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# Three-Stage Distributed State Estimation for AC-DC Hybrid Distribution Network Under Mixed Measurement Environment

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**ABSTRACT** The ac–dc hybrid distribution network is a credible path for the future evolution of distribution network. State estimation is a paramount foundation for the safe and stable operation of the complex distribution network. The application of the centralized state estimation method in an ac–dc distribution network has some obstacles, such as low computational efficiency, large communication capacity, and privacy protection problem. Based on the three-stage state estimation theory, this paper established a three-stage state estimation model for the ac–dc hybrid distribution network integrating supervisory control and data acquisition system and phasor measurement unit. The proposed three-stage estimation model achieves linearization of the nonlinear state estimation for the ac–dc hybrid distribution network. That is, the first and third stages simply solve the linear state estimation problem, and the second stage is a one-step nonlinear transform. Moreover, the alternating direction method of multipliers (ADMM) is applied to solving the distributed problem. Thus, an accurate and efficient three-stage distributed state estimation method for the ac–dc hybrid distribution network is proposed. In this method, ac subsystems and dc subsystems execute state estimation tasks based on local information and eventually achieve the overall consistency of the system state estimation through the transfer iteration of the boundary information, respectively. The combination of bilinear theory and ADMM ensures the convergence of distributed state estimation while improving computational efficiency. The simulation results verified the advantage of the proposal over the existing methods in many aspects.

**INDEX TERMS** Distributed state estimation, ac-dc hybrid distribution network, phasor measurement unit (PMU), three-stage state estimation modeling, alternating direction method of multipliers (ADMM).

## I. INTRODUCTION

### A. MOTIVATIONS

With the increasing penetration of distributed generation and electric vehicles, more and more DC equipment will be embedded into the distribution network in the future. The traditional AC distribution network will face severe challenges such as power quality and power supply reliability. Considering that the distribution network is still dominated by AC loads, AC-DC hybrid distribution networks would become an important form of smart distribution network [1]–[4]. Against the background of the present AC/DC hybrid mode in the current distribution network, it is

crucial to grasp the actual state of the dynamic source and load in the distribution network. State estimation is the basis of power system operation analysis and coordination control, which is a key part of energy management system [5], [6]. It is necessary to study the state estimation technology of AC-DC hybrid power distribution network.

Unlike traditional distribution networks, AC-DC hybrid distribution networks are interconnected by multiple AC and DC subsystems, therefore the topology and operating mode of AC-DC distribution network are decentralized [7]. Compared to the centralized method, distributed methods have advantages in computational efficiency, communication

pressure, and information protection [8], [9]. Currently, some studies have been made on the decentralized operation control of AC-DC hybrid distribution network [10]–[12]. However, there is a lack of extensive and in-depth discussion on the application of decentralized state estimation methods in AC-DC hybrid distribution network.

References [13] and [14] study centralized state estimation for AC-DC hybrid power grids, but when the system is large, computational efficiency will limit its application. The literature [8] has studied the distributed state estimation method of the AC-DC microgrid, but this method is only applicable to the case that the measurement equation is linear. With nonlinear measurements, literature [15] proposed a fully distributed robust state estimation method that is applicable to multiarea power systems. The proposed method was proved of good robustness and convergence, but it did not discuss its applicability in AC-DC hybrid power grids. The literature [16]–[18] use a decomposition-coordinated two-level approach to distributed state estimation, but generally, only sub-optimal solutions can be obtained [19]. Compared with the two-level distributed method, the literature [20] proposes a distributed method that directly exchanges boundary bus information in adjacent areas and does not require a central coordination side based on the Lagrangian relaxation optimization theory. This kind of distributed method is easier to implement and its estimation result is closer to the integral method, but the convergence performance is weaker [19]. Based on Lagrangian Relaxation method, distributed state estimation methods for AC/DC network were proposed in [21]. However, the proposed method requires relatively large number iterations to converge and the convergence would become worse when the system is larger.

In addition, the configuration of the phasor measurement unit (PMU) facilitates the development of distributed state estimation. The sampling frequency and measurement accuracy of PMU are better than that of the SCADA system [22]. However, the higher cost currently restricts its overall configuration in the distribution network. Thus the mixed measurement environment in which the SCADA system is combined with the proper deployment of the PMU is in line with practical engineering applications. The combination of PMU would improve the accuracy of the state estimation. The distributed approach of [23] and [24] take into account SCADA systems and PMU mixed measurements, but the main focus is on transmission network. The literature [25] uses PMU measurement to convert the state estimation into a linear least-squares problem. It is solved by the alternating direction multiplier method (ADMM). This method has higher estimation accuracy and convergence speed. But the model assumes that all the measurements are obtained from PMU, so the engineering application background is not strong. Overall, although the modern power grid is equipped with a certain number of PMU measurements, it is still difficult to ensure that the entire network can be observed, so taking into account the mixed measurement of SCADA systems and PMU state estimation has more engineering research value.

This paper aims to propose an efficient and robust distributed state estimation method for AC/DC distribution networks under mixed measurement environment.

## B. CONTRIBUTIONS

The contributions of this paper include the following:

1. This paper establishes a three-stage state estimation model of AC-DC hybrid distribution network with voltage converters under mixed measurement environment integrating SCADA and PMU. The bilinearization of the state estimation for AC-DC hybrid distribution network, which is a nonlinear problem, is realized in the proposed model.
2. This paper proposes a distributed state estimation method based on the proposed three-stage model and ADMM. ADMM is used to realize the decoupling of AC-DC system and to solve the distributed state estimation. Compared with the centralized distributed state estimation method, the proposed method has higher computational efficiency. Moreover, the proposed method has higher accuracy and better convergence characteristics compared with existing distributed state estimation method.

## II. THREE-STAGE STATE ESTIMATION MODEL FOR AC-DC HYBRID DISTRIBUTION NETWORK

A typical AC-DC hybrid distribution network structure is shown in Figure 1: AC subsystem and DC subsystem are connected through the voltage converter for the coordinated operation between subsystems. Meanwhile, each subsystem has strong independence and autonomy. Among them, the equivalent circuit of the voltage converter is also shown in Figure 1.

