

Received April 11, 2018, accepted May 15, 2018, date of publication June 21, 2018, date of current version July 30, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2840539

# Comparative Analysis of Sequential and Combinatorial Auctions Based on Petri Nets

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This work was supported by the Science and Technology Development Fund, MSAR, under Grant 122/2107/A3.

**ABSTRACT** An auction is a crucial resource allocation mechanism for a supply chain and includes the purchasing price at the winner's bid value, eligible suppliers in winning bidder sets, and business payment terms. A multi-object auction usually involves two types of mechanisms: sequential and combinatorial auctions (CAs). A literature review reveals that the research on CAs still has some treasures waiting to be discovered. Using the framework of Petri nets, this paper employs timed and colored tokens representing every bidder's bidding data and applies transition nodes to execute bidding rules such as combinatorial discriminate analysis, bidder's bid decision-making, and the auctioneer's winner decision-making. Based on the contribution of workflows in Petri net theory, we present a case-based CA Petri net model and a sequential auction Petri net model. We compare the supply chain coordination performance of these two types of auctions and reach the conclusion under given assumptions. This paper is an attempt to apply Petri net theory to auctions and provides valuable insights for organizers to establish scientific and efficient bidding processes.

**INDEX TERMS** Auction, combinatorial auction, sequential auction, substitution, supply chain, Petri net.

## I. INTRODUCTION

As the most important resource allocation method, bidding auctions are gradually becoming a widely used and important tool for supply chain procurement. Many auctions involve multiple heterogeneous objects. Examples include Federal Communications Commission (FCC) spectrum auctions and auctions for material allocation, network routing, railroad segments and logistics routes. Due to the existence of substitution or complementarities among multiple heterogeneous goods, bidders might prefer parcels of items more than individual items. An auction mechanism that accommodates the above features is called a combinatorial auction (CA), in which bidders can bid for the bundles of items that they prefer.

With the development of e-business, CAs are being increasingly frequently employed in business-to-business (B2B), business-to-customer (B2C), customer-to-customer (C2C), and even online-to-offline (O2O) transactions.

The first CA was first conducted in 1993/1994 by Net Exchange [www.nex.com](http://www.nex.com), and it allocated transportation services resulting from the CA [1]. At present, CAs, functioning as purchasing "specialists," have been introduced into a broader scope of business arenas that also include the B2B/B2C/C2C/O2O arenas (e.g., [www.taobao.com](http://www.taobao.com), [www.combinenet.com](http://www.combinenet.com)), industry, logistics, information technology and solution support (IBM: [www.ibm.com](http://www.ibm.com), Google: [www.google.com](http://www.google.com)). Following the emergence of scientific research on the topic, the practical business applications of CAs have been adopted with amazing speed as analysis and decision-making tools [2].

Note that CAs involve interdisciplinary knowledge of auction theory, game theory, combinatorial optimization, mathematical programming, and related computer technology. Three core problems involved in CAs are their mechanism design, bidding languages, and the winner determination problem (WDP) [3]. Most existing academic studies on CAs

focus on narrow technical issues [4]: computer specialists are concerned with exploiting faster heuristics and simplifying the complexity of the WDP; some economists study actual CAs according to an overview of the banks; operation research investigators mostly use integer programming; and game theorists generally simplify the models to reach theoretically desirable properties. In order to produce a better understanding of bidding behaviors in a CA, this study begins with the actual context, and a case study is presented on the conditions for several heterogeneous objects. Petri net models of a sequential auction and a CA demonstrate every process of bidding and vividly display every behavior of the bidders and the auctioneers, including mechanism design, the discrimination of complementarities and substitution, the bidder's bid decision-making algorithm, and the winner decision-making algorithm.

## II. LITERATURE REVIEW

### A. SUPPLY CHAINS AND AUCTIONS/BIDDING

Supply chains operating as a many-to-many-based network structures are called supply chain networks (SCNs) and are made up of complex interconnections among various sellers and buyers. The entities providing the services, raw materials, and solutions generally situate themselves upstream of SCNs, the core manufactures are in the middle of the SCNs, and the distributors, the retailers and customers are downstream of SCNs [5]. A supply chain is a group of business entities that work together for the same final product or service and assume its complementary contribution, cost and profit. At present, business competition is no longer constituted by autonomous entities alone but rather by a win-win business team, in which each entity pursues common economic interests, which is termed supply chain coordination in supply chain management (SCM) [6]. Most auctions are price-driven and related by cost to logistics, capacity management, inventories, the credit period, and price-dependent demand. The results of auctions answer the 6W+2H questions of SCM such as who is the seller/partner, what is the corresponding cooperative product, when, where, how many, at what price and by which condition the contract is executed, and why a buyer chooses these sellers. As a result, auctions have become one of the most important activities in SCM.

Auction theory has been widely applied in practical, empirical and theoretical areas. For instance, a deluge of economic transactions are implemented through auctions. Auction theory provides considerable experimental samples for economic theory. Furthermore, much of the fundamental theoretical work is conducted based on auction theory, including posted prices/published prices and negotiations [7]. Auctions are defined in [8] as follows: an auction is a market resource reorganization mechanism that allocates the market resource at the price of a bid with a series of supply chain collaboration items though a set of special auction rules. Although many auctions define an environment in which multiple bidders purchase from one seller, it is more common

in actual transactions that a single buyer purchases a commodity from multiple bidders. This study considers an environment of the latter type. In other words, a single buyer chooses optimal sellers from various auction participants in a supply chain.

In the marketplace, four basic auction mechanisms are widely used [7]. The first two mechanisms are the ascending-bid auction and the descending-bid auction, and the latter two mechanisms are the first-price sealed-bid auction and the second-price sealed-bid auction. In an ascending-bid auction, the price is gradually increased until only one bidder remains and wins the object at the final price. A descending-bid auction operates in exactly the opposite way. A first-price sealed-bid auction and a second-price sealed-bid auction are auctions in which each bidder individually submits a single private bid without receiving any information from others, and the object is sold to the bidder who makes the highest bid (the first-price) and the second-highest bid (the second-price), respectively. In the first two mechanisms, bidders can obtain some information, but they cannot in the latter two mechanisms.

Excluding single-object or homogeneous multi-object auctions, this study addresses heterogeneous multi-object auctions. There are two types of heterogeneous objects: the first is a physical object, namely, a concrete entity with a specific color, shape, and property, and the second is an abstract object representing nonobjective or nonrepresentational objects such as routing, smartphone design, and management solutions. This study works with the former type and constructs corresponding models. Examples include FCC spectrum auctions, transportation routing, manufacturing materials and solution auctions. There are many issues addressed in multi-object heterogeneous bidding auctions, beginning with the focus on sequential auctions [9]. In multi-object heterogeneous auctions, objects are sold or bought sequentially due to a bidder's interest in only one item, but in some settings, the seller focuses on an individual object and maximizing revenue but rather on (strong) efficiency and an object bundle [10]. Palfrey [3] finds that an auctioneer tends to parcel objects together and sells/buys them as a single item to two bidders, but when there is a large number of bidders, a seller tends toward separate auctions. The preferences of the bidders depend on the complementarities and substitution, which will be addressed in greater detail below.

Many auction topics were explored in the 1960s, including optimal auctions, revenue equivalence, marginal revenues, risk preference, information effects, and the probability of winning, and there are many related studies based on the relaxation of various constraints. The earliest auction model is investigated in [11]. It is considered one of the most important bidding models of first-price sealed-bid auctions. Many studies, however, have found it too restrictive and constructed a variety of models with relaxed constraints [12]. There are some improved models in [9], [13]–[15], and [16]. Skitmore [17] tests four of the leading models through an empirical study of three large samples of truthful architecture

contract auction data and concludes that all the models produce poor predictions in both one-out and one-on models. From the CA perspective, mechanism design, bidding languages, and the WDP are most popular topics in this area. In this work, we model a first-price CA process based on the results of these three topics and illustrate a timed and dynamic event process of auctions.

