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On the First Hyperchaotic Hyperjerk System With No Equilibria: A Simple Circuit for Hidden Attractors

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ABSTRACT A fourth-order hyperchaotic hyperjerk system with no equilibria has never been reported, whereas a search for its existence has been an open research problem. A new system in a rare type of 4D seven-term no-equilibrium hyperchaotic systems is proposed and also compared with two existing systems of such a rare type. The proposed system offers 10 concurrent advantages, six of which appear to be superior to both existing systems, that is; 1) the first report of a fourth-order hyperchaotic hyperjerk system with no equilibria; 2) a much simpler circuit based on only 21 electronic components; 3) a higher value of the Lyapunov dimension at 3.2280; 4) a higher value of the largest Lyapunov exponent at 0.2525; 5) a large two-parameter space of hyperchaos; and 6) a boostable variable for offset control. The other four advantages offer either equal or better features, that is; 7) the number of nonlinear terms is two; 8) no potential dangers of multistability; 9) hyperchaos, chaos, and periodic behavior are possible; and 10) attractors are readily hidden attractors. The system is unique in the sense that there are no equilibria in both dynamical and hyperjerk forms.

INDEX TERMS Hidden attractor, hyperchaos, hyperjerk, no equilibrium.

I. INTRODUCTION

Chaos has attracted much attention owing to its practical applications to various fields of science and engineering, e.g., [1]–[3]. A three dimensional (3D) chaotic system is described by a set of three coupled first-order ordinary differential equations (ODEs) in three phase space variables ($\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$) [4]. Such coupled ODEs may be recast into a single third-order ODE known as a jerk ODE of the form $\dddot{x} = f(x, \dot{x}, \ddot{x})$ [5]. As the expansion, an n dimensional chaotic system for $n > 3$ is described by a set of n coupled first-order ODEs, which may be recast into a single nth-order ODE known as a hyperjerk ODE [6]. For $n = 4$, a fourth-order hyperjerk ODE of the form $\ddddot{x} = f(x, \dot{x}, \ddot{x}, \dddot{x})$ [7] may exhibit either chaos e.g., [7], [8], or hyperchaos e.g., [8], [9], where chaos has one positive Lyapunov exponent (LE), and hyperchaos has (at least) two positive LEs.

Jerk and hyperjerk systems have been of interest because of their simplicity and rich dynamics [9], [10]. In particular, hyperjerk systems have been prototypical examples of complex dynamical systems in a high-dimensional phase space [8], and therefore provide alternative methods to the study of chaos and hyperchaos. In practice, hyperjerk systems have demonstrated many engineering applications such as in the design of intermittent-motion mechanisms, e.g., cams and Geneva drives [11] and robotic arms [12].

In 2010, the first hidden attractor has been revealed [13] and therefore attractors are typically classified as self-excited and hidden attractors [14] depending on the basin of attraction, which is a set of initial points whose trajectories tend to the attractor. A self-excited attractor has a basin of attraction that intersects with a neighborhood of an equilibrium point, whereas a hidden attractor has a basin of attraction that does not intersect with a neighborhood of an equilibrium point [15]. As a result, attractors of dissipative flows with no equilibria [16] will readily be hidden attractors as their basins of attraction will never intersect with any equilibria. Hidden attractors have been displayed in various systems, e.g., nonlinear oscillators [17], convective fluid motion in rotating cavity [15], and a multilevel DC/DC converter [18].

On the one hand, most existing fourth-order hyperjerk systems for either chaos e.g., [7], [8], [19], or hyperchaos e.g., [8] have exhibited self-excited attractors with a finite number of equilibrium points. On the other hand, a fourth-order hyperchaotic hyperjerk system with no equilibria has never been reported in the literature, although chaotic jerk [20] and chaotic hyperjerk [21] systems with no equilibria have been proposed. It is naturally interesting to ask an open research question [22] whether there exists a hyperchaotic hyperjerk system with no equilibria.

Recently, a technique of a boostable variable [23] has been suggested to transform a bipolar chaotic signal to a unipolar chaotic signal or vice versa. Such a technique is useful in practical applications where a unipolar signal is required, e.g., to reduce hardware for a desired voltage level for signal transmission [24]. The technique has however never been employed for a fourth-order hyperchaotic hyperjerk system with no equilibria.