The power  $P_{VSC_k}^{AC}$ ,  $Q_{VSC_k}^{AC}$ ,  $P_{VSC_k}^{DC}$  transmitted between the AC and DC systems in Figure 1 can be described in equations (1)–(3):

$$P_{VSC_k}^{AC} = \frac{\mu_k M_k}{\sqrt{2}} V_{VSC_k}^{AC} V_{VSC_{kl}}^{DC} Y_k \sin(\theta_{VSC_k}^{AC} - \alpha_k) + \left(V_{VSC_k}^{AC}\right)^2 Y_k \sin \alpha_k \quad (1)$$

$$Q_{VSC_k}^{AC} = \frac{-\mu_k M_k}{\sqrt{2}} V_{VSC_k}^{AC} V_{VSC_k}^{DC} Y_k \cos(\theta_{VSC_k}^{AC} - \alpha_k) + \left(V_{VSC_k}^{AC}\right)^2 Y_k \cos \alpha_k - \left(V_{VSC_k}^{AC}\right)^2 / X_{k-c} \quad (2)$$

$$P_{VSC_k}^{DC} = \frac{\mu_k M_k}{\sqrt{2}} V_{VSC_k}^{AC} V_{VSC_k}^{DC} Y_k \sin(\theta_{VSC_k}^{AC} + \alpha_k) - \left(\frac{\mu_k^2 M_k^2}{2}\right) \left(V_{VSC_k}^{DC}\right)^2 Y_k \sin \alpha_k \quad (3)$$

In the equations above,  $Y_k = 1/\sqrt{R_k^2 + X_{k-l}^2}$  and  $M_k$  denote the voltage converter variable ratio  $\mu_k$  represents the voltage utilization of voltage converter. In the hybrid system, the state variables of AC subsystem are  $\mathbf{x}_{AC_k} = [V_{AC_k}, \theta_{AC_k}]^T$ , and the state variables of DC subsystem are  $\mathbf{x}_{DC} = [V_{DC}]^T$ . The state variables of system are  $\mathbf{x} = [x_{AC_1}^T, \dots, x_{AC_K}^T, x_{DC}^T]^T$ , where  $K$  denotes the total number of the subsystem in the hybrid system.

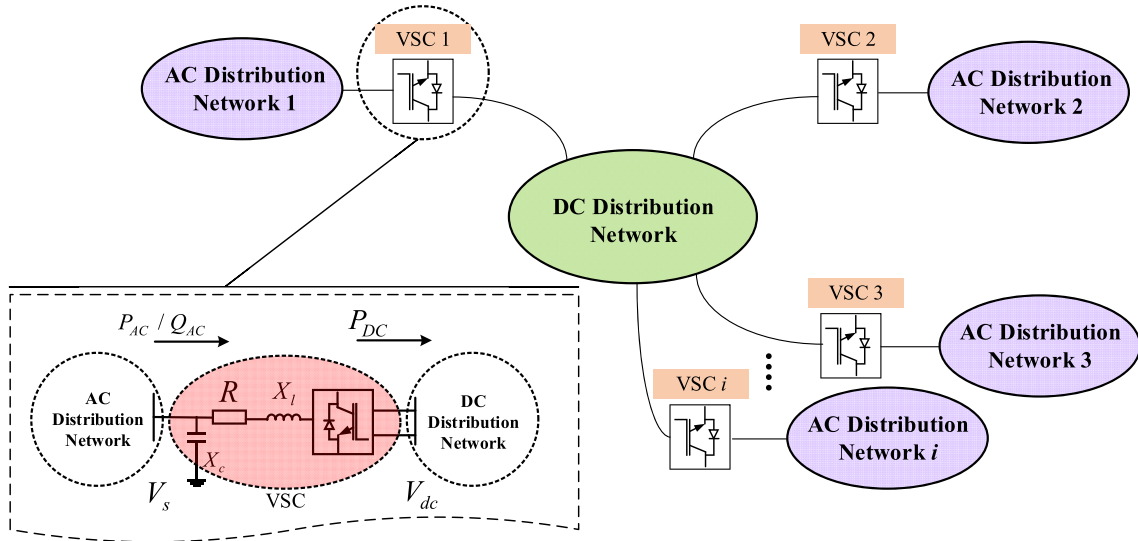


FIGURE 1. AC and DC distribution network structure and voltage converter equivalent circuit diagram.

System measurement variables, including node voltage amplitude measurement, injection power measurement, branch current measurement and branch power measurement, etc., can be divided into AC subsystem measurement, DC subsystem measurement and the measurement of converter position. This paper extends the converter position measurement to the DC subsystem. Thus the system measurements can be uniformly expressed as the following forms:

$$z = \begin{bmatrix} z_{AC_1} \\ \vdots \\ z_{AC_K} \\ z_{DC} \end{bmatrix} = \begin{bmatrix} h_{AC_1}(x_{AC_1}) \\ \vdots \\ h_{AC_K}(x_{AC_K}) \\ h_{DC}(x) \end{bmatrix} + \begin{bmatrix} r_{AC_1} \\ \vdots \\ r_{AC_K} \\ r_{DC} \end{bmatrix} \quad (4)$$

where  $z_{AC_k}$ ,  $h_{AC_k}$  and  $r_{AC_k}$  denote the measurement, the measurement equation and the measurement residual of AC subsystem  $k$ , respectively.  $z_{DC}$ ,  $h_{DC}$  and  $r_{DC}$  denote the measurement, the measurement equation and the measurement residual of DC subsystem, respectively. Based on weighted least square method, the state estimation model of the hybrid system is established as:

$$\min J_{DC}(\mathbf{x}) + \sum_{k=1}^K J_{AC_k}(\mathbf{x}_{AC_k}) \quad (5)$$

$$J_{DC}(\mathbf{x}) = \mathbf{r}_{DC}^T \mathbf{R}_{DC}^{-1} \mathbf{r}_{DC} \quad (6)$$

$$J_{AC_k}(\mathbf{x}_{AC_k}) = \mathbf{r}_{AC_k}^T \mathbf{R}_{AC_k}^{-1} \mathbf{r}_{AC_k} \quad (7)$$

where  $\mathbf{R}_{AC_k}$  and  $\mathbf{R}_{DC}$  are the measurement covariance matrix of AC subsystems and DC subsystems, respectively. The measurement variable  $z_{AC_k}$  of AC subsystems can be represented by the state variable  $\mathbf{x}_{AC_k}$ . As for the DC subsystems measurement  $z_{DC}$ , however, the measurement variable of voltage converter from is related to both the state variables of DC subsystems and AC subsystems. Resulted from the weak coupling between the state variables of DC subsystems

and AC subsystems, the model of the hybrid distribution network can not be solved directly in a distributed form. Also, under normal circumstances, the measurement equation in the power system is not linear with the state variables, and the state estimation model is non-convex. If the distributed optimization algorithm is used, the convergence can not be guaranteed.

