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Cooperative Control of Multiple Nonholonomic Robots for Escorting and Patrolling Mission Based on Vector Field

SONG GAO[1,2], RUI SONG[1], (Member, IEEE), AND YIBIN LI[1], (Member, IEEE)
[1]School of Control Science and Engineering, Shandong University, Jinan 250061, China
[2]School of Mechanical and Electric Engineering, Shandong Yingcai University, Jinan 250104, China

Corresponding author: Rui Song (rsong@sdu.edu.cn)

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ABSTRACT This paper investigates the cooperative control problem of multiple nonholonomic robots for an escorting and patrolling mission with respect to a target with a time-varying velocity. The goal of the multi-robot system is to orbit a moving target in a common circle with a prescribed radius and circular velocity, while maintaining even spacing along the perimeter of the circle. This paper proposes a distributed control strategy based on the vector field that does not require each robot to know the full state of the target. A distributed estimation law is developed to enable each robot to estimate the position of the target, and a tracking differentiator is used to estimate the velocity of the target. Based on these estimates of the target, the distributed control law is designed to ensure the asymptotic convergence of the multi-robot system to achieve the desired motion. Under some mild assumptions about the interaction graph among the target and the robots, and the velocity of the target, explicit stability and convergence analyses are presented using Lyapunov tools. Simulation results from test cases of a group of nonholonomic mobile robots verify the effectiveness of the proposed distributed control algorithms.

INDEX TERMS Cooperative control, escorting and patrolling, nonholonomic robots, vector field.

I. INTRODUCTION

In recent years, cooperation in multi-robot systems has received increasing attention. Existing literature addresses a broad range of topics including consensus [1], formation control [2], swarming and flocking [3], coverage [4]–[6], and cooperative target tracking [7], [8]. Escorting and patrolling missions are a typical application of cooperative target tracking and formation control.

Multiple-robot escorting and patrolling missions have numerous applications including military robotics, robotic surveillance security systems, and entertainment robotics [9]. The mission of escorting is the task of surrounding and maintaining a formation around a target. As the target moves, the formation also moves to keep the target at its center, maintaining a set distance between the robots and the target and evenly distributing the robots around the target. The patrolling task can be defined as an extension of escorting in which the formation rotates around the target for surveillance purposes.

Studies [4], [10]–[13] investigated multiple robots uniformly distributed in a circle orbiting a stationary target. However, these approaches cannot be directly extended to the scenario of a moving target. Other studies that relied on a single or double integrator investigated the case of a moving target [14]–[16]. However, they have had limited success when applied to multi-robot systems owing to nonlinearity and nonholonomic constraints. Zhu *et al.* [17] studied the case where the target was moving while all vehicles maintained constant linear velocities.

When the velocity of the target is time-varying, the escorting and patrolling control problem becomes increasingly challenging. Lan *et al.* [18] proposed a distributed reconfigurable control law for unicycles to surround a moving target for an escorting and patrolling mission, but the tracking errors were locally uniformly bounded. In [19], circle formation tracking of a time-varying center was studied, but the proposed control law depended on the initial conditions of the robots and target. In [20], a translation control was designed to achieve the global asymptotic stabilization.

Most of the existing cooperative control schemes for multiple nonholonomic robots on an escorting and patrolling mission, require access to the full state of the target.

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However, owing to limitations in communication bandwidth and range, it is reasonable to assume that target information is only available to a subset of the robots.

This paper proposes a distributed estimation law that enables each robot to estimate the position of the target. If the speed of the target is unknown, we apply a tracking differentiator to estimate the velocity of the target. Then, based on the vector field, we design the distributed control law, which achieves cooperative control of the escorting and patrolling mission. The idea of introducing the vector field is drawn from the vector field approach for single and double integrators applied in [21]. We address the missions using multiple nonholonomic robots that rotate around a central moving target at a prescribed radius and circular velocity with equal angle inter-robot spacing. The proposed distributed control strategy enables the multi-robot system to achieve asymptotic convergence to the desired formation. The stability and the convergence analyses are presented using Lyapunov tools.

The remainder of this paper is organized as follows: Section II states algebraic graph theory and distributed estimation of the target's state. Section III presents the problem formulation. Section IV introduces the distributed control algorithm and provides the main results. Simulation results illustrating the effectiveness of estimator and the proposed controller are detailed in Section V. Section VI concludes the paper.

II. PRELIMINARIES

A. ALGEBRAIC GRAPH THEORY

Before presenting the definition of the escorting and patrolling task and the problem statement, we introduce some elements and results from graph theory.

Let $G = \{v, \varepsilon, A\}$ be a digraph with a set of vertices $v = \{1, 2, \ldots n\}$, a set of edges $\varepsilon = \{(j, i) : j \neq i, i, j \in v\}$, an adjacency matrix $A = \left[a_{ij}\right]_{n\times n} \in \mathbb{R}^{n\times n}$. The adjacency matrix A is defined as follows: if there is a directed link from node j to node i, then $a_{ij} > 0$, otherwise $a_{ij} = 0$. We only consider the case of simple digraph without self-edges, thus $a_{ii} = 0$ for all i. A graph is undirected if $a_{ij} > 0$ implies $a_{ji} > 0$ for all $i, j \in v$. If $a_{ij} = a_{ji}$ for all $i, j \in v$, then the weights are symmetric. The set neighbors of node i are denoted as $N_i = \{j \in v|(i, j) \in \varepsilon\}$, which, in the case of an undirected graph, results in a mutual adjacency relationship among vertices, that is, $i \in N_j \Leftrightarrow j \in N_i$. An undirected graph is said to be connected if there are undirected paths from every node to every other node. The Laplacian matrix $L = \left[l_{ij}\right]_{n\times n}$ associated with the adjacency matrix A is defined as

$$l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{j=1, j\neq i}^{n} a_{ij} & i = j \end{cases} \tag{1}$$

We assume that the robots are labeled as agents 1 to n and the target is labeled as agent 0. The interactions among the target and the robots can be characterized by the adjacency weights $a_{i0} : a_{i0} > 0$ if the target is a neighbor of robot i, otherwise, $a_{i0} = 0$. Denote $a = [a_{10}, a_{20}, \ldots, a_{n0}]^T$ and define matrix $H \in \mathbb{R}^{n\times n}$ as

$$H = L + diag(a) \tag{2}$$

Based on the results in [22], we make the following claim on the matrix H:

Lemma 1: Matrix H is symmetric positive definite if, and only if, the undirected graph G is connected, and at least one $a_{i0} > 0$.