Most existing four dimensional (4D) hyperchaotic systems for either self-excited attractors e.g., [25], [26] or hidden attractors with no equilibria e.g., [16], [27] are relatively complicated as the number of algebraic terms in the set of four coupled first-order ODEs is more than 7 including two or more terms of nonlinearity. However, two cases of a rare type of 4D seven-term no-equilibrium hyperchaotic systems have been reported for hidden attractors based on two [28] and three [29] terms of nonlinearity. Both cases [28] and [29] have several unattractive disadvantages. For example, both are not hyperjerk systems, nor do both demonstrate the technique of a boostable variable. In particular, as will be described, both cases have suffered from a narrow range of hyperchaos. In addition, other individual disadvantages of [28] and [29] are as follows.

The first case [28] has exhibited multistability, though without circuit realization, with a low value of the largest Lyapunov exponent (LLE) at 0.0704 and a relatively low value of the Lyapunov dimension (D_L) at 3.0768. As multistable systems potentially allow unexpected disasters in various systems ranging from e.g., sudden climate changes to serious diseases, financial crises and disasters of commercial devices [30], it is important that a multistable system be avoided to prevent serious catastrophes. The second case [29] has required lots of (i.e., 46) electronic components with a low value of LLE = 0.064 and a relatively low value of $D_L = 3.089$.

In this paper, a fourth-order hyperchaotic hyperjerk system with no equilibria is proposed for the first time. Its single fourth-order ODE is based on a new set of four coupled first-order ODEs in the rare type of 4D seven-term no-equilibrium hyperchaotic systems. The proposed system is compared with the two existing systems [28] and [29] in this type, and introduces not only six superior advantages, but also four advantages of either similar or improved features. All ten advantages are simultaneously demonstrated by only a single system.

II. THE FIRST HYPERCHAOTIC HYPERJERK SYSTEM WITH NO EQUILIBRIA

A. A NEW 4D SEVEN-TERM NO-EQUILIBRIUM SYSTEM

Existing 3D minimum-five-term chaotic systems [31], [32] based on a diffusionless Lorenz system can be modified through a technique of linear feedback control resulting in a new simple 4D, seven-term, no-equilibrium system of the form

$$\begin{cases} \dot{x} = y - x + w \\ \dot{y} = -axz \\ \dot{z} = xy - 1 \\ \dot{w} = -bx \end{cases} \tag{1}$$

As will be described, (1) will exhibit hyperchaos. For nonzero parameters a and b, (1) has no equilibria and therefore attractors are readily hidden. Equation (1) is simple in the sense that it consists of seven algebraic terms, which appear to be the minimum number of terms for 4D seven-term no-equilibrium hyperchaotic systems. In addition, (1) consists of two terms of nonlinearity, which also appear to be the minimum number of nonlinear terms for 4D seven-term no-equilibrium hyperchaotic systems.

B. A NEW NO-EQUILIBRIUM HYPERJERK SYSTEM

The 4D dynamical model in (1) can be equivalently transferred to a single fourth-order no-equilibrium hyperjerk ODE with seven terms of the form $w = f_1(w, \dot{w}, \ddot{w}, \dddot{w})$ as

$$\ddddot{w} = \left(\frac{\ddot{w}}{\dot{w}} - 1\right)\dddot{w} + \left(\frac{\ddot{w}}{\dot{w}} - k_3\dot{w}^2\right)\ddot{w} - \left(k_3\dot{w}^2 + k_2 w\dot{w} - k_1\right)\dot{w} \tag{2}$$

where constants k_1, k_2 and k_3 are defined as

$$[k_1 \quad k_2 \quad k_3] = \left[a \quad a/b \quad a/b^2\right] \tag{3}$$

Equation (2) appears to be the first report of a hyperjerk system with no equilibria for hyperchaos. The phase space variables x, y, and z in (1) can be alternatively represented by individual functions of the form $f(w, \dot{w}, \ddot{w}, \dddot{w})$ as

$$x = \frac{\dot{w}}{-b} = f(\dot{w}) \tag{4}$$

$$y = \frac{-\ddot{w} - \dot{w} - bw}{b} = f(\ddot{w}) \tag{5}$$

$$z = \frac{-\dddot{w} - \ddot{w} - b\dot{w}}{a\dot{w}} = f(\dddot{w}) \tag{6}$$

where x, y, and z in (4) to (6) are, for simplicity, denoted as $f(\dot{w})$, $f(\ddot{w})$, and $f(\dddot{w})$, respectively.

