyk algorithm is proposed to optimize this model. Furthermore, power flow constraint is introduced into the original equations of the interval state estimation model to improve the computation speed and accuracy. Modified IEEE 57-bus system is used to verify the effectiveness of the proposed method. Taking the results of Monte Carlo simulation as actual values, the proposed method performs better both in convergence and estimation accuracy compared with the existing unconstrained interval solving method.

INDEX TERMS Interval estimation, power flow constraint, network uncertainties, Krawczyk algorithm.

I. INTRODUCTION

With the distributed generations(DG), electric vehicles(EV), other active loads and a large number of measurement devices connected to the network, the penetration of renewable energy in the distribution network is getting higher and higher, resulting in that the traditional radial distribution network is gradually transforming to the active distribution network [1]. The traditional state estimation becomes difficult to meet the current development needs either in calculation speed or estimation accuracy [2]. Therefore, it is very important to build a new state estimation model to provide reliable data for the active distribution network [3], [4].

Recently, many achievements have been made on the state estimation of distribution network [5]–[9]. The transmission grid state estimation model was proposed and introduced into the distribution network in [5]–[6]. The application of branch current as state variable and its correction algorithm in distribution network state estimation was studied in [7] and [8]. A state estimation method using the square of branch current amplitude and the branch injection power as state variables was proposed in [9], which was capable of resisting bad data by using the exponential objective function.

However, the changes in the field environment and operation conditions, equipment aging and other reasons can bring the error of the network parameters, which results in the inaccuracy of the state estimation. In addition, large-scale EV random charging and high-permeability DG intermittent connected further increase the randomness of the distribution network and the difficulty of estimating the uncertain variables. The aforementioned factors make it non-trivial in practice, so state estimation of distribution network needs to have the ability of handling uncertainties [10]–[15]. Therefore, it is necessary to propose reasonable mathematical model for the uncertainties of distribution network and find a suitable solution algorithm.

The uncertainties of distribution network have always been a difficult part in state estimation. There are three main modeling methods for the uncertain variables in the distribution network:

(1) Stochastic state estimation method, which uses the probabilistic model to deal with the random information [16], [17];

(2) Fuzzy mathematics method, which establishes the distribution network state estimation model with fuzzy numbers, and handles the uncertain information with the fuzzy membership functions [18], [19];

(3) Interval analysis method, which uses the interval mathematics to solve the problem. Although the exact values of network parameters and measurements cannot be obtained, in most cases the upper and lower limits are available. Thus, after describing and modeling the uncertain variables in the form of interval numbers, the results obtained by state estimation are also interval forms, which can provide more intuitive information as upper and lower bounds of system state [20]–[22].

The interval analysis method was first proposed by Moore in 1966, which has been becoming a very important branch of computational mathematics [20]. Compared with the stochastic state estimation method, the interval analysis method has less computational cost. Meanwhile, there is no predefined fuzzy membership function compared to the fuzzy mathematics method. Therefore, the results are not affected by any hand-craft factors. Interval analysis method as a possibility method describing the uncertainties of distribution network with the interval numbers, has established a complete set of interval operation methods and rules [20]. Considering the advantages of this method, it has been widely applied in the state estimation of distribution network [21]. Based on the upper and lower limits of hybrid measurement data, a linear state estimation model was proposed and solved by interval Gaussian elimination(IGE) method in [22]. Although the IGE solving method certainly contains all the feasible solutions, the results are always the extended intervals, which are larger than the intervals of feasible values. This is called the conservatism of interval solving method. The conservative problem is the key factor that restricts the accuracy of interval state estimation, so it is necessary to find out a proper method to cope with this problem.

Due to the connection of DG and EV, the uncertainties of distribution network are increasing, which results in the aggravation of the conservative problem in the interval state estimation. Thus, partial results of interval state estimation usually do not satisfy the constraint of the power flow equations in the solving process [23]–[25]. At the same time, the accuracy and the calculation efficiency of the interval state estimation are not high enough, which is not suitable for practical application. In order to handle with these problems, an interval state estimation algorithm based on the Krawczyk operator and power flow constraint is employed to deal with the above-mentioned challenges. The main contributions of this paper can be summarized as follows:

(1) A linear interval state estimation model is proposed for hybrid measurement data, which considers the uncertainties of network parameters, distributed generations, electric vehicles and measurement data.

(2) To relieve the conservative problem in the interval state estimation, the Krawczyk algorithm is applied to solve the interval state estimation model, which can reduce the conservatism and obtain more accurate interval solution set. Moreover, to reduce the required iteration number, the solution of IGE algorithm is taken as the initialization of the Krawczyk algorithm.

(3) To mitigate the problem that partial results exceed the interval of feasible values and to ensure convergence speed, the power flow equations are used as the constraint condition of the interval state estimation algorithm.

This paper is organized as follows: In Section II, the traditional state estimation model with hybrid measurement data is demonstrated. In Section III, the state estimation model with interval set is proposed. In Section IV, the interval state estimation with Krawczyk algorithm and power flow constraint is studied. In Section V, a precision evaluation method is introduced. In Section VI, the case study of revised IEEE 57-bus system and numerical results are presented. The conclusions are noted in Section VII.

II. STATE ESTIMATION MODEL WITH HYBRID MEASUREMENT DATA

A. SCADA AND PMU HYBRID MEASUREMENT

The current measurement data in medium voltage distribution network are mainly PMU and SCADA. SCADA measurements are the bus voltage phasor, branch power and bus injection power, while PMU measurements are the bus voltage phasor and branch current phasor [26].