Given n states, $x_i \in \mathbb{R}^m$, $i = 1, 2, \ldots, n$, the following results of graph theory, which can be found in [22] and [23], are used in this paper.

Proposition 1: For an undirected graph with symmetric weights, the following fact can be proven

$$\sum_i \sum_j a_{ij} x_i^T \left(x_i - x_j\right) = \frac{1}{2} \sum_i \sum_j a_{ij} \left\|x_i - x_j\right\|^2$$

Proposition 2: For a connected undirected graph, the following result holds

$$\sum_i \sum_j a_{ij} \left\|x_i - x_j\right\|^2 = 0 \Leftrightarrow x_i = x_j, \quad \forall i,\ j \in v$$

B. DISTRIBUTED ESTIMATION OF TARGET'S STATE

Consider a system of n multiple nonholonomic robots, the kinematics of each robot are given as

$$\begin{cases} \dot{x}_i = v_i \cos\theta_i \\ \dot{y}_i = v_i \sin\theta_i \\ \dot{\theta}_i = w_i \end{cases} \tag{3}$$

where $i = 1, 2, \ldots n$, $q_i := [x_i, y_i]^T \in \mathbb{R}^2$ is the absolute position and $\theta_i \in [-\pi, \pi]$ is the heading angle in the inertial frame. The linear velocity and angular velocity of the robot are $v_i \in \mathbb{R}$ and $w_i \in \mathbb{R}$, respectively.

Each robot has access to its own current state information and the state of its neighbor robots. Let $G = \{v, \varepsilon, A\}$ be the undirected graph characterizing the interactions among n robots. Let $a = [a_{10}, a_{20}, \ldots, a_{n0}]^T$ be the weights representing the interactions among the target and the robots corresponding to G. As in most literature on distributed control of multi-robot systems, we operate under the following assumption:

Assumption 1: The undirected graph G is connected with symmetric weights, and at least one $a_{i0} > 0$.

Given a target with position $q_t = [x_t, y_t]^T \in \mathbb{R}^2$, we denote $q_{it} = [x_{it}, y_{it}]^T$ as the estimation of q_t obtained by each robot i. To make the estimation algorithms fully distributed, the estimation laws must follow the graph topology and can only use the local neighborhood information of that robot.

The neighborhood position estimation errors of robot i are defined as

$$e_{iq} = \sum_{j=1}^{n} a_{ij} \left(q_{it} - q_{jt}\right) + a_{i0} \left(q_{it} - q_t\right) \tag{4}$$

With the neighborhood errors e_{iq}, we consider the following estimation algorithm for q_{it}.

$$\dot{q}_{it} = \frac{1}{\lambda_i}\left(-\Gamma_i e_{iq} + \sum_{j=1}^{n} a_{ij}\dot{q}_{jt} + a_{i0}\dot{q}_t\right) \tag{5}$$

where $\lambda_i = \sum_{j=1}^{n} a_{ij} + a_{i0}$ and $\Gamma_i \in \mathbb{R}^2$ is a symmetric positive definite matrix.

For the distributed estimation laws of q_{it}, we have the following result.

Theorem 1: Consider the estimation law (5) if Assumption 1 holds, then q_{it} exponentially converges to q_t for all i.

Proof: Taking the time derivative of e_{iq}, we obtain

$$\begin{aligned}\dot{e}_{iq} &= \sum_{j=1}^{n} a_{ij}\left(\dot{q}_{it} - \dot{q}_{jt}\right) + a_{i0}\left(\dot{q}_{it} - \dot{q}_t\right)\\ &= \lambda_i \dot{q}_{it} - \sum_{j=1}^{n} a_{ij}\dot{q}_{jt} - a_{i0}\dot{q}_t\\ &= -\Gamma_i e_{iq}\end{aligned} \tag{6}$$

As Γ_i is symmetric positive definite, it follows that e_{iq} exponentially converges to zero. Moreover, if we denote $e_q = \left[e_{1q}^T, e_{2q}^T, \ldots e_{iq}^T\right]^T$, $\tilde{q} = \left[\tilde{q}_1^T, \tilde{q}_2^T, \ldots \tilde{q}_n^T\right]^T$ with $\tilde{q}_i = q_{it} - q_t$, then (4) can be rewritten as

$$e_q = (H \otimes I_2)\,\tilde{q} \tag{7}$$

where $H \in \mathbb{R}^{n\times n}$ is a matrix defined by $H = L + diag(a)$. Because H is a symmetric positive definite matrix, when Assumption 1 holds by applying lemma 1, e_q exponentially converges to zero, which implies that $\tilde{q}$ also exponentially converges to zero.

Therefore, each robot obtains access to the position of the target by the distributed estimation law, but the speed of the target remain unknown.

We can use the "tracking differentiator" from [24] to estimate the velocity of the target, with $v_0(t)$ as the input signal to be differentiated, where x_1 tracks $v_0(t)$, x_2 tracks $\dot{v}_0(t)$, and the parameter γ determines the speed. Thus, $v_0(t) = q_{it}$ and $\dot{v}_0(t) = \dot{q}_{it} = [\dot{x}_{it}, \dot{y}_{it}]^T$ are obtained from (8).