Therefore, based on the three-stage state estimation theory, a three-stage state estimation model for AC-DC hybrid distribution network is constructed in this paper by introducing intermediate variables and constructing linear measurement equations.

### A. THE FIRST-STAGE STATE ESTIMATION MODEL

In this stage, the state estimation is based on the system measurement  $\mathbf{z}$ . It is defined that the first-stage state variables of AC subsystem and DC subsystem are  $\mathbf{y}_{AC_k}$  and  $\mathbf{y}_{DC}$  respectively. The first-stage state variable  $\mathbf{y}$  of the system is the union of  $\mathbf{y}_{AC_k}$  and  $\mathbf{y}_{DC}$ .

$$\mathbf{y}_{AC_k} = \{U_{AC_k,i}; K_{AC_k,ij}; L_{AC_k,ij}\}_{i,j \in AC_k} \quad (8)$$

$$\begin{cases} K_{AC_k,ij} = V_{AC_k,i} V_{AC_k,j} \cos \theta_{AC_k,ij} \\ L_{AC_k,ij} = V_{AC_k,i} V_{AC_k,j} \sin \theta_{AC_k,ij} \\ U_{AC_k,i} = V_{AC_k,i}^2 \end{cases} \quad (9)$$

$$\mathbf{y}_{DC} = \{U_{DC,i}; K_{DC,ij}; K_{VSC_k}; L_{VSC_k}\}_{i,j,k \in DC} \quad (10)$$

$$\begin{cases} K_{DC,ij} = V_{DC,i} V_{DC,j} \\ U_{DC,i} = V_{DC,i}^2 \\ K_{VSC_k} = V_{VSC_k}^{DC} V_{VSC_k}^{AC} \cos \theta_{VSC_k}^{AC} \\ L_{VSC_k} = V_{VSC_k}^{DC} V_{VSC_k}^{AC} \sin \theta_{VSC_k}^{AC} \end{cases} \quad (11)$$

$$\mathbf{y} = [y_{AC_1}^T, \dots, y_{AC_K}^T, y_{DC}^T]^T \quad (12)$$

It is not difficult to verify that any element in the measurement  $\mathbf{z}$  can be expressed linearly by  $\mathbf{y}$ , that is,  $\mathbf{z} = \mathbf{A}\mathbf{y} + \mathbf{r}$ .

The first-stage state estimation model based on the weighted least square method has the following form:

$$\min J^f(\mathbf{y}) = \mathbf{r}^T \mathbf{W} \mathbf{r} \quad (13)$$

In the formula, the  $W$  is the weight coefficient matrix of the measurement,  $W = R^{-1}$ . Because  $R$  is diagonal matrix and  $A$  is full rank array, the model is convex. Let the calculation result of the model be  $\mathbf{y}$ , the gain matrix and the covariance formula can be described as:

$$\mathbf{G}_y = \mathbf{A}^T \mathbf{R}^{-1} \mathbf{A} \quad (14)$$

$$\text{cov}(\mathbf{y}) = \mathbf{G}_y^{-1} \quad (15)$$

### B. THE SECOND-STAGE NONLINEAR TRANSFORMATION

In this stage, the calculation result  $\mathbf{y}$  of the first-stage model is nonlinearly transformed to obtain the second-stage intermediate variable  $\mathbf{u}$ . Among them, the conversion process of AC subsystems and DC subsystems is shown in equation; intermediate variables  $\mathbf{u}$  is expressed in equation (16)-(18).

$$\begin{cases} \varphi_{AC_k,i} = \ln U_{AC_k,i} \\ \varphi_{AC_k,ij} = \ln \left( (K_{AC_k,ij})^2 + (L_{AC_k,ij})^2 \right) \\ \theta_{AC_k,ij} = \theta_{AC_k,i} - \theta_{AC_k,j} \end{cases} \quad (16)$$

$$\begin{cases} \varphi_{DC,i} = \ln U_{DC,i} \\ \varphi_{DC,ij} = \ln (K_{DC,ij})^2 \\ \varphi_{VSC_k} = \ln \left( (K_{VSC_k})^2 + (L_{VSC_k})^2 \right) \\ \theta_{VSC_k}^s = ac \tan (L_{VSC_k} / K_{VSC_k}) \end{cases} \quad (17)$$

$$\mathbf{u} = \left\{ \varphi_{AC_k,i}, \theta_{AC_k,ij}, \varphi_{AC_k,ij}, \theta_{VSC_k}^{AC}, \varphi_{DC,i}, \varphi_{DC,ij} \right\} \quad (18)$$

For a more convenient description, the non-linear conversion process is expressed as a non-linear function  $\mathbf{u} = f(\mathbf{y})$ . Let  $\mathbf{F}_u$  be the Jacobi matrix of  $\mathbf{f}(\mathbf{y})$  at  $\mathbf{y}$ , then the covariance matrix of  $\mathbf{u}$  and its gain matrix  $\mathbf{W}_u$  can be obtained by equation (19),(20).

$$\text{cov}(\mathbf{u}) = \mathbf{F} \text{cov}(\mathbf{y}) \mathbf{F}^T \quad (19)$$

$$\mathbf{W}_u = \text{cov}^{-1}(\mathbf{u}) = \mathbf{F}_u^{-T} \mathbf{G}_y \mathbf{F}_u \quad (20)$$

### C. THE THIRD-STAGE LINEAR STATE ESTIMATION MODEL

In this stage, the state estimation is based on the calculation result  $\mathbf{u}$  of the second-stage model. Let the third-stage state variables of AC subsystem and DC subsystem be  $\mathbf{x}_{AC_k}^t$  and  $\mathbf{x}_{DC}^t$ . The state variables of hybrid system is defined as  $\mathbf{x}^t$ .

$$\mathbf{x}_{AC_k}^t = \{ \ln V_{AC_k,i}; \theta_{AC_k,i} \} \quad (21)$$

$$\mathbf{x}_{DC}^t = \{ \ln V_{DC,i} \} \quad (22)$$

$$\mathbf{x}^t = \left[ \mathbf{x}_{AC_1}^{tT}, \dots, \mathbf{x}_{AC_T}^{tT}, \mathbf{x}_{DC}^{tT} \right]^T \quad (23)$$

It is not difficult to verify that the intermediate variable  $\mathbf{u}$  can be expressed linearly by  $\mathbf{x}$ , that is  $\mathbf{u} = \mathbf{C}\mathbf{x}^t + \mathbf{r}_u$ . Therefore, the third-stage state estimation model can be established

in equation (24) and (25).

