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Synchronization of Nonlinear Complex Spatio-Temporal Networks Using Adaptive Boundary Control and Pinning Adaptive Boundary Control

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ABSTRACT There are many behaviors with spatio-temporal characteristics in nature. Complex spatio-temporal networks (CSNs)-based on partial differential equations are to process such issues. This paper studies two synchronization boundary control methods. An adaptive boundary controller is first studied to ensure synchronization of the CSNs. Furthermore, a pinning adaptive boundary controller is proposed to achieve synchronization of the CSNs. Two sufficient criteria of synchronization and pinning synchronization are respectively obtained. Finally, two numerical examples demonstrate the effectiveness of the proposed theoretical results.

INDEX TERMS Complex networks, adaptive control, nonlinear systems.

I. INTRODUCTION

Complex networks (CNs) extensively exist in real life, such as social interaction networks, traffic networks, electrical power grids, ecosystems, coupled biological and chemical systems [1]–[4]. Over the past few decades, synchronization, as one of the most important behaviors of CNs, has received a great attention due to its wide applications in many areas, e.g., spacecraft formation [5], [6], multimedia [7], multi-agent cooperation [8]–[11], image encryption [12] and secure communication [13]. Many effective control methods for synchronization of CNs have been extensively reported in previous literatures [14]–[18]. Most of them naturally assumed the states of CNs only rely on the time.

In real practice, food webs, reaction-diffusion neural networks, biological systems, chemical process, neurophysiology and many other social networks, rely not only on the time but also on the spatial positions [19], [20]. These processes are usually modeled by CNs described as partial differential equations (PDEs) with spatio-temporal characteristics, named complex spatio-temporal networks (CSNs). The research of synchronization control of CSNs is challenging due to the infinite dimensional spatial characteristics of states [21].

Over the past few years, an array of important synchronization control methods of CSNs have been studied, e.g., scalar proportional control [22], [23], matrix proportional control [24]–[28], impulsive control [29], stochastic sampled-data control [30], intermittent control [31], P-sD control [32]. In real practice, sufficient large feedback gains may be taken, which is possibly much larger than the actual values in need. Adaptive control method can real time modify the feedback gains to avoid this problem [27]. Therefore, many researchers constructed adaptive controllers for CSNs. For example, Wang *et al.* proposed several adaptive controllers for synchronization, H_∞ synchronization and pinning synchronization of linearly coupled reaction-diffusion neural networks without and with hybrid coupling [19], [33]. In [34], adaptive control was developed for synchronization of stochastic reaction–diffusion neural networks with mixed time delays. In [35], an adaptive control algorithm was studied for synchronization in diffusively–coupled systems. In [36], adaptive control was developed for anti–synchronization and H_∞ anti-synchronization for memristive reaction-diffusion neural networks with mixed time delays. As a whole, the offered controllers above are feedback of the whole spatial states.

Boundary control, firstly applied to stability of PDE-based systems [37]–[39] and then to synchronization of CSNs, is implemented by only controlling the spatial boundary positions. For examples, state feedback boundary control based on distributed measurement is developed to achieve synchronization of CSNs [40]; using backstepping methods, boundary control based on distributed measurement was studied for synchronization of a coupled linear partial differential system [41]; using Lyapunov methods, boundary control based on distributed measurement was proposed for asymptotical synchronization of linear coupled time-delayed partial differential systems [42]; In [43], a boundary control based on boundary measurement method was studied for synchronization of CSNs. However, adaptive boundary control based on boundary measurement for synchronization of CSNs has not yet been considered. This is the first problem to be dealt with in this paper.

Actually, a real-world CN usually consists of a great many nodes, even thousands upon thousands. Therefore, it is hard to apply control actions to all nodes in practice. With the merit of overcoming this difficulty, pinning control approaches have attracted much interest in the past few years [44]–[48]. The strategy is effective since it controls only a small fraction nodes of networks. For CSNs, there are also many important literatures on pinning control in recent years. In [49], scalar proportional control method was studied for pinning synchronization of unbounded time delays reaction–diffusion neural networks. Aperiodically intermittent pinning controller was developed for synchronization of reaction-diffusion neural networks with hybrid coupling and time-varying delays [50]. In [51], a pinning-impulsive controller was studied for synchronization of reaction-diffusion neural networks with time-varying delays. As a whole, the offered control methods above are feedback of the whole spatial states [49]–[51]. Therefore, when there are many nodes while only spatial boundary states could be measured, it is important to further establish pinning adaptive boundary control based on boundary measurement, implemented by only controlling and measuring spatial boundary positions of a small fraction of nodes chosen before. This is the second main contribution of this paper.

