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An Improved Reciprocally Convex Inequality and Application to Stability Analysis of Time-Delay Systems Based on Delay Partition Approach

YU XUE, HAIFANG LI, AND XIAONA YANG
School of Mathematical Science, Heilongjiang University, Harbin 150080, China
Heilongjiang Provincial Key Laboratory of the Theory and Computation of Complex Systems, Heilongjiang University, Harbin 150080, China

Corresponding author: Xiaona Yang (yangxiaona3533@163.com)

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ABSTRACT In this paper, the problem of stability analysis for linear continuous-time systems with constant discrete and distributed delays is investigated. First, an improved reciprocally convex lemma is presented, which is a generalization of the existing reciprocally convex inequalities and can be directly applied in the case that of the delay interval is divided into $N \geq 2$ subintervals. Second, combining this with the auxiliary functions-based integral inequalities and the delay partition approach, a novel stability criterion of delay systems is given in terms of linear matrix inequalities. Finally, three numerical examples are given and their results are compared with the existing results. The comparison shows that the stability criterion proposed in this paper can provide larger upper bounds of delay than the other ones.

INDEX TERMS Time-delay systems, reciprocally convex inequality, delay partition approach, Lyapunov–Krasovskii functional.

I. INTRODUCTION

Due to time delays being frequently encountered in a variety of dynamic systems and often resulting in poor performance and/or instability, many efforts have been made to establish the stability criteria of time-delay systems, such as [1]–[25]. Generally, the stability criteria of time-delay systems are classified as delay-independent conditions or delay-dependent ones. Since the delay-dependent stability conditions contain the information of time-delay, the delay-dependent conditions are less conservative than delay-independent ones.

As is well known, constructing the delay-dependent Lyapunov–Krasovskii functionals (LKFs) is a foundation for stability analysis of delay systems. Additionally, utilizing the appropriate enlargement technique to estimate the upper bound of the derivative of LKF is a key step in reducing the conservativeness of stability criteria. To date, a series of effective approaches to estimating the derivative of LKF have been proposed, such as using matrix inequalities [3], [7], [10], [15], free-weighting matrices [5], the reciprocally convex approach [9], [11], [20], [22]–[27] and the delay partition approach ([17], [28]–[30]). Actually, a combination of several approaches is used in most stability analysis results.

Note that the integral quadratic terms are usually produced by computing the derivative of LKF. To address these integral quadratic terms, the reciprocally convex approach is usually employed by combining it with integral inequalities and the delay partition approach. First, by applying the delay partition approach, the delay interval is divided into two subintervals. Then, the integral inequalities (Jensen's inequality, Wirtinger inequality, or Bessel-Legendre inequality) are used to estimate these two integral quadratic terms. Finally, based on the reciprocally convex combination method [9], [11], [20], [22]–[27], the less conservative upper bound of these integral quadratic terms can be obtained. This is the most popular approach to estimating these integral quadratic terms, and then less conservative stability criteria are derived. However, it is worth stressing that these reciprocally convex inequalities (RCIs) [9], [11], [20], [22]–[27] are only in the case in which the delay interval is divided into only two subintervals. Applying the delay partition approach to address the stability analysis of delay systems, the delay interval need to be divided into N subintervals ($N > 2$), the existing RCIs cannot be applied directly. Therefore, it is necessary and more challenging to extend the classical reciprocally convex method to address the case of $N > 2$ subintervals in the delay interval, which is the main motivation of this paper.

In this paper, we will propose a novel stability criterion for time-delay systems by using the general reciprocally convex combination approach and the delay partition technique, and we will also seek to improve upon the existing works. First, an improved RCI is derived (see Lemma 1 below), which is a generalization of the existing reciprocally convex combination inequalities. Second, by combining the improved RCI with the auxiliary functions-based integral inequalities (see Lemma 2 below), the delay partition approach and the improved RCI, a new stability criterion for time-delay systems is proposed, which is less conservative than the existing results. Finally, three numerical examples are given to demonstrate that the proposed stability criterion is less conservative than the existing ones.

Compared with the existing results, the main contributions of this work are as follows: (1) The proposed improved RCI generalizes the existing reciprocally convex combination methods, that is, when the parameters of RCIs are selected as several special values, the existing results are special cases of Lemma 1. (2) To overcome the main obstacle, the improved RCIs is presented and applied to address the case of more than two subintervals in the delay interval, which is useful when utilizing the delay partition approach.

The organization of the rest of this article is as follows: An improved reciprocally convex combination lemma is introduced in Section II. In Section III, the novel stability analysis result is presented and elaborated. Three numerical examples are provided in Section IV; and finally, we conclude this paper in Section V.

Notation: Throughout this paper, the set of real numbers will be denoted by $\mathbb{R}$. Let $\mathbb{R}^{n\times m}$ represent the set of all $n \times m$ matrices over $\mathbb{R}$, and denote $\mathbb{R}^n = \mathbb{R}^{n\times 1}$. If A and B are symmetric matrices, by $A > B$ and $A \geq B$ we means that $A - B$ is real symmetric positive and semi-positive definite, respectively. A^{T} denotes the transpose of matrix A. For a square matrix, sym(X) means the sum of X and its transpose matrix X^{T}. $*$ in a matrix represents the elements below the main diagonal of a symmetric matrix. $\langle l\rangle = \{1, 2, \ldots, l\}$ for any positive integer l.

II. PRELIMINARIES

First, we will provide an improved result on the basis of the reciprocally convex inequality, which will be helpful in the stability analysis.

