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Identification for FIR Systems With Scheduled Binary-Valued Observations

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ABSTRACT This paper studies the identification of finite impulse response (FIR) systems whose output observations are subject to both the binary-valued quantization and the scheduling scheme. By utilizing the statistical property of the system noise and the scheduling policy, an empirical-measure-based identification algorithm is proposed. Under periodical inputs, it is proved that the estimation from the algorithm can converge to the real parameters. The mean-square convergence rate of the estimation error is established, based on which and the Cramér-Rao lower bound, the asymptotic efficiency of algorithm is proved. Moreover, the communication rate is derived and the input design problem is discussed. A numerical example is given to illustrate the main results obtained.

INDEX TERMS Identification, FIR systems, binary-valued quantization, scheduling policy, convergence, Cramér-Rao lower bound.

I. INTRODUCTION

With the extensive use of digital electronics and microsensors in cyber-physical systems, networked control systems and wireless sensor networks, the constraint of communication resources must be considered, when estimating the system parameter remotely in a network environment. For example, in wireless sensor networks, large amounts of sensors often share a public communication channel and they are powered by batteries [1]. This causes both the network bandwidth and sensor energy are limited. The shortage of bandwidth means nodes cannot transmit a mass of data, meanwhile, the limitation of sensor energy results in limited times of transmission. In the highly developed information technology era, traditional system identification has to face these new challenges [2]. To overcome the constraints, identification methods should be improved in both temporal and spatial context.

The scheduled communication scheme is one of effective ways to deal with the transmission constrain. More specifically, the sensor only transmits its sample data to the remote estimator only when a scheduling condition is satisfied, otherwise, the sensor remains silence. This kind of schemes reduces the communication obviously, meanwhile, it has a better performance than just dropping out packets by a deterministic rule or randomly. Because the not-trigged condition can be known when there is no communication, in this way, the remote estimator can get extra information naturally. It has attracted lots of attention and been extensively introduced to the field of state estimation, feedback control and system identification [3]–[11]. In [3], a novel scheduling scheme for estimating system state remotely with multi-channel transmission was discussed. Reference [4] proposed an optimal communication scheduled remote estimation algorithm over an additive noise channel. References [5] and [6] investigated the scheduled transmission in networked control systems. Meanwhile, [7] considered hybrid scheduling and quantized output feedback control. An asymptotically optimal parameter estimation under communication constraints was addressed by [8]. And [9] proposed an ML (Maximum Likelihood)-based method to estimate parameters with scheduled measurements. The parameter estimation of linear systems is discussed under controlled communication in [10] and under stochastic packet scheduling in [11].

On the other hand, usually only quantized output observations are available in modern control systems due to sensor limitations, signal quantization by analog-to-digital (A/D) converters, or coding for communications. And the identification with quantized data has also received much research attention, and much fruitful progress has been made [12]–[20]. Under full rank periodic inputs, [12] and [13] studied the system identification with binary-valued outputs, and discussed quantized identification algorithms and their key convergence properties in both stochastic and deterministic frameworks. In [14], a weighted Least-Squares approach was proposed for FIR systems. Reference [15] considered the input design in worst-case system parameter estimation with quantized observations. [16] dealt with the approximation problem of fixed-order FIR systems subjected to quantized input/output observations. With general quantized inputs and quantized output observations, [17] gave an asymptotically efficient identification algorithm for FIR systems. The interest of research also expanded to Wiener and Hammerstein systems [18]–[20].

For reducing communication burden while guaranteeing the system performance, taking both quantization and transmission scheduling into account is an intuitive way. However, the scheduling policy breaks the completeness of the observation data, and the quantization makes the relationships between the measured quantized signals and the output to be essentially nonlinear. All these characteristics bring difficulties to the design of parameter estimation algorithms and the analysis of convergence performances, and lead to less results on system identification with both scheduled and quantized output observations.

This paper focuses on FIR systems to study the identification with scheduled binary-valued observations. Firstly, the formulation of FIR systems is given with binary-valued output observations and the per-specified scheduling policy. By utilizing the statistical property of the system noise, we introduce an empirical-measure-based identification algorithm. Secondly, by use of Bayes' Law, total probability formula and the law of large numbers, under periodical input the strong convergence is proved, the mean-square convergence rate and the CR (Cramér-Rao) lower bound are established, and the asymptotic efficiency is also illustrated. Finally, the communication rate is derived and the input design is studied for the minimum communication rate.

The coming sections of this paper are arranged as follows. Section II describes the system set-up and the identification problem with scheduled binary-valued observations. Section III proposes an algorithm to identify the system parameters. Section IV establishes the strong convergence of the estimates and the mean-square convergence rate of the estimation error, together with the asymptotic efficiency and the communication rate. Section V uses a numerical example to simulate the main theoretical results obtained and show the the effectiveness of the algorithm. Section VI summarizes the conclusion of the paper and gives the future works.