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -\gamma sign(x_1 - v_0(t) + \frac{x_2|x_2|}{2\gamma})\end{cases} \tag{8}$$

III. PROBLEM FORMULATION

Based on the estimated state of the target, the escorting and patrolling task requires all robots to orbit the target in a circle with precribed radius R. As the target moves, all robots maintain the target q_t as the center and evenly space themselves along the circle perimeter, as depicted in Fig. 1.

For the escorting and patrolling task, each robot is assigned to a specific subset N_i of the robot team that comprises the robots with which it can communicate to achieve the desired goal.

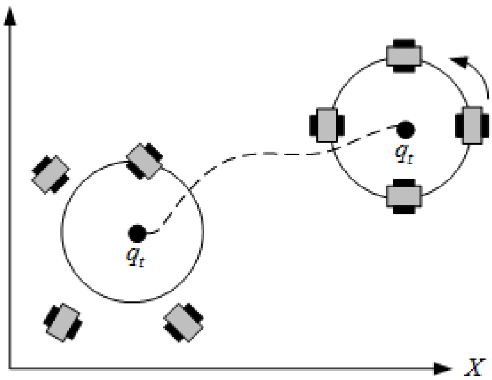

FIGURE 1. Schematic diagram of the escorting and patrolling mission.

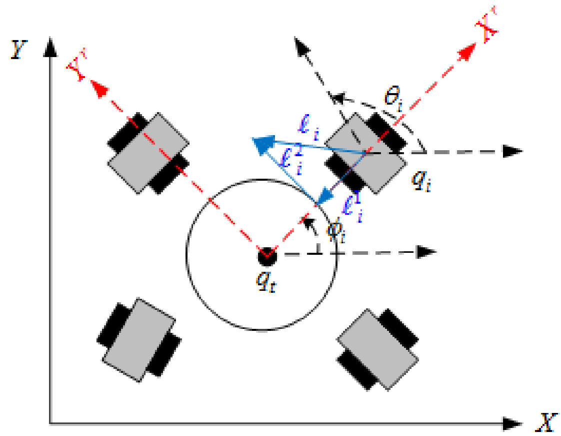

FIGURE 2. The escorting and patrolling mission with multiple nonholonomic robots.

Based on the vector field theory, the spatial velocity field is composed of multiple velocity vectors. The velocity vector ℓ_i can be composed of ℓ_i^1 and ℓ_i^2 in the plane, where ℓ_i^1 is the shortest distance from any point in space to the specified trajectory and ℓ_i^2 is the tangent vector of the trajectory curve. Fig. 2 shows the application of the concept of the velocity field to the two-dimensional plane. We construct a new frame $X^r - q_t - Y^r$, with its origin at q_t and X^r axis coincident with ϕ_i, which is the angle between robot i and the target.

In the inertial coordinate, the relative dynamics between robot i and the target can be written as

$$\tilde{q}_i = \begin{bmatrix} x_i - x_{it}\\ y_i - y_{it}\end{bmatrix} = \begin{bmatrix}\tilde{x}_i\\ \tilde{y}_i\end{bmatrix} \tag{9}$$

$$\dot{\tilde{q}}_i = \begin{bmatrix} \dot{x}_i - \dot{x}_{it}\\ \dot{y}_i - \dot{y}_{it}\end{bmatrix} = \begin{bmatrix}\dot{\tilde{x}}_i\\ \dot{\tilde{y}}_i\end{bmatrix} \tag{10}$$

In the frame of $X^r - q_t - Y^r$, $\tilde{q}_i$ and $\dot{\tilde{q}}_i$ are turned into $\tilde{q}_i^r$ and $\dot{\tilde{q}}_i^r$.

$$\tilde{q}_i^r = \begin{bmatrix}\tilde{x}_i^r\\ \tilde{y}_i^r\end{bmatrix} = \begin{bmatrix}\cos\phi_i & \sin\phi_i\\ -\sin\phi_i & \cos\phi_i\end{bmatrix}\begin{bmatrix}\tilde{x}_i\\ \tilde{y}_i\end{bmatrix} \tag{11}$$

$$\dot{\tilde{q}}_i^r = \begin{bmatrix}\dot{\tilde{x}}_i^r\\ \dot{\tilde{y}}_i^r\end{bmatrix} \tag{12}$$

Obviously, $\tilde{y}_i^r = 0, \dot{\tilde{y}}_i^r = 0$.

Thus,

$$\dot{\tilde{x}}_i^r = v_i \cos(\theta_i - \phi_i) - (\dot{x}_t \cos\phi_i + \dot{y}_t \sin\phi_i) = v_i \cos\alpha_i - \Psi_i \quad (13)$$

and

$$\tilde{x}_i^r \dot{\phi}_i = v_i \sin(\theta_i - \phi_i) - (\dot{y}_t \cos\varphi_i - \dot{x}_t \sin\phi_i) = v_i \sin\alpha_i - \Phi_i \quad (14)$$

where $\Psi_i = \dot{x}_t \cos\phi_i + \dot{y}_t \sin\phi_i$, $\Phi_i = \dot{y}_t \cos\phi_i - \dot{x}_t \sin\phi_i$, $\alpha_i = \theta_i - \phi_i$.

The goal of cooperative control in the escorting and patrolling problem is to design v_i and w_i for each robot i, $i = 1, 2, \ldots, n$ such that

$$\lim_{t\to\infty} \tilde{x}_i^r(t) = R, \quad \forall i \in N \quad (15)$$

$$\lim_{t\to\infty} \dot{\phi}_i(t) = w_d, \quad \forall i \in N \quad (16)$$

$$\lim_{t\to\infty} \phi_i(t) - \phi_j(t) = \delta_{ij}, \quad \forall i \in N,\ j \in N_i \quad (17)$$

IV. MAIN RESULTS

First, we introduce a nonlinear function $h(e_i)$ with

$$e_i = \sum_{j\in N_i} a_{ij} \left(\phi_i - \phi_j - \delta_{ij}\right) \quad (18)$$

As previously explained, $a_{ij} = a_{ji} > 0$, and $h(e_i)$ is a nonlinear function that assumes the following:

Assumption 2: For all $\kappa \in \mathbb{R}$, function $h(.)$ processes the following three properties:

(i) $\exists h_0 > 0, |h(\kappa)| \le h_0$

(ii) $h(\kappa)\kappa \ge 0$

(iii) $h'(\kappa) > 0$

Remark 1: Function $h(e_i)$ is responsible for angular spacing. Upon achieving the angular spacing, $h(e_i)$ becomes zero. Assumption 2 implies that $h(.)$ is a bounded and monotone increasing function that satisfies $h(0) = 0$.