Compared with the existing SCADA measurement, PMU can realize the direct observation of bus with the characteristic of high measurement precision, short update cycle and small transmission delay, which can better monitor the changes of system state. However, due to technical and economic constraints, PMU is only configured in some key buses, which cannot satisfy the requirements of complete observation. Therefore, it is necessary to cooperate SCADA data with PMU data to realize the observable in state estimation [27], [28].

SCADA measurement types include the bus injection active power measurement P_i and reactive power measurement Q_i, active power measurement P_{ij} and reactive power measurement Q_{ij} of branch i-j, and voltage amplitude measurement $|\boldsymbol{U}_i|$ of bus i. The conversion formulae of different measurement types can be written as follows:

$$\boldsymbol{I}_{ij} = \frac{P_{ij}e_i + Q_{ij}f_i}{|\boldsymbol{U}_i|^2} + j\frac{P_{ij}f_i - Q_{ij}e_i}{|\boldsymbol{U}_i|^2} \tag{1}$$

$$I_i = \frac{P_i e_i + Q_i f_i}{|\boldsymbol{U}_i|^2} + j\frac{P_i f_i + Q_i e_i}{|\boldsymbol{U}_i|^2} \tag{2}$$

where $\boldsymbol{I}_{ij}$ is the current phasor of branch i-j; $\boldsymbol{I}_i$ is the current phasor of bus i; e_i and f_i are the real part and imaginary part of $\boldsymbol{U}_i$ respectively, meanwhile satisfying $U_i = e_i + jf_i$.

PMU measurement types are the amplitude and phase of bus voltages $\boldsymbol{U_i}$ and the amplitude and phase of branch current $\boldsymbol{I_{ij}}$, which can be expressed as:

$$\begin{aligned}\boldsymbol{I_{ij}} &= \left[e_i g_{ij} - f_i\left(b_{ii} + b_{ij}\right) - e_j g_{ij} + f_j b_{ij}\right] \\ &\quad + j\left[f_i g_{ij} + e_i\left(b_{ii} + b_{ij}\right) - e_j b_{ij} - f_j g_{ij}\right] \quad (3)\\ \boldsymbol{U_i} &= [0, \cdots 0, 1, 0, \cdots, 0]\boldsymbol{U} \quad (4)\end{aligned}$$

where b_{ii} is the imaginary part of self admittance phasor of bus i, g_{ij} and b_{ij} are the real part and imaginary part of admittance phasor of branch i-j respectively. $\boldsymbol{U} = [\boldsymbol{U}_1, \boldsymbol{U}_2, \cdots, \boldsymbol{U}_n]^{\mathrm{T}}$ is the node voltage vector and n is the number of buses. $[0, \cdots 0, 1, 0, \cdots, 0]$ is a vector whose i^{th} element is 1 and the remaining elements are 0.

For the actual network with SCADA and PMU measurements, the SCADA measurement can be converted into current phase measurements by using (1) and (2) for state estimation. Meanwhile, the current phasor measurement weight and the original measurement weight satisfy the function transfer relationship [29].

B. STATE ESTIMATION MODEL

In general, for any given topology, network parameters and hybrid measurement data, the mathematical model of state estimation can be expressed as:

$$z = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v} \quad (5)$$

where $z = [z_1, \cdots, z_m]^{\mathrm{T}}$ is the vector of all measurements with m-dimensional. H is the measurement coefficient matrix depending on the network parameters, with the size $m \times (2n-1)$. $x = [\boldsymbol{x}_1, \cdots, \boldsymbol{x}_{2n-1}]^{\mathrm{T}}$ is the vector of all state variables with $(2n-1)$-dimensional, usually $m \geq (2n-1)$. $\boldsymbol{v}$ represents the error vector of the measurements with m-dimensional.

When the amplitude and phase measurement of each bus voltage and each branch current are known, equation (5) can be expanded as the form of block matrix:

$$\begin{bmatrix} z_e \\ z_f \\ z_{Ire} \\ z_{Iim} \end{bmatrix} = \begin{bmatrix} \frac{\partial z_e}{\partial e} & \frac{\partial z_e}{\partial f} \\ \frac{\partial z_f}{\partial e} & \frac{\partial z_f}{\partial f} \\ \frac{\partial z_{Ire}}{\partial e} & \frac{\partial z_{Ire}}{\partial f} \\ \frac{\partial z_{Iim}}{\partial e} & \frac{\partial z_{Iim}}{\partial f} \end{bmatrix} \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{f} \end{bmatrix} + \begin{bmatrix} \boldsymbol{v_e} \\ \boldsymbol{v_f} \\ \boldsymbol{v_{Ire}} \\ \boldsymbol{v_{Iim}} \end{bmatrix} \quad (6)$$

where z_e and z_f are the real part and imaginary part of bus voltage phasor measurements. $\boldsymbol{z_{Ire}}$ and $\boldsymbol{z_{Iim}}$ are the real part and imaginary part of branch current phasor measurements. $\boldsymbol{e}$ are the real part of voltage variables of reference bus and the other buses. $\boldsymbol{f}$ are the imaginary part of voltage variables of the other buses. $\boldsymbol{v_e}$ and $\boldsymbol{v_f}$ are the real part and imaginary part of voltage measurement error vector. $\boldsymbol{v_{Ire}}$ and $\boldsymbol{v_{Iim}}$ are the real part and imaginary part of current measurement error vector.