Notations: The notations will be used as follows. I_n denotes identity matrix of order n. Matrix M is negative definite denoted by $M < 0$. $W_2^2([0, L]; R^{Nn})$ is a Sobolev space of absolutely continuous Nn-dimensional vector functions $\omega(x) : [0, L] \to \mathbb{R}^{Nn}$ with square integrable $\frac{d^k\omega(x)}{d\omega^k}$ of the order $k \geq 1$. $\lambda_{\max}(\cdot)$ stands for the maximum eigenvalue of a square matrix. $\|\cdot\|_2$ denotes the Euclidean norm. $\otimes$ is the Kronecker product for matrices.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a nonlinear CSN modeled by semi-linear parabolic PDEs, the dynamics of the i-th ($i \in \{1, 2, \cdots, N\}$) node described as

$$\begin{cases} \dfrac{\partial y_i(x,t)}{\partial t} = \alpha \dfrac{\partial^2 y_i(x,t)}{\partial x^2} + Ay_i(x,t) \\ \qquad\qquad + Bf(y_i(x,t)) + c\displaystyle\sum_{j=1}^{N} g_{ij} y_j(x,t), \\ \dfrac{\partial y_i(x,t)}{\partial x}\Big|_{x=0} = 0, \quad \dfrac{\partial y_i(x,t)}{\partial x}\Big|_{x=L} = u_i(t), \\ y_i(x,0) = y_i^0(x), \end{cases} \tag{1}$$

where $(x, t) \in [0, L] \times [0, \infty)$ respectively stand for the spatial variable and the time variable; $y_i(x, t) = [y_{i1}(x, t), y_{i2}(x, t), \cdots, y_{in}(x, t)]^T$ and $u_i(t) = [u_{i1}(t), u_{i2}(t), \cdots, u_{in}(t)]^T \in \mathbb{R}^n$ are the states and control inputs, respectively; $y_i^0(x)$ are bounded and continuous initial functions; $\alpha > 0$ is a known scalar; A and B are known $n \times n$ matrices; the nonlinear term $f(y_i(x, t)) = [f(y_{i1}(x, t)), f(y_{i2}(x, t)), \ldots, f(y_{in}(x, t))]^T \in \mathbb{R}^n$ is a sufficiently smooth nonlinear function; and a known scalar c is coupling strength. Assume that the topological structure $G = (g_{ij})_{N\times N}$ is defined as: $g_{i,j} > 0 (i \neq j)$ if i is connected to j , otherwise $g_{i,j} = 0 (i \neq j)$; and $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}$, $i, j \in \{1, 2, \cdots, N\}$.

Remark 1: The coupling configuration matrix G represents the topological structure of CSN (1). It can be a directed graph or an undirected graph. In this paper, G is not assumed to be symmetric or irreducible, and therefore it may have complex eigenvalues.

The isolated node of the CSN (1) is given as

$$\begin{cases} \dfrac{\partial s(x)}{\partial t} = \alpha \dfrac{\partial^2 s(x,t)}{\partial x^2} + As(x,t) + Bf(s(x,t)), \\ \dfrac{\partial s(x,t)}{\partial x}\Big|_{x=0} = \dfrac{\partial s(x,t)}{\partial x}\Big|_{x=L} = 0, \\ s(x,0) = s^0(x), \end{cases} \tag{2}$$

where $s(x, t) = [s_1(x, t), s_2(x, t), \cdots, s_n(x, t)]^T$ is the state; $s^0(x)$ is the bounded and continuous initial function.

Let $e_i(x, t) \triangleq y_i(x, t) - s(x, t)$. The synchronization error system can be obtained from (1) and (2) that

$$\begin{cases} \dfrac{\partial e_i(x,t)}{\partial t} = \alpha \dfrac{\partial^2 e_i(x,t)}{\partial x^2} + Ae_i(x,t) \\ \qquad\qquad + BF(e_i(x,t)) + c\displaystyle\sum_{j=1}^{N} g_{ij} e_j(x,t), \\ \dfrac{\partial e_i(x,t)}{\partial x}\Big|_{x=0} = 0, \quad \dfrac{\partial e_i(x,t)}{\partial x}\Big|_{x=L} = u_i(t), \\ e_i(x,0) = e_i^0(x), \end{cases} \tag{3}$$

where $e_i^0(x) \triangleq y_i^0(x) - s^0(x)$ and $F(e_i(x,t)) \triangleq f(y_i(x,t)) - f(s(x,t))$, in a compact way as

$$\begin{cases} \dfrac{\partial e(x,t)}{\partial t} = \alpha \dfrac{\partial^2 e(x,t)}{\partial x^2} + \bar{A}e(x,t) \\ \qquad\qquad + (I_N \otimes B)F(e(x,t)), \\ \dfrac{\partial e(x,t)}{\partial x}\Big|_{x=0} = 0, \quad \dfrac{\partial e(x,t)}{\partial x}\Big|_{x=L} = u(t), \\ e(x,0) = e^0(x), \end{cases} \tag{4}$$

where $e(x,t) \triangleq [e_1^T(x,t), e_2^T(x,t), \cdots, e_N^T(x,t)]^T$, $u(t) \triangleq [u_1^T(t), u_2^T(t), \cdots, u_N^T(t)]^T$, $\bar{A} \triangleq I_N \otimes A + cG \otimes I_n$, and $e^0(x) \triangleq [e_1^{0,T}(x), e_2^{0,T}(x), \cdots, e_N^{0,T}(x)]^T$.