C. SIMPLE CIRCUIT REALIZATION

Circuit realization of (1) is shown in Fig. 1. The circuit is simple in the sense that the number of components is 21, which includes the required DC voltage V_b but excludes the traditional power supply. The number of 21 appears to be the minimum number of components ever reported not only for the type of 4D seven-term no-equilibrium hyperchaotic

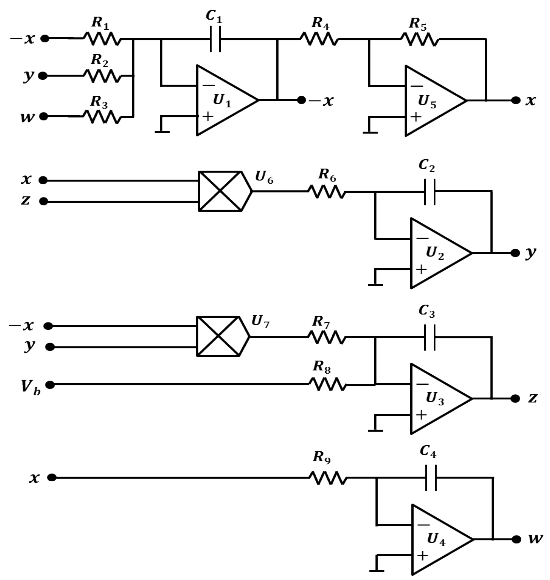

FIGURE 1. A simple 4D seven-term no-equilibrium hyperchaotic circuit, or a simple fourth-order no-equilibrium hyperchaotic hyperjerk circuit.

circuits, but also for the type of fourth-order no-equilibrium hyperchaotic hyperjerk circuits. The proposed circuit therefore represents the simplest circuit for such types.

The circuit consists of four op-amps U_1 to U_4 for four integration channels, an op-amp U_5 for an inverting amplifier, and two analog multipliers U_6 and U_7 (using AD633) for the two terms of quadratic nonlinearity. All op-amps are μ741 powered by $\pm$15 V. The DC voltage V_b is +1 V. A set of four coupled first-order ODEs of the circuit is

$$\begin{cases} \dot{x} = -\dfrac{1}{R_4C_1}(\dfrac{R_5}{R_1}x - \dfrac{R_5}{R_2}y - \dfrac{R_5}{R_3}w) \\ \dot{y} = -\dfrac{1}{R_6C_2}(\dfrac{xz}{10}) \\ \dot{z} = \dfrac{1}{R_7C_3}(\dfrac{xy}{10}) - \dfrac{1}{R_8C_3}(V_b) \\ \dot{w} = -\dfrac{1}{R_9C_4}(x) \end{cases} \tag{7}$$

Equation (7) corresponds to (1), where the phase space variables x, y, z, and w represent the output voltages of the four integration channels ($U_1 - U_4$) and the inverting amplifier (U_5). Coefficients in (1) and (7) are compared as $R_5/(R_4R_1C_1) = R_5/(R_4R_2C_1) = R_5/(R_4R_3C_1) = 1$, $a = 1/(10R_6C_2)$, $1/(10R_7C_3) = 1$, $V_b/(R_8C_3) = 1$, and $b = 1/R_9C_4$. For clarity, all coefficients in (7) will be scaled by a certain factor, e.g., 10×10^3.

III. NUMERICAL AND EXPERIMENTAL RESULTS

A. HYPERCHAOS AND A HIDDEN ATTRACTOR

The proposed 4D seven-term no-equilibrium system in (1) is numerically simulated by using the fourth-order Runge-Kutta integrator with an adaptive step size (time step $\leq$ 0.001). As mentioned earlier, for non-zero parameters a and b, system (1) is a no-equilibrium system and attractors are readily hidden. As will be described in this section, parameters $a = 18.13$ and $b = 0.994$ are chosen so as to enable the maximum hyperchaos.

Numerical trajectories of a hidden attractor are illustrated in Figs. 2(a)–2(d) on (x, y), (x, z), (x, w), and (y, z) planes, respectively, where (x, y, z) refer to $[f(\dot{w}), f(\ddot{w}), f(\dddot{w})]$ as shown in (4) to (6). Three colored trajectories in red, blue and green indicate positive, negative and zero values