There are five basic stages in an auction. The first is the bidding invitation process that calls for all of the potential suppliers to discuss their agreement intention regarding the threshold price, credit period, capacity, and lead time. The second is the bid process that includes the bid decision-making and related data. The third is the close-of-bidding process, which specifies the rule for ending the second process such as the tender deadline or other rules. The fourth is the open bid process that ensures the openness and fairness of uncovering every bidder's bid, and the last is a bidding evaluation process that is executed by an auctioneer to decide the winner set. Note that the bidding object is a trade between a buyer and sellers. In other words, the winner's bid is the bidding object's price at which vendors sell to the buyer. This study addresses two heterogeneous multi-object auction mechanisms to compare their supply chain coordination performance and reaches a conclusion under given assumptions.

## B. COMBINATORIAL AUCTIONS AND SUPPLY CHAINS

Auctions are usually called CAs if bidders are allowed to submit their bids on combinations or bundles of objects [2]. CAs are simultaneous multi-object auctions that allow submissions in the form of "all-or-nothing bids" for bundles or combinations of the items being sold/bought [4]. According to the work by Gujar and Narahari [18], a CA is a mechanism whereby bidders can submit bids on combinations of items. CAs are defined as those auctions in which suppliers can bid for bundles of objects, and this type of auction has attracted considerable interest in procurement applications [19]. According to [20], CAs have increased in recent years, becoming an increasingly important mechanism, offering a considerable advantage in e-business applications such as e-selling/procurement, e-logistics, supply chain structure formation, and B2B/B2C/C2C exchanges. They are popularly employed in fields such as carrier selection [1], delivery/transportation/logistics routes, meal supply, bus routes, and space shuttles. Combinations, packages, simultaneous and bundles are the core concepts of CAs. Based on these features, CAs have become a mechanism, by which bidders can convey their preferences freely and resources can be allocated legitimately in a supply chain. The main advantage of CAs [19] is that they induce suppliers/bidders to coordinate the cost among objects in the CA process, which often leads to a lower purchasing cost for the buyer/auctioneer.

Over the past two decades, supply chain coordination, representing the main advantage of supply chains, has received considerable attention. Decision-making considered in a globally optimal supply chain can benefit all the parties involved in the supply chain, as opposed to each party

individually making its own decisions [21]. The mechanisms affecting supply chain coordination are numerous and include price discounts, credit periods, and quantity discounts. The aim of this study is to prove the effectiveness of the CA mechanism in supply chain coordination. However, there are three key issues to be addressed: CA mechanism design, bidding languages, and the WDP [22].

CA mechanism designs broadly create the rules, by which the bidding process operates. And the rules can be represented as single-round auctions, first-price & sealed-bid auctions, Vickrey-Clarke-Groves (VCG) auctions, uniform & market-clearing price auctions, and iterative CAs [4]. The aims of CA designs are to solve the exposure problems, threshold problems, unreasonable bidding action and moving, resolving bugs, and the complexity of management. A good mechanism design should arouse bidders' attendant desire and satisfy the mechanism designer's profit/utility maximization. Bidding languages are capable of bidding expression. On the basis of a large number of bids, to make every bid standardized and precise, every bidder encodes its bids/preferences to be understood by the auctioneer. This is the efficient way to conduct a CA. A fully expressive bidding language entails that bidders are allowed to submit "OR of XOR bids," where OR and XOR display the logical "OR" and the exclusive "OR" operations, respectively. Winner determination is the most difficult problem to solve, and it is a popular topic in operation research and computer science. The WDP results from large numbers of packages and the speed of finding the optimal bid from them. There are many available methods, such as total unimodularity, perfect matrices, balanced matrices, graph-theoretic methods, and using preferences [2]. Because a WDP is NP-hard, the bidder's freedom to fully convey its preference is limited. Hence, this work addresses the limit of combinatorial bids to vividly describe every behavior in a CA.

Although our research topic and method are similar to those of Giovannucci *et al.* [23], [24], [26], [27] and Vinyals *et al.* [25], the difference lies in that most of Andrea's studies on combinatorial auctions make use of weighted transition Petri nets (WTPNs), whereas this work uses colored timed Petri nets (CTPNs) as the methodology. Indeed, the issue of this work is completely different from those of previous studies. This paper addresses CAs as a workflow with several auction sub-processes optimizing the winner of a CA using CTPNs.

## C. MODELING TOOL: PETRI NETS

Petri nets [28] have evolved into a formalism employed in various fields such as workflow [29], evaluation and event management [30], communications [31], electronics [31], chemistry logistics [32], single-arm cluster tool with wafer revisiting [33], manufacturing systems [34]–[55] and supervisory control of discrete event systems [56]–[59], [74], [75]. Due to the limitations of the original paradigm of Petri nets, various extensions have been made, including the concept of time, colors and hierarchical levels. A variety of types of Petri nets have been derived, including generalized stochastic Petri

nets (GSPNs) [60], timed Petri nets (TPNs) [61], colored Petri nets (CPNs) [62], CTPNs [63], batch deterministic stochastic Petri nets (BDSPNs) [6] and deterministic and stochastic Petri nets (DSPNs) [64]. Accordingly, many simulation tools have been compiled such as INA [65], TINA [66], CPN [62], and ExSpect [67].

Specifically, CPNs are conducted with high-level programming languages based on the ability of basic Petri nets. They are widely used in modeling systems in which communication, synchronization and resource sharing play a critical role. Given a place node, all tokens must have token colors that belong to a specified type that is called a *color* [68]. In a CPN, each token is bundled with a color, presenting the common identity of tokens. A transition executing some behavior can fire the token/tokens to the output place based on its related firing rules. The color affixed to a token may be changed by a transition firing, and it often represents a complex data value [69].

According to [69], a CPN is defined as a 5-tuple  $CPN = (P, T, C, W, M_0)$ , where

- 1)  $P$ : a set of places;
- 2)  $T$ : a set of transitions;
- 3)  $P \cap T = \emptyset, P \cup T \neq \emptyset$ ;
- 4)  $C$  is the colored-function defined from  $P \cup T$  into nonempty sets;
- 5)  $W$  is the incidence-function defined on  $P \times T$  such that  $W(p, t) \in [C(p) \rightarrow [C(p) \rightarrow \mathbb{Z}]_f]$  for all  $(p, t) \in P \times T$ , where  $\mathbb{Z}$  denotes integers; and
- 6)  $M_0$ , the initial marking, is a function defined on  $P$ , such that  $M_0(p) \in [C(p) \rightarrow \mathbb{N}]_f$  for all  $p \in P$ , where  $\mathbb{N}$  denotes the set of nonnegative integers.

Elements of  $C(p)$  and  $C(t)$  are called colors. A place  $p$  is an input place (output place) for a transition  $t$ . If  $W(p, t)(c')(c'') < 0$  ( $W(p, t)(c')(c'') > 0$ ) for at least one pair of colors  $c' \in C(t)$  and  $c'' \in C(p)$ . To formalize the firing rule, some definitions are required: a weighted set of transitions is a function defined on  $T$  such that  $X(t) \in [C(t) \rightarrow \mathbb{Z}]_f$  for all  $t \in T$ .

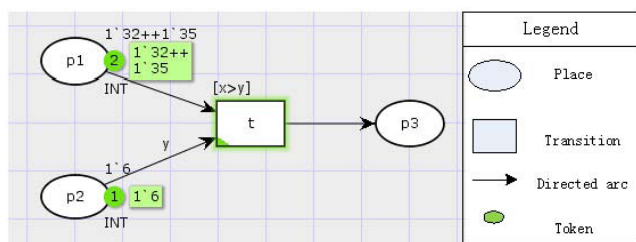


FIGURE 1. A colored Petri net.