Obviously, the measurement coefficient matrix $\boldsymbol{H}$ is a constant matrix and the expression of each element can be obtained by (3) and (4). Thus, equation (6) is the linear system of equations.

C. WEIGHTED LEAST SQUARES(WLS) SOLUTION METHOD

The aim of state estimation based on WLS criteria is to find the solution that minimizes the following objective function developed from (5):

$$\min \boldsymbol{f}(\boldsymbol{x}) = [z - \boldsymbol{H}\boldsymbol{x}]^{\mathrm{T}} \cdot \boldsymbol{W}^{-1} \cdot [z - \boldsymbol{H}\boldsymbol{x}] \quad (7)$$

where $\boldsymbol{W}^{-1} = \boldsymbol{diag}(1/\sigma_1^2, 1/\sigma_2^2, \cdots, 1/\sigma_m^2)$ is the m-order weight diagonal square of the measurements which is generally assigned to the reciprocal of the corresponding measured variance.

The optimal estimate $\boldsymbol{x}^*$ of this unconstrained optimization problem is given by:

$$\boldsymbol{x}^* = [\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{W}^{-1}\boldsymbol{H}]^{-1}\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{W}^{-1}\boldsymbol{z} \quad (8)$$

III. INTERVAL STATE ESTIMATION MODELWITH HYBRID MEASUREMENT DATA

The difference between the interval state estimation established in this paper and the traditional state estimation is: The traditional state estimation algorithm always tries to find the optimal solution under WLS or other estimation criteria, and applies the optimal solution as the ''surrogate value'' of the true system state. However, the selection of such estimation criteria is subjective, often based on certain assumptions, such as normal distribution of measurement errors [30]. As a result, the information obtained by this way is not necessarily established in practice.

In order to better reflect the influence of uncertain factors on the state estimation results, and to improve the engineering application value, the interval state estimation model is established. In this model, all buses power injection pseudo measurement and measurement error no longer follow the normal distribution but are described as interval numbers. For real-time measurement information, due to the high precision, the interval values are obtained by adding a small $\pm\delta\%$ deviation to the measurements. For pseudo measurements such as distributed generations and loads, we use neural network model [31] to predict the state and also add an $\pm$ average prediction error to generate the interval values. The solution results are also interval numbers, which can provide the dispatchers with effective information on the ''boundary'' of system state.

Therefore, once all variables are expressed by interval numbers, the traditional state estimation solution algorithm such as WLS method is no longer applicable.

A. INTERVAL ARITHMETIC

In this section, the uncertainties of parameters and measurements are expressed as the upper and lower limits [32].

For convenience, the following square brackets variables are interval variables and can be mathematically defined as:

$$[\boldsymbol{x}] = [\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}] = \{\boldsymbol{x} \in \boldsymbol{R} | \underline{\boldsymbol{x}} \leq \boldsymbol{x} \leq \overline{\boldsymbol{x}}\} \quad (9)$$

where $\underline{x}, \bar{x}$ are the lower bound and upper bound of the interval $[x]$ respectively. Especially, when $\underline{x} = \bar{x}$, $[x]$ becomes point number, that is, the normal real number. The common operations of the interval number are:

$$[\boldsymbol{x}]\#[\boldsymbol{y}] = \{\boldsymbol{x}\#\boldsymbol{y} | \boldsymbol{x} \in [x], \boldsymbol{y} \in [\boldsymbol{y}]\} \tag{10}$$

where $\# \in \{+, -, \times, \div, \leq, \cup, \cap\}$.

In addition, the interval width of $[\boldsymbol{x}]$ refers to $\boldsymbol{Wid}([\boldsymbol{x}])$ and the absolute value refers to $|[\boldsymbol{x}]|$, which are defined as:

$$\boldsymbol{Wid}([\boldsymbol{x}]) = \bar{\boldsymbol{x}} - \underline{\boldsymbol{x}}, \quad |[\boldsymbol{x}]| = \max(|\bar{\boldsymbol{x}}|, |\underline{\boldsymbol{x}}|) \tag{11}$$

Similar as the real vector, the interval vector is defined to be a vector with interval components. For an n-dimensional interval vector, its value space can be denoted by $\boldsymbol{IR^n}$. According to [33], a norm of an interval vector $[\boldsymbol{x}] \in \boldsymbol{IR}^{n\times 1}$ is given by:

$$\|[\boldsymbol{x}]\| = \max\{|[\boldsymbol{x}_i]| : i = 1, 2, \ldots, k\} \tag{12}$$

where k is the dimension of the interval vector.