In this paper, two forms of boundary control methods, adaptive boundary control and pinning adaptive boundary control based on boundary measurement, will be respectively studied for synchronization of the CSN (1) with the isolated node (2).

Case I: To achieve synchronization of the CSN (1), the following adaptive boundary controller based on boundary measurement are first studied:

$$\begin{aligned} u_i(t) &= -d_i(t)e_i(L,t), \quad i \in \{1,2,\cdots,N\}, \\ \dot{d}_i(t) &= k_i e_i^T(L,t)e_i(L,t), \quad i \in \{1,2,\cdots,N\}, \end{aligned} \tag{5}$$

in which $d_i(t)$ is the feedback strength and k_i is an arbitrary positive constant.

Remark 2: Only the spatial boundary state $e_i(L,t)$ is used in the adaptive boundary controller (5). Therefore, the merit of the proposed controller (5) lies in that it only requires one sensor and actuator to locate at the spatial boundary position $x = L$.

Remark 3: In [40]–[42], the boundary controller is defined as $u_i(t) = \int_0^L Ke_i(x,t)dx$. The whole state $e_i(x,t)$ used in that controller, therefore it requires sensors to distribute the whole spatial position $x \in [0, L]$. If only the spatial boundary states could be measured, the boundary controller based on boundary measurement (5) is more suitable.

Case II: When the number of nodes in networks is large, pinning synchronization of the CSN (1) is to be achieved by the pinning adaptive boundary controller as

$$\begin{aligned} u_i(t) &= \begin{cases} -d_i(t)e_i(L,t), & i \in \{1,2,\cdots,l\}, \\ 0, & i \in \{l+1, l+2, \cdots, N\}, \end{cases} \\ \dot{d}_i(t) &= k_i e_i^T(L,t)e_i(L,t), \quad i \in \{1,2,\cdots,l\}, \end{aligned} \tag{6}$$

in which $d_i(t)$ is the feedback strength, k_i is an arbitrary positive constant and l is the number of controlled nodes with the constraint $1 \le l \le N$.

Remark 4: To implement the pinning adaptive boundary controller (6), firstly, a part of nodes to be controlled is chosen according to rules. And then, only one actuator and only one sensor are located at the boundary position $x = L$ of a fraction of nodes chosen in the first step. As a result, compared with the adaptive boundary controller (5), it is easier to use the pinning adaptive boundary controller (6) in practice when the number of nodes is large.

The equivalent state-space description of the synchronization error system (4) is given by the following nonlinear abstract differential equation on the state space $\mathcal{H}^{Nn}$:

$$\dot{e}(t) = \mathcal{A}e(t) + \bar{A}e(t) + B\tilde{F}(e(t)), \quad e(0) = e^0(\cdot) \in \mathcal{H}^{Nn}, \tag{7}$$

processing the dense domain

$$\mathcal{D}(\mathcal{A}) = \{e \in \mathcal{W}_2^2([0,L]; R^{Nn}) : \frac{\partial e}{\partial x}\Big|_{x=0} = 0, \; \frac{\partial e}{\partial x}\Big|_{x=L} = u(t)\} \tag{8}$$

and the nonlinear term $\tilde{F}$ is chosen as $\tilde{F}(e(t)) = F(e(\cdot,t))$.

In this paper, we assume that there exists an admissible control input $u(t)$ such that $\mathcal{A}$ is the infinitesimal generator of a C_0 semigroup. According to [45, Exercise 3.15, p. 135], we can easily verify the above assumption when $u(t)$ takes the form of (5) or (6), the operator generates a C_0 semigroup on $\mathcal{H}^{Nn}$. Utilizing [46, Th. 1.7, Ch. 6], the local existence of the classical solution to the synchronization error system (4) can be easily obtained when $e^0(\cdot) \in \mathcal{D}(\mathcal{A})$.

The following definition, lemmas and assumption will be used for the subsequent development.

Definition 1: For the CSN (1) and the isolated node (2) with any initial conditions, if

$$lim_{t\to\infty}||y_i(x,t) - s(x,t)||_2 \to 0 \tag{9}$$

for any $i \in \{1,2,\cdots,N\}$, then the CSN (1) synchronizes the isolated node (2).

Lemma 1 [52]: For any square integrable vector $z(s)$ with $z(0) = 0$ or $z(L) = 0$, the following inequality holds:

$$\int_0^L z^T(s)z(s)ds \le 4L^2\pi^{-2}\int_0^L \dot{z}^T(s)\dot{z}(s)ds. \tag{10}$$

Remark 5: Lemma 1 is Wirtinger's inequality, illustrating the relationship between states and their derivatives. It is hard to directly process the derivative terms in the proof of theorems after, while the derivative terms can be minimized to be the state terms easier to be processed according to Lemma 1.