Fig. 1 illustrates a CPN with a net structure, colored tokens, and transition rules.

In the net structure, the initial marking is (2, 1, 0), where two colored tokens including 32 and 35 are in place  $p_1$ , and one token is in place  $p_2$ . Firing transition  $t$  will remove tokens from places  $p_1$  and  $p_2$ , and then deposit the token valued as

$(x - y)$  into the output place  $p_3$  conditional on the guard of  $[x > y]$ .

When timing information is added to a CPN model, CTPNs can be constructed. If a deluge of tokens with the same type of color is added to a Petri net, a CTPN can create a system working in a very efficient manner based on validated real time that reflects a real timing event operation system. In addition, the CTPN is an extension of Petri nets that can cope with multiple processes and time constraints with several important characteristics including concurrency, distributed nature, and synchronization [70]. A CTPN is differentiated from a CPN by the former's time-dependent manner with a second color value of tokens, called a timestamp. In other words, the marking of a place is a timed multiset if tokens are attached with timestamps. By removing all timestamp information on arc and transition nodes, a CTPN can be transformed into an untimed CPN. In the conduct process of the CTPN and CPN, turning an untimed CPN model into a timed model cannot create new behavior in the form of new occurrence sequences.

Zhang et al. [68] present a detailed literature review on Petri nets' applications to SCM, including strategic competitiveness, supply chain tactics (firm-focused tactics), and efficiency of supply chain operation. Strategic competitiveness includes the design of supply chains and competitive advantage assessment. Relationship development between every entity in supply chains, integrated operations, logistics and transportation, and collaboration are covered in firm-focused tactics. Operational efficiency involves inventory management and control, production, planning and scheduling, information sharing, coordination and monitoring, and supply chain risk management. Nandula and Murali [71] study an auction Petri net model with emphasis on manufacturing systems. Thus far, supply chain auctions have barely been explored using Petri nets. This study makes an attempt to model, analyze, and evaluate the performance of an auction process.

As demonstrated in [72], Petri net models have a good track record for performance validation of time-dependent concurrent processes, such as communication and messaging protocols. Due to the advantages of Petri nets, they are a suitable approach to connect logistics, information flow and cash flow. In addition, they can accurately represent auction behavioral rules and implied event sequencing. The characteristics of concurrency, asynchronism, dynamics, and timed colored tokens demonstrate that Petri nets are an ideal model to express the dynamic online auctions that can be indicated visually and understandably based on the monitoring of every time unit like a vidicon. In this study, colored tokens provide an excellent representation of the many different and changing bidding data states of every bidder. In addition, colored tokens make a Petri net model structurally smaller than a non-colored one, which would be complex and unreadable. Simulations based on Petri nets can reflect bidding information interactivity, and timeliness, just as they happen in a real scenario.



### III. CONTRIBUTION

This study models a first-price sealed-bid CA using colored timed Petri nets. Compared with a first-price sealed-bid heterogeneous multi-object sequential auction, this work reaches a conclusion with respect to supply chain coordination. It is the first attempt to model, analyze, control, and evaluate the performance of a CA based on Petri nets. This work simulates a smartphone material/module auction and achieves an auction mechanism with better supply chain coordination performance by using the CA mechanism designs, bidding languages, and winner determination optimization.

Although CAs are widely employed in many applications, crucial problems in such auctions still exist such as NP-hard WDP, bidding data security, and relevance issues of the items. Moreover, depending on the bidding environment, bidding data, and bidding rules, CAs have entirely different solutions and optimal results. However, numerous studies on CAs are mainly concerned with mechanism design, bidding languages, and the WDP. There are few case-based studies on CA system embedding in mechanism design, advantages and disadvantages, bidding languages, and winner decision making based on CTPNs. This work attempts to fill the gap in the literature on this problem.

The remainder of this work is organized as follows. In Section IV, a first-price sealed-bid CA model is presented with three crucial transitions embedded in discriminate algorithms for complementarities and substitution, a combinatorial bid decision-making algorithm, and a combinatorial winner decision-making algorithm. In Section V, we analyze a first-price sealed-bid heterogeneous multi-object sequential auction embedded in a bidder's bid decision-making algorithm and a winner decision-making algorithm for an individual object. Section VI compares the coordination performance of the two types of above-mentioned auctions, and Section VII concludes this study.

## IV. A FIRST-PRICE SEALED-BID COMBINATORIAL AUCTION

### A. ASSUMPTIONS

Assumptions for a first-price sealed-bid CA are basically consistent with general standard conditions given by classical auction theory. In order to avoid leaving some bids for objects that none of the bidders are interested in empty, the assumptions are stated as below:

1) There is information symmetry between bidders and sellers and asymmetry between sellers and buyers; that is every bidder only holds its private information (private value, e.g., complementarities and substitution), which cannot be obtained by any other rivals. An auctioneer knows some information that is unknown to bidders, however, such as the number of bidders, bidders' capacity, and the preference of every bidder.

2) The objective of every bidder is profit maximization.

3) The objective of the auctioneer is cost minimization.

4) Each rival is likely to bid as he has done in the past, and this behavior is imperceptible and never changed by others.

5) Every bid is independent.

6) Multi-object with same multi-units: bidding objects are a set including a variety of distinct items, and in addition, each of them has the same bidding quantity.

7) Each bidder is risk neutral.

8) Existence of objects' complementarities: the complementarities are dependent on the profit /cost variance of combination solution.

9) Every bidder's bid must cover all of the objects in which a package or individual bid is permitted. The only difference between every package is the ladder quantity and the corresponding price. For example, if the auction quantity is 500, the package could be 200 for ladder 1 and 300 for ladder 2 at the price of \$3.5 and \$3.1, respectively.

10) More than two ladders in a single subset are not permitted.

11) Winner's capacity is sufficient for auction objects.

12) Make-to-order (MTO) mechanism [73]: the legal order is placed according to the auctioneer's truthful needs. The order quantity is probably smaller than the auction quantity.

13) Each bidder tends to allocate the specific costs in the first ladder with the objective of profit maximization.

14) Same specific costs for every object: package-specific costs are equal to the sum of all individual object's costs.

Assumptions 1-5 and 7 describe the characteristics of bidders and auctioneer. Assumptions 6 and 8 represent the quantitative ratio and complementary characteristics of multi-object. The rest of Assumptions provide rules of bidding and cost calculation.

### B. NOTATION

$N$ : number of bidders with  $ID = 1, 2, \dots, i, \dots, n$ .

$M$ : number of objects with  $ID = 1, 2, \dots, j, \dots, m$ .

$S = \{s_{ik} | i = 1, 2, \dots, n; k = 1, 2, \dots, a\}$ : a set representing bidder's bid subsets. Note that a subset's format can be bundled or an individual item.

$P_{aij} = \{p_{aij} | i = 1, 2, \dots, n; j = 1, 2, \dots, m\}$ : a set representing bidders' estimated prices of rivals that are called average bidders' bid.

$C = \{c_{ij}, c(s_{ik}) | i = 1, 2, \dots, n; j = 1, 2, \dots, m; k = 1, 2, \dots, a\}$ : a set representing the industry average costs assessed by bidders.

$C_0 = \{c_{0ij}, c_0(s_{ik}) | i = 1, 2, \dots, n; j = 1, 2, \dots, m; k = 1, 2, \dots, a\}$ : a set representing the actual costs of bidders considering the fixed costs and variable costs.

$C_1 = \{c_{1ij}, c_1(s_{ik}) | i = 1, 2, \dots, n; j = 1, 2, \dots, m; k = 1, 2, \dots, a\}$ : a set representing raw material costs for a package or individual object.