B. INTERVAL STATE ESTIMATION MODEL

According to the interval arithmetic, the traditional state estimation model can be transformed into an interval model. In this section, the uncertainties of measurement data, network parameters and distributed generations are considered, which can be mathematically described as:

$$\begin{aligned}[z] &= \left[[z_e], [z_f], [z_{\boldsymbol{Ire}}], [z_{\boldsymbol{Iim}}]\right]^{\mathrm{T}} \\ &= \{z \in \boldsymbol{R}^{m\times 1} : \underline{z}_j \leq z_j \leq \bar{z}_j, j = 1, 2, \ldots, m\} \end{aligned} \tag{13}$$

$$\begin{aligned}[\boldsymbol{H}] &= \begin{bmatrix} [\boldsymbol{h}_{1,1}] & \cdots & [\boldsymbol{h}_{1,2n-1}] \\ \vdots & \ddots & \vdots \\ [\boldsymbol{h}_{m,1}] & \cdots & [\boldsymbol{h}_{m,2n-1}] \end{bmatrix} \\ &= \{\boldsymbol{h} \in \boldsymbol{R}^{m\times(2n-1)} : \underline{\boldsymbol{h}}_{i,j} \leq \boldsymbol{h}_{i,j} \leq \overline{\boldsymbol{h}}_{i,j}, \\ &\qquad i = 1, 2, \ldots, 2n-1, j = 1, 2, \ldots, m\} \end{aligned} \tag{14}$$

$$[\boldsymbol{x}] = [[\boldsymbol{e}], [\boldsymbol{f}]]^{\mathrm{T}} = \{\boldsymbol{x} \in \boldsymbol{R}^{(2n-1)\times 1} : \boldsymbol{Hx} \in [z]\} \tag{15}$$

where $\boldsymbol{h_{i,j}}$ is each element of matrix $\boldsymbol{H}$.

Equation (13) is the expression of measurement vector in the interval state estimation model, which represents the measurement information of node voltage and branch current depicted with interval numbers. Equation (14) is the expression of measurement coefficient matrix in the interval model related to network parameters. Equation (15) represents that the real and the imaginary part of node voltage is taken as the state variables in the interval model.

When all variables are expressed by interval numbers, the objective function in (7) is not applicable. This is because the measurement error of the buses in the interval state estimation model no longer obeys specific distribution. It has been quantitatively analyzed and described by the interval numbers. Meanwhile, the state variables are also interval numbers, rather than the optimal solution under WLS or other estimation criteria. Thus, the whole interval state estimation model in this paper can be formed as:

$$[\boldsymbol{H}][\boldsymbol{x}] = [z] \tag{16}$$

Since the dimension of the measurements is larger than the dimension of the system state variables, this kind of problem is subordinate to the modeling and solution of interval over-determined equations. It is generally believed that establishing a unified analytical expression and standard analysis method is difficult. A method proposed in [34] is applied to convert the over-determined equations into the following equations with square matrix as coefficient matrix:

$$\begin{bmatrix} [\boldsymbol{H}] & -\boldsymbol{1} \\ \boldsymbol{0} & [\boldsymbol{H}]^{\mathrm{T}}\boldsymbol{W}^{-1} \end{bmatrix} \begin{bmatrix} [\boldsymbol{x}] \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} [\boldsymbol{Z}] \\ \boldsymbol{0} \end{bmatrix} \tag{17}$$

where $-\boldsymbol{1}$ and $\boldsymbol{0}$ indicate an identity matrix and a zero matrix of appropriate dimension respectively.

Equation (17) can be thought as the following form which is linear equation with interval element:

$$[\boldsymbol{A}][\boldsymbol{x}] = [\boldsymbol{b}] \tag{18}$$

where $[\boldsymbol{A}]$ is a square matrix of size $(m + 2n - 1) \times (m + 2n - 1)$. $[\boldsymbol{x}]$ and $[\boldsymbol{b}]$ are both a vector with $(m + 2n - 1)$-dimensional.

C. IGE SOULUTION METHOD OF INTERVAL ARITHMETIC

Currently, the solution algorithm of the interval linear equation mainly includes the optimization method [35], the linear programming method [36] and the interval analysis method [37]. However, due to the first two methods need to focus on the objective function, the solving process is relatively complex. The interval analysis method only needs one calculation to complete the interval estimation of state quantities, so it has been widely studied. In the previous research literatures [38], [39], IGE was taken as the interval analysis solving method. This method is based on the traditional Gaussian method, where the interval numbers are employed to replace the point value. The coefficient matrix can be formed and converted to the upper triangular matrix in the usual way but with interval arithmetic. The whole process can be divided into three steps:

(1) Equation (18) can be expanded as the following form:

$$\begin{bmatrix} [\boldsymbol{a}_{1,1}] & \cdots & [\boldsymbol{a}_{1,\boldsymbol{m}+2\boldsymbol{n}-1}] \\ \vdots & \ddots & \vdots \\ [\boldsymbol{a}_{\boldsymbol{m}+2\boldsymbol{n}-1,1}] & \cdots & [\boldsymbol{a}_{\boldsymbol{m}+2\boldsymbol{n}-1,\boldsymbol{m}+2\boldsymbol{n}-1}] \end{bmatrix} \begin{bmatrix} [\boldsymbol{x}_1] \\ \vdots \\ [\boldsymbol{x}_{\boldsymbol{m}+2\boldsymbol{n}-1}] \end{bmatrix} = \begin{bmatrix} [\boldsymbol{b}_1] \\ \vdots \\ [\boldsymbol{b}_{\boldsymbol{m}+2\boldsymbol{n}-1}] \end{bmatrix} \tag{19}$$

(2) Equation (19) can be converted to the following upper triangular form by matrix row and column transformation:

$$\begin{bmatrix} [\boldsymbol{a}'_{1,1}] & \cdots & [\boldsymbol{a}'_{1,\boldsymbol{m}+2\boldsymbol{n}-1}] \\ \vdots & \ddots & \vdots \\ 0 & \cdots & [\boldsymbol{a}'_{\boldsymbol{m}+2\boldsymbol{n}-1,\boldsymbol{m}+2\boldsymbol{n}-1}] \end{bmatrix} \begin{bmatrix} [\boldsymbol{x}_1] \\ \vdots \\ [\boldsymbol{x}_{\boldsymbol{m}+2\boldsymbol{n}-1}] \end{bmatrix} = \begin{bmatrix} [\boldsymbol{b}'_1] \\ \vdots \\ [\boldsymbol{b}'_{\boldsymbol{m}+2\boldsymbol{n}-1}] \end{bmatrix} \tag{20}$$