Lemma 1: For given scalars $\alpha_i \in \mathbb{R}$ satisfying $\alpha_i > 0$ and $\sum_{i=1}^{m}\alpha_i = 1$, symmetric matrices $R_i > 0$ and $M_i > 0$, and any matrices W_i, $i \in \langle m\rangle$, if the following matrix inequalities are satisfied

$$\begin{bmatrix} R_i - \alpha_i M_i & W_{ij}(\alpha_i, \alpha_j) \\ * & R_j - \alpha_j M_j \end{bmatrix} \geq 0, \quad 1 \leq i < j \leq m, \tag{1}$$

then the following inequality holds:

$$\Phi_m := \mathrm{diag}(\frac{1}{\alpha_1}R_1, \frac{1}{\alpha_2}R_2, \ldots, \frac{1}{\alpha_m}R_m) \geq \Xi_m, \tag{2}$$

where

$$W_{ij}(\alpha_i, \alpha_j) = \alpha_i W_i + \alpha_j W_j,$$

and Ξ_m is defined in (3), as shown at the top of the next page.

Proof: According to inequalities (1), it is not difficult to obtain

$$\begin{aligned}\Xi_{ij} := {} & \epsilon_i^{\mathrm{T}}(R_i - \alpha_i M_i)\epsilon_i + \epsilon_i^{\mathrm{T}} W_{ij}(\alpha_i, \alpha_j)\epsilon_j \\ & + \epsilon_j^{\mathrm{T}} W_{ij}^{\mathrm{T}}(\alpha_i, \alpha_j)\epsilon_i + \epsilon_j^{\mathrm{T}}(R_j - \alpha_j M_j)\epsilon_j \geq 0, \\ & \text{for all} \quad 1 \leq i < j \leq m, \end{aligned} \tag{4}$$

where $R_i \in \mathbb{R}^{n_i\times n_i}$, $i \in \langle m\rangle$, and

$$\epsilon_i = [0_{n_1} \quad \cdots \quad 0_{n_{i-1}} \quad I_{n_i} \quad 0_{n_{i+1}} \quad \cdots \quad 0_{n_m}].$$

Denote

$$\Omega_{ij} = \frac{\sqrt{\alpha_j}}{\sqrt{\alpha_i}}\epsilon_i - \frac{\sqrt{\alpha_i}}{\sqrt{\alpha_j}}\epsilon_j + \sum_{\substack{k=1\\k\neq i,j}}^{m}\epsilon_k. \tag{5}$$

From (4), it follows that

$$\begin{aligned}\Gamma_{ij} := {} & \frac{\alpha_j}{\alpha_i}\epsilon_i^{\mathrm{T}}(R_i - \alpha_i M_i)\epsilon_i - \epsilon_i^{\mathrm{T}} W_{ij}(\alpha_i, \alpha_j)\epsilon_j \\ & - \epsilon_j^{\mathrm{T}} W_{ij}^{\mathrm{T}}(\alpha_i, \alpha_j)\epsilon_i + \frac{\alpha_i}{\alpha_j}\epsilon_j^{\mathrm{T}}(R_j - \alpha_j M_j)\epsilon_j \\ = {} & \Omega_{ij}^{\mathrm{T}}\Xi_{ij}\Omega_{ij} \\ \geq {} & 0, \end{aligned} \tag{6}$$

then, we have

$$\Gamma_m := \sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\Gamma_{ij} \geq 0. \tag{7}$$

This, together with $\Phi_m - \Xi_m = \Gamma_m$, completes the proof. ∎

Remark 1: When the scalar m and matrices R_i, M_i and W_i are selected as several special values and matrices, the existing results are special cases of Lemma 1: when $m = 2$, Lemma 1 is equivalent to [27, Lemma 2] and [25, Lemma 2]. When $M_1 = R_1 - W_1 M_2^{-1} W_1^{\mathrm{T}}$ and $M_2 = R_2 - W_2^{\mathrm{T}} M_1^{-1} W_2$, Lemma 1 directly reduces to [20, Th. 1] and [22, Lemma 4]. When $R_1 = R_2 = R$, $W_{12} = S$, $M_1 = R - SR^{-1}S^{\mathrm{T}}$ and $M_2 = R - S^{\mathrm{T}}R^{-1}S$, Lemma 1 reduces to [24, Lemma 3]. Additionally, [27, Lemma 2], [25, Lemma 2], [20, Th. 1], [22, Lemma 4] and [24, Lemma 3] are special cases of Lemma 1. Moreover, if taking $M_1 = M_2 = 0$ and $W_1 = W_2 = 0$, Lemma 1 reduces to the popular reciprocally convex inequality given in [11, Lemma 2.2] and [23, Lemma 1] in the case of $n = 1$. Furthermore, setting $\kappa = \frac{\alpha_2}{\alpha_1}$, we can obtain [13, Lemma 1]. When $m = N$, $R_i = R$, $M_i = 0$ and $W_{ij}(\alpha_i, \alpha_j) = W_{ij}$, Lemma 1 reduces to [31, Lemma 2]. In general, Lemma 1 generalizes and improves the existing reciprocally convex inequalities.

$$\Xi_m := \begin{bmatrix} R_1+(1-\alpha_1)M_1 & W_{12}(\alpha_1,\alpha_2) & W_{13}(\alpha_1,\alpha_3) & \cdots & W_{1m}(\alpha_1,\alpha_m) \\ * & R_2+(1-\alpha_2)M_2 & W_{23}(\alpha_2,\alpha_3) & \cdots & W_{2m}(\alpha_2,\alpha_m) \\ * & * & R_3+(1-\alpha_3)M_3 & \cdots & W_{3m}(\alpha_3,\alpha_m) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ * & * & * & \cdots & R_m+(1-\alpha_m)M_m \end{bmatrix}. \quad (3)$$