$C_2 = \{c_{2ij}, c_2(s_{ik}) | i = 1, 2, \dots, n; j = 1, 2, \dots, m; k = 1, 2, \dots, a\}$ : a set representing the R&D fees and market costs for a package or individual object.

$C_3 = \{c_{3ij}, c_3(s_{ik}) | i = 1, 2, \dots, n; j = 1, 2, \dots, m; k = 1, 2, \dots, a\}$ : a set representing the costs of special tools for a package or individual object (e.g., mold cost).

$Q$ : a variable for an object's quantity. On the basis of Assumption 6, every object has the same bidding quantity as  $Q$ .

$Q_1 = \{q_{1ij}, q_1(s_{ik}) | i = 1, 2, \dots, n; j = 1, 2, \dots, m; k = 1, 2, \dots, a\}$ : a set representing the ladder quantities of raw material supported by the bidders' vendor, where  $Q_1 < Q$ .

$Q_2 = \{q_{2ij}, q_2(s_{ik}) | i = 1, 2, \dots, n; j = 1, 2, \dots, m; k = 1, 2, \dots, a\}$ : a set representing the estimated quantities of truthful orders by bidders, where  $Q_2 < Q$ .

$Q^* = \{q^*(s_{ik}) | i = 1, 2, \dots, n; k = 1, 2, \dots, a\}$ : a set representing the ladder quantities of a package subset (e.g.,  $q^*(s_{11}) = 200$  indicates that the first ladder is  $0 \sim 200$  and the second ladder is  $201 \sim 500$  if  $Q = 500$ ).

$G = \{g_i | i = 1, 2, \dots, n\}$ : a set representing the fixed costs of bidders;

$Y = \{y_i(\cdot) | i = 1, 2, \dots, n\}$ : a set representing yields for a given quantity of a subset (e.g.,  $y_i(Q_{1i}), y_i(Q_{2i}), y_i(Q)$ ), where  $0 \leq Y \leq 1$ .

$P(X)$ : a set representing the probability of winning a given bid of  $X$ .

$O = \{o_{ij} | i = 1, 2, \dots, n; j = 1, 2, \dots, m\}$ : a set representing the negotiation prices.

$\lambda_i(\cdot) = \{\lambda_i | i = 1, 2, \dots, n\}$ : a set representing the bidders' estimated numbers of participants for every subset.

$f(\cdot)$ : a variable representing the auctioneer/buyer's specific costs for every object. If the winner set is individual object, this variable is the sum of the every object's specific costs. Otherwise, it is equal to one object's specific cost.

$X(S_{ik}) = \{x(s_{ik}) | i = 1, 2, \dots, n; k = 1, 2, \dots, a\}$ : a set representing the bidders' bids of a subset (decision variable of a bid decision-making algorithm). The representation format of  $X(S_{ik})$  takes the form:  $X(S_{ik}) = S_{ik}(X_1, X_2)$ , where  $X_1$  and  $X_2$  are the price for  $Q_1$  and the remaining ( $Q-Q_1$ ) quantity, respectively. The valuation of  $X(S_{ik})$  can be calculated as  $X(S_{ik}) = \frac{X_1 * Q_1 + X_2 * (Q - Q_1)}{Q}$ .

$H_i(\cdot) = \{h_i | i = 1, 2, \dots, n\}$ : a set representing bidders' winning shares of subset (decision variable of winner decision-making algorithm). Based on Assumption 11,  $H$  is a binary integer.

**C. PROCESS DESCRIPTION OF A COMBINATORIAL AUCTION**

To illustrate the entire process of a first-price sealed-bid combinatorial auction (FpSbCA), a flowchart is represented in Fig. 2:

Fig. 2 shows the entire process operation logic of a FpSbCA. It has the same overall framework as a sequential auction embedded in different sub-processes (white part), and further interpretation is given below:

*Step 1:* To organize a valid auction, an auctioneer conveys his/her bidding object information to every potential bidder, including the objects' quantity, quality, property, delivery requirement, anticipated price, and auction mechanism.

*Step 2:* After obtaining the auction information, bidders begin to summarize their preferences by analyzing objects'

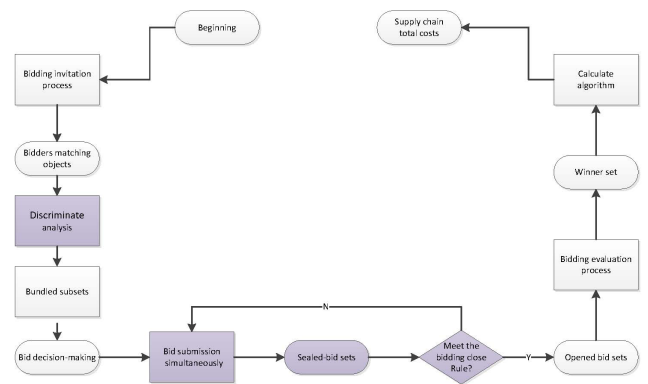


FIGURE 2. An auction process of FpSbCA.

complementarities and substitutions, which mainly result from cost-saving or revenue-increasing considerations. This phase is also the ready process.

*Step 3:* Bidders submit their bid sets simultaneously until the auction deadline is met. This is a bidding process.

*Step 4:* Bid sets are private information until an auctioneer unpacks them in an open and fair environment. This is an open bidding process.

*Step 5:* To compare the supply chain coordination of a CA with heterogeneous multi-object sequential auctions, an algorithm for a two-stage supply chain coordination is given.

**1) DISCRIMINATE ANALYSIS OF COMPLEMENTARITIES AND SUBSTITUTION (C&S)**

Complementarities of multi-objects refer to the fact that the submitted bids on combinations of objects are more profitable to bidders than the sum of the values of bids for the individually submitted items in the combination [4]. In other words, complementarities usually mean cost savings or revenue increases. As a result, complementarities of multi-objects become an endogenous and crucial factor for arousing interest in CAs. In contrast, some bidders would rather choose one item or only some of the objects resulting from the substitution of multi-objects. Some examples of substitution are capacity conflict, business competition, increasing margin cost, technological limitations, and policy or capital. Bidders prefer multi-object CAs when they admit the existence of complementarities, whereas when substitution is the case, they prefer the sequential auction. This is echoed by the CAs' endogenous factor mentioned in the last paragraph.

Before a CA Petri net model is presented, a flowchart for the discrimination of complementarities is illustrated in Fig. 3.

Note that the discrimination of multi-object complementarities and substitution are so important that they determine the preference of bidders as a bidding subset. This case-based paper will use auctions involving several heterogeneous objects and bidders in the manufacturing industry to illustrate the detailed discrimination process for complementarities and substitution.

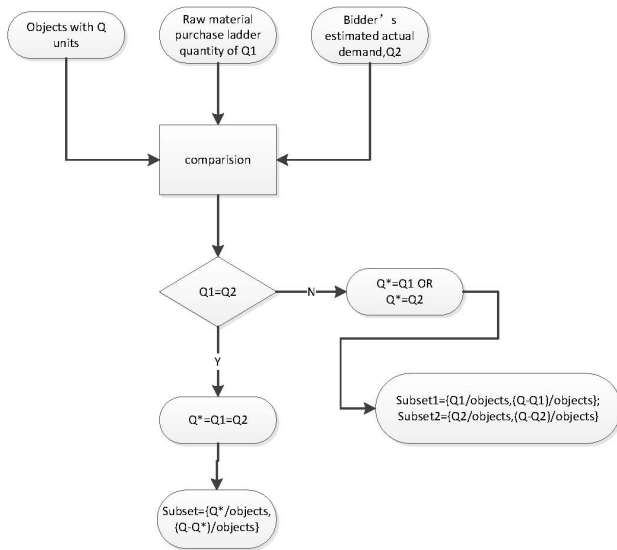


FIGURE 3. A flowchart for discriminating C&S.