Based on the vector field, we obtain

$$\ell_i^1 = -k_1 \left(\tilde{x}_i^r - R\right) = -k_1 \chi_i \quad (19)$$

$$\ell_i^2 = R\left(w_d - h(e_i)\right) \quad (20)$$

where $k_1 > 0$ is a positive constant and $\chi_i = \tilde{x}_i^r - R$.

We assume that the linear velocity v_i and the angle deviation s_i satisfy the following equations.

$$v_i \cos(\alpha_i - s_i) - \Psi_i = -k_1 \chi_i \quad (21)$$

$$v_i \sin(\alpha_i - s_i) - \Phi_i = R\left(w_d - h(e_i)\right) \quad (22)$$

Then, v_i and β_i, which is the desired value of α_i, can be obtained as

$$v_i = \sqrt{(-k_1\chi_i + \Psi_i)^2 + [R(w_d - h(e_i)) + \Phi_i]^2} \quad (23)$$

$$\beta_i = \alpha_i - s_i = a\tan 2\frac{R(w_d - h(e_i)) + \Phi_i}{-k_1\chi_i + \Psi_i} \quad (24)$$

Substituting (21) and (22) into (13) and (14), we establish the error dynamics as follows:

$$\dot{\tilde{x}}_i^r = -k_1\chi_i + v_i[\cos\alpha_i - \cos(\alpha_i - s_i)] = -k_1\chi_i + v_i s_i \frac{\cos\alpha_i(1-\cos s_i) - \sin\alpha_i \sin s_i}{s_i} = -k_1\chi_i + v_i s_i g_i \quad (25)$$

where $g_i = \frac{\cos\alpha_i(1-\cos s_i) - \sin\alpha_i \sin s_i}{s_i}$

Because $\frac{\sin s_i}{s_i} = \int_0^1 \cos(xs_i)dx$, $\frac{1-\cos s_i}{s_i} = \int_0^1 \sin(xs_i)dx$, it can be concluded that g_i is a smooth and bounded function in s_i.

$$\tilde{x}_i^r \dot{\phi}_i = R(w_d - h(e_i)) + v_i[\sin\alpha_i - \sin(\alpha_i - s_i)] \quad (26)$$

$$\dot{s}_i = \dot{\alpha}_i - \dot{\beta}_i = w_i - \dot{\phi}_i - \dot{\beta}_i \quad (27)$$

Using (27), we can design the angular velocity w_i as

$$w_i = \dot{\phi}_i + \dot{\beta}_i - k_2 s_i - k_3 v_i g_i \chi_i \quad (28)$$

where $k_2 > 0$ and $k_3 > 0$ are positive constants. To ensure that v_i is always a nonzero value, and that $\dot{\beta}_i$ always exists, the following assumption is made:

Assumption 3: The function of the target $|\Psi_i(t)|$ is smooth and bounded and h_0 is chosen such that

$$R\left(|w_d| - h_0\right) > \max_{t\ge0} |\Psi_i(t)| \quad (29)$$

Under Assumption 3, the following fact is applied:

$$v_i \ge |R(w_d - h(e_i)) + \Phi_i| - |-k_1\chi_i + \Psi_i(t)| \ge |R(w_d - h(e_i))| - |\Psi_i(t)| \ge |R(w_d - h_0)| - |\Psi_i(t)|$$

Thus, it can be verified that the linear velocity v_i is always nonzero, and hence, $\dot{\beta}_i$ is bounded. We have the following result:

Theorem 1: Consider the system (3) with the control laws (23) and (28). If Assumptions 2 and 3 hold, then the error dynamics (25) and (27) are asymptotically stable and, ultimately, the objectives in (15)-(17) are achieved.

Proof: Consider the following Lyapunov function:

$$V_1 = \frac{1}{2}\sum_i k_3\chi_i^2 + \frac{1}{2}\sum_i s_i^2 \quad (30)$$

The time derivative of V_1 along (25) and (27) is:

$$\dot{V}_1 = \sum_i k_3\chi_i\dot{\chi}_i + \sum_i s_i\dot{s}_i = \sum_i k_3\chi_i\dot{\tilde{x}}_i^r + \sum_i s_i(-k_2 s_i - k_3 v_i g_i \chi_i) = \sum_i k_3\chi_i(-k_1\chi_i + v_i s_i g_i) + \sum_i s_i(-k_2 s_i - k_3 v_i g_i \chi_i) = -\sum_i k_3 k_1 \chi_i^2 - \sum_i k_2 s_i^2 \quad (31)$$

Hence, χ_i and s_i are bounded and exponentially convergent to zero. We have $\tilde{x}_i^r$ approaching to R, which demonstrates that (15) holds.

Moreover, using the (26), we obtain

$$\tilde{x}_i^r \dot{\phi}_i = R(w_d - h(e_i)) + v_i[\sin\alpha_i - \sin(\alpha_i - s_i)] = R(w_d - h(e_i)) + \zeta_i \quad (32)$$

where $\zeta_i = v_i[\sin\alpha_i - \sin(\alpha_i - s_i)]$

Because s_i converges exponentially to zero, we have ζ_i exponentially convergent to zero.