Lemma 2: [15] For given a real symmetric positive definite matrix $R \in \mathbb{R}^{n\times n}$, and an integral function $\omega : [a,b] \to \mathbb{R}^n$, the following inequalities hold:

$$(b-a)\int_a^b w^{\mathrm{T}}(s)Rw(s)\mathrm{d}s \geq \bar{\Omega}^{\mathrm{T}}\bar{R}\bar{\Omega}, \quad (8)$$

$$\frac{(b-a)^2}{2}\int_a^b\int_s^b w^{\mathrm{T}}(u)Rw(u)\mathrm{d}u\mathrm{d}s \geq \hat{\Omega}^{\mathrm{T}}\hat{R}\hat{\Omega}, \quad (9)$$

where

$$\begin{aligned} \bar{R} &= \mathrm{diag}(R, 3R, 5R), \quad \hat{R} = \mathrm{diag}(R, 8R), \\ \bar{\Omega} &= \mathrm{col}(\bar{\Omega}_1, \bar{\Omega}_2, \bar{\Omega}_3), \quad \hat{\Omega} = \mathrm{col}(\hat{\Omega}_1, \hat{\Omega}_2), \\ \bar{\Omega}_1 &= \int_a^b w(s)\mathrm{d}s, \quad \hat{\Omega}_1 = \int_a^b\int_s^b w(u)\mathrm{d}u\mathrm{d}s, \\ \bar{\Omega}_2 &= \bar{\Omega}_1 - \frac{2}{b-a}\hat{\Omega}_1, \\ \bar{\Omega}_3 &= \bar{\Omega}_1 - \frac{6}{b-a}\hat{\Omega}_1 + \frac{12}{(b-a)^2}\int_a^b\int_s^b\int_u^b w(v)\mathrm{d}v\mathrm{d}u\mathrm{d}s, \\ \hat{\Omega}_2 &= \hat{\Omega}_1 - \frac{3}{b-a}\int_a^b\int_s^b\int_u^b w(v)\mathrm{d}v\mathrm{d}u\mathrm{d}s. \end{aligned}$$

Remark 2: Compared with [7], [32], the integral inequalities (8) and (9) in Lemma 2 give much tighter lower bounds than the Jensen's inequalities do.

III. STABILITY ANALYSIS OF TIME-DELAY SYSTEMS

Discrete and distributed time-delays exist widely in the fields of chemistry, physics, biology, population dynamics, and so on. They also exist in the systems of network control, communication and other control systems [33]. Discrete and distributed time-delays are widely applied in biological systems for describing biology dynamical behaviors.

In this paper, we consider the following linear system with the discrete and distributed delays:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-h) + A_D \int_{t-h}^t x(s)\mathrm{d}s, \\ x(t) = \phi(t), \quad t \in [-h, 0], \end{cases} \quad (10)$$

where $x : [-h, +\infty) \to \mathbb{R}^n$ is the state vector; $\phi : [-h, 0] \to \mathbb{R}^n$ is the continuous initial vector function; A, A_d and A_D represent real constant matrices of appropriate dimensions; h is a time-invariant delay satisfying $h \in [h_{min}, h_{max}]$. For given a positive number t, we define the function, $x_t : [-h, 0] \to \mathbb{R}^n$, by

$$x_t(s) = x(t+s), \quad s \in [-h, 0]. \quad (11)$$

The problem of this paper is to establish a novel stability criterion for a time-delay system (10) by applying the improved reciprocally convex inequality given in Lemma 1 and the delay partition approach.

Remark 3: Note that combining the reciprocally convex inequality with the integral inequalities and the delay partition approach is a popular method for reducing the conservativeness of delay-dependent stability criteria. However, the existing reciprocally convex methods in [9], [11], [20], and [22]–[27] are only applicable to the case in which the delay interval is divided into two subintervals. For the other cases, the above RCIs cannot be directly applied. To overcome this difficulty, the improved RCIs are proposed, which can be applied when the number of subintervals in the delay interval is more than two.

For convenience, the interval $[0, h]$ is divided into $r+1$ subintervals $[h_{i-1}, h_i]$, $i \in \langle r+1 \rangle$, where $h_0 = 0$ and $h_{r+1} = h$. Also, let $\delta_i = h_i - h_{i-1}$, $i \in \langle r+1 \rangle$. Before introducing the main results, the following notations are denoted:

$$\begin{aligned} \eta_1(x_t) &= \mathrm{col}\left(x_t(-h_1), x_t(-h_2), \ldots, x_t(-h_{r+1})\right), \\ \eta_2(x_t) &= \mathrm{col}\Bigg(\int_{-h_1}^0 x_t(s)\mathrm{d}s, \int_{-h_2}^{-h_1} x_t(s)\mathrm{d}s, \ldots, \\ &\qquad \int_{-h}^{-h_r} x_t(s)\mathrm{d}s\Bigg), \\ \eta_3(x_t) &= \mathrm{col}\Bigg(\int_{-h_1}^0\int_s^0 x_t(u)\mathrm{d}u\mathrm{d}s, \\ &\qquad \int_{-h_2}^{-h_1}\int_s^{-h_1} x_t(u)\mathrm{d}u\mathrm{d}s, \ldots, \\ &\qquad \int_{-h}^{-h_r}\int_s^{-h_r} x_t(u)\mathrm{d}u\mathrm{d}s\Bigg), \\ \eta_4(x_t) &= \mathrm{col}\Bigg(\int_{-h_1}^0\int_\theta^0\int_s^0 x_t(v)\mathrm{d}v\mathrm{d}s\mathrm{d}\theta, \\ &\qquad \int_{-h_2}^{-h_1}\int_\theta^{-h_1}\int_s^{-h_1} x_t(v)\mathrm{d}v\mathrm{d}s\mathrm{d}\theta, \ldots, \\ &\qquad \int_{-h}^{-h_r}\int_\theta^{-h_r}\int_s^{-h_r} x_t(v)\mathrm{d}v\mathrm{d}s\mathrm{d}\theta\Bigg), \\ \eta(x_t) &= \mathrm{col}\Big(x_t(0),\ \eta_1(x_t),\ \eta_2(x_t),\ \eta_3(x_t),\ \eta_4(x_t)\Big), \\ \xi(x_t) &= \mathrm{col}\Big(x_t(0),\ \eta_2(x_t),\ \eta_3(x_t),\ \eta_4(x_t)\Big), \\ \zeta(t) &= \mathrm{col}(x(t),\ \dot{x}(t)). \end{aligned}$$