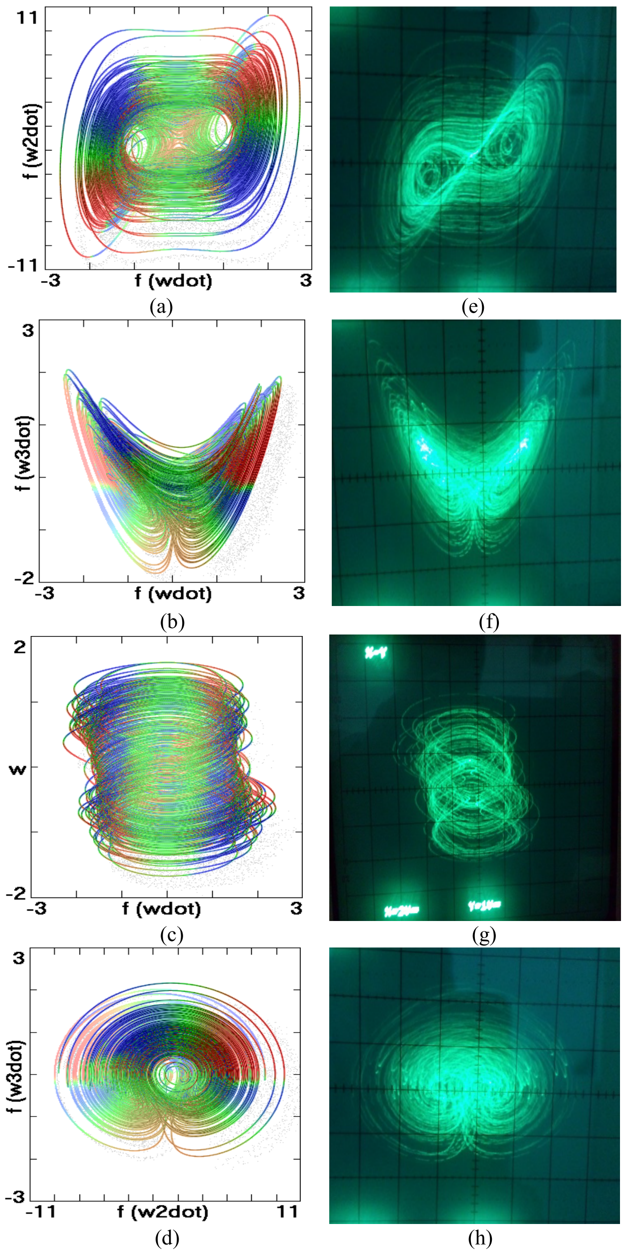

FIGURE 2. Trajectories of system (1) on (x, y), (x, z), (x, w), and (y, z) planes respectively for (a)–(d) on numerical results, and for (e)–(h) on oscilloscope traces (H. 2V/cm, and V. 5V/cm), where (x, y, z) refer to $[f(\dot{w}), f(\ddot{w}), f(\dddot{w})]$ as shown in (4) to (6). Three colored trajectories in red, blue and green indicate positive, negative and zero values of local LLEs, respectively.

of local LLEs, respectively. Although initial conditions are not critical, they are chosen at $(x_0, y_0, z_0, w_0) = (1, -1, 1, -1)$ close to the attractors in order to reduce initial transients. Similarly, for $a = 18.13$ and $b = 0.994$, oscilloscope traces are illustrated in Figs. 2(e)−2(h) on (x, y), (x, z), (x, w), and (y, z) planes, respectively, using $C_1 = C_2 = C_3 = C_4 = 10$ nF, $R_1 = R_2 = R_3 = R_4 = R_5 = R_8 = R_9 = 10$ kΩ, $R_6 = 55$ Ω, and $R_7 = 1$ kΩ. The experimental results are in good agreement with the numerical results.

Based on the Wolf algorithm [33] at $b = 0.994$, Fig. 3 illustrates the spectrum of LEs (L_1, L_2, L_3, L_4), ordered from large to small values, versus parameter 'a' from 2 to 26. The resistor R_6 is a potentiometer between 38 Ω to 100 Ω in order to adjust the parameter 'a' for a wide-range bifurcation from $a = 2$ to 26. The step size of parameter 'a' is 0.015, and the initial conditions are at $(x_0, y_0, z_0, w_0) = (1, -1, 1, -1)$. The calculations also employ the fourth-order Runge-Kutta integrator with an adaptive step size (time step ≤ 0.001). To ensure that chaos or hyperchaos is neither transients nor numerical artifacts, the calculations take a sufficiently longer time up to time $t = 1 \times 10^8$. In Fig. 3, two positive LEs (L_1, L_2) are evidenced and therefore system (1) is a hyperchaotic system with no equilibria for $10.0 \leq a \leq 26.0$, indicating a wide range of parameter space, with a small window of periodic behavior. An existing criterion in [28] has suggested that the maximum hyperchaos occurs where the ratio of L_2 (the second largest LE) to L_4 (the most negative LE) is maximum. Following such a criterion, the maximum hyperchaos occurs at $a = 18.13$ and $b = 0.994$ and becomes the reason for choosing both values for Fig. 2.