Clearly,

$$\dot{\phi}_i = \frac{R\,(w_d - h\,(e_i)) + v_i[\sin\alpha_i - \sin(\alpha_i - s_i)]}{\tilde{x}_i^r} = w_d - h\,(e_i) + \xi_i \tag{33}$$

where $\xi_i = \frac{\zeta_i - \chi_i(w_d - h(e_i))}{\tilde{x}_i^r}$

Because χ_i and ζ_i are converging exponentially to zero, we have ξ_i exponentially convergent to zero.

Moreover, following the definition in (18), we have

$$\dot{e}_i = -\sum_{j\in N_i} a_{ij}[h\,(e_i) - h(e_j) - (\xi_i - \xi_j)] \tag{34}$$

We consider a Lyapunov function candidate

$$V_2 = \sum_i 2\int_0^{e_i} h(r)dr \tag{35}$$

Taking the time derivative of V_2 as

$$\dot{V}_2 = 2\sum_i h(e_i)\dot{e}_i = -2\sum_i\sum_{j\in N_i} a_{ij}h(e_i)[h(e_i) - h(e_j)] + 2\sum_i\sum_{j\in N_i} a_{ij}h(e_i)(\xi_i - \xi_j) \tag{36}$$

and applying Proposition 1, we have

$$\begin{aligned}\dot{V}_2 &= -\sum_i\sum_{j\in N_i} a_{ij}[h(e_i) - h(e_j)]^2 + 2\sum_i\sum_{j\in N_i} a_{ij}h(e_i)\left(\xi_i - \xi_j\right)\\ &\leq -\sum_i\sum_{j\in N_i} a_{ij}[h(e_i) - h(e_j)]^2 + 2\sum_i\sum_{j\in N_i} a_{ij}h_0(|\xi_i| + |\xi_j|)\end{aligned} \tag{37}$$

Integrating the inequality from 0 to t, yields

$$V_2(t) + \int_0^t \sum_i\sum_{j\in N_i} a_{ij}[h(e_i) - h(e_j)]^2 dr \leq V_2(0) + 2\int_0^t \sum_i\sum_{j\in N_i} a_{ij}h_0\left(|\xi_i| + |\xi_j|\right) dr \tag{38}$$

Because function $h(.)$ satisfies Assumption 2, $V_2 \geq 0$ always holds. Because ξ_i exponentially converges to zero for all i, then $\int_0^t \sum_i \sum_{j\in N_i} a_{ij}h_0(|\xi_i| + |\xi_j|)dr$ is bounded. With these facts, inequality (38) implies that both $V_2(t)$ and $\int_0^t \sum_i \sum_{j\in N_i} a_{ij}[h(e_i) - h(e_j)]^2 dr$ are bounded. Thus, by applying the Barbalat's lemma, we have

$$\lim_{t\to\infty} \sum_i\sum_{j\in N_i} a_{ij}[h(e_i) - h(e_j)]^2 = 0 \tag{39}$$

Following Proposition 2 and (39), we have $h\,(e_i) = h\left(e_j\right)$ for all i, j when $t \to \infty$. For all i, j, $h\,(e_i) = h\left(e_j\right)$ gives $e_i = e_j$ using the monotonicity of function $h(.)$. Because $\sum_i e_i = 0$, $e_i = e_j$, $\forall i \in N, j \in N_i$ implies $e_i = 0$,

$\forall i$. For all i and the undirected connected graph G, $e_i = \sum_{j\in N_i} a_{ij}\left(\phi_i - \phi_j - \delta_{ij}\right)$ implies $\phi_i - \phi_j = \delta_{ij}$, $\forall i \in N$, $j \in N_i$. Finally, when e_i tends to zero, hence $h\,(e_i)$ tends to zero, then $\dot{\phi}$ tends to w_d following (33), and it is shown that (16) and (17) hold. This completes the proof.

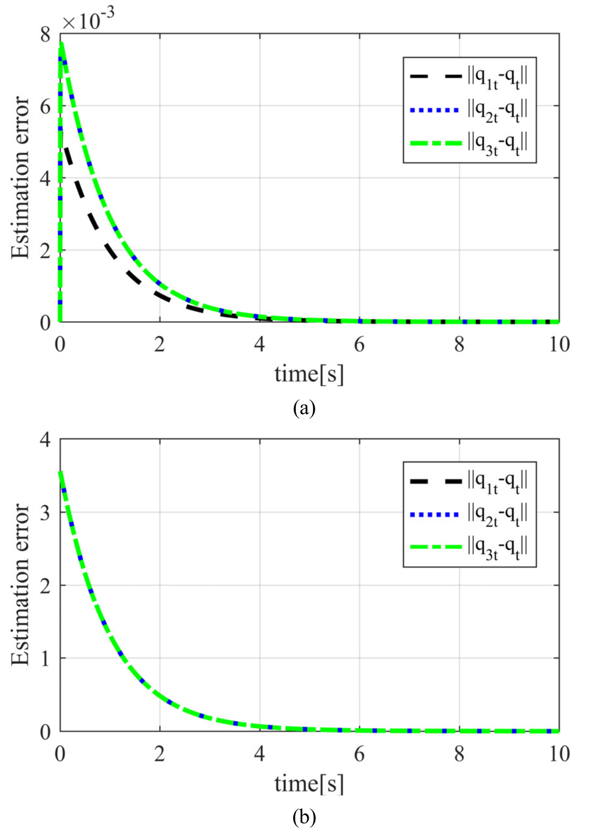

FIGURE 3. Estimation errors $\left\|q_{it} - q_t\right\|$. (a) Estimation errors $\|q_{it} - q_t\|$ in case 1. (b) Estimation errors $\left\|q_{it} - q_t\right\|$ in case 2.

V. SIMULATION RESULTS

In this section, we present some simulations to illustrate the validity of the proposed estimation law and control algorithm. We consider a multi-robot system comprised of three robots with an undirected communication graph. The adjacency matrix of the communication graph among the robots, and the adjacency weights among the target and the robots are respectively set as

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

and

$a = [1, 0, 0]^T$.