II. PROBLEM FORMULATION

Consider an SISO (single-input-single-output) finite impulse response (FIR) system described by

$$y_k = a_1 u_k + \cdots + a_n u_{k-n+1} + d_k = \phi_k^T \theta + d_k, \tag{1}$$

where $\theta = [a_1, \ldots, a_n]^T \in \mathbb{R}^n$ is the unknown parameters, $\phi_k = [u_k, \ldots, u_{k-n+1}]^T$ is the regression vector, and d_k is the system noise. Here, the symbol ^{T} is used to stand for a vector' (or a matrix') transpose. The output y_k cannot be measured exactly, but done by a binary-valued sensor whose threshold is $C \in (-\infty, +\infty)$. We use an indicator function to represent such binary-valued observation as

$$s_k = I_{\{y_k \le C\}} = \begin{cases} 1, & \text{if } y_k \le C; \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

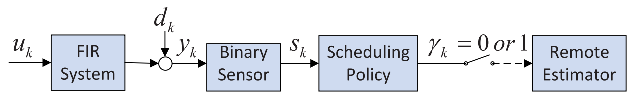

FIGURE 1. System set-up.

As shown by Figure 1, s_k is transmitted to a remote estimator through a communication channel/network. To reduce the number of transmission, a scheduling policy is implemented, that is,

$$\gamma_k = I_{\{|s_k - \tau_k| \ge \delta_k\}} = \begin{cases} 1, & \text{if } |s_k - \tau_k| \ge \delta_k; \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $\{\tau_k\}$ and $\{\delta_k\}$ are to be designed to balance the system performance and the communication bandwidth utilization. When $\gamma_k = 1$, a transmission is triggered and then s_k is transmitted to the receiver. When $\gamma_k = 0$, the estimator can not receive anything from the channel. As a consequence, the available information for the estimator is $\{\gamma_k s_k, \gamma_k\}$ at time k.

For such a system set-up, this paper aims to investigate the corresponding identification problem. Two essential issues will be discussed: 1) How to construct algorithms to estimate θ based on the input$\{u_k\}$ and the scheduled binary-valued observation $\{\gamma_k s_k, \gamma_k\}$? 2) How $\{\tau_k\}$ and $\{\delta_k\}$ (namely, the scheduling policy) affect the performance of the algorithm and save the communication resource?

Assumption 1: The noise sequence $\{d_k\}$ follows that: 1) it is independent and identically distributed (i.i.d.). 2) The cumulative distribution function and the probability density function of d_1 is denoted by $F(\cdot)$ and $f(\cdot)$. $F(\cdot)$ is invertible and its inverse function $F^{-1}(\cdot)$ is twice continuously differentiable. $f(\cdot)$ is bounded. 3) The moment generating function of d_1 exists.

III. IDENTIFICATION ALGORITHM

For simplicity of algorithm design and analysis, it is assumed that the input is n-periodic, that is, $u_{k+n} = u_k$ for all k. This is extensively used in quantized identification and the advantages can be seen in [13]. For non-periodic signals, it can still work (see [21, pp. 81-82]).

Suppose that one-period of $\{u_k\}$ is $u_1 = v_1, u_2 = v_2, \ldots, u_n = v_n$, and denote $\pi_1 = \phi_1^T, \ldots, \pi_n = \phi_n^T$. Then the circulant matrix generated by $v_1, \ldots, v_n$ is

$$\Phi = [\pi_1^T, \ldots, \pi_n^T]^T. \tag{4}$$

The n-periodic input $\{u_k\}$ is said to be *full rank* if the n-dimensional matrix Φ is invertible.

At N (observation length), define

$$N_i = \sum_{j=0}^{L_N-1} I_{\{|1-\tau_{i+jn}|\geq\delta_{i+jn}\}} = \sum_{\substack{j=0,\ldots,L_N-1\\ |1-\tau_{i+jn}|\geq\delta_{i+jn}}} 1, \quad i=1,\ldots,n, \tag{5}$$

where $L_N = \lfloor \frac{N}{n} \rfloor$ represents the largest integer less than or equal to $\frac{N}{n}$. We introduce an algorithm to estimate θ as follows,

$$\xi_{i,N} = \frac{1}{N_i}\sum_{j=0}^{L_N-1} \gamma_{i+jn}s_{i+jn}, \tag{6}$$

$$\zeta_{i,N} = C - F^{-1}(\xi_{i,N}), \tag{7}$$

$$\widehat{\theta}_N = \Phi^{-1}\left(\zeta_{1,N},\ldots,\zeta_{n,N}\right)^T, \tag{8}$$

where $\widehat{\theta}_N$ denotes the estimate of θ at N. C is the threshold of the binary-valued sensor in (2). The distribution function $F(\cdot)$ is given in Assumption 1.