(3) Equation (20) can be solved as follow interval algebraic equations:

$$\begin{cases} [x_{m+2n-1}] = [b'_{m+2n-1}]/[a'_{m+2n-1,m+2n-1}] \\ \quad\vdots \\ [x_1] = \dfrac{([b'_1] - [a'_{1,2}][x_2] - \cdots - [a'_{1,m+2n-1}][x_{m+2n-1}])}{[a'_{1,1}]} \end{cases} \tag{21}$$

D. CONSERVATISM OF INTERVAL ARITHMETIC

It is important to note that the solution of the interval linear equations is completely different from that of the ordinary linear equations. First, the solution set for interval linear equations has very complex non-convex structure that cannot be easily characterized by interval vector. Figure 1 illustrates the solution of two-dimensional interval equations. The blue area in Figure 1 is the exact feasible solution set $[\boldsymbol{S}]$. It can be seen from the figure that the set $[\boldsymbol{S}]$ cannot be characterized as an interval. However, it is possible to find the shell of $[\boldsymbol{S}]$, where the shell is defined as the minimum interval vector that contains $[\boldsymbol{S}]$, which is illustrated as the square area surrounded by the black dotted line in Figure 1. The results of the IGE method and the Krawczyk algorithm are also represented in Figure 1 with the red dotted line and the green dotted line respectively.

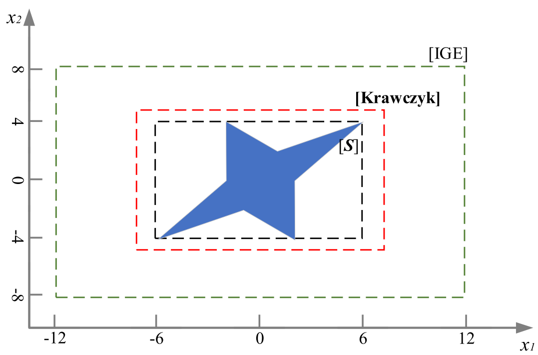

FIGURE 1. Solution of 2 by 2 interval equations.

It can be seen that the interval solution set obtained by IGE method is much larger than that by Krawczyk algorithm. Considering the solving target of the interval analysis method is to obtain the minimum interval vector containing $[\boldsymbol{S}]$ as much as possible, obviously, IGE method is always too conservative to maintain its results high precision.

IV. INTERVAL STATE ESTIMATION ALGORITHM WITH KRAWCZYK OPERATOR AND POWER FLOW CONSTRAINT

In this section, the improvement of the interval analysis method by using Krawczyk algorithm is discussed. In addition, the power flow constraint is considered to further meliorate the accuracy and efficiency of the algorithm.

A. POWER FLOW CONSTRAINT

In practice, the linear interval state estimation with Krawczyk operator are faced with severe challenges in terms of accuracy and speed when the network is complex. The width of the solution set computed by Krawczyk algorithm is always larger than the exact feasible solution $[\boldsymbol{S}]$. Thus, partial results in the solution set cannot satisfy the constraint of the power flow equations. In order to solve this problem, the relation information of state variables in power flow equations is added into the interval state estimation. More specifically, the equality power flow constraint is introduced into the interval state estimation.

Therefore, the state estimation model of distribution network with power flow constraint is adopted:

$$\begin{bmatrix} [\boldsymbol{H}] & -\boldsymbol{1} \\ \boldsymbol{0} & [\boldsymbol{H}]^T \boldsymbol{W}^{-1} \end{bmatrix} \begin{bmatrix} [\boldsymbol{x}] \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} [\boldsymbol{Z}] \\ \boldsymbol{0} \end{bmatrix} \quad \text{s.t.} \ \boldsymbol{c}([\boldsymbol{x}]) = \boldsymbol{0} \tag{22}$$

The equality constraint (22) represents the AC power flow equations of the distribution network, and its two important functions are summarized as follows:

(1) Increase the accuracy of the interval solving method. The electrical relationship implied by each state variable can be included in the state estimation process by adding the power flow constraint. In the iterative process, as long as the solution conforms to the power flow constraint, it belongs to the solution set $[\boldsymbol{S}]$. Under the constraint of power flow equations, the deviation of the solution is limited to a certain space, and the estimated results are closer to the actual operation state of the distribution network. These factors are supposed to jointly improve the accuracy of the interval state estimation.

(2) Improve the speed of the interval solving method. Compared with the iterative process that only uses Krawczyk operator, adding power flow constraint theoretically filters out some infeasible solution that does not conforms to the actual constraint. It accelerates the convergence of the iterative interval set. Therefore, the less limited iteration step and the faster calculation speed given the same accurate interval solution can be achieved.

B. KRAWCZYK SOLUTION OF INTERVAL ARITHMETIC

The Krawczyk algorithm was developed by Krawczyk in 1969. The main idea of this algorithm is utilizing the Krawczyk operator for iteration to approximate the solution shell. Due to the iteration, the conservatism of Krawczyk algorithm is much smaller than IGE method, which means the results of the Krawczyk algorithm can be more accurate [40]–[45].