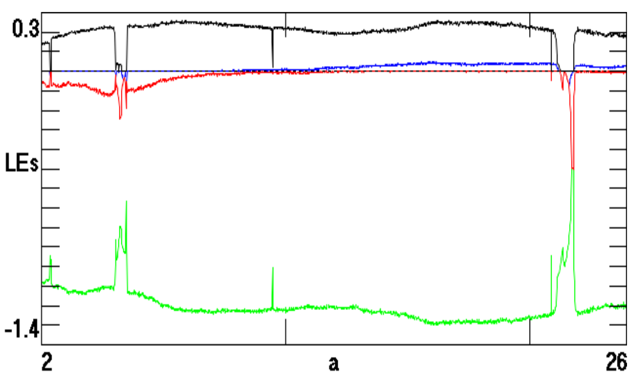

FIGURE 3. The spectrum of LEs (L_1, L_2, L_3, L_4) of system (1), ordered from large to small values, against parameter 'a' showing hyperchaos.

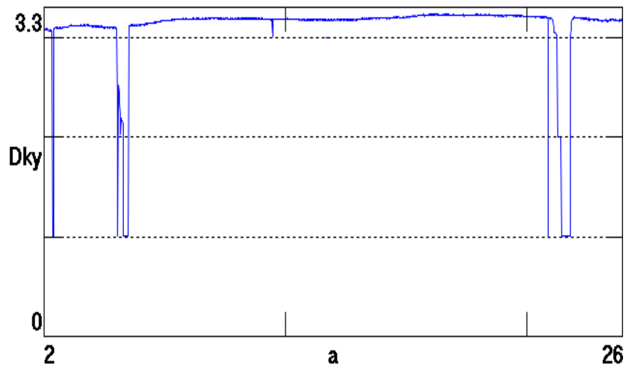

FIGURE 4. Plots of the Lyapunov dimension (D_L) against parameter 'a' showing the highest value at $D_L = 3.2280$ where $a = 18.13$, for $b = 0.994$.

B. HIGHER VALUES OF D_L AND LLE

At b=0.994, the Lyapunov dimension D_L (or the Kaplan-Yorke dimension D_{ky}) of the proposed system (1) is numerically shown in Fig. 4 versus parameter 'a' from 2 to 26. In Fig. 4, the highest value is at $D_L = 3.2280$, where $a = 18.13$ and the spectrum of LEs is $(L_1, L_2, L_3, L_4) = (0.2525, 0.0428, 0, -1.2953)$ where L_1 is the LLE. The sum of the calculated LEs is −1 as required by the trace of the Jacobian of Eq. (1). Although the values of (D_L, L_1) are normally not critical, it is interesting to directly compare them (without changing any scale through any multiplication) with values of (D_L, L_1) of the two existing 4D seven-term no-equilibrium hyperchaotic systems [28] and [29].

As a result, this paper offers relatively higher values of $(D_L, L_1) = (3.2280, 0.2525)$, compared to $(D_L, L_1) = (3.0768, 0.0704)$ of [28], and $(D_L, L_1) = (3.089, 0.064)$ of [29]. In particular, most values of D_L in Fig. 4 are relatively high and maintained near 3.2280 for $6 \leq a \leq 20$. At $b = 0.994$, a bifurcation diagram of the vertex of x (x_m) is numerically shown in Fig. 5 against parameter 'a' from 2 to 26. Two tiny windows of periodic behavior are observed.

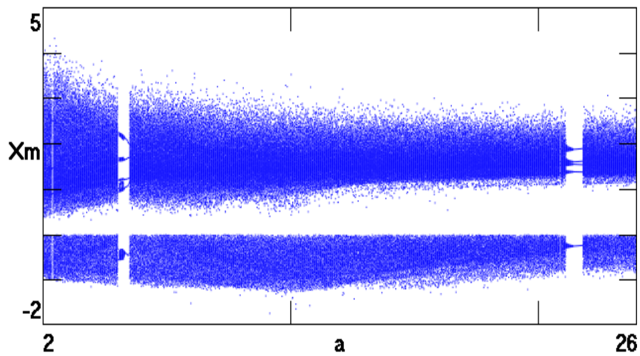

FIGURE 5. A bifurcation diagram of the vertex of x (x_m) versus parameter 'a'.