$$\min J^t(\mathbf{x}) = (\mathbf{u} - \mathbf{C}\mathbf{x})^T \mathbf{W}_u (\mathbf{u} - \mathbf{C}\mathbf{x}) \quad (24)$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ |\mathbf{A}^T| & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_r^T \end{bmatrix} \quad (25)$$

### D. THREE-STAGE STATE ESTIMATION MODEL CONSIDERING PMU MEASUREMENT

Compared with the traditional SCADA system measurement, PMU can directly measure the phase angle of the bus voltage. In the distributed state estimation, the PMU measurement can be configured on the reference bus in each area to achieve the conversion from the reference bus to the reference bus of each area. However, although the PMU measurement accuracy is high, there is also some random noise. It is not precise enough to directly use the PMU measured value of each sub-area reference bus as the true value, and multiple sub-zones may be configured with PMU measurements. Therefore, in addition to the reference bus of the whole network, it is necessary to participate in the state estimation together with the measurement of the PMU and the SCADA system measurement. Based on the above bilinear model, the PMU measurement can be considered in the third stage:

$$\begin{bmatrix} \mathbf{u} \\ \theta_P \end{bmatrix} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\theta_P \end{bmatrix} \mathbf{x}^t \quad (26)$$

where:  $\theta_P$  is the measured value of PMU, and  $\mathbf{C}\theta_P$  is a constant matrix composed of 0 and 1, which guarantees the linear relationship between the measurement variables and the state variables in the third stage.

## III. ADMM BASED DISTRIBUTED STATE ESTIMATION METHOD FOR AC-DC HYBRID DISTRIBUTION NETWORK

### A. THE FIRST-STAGE DISTRIBUTED STATE ESTIMATION

In the first stage, the measuring equations of AC subsystem and DC subsystem are expressed in equation (27)-(28):

$$z_{AC_k} = \mathbf{A}_{AC_k} \mathbf{y}_{AC_k} + r_{AC_k} \quad (27)$$

$$\mathbf{z}_{DC} = \sum_{k=1}^K U_{VSC_k}^{AC} \begin{bmatrix} 0 \\ \vdots \\ Y_k \sin \alpha_k \\ Y_k \cos \alpha_k - 1/X_{k-c} \\ 0 \\ \vdots \end{bmatrix} + \mathbf{A}_{DC} \mathbf{y}_{DC} + r_{DC} \quad (28)$$

where  $z_{AC}$  is linearly related to  $y_{AC}$ , and  $z_{DC}$  is linearly related to  $\{y_{DC}, U_{VSC_k}^{AC}\}$ . Define new variables  $U_{VSC_k}^{AC'} = U_{VSC_k}^{AC}$  and the system variable  $y_{DC}$  can be extended to  $y_{DC+} = [U_{VSC_1}^{AC'}, \dots, U_{VSC_K}^{AC'}, y_{DC}]^T$ . Thus the first-stage state esti-

mate is equivalent to the following form:

$$\min \sum_{k=1}^K J_{AC_k}^f(\mathbf{y}_{AC_k}) + J_{DC}^f(\mathbf{y}_{DC+})$$

$$s.t. U_{VSC_k}^{AC'} = U_{VSC_k}^{AC}, \quad \forall k \quad (29)$$

$$J_{AC_k}^f(\mathbf{y}_{AC_k}) = \mathbf{r}_{AC_k}^T \mathbf{R}_{AC_k}^{-1} \mathbf{r}_{AC_k} \quad (30)$$

$$J_{DC}^f(\mathbf{y}_{DC+}) = \mathbf{r}_{DC}^T \mathbf{R}_{DC}^{-1} \mathbf{r}_{DC} \quad (31)$$

In equation (29), the overlap  $U_{VSC_k}^{AC}$  in  $y_{AC}$  and  $y_{DC+}$  is included in the state estimation model as an equality constraint. In the objective function of this model, the AC-DC subsystem is decoupled, and the equality constraint is the only reason for the coupling of the AC-DC subsystem state estimation. In the theory of decentralized optimization, the constraints that make the optimization problem difficult to disperse are generally called “hard constraints”.

As a decentralized optimization algorithm, alternating direction multiplier method (ADMM) is to make “hard constraint” relaxed into the objective function, so that the original problem is decomposed into multiple sub-problems for parallel solution [26]–[28]. When the original problem is convex, ADMM theoretically can eventually converge. The Lagrange function is constructed using the ADMM as follows:

$$\min LR(\mathbf{y}_{AC}, \mathbf{y}_{DC}, \lambda) = J_{AC,1}(\mathbf{y}_{AC}) + J_{DC,1}(\mathbf{y}_{DC})$$

$$+ \lambda(U_{VSC_k}^{AC'} - U_{VSC_k}^{AC}) + \frac{\rho}{2} \|U_{VSC_k}^{AC'} - U_{VSC_k}^{AC}\|_2^2 \quad (32)$$

$$\lambda^k = \lambda^{k-1} + \rho \left( \mathbf{E}_{AC} \mathbf{y}_{AC}^k - C_A + \eta \left( \mathbf{E}_{DC} \mathbf{y}_{DC}^k - C_D \right) \right) \quad (33)$$

$\lambda$  is the Lagrange multiplier, which is updated based on the second derivative method. Its role is to coordinate  $\mathbf{y}_{AC}$  and  $\mathbf{y}_{DC}$  to ensure that the relaxation of the equation constraints. And  $\rho$  represents the penalty factor. In this paper, the subderivative method is used to iterate the multiplier. The concrete steps are as follows:

$$\mathbf{y}_{AC_k}^m := \arg \min J_{AC_k}^f(\mathbf{y}_{AC_k}) - \lambda_k^{m-1} U_{VSC_k}^{AC} \quad (34)$$

$$\mathbf{y}_{DC+}^m := \arg \min J_{DC}^f(\mathbf{y}_{DC+}) + \lambda_k^{m-1} U_{VSC_k}^{AC'} \quad (35)$$

$$\lambda_k^m = \lambda_k^{m-1} + \rho \lambda \left( U_{VSC_k}^{AC'-m} - U_{VSC_k}^{AC-m} \right) \quad (36)$$

In equation (34)–(36), the superscript  $m$  indicates the number of iterations. Equation (34) and equation (35) are state estimation subproblems, where  $\lambda_k^{m-1}$  is a known value and the subproblems therefore can be solved in parallel in AC and DC subsystem. Equation (36) represents the Lagrange multiplier iterative formula, involving the amount of the boundary  $U_{VSC_k}^{AC'}$  and  $U_{VSC_k}^{AC}$ . The multiplier update can be implemented in two ways. The first is to set up an intermediate coordination mechanism between the AC and DC subsystems. The subsystem transmits the boundary information to the intermediate coordination mechanism. The intermediate coordination mechanism updates the multiplier and sends it to the related

subsystem. The other is that without the intermediate coordination mechanism, the boundary between the AC and DC subsystems is transferred to each other, and the subsystem respectively performs multiplier update. This paper utilizes the latter way as an implementation. Iteration convergence condition is set to (for a very small amount) be as follows:

$$\|\lambda^m - \lambda^{m-1}\| \leq \varepsilon_\lambda \quad (37)$$

The realization of the first stage distributed state estimation mechanism is shown in Figure 2.