Here, it is obvious to see that $x_t(0) = e_1\eta(x_t)$, $\dot{x}_t(0) = \chi\eta(x_t)$, $\xi(x_t) = \Gamma\eta(x_t)$ and $\dot{\xi}(x_t) = \Gamma_d\eta(x_t)$. Based on the previous preparation, a new stability criterion for the time-delay system (10) is proposed as follows.

Theorem 1: For given scalars $0 = h_0 < h_1 < h_2 < \cdots < h_r < h_{r+1} = h$, the time-delay system (10) is asymptotically stable if there exist real symmetric positive definite matrices $P \in \mathbb{R}^{(3r+4)n\times(3r+4)n}$, $Q_1 \in \mathbb{R}^{n\times n}$, $Q_2 \in \mathbb{R}^{2n\times 2n}$ and $Q_3 \in \mathbb{R}^{n\times n}$, appropriately dimensioned symmetric matrices M_i, N_i, $i \in \langle r+1\rangle$ and T_i, $i \in \langle r\rangle$, and appropriately dimensioned matrices X_{ij}, Y_{ij}, $i,j \in \langle r\rangle$ and Z_{ij}, $i,j \in \langle r-1\rangle$, such that the following LMIs hold:

$$\begin{bmatrix} \bar{Q}_2 - \beta_i M_i & X_{ij} \\ * & \bar{Q}_2 - \beta_j M_j \end{bmatrix} \geq 0, \quad 1 \leq i < j \leq r+1, \tag{12}$$

$$\begin{bmatrix} \beta_i^{-1}\hat{Q}_3 - \beta_i N_i & Y_{ij} \\ * & \beta_j^{-1}\hat{Q}_3 - \beta_j N_j \end{bmatrix} \geq 0, \quad 1 \leq i < j \leq r+1, \tag{13}$$

$$\begin{bmatrix} \gamma_i \bar{Q}_3 - \xi_i T_i & Z_{ij} \\ * & \gamma_j \bar{Q}_3 - \xi_j T_j \end{bmatrix} \geq 0, \quad 1 \leq i < j \leq r, \tag{14}$$

$$\Psi := \Psi_0 - D_2^{\mathrm{T}}\Theta_1 D_2 - D_3^{\mathrm{T}}\Theta_2 D_3 - D_4^{\mathrm{T}}\Theta_3 D_4 < 0, \tag{15}$$

where

$$\begin{aligned}
\delta_i &= h_i - h_{i-1}, \quad \beta_i = \frac{\delta_i}{h}, \ i \in \langle r+1\rangle, \\
\gamma_j &= \frac{h^2(h-h_j)}{2h_r}, \quad \xi_j = \frac{h_j - h_{j-1}}{h_r}, \ j \in \langle r\rangle, \\
\bar{Q}_j &= \mathrm{diag}(Q_j, 3Q_j, 5Q_j), \quad j = 2, 3, \\
\hat{Q}_3 &= \mathrm{diag}(Q_3, 8Q_3),
\end{aligned}$$