C. NO MULTISTABILITY

On the one hand, multistability may offer flexibility in system performance, through proper control of initial conditions without changes in system parameters, by inducing proper switching between different coexisting states. On the other hand, as mentioned earlier in Section I, multistability may potentially allow unexpecteed disasters in various systems [30] and therefore, without the proper control, multistability should be avoided to prevent serious catastrophes [30]. It can be shown that the proposed system (1) does not exhibit multistability and therefore does not encounter potential dangers associated with the multistability.

For example, in the hyperchaos at $a = 18.13$, and $b = 0.994$, Fig. 6 simultaneously visualizes both the Poincaré section in red and its basin of attraction in yellow, on the same (x, y) plane at $z = 0$. Fig. 6 shows that only a single basin of attraction in yellow emerges and there is no other basins of attraction (in other colors) in parallel with the yellow. In addition, only a single Poincaré section in red appears and there is no other Poincaré sections (in other colors) in parallel with the red. The system therefore does not exhibit multistability. In addition, Fig. 7 visualizes the Poincaré section on a (y, z) plane at $w = 0$ with initial conditions $(1, -1, 1, -1)$ and, unlike Fig. 6, the basin of attraction is omitted for clarity. Fig. 7 shows that system (1) has rich dynamical behavior on this plane as there is no regular limbs [16].

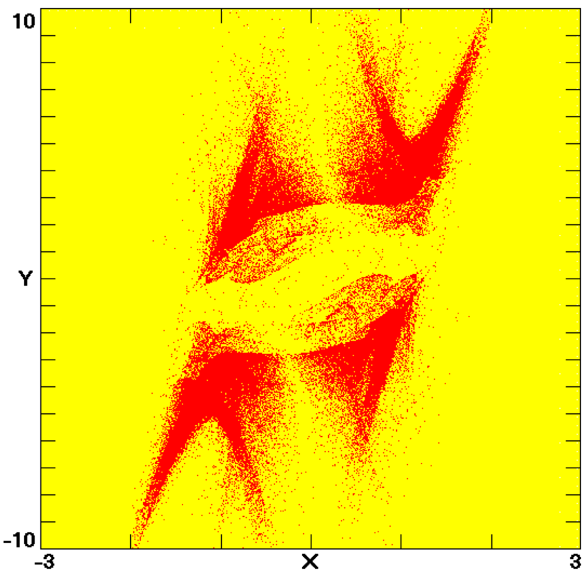

FIGURE 6. **The Poincare section in red and its basin of attraction in yellow on the same (*x, y*) plane at *z* = 0 for *a* = 18.13, b = 0.994.**

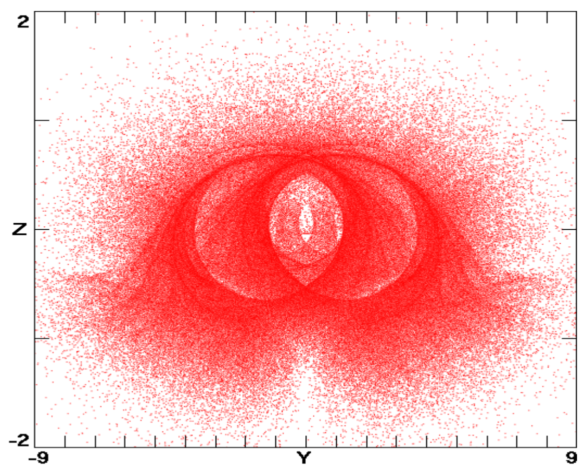

FIGURE 7. **The Poincare section on a (*y, z*) plane at *w* = 0 for *a* = 18.13, *b* = 0.994 and initial conditions (1, −1, 1, −1).**

D. A LARGE TWO-PARAMETER SPACE OF HYPERCHAOS

Based on the spectrum of LEs (L_1, L_2, L_3, L_4), Fig. 8 illustrates dynamical behavior of system (1) in three color pixels on an (a, b) plane of a two-parameter space where $10 \leq a \leq 26$ and $0.1 \leq b \leq 1$. Pixels in red, black, and blue represent hyperchaos, chaos, and periodic behavior, respectively. Initial conditions are chosen from a Gaussian distribution with zero mean and unit variance in every parameter space of 400×400 pixels. For $b = 0.994$ and $10 \leq a \leq 26$ in Fig. 8, the red pixels refer to the hyperchaos described in Fig. 3. It can be seen from Fig. 8 that the red pixels of hyperchaos occupy a large two-parameter space compared with a smaller area of the black pixels of chaos. In addition, the blue pixels of periodic behavior form a much smaller area surrounded by the black and red pixels.