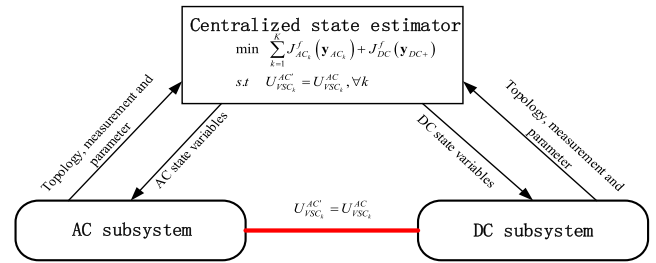


FIGURE 2. The mechanism of the first-stage state estimation.

The state estimator is located in each subsystem to collect the topology, parameters, measurement information and the bounds of the connected subsystems. And the state estimation subproblems are calculated and the Lagrange multipliers are updated. AC subsystems and DC subsystems respectively execute state estimation tasks based on local information, and eventually achieve the overall consistency of the system state estimation through the transfer iteration of the boundary information.

In this process, the AC-DC subsystem state estimator computes in parallel, which improves the computational efficiency and the local information of the subsystem does not need to be uploaded uniformly, so the traffic is significantly reduced.

Let the results be  $y_{AC}$  and  $y_{DC}$ , and within the allowable range of error,  $U_{VSC_k}^{AC'} = U_{VSC_k}^{AC}$ .

### B. THE SECOND-STAGE DISTRIBUTED STATE ESTIMATION

The intermediate variable  $\mathbf{u}$  in the centralized state estimation is split into the AC part and the DC part, and the intermediate variables of the AC subsystem and the DC subsystem are  $\mathbf{u}_{AC_k}$  and  $\mathbf{u}_{DC}$ , respectively.

$$\mathbf{u} = \left[ u_{AC_1}^T, \dots, u_{AC_k}^T, u_{DC}^T \right]^T \quad (38)$$

$$\mathbf{u}_{AC_k} = \{ \varphi_{AC_k,i}, \theta_{AC_k,ij}, \varphi_{AC_k,ij} \} \quad (39)$$

$$\mathbf{u}_{DC} = \{ \varphi_{DC,i}, \varphi_{DC,ij}, \varphi_{VSC_k}, \theta_{VSC_k}^{AC} \} \quad (40)$$

From Section 2.2, it can be seen that the non-linear conversion steps of the AC subsystem and the DC subsystem can be represented as the equation (41) and equation (42), respectively. When the subsystems perform non-linear transformations, they do not need to transfer information and

can be directly solved according to the local first-stage state estimation results.

$$\mathbf{u}_{AC_k} = f_{AC_k}(\mathbf{y}_{AC_k}) \quad (41)$$

$$\mathbf{u}_{DC} = f_{DC}(\mathbf{y}_{DC}) \quad (42)$$

### C. THE THIRD-STAGE DISTRIBUTED STATE ESTIMATION

In AC subsystem,  $\mathbf{u}_{AC_k}$  can be expressed linearly by  $\mathbf{x}_{AC_k}$ :  $\mathbf{u}_{AC_k} = C_{AC_k}x_{AC_k} + r_{AC_k}^t$ . In DC subsystem, in addition to the linear correlation with  $\mathbf{x}_{DC}$ ,  $\mathbf{u}_{DC}$  is also associated with elements  $\left\{ \ln V_{VSC_k}^{AC}; \theta_{VSC_k}^{AC} \right\}$  from  $\mathbf{x}_{AC_k}$ . Define new variables  $\ln V_{VSC_k}^{AC'}$  and  $\theta_{VSC_k}^{AC'}$ , where  $\ln V_{VSC_k}^{AC} = \ln V_{VSC_k}^{AC'}$ ,  $\theta_{VSC_k}^{AC} = \theta_{VSC_k}^{AC'}$ .

Expand  $\mathbf{x}_{DC}$  to  $\mathbf{x}_{DC+}$ , then  $\mathbf{u}_{DC}$  can be linearly represented by  $\mathbf{x}_{DC+}$ .

$$\mathbf{x}_{DC+}^t = \left\{ \ln V_{DC+,i}; \ln V_{VSC_k}^{AC'}; \theta_{VSC_k}^{AC'} \right\} \quad (43)$$

$$\mathbf{u}_{DC} = C_{DC}x_{DC+} + r_{DC}^t \quad (44)$$

According to the derivation of  $\mathbf{W}_u$  in the appendix, the specific expression of  $J^t(\mathbf{x})$  is as follows:

$$\begin{aligned} J^t(\mathbf{x}) &= \sum_{k=1}^K \left( r_{AC_k}^{tT} W_u^{AC_k} r_{AC_k}^t \right) + r_{DC}^{tT} W_u^{DC} r_{DC}^t \\ &+ 2 \sum_{k=1}^K \left( \ln V_{VSC_k}^{AC'} - \ln V_{VSC_k}^{AC} \right) \left\{ \left( \theta_{VSC_k}^{AC} - \theta_{VSC_k}^{AC'} \right) \cdot \theta V \right. \\ &\left. + \left( \varphi_{VSC_k} - \ln V_{VSC_k}^{AC'} - \ln V_{VSC_k}^{DC} \right) \cdot \varphi V \right\} \quad (45) \end{aligned}$$

The sub-objective functions of the AC and DC subsystem are defined in equation (46) and equation (47), respectively. The third-stage system state estimation model is equivalent to the following problem in equation (48). Among them, the overlap between the state variables of AC and DC subsystem is included in the state estimation model in the form of equality constraint.