Lemma 2 [53]: Assume $M \in \mathbb{R}^{N\times N}$ is a symmetric matrix and $D = diag\{d_1, \ldots, d_r, \underbrace{0, \ldots, 0}_{N-r}\}$ with $d_i > 0$. When $d_i (1 \le i \le r)$ is sufficiently large, $M - D < 0$ is if and only if $M_l < 0$, where M_l is the minor matrix of M by removing its first r row-column pairs.

Assumption 1: Assume f satisfies the Lipschitz condition, i.e., for any sclars s_1 and s_2, there exist scalars $\chi > 0$ such that

$$|f(s_1) - f(s_2)| \le \chi|s_1 - s_2|. \tag{11}$$

Lipschitz condition has been widely used in many kinds of systems and networks.

III. SYNCHRONIZATION VIA ADAPTIVE BOUNDARY CONTROL

For Case I, employing the adaptive boundary controller (5) for the CSN (1), the following result can be obtained.

Theorem 1: Under Assumption 1, using the adaptive boundary controller (5), the CSN (1) synchronizes the isolated node (2) if

$$\lambda_{max}(W) < 0, \tag{12}$$

where

$$W \triangleq I_N \otimes (\frac{A+A^T}{2} + BB^T) + c\frac{G+G^T}{2} \otimes I_n + (\chi^2 - 0.25L^{-2}\pi^2\alpha)I_{Nn}. \tag{13}$$

Proof: Construct the Lyapunov functional candidate as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}\int_0^L e_i^T(x,t)e_i(x,t)dx + \sum_{i=1}^{N}\frac{\alpha}{2k_i}(d_i(t)-d_i^*)^2, \tag{14}$$

where $d_i^* > 0$ is a real scalar to be determined and α is defined in Eq. (1).

Substituting (3) and (5) into the time derivative of $V(t)$,

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N}\int_0^L e_i^T(x,t)\frac{\partial e_i(x,t)}{\partial t}dx + \sum_{i=1}^{N}\frac{\alpha}{k_i}(d_i(t)-d^*)\dot{d}_i(t)\\ &= \alpha\int_0^L\sum_{i=1}^{N}e_i^T(x,t)\frac{\partial^2 e_i(x,t)}{\partial x^2}dx\\ &\quad+\int_0^L\sum_{i=1}^{N}e_i^T(x,t)Ae_i(x,t)dx\\ &\quad+\int_0^L\sum_{i=1}^{N}e_i^T(x,t)BF(e_i(x,t))dx\\ &\quad+c\int_0^L\sum_{i=1}^{N}e_i^T(x,t)\sum_{j=1}^{N}g_{ij}e_j(x,t)dx\\ &\quad+\sum_{i=1}^{N}\frac{\alpha}{k_i}(d_i(t)-d_i^*)\dot{d}_i(t).\end{aligned} \tag{15}$$

Using integrating by parts and the boundary condition of (3),

$$\begin{aligned}\alpha\int_0^L\sum_{i=1}^{N}e_i^T(x,t)\frac{\partial^2 e_i(x,t)}{\partial x^2}dx &= -\alpha d_i(t)\sum_{i=1}^{N}e_i^T(L,t)e_i(L,t)\\ &\quad-\alpha\int_0^L\sum_{i=1}^{N}\frac{\partial e_i^T(x,t)}{\partial x}\frac{\partial e_i(x,t)}{\partial x}dx,\end{aligned} \tag{16}$$

in which, by using Lemma 1,

$$\begin{aligned}-\alpha\int_0^L\sum_{i=1}^{N}\frac{\partial e_i^T(x,t)}{\partial x}\frac{\partial e_i(x,t)}{\partial x}dx &\le -0.25L^{-2}\pi^2\alpha\int_0^L[e_i(x,t)-e_i(L,t)]^T\\ &\quad\times[e_i(x,t)-e_i(L,t)]dx,\end{aligned} \tag{17}$$

Considering (5), it follows from (15) that

$$\begin{aligned}\sum_{i=1}^{N}\frac{\alpha}{k_i}(d_i(t)-d^*)\dot{d}_i(t) &= \sum_{i=1}^{N}\frac{\alpha}{k_i}(d_i(t)-d^*)k_ie_i^T(L,t)e_i(L,t)\\ &= \alpha\sum_{i=1}^{N}(d_i(t)-d^*)e_i^T(L,t)e_i(L,t).\end{aligned} \tag{18}$$