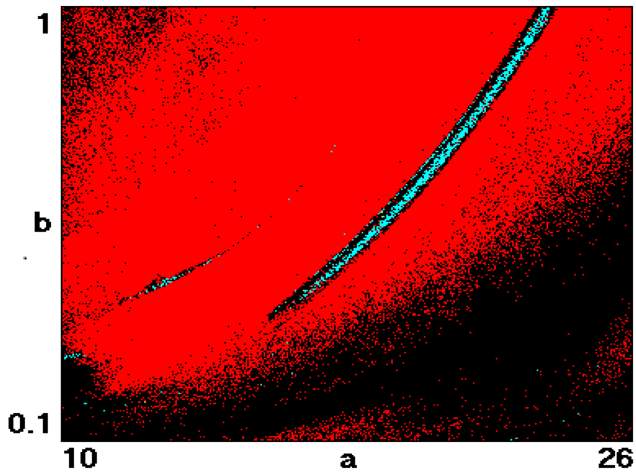

FIGURE 8. **Dynamical behavior of system (1) with pixels in red, black and blue for hyperchaos, chaos and periodic behavior, respectively.**

E. A BOOSTABLE VARIABLE WITH A SINGLE CONTROL CONSTANT

As the phase space variable z appears only once in (1), z can be conveniently controlled as an offset boostable variable by replacing z with $z + k$ where k is a single control constant. System (1) can be rewritten as

$$\begin{cases} \dot{x} = y - x + w \\ \dot{y} = -ax(z + k) \\ \dot{z} = xy - 1 \\ \dot{w} = -bx \end{cases} \tag{8}$$

Fig. 9 illustrates three examples of hyperchaotic trajectories of z with $a = 18.13$ and $b = 0.994$: (*i*) a bipolar trajectory z in blue at the center for $k = 0$, (*ii*) a positively unipolar (shifted-up) trajectory z in black at the upper half for $k = -1.9$, and (*iii*) a negatively unipolar (shifted-down) trajectory z in red at the lower half for $k = 1.9$. Fig. 10 shows mean values of x, y, z and w versus the single control constant k. It can be seen from Fig. 10 that k only changes the mean value of z, but does not influence the mean values of x, y and w.

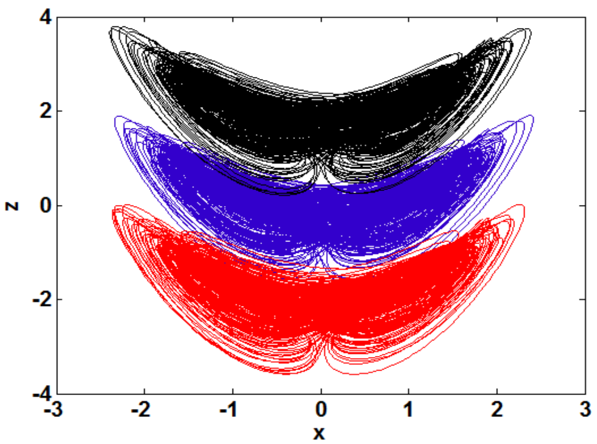

FIGURE 9. A bipolar signal in blue for $k = 0$, a positively unipolar (shifted-up) signal in black for $k = -1.9$, and a negatively unipolar (shifted-down) signal in red for $k = 1.9$.

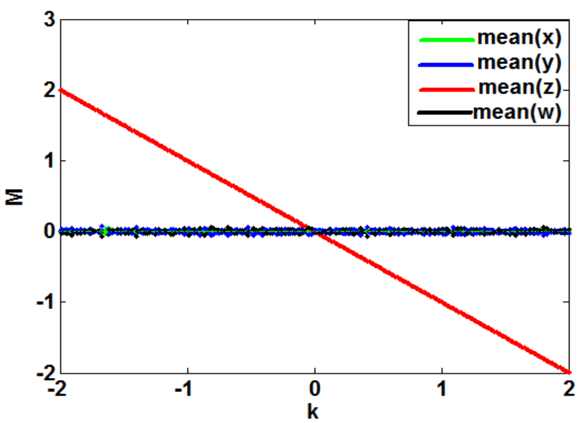

FIGURE 10. Mean values of x, y, z and w versus the single control constant k from -2 to 2, for $a = 18.13$, $b = 0.994$.