IV. CONVERGENCE PERFORMANCE

This section will establish the strong convergence of the algorithm. The mean-square convergence rate of the estimation error will be given, together with the CR lower bound. Then it will be shown that the algorithm is asymptotically efficient. Moreover, the communication rate will be obtained.

A. CONVERGENCE AND CONVERGENCE RATE

Theorem 2: Consider system (1) with binary-valued observations (2) and scheduling mechanism (3) under Assumption 1. If the input $\{u_k\}$ is full rank and $\min_{1\leq i\leq n} N_i \to \infty$ as $N \to \infty$ (N_i is the one in (5)), then the parameter estimate $\widehat{\theta}_N$ provided by algorithm (6)-(8) can strongly converge to the real value θ, i.e.,

$$\widehat{\theta}_N \to \theta, \quad w.p.1. \quad \text{as } N\to\infty.$$

Proof: If $r_k = 0$, then we have $\gamma_k s_k = 0$. If $r_k = 1$ and $|1-\tau_k| < \delta_k$, then it indicates that $s_k = 0$ by (3), and we also have $\gamma_k s_k = 0$. Therefore, it is known that

$$\gamma_k s_k = 0 \ \text{ if } \ |1-\tau_k| < \delta_k. \tag{9}$$

According to Bayes' law and (3), one can have

$$\Pr(\gamma_k = 1, s_k = 1) = \Pr(s_k = 1)\Pr(|1-\tau_k| \geq \delta_k | s_k = 1).$$

Considering that $\Pr(s_k = 1) = \Pr(y_k \leq C) = F(C-\phi_k^T\theta)$, it can be seen that

$$E\gamma_k s_k = \Pr(\gamma_k = 1, s_k = 1) = F(C-\phi_k^T\theta) \quad \text{if } \ |1-\tau_k| \geq \delta_k. \tag{10}$$

Noticing that $\phi_{i+jn}^T = \pi_i$ for any positive integer j, which and (10) can give

$$E\gamma_{i+jn}s_{i+jn} = F(C-\pi_i\theta) \quad \text{if } \ |1-\tau_{i+jn}| \geq \delta_{i+jn}, \quad i=1,\ldots,n. \tag{11}$$

Due to (6) and (9), it can be derived that

$$\begin{aligned}\xi_{i,N} &= \frac{1}{N_i}\sum_{\substack{j=0,\ldots,L_N-1\\ |1-\tau_{i+jn}|\geq\delta_{i+jn}}} \gamma_{i+jn}s_{i+jn} + \frac{1}{N_i}\sum_{\substack{j=0,\ldots,L_N-1\\ |1-\tau_{i+jn}|<\delta_{i+jn}}} \gamma_{i+jn}s_{i+jn}\\ &= \frac{1}{N_i}\sum_{\substack{j=0,\ldots,L_N-1\\ |1-\tau_{i+jn}|\geq\delta_{i+jn}}} \gamma_{i+jn}s_{i+jn}.\end{aligned} \tag{12}$$

Since $\min_{1\leq i\leq n} N_i \to \infty$ if and only if $N_i \to \infty$ for $i = 1,\ldots,n$, under hypothesis and by the law of large numbers, (11) and (12) can yield that $\xi_{i,N} \to F(C-\pi_i\theta)$, which implies that

$$\zeta_{i,N} \to \pi_i\theta, \ \ w.p.1 \ \text{ as } N\to\infty, \ i=1,\ldots,n. \tag{13}$$

By (7), (8), (13) and (4), we can know that

$$\widehat{\theta}_N \to \Phi^{-1}\begin{pmatrix}\pi_1\\ \vdots\\ \pi_n\end{pmatrix}\theta = \theta, \ \ w.p.1. \ \text{ as } N\to\infty.$$

Hence, the theorem is proved. □

Let $\Sigma(N;\theta)$ represents the covariance matrix of the estimation error of $\widehat{\theta}_N$, i.e.,

$$\Sigma(N;\theta) = E(\widehat{\theta}_N - \theta)(\widehat{\theta}_N - \theta)^T,$$

and denote $\sigma_i = \frac{F(C-\pi_i\theta)(1-F(C-\pi_i\theta))}{f^2(C-\pi_i\theta)}$ for $i=1,\ldots,n$, where $E(\cdot)$ is the expectation.