$$\begin{aligned}
\Psi_0 &:= \mathrm{sym}(\Gamma^{\mathrm{T}} P \Gamma_d) + e_1^{\mathrm{T}} Q_1 e_1 - e_{r+2}^{\mathrm{T}} Q_1 e_{r+2} \\
&\quad + h^2 D_1^{\mathrm{T}} Q_2 D_1 + \frac{h^4}{4}\chi^{\mathrm{T}} Q_3 \chi, \\
\Gamma &= \mathrm{col}(e_1,\ e_{r+3},\ e_{r+4},\ \ldots,\ e_{4r+5}), \\
\Gamma_d &= \mathrm{col}\Big(\chi,\ e_1 - e_2,\ \ldots,\ e_{r+1} - e_{r+2}, \\
&\quad \delta_1 e_1 - e_{r+3},\ \delta_2 e_2 - e_{r+4}, \\
&\quad \ldots,\ \delta_{r+1} e_{r+1} - e_{2r+3}, \\
&\quad \frac{\delta_1^2}{2} e_1 - e_{2r+4},\ \frac{\delta_2^2}{2} e_2 - e_{2r+5}, \\
&\quad \ldots,\ \frac{\delta_{r+1}^2}{2} e_{r+1} - e_{3r+4}\Big), \\
\chi &= A e_1 + A_d e_{r+2} + A_D \sum_{j=r+3}^{2r+3} e_j, \\
D_1 &= \mathrm{col}(e_1,\ \chi), \\
D_j &= \mathrm{col}(D_{j1},\ D_{j2},\ \ldots,\ D_{j\,r+1}), \quad j = 2, 3, \\
D_{2i} &= \mathrm{col}(e_{r+2+i},\ e_i - e_{i+1}, e_{r+2+i} - \frac{2}{\delta_i} e_{2r+3+i}, \\
&\quad -e_i - e_{i+1} + \frac{2}{\delta_i} e_{r+2+i}, e_{r+2+i} - \frac{6}{\delta_i} e_{2r+3+i} \\
&\quad + \frac{12}{\delta_i^2} e_{3r+4+i}, e_i - e_{i+1} + \frac{6}{\delta_i} e_{r+2+i} \\
&\quad - \frac{12}{\delta_i^2} e_{2r+3+i}), \\
D_{3i} &= \mathrm{col}(\delta_i e_i - e_{r+2+i}, \\
&\quad -\frac{1}{2}\delta_i e_i - e_{r+2+i} + \frac{3}{\delta_i} e_{2r+3+i}), \quad i \in \langle r+1\rangle, \\
D_4 &= \mathrm{col}(D_{41},\ D_{42},\ \ldots,\ D_{4r}), \\
D_{4j} &= \mathrm{col}(e_j - e_{j+1},\ -e_j - e_{j+1} + \frac{2}{\delta_j} e_{r+2+j}, \\
&\quad e_j - e_{j+1} + \frac{6}{\delta_j} e_{r+2+j} - \frac{12}{\delta_j^2} e_{2r+3+j}), \quad j \in \langle r\rangle, \\
e_k &= [0_{n\times(k-1)n}\ I_n\ 0_{n\times(4r+5-k)n}], \quad k \in \langle 4r+5\rangle.
\end{aligned}$$

Proof: For the linear time-delay system (10), we define the Lyapunov-Krasovskii functional candidate as follows:

$$\begin{aligned}
V(x_t) &= \xi^{\mathrm{T}}(x_t) P \xi(x_t) + \int_{-h}^{0} x_t^{\mathrm{T}}(s) Q_1 x_t(s) \mathrm{d}s \\
&\quad + h \int_{-h}^{0}\int_{s}^{0} \zeta_t^{\mathrm{T}}(u) Q_2 \zeta_t(u) \mathrm{d}u \mathrm{d}s \\
&\quad + \frac{h^2}{2}\int_{-h}^{0}\int_{\theta}^{0}\int_{s}^{0} x_t^{\mathrm{T}}(v) Q_3 x_t(v) \mathrm{d}v \mathrm{d}s \mathrm{d}\theta,
\end{aligned} \tag{16}$$

where $P > 0$ and $Q_i > 0 (i = 1, 2, 3)$ are taken from the feasible solutions to (12)-(15). Then, the time derivative of $V(x_t)$ along the trajectories of system (10) can be easily obtained as follows:

$$\begin{aligned}
\dot{V}(x_t) &= \eta^{\mathrm{T}}(x_t)\Psi_0 \eta(x_t) - h\int_{-h}^{0} \zeta_t^{\mathrm{T}}(s) Q_2 \zeta_t(s) \mathrm{d}s \\
&\quad - \frac{h^2}{2}\int_{-h}^{0}\int_{s}^{0} \dot{x}_t^{\mathrm{T}}(u) Q_3 \dot{x}_t(u) \mathrm{d}u \mathrm{d}s.
\end{aligned} \tag{17}$$

Since $0 = h_0 < h_1 < h_2 < \cdots < h_r < h_{r+1} = h$, it is not difficult to obtain that

$$-h\int_{-h}^{0} \zeta_t^{\mathrm{T}}(s) Q_2 \zeta_t(s) \mathrm{d}s = -h \sum_{i=1}^{r+1} U_i \tag{18}$$

and

$$\begin{aligned}
&-\frac{h^2}{2}\int_{-h}^{0}\int_{s}^{0} \dot{x}_t^{\mathrm{T}}(u) Q_3 \dot{x}_t(u) \mathrm{d}u \mathrm{d}s \\
&= -\frac{h^2}{2}\sum_{i=1}^{r+1}\int_{-h_i}^{-h_{i-1}}\int_{s}^{0} \dot{x}_t^{\mathrm{T}}(u) Q_3 \dot{x}_t(u) \mathrm{d}u \mathrm{d}s \\
&= -\frac{h^2}{2}\sum_{i=1}^{r+1}\left(V_i + \delta_i \int_{-h_{i-1}}^{0} \dot{x}_t^{\mathrm{T}}(u) Q_3 \dot{x}_t(u) \mathrm{d}u\right) \\
&= -\frac{h^2}{2}\sum_{i=1}^{r+1} V_i - \sum_{i=1}^{r}\frac{h^2\delta_{i+1}}{2}\sum_{j=1}^{i} W_j \\
&= -\frac{h^2}{2}\sum_{i=1}^{r+1} V_i - \sum_{j=1}^{r}\sum_{i=j}^{r}\frac{h^2\delta_{i+1}}{2} W_j \\
&= -\frac{h^2}{2}\sum_{i=1}^{r+1} V_i - \sum_{j=1}^{r}\frac{h^2(h-h_j)}{2} W_j,
\end{aligned} \tag{19}$$

$$\Theta_1 := \begin{bmatrix} \bar{Q}_2+(1-\beta_1)M_1 & X_{12} & X_{13} & \cdots & X_{1r+1} \\ * & \bar{Q}_2+(1-\beta_2)M_2 & X_{23} & \cdots & X_{2r+1} \\ * & * & \bar{Q}_2+(1-\beta_3)M_3 & \cdots & X_{3r+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ * & * & * & \cdots & \bar{Q}_2+(1-\beta_{r+1})M_{r+1} \end{bmatrix} \geq 0,$$