According to Assumption 9 above, a bidder’s bid subset must cover all the auction objects. If necessary, ladder quantity and the corresponding price can differentiate between bundles. Our work will solve for the ladder quantity as the valuation of  $Q^*$  for every package subset. The constraints that mainly impact  $Q^*$  vary from the upstream supplier’s ladder quantity to the bidder’s estimated truthful demand for legal order quantity. The bidder’s aim is profit maximization by employing these factors during the bidding period. As a result,  $Q^*$  valuation can be represented by

$$Q^* = \begin{cases} Q^* = Q_1 = Q_2 & Q_1 = Q_2 \\ Q^* = Q_1 \text{ OR } Q_2 & \text{otherwise} \end{cases}$$

The expression above is derived from the bidder’s risk control based on an MTO supply chain mechanism. There are two cases. If  $Q_1 = Q_2$ , then  $Q^* = Q_1 = Q_2$ . Otherwise,  $Q^* = Q_1$  or  $Q^* = Q_2$ . This means that as a risk-neutral bidder, one tends to take all the specific costs back during its estimated quantity,  $Q_2$ . As a result  $Q^* = Q_2$  is always the bidder’s preference. However, if  $Q_1 < Q_2$ , the upstream supplier holds a more conservative estimate than the bidder’s; following risk control logic,  $Q^*$  could be equal to  $Q_1$ . If  $Q_1 > Q_2$ , the bidder may hold a preference of  $Q^* = Q_1$  at the ladder price depending on the specific cost allocation for  $Q_1$  or  $Q_2$ . Based on Assumption 10, using bidding languages, the bidding preference of bidders can be represented by:

$$S_{ij} = \left\{ \underbrace{Q_i^*/objects}_{1^{st} \text{ ladder}}, \underbrace{(Q - Q_i^*)/objects}_{2^{nd} \text{ ladder}} \right\}$$

A bidder’s bid can be represented as

$$X(S_{ij}) = (S_{ij} : 1^{st} \text{ bid}, 2^{nd} \text{ bid})$$

A case-based example is illustrated below:

$$S_{11} = \left\{ \underbrace{100/(a, b, c)}_{1^{st} \text{ Ladder}}, \underbrace{300/(a, b, c)}_{2^{nd} \text{ Ladder}} \right\}$$

and its bid is represented as  $X(S_{11}) = (S_{11} : \$6.5, \$6)$ . An example for  $Q^*$  valuation with three objects of  $a, b, c$  is presented in Table 1.

TABLE 1. An example for  $Q^*$  valuation.

$i$	$Q$	$Q_1$	$Q_2$	$Q^*$	subset( $S_{ik}$ )
1	500	100	300	100 or 300	$S_{11}=100/a, b, c; 400/a, b, c$ $S_{12}=300/a, b, c; 200/a, b, c$
2	500	100	100	100	$S_{21}=100/a, b, c; 400/a, b, c$
3	500	200	150	150 or 200	$S_{31}=150/a, b, c; 350/a, b, c$ $S_{32}=200/a, b, c; 300/a, b, c$
4	500	150	500	150 or 500	$S_{41}=150/a, b, c; 350/a, b, c$ $S_{42}=500/a, b, c$

On the condition of multi-object ( $a, b, c$ ) with the constant quantity of 500, the first bidder’s upstream supplier’s ladder quantity is 100, and its own estimated quantity for actual demand is 300. According to the expression for  $Q^*$ , the first bidder can obtain two bidding subsets such as  $S_{11} = \{100/(a, b, c); 400/(a, b, c)\}$  that divide 500 into 100 and 400 with their own bids, respectively. From the third bidder’s perspective, its  $Q_1$  and  $Q_2$  are equal to 200 and 150, respectively. As a result, one solution extracts cost from its estimated demand as  $S_{31} = \{150/(a, b, c); 350/(a, b, c)\}$ . Similar to a bet, if a bidder provides its preference of  $S_{32} = \{200/(a, b, c); 300/(a, b, c)\}$ , the bidder could withdraw its specific cost or lose some cost while selecting 150 and 200, which could be decided by a bid decision-making algorithm. Moreover, an example of a subset’s actual cost calculation is presented in Table 2.

TABLE 2. An example of subset phase cost calculation ( $Q = 50$ ).

object	1st Bidder						$Q^*$	
	$C_1(Q_1)$	$C_1(Q - Q_1)$	$C_2$	$C_3$	$G(Q_1)$	$Q_2$		
$a$	2.3	2	1.6	5	30	10	20	10 OR 20
$b$	0.7	0.5	1.6	5	30	10	20	10 OR 20
$c$	1.5	1.2	1.6	5	30	10	20	10 OR 20
$(a, b, c)$	4.5	3.7	4.8	15	30	10	20	10 OR 20
$S_{ik}$	$S_{11}/S_{13}=(10/a, b, c; 40/a, b, c)$ OR $S_{12}=(20/a, b, c; 30/a, b, c)$							
	$S_{11}$		$S_{12}$		$S_{13}$			
$C_0(Q^*)$	9.5		6.59		8.49			
$C_0(Q - Q^*)$	4.45		4.7		4.6975			

According to the notation above,  $C_1(Q_1)$  represents the raw material costs for a quantity of  $Q_1$ , and we have an analogous interpretation for  $C_1(Q_2)$ . Based on Assumption 9, bundled subsets are only one format that differs from the ladder quantity such as  $S_{11}$  and  $S_{12}$ . The first group of actual costs is  $C_0(S_{11}) = \{\$9.5, \$4.45\}$ , with all of the specific cost being allocated to a quantity of  $Q_1$  of 10 units; the second

group of actual costs is  $C_0(S_{13}) = \{\underbrace{\$8.49}_{1st}, \underbrace{\$4.6975}_{2nd}\}$ , with part of the specific costs (part ratio =  $\frac{Q_1}{Q_2}$ ) being allocated to 10 units and the remaining costs distributed into 40 units.

2) A PETRI NET MODEL FOR COMPLEMENTARITIES AND SUBSTITUTION

Using a CTPN, colored timed tokens represent the valuations of  $Q, Q_1, Q_2,$  and  $Q^*$  and real-time data input and output in the Petri net. Transition nodes act based on the rules of comparison and calculation. In a complementarity and substitution Petri net, there are two logic paths conditional on  $Q_1 = Q_2$  and  $Q_1 \neq Q_2$ , leading to the final results that follow two types of subset setting and actual cost calculation. A Petri net for complementarities and substitution is shown in Fig. 4.

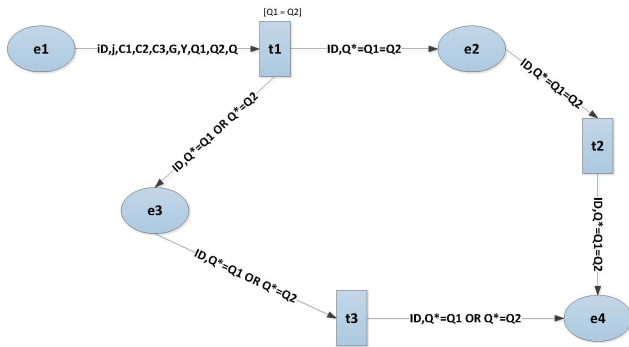


FIGURE 4. A Petri net model for C&S.