Theorem 3: Under the condition of Theorem 2, if

$$N_i/N \to \lambda_i \ \text{ as } N\to\infty, \quad i=1,\ldots,n, \tag{14}$$

then the mean-square convergence rate of the estimate $\widehat{\theta}_N$ given by (8) is

$$N\Sigma(N;\theta) \to \Phi^{-1}diag\left(\frac{\sigma_1}{\lambda_1},\ldots,\frac{\sigma_n}{\lambda_n}\right)\Phi^{-T} \text{ as } N\to\infty. \tag{15}$$

Proof: Denoting $\widetilde{\zeta}_{i,N} = \zeta_{i,N} - \pi_i\theta$, from [13] and (13) we can have $N_iE\widetilde{\zeta}_{i,N}^2 \to \sigma_i$, $\sqrt{N_i}E\widetilde{\zeta}_{i,N} \to 0$, $i=1,\ldots,n$, which together with (14) implies that

$$NE\left[(\widetilde{\zeta}_{1,N},\ldots,\widetilde{\zeta}_{n,N})^T(\widetilde{\zeta}_{1,N},\ldots,\widetilde{\zeta}_{n,N})\right] \to diag\left(\frac{\sigma_1}{\lambda_1},\ldots,\frac{\sigma_n}{\lambda_n}\right) \text{ as } N\to\infty. \tag{16}$$

By (4), it can be verified that $\theta = \Phi^{-1}(\pi_1\theta,\ldots,\pi_n\theta)^T$, which together with (8) and (16) can lead to

$$\begin{aligned}&N\Sigma(N;\theta)\\ &= N\Phi^{-1}E\left[(\widetilde{\zeta}_{1,N},\ldots,\widetilde{\zeta}_{n,N})^T(\widetilde{\zeta}_{1,N},\ldots,\widetilde{\zeta}_{n,N})\right]\Phi^{-T}\\ &\to \Phi^{-1}diag\left(\frac{\sigma_1}{\lambda_1},\ldots,\frac{\sigma_n}{\lambda_n}\right)\Phi^{-T} \text{ as } N\to\infty.\end{aligned}$$

The proof is obtained. □

$$\Sigma_{CR}(N;\theta) = \left(\Phi^T diag\left(\frac{N_1}{\sigma_1}, \ldots, \frac{N_n}{\sigma_n}\right)\Phi + \sum_{\substack{k=nL_N+1,\ldots,N \\ |1-\tau_k|\geq\delta_k}} \frac{\phi_k f^2(C-\phi_k^T\theta)\phi_k^T}{F(C-\phi_k^T\theta)(1-F(C-\phi_k^T\theta))} \right)^{-1}. \quad (17)$$

B. ASYMPTOTICALLY EFFICIENCY

Lemma 4: Based on $\{\gamma_k s_k : 1 \leq k \leq N\}$, the CR lower bound for estimating θ is [see (17) at the top of this page].

Proof: Let z_k be some possible sample value of $\gamma_k s_k$. In view of (9), it is known that $z_k = 0$ for $k \in \{\iota : 1 \leq \iota \leq N, |1-\tau_\iota| < \delta_\iota\}$. From this and (10), the likelihood function of $\gamma_1 s_1, \ldots, \gamma_N s_N$ taking values $z_1, \ldots, z_N$ conditioned on θ is

$$\begin{aligned} &\ell(z_1,\ldots,z_N;\theta) \\ &\quad = \Pr(\gamma_1 s_1 = z_1, \ldots, \gamma_N s_N = z_N; \theta) \\ &\quad = \prod_{\substack{k=1,\ldots,N \\ |1-\tau_k|\geq\delta_k}} F^{z_k}(C-\phi_k^T\theta)(1-F(C-\phi_k^T\theta))^{1-z_k} \\ &\quad = \prod_{i=1}^{n}\prod_{j\in\Omega_i} F^{z_{i+jn}}(C-\pi_i\theta)(1-F(C-\pi_i\theta))^{1-z_{i+jn}} \\ &\qquad + \prod_{k\in\Upsilon} F^{z_k}(C-\phi_k^T\theta)(1-F(C-\phi_k^T\theta))^{1-z_k}, \end{aligned}$$

where $\Omega_i = \{\iota : 0 \leq \iota \leq L_N - 1, |1-\tau_{i+\iota n}| \geq \delta_{i+\iota n}\}$ and $\Upsilon = \{\iota : nL_N + 1 \leq \iota \leq N, |1-\tau_\iota| \geq \delta_\iota\}$.