The detail process of Krawczyk algorithm can be summarized as follows:

(1) Take any $\boldsymbol{A} \in [\boldsymbol{A}]$ and $\boldsymbol{b} \in [\boldsymbol{b}]$, $\boldsymbol{A}^{-1}\boldsymbol{b} \in [\boldsymbol{x}]$ according to (18).

(2) Then according to [34], a specific $\boldsymbol{C} \in \boldsymbol{R}^{(m+2n-1)\times(m+2n-1)}$ can be found that makes $\boldsymbol{A}^{-1}\boldsymbol{b}$ can be further

expanded to (23).

$$A^{-1}b = Cb - (CA - I)A^{-1}b \tag{23}$$

where I is a $(m+2n-1) \times (m+2n-1)$ unit matrix.

(3) Here, C can be the inverse of the midpoint matrix of $[A]$:

$$C = (Mid([A]))^{-1} \tag{24}$$

$$Mid([A]) = \begin{bmatrix} Mid([a_{1,1}]) & \cdots & Mid([a_{1,m+2n-1}]) \\ \vdots & \ddots & \vdots \\ Mid([a_{m+2n-1,1}]) & \cdots & Mid([a_{m+2n-1,m+2n-1}]) \end{bmatrix} \tag{25}$$

where $Mid()$ is the median function of interval number.

(4) When equation (23) satisfies the following condition: $A^{-1}b = Cb - (CA - I)A^{-1}b \in C[b] - (C[A] - I)[x]$, the Krawczyk operator $K_{operator}$ can be used to obtain the following iterative equation that can approximate the solution set $[S]$:

$$\begin{aligned} K_{operator} &= (C[b] - (C[A] - I)[x^k]) \\ [x^{k+1}] &= K_{operator} \cap [x^k] \end{aligned} \tag{26}$$

where $[x^k]$ is the solution of iteration k.

Thus, substituting (17), the iterative equation can be expressed as follow:

$$[x^{k+1}] = (C\begin{bmatrix}[z] \\ 0\end{bmatrix} - (C\begin{bmatrix}[H] & -1 \\ 0 & [H]^T W^{-1}\end{bmatrix} - I)[x^k]) \cap [x^k] \tag{27}$$

According to [46], Krawczyk operator at iteration k is a set containing all feasible solutions and the interval width is always less than that of $[x^{k-1}]$. Therefore, with the iteration of (27), the interval width of the solution set $[x]$ decreases and gradually approaches the shell of $[S]$.

(5) When the amplitude of the infinite norm of the interval solution vector $[x^k]$ decreases to the convergence criterion, the iteration will be stopped, which is:

$$\sum_{i=1}^{n} ||Wid([x^k])|| - \sum_{i=1}^{n} ||Wid([x^{k+1}])|| < \varepsilon \tag{28}$$

where $||Wid([x])||$ is the interval width of $[x]$. ε is a given small positive number, usually taking 10^{-6}.

In addition, it can be noted that the initial value should be set to cover the entire feasible solutions. Due to the common equation for the initial value of Krawczyk algorithm is relatively complex, and considering the conservatism of IGE method, the state calculated by IGE method can be directly taken as the initial value of the Krawczyk algorithm in order to start iteration more quickly.

C. ALGORITHM FLOW

In summary, the detailed steps of the interval state estimation with Krawczyk operator and power flow constraint are shown in Figure 2.

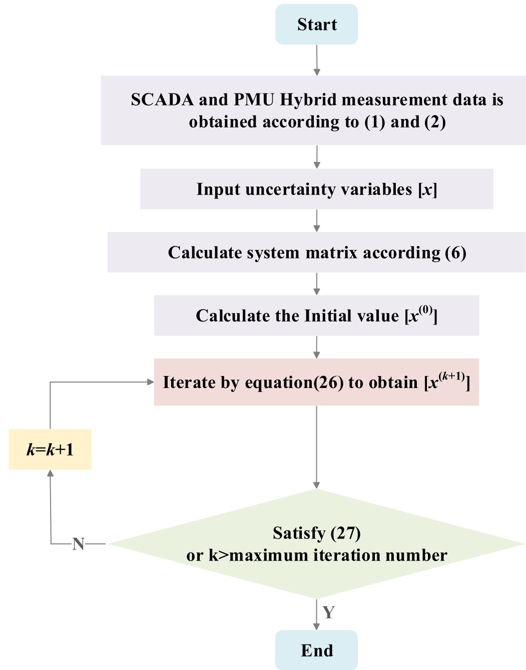

FIGURE 2. Flow chart of interval state estimation.

Step 1: Get the fusion measurement data according to the equation (1) and (2), and input the measurement interval values of the original parameters.

Step 2: Select the state variables, which is the bus voltage phasor in this paper.

Step 3: Calculate the interval expression of (7) for state estimation of distribution system.

Step 4: Calculate the initial value $[x^0]$ by IGE method and substitute it into the iterative equation (27) to obtain $[x^{k+1}]$.

Step 5: Determine whether the iteration has converged using the convergence criterion. In this paper, the criterion of convergence is given in (28).

Step 6: If not convergent, $[x^{k+1}]$ is used as the initial approximate value of the iterative equation, and the next iteration is started from Step 4. The process ends when the convergence criterion is reached. Finally, the estimated value of the interval state estimation of distribution network can be obtained.