TABLE 3. Interpretation of the places in the Petri net in Fig. 4.

place	description
e1	Every object’s cost items for the bidder
e2	Ladder quantity conditional on $Q_1 = Q_2$
e3	Ladder quantity conditional on $Q_1 \neq Q_2$
e4	The combinatorial subset of every bidder

TABLE 4. Table 4: Interpretation of the transitions in the Petri net in Fig. 4.

transition	description
t1	Comparing $Q_1$ and $Q_2$
t2	Enabled if $Q_1 = Q_2$ , transfer token with $Q^* = Q_2 = Q_1$
t3	Enabled if $Q_1 \neq Q_2$ , transfer token with $Q^* = Q_1$ or $Q^* = Q_2$

TABLE 5. Colored value interpretation of the places in the Petri net in Fig. 4.

place	colored token value
e1	$1^*(i, j, C_1, C_2, C_3, G, Y, Q_1, Q_2, Q)++$
e2	$1^*(i, j, Q_1, Q_2)++$
e3	$1^*(i, j, Q_1, Q_2)++$
e4	$1^*(S_{ik})++$

Based on the Petri net model in Fig. 4 and the related interpretations shown by Tables 3, 4, 5, and 6, a detailed understanding of an entire process of complementarities and

TABLE 6. Colored value interpretation of the places in the Petri net in Fig. 4.

transition	colored token value
t1	if $Q_{i1} = Q_{i2}$ , transfer tokens to e2, else transfer tokens to e3
t2	If $Q_{i1} = Q_{i2}, Q^* = Q_2 = Q_1$ , else empty;
t3	If $Q_{i1} \neq Q_{i2}, Q^* = Q_1$ or $Q^* = Q_2$ , else empty;

substitutions can be obtained. Every bidder holding its cost items for every heterogeneous object (place e1) is invited to an auction. After one compares (transition t1 with a guard of  $Q_1 = Q_2$ ) the two ladder quantities given by its suppliers and its own estimation of truthful demand, there are two results that could be reached (places e2 and e3). Transition node t2 or t3 is enabled conditional on the presence of the token in place e2 or e3 and transfers the token to the combinatorial subset of every bidder (place e4).

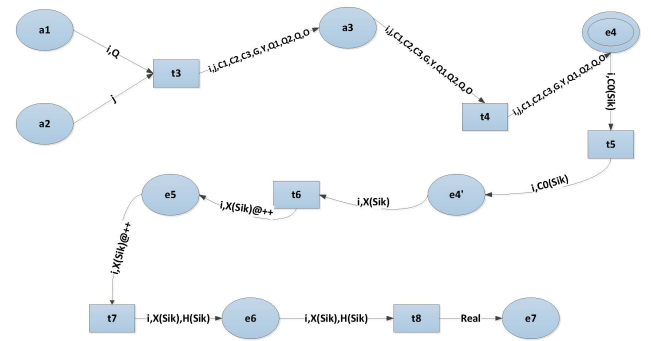


FIGURE 5. A HPN model for a FpSbCA.

D. A HIERARCHAL PETRI NET (HPN) MODEL FOR A COMBINATORIAL AUCTION

Similar to the Petri net of complementarities and substitutions, using the contribution of workflows in Petri net theory, colored timed tokens are valued with every set of bidding data, which represent the conditions or results in the bidding process, while every action changes the auction conditions, and the results employ the transition rules. Note that place e4 in Fig. 5 is the output place in Fig. 4. A FpSbCA Petri net is a hierarchal net, as shown in Fig. 5, and the related interpretations for Fig. 4 are presented in Tables 7, 8, 9, and 10.

TABLE 7. Interpretation of the places in the Petri net in Fig. 5.

place	description
a1	Multi-object resource
a2	Bidder resource
a3	Bidders’ initial intention after communication before bidding
e4	The output place e4 in Fig. 4, bidding subset with actual costs
e4'	Bid valuation for a subset
e5	Bid subset is submitted from every bidder (timed)
e6	Winner set
e7	A supply chain total cost

An auctioneer organizes an auction involving heterogeneous multi-objects (place a1) and invites several bidders (place a2). After the bidders match (transition t3) their private information, the raw material costs, the R&D fees



TABLE 8. Interpretation of the transitions in Fig. 5.

transition	description
$t3$	Bidding invitation process
$t4$	Discriminate analysis
$t5$	Bidding process
$t6$	Bidding close process
$t7$	Bidding evaluation process
$t8$	Calculating algorithm

TABLE 9. Colored value interpretation of the places in Fig. 5.

place	colored token value
$a1$	$1'(j, Q)++$
$a2$	$1'(i) ++$
$a3$	$1'(i, j, C_1, C_2, C_3, G, Y, Q_1, Q_2, Q, O)++$
$e4$	$1'(C_0(S_{ik}))++$
$e4'$	$1'(X(S_{ik}))++$
$e5$	$1'(i, X(S_{ik})) @++$
$e6$	$1'(i, H(S_{ik})) ++$
$e7$	Real

TABLE 10. Colored and timed interpretation of the transitions in Fig. 5.

transition	transition description
$t3$	Matching private information by bidders
$t4$	Equation1 execution
$t5$	Bidder's bid decision-making
$t6$	Deadline of bidding close process (timed)
$t7$	Auctioneer's winner decision-making
$t8$	Calculate total cost of the supply chain

and market costs, special costs, the yields and the fixed costs, the ladder quantities of raw material supported by the bidders' vendors and the estimated quantities of truthful orders by bidders can be reached (place  $a3$ ). With the comparison (transition  $t4$ ) of the ladder quantities between  $Q_1$  and  $Q_2$ , the output of Fig. 4 (place  $e4$ ) is given. Conditional on these bidding data, the bidders make (transition  $t5$ ) a bid decision (place  $e4'$ ). Furthermore, when the bidding close rule is reached (transition  $t6$ ), the bid set (place  $e5$ ) is submitted on time. After the auctioneer decides the winner (transition  $t7$ ), the winner set (place  $e6$ ) can be obtained, and the supply chain's total costs (place  $e7$ ) can be calculated (transition  $t8$ ).

### 1) BIDDER'S BID DECISION-MAKING

Based on Assumption 2, the objective of every bidder is profit maximization. From the bidder's perspective, the actual cost of every object is equal to the sum of the raw material cost and the allocation of the other costs based on the ladder quantity of package bid subset ( $Q^*$ ). A bidder faces two cases. The first case is that the ladder quantity of the package bid subset is not less than the estimated quantity of actual demand ( $Q^* \geq Q_2$ ), and the second case is the converse ( $Q^* < Q_2$ ). A risk-

neutral bidder prefers to allocate the special cost for the bidding object to the smallest ladder quantity that guarantees the special cost can be withdrawn in the shortest period. However, the bidder actually does not choose the smallest quantity due to the desire to submit the winning bid, which leads the bidder to offer a competitive cost. Strictly speaking, this work makes an assumption for these two cases:

*Case I* ( $Q^* \geq Q_2$ ): Every bidder allocates its special cost ( $C_2 + C_3$ ) to a quantity of  $Q^*$ . In other words, the first ladder price is equal to the sum of the corresponding raw material cost and the allocation of the special cost and the fixed cost to  $Q^*$ . However, the second ladder price is different from the first one with no special cost allocation. Thus, the actual cost function can be represented by

$$C_0(S_{ik}) = \begin{cases} C_1 + \frac{G + C_2 + C_3}{Q^*} & \text{First ladder price} \\ C_1 + \frac{G}{Q - Q^*} & \text{Second ladder price} \end{cases}$$

*Case II* ( $Q^* < Q_2$ ): In Case II, every bidder also allocates its special cost to the quantity of  $Q^*$ , but the difference is that the special cost is allocated to two ladder quantities: the first one is  $Q_2$ , and the second one is  $(Q_2 - Q^*)$ . Thus, the actual cost in Case II can be represented as follows:

$$C_0(S_{ik}) = \begin{cases} C_1 + \frac{G + (C_2 + C_3) * \frac{Q^*}{Q_2}}{Q^*} & \text{First ladder price} \\ C_1 + \frac{G + (C_2 + C_3) * (1 - \frac{Q^*}{Q_2})}{Q - Q^*} & \text{Second ladder price} \end{cases}$$

Unit profit is equal to the difference between the bid price and the actual cost,

$$\text{which can be represented as unit profit} = (X - C_0) \quad (1)$$

According to Friedman's probability of winning [11], the function  $P(x)$  is represented as

$$P(x) = \exp[-\lambda(1 - \sum_{v=0}^b \frac{1}{v!} \left\{ \frac{ax}{C} \right\}^v e^{-ax/c})] \quad (2)$$

where  $a$  and  $b$  are constants by the probability density function of the ratio of the average bidder's bid to one bidder's cost estimate, which has a gamma distribution. The "average bidder" is found by combining all previous ratios of an opposition bid to some bidder's cost estimate and obtaining one distribution function for a definite object.