The initial position of the three robots and the target respectively are

$$q_1(0) = [-2, -6]^T\text{m}, \quad q_2(0) = [-5, -2]^T\text{m}, \quad q_3(0) = [-3, -8]^T\text{m}, \quad q_t(0) = [0, 0]^T\text{m}.$$

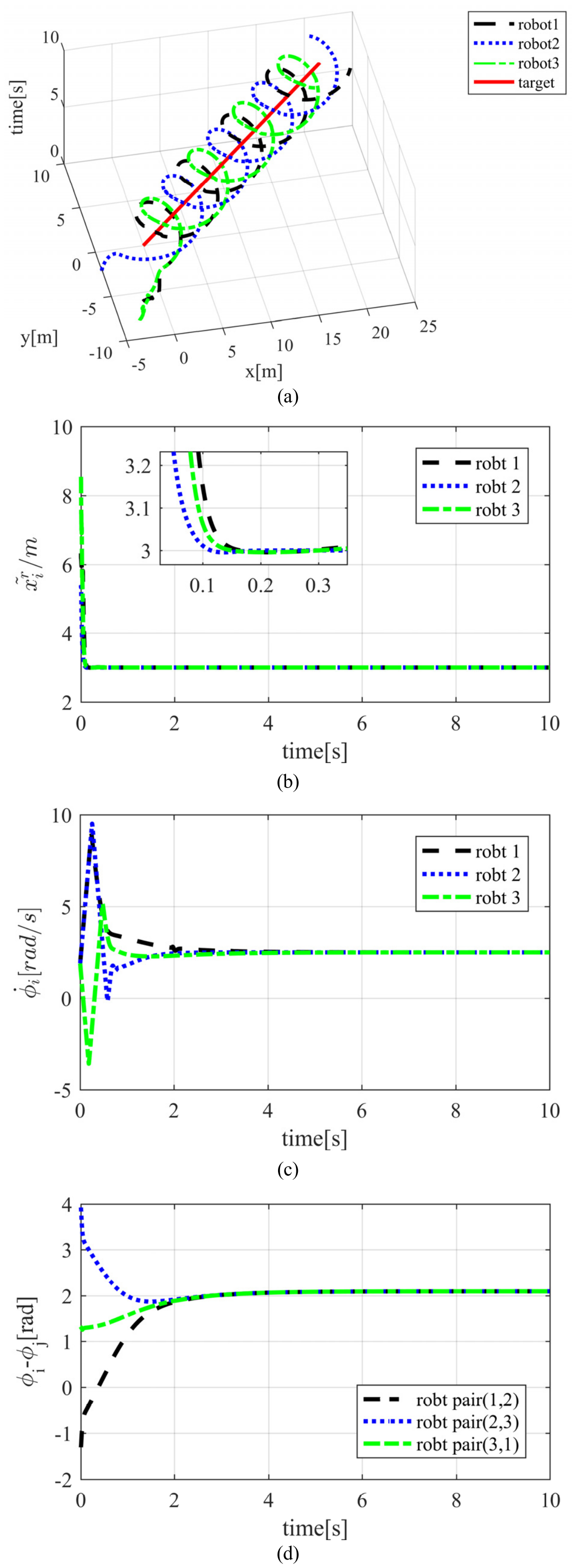

FIGURE 4. Simulation results for case 1. (a) Robot's trajectories. (b) Distance between robot and target $\tilde{x}_i^r$. (c) The circular velocity of robot $\dot{\phi}_i$. (d) The inter-robot angular spacing $\phi_i - \phi_j$.