Combining (16)-(18),

$$\begin{aligned}&\alpha\int_0^L\sum_{i=1}^{N}e_i^T(x,t)\frac{\partial e_i^2(x,t)}{\partial x^2}dx + \sum_{i=1}^{N}\frac{\alpha}{k_i}(d_i(t)-d^*)\dot{d}_i(t)\\ &\le -0.25L^{-2}\pi^2\alpha\int_0^L\sum_{i=1}^{N}[e_i(x,t)-e_i(L,t)]^T\\ &\qquad\times[e_i(x,t)-e_i(L,t)]dx\\ &\quad-d^*\alpha\sum_{i=1}^{N}e_i^T(L,t)e_i(L,t)\\ &= -0.25L^{-2}\pi^2\alpha\int_0^L e^T(x,t)e(x,t)dx\\ &\quad+0.5L^{-2}\pi^2\alpha\int_0^L e^T(x,t)e(L,t)dx\\ &\quad-0.25L^{-2}\pi^2\alpha\int_0^L e^T(L,t)e(L,t)dx\\ &\quad-\int_0^L e^T(L,t)De(L,t)dx,\end{aligned} \tag{19}$$

where $D \triangleq \frac{\alpha}{L}diag\{d_1^*, d_2^*, \ldots, d_N^*\} \otimes I_n$. Using Assumption 1,

$$\begin{aligned}&\int_0^L\sum_{i=1}^{N}e_i^T(x,t)BF(e_i(x,t))dx\\ &\le\int_0^L\sum_{i=1}^{N}e_i^T(x,t)BB^Te_i(x,t)dx\\ &\quad+\int_0^L\sum_{i=1}^{N}F^T(e_i(x,t))F(e_i(x,t))dx\\ &\le\int_0^L\sum_{i=1}^{N}e_i^T(x,t)BB^Te_i(x,t)dx\\ &\quad+\chi^2\int_0^L\sum_{i=1}^{N}e_i^T(x,t)e_i(x,t)dx.\end{aligned} \tag{20}$$

According to the Kronecker product for matrices and (20),

$$\begin{aligned}&\int_0^L\sum_{i=1}^{N}e_i^T(x,t)Ae_i(x,t)dx\\ &\quad+\int_0^L\sum_{i=1}^{N}e_i^T(x,t)BF(e_i(x,t))dx\end{aligned}$$

$$+c\int_0^L \sum_{i=1}^N e_i^T(x,t)\sum_{j=1}^N g_{ij}e_j(x,t)dx$$
$$\leq \int_0^L e^T(x,t)[I_N \otimes (\frac{A+A^T}{2}+BB^T)$$
$$+c\frac{G+G^T}{2}\otimes I_n + \chi I_{Nn}]e(x,t)dx. \quad (21)$$

Substituting (19) and (21) into (15),

$$\dot{V}(t) \leq -0.25L^{-2}\pi^2\alpha\int_0^L e^T(x,t)e(x,t)$$
$$+0.5L^{-2}\pi^2\alpha\int_0^L e^T(x,t)e(L,t)dx$$
$$-0.25L^{-2}\pi^2\alpha\int_0^L e^T(L,t)e(L,t)dx$$
$$-\int_0^L e^T(L,t)De(L,t)dx$$
$$+\int_0^L e^T(x,t)[I_N\otimes(\frac{A+A^T}{2}+BB^T)$$
$$+(c\frac{G+G^T}{2})\otimes I_n+\chi I_{Nn}]e(x,t)dx$$
$$=\int_0^L \tilde{e}^T(x,t)\Psi\tilde{e}(x,t)dx, \quad (22)$$

where $\tilde{e}(x,t) \triangleq [e^T(L,t), e^T(x,t)]^T$, and

$$\Psi \triangleq \begin{bmatrix} -0.25L^{-2}\pi^2\alpha I_{Nn} - D & 0.25L^{-2}\pi^2\alpha I_{Nn} \\ 0.25L^{-2}\pi^2\alpha I_{Nn} & W \end{bmatrix}, \quad (23)$$

in which W is defined in (13). According to (12), if $\lambda_{max}(W) < 0$, then

$$W < 0. \quad (24)$$

By Lemma 2, the inequality (24) is equivalent to

$$\Psi < 0, \quad (25)$$

for large enough $d_i^*, i \in \{1, 2, \ldots, N\}$. It follows from (22) and (25) that $\dot{V}(t) \leq 0$ and $\dot{V}(t) = 0$ if and only if $\tilde{e}_i(x,t) = 0$. According to Definition 1, the CSN (1) synchronizes the isolated node (2) if the condition (12) holds.

This completes the proof. □

Remark 6: For controlled CSNs, sufficient large feedback gains may be taken, which is possibly much larger than the actual values in need. With merits of the adaptive control, the adaptive boundary controller (5) can real time modify the feedback gains.

IV. SYNCHRONIZATION VIA A PINNING ADAPTIVE BOUNDARY CONTROLLER

For case II, when the number of nodes is very large, the pinning adaptive boundary controller (6) is employed and the following conclusion can be obtained.