$$\Theta_2 := \begin{bmatrix} \beta_1\hat{Q}_3+(1-\beta_1)N_1 & Y_{12} & Y_{13} & \cdots & Y_{1r+1} \\ * & \beta_2\hat{Q}_3+(1-\beta_2)N_2 & Y_{23} & \cdots & Y_{2r+1} \\ * & * & \beta_3\hat{Q}_3+(1-\beta_3)N_3 & \cdots & Y_{3r+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ * & * & * & \cdots & \beta_{r+1}\hat{Q}_3+(1-\beta_{r+1})N_{r+1} \end{bmatrix} \geq 0,$$

$$\Theta_3 := \begin{bmatrix} \gamma_1\bar{Q}_3+(1-\xi_1)T_1 & Z_{12} & \cdots & Z_{1r} \\ * & \gamma_2\bar{Q}_3+(1-\xi_2)T_2 & \cdots & Z_{2r} \\ \cdots & \cdots & \cdots & \cdots \\ * & * & \cdots & \gamma_r\bar{Q}_3+(1-\xi_r)T_r \end{bmatrix} \geq 0,$$

where

$$U_i = \int_{-h_i}^{-h_{i-1}} \zeta^{\mathrm{T}}(x_t(s))Q_2\zeta(x_t(s))\mathrm{d}s,$$
$$V_i = \int_{-h_i}^{-h_{i-1}}\int_{s}^{-h_{i-1}} \dot{x}_t^{\mathrm{T}}(u)Q_3\dot{x}_t(u)\mathrm{d}u\mathrm{d}s,$$
$$W_j = \int_{-h_j}^{-h_{j-1}} \dot{x}_t^{\mathrm{T}}(u)Q_3\dot{x}_t(u)\mathrm{d}u.$$

Also, by using the inequalities (8) and (9) given in Lemma 2, we have

$$U_i \geq \frac{1}{\delta_i}\eta^{\mathrm{T}}(x_t)D_{2i}^{\mathrm{T}}\bar{Q}_2D_{2i}\eta(x_t),$$
$$V_i \geq \frac{2}{\delta_i^2}\eta^{\mathrm{T}}(x_t)D_{3i}^{\mathrm{T}}\hat{Q}_3D_{3i}\eta(x_t),$$
$$W_j \geq \frac{1}{\delta_j}\eta^{\mathrm{T}}(x_t)D_{4j}^{\mathrm{T}}\bar{Q}_3D_{4j}\eta(x_t).$$

Then we can obtain

$$\begin{aligned} h\sum_{i=1}^{r+1}U_i &\geq h\sum_{i=1}^{r+1}\frac{1}{\delta_i}\eta^{\mathrm{T}}(x_t)D_{2i}^{\mathrm{T}}\bar{Q}_2D_{2i}\eta(x_t) \\ &= \eta^{\mathrm{T}}(x_t)D_2^{\mathrm{T}}\Phi_1D_2\eta(x_t), \\ \frac{h^2}{2}\sum_{i=1}^{r+1}V_i &\geq \frac{h^2}{2}\sum_{i=1}^{r+1}\frac{2}{\delta_i^2}\eta^{\mathrm{T}}(x_t)D_{3i}^{\mathrm{T}}\hat{Q}_3D_{3i}\eta(x_t) \\ &= \eta^{\mathrm{T}}(x_t)D_3^{\mathrm{T}}\Phi_2D_3\eta(x_t), \\ &\sum_{j=1}^{r}\frac{h^2(h-h_j)}{2}W_j \\ &\geq \sum_{j=1}^{r}\frac{h^2(h-h_j)}{2\delta_j}\eta^{\mathrm{T}}(x_t)D_{4j}^{\mathrm{T}}\bar{Q}_3D_{4j}\eta(x_t) \\ &= \eta^{\mathrm{T}}(x_t)D_4^{\mathrm{T}}\Phi_3D_4\eta(x_t), \end{aligned}$$

where

$$\Phi_1 = \mathrm{diag}\left(\frac{1}{\beta_1}\bar{Q}_2, \frac{1}{\beta_2}\bar{Q}_2, \cdots, \frac{1}{\beta_{r+1}}\bar{Q}_2\right),$$
$$\Phi_2 = \mathrm{diag}\left(\frac{1}{\beta_1^2}\hat{Q}_3, \frac{1}{\beta_2^2}\hat{Q}_3, \cdots, \frac{1}{\beta_{r+1}^2}\hat{Q}_3\right),$$
$$\Phi_3 = \mathrm{diag}\left(\frac{\gamma_1}{\xi_1}\bar{Q}_3, \frac{\gamma_2}{\xi_2}\bar{Q}_3, \cdots, \frac{\gamma_r}{\xi_r}\bar{Q}_3\right),$$
$$\sum_{i=1}^{r+1}\beta_i = 1, \quad \sum_{j=1}^{r}\xi_j = 1.$$

This, together with Lemma 1 and (12)–(14), implies that

$$h\sum_{i=1}^{r+1}U_i \geq \eta^{\mathrm{T}}(x_t)D_2^{\mathrm{T}}\Theta_1D_2\eta(x_t), \tag{20}$$
$$\frac{h^2}{2}\sum_{i=1}^{r+1}V_i \geq \eta^{\mathrm{T}}(x_t)D_3^{\mathrm{T}}\Theta_2D_3\eta(x_t), \tag{21}$$
$$\sum_{j=1}^{r}\frac{h^2(h-h_j)}{2}W_j \geq \eta^{\mathrm{T}}(x_t)D_4^{\mathrm{T}}\Theta_3D_4\eta(x_t). \tag{22}$$