Replace the particular realizations z_k by their corresponding random variables $\gamma_k s_k$, and denote the resulting quantity by $\ell = \ell(\gamma_1 s_1, \ldots, \gamma_N s_N; \theta)$. Let $\chi_{i,j} = \gamma_{i+jn} s_{i+jn}$. Then, we have

$$\begin{aligned} \ell = &\prod_{i=1}^{n}\prod_{j\in\Omega_i} F^{\chi_{i,j}}(C-\pi_i\theta)(1-F(C-\pi_i\theta))^{1-\chi_{i,j}} \\ &\quad + \prod_{k\in\Upsilon} F^{\gamma_k s_k}(C-\phi_k^T\theta)(1-F(C-\phi_k^T\theta))^{1-\gamma_k s_k}, \end{aligned}$$

which results in

$$\begin{aligned} \log\ell = &\sum_{i=1}^{n}\sum_{j\in\Omega_i}\big(\chi_{i,j}\log F(C-\pi_i\theta) \\ &+(1-\chi_{i,j})\log(1-F(C-\pi_i\theta))\big) \\ &+\sum_{k\in\Upsilon}\big(\gamma_k s_k \log F(C-\phi_k^T\theta) \\ &+(1-\gamma_k s_k)\log(1-F(C-\phi_k^T\theta))\big) \end{aligned}$$

and

$$\begin{aligned} \frac{\partial\log\ell}{\partial\theta} = &\sum_{i=1}^{n}\sum_{j\in\Omega_i}\left(-\chi_{i,j}\pi_i^T\frac{f(C-\pi_i\theta)}{F(C-\pi_i\theta)}\right. \\ &\left.+(1-\chi_{i,j})\pi_i^T\frac{f(C-\pi_i\theta)}{1-F(C-\pi_i\theta)}\right) \\ &+\sum_{k\in\Upsilon}\left(-\gamma_k s_k\phi_k\frac{f(C-\phi_k^T\theta)}{F(C-\phi_k^T\theta)}\right. \\ &\left.+(1-\gamma_k s_k)\phi_k\frac{f(C-\phi_k^T\theta)}{(1-F(C-\phi_k^T\theta))}\right). \end{aligned}$$

In terms of (10), (5) and (4), it follows that

$$\begin{aligned} E\frac{\partial^2\log\ell}{\partial\theta^2} = &-\sum_{i=1}^{n}\sum_{j\in\Omega_i}\frac{\pi_i^T\pi_i}{\sigma_i} \\ &-\sum_{k\in\Upsilon}\frac{\phi_k f(C-\phi_k^T\theta)\phi_k^T}{F(C-\phi_k^T\theta)(1-F(C-\phi_k^T\theta))} \\ = &-\Phi^T diag\left(\frac{N_1}{\sigma_1},\ldots,\frac{N_n}{\sigma_n}\right)\Phi \\ &-\sum_{k\in\Upsilon}\frac{\phi_k f^2(C-\phi_k^T\theta)\phi_k^T}{F(C-\phi_k^T\theta)(1-F(C-\phi_k^T\theta))}. \end{aligned}$$

Hence, the lemma is proved. □

Theorem 5: Under the condition of Theorem 3, the asymptotic efficiency of the estimate $\widehat{\theta}_N$ from (8) can be given by

$$N\Sigma(N;\theta) - N\Sigma_{CR}(N;\theta) \to 0 \text{ as } N \to \infty.$$

Proof: By virtue of Lemma 4, one can have

$$N\Sigma_{CR}(N;\theta) = \left(I_{1,N} + I_{2,N}\right)^{-1}, \quad (18)$$

where $I_{1,N} = \frac{1}{N}\Phi^T diag\left(\frac{N_1}{\sigma_1},\ldots,\frac{N_n}{\sigma_n}\right)\Phi$ and $I_{2,N} = \frac{1}{N}\sum_{k\in\Upsilon}\frac{\phi_k f^2(C-\phi_k^T\theta)\phi_k^T}{F(C-\phi_k^T\theta)(1-F(C-\phi_k^T\theta))}$. By (14), it can be seen that

$$I_{1,N} \to \Phi^T diag\left(\frac{\lambda_1}{\sigma_1},\ldots,\frac{\lambda_n}{\sigma_n}\right)\Phi \text{ as } N \to \infty. \quad (19)$$

From Assumption 1, we know that $f(\cdot)$ is bounded and then there exists a real number M such that $f(z) \leq M$ for $x \in \mathbb{R}$. Note that $\|\phi_k\| \leq \rho$ and $C - \phi_k^T\theta \in [-\eta, \eta]$, where $\rho = n\max_{1\leq i\leq n}|v_i|$ and $\eta = C + \rho\|\theta\|^2$ are two constants, and $\|\cdot\|$ is the Euclidean norm of a vector or the Frobenius norm of a matrix. As a result, it can be concluded that

$$\begin{aligned} \left\|\frac{\phi_k f^2(C-\phi_k^T\theta)\phi_k^T}{F(C-\phi_k^T\theta)(1-F(C-\phi_k^T\theta))}\right\| &\leq \frac{\rho^2 M^2}{F(-\eta)(1-F(\eta))} \\ &< \infty. \end{aligned}$$

Together with $\sum_{k\in\Upsilon} 1 < n$, the above gives rise to

$$\|I_{2,N}\| \leq \frac{1}{N}\frac{n\rho^2 M^2}{F(-\eta)(1-F(\eta))} \to 0 \text{ as } N \to \infty. \quad (20)$$