Theorem 2: Under Assumption 1, using the pinning adaptive boundary controller (6), the CSN (1) synchronizes the isolated node (2) if

$$\lambda_{max}(W+\Lambda) < 0, \quad (26)$$

where W is defined in (13) and

$$\Lambda \triangleq \begin{bmatrix} 0_{ln} & 0 \\ 0 & 0.25L^{-2}\pi^2\alpha I_{(N-l)n} \end{bmatrix}. \quad (27)$$

Proof: The proof is similar to that of Theorem 1. To avoid unnecessary duplications, only a part of the proof is given.

Choosing the same Lyapunov functional candidate to that in Theorem 1 defined by (14), after similar calculations as the proof of Theorem 1,

$$\dot{V}(t) \leq \int_0^L \tilde{e}^T(x,t)\Pi\tilde{e}(x,t)dx, \quad (28)$$

where

$$\Pi \triangleq \begin{bmatrix} -0.25L^{-2}\pi^2\alpha I_{Nn} - D^* & 0.25L^{-2}\pi^2\alpha I_{Nn} \\ 0.25L^{-2}\pi^2\alpha I_{Nn} & W \end{bmatrix},$$
$$D^* \triangleq \frac{\alpha}{L}diag\{d_1^*, d_2^*, \ldots, d_l^*, \underbrace{0, \ldots, 0}_{N-l}\} \otimes I_n.$$

According to (28), if $\lambda_{max}(W+\Lambda) < 0$, then

$$W + \Lambda < 0. \quad (29)$$

By Schur complement, the inequality (29) is equivalent to that

$$\bar{\Pi} \triangleq \begin{bmatrix} -0.25L^{-2}\pi^2\alpha I_{(N-l)n} & \bar{\Pi}_{12} \\ \bar{\Pi}_{12}^T & W \end{bmatrix} < 0, \quad (30)$$

where $\bar{\Pi}_{12} \triangleq 0.25L^{-2}\pi^2\alpha[0_{(N-l)n,ln} \quad I_{(N-l)n}]$. Using Lemma 2, take large enough $d_i^* > 0$ such that the inequality (30) is equivalent to that

$$\Pi < 0 \quad (31)$$

From (28) and (31), $\dot{V}(t) \leq 0$ and $\dot{V}(t) = 0$ if and only if $\tilde{e}_i(x,t) = 0$. According to Definition 1, the CSN (1) synchronizes the isolated node (2). This completes the proof. □

Remark 7: As been defined in Eq. (6), l is the number of controlled nodes. When $l = N$ (N is the number of nodes in the CSN), all nodes are controlled. That is the main research of Section III. It can also be seen the consistency of boundary control with and without pinning strategy. When $l = N$, Λ defined in (27) is a zero matrix and thereby Theorem 2 degenerates into Theorem 1.

Remark 8: It is obvious that pinning synchronization of the CSN (1) implies synchronization of the CSN (1) because of inequality (26) implying inequality (12).

Remark 9: Pinning controllers in [49]–[51] require actuators and sensors distributing the whole spatial domain. What is different from those controllers is that the pinning adaptive boundary controller (6) require sensors and actuators only locating at the spatial boundary positions.

V. NUMERICAL SIMULATION

Example 1: In order to show the effectiveness of Theorem 1, consider a nonlinear CSN (1) composed of 4 nodes with coefficients listed as

$$\alpha = 2, \quad A = \begin{bmatrix} 2.7 & -1 \\ 1 & 2.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & -0.1 \\ 0.25 & 1 \end{bmatrix},$$

$$f(y_i(x,t)) = [\tanh(y_{i1}(x,t)), -\tanh(y_{i2}(x,t))]^T,$$
$$c = 0.2, \quad L = 1, \; n = 2, \; i \in \{1, 2, \cdots, 6\}, \tag{32}$$

with the coupling matrix

$$G \triangleq \begin{bmatrix} -4 & 0 & 3 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 0 & -2 \end{bmatrix},$$

and the initial conditions

$$\begin{aligned} y_1^0(x) &= [0.5 + 0.3\cos(2\pi x), -2 - 0.2\cos(\pi x + \pi/4)]^T, \\ y_2^0(x) &= [0.4 + 0.1\sin(2\pi x + \pi/6), 0.1 + 0.2\cos(\pi x)]^T, \\ y_3^0(x) &= [0.3 - 0.2\cos(3\pi x), 5\sin(\pi x + \pi/12)]^T, \\ y_4^0(x) &= [-3 + 0.3\cos(\pi x), -0.3 + 0.2\cos(5\pi x)]^T, \\ s^0(x) &= [0.2\cos(\pi x), 0.3\sin(\pi x)]^T. \end{aligned} \tag{33}$$