$$J_{AC_k}^t(x_{AC_k}) = (r_{AC_k}^t)^T W_u^{AC_k} (r_{AC_k}^t) \quad (46)$$

$$\begin{aligned} J_{DC}^t(x_{DC}) &= (r_{DC}^t)^T W_u^{DC} (r_{DC}^t) \\ &+ 2 \sum_{k=1}^K \left( \ln V_{VSC_k}^{AC} - \ln V_{VSC_k}^{AC'} \right) \\ &\times \left\{ \left( \theta_{VSC_k}^{AC} - \theta_{VSC_k}^{AC'} \right) \beta_k \right. \\ &\left. + \left( \varphi_{VSC_k} - \ln V_{VSC_k}^{AC'} - \ln V_{VSC_k}^{DC} \right) \gamma_k \right\} \quad (47) \end{aligned}$$

$$J^t(\mathbf{x}) = \sum_{k=1}^K J_{AC_k}^t(\mathbf{x}_{AC_k}) + J_{DC}^t(\mathbf{x}_{DC})$$

$$\begin{aligned} s, t \ln V_{VSC_k}^{AC} &= \ln V_{VSC_k}^{AC'}, \quad \forall k \\ \theta_{VSC_k}^{AC} &= \theta_{VSC_k}^{AC'} \quad (48) \end{aligned}$$

It can be found that the state estimation model in the third stage is similar to the first stage, so a distributed method

similar to the first stage can be used. The specific solution process is as follows:

$$\begin{aligned} x_{AC_k}^m &:= \arg \min J_{AC_k}^t(x_{AC_k}) - \pi^{m-1} \ln V_{VSC_k}^{AC} \\ &- v^{m-1} \theta_{VSC_k}^{AC} \quad (49) \end{aligned}$$

$$\begin{aligned} x_{DC}^m &:= \arg \min J_{DC}^t(x_{DC}) + \pi^{m-1} \ln V_{VSC_k}^{AC'} \\ &+ v^{m-1} \theta_{VSC_k}^{AC'} \quad (50) \end{aligned}$$

$$\pi_k^m = \pi_k^{m-1} + \rho_\pi \left( \ln V_{VSC_k}^{AC'-m} - \ln V_{VSC_k}^{AC-m} \right) \quad (51)$$

$$v_k^m = v_k^{m-1} + \rho_v \left( \theta_{VSC_k}^{AC'-m} - \theta_{VSC_k}^{AC-m} \right) \quad (52)$$

The final iteration convergence condition is set to:

$$\left\| \pi_k^m - \pi_k^{m-1} \right\| + \left\| v_k^m - v_k^{m-1} \right\| \leq \varepsilon_{\pi v} \quad (53)$$

From the analysis of above process, the AC and DC subsystem state estimator estimate the state of the subsystem according to the equation (49) and the equation (50), and transfer the boundary values  $\left\{ \ln V_{VSC_k}^{AC}; \theta_{VSC_k}^{AC} \right\}$  and  $\left\{ \ln V_{VSC_k}^{AC'}; \theta_{VSC_k}^{AC'} \right\}$  to each other. The subsystem state estimators update the iterative multipliers according to Equation (51) and (52), and so on until it converges.

This section mainly analyzes the distributed state estimation method of AC-DC hybrid distribution network. The method is divided into three stages. The first stage and the third stage are the state estimation processes. The AC-DC subsystem is decoupled based on the ADMM, which achieves the distributed solution to the system state estimation. The second stage is a nonlinear transformation. In this stage, AC-DC subsystem calculates the value of the intermediate variable of the subsystem based on the results of the first stage respectively, which is used as the equivalent measurement of the third stage of the subsystem. The specific process of the third-stage distributed state estimation is shown in Figure3.

## IV. SIMULATION AND RESULTS

In order to verify the validity of the proposed method, the system in the literature [27] is used as a simulation example. The system includes a 7-node DC distribution network. The DC distribution network is connected to nodes #6, #7, and #8 of the IEEE 33 node distribution network through voltage converters at three nodes #1, #3 and #5 respectively. Figure 4 and Figure 5 were simulated system diagram and 33-node distribution network diagram. The system parameter setting is consistent with the literature.

The measurement data is generated by superimposing a normal distribution noise with an average value of 0 on the basis of the load flow calculation. The voltage amplitude measurement error is 0.001, the power measurement error is 0.001, the current amplitude measurement error is 0.001, and the PMU voltage phase angle measurement error is 0.0005. Table 1 shows the system measurement configuration. In the absence of special instructions,  $\rho$  is taken as 20 and  $\varepsilon$  is taken as  $1e10^{-5}$ . The simulation results

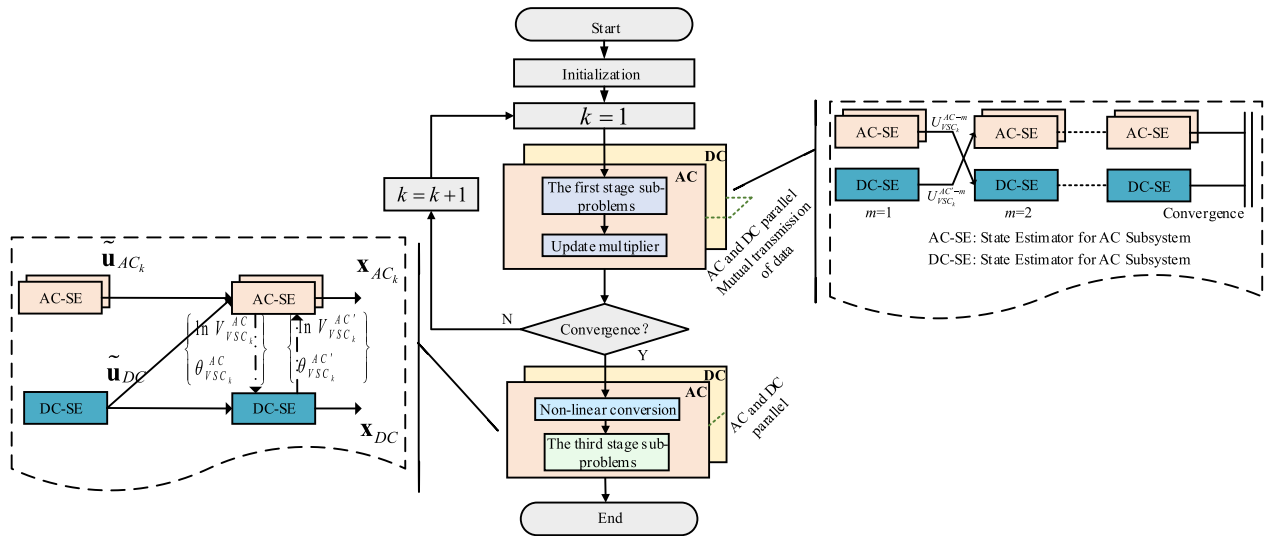


FIGURE 3. The entire flow chart of the three-stage distributed state estimation method.