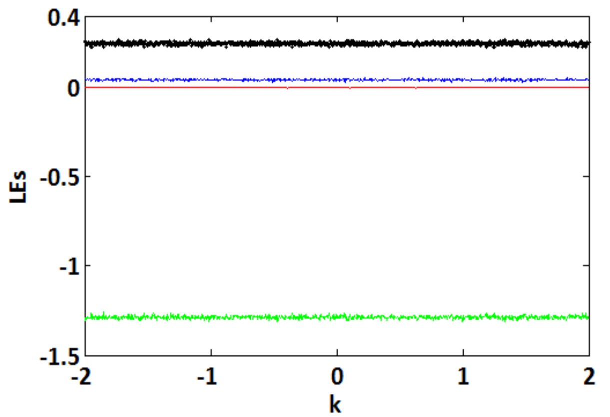

FIGURE 11. The relatively unchanged spectrum of LEs (L_1, L_2, L_3, L_4) versus the single control constant from -2 to 2, for a = 18.13, b = 0.994.

TABLE 1. A comparison of 4D 7-term no-equilibrium hyperchaotic systems.

No.	Terms of Comparison	Reference		
		[28]	[29]	This paper
1	A single fourth-order hyperjerk ODE	×	×	✓
2	A large two-parameter area of hyperchaos	×	×	✓
3	A boostable variable	×	×	✓
4	The number of circuit components	–	46	21
5	Lyapunov dimension (D_L)	3.0768	3.089	3.2280
6	The largest Lyapunov exponent LLE (L_1)	0.0704	0.064	0.2525
7	The number of nonlinear terms	02	03	02
8	No potential danger of multistability	×	✓	✓
9	Hyperchaotic, chaotic, periodic Attractors	✓	✓	✓
10	Hidden attractors	✓	✓	✓

In particular, the spectrum of LEs (L_1, L_2, L_3, L_4) remains relatively unchanged for $-2 \leq k \leq 2$, as shown in Fig. 11. As a result, the change of k introduces no effects on the dynamics but provides a controllable level shift for practical applications where a unipolar signal is required, e.g., [34].

IV. A COMPARISON OF 4D SEVEN-TERM NO-EQUILIBRIUM HYPERCHAOTIC SYSTEMS

Table 1 shows a comparison of 4D seven-term no-equilibrium hyperchaotic systems between the proposed system (1) and the two existing systems [28], [29]. Table 1 shows that this paper offers 10 simultaneous advantages, whereas each of the existing systems [28] and [29] offers less than 10. As shown in Table 1, six advantages in this paper appear to be superior to [28] and [29], i.e., the less (and minimum) number of circuit components, the larger values of Lyapunov dimension (D_L) and LLE (L_1), the large two-parameter space of hyperchaos, and the boostable variable.

In particular, another advantage is the first report of a single fourth-order hyperchaotic hyperjerk ODE in a system with no equilibria, as shown in (2). Such a single ODE enables an alternative model for the study of a no-equilibrium system for hyperchaos. The other four advantages in this paper present equal or superior features compared to those of [28] and [29], i.e., the number of nonlinear terms, no multistability (and its potential dangers), and the hidden attractors are possible for hyperchaos, chaos, and periodic behavior.

V. CONCLUSIONS

A new set of four coupled first-order ODEs for a rare type of 4D seven-term no-equilibrium hyperchaotic systems has been presented and compared with the two existing systems [28] and [29] in the same type. The six advantages, which are superior to the existing systems, are the first report of a fourth-order hyperchaotic hyperjerk system with no equilibria, the simple circuit realization using only 21 electronic components, the Lyapunov dimension of 3.2280, the largest Lyapunov exponent of 0.2525, the large two-parameter space of hyperchaos, and the technique of a boostable variable.

In addition, the four advantages, which are either similar or better characteristics compared to the existing systems, are that the number of nonlinear terms is two, the multistability does not exist, the hyperchaotic, chaotic and periodic attractors are feasible, and all attractors are hidden attractors due to the absence of equilibria. All of the aforementioned advantages enable the proposed system and circuit to be well-suited for chaotic applications such as chaos-based communications.

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