Combining (18)-(20), we have

$$\begin{aligned} N\Sigma_{CR}(N;\theta) &\to \left(\Phi^T diag\left(\frac{\lambda_1}{\sigma_1},\ldots,\frac{\lambda_n}{\sigma_n}\right)\Phi\right)^{-1} \\ &= \Phi^{-1} diag\left(\frac{\sigma_1}{\lambda_1},\ldots,\frac{\sigma_n}{\lambda_n}\right)\Phi^{-T} \\ &\quad \text{as } N \to \infty, \end{aligned}$$

which and (15) complete the proof. □

$$
\begin{aligned}
E\gamma_{i+jn} &= \Pr(\gamma_{i+jn}=1)\\
&= \Pr\left(|s_{i+jn}-\tau_{i+jn}| \ge \delta_{i+jn}\right)\\
&= \Pr\left(|s_{i+jn}-\tau_{i+jn}| \ge \delta_{i+jn} | s_{i+jn}=1\right)\Pr\left(s_{i+jn}=1\right)\\
&\quad + \Pr\left(|s_{i+jn}-\tau_{i+jn}| \ge \delta_{i+jn} | s_{i+jn}=0\right)\Pr\left(s_{i+jn}=0\right)\\
&= I_{\{|1-\tau_{i+jn}|\ge\delta_{i+jn}\}}F(C-\pi_i\theta) + I_{\{|\tau_{i+jn}|\ge\delta_{i+jn}\}}(1-F(C-\pi_i\theta)).
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
\lim_{N\to\infty}\frac{1}{L_N}\sum_{j=0}^{L_N-1}\gamma_{i+jn} &= \lim_{N\to\infty}\frac{1}{L_N}\sum_{j=0}^{L_N-1}\left(I_{\{|1-\tau_{i+jn}|\ge\delta_{i+jn}\}}F(C-\pi_i\theta) + I_{\{|\tau_{i+jn}|\ge\delta_{i+jn}\}}(1-F(C-\pi_i\theta))\right)\\
&= \left(\lim_{N\to\infty}\frac{N_i}{L_N}\right)F(C-\pi_i\theta) + \left(\lim_{N\to\infty}\frac{1}{L_N}\sum_{j=0}^{L_N-1} I_{\{|\tau_{i+jn}|\ge\delta_{i+jn}\}}\right)(1-F(C-\pi_i\theta))\\
&= n\lambda_i F(C-\pi_i\theta) + n\mu_i(1-F(C-\pi_i\theta)).
\end{aligned} \tag{25}
$$

C. COMMUNICATION RATE

To describe the capacity of the scheduling mechanism in saving the communication resources, we define the communication rate as

$$\overline{\gamma} = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\gamma_k. \tag{21}$$

Theorem 6: Under the condition of Theorem 3, if

$$\frac{1}{N}\sum_{j=0}^{L_N-1} I_{\{|\tau_{i+jn}|\ge\delta_{i+jn}\}} \to \mu_i, \quad i=1,\ldots,n, \tag{22}$$

then the communication rate $\overline{\gamma}$ from (21) can be given by

$$\overline{\gamma} = \sum_{i=1}^{n}\left(\lambda_i F(C-\pi_i\theta) + \mu_i(1-F(C-\pi_i\theta))\right). \tag{23}$$

Proof: By (3) and the law of total probability, it can be seen that [see (24) at the top of this page].

Note that $\lim_{N\to\infty} N/L_N = n$. By (5), (24) and the law of large numbers, we have [see (25) at the top of this page], which implies that

$$
\begin{aligned}
\lim_{N\to\infty}\frac{1}{N}\sum_{j=0}^{L_N-1}\gamma_{i+jn} &= \lim_{N\to\infty}\frac{L_N}{N}\cdot\frac{1}{L_N}\sum_{j=0}^{L_N-1}\gamma_{i+jn}\\
&= \lambda_i F(C-\pi_i\theta) + \mu_i(1-F(C-\pi_i\theta)).
\end{aligned} \tag{26}
$$

Considering $\frac{1}{N}\sum_{i=1}^{N}\gamma_i = \frac{1}{N}\sum_{i=1}^{nL_N}\gamma_i + \frac{1}{N}\sum_{i=nL_N+1}^{N}\gamma_i$ and $\frac{1}{N}\sum_{i=nL_N+1}^{N}\gamma_i \to 0$ as $N\to\infty$, from (26) we have

$$
\begin{aligned}
\overline{\gamma} &= \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{nL_N}\gamma_i\\
&= \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{n}\sum_{j=0}^{L_N-1}\gamma_{i+jn}\\
&= \sum_{i=1}^{n}\left(\lim_{N\to\infty}\frac{1}{N}\sum_{j=0}^{L_N-1}\gamma_{i+jn}\right)\\
&= \sum_{i=1}^{n}\left(\lambda_i F(C-\pi_i\theta) + \mu_i(1-F(C-\pi_i\theta))\right).
\end{aligned}
$$