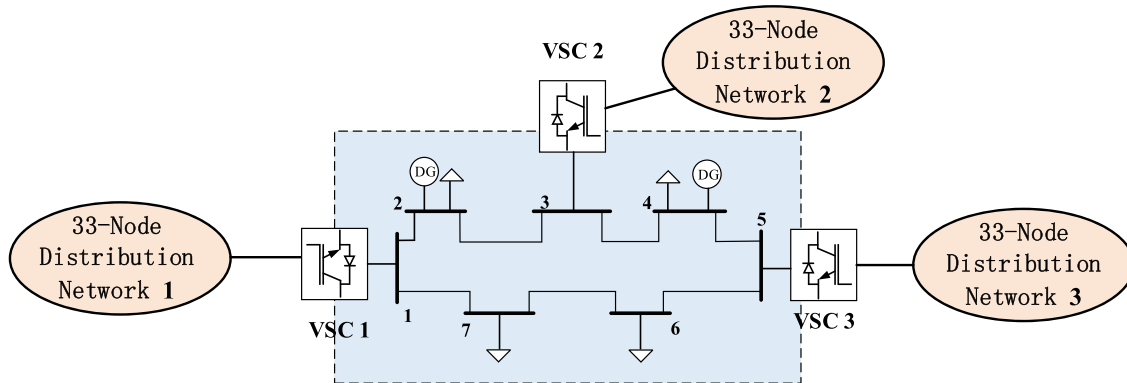


FIGURE 4. The simulated system.

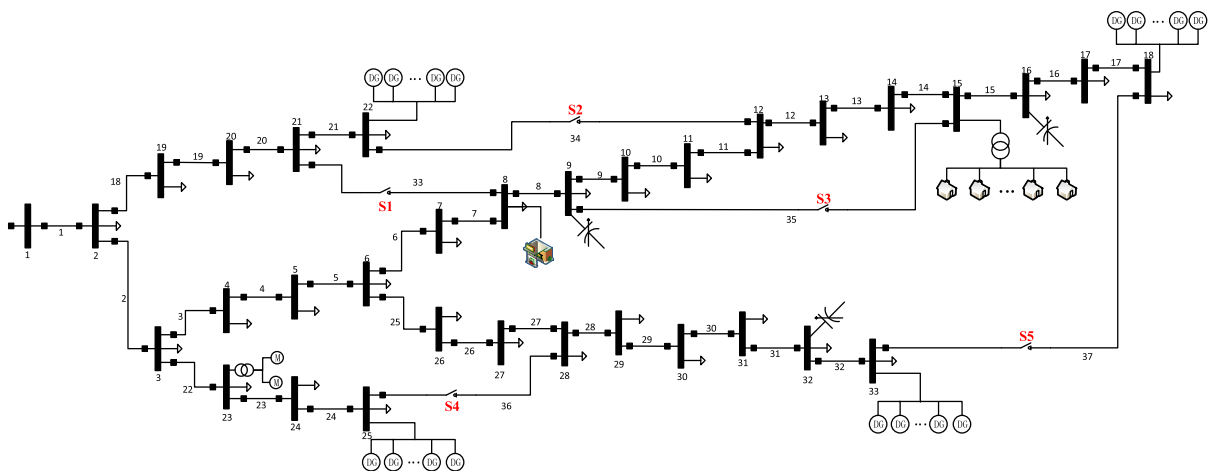


FIGURE 5. The 33-node distribution network diagram.

were obtained on a desktop equipped with an Intel dual-core 2.4 GHz CPU and 2G of memory, and the optimization subproblem was solved using Matlab to call CPLEX.

The results of the absolute errors of voltage amplitude and phase angle are shown in Figure 6 and Figure 7, respectively.

TABLE 1. System measurement configuration.

System	Measurement type	Measurement number
AC subsystem	Inject active power	33
	Injection reactive power	33
	Branch active power	7
	Branch reactive power	7
	Branch current	10
	Voltage amplitude	12
DC subsystem	Voltage phase angle	6
	Injection power	4
	Branch power	2
	Branch current	3
	Voltage amplitude	7

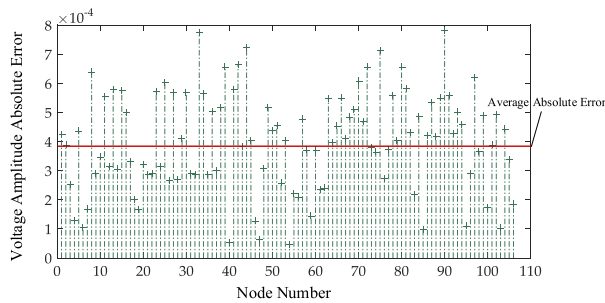


FIGURE 6. Voltage amplitude error obtained from the proposed method.

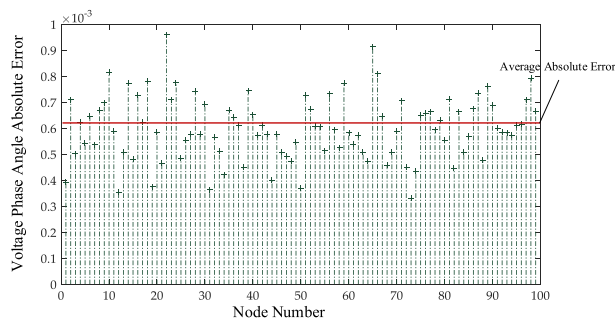


FIGURE 7. Voltage phase angle error obtained from the proposed method.

The maximum absolute errors of the voltage amplitude and phase angle are  $7.82e^{-4}$  and  $9.67e^{-4}$ , respectively. And the average absolute errors are  $3.81e^{-4}$  and  $6.17e^{-4}$ , respectively.

Meanwhile, the comparison between the proposed method and Lagrangian Relaxation method is analyzed. The results of the absolute errors of voltage amplitude and phase angle are shown in Figure 8 and Figure 9, respectively. The maximum absolute errors of the voltage amplitude and phase angle are  $1.38e^{-2}$  and  $4.83e^{-3}$ , respectively. And the average absolute errors are  $5.74e^{-3}$  and  $1.62e^{-3}$ , respectively.