By Profit = (unit profit)\* (probability of winning) and Eqs. (1) and (2), the profit function can be represented as:

$$\Pi = \{ \exp[-\lambda(1 - \sum_{v=0}^b \frac{1}{v!} \left\{ \frac{ax}{C} \right\}^v e^{-ax/c})] \} * (X - C_0) * H \quad (3)$$

TABLE 11. Simulation results of a combinatorial auction(Q = 50).

object	1st bidder								2nd bidder								3rd bidder							
	C <sub>1-1</sub>	C <sub>1-2</sub>	C <sub>2</sub>	C <sub>3</sub>	G	Q <sub>1</sub>	Q <sub>2</sub>	Q*	C <sub>1-1</sub>	C <sub>1-2</sub>	C <sub>2</sub>	C <sub>3</sub>	G	Q <sub>1</sub>	Q <sub>2</sub>	Q*	C <sub>1-1</sub>	C <sub>1-2</sub>	C <sub>2</sub>	C <sub>3</sub>	G	Q <sub>1</sub>	Q <sub>2</sub>	Q*
a	2.3	2.0	1.6	5.0	30	10	20	10 OR 20	2.2	1.9	1.5	3.0	25	15	15	15	2.5	2.1	1.8	5.2	45	30	25	25 OR 30
b	0.7	0.5	1.6	5.0	30	10	20	10 OR 20	0.7	0.5	1.5	3.0	25	15	15	15	0.9	0.6	1.8	5.2	45	30	25	25 OR 30
c	1.5	1.2	1.6	5.0	30	10	20	10 OR 20	1.3	1.1	1.5	3.0	25	15	15	15	1.6	1.2	1.8	5.2	45	30	25	25 OR 30
(a, b, c)	4.5	3.7	4.8	15.0	30	10	20	10 OR 20	4.2	3.5	4.5	9.0	25	15	15	15	5.0	3.9	5.4	15.6	45	30	25	25 OR 30
S <sub>ij</sub>	S <sub>11</sub>		S <sub>12</sub>		S <sub>13</sub>								S <sub>21</sub>				S <sub>31</sub>				S <sub>32</sub>			
C <sub>0</sub>	(9.5,4.45)		(6.59,4.7)		(8.49,4.6975)								(6.77,4.21)				(7.64,5.92)				(7.2,6.15)			
X(S)	(11.3,5.25)		(7.79,5.5)		(10.09,5.498)								(7.97,5.01)				(9.04,6.92)				(8.6,7.35)			
Average	6.46		6.416		6.4164								5.898				7.98				8.1			
H	0		0		0								1				0				0			

Thus, this Model (Model 1) can be represented as:

$$\text{Maximize } \Pi = \{exp[-\lambda (1 - \sum_{v=0}^b \frac{1}{v!} \left\{ \frac{ax}{C} \right\}^v e^{-ax/c})]\} \\ * (X - C_0) * H$$

subject to:  $X > C_0;$   
 $X \leq O.$

The first constraint illustrates that for a risk-neutral bidder, the bid price should be larger than its actual cost, and the second constraint conveys a business rule that a bidder’s price cannot be larger than the negotiation price. Note that the decision variable X can represent the ladder prices, X<sub>1</sub>, X<sub>2</sub>, and X. These three ladder prices can be optimized step by step, which is how to first optimize the values of X<sub>1</sub>, then X<sub>2</sub> and finally X.

2) AUCTIONEER’S WINNER DETERMINATION

The precise formulation of an auctioneer’s winner decision-making depends on the objectives of the auctioneer. According to Assumption 3, in an FpSbCA, the auctioneer’s objective is cost minimization. Note that the total cost of the auctioneer can be represented as follows:

$$TC = X(S_{ik}) * H(S_{ik}) \tag{4}$$

As a result, an auctioneer’s winner determination method can be represented as:

$$\text{min } TC = X(S_{ik}) * H(S_{ik})$$

subject to:  $H(S_{ik}) = 0, 1 \quad i \in N, k = 1, 2;$   
 $X(S_{ik}) < C_0(S_{ik}), \quad i \in N, k = 1, 2;$   
 $X(S_{ik}) \leq O(S_{ik}), \quad i \in N, k = 1, 2;$

According to the definition of H, H is a binary integer. The second and the third constraints are executed in a similar manner to those in the bidder’s bid decision algorithm.

3) SIMULATION RESULTS

Given three objects and three bidders, the preference of every bidder is permitted. Thus, six preferences from all bidders appear in Table 11. Conditional on the first bidder’s Q<sub>1</sub> ≠ Q<sub>2</sub>, there are three preference parcels including S<sub>11</sub>, S<sub>12</sub> and S<sub>13</sub>, in which the first ladder quantity set is 10, 20, and 10, respectively. Take S<sub>11</sub> as an example. The actual cost within 10 units

can be calculated as 9.5 and 4.45 for 40 units. Taking advantage of the bidder’s bid decision-making algorithm, the bid for S<sub>11</sub> can be optimized as a price of 11.3 for 10 units and 5.25 for 40 units.

In Table11, we have Q = 50, S<sub>11</sub> = 10/(a, b, c); 40/(a, b, c), S<sub>12</sub> = 20/(a, b, c); 30/(a, b, c), S<sub>13</sub> = S<sub>11</sub> = 10/(a, b, c); 40/(a, b, c); S<sub>21</sub> = 15/(a, b, c); 35/(a, b, c); S<sub>31</sub> = 25/(a, b, c); 25/(a, b, c), and S<sub>32</sub> = 30/(a, b, c); 20/(a, b, c).

V. A FIRST-PRICE SEALED-BID HETEROGENEOUS MULTI-OBJECT SEQUENTIAL AUCTION (FpSbMoSA) MODEL

A. ASSUMPTIONS & NOTATION

The assumptions of a sequential auction model are similar to those in a CA (Section 4.1) excluding items 8, 9, 10, and 13. With respect to assumptions, a sequential auction has several similarities with a CA that make the two formats comparable. Compatible with the notation of a CA (Section 4.2), the only difference between them is that every set for a sequential auction simply represents the valuation of individual object.

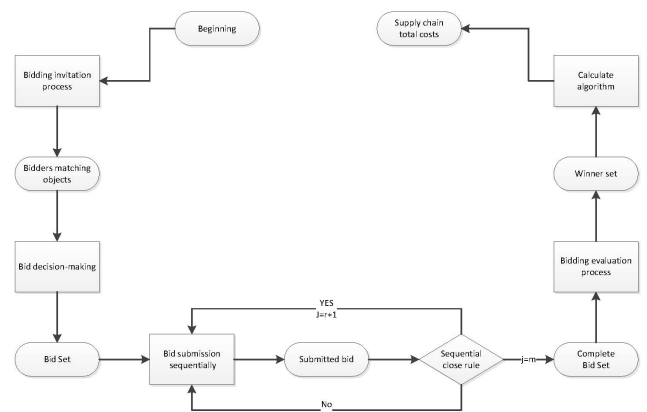


FIGURE 6. A FpSbMoSA process.