The theorem is obtained. □

For a given scheduling policy, by (23) we know that $\overline{\gamma}$ is a function of the system input, which can be represented by $\overline{\gamma} = \overline{\gamma}(\pi_1,\ldots,\pi_n)$. Then an interesting problem is how to design the input such that $\overline{\gamma}(\pi_1,\ldots,\pi_n)$ achieves a minimum value, which can be stated as a constrained minimization problem

$$
\begin{aligned}
&\min\nolimits_{\pi_1,\ldots,\pi_n}\ \overline{\gamma}(\pi_1,\ldots,\pi_n)\\
&\text{s.t. } \Phi \text{ is full rank,}
\end{aligned}
$$

where "s.t." denotes "subject to".

By (23) again, it can be derived that

$$\overline{\gamma} = \sum_{i=1}^{n}\left((\lambda_i-\mu_i)F(C-\pi_i\theta) + \mu_i\right). \tag{27}$$

If $\lambda_i \ge \mu_i$, then we can have $\min_{\pi_i}\{(\lambda_i-\mu_i)F(C-\pi_i\theta) + \mu_i\} = \mu_i$. If $\lambda_i < \mu_i$, then it follows that $\min_{\pi_i}\{(\lambda_i-\mu_i)F(C-\pi_i\theta) + \mu_i\} = \lambda_i$. As a consequence, by (27) it is known that

$$\min_{\pi_1,\ldots,\pi_n}\overline{\gamma} \ge \sum_{\substack{i=1,\ldots,n\\ \lambda_i\ge\mu_i}}\mu_i + \sum_{\substack{i=1,\ldots,n\\ \lambda_i<\mu_i}}\lambda_i.$$

The above provides a lower bound of the minimum communication rate that can be achieved by the input design.

Remark 7: In light of (5), (14) and (22), one can see that N_i, λ_i and μ_i are all generated by the scheduled sequences $\{\tau_k\}$ and $\{\delta_k\}$. Therefore, $\min_{1\le i\le n} N_i \to \infty$ in Theorem 2, (14) in Theorem 3 and (22) in Theorem 6, in fact, present the conditions that should be met by the scheduling policy (3) to ensure the strong convergence and the asymptotic efficiency of the identification algorithm (6)-(8) and the existence of the communication rate (21).

V. NUMERICAL SIMULATION

Consider an FIR system $y_k = a_1 u_k + a_2 u_{k-1} + d_k = \phi_k^T\theta + d_k$, where the parameter $\theta = [7,3]^T$ is unknown, but within the range of $\Theta = \{(x,y) : 0 \le x \le 18, 0 \le y \le 5\}$. $\{d_k\}$ is i.i.d. with zero mean and standard deviation $\sigma = 80$. The binary-valued observation $s_k = I_{\{y_k\le C\}}$, where the threshold $C = 8$. The input signal $\{u_k\}$ is 2-periodic with one period $[v_1, v_2] = [23, 11]$. The scheduling transmission mechanism is

$$\gamma_k = I_{\{|s_k-\tau_k|\geq\delta_k\}} = \begin{cases} 1, & \text{if } |s_k-\tau_k| \geq \delta_k; \\ 0, & \text{otherwise,} \end{cases} \tag{28}$$

where $\tau_k = F(C - \phi_k^T\bar{\theta})$ with $\bar{\theta} = [9, 2.5]^T$, $\delta_k = 0.4M_k + 0.3$, and $\{M_k\}$ is a maximum length sequence (the coefficients of its feedback function are 0010101001001 and its initial values are 0010010101011).

For the observation length $N = 3000$, the identification algorithm (6)-(8) is employed to generate the estimate $\widehat{\theta}_N$ of θ. Fig. 2 displays a trajectory of $\widehat{\theta}_N$, where $\widehat{\theta}_N$ can indeed converge to the real value and this is accord with

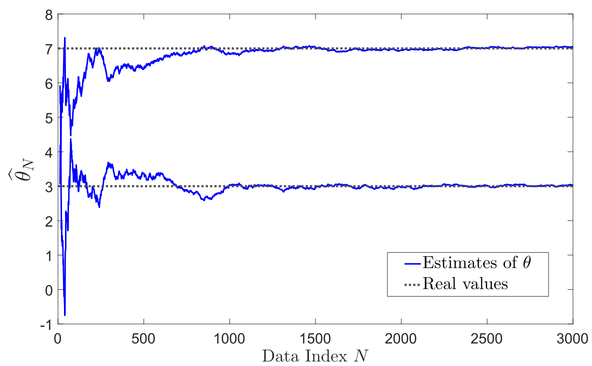

FIGURE 2. Convergence of $\widehat{\theta}_N$.