Table 2 shows the average absolute error between the estimated values and the true values of voltage magnitude and phase angle under different method. As can be seen from Table 2, the accuracy of the centralized state estimation is the highest, and the results of the proposed method are very

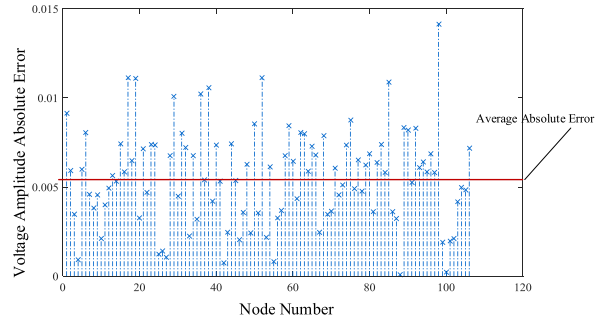


FIGURE 8. Voltage amplitude error obtained from Lagrange Relaxation method.

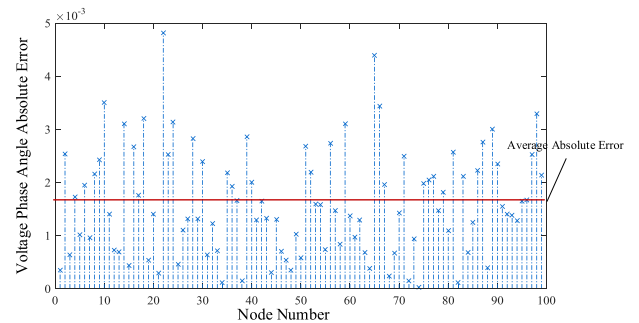


FIGURE 9. Voltage phase angle error obtained from Lagrange Relaxation method.

TABLE 2. The comparison of different state estimation method.

Method Type	The average absolute error of voltage amplitude	The average absolute error of voltage phase angle
WLS	$2.0138e^{-4}$	$6.3294e^{-4}$
Lagrange Relaxation method	$5.7401e^{-3}$	$1.6230e^{-3}$
The proposed method	$3.8139e^{-4}$	$6.1723e^{-4}$

similar to those of the centralized state estimation. Compared with the existing distributed optimization methods, the proposed method has higher accuracy in both voltage amplitude and voltage phase angle.

The reason is that this method does not require a central coordination side. From the mathematical point of view, when the equality constraints in formula (29) are satisfied, the distributed method’s estimation results are equivalent to the centralized method. However, the sub-regions and coordination sides of the existing distributed methods are solved separately, and their estimation results are generally sub-optimal solutions, which are difficult to be consistent with the overall method [19], [22].

As for the convergence of the proposed method, the iterative diagrams of the first stage and the third stage are shown in Figure 10 and Figure 11, respectively.

The consistency of the boundary variables can be achieved quickly, and due to the similarity between the first stage and the third stage state estimation models, the convergence



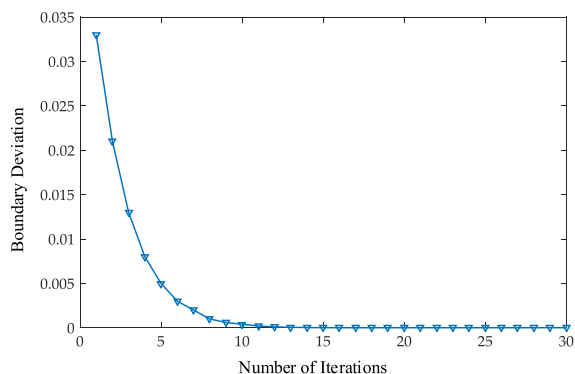


FIGURE 10. The iteration analysis of the first stage.

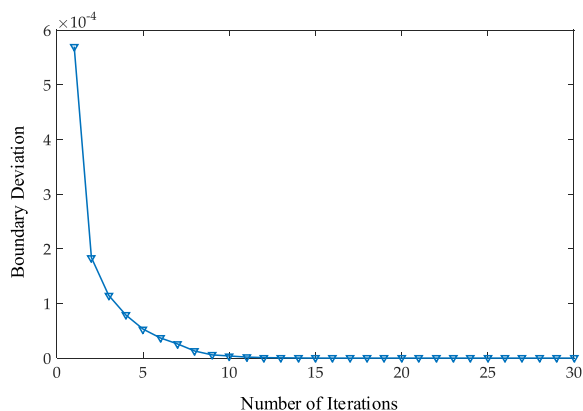


FIGURE 11. The iteration analysis of the third stage.

TABLE 3. The comparison of efficiency.

Method Type	Calculation Time
WLS	4.83
Lagrange Relaxation method	1.31
The proposed method	1.14

characteristics of the iterative convergence curves are also similar.

Table 3 compares the distributed state estimation time of this article with the centralized state estimation calculation time. From Table 3, we can see that the calculation time of the distributed state estimation in the test AC-DC distribution network system only needs 1.14s. Compared with the centralized state estimation method that uses 4.83s, the calculation time is significantly reduced. In the distributed state estimation method proposed in this paper, each subsystem performs the computation task in parallel, so the computational efficiency is improved very effectively compared to the centralized state estimation. Under normal circumstances, the calculation time of the state estimation increases exponentially with the increase of the system scale. Therefore, when the AC-DC hybrid system is large, the dispersion state estimation will have a greater advantage.

Compared with the centralized state estimation, the distributed state estimation has other obvious advantages. As described above, the distributed state estimation mechanism makes each subsystem relatively independent, and the privacy of data is ensured. Moreover, the measurement of each subsystem does not need to be centrally uploaded to the computing center. With the increasing number of measurement devices, distributed state estimation is used. The method can greatly reduce the traffic and relieve the communication pressure.

### V. CONCLUSIONS

This paper establishes a three-stage state estimation model for AC/DC hybrid distribution network, and proposes a distributed state estimation method for hybrid systems based on ADMM. Through simulation analysis, the following conclusions can be drawn:

(1) In the distributed state estimation process, the AC-DC subsystem only needs to transfer the boundary information to each other, and update the Lagrangian multiplier according to the interaction information. The communication mechanism is simple and easy, and the communication volume is greatly reduced;

(2) Compared with centralized state estimation, the computational efficiency of the distributed state estimation is significantly improved, and it is more efficient as the system scale increases;

(3) The accuracy of the three-stage state estimation results is exceedingly high, which is similar to the results of the traditional state estimation method and has outstanding practical engineering application value.

(4) The proposed method in this paper is suitable for the distributed state estimation of AC-DC hybrid distribution networks with single or mixed measurement environment.

The next research work will focus on the dynamic distributed state estimation based on Extended Kalman Filter, which can have better real-time performance in tracking the running state of the distribution network, and have relatively high accuracy of system state prediction capability.

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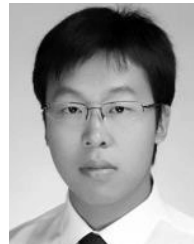
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