V. PRECISION EVALUATION METHOD

In this paper, Monte Carlo method is employed to evaluate the precision of the algorithm [47], [48]. The Monte Carlo method first samples the uncertain parameters to carry out the point state estimation, and the result is considered as one feasible value. Then the sampling process is taken for many times and it can get the corresponding state estimation results. Finally, the intervals which can include all those estimation results are defined as the optimal intervals. The accuracy of two interval state estimation methods with and without power flow constraints are compared respectively. More concretely, the following two indicators are used to evaluate the precision of the results:

$$W_1 = \frac{1}{2n-1}\sum_{i=1}^{2n-1}(\overline{x_i} - \underline{x_i}) \tag{29}$$

$$W_2 = \max(\overline{x_i} - \underline{x_i}) \tag{30}$$

W_1 is the average value of the interval width and W_2 is the maximum value of the interval width. The smaller the W_1 and W_2 are, the more accurate the interval algorithm is.

VI. CASE STUDIES

A. TEST SYSTEM

In order to verify the accuracy of the proposed method in distribution network, the interval estimation is conducted on the IEEE 57-bus system as shown in Figure 3. Bus 19 and Bus 35 are connected to a PV station with rated capacity of 500kW, Bus 22 and Bus 53 is connected to a wind turbine with rated capacity of 1000kW.

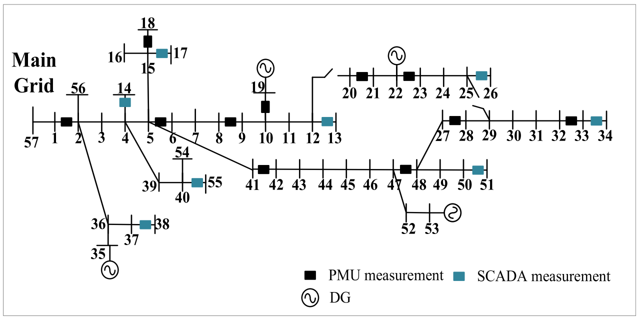

FIGURE 3. IEEE 57-bus test system.

In practice, the uncertainty of measurement and parameters are various. The uncertainty variables considered in this paper include voltage, current and power data, network parameters and distributed generations. The standard deviations for each type are listed as follows [22]:

(1) For the PMU measurement data, the standard deviation of current measurement is 0.4% with respect to(w.r.t.) the measured values. The standard deviation of voltage measurement is 0.8% w.r.t. the measured values.

(2) For the SCADA measurement data, the standard deviations of active and reactive power measurement are 1% and 1.5% w.r.t. the measured values respectively. The standard deviation of voltage amplitude measurement is 1% w.r.t. the measured values.

(3) For network parameters, the standard deviation of conductance is 2% w.r.t. the measured values. The standard deviation of susceptance is 3% w.r.t. the measured values.

(4) For other pseudo measurement data such as distributed generations and loads, the standard deviation is 8% w.r.t. the actual values [49].

B. RESULTS COMPARISON BETWEEN IGE AND KRAWCZYK

In this section, the comparison is made by considering the real part of voltage. In addition, Monte Carlo results are taken as the actual values. The indicators used here are formulated in (29) and (30) to measure the accuracy of the results obtained from the IGE and Krawczyk methods respectively. The evaluation index of different methods is shown in Table 1.

TABLE 1. Comparison between interval state estimation with IGE and Krawczyk methods.

	Krawczyk method without power flow constraint		IGE method	
Evaluation index	W_1/p.u	W_2/p.u	W_1/p.u	W_2/p.u
	0.0147	0.0187	0.0214	0.0259

It can be observed from the evaluation index that W_1 is 0.0214 and W_2 is 0.0259 when using IGE method whereas W_1 and W_2 are 0.0147 and 0.0187 respectively when using Krawczyk algorithm without power flow constraint. The results indicate that compared with IGE method, Krawczyk algorithm can obtain a more accurate solution of the interval equations.

C. THE EFFECT OF POWER FLOW CONSTRAINT

In this section, the influence of the power flow constraint on the accuracy is analyzed. Taking Bus 2 as an example, the interval voltage obtained by Monte Carlo method is $[1.009, 1.011]\angle[-13.003, -12.455]$, and the interval width of amplitude is 0.002. The interval voltage computed by the Krawczyk algorithm without power flow constraint is $[1.004, 1.017]\angle[-13.247, -12.205]$, and the interval width of amplitude is 0.013. The interval voltage by Krawczyk algorithm with power flow constraint is $[1.008, 1.012]\angle[-13.127, -12.320]$, and the interval width of amplitude is 0.004. Compared to the Krawczyk algorithm without power flow constraint, the interval width of the Krawczyk algorithm with power flow constraint reduces 69.23% relatively.

The evaluation index is shown in Table 2.

TABLE 2. Comparison between interval state estimation with and without power flow constraint.

	Krawczyk method with power flow constraint		Krawczyk method without power flow constraint	
Evaluation index	W_1/p.u	W_2/p.u	W_1/p.u	W_2/p.u
	0.0082	0.0119	0.0147	0.0187

According to the Table 2, when using the Krawczyk algorithm without power flow constraint, W1 is 0.0147 and W2 is 0.07, whereas W1 and W2 are 0.0082 and 0.0119 respectively for the Krawczyk algorithm with power flow constraint.

It is obvious that among the aforementioned two methods, the Krawczyk algorithm with power flow constraint generates more accurate results w.r.t. the actual values, which indicates the Krawczyk algorithm with power flow constraint performs better for interval state estimation of distribution network.