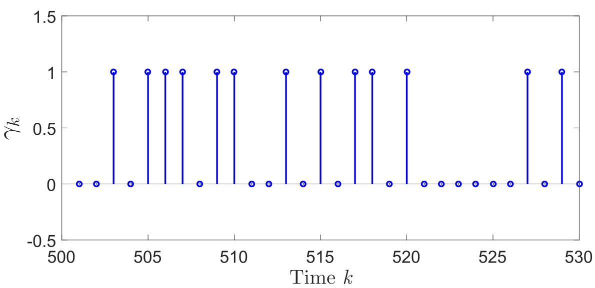

FIGURE 3. A realization of γ_k on the horizon [500, 530].

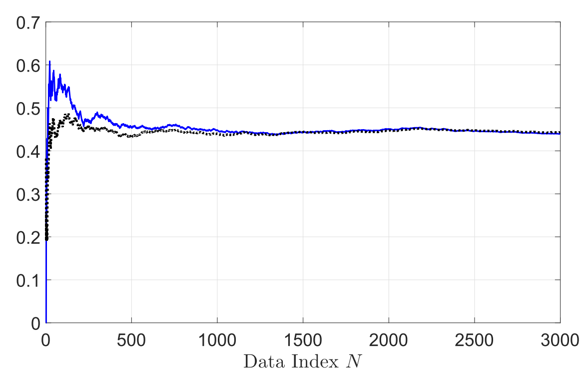

FIGURE 4. Communication rate (The blue solid line represents $\frac{1}{N}\sum_{k=1}^{N}\gamma_k$, and the black dotted line does $\sum_{i=1}^{n}(\frac{N_i}{N}F(C-\pi_i\theta)+\frac{1}{N}\sum_{j=0}^{L_N-1}I_{\{|\tau_{i+jn}|\geq\delta_{i+jn}\}}(1-F(C-\pi_i\theta)))$.

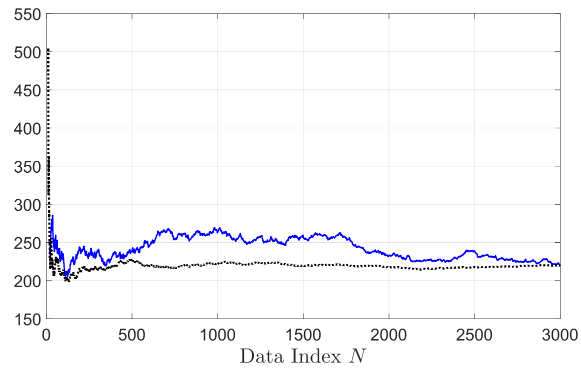

FIGURE 5. Asymptotic efficiency of $\widehat{\theta}_N$: The blue solid line is the average of 200 trajectories of $\|N(\widehat{\theta}_N-\theta)(\widehat{\theta}_N-\theta)^T\|$ and the black dash one is $\|N\Sigma_{CR}(N;\theta)\|$, where $\|\cdot\|$ is the Frobenius norm.

Theorem 2. The transmission time is demonstrated in Fig. 3. It can be seen that only 13 measurements are send to the remote estimator in the interval [500, 530], which indicates the ability of (28) to reduce the communication consumption. Fig. 4 shows $\frac{1}{N}\sum_{k=1}^{N}\gamma_k$ and $\sum_{i=1}^{n}(\frac{N_i}{N}F(C-\pi_i\theta) + \frac{1}{N}\sum_{j=0}^{L_N-1}I_{\{|\tau_{i+jn}|\geq\delta_{i+jn}\}}(1-F(C-\pi_i\theta)))$ with respect to N, where their limits are the same and this is consistent with (23) by (5) and (22). Fig. 5 gives the relationship between the covariance matrix of the estimation error and the CR lower bound, which illustrates the asymptotic efficiency of $\widehat{\theta}_N$ and Theorem 5.

VI. CONCLUDING REMARKS

As the wide applications of networked control systems in the information field, how to save communication resources brings new challenges to the conventional system identification. This paper proposes an identification method for FIR systems with scheduled binary-valued observations. By use of the statistical property of the system noise and the scheduling policy, an empirical-measure-based off-line identification algorithm is provided and proved to be convergent under a periodical input sequence. The convergence rate of the estimation error and the asymptotically efficiency are also obtained. Moreover, the communication rate is discussed.

For the future study, extending the proposed algorithm into multi-level quantized observations, nonlinear models, and systems with structural uncertainties are all interesting issues.

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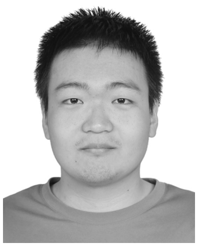

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