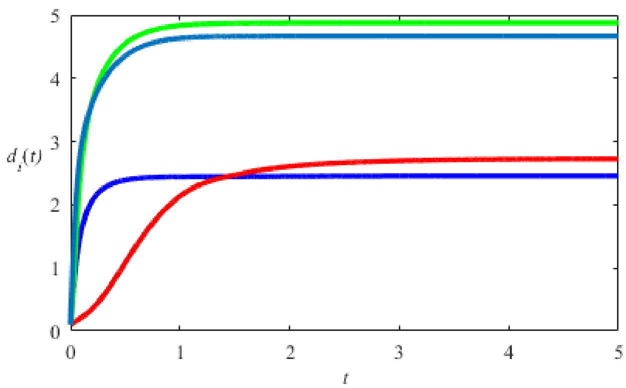

FIGURE 1. Trajectories of control gains $d_i(t)$ in Example 1.

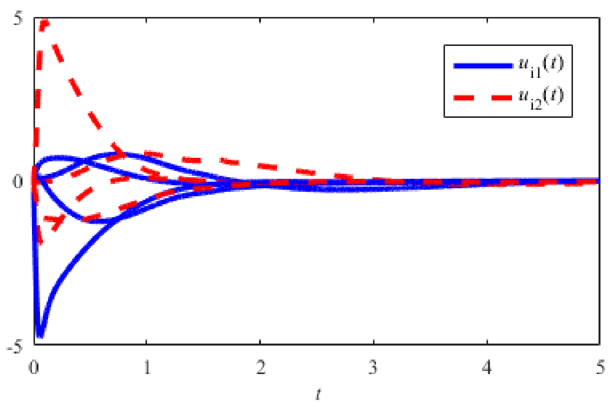

FIGURE 2. Trajectories of control inputs $u_i(t)$ in Example 1.

It is not difficult to verify that $f(y_i(x,t))$ satisfies the Lipschitz condition with $\chi = 1$. According to Theorem 1, applying the adaptive boundary controller (5) with $k_i = 5$, $\lambda_{max}(W) = -0.3481 < 0$ is obtained. The trajectories of the feedback gains and the control inputs of the controller (5) are respectively illustrated in Figures 1 and 2. Applying the controller (5) with the feedback gains shown in Figure 1, $e_i(x,t)$ is obtained as shown in Figure 3. Obviously, the proposed controller (5) can guide the CSN (1) to synchronize the isolated node (2).

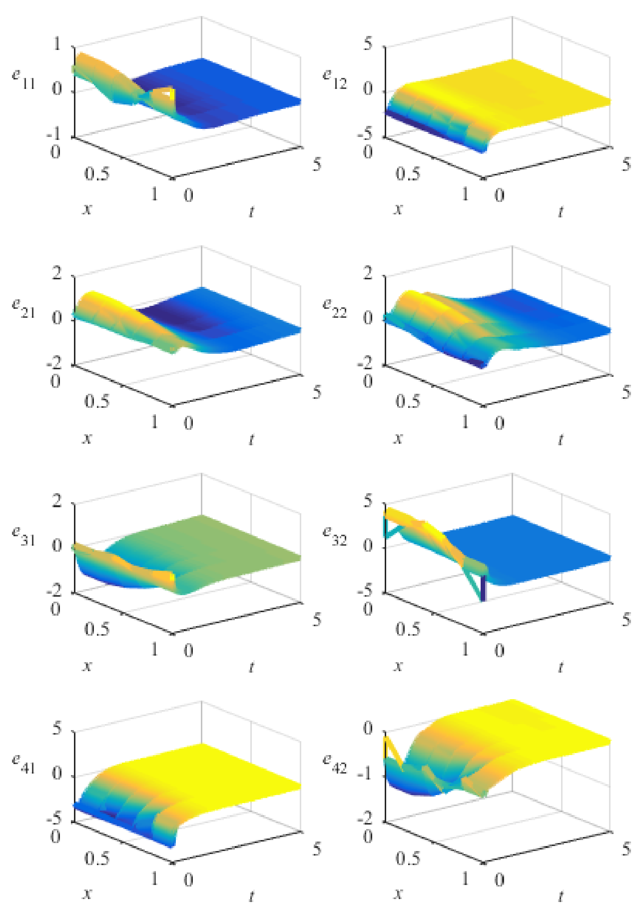

FIGURE 3. Profiles of the synchronization error $e_i(x,t)$ with 4 nodes being controlled in Example 1.