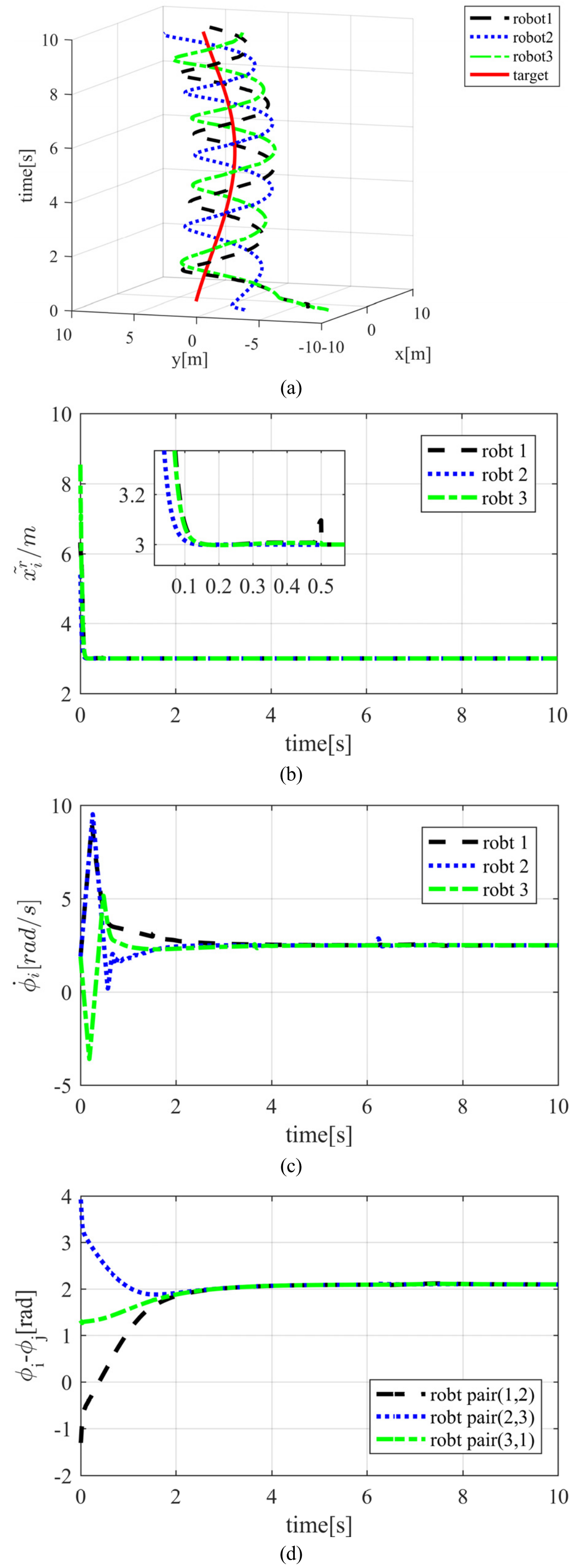

FIGURE 5. Simulation results for case 2. (a) Robot's trajectories. (b) Distance between robot and target $\tilde{x}_i^r$. (c) The circular velocity of robot $\dot{\phi}_i$. (d) The inter-robot angular spacing $\phi_i - \phi_j$.

The specified radius and the circular velocity of the circle around the target are $R = 3$m, $w_d = 2.5$rad/s. The inter-robot angular spacing is specified as $\delta_{12} = \delta_{23} = \frac{2\pi}{3}$. The nonlinear function h is set as

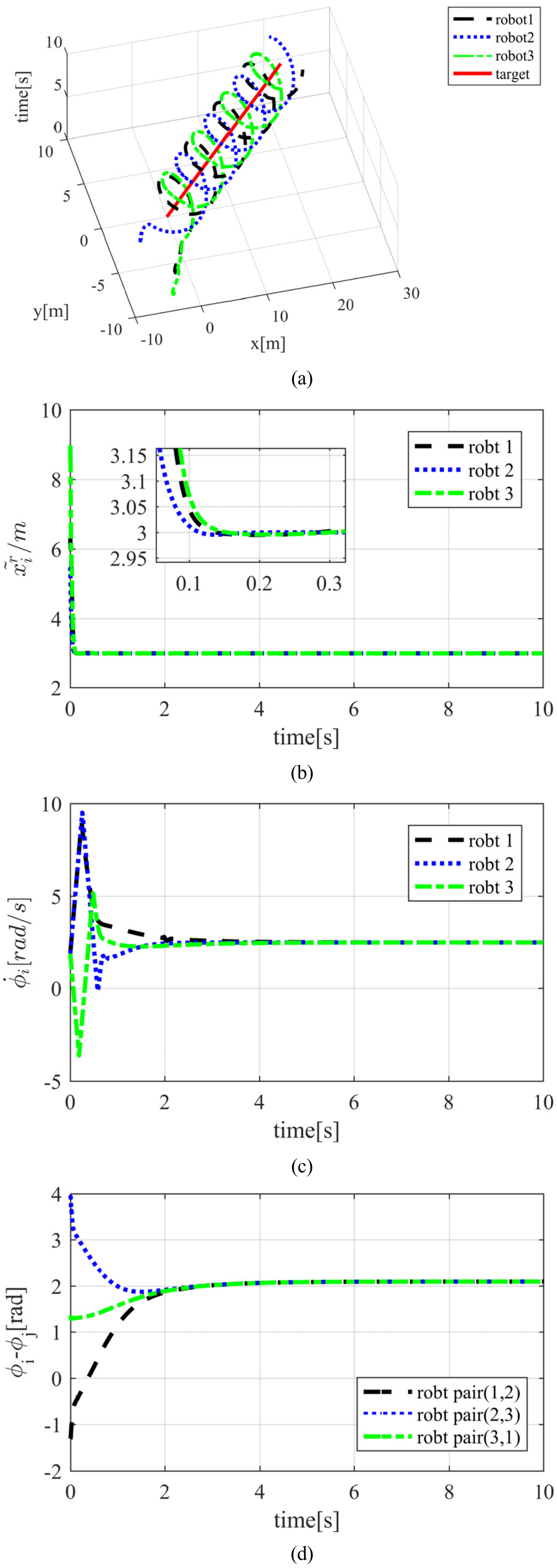

FIGURE 6. Simulation results for case 3. (a) Robot's trajectories. (b) Distance between robot and target $\tilde{x}_i^r$. (c) The circular velocity of robot $\dot{\phi}_i$. (d) The inter-robot angular spacing $\phi_i - \phi_j$.