D. ACCURACY UNDER VARIOUS PMU AND SCADA DATA ERRORS

The influence brought by PMU and SCADA measurement accuracy on the performance of the algorithm is also analyzed. The errors of PMU and SCADA data are set to be the two times, four times, six times and eight times the width of the uncertainty interval respectively. The evaluation results are shown in Table 3.

TABLE 3. Comparison under various PMU and SCADA data errors.

	Measurement Accuracy (the width of uncertainty interval)	W_1/p.u	W_2/p.u
Krawczyk method without power flow constraint	[$-2\delta,+2\delta$]	0.0208	0.0511
	[$-4\delta,+4\delta$]	0.0495	0.0941
	[$-6\delta,+6\delta$]	0.0759	0.1680
	[$-8\delta,+8\delta$]	0.1511	0.2717
Krawczyk method with power flow constraint	[$-2\delta,+2\delta$]	0.0174	0.0367
	[$-4\delta,+4\delta$]	0.0323	0.0689
	[$-6\delta,+6\delta$]	0.0492	0.1063
	[$-8\delta,+8\delta$]	0.1204	0.1942

(δ is the initial error width of every measurement.)

It can be noticed that the increase of uncertainties enlarges the variations of the estimated values of state variables. Even so, compared to the Krawczyk method without constraint, the proposed method still gives tighter outer solutions in this test case.

E. THE ACCURACY UNDER VARIOUS DG DATA ERRORS

In order to investigate the influence of DG pseudo measurement accuracy on the state estimation of distribution network, the maximum interval width of bus voltage amplitude is reported in Table 4 where the measurement errors are 25%, 50% and 75%, respectively.

TABLE 4. Maximum Interval Width(W_2) under Various DG Data Errors.

Measurement error	Krawczyk method without power flow constraint	Krawczyk method with power flow constraint
25%	0.15	0.12
50%	0.35	0.27
75%	0.54	0.41

Compared with the method without power flow constraint, using power flow equations as the equality constraint, the interval width is narrower, so the conservatism is lower, and the accuracy of the estimation results is higher. However, with the growth of the measurement error, the uncertain region gradually increases, which brings in more difficulty for convergence of the interval state estimation. It can be seen from Table 4 that when the error is 75%, the maximum interval width is beyond the normal range. Therefore, the interval state estimation algorithm proposed in this paper perform better as DG data errors decrease. Basing on above observation, we claim that if some PMUs can be installed at the DG buses in real application, our algorithm is supposed to be more practical.

F. PERFORMANCE ANALYSIS

In order to verify the universality of the algorithm for different types and different scales of power grid, the performance of the algorithm is tested on IEEE 57-bus distribution network and IEEE 300-bus system. The results accuracy, iteration numbers and the time consumption are calculated respectively under the uncertainty width varying from twice to eight times of the original set width. The results are shown in Table 5.

TABLE 5. Performance analysis.

Case		The width of uncertainty interval	W_1 /p.u	Iteration numbers	Calculating time/ms
IEEE57	Krawczyk method without power flow constraint	[$-2\delta,+2\delta$]	0.0208	4	237.68
		[$-4\delta,+4\delta$]	0.0495	7	250.56
		[$-6\delta,+6\delta$]	0.0759	10	281.04
		[$-8\delta,+8\delta$]	0.1511	13	321.23
	Krawczyk method with power flow constraint	[$-2\delta,+2\delta$]	0.0174	4	156.29
		[$-4\delta,+4\delta$]	0.0323	6	193.26
		[$-6\delta,+6\delta$]	0.0492	9	221.49
		[$-8\delta,+8\delta$]	0.1204	11	279.37
IEEE300	Krawczyk method without power flow constraint	[$-2\delta,+2\delta$]	0.0287	7	1821.89
		[$-4\delta,+4\delta$]	0.0510	10	2121.88
		[$-6\delta,+6\delta$]	0.1072	12	2591.23
		[$-8\delta,+8\delta$]	0.2141	16	3014.46
	Krawczyk method with power flow constraint	[$-2\delta,+2\delta$]	0.0197	5	1072.96
		[$-4\delta,+4\delta$]	0.0345	8	1342.21
		[$-6\delta,+6\delta$]	0.0516	11	1674.16
		[$-8\delta,+8\delta$]	0.1323	13	2005.28

It can be observed that the computational complexity of the algorithm is reduced and the speed of the interval state estimation is improved by adding the power flow constraint. Also, compared with the method without power flow constraint, the estimation results of the algorithm with power flow constraint always maintains high precision.

VII. CONCLUSIONS

In this paper, the problem of network uncertainties and low redundancy in state estimation of distribution network has been studied. The interval state estimation algorithm based on Krawczyk operator and power flow constraint is proposed. The proposal has several advantages as follows:

The algorithm adopts SCADA and PMU hybrid measurement data which can make full use of existing measurements. Compare with IGE method that is too conservative to solve the interval state estimation model, adding our proposer greatly improve the accuracy of the estimation results. Compared with the method that do not constraint with power flow equations, our proposer improve not only the estimation accuracy but also the calculation speed.

In future work, we will further analyze and model for the situation of three phase unbalanced distribution network, system measurement outliers (bad data) and dynamic interval state estimation [50], [51]. We will also focus on speeding up online, real-time and practical application of interval state estimation for distribution network.

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