Example 2: To illustrate the effectiveness of Theorem 2, consider a nonlinear CSN (1) composed of 100 nodes with coefficients listed as

$$\alpha = 5, \quad A = \begin{bmatrix} 1 & -.2 \\ 0.2 & -1.5 \end{bmatrix}, \; B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$
$$f(y_i(x,t)) = [\tanh(y_{i1}(x,t)), -\tanh(y_{i2}(x,t))]^T,$$
$$c = 2, \quad L = 1, \; n = 2, \; i \in \{1, 2, \cdots, 100\}, \tag{34}$$

with the initial conditions

$$\begin{aligned} y_{i,0}(x) &= [0.5 + 0.4\sin(\pi x) + rand(1), \\ &\qquad 0.2\cos(\pi x) + rand(1)]^T, \\ s_0(x) &= [0.2\cos(\pi x) + 0.2, 0.3\sin(\pi x) - 0.1]^T. \end{aligned} \tag{35}$$

where $rand(1)$ is a random fraction between 0 and 1 according to uniform distribution. The topological structure G is firstly induced according to an Erdos-Renyi random graph with probability $p = 0.5$; and then $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}$.

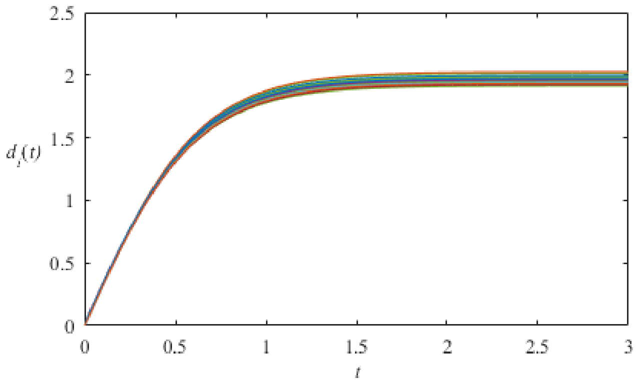

FIGURE 4. Trajectories of control gains $d_i(t)$ in Example 2.

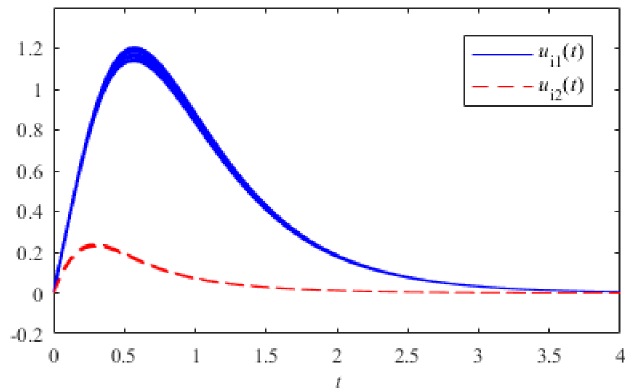

FIGURE 5. Trajectories of control inputs $u_i(t)$ in Example 2.

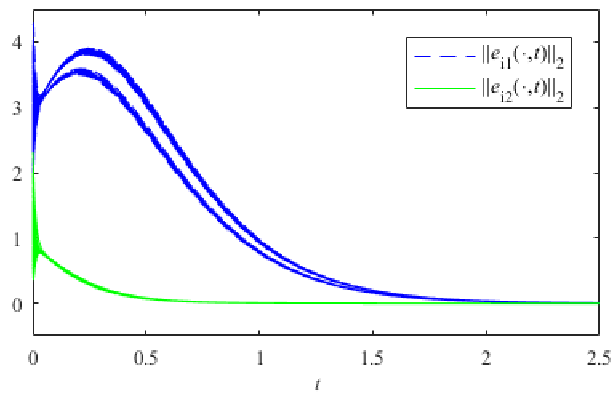

FIGURE 6. Trajectories of synchronization error $\|e_i(\cdot,t)\|_2$ in Example 2.

$\chi = 1$ can be obtained similar to Example 1. Next, 30 nodes are randomly selected from 100 nodes to be controlled, i.e., $l = 30$. Applying the pinning adaptive boundary controller (6) with $k_i = 2$, $\lambda_{max}(W) = -0.6235 < 0$ is got by Theorem 2. The trajectories of the feedback gains and the control inputs $u_i(t)$ are respectively illustrated in Figures 4 and 5. Applying the controller (6) with the feedback gains shown in Figure 5, trajectories of synchronization errors $\|e_i(\cdot,t)\|_2$ are obtained as shown in Figure 6. Obviously, under the proposed controller (6), the CSN (1) synchronizes the isolated node (2).

VI. CONCLUSIONS

Synchronization and pinning synchronization are respectively studied for a nonlinear CSN modeled by semi-linear parabolic PDEs. Two boundary control methods that adaptive boundary control and pinning adaptive boundary control have been investigated. By using Lyapunov direct method and some inequalities, two sufficient synchronization criteria are derived. The obtained criteria are independent on spatial positions. Simulation results of numerical examples respectively verify the effectiveness of the proposed adaptive boundary control and pinning adaptive boundary control methods. One interesting topic in future is to further study pinning synchronization of CSNs by constructing an impulse adaptive boundary controller.

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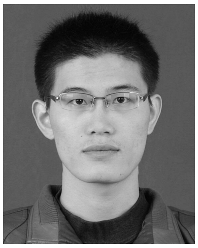

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