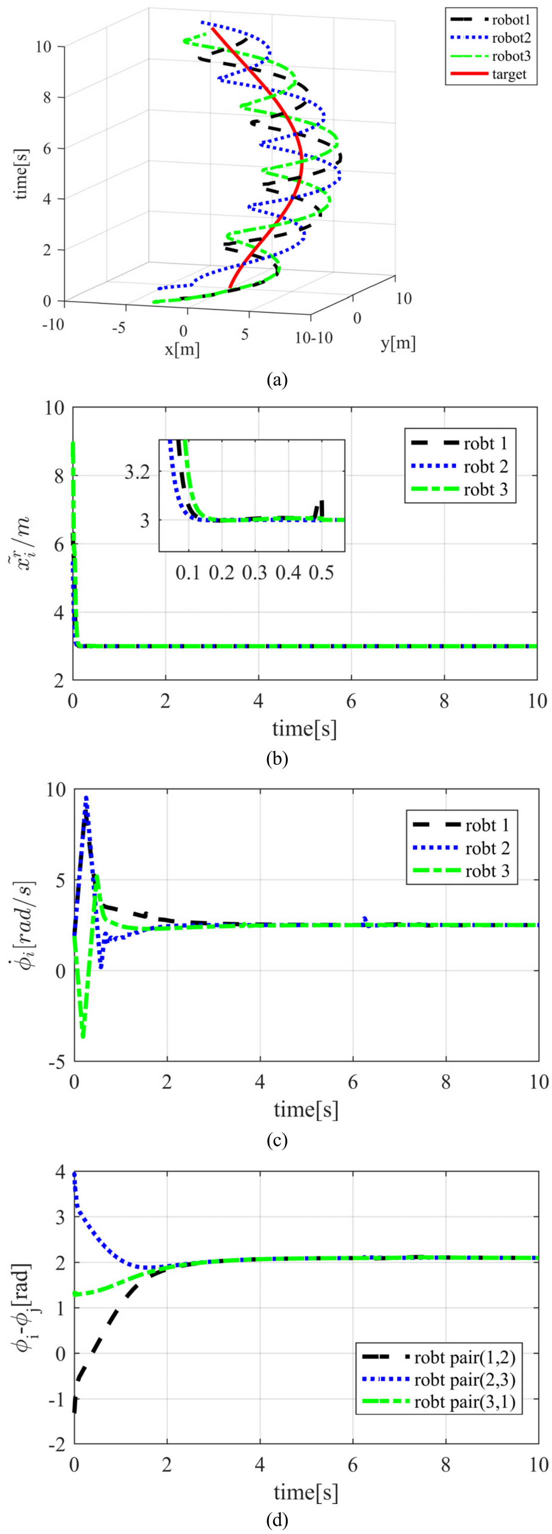

FIGURE 7. Simulation results for case 4. (a) Robot's trajectories. (b) Distance between robot and target $\tilde{x}_i^r$. (c) The circular velocity of robot $\dot{\phi}_i$. (d) The inter-robot angular spacing $\phi_i - \phi_j$.

$h(e_i) = 0.8 \tanh(e_i)$, and the control gains are chosen as $k_1 = 5, k_2 = 5, k_3 = 2$.

We first investigate the performance of the estimation algorithm. Two cases are simulated:

(i) $q_t(t) = [2t, 0.5t]^T$

(ii) $q_t(t) = \left[5\sin\left(0.4t - \frac{\pi}{8}\right), 3 - 0.7\tanh(-0.3t)^2\right]^T$

Second, based on the estimators, the following four cases are simulated:

(i) Target with constant velocity and the initial orientations of robots are zero.

$$q_t(t) = [2t, 0.5t]^T$$
$$\theta_1(0) = 0, \theta_2(0) = 0, \theta_3(0) = 0$$

(ii) Target with time-varying velocity and the initial orientations of robots are zero.

$$q_t(t) = \left[5\sin\left(0.4t - \frac{\pi}{8}\right), 3 - 0.7\tanh(-0.3t)^2\right]^T$$
$$\theta_1(0) = 0, \quad \theta_2(0) = 0, \ \theta_3(0) = 0$$

(iii) Target with constant velocity and the initial orientations of robots are nonzero.

$$q_t(t) = [2t, 0.5t]^T$$
$$\theta_1(0) = \frac{\pi}{2}, \theta_2(0) = \frac{\pi}{2}, \quad \theta_3(0) = -\frac{\pi}{2}$$

(iv) Target with time-varying velocity and the initial orientations of robots are nonzero.

$$q_t(t) = \left[5\sin\left(0.4t - \frac{\pi}{8}\right), 3 - 0.7\tanh(-0.3t)^2\right]^T$$
$$\theta_1(0) = \frac{\pi}{2}, \quad \theta_2(0) = \frac{\pi}{2}, \ \theta_3(0) = -\frac{\pi}{2}$$

As shown in Fig. 3, we find that the estimation errors $\|q_{it} - q_t\|$ exponentially converge to zero. Using control algorithms (23) and (28), the simulation results are described in Figs. 4- Figs. 7. As shown in Figs. 4(a)–(d) and Figs. 5(a)–(d), when the initial orientations of all robots are zero, the robots implement the objectives of the escorting and patrolling mission prescribed in (15)–(17), under conditions of constant and time-varying target's speed, respectively. As described in Figs. 6(a)–(d) and Figs. 7(a)–(d), the robots achieve the objectives when the initial orientations are nonzero. The simulation results demonstrate good control performance of the proposed control laws.

VI. CONCLUSION

This paper discussed the escorting and patrolling problem of multiple nonholonomic robots such that they can travel as a common circle centered on a moving target and maintain even spacing along the circle perimeter. By estimating the position and the velocity of the target, and coordinating with other robots, the control law based on vector field for each robot was designed to function without information on the full state of the target. Simulation results of different cases verified the effectiveness of the proposed algorithms. Our future work will focus on extending the proposed control laws to robots with mechanical dynamics to generate the real torque or force control inputs. It is also interesting to consider the estimation of the position and velocity of the target using distance measurement in the proposed control strategy.

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SONG GAO received the B.S. degree in automation from the Chongqing University of Technology, Chongqing, China, in 2005, and the M.S. degree from the School of Automation, University of Chongqing, Chongqing, in 2008. She is currently pursuing the Ph.D. degree with the School of Control Science and Engineering, Shandong University, Jinan, China. Her research interests include the control of multiple mobile robots and multi-robot formation.

RUI SONG received the B.E. degree in industrial automation in 1998, the M.S. degree in control theory and control engineering from the Shandong University of Science and Technology in 2001, and the Ph.D. degree in control theory and control engineering from Shandong University, Jinan, China, in 2011. He is currently an Associate Professor with the School of Control Science and Engineering, Shandong University, and one of the directors of the Center of Robotics of Shandong University. He is involved in research on intelligent sensor networks, intelligent robot technology, and intelligent control systems. His research interests include medical robots and quadruped robots.

YIBIN LI received the B.S. degree in automation from Tianjin University, China, in 1982, the M.S. degree in electrical automation from the Shandong University of Science and Technology, China, in 1990, and the Ph.D. degree in automation from Tianjin University in 2008. From 1982 to 2003, he was with the Shandong University of Science and Technology. Since 2003, he has been the Director of the Center for Robotics, Shandong University. His research interests include bionic robots, control of intelligent robots, machine learning, and intelligence architecture.