Moreover, the combination of (17)–(22) yields

$$\dot{V}(t, x_t) \leq \eta^{\mathrm{T}}(x_t)\Psi\eta(x_t).$$

Then, it follows from (15), that $\dot{V}(t, x_t) < 0$, and hence the time-delay system (10) is asymptotically stable. The proof is completed. ■

Remark 4: In this paper, the interval $[0, h]$ is divided into $r+1$ subintervals, $[h_{i-1}, h_i]$, $i \in \langle r+1\rangle$, whose lengths need not be the same. Note that the new LKF (16) is constructed, which depends on all these subintervals. Then, the improved RCIs and delay partition approach are introduced to estimate the derivative of the LKF. As a result, a less conservative stability criterion in the form of LMIs is yielded, which will be illustrated by three examples in the next Section.

Remark 5: It is worth stressing that the total number of scalar decision variables is increasing with the number r, which should be helpful in reducing the conservatism of the resulting stability criterion at the price of increasing the computational burden. By specially choosing the matrices in Theorem 1, for example diagonal matrices, the number of decision variables can be decreased and the similar results can be obtained (see Example 1 and 2).

IV. NUMERICAL EXAMPLES

In this section, three numerical examples are provided to demonstrate the effectiveness of theory results in this paper.

Example 1: Consider system (10) with:

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\quad A_d=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\ A_D=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}.$$

This example is a well-known example that is frequently used to check the conservatism of delay-dependent stability criteria. The maximum allowable delay bound of this system is $h_{max} = 6.1725$, which is obtained from a so-called eigenvalue analysis method. For $r = 1$ and $r = 2$, the upper bounds of h are obtained by applying Theorem 1 and other methods in [1], [4], [6], [10], [12], [16], [18], [21], and [34]. The comparison results are listed in Table 1.

TABLE 1. The upper bounds *h*.

Method	h_{max}	Decision variables
[1]($N=1$)	6.059	$7.5n^2+2.5n$
[1]($N=2$)	6.165	$9.5n^2+3.5n$
[4]	4.472	$11.5n^2+4.5n$
[6]	6.1107	$1.5n^2+9n+9$
[10]	6.059	$3n^2+2n$
[12]($N=2$)	3.21	$9n^2+3n$
[12]($N=4$)	5.28	$19n^2+4n$
[12]($N=6$)	6.12	$33n^2+5n$
[16]	6.1664	$17.5n^2+2.5n$
[18]	6.168	$27n^2+4n$
[21]	6.1719	$29n^2+3n$
[34]($m=8$)	5.0998	$108.5n^2+8.5n$
[34]($m=14$)	5.7410	$315.5.5n^2+14.5n$
Theorem 1 ($r=1, h_1=\frac{3}{4}h$)	6.1719	$107.5n^2+13.5n$
Theorem 1* ($r=1, h_1=\frac{3}{4}h$)	6.1717	$28.5n^2+29.5n$
Theorem 1 ($r=2$, $h_1=\frac{1}{4}h, h_2=\frac{3}{4}h$)	6.1724	$247n^2+20n$
Theorem 1** ($r=2$, $h_1=\frac{1}{4}h, h_2=\frac{3}{4}h$)	6.1724	$214.5n^2+31.5n$
Eigenvalue Analysis Method	6.1725	

* $Q_1, Q_2, Q_3, M_1, M_2, N_1, N_2$ and X are diagonal matrices;
** $Q_3, Y_{12}, Y_{13}, Y_{23}, N_1, N_2, N_3, Z_{12}, T_1$ and T_2 are diagonal matrices.

The results of Example 1 state that, when the number r increases, the allowable delay-varying range increases (i.e., the conservativeness is reduced by increasing the number r). In the case of $r = 1$, the maximum allowable delay bound is $h_{max} = 6.1719$. And in the case of $r = 2$, the maximum allowable delay bound is $h_{max} = 6.1724$. However, it is noted that the reduction of conservativeness of the stability criterion is at the price of increasing the number of scalar decision variables. When $r = 1$, the number of decision variables is $107.5n^2 + 13.5n$. And when $r = 2$, the number of decision variables is $247n^2 + 20n$. The number of decision variables in the case of $r = 1$ is more than the one in [21], which has the same result with Theorem 1.

In order to decrease the number of decision variables, the matrices in Theorem 1 can be specially selected. when $r = 1$, choose Q_1, Q_2, Q_3, M_1, M_2, N_1, N_2 and X be diagonal matrices, the maximum allowable delay bound is $h_{max} = 6.1717$ which is slightly smaller than 6.1719 in [21], and the number of decision variables is $28.5n^2 + 29.5n$, which is close to $29n^2 + 3n$ in [21]; when $r = 2$, letting Q_3, Y_{12}, Y_{13}, Y_{23}, N_1, N_2, N_3, Z_{12}, T_1 and T_2 be diagonal matrices, the maximum allowable delay bound is $h_{max} = 6.1724$, which is approach to the precise value 6.1725.

It is clear that the results of Theorem 1 are very close to the theoretical value and larger than the ones in the literature, which shows the lower conservation of Theorem 1.

Remark 6: Many researcher engage in gaining the more less conservativeness of the stability criteria for time-delay systems by using different approaches. By improving the integral inequalities, the less conservativeness of the stability criteria with smaller computational complexity is obtained in [18]. Although the computational complexity of our approach is bigger than the ones of [18], our method can be applied to a more larger range. For example, when the delay partition approach is employed to address the stability analysis of delay systems, it is possible that the delay interval need to be divided into N subintervals ($N > 2$). In this case, the existing reciprocally convex inequalities cannot be applied directly. In this paper, we aim to solve the problem and extend the classical reciprocally convex method to address the case of $N > 2$.

Example 2: Consider system (10) with:

$$A=\begin{bmatrix}0.2 & 0\\ 0.2 & 0.1\end{bmatrix},\quad A_d=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\ A_D=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}.$$

TABLE 2. The lower and upper bounds *h*.

Method	$[h_{min}, h_{max}]$	Decision variables
[2]	[0.2001,1.6339]	$19.5n^2+4.5n$
[14]	[0.2000,1.9504]	$12.5n^2+4.5n$
Theorem 1 ($r=1, h_1=\frac{1}{4}h$)	[0.2001,2.0409]	$107.5n^2+13.5n$
Theorem 1 ($r=1, h_1=\frac{1}{3}h$)	[0.2000,2.0409]	$107.5n^2+13.5n$
Theorem 1 ($r=1, h_1=\frac{2}{3}h$)	[0.2001,2.0407]	$107.5n^2+13.5n$
Theorem 1* ($r=1, h_1=\frac{1}{4}h$)	[0.2001,2.0409]	$28.5n^2+29.5n$
Eigenvalue Analysis Method	[0.2000,2.04]	

* $Q_1, Q_2, Q_3, M_1, M_2, N_1, N_2$ and X are diagonal matrices.

When the so-called eigenvalue analysis method is used to analyze the stability of this system, we derive a maximum allowable delay-varying interval of [0.2000, 2.04]. Table 2 shows the results obtained by Theorem 1 and the methods in [2] and [14]. When $r = 1, h_1 = \frac{1}{4}h$, the maximum allowable delay interval is [0.2001, 2.0409] and the number of decision variables is $107.5n^2 + 13.5n$. The maximum allowable delay interval is very close to the precise value. As in Example 1, choose Q_1, Q_2, Q_3, M_1, M_2, N_1, N_2 and X be diagonal matrices, the maximum allowable delay interval is [0.2001, 2.0409] and the number of decision variables is $28.5n^2 + 29.5n$, which is much less than $107.5n^2 + 13.5n$.

From Table 2, it is clear that the results obtained by Theorem 1 can provide larger upper bounds than the other results. It shows that Theorem 1 has a less conservative stability criterion than those in [2] and [14].

Example 3: Consider system (10) with:

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 0.1 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, A_D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

TABLE 3. The lower and upper bounds *h*.

Method	$[h_{min}, h_{max}]$	Decision variables
[35]	[0.102,1.424]	$24.5n^2 + 3.5n$
[14]	[0.1002,1.5954]	$12.5n^2 + 4.5n$
Theorem 1 ($r = 1, h_1 = \frac{1}{3}h$)	[0.1002,1.7174]	$107.5n^2 + 13.5n$
Theorem 1 ($r = 1, h_1 = 0.3h$)	[0.1002,1.7175]	$107.5n^2 + 13.5n$
Eigenvalue Analysis Method	[0.1002,1.7178]	

When $h = 0$, we know that this system is unstable, since $\mathrm{Re}(\mathrm{eig}(A + A_d)) = 0.05 > 0$. Based on Jensen's inequality, classical Lyapunov-Krasovskii approaches cannot provide the stable delay range [11]. The results obtained by Theorem 1 and some existing results are listed in Table 3. From Table 3, it shows that for both $r = 1$ and $r = 2$ the results obtained by Theorem 1 are very close to the theoretical value.

V. CONCLUSIONS

In this study, we have studied the stability problem for linear systems with constant discrete and distributed time-delays. First, a novel reciprocally convex lemma has been introduced, which is a generalization of the existing reciprocally convex approaches. Second, a new delay-dependent LKF has been constructed to establish stability analysis, and the improved RCIs and delay partition technique have been employed to estimate the derivative of LKF. Third, a less-conservative stability criterion has been derived. Finally, three numerical examples have been provided to illustrate the advantage of the proposed results. Our future work will focus on finding the new methods or integral inequalities to reduce the conservativeness of the stability criteria for time-delay systems.

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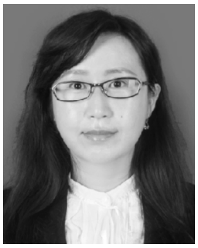

YU XUE received the B.S. degree in automation from the Harbin University of Science and Technology, China, in 2002, and the Ph.D. degree in control theory and control engineering from the Harbin Institute of Technology, China, in 2007. From 2007 to 2009, she was an Engineer with the Sichuan Academy of Aerospace Technology. From 2009 to 2013, she was a Senior Engineer with the Sichuan Academy of Aerospace Technology. In 2013, she joined Heilongjiang University. Her research interests include the stability analysis of delayed dynamic systems, robust control, and genetic regulatory networks.

HAIFANG LI received the B.S. degree from the School of Mathematical Science, Hulunbeir College, Hulunbeir, China, in 2016. She is currently pursuing the M.S. degree with Heilongjiang University, Harbin, China. Her current research interests include robust control and stability analysis of delayed dynamic systems.

XIAONA YANG received the B.S. and M.S. degrees from the School of Mathematical Science, Heilongjiang University, Harbin, China, in 2008 and 2011, respectively, and the Ph.D. degree in statistics from Nankai University, Tianjin, China, in 2017. He is currently a Lecturer with the School of Mathematical Science, Heilongjiang University. Her research interests include system science, statistical process control, and change-point and outlier detection.