B. AUCTION PROCESS DESCRIPTION

To illustrate the entire process of a first-price sealed-bid multi-object sequential auction, a flowchart is visualized in Fig. 6.

In Fig. 6, a bidder sequentially submits m heterogeneous objects satisfying the sequential bidding close rule. The variable j represents the number of submitted bid objects, and the variable r is the present sequential number of the submitting

object. For example, if a bidder submits the bid of an object with label 1 in a heterogeneous multi-object sequential auction involving three objects, the values of  $m$ ,  $j$  and  $r$  are 3, 1 and 1, respectively.

On the condition of successful submission of an object with label 1, the next object submission is required, denoted by  $j = r + 1$ . In other words, a bidder should submit the bid of an object with label 2 and operate in the same manner until the last object is submitted.

Fig. 6 shows the entire process logic of a first-price sealed-bid multi-object sequential auction, and the additional interpretation is shown below:

*Step 1:* Same as Step 1 in Section IV.C.

*Step 2:* With the auction information mentioned above, bidders sequentially submit their sealed bids before the auction deadlines. A sequential submission is permitted until all of the objects have been completed. This is a bidding process. During this process, bidders should optimize their bids using a bid decision-making algorithm.

*Step 3:* Bid results are private information until the auction deadline is reached. The auctioneer can obtain every bidder's bid in an open and fair environment. This is a bidding close process.

*Step 4:* The auctioneer determines the winner set of every object. This is a bidding evaluation process. During the process, the auctioneer decides the winner by employing the winner decision-making algorithm.

*Step 5:* To compare the supply chain coordination between sequential auctions and CAs, a calculation for a two-stage supply chain is given.

The main difference between a first-price sealed-bid heterogeneous multi-object sequential auction and a CA is the method of bid submission, by which a series of bidding decisions are changed.

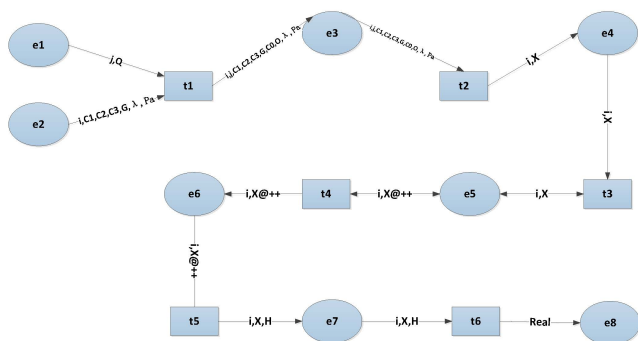


FIGURE 7. A Petri model for a FpSbMoSA.

**C. A PETRI NET MODEL FOR A FIRST-PRICE SEALED-BID HETEROGENEOUS MULTI-OBJECT SEQUENTIAL AUCTION**

A Petri net model for a first-price sealed-bid heterogeneous multi-object sequential auction can be seen in Fig. 7, and its related interpretation is listed in Tables 12, 13, 14, and 15.

Based on the bidding process introduction of a first-price sealed-bid heterogeneous multi-object sequential auction previously mentioned, a combinatorial bid is not permitted

TABLE 12. Interpretation of the places in the Petri net in Fig. 7.

place	description
e1	Multi-object resource
e2	Bidders' resource
e3	Bidders' intention after communication before bidding
e4	Bid set of every bidder
e5	Sequential bid set
e6	Complete bids
e7	Winner set
e8	Supply chain total cost

TABLE 13. Interpretation of the transitions in the Petri net in Fig. 7.

transition	description
t1	Bidding invitation process
t2	Bid decision-making
t3	Sequential bid submission
t4	Sequential bidding close process
t5	Bidding evaluation process
t6	Calculating algorithm

TABLE 14. Colored value interpretation of the places in the Petri net in Fig. 7

place	colored token value
e1	$1'(j, Q)++$
e2	$1'(i, C_1, C_2, C_3, G, \lambda, Pa)++$
e3	$1'(i, j, C_1, C_2, C_3, G, C_0, O, \lambda, Pa)++$
e4	$1'(i, X)$
e5	$1'(i, X)@++(\text{timed})$
e6	$1'(i, X)@++(\text{timed})$
e7	$1'(X_{ij}, H_{ij})$
e8	$1'(\text{Real})$

TABLE 15. Colored value interpretation of the transitions in the Petri net in Fig. 7.

transition	transition description
t1	Matching private information by bidders
t2	Bid decision-making
t3	"j" is transmitted & next number "j+1" is received (timed)
t4	"j" is received & next number "j+1" is requested (timed)
t5	Winner decision-making
t6	Calculate total costs of the supply chain

in a sequential auction. In fact, the colored timed token and the transition node are basically used in the same manner in both a heterogeneous multi-object sequential auction and a CA. In the Petri net model for a first-price sealed-bid heterogeneous multi-object sequential auction, the transition node acting as the discriminate analysis of complementarities and substitution is canceled, and its related input and output place are correspondingly deleted. As a consequence, the concrete action conducted by transition t3 is sequential, and the colored timed token value in the output place is changed.

1) BIDDER'S BID DECISION-MAKING

Based on Assumption 2, as a risk-neutral bidder, its objective is profit maximization. First, the actual cost of every object can be calculated by Eq. (5), which illustrates that the actual cost is equal to the sum of the raw material cost and the other special cost allocation. The profit of every object can be represented by Eq. (6), which implies that the profit ( $\Pi$ ) is equal to the product of the probability of winning ( $P(X_{ij})$ ) and the difference between the bid price and actual cost ( $X_{ij} - C_{0ij}$ ). We have

$$C_{0ij} = C_{1ij} + \frac{C_{2ij} + C_{3ij} + G_i}{Q} \quad (5)$$

$$\Pi = (X_{ij} - C_{0ij})P(X_{ij}) \quad (6)$$

where  $P(X_{ij})$ 's valuation is the same as that mentioned in Section 4.4.1.

Thus, the model can be represented as

$$\begin{aligned} &\text{maximize } \Pi = \{ \exp[-\lambda (1 - \sum_{v=0}^b \frac{1}{v!} \left\{ \frac{ax}{C} \right\}^v e^{-ax/c})] \} \\ &\quad * (X_{ij} - C_{0ij}) \\ &\text{subject to: } x_{ij} > C_{0ij}; \\ &\quad H_{ij} = 0, 1; \\ &\quad X_{ij} \leq O_{ij} \end{aligned}$$

Note that the interpretation of the constraints above is the same as in Section IV.D.2).

2) AUCTIONEER'S WINNER DECISION-MAKING

According to Step 3, every bidder's bid is known by the auctioneer; thus, the value of  $X_{ij}$  is sequentially obtained, and the winning share labeled  $H$  is a decision variable. The buyer's total cost, denoted by  $TC$ , is equal to the sum-product of the bid price and the probability of winning, and the function can be represented as Eq. (7):

$$TC = \sum_{j=1}^m \sum_{i=1}^n X_{ij} H_{ij} \quad (7)$$

Hence, the auctioneer's winner decision-making method can be represented as follows based on the auctioneer's objective of cost minimization.

$$\begin{aligned} &\text{minimize } TC = \sum_{j=1}^m \sum_{i=1}^n X_{ij} H_{ij} \\ &\text{subject to } x_{ij} > C_{0ij} \\ &\quad h_{ij} = 0, 1 \quad \forall i \in n, j \in m; \\ &\quad \sum_{i=1}^n h_j = 1 \quad \forall i \in n, j \in m; \\ &\quad X_{ij} \leq O_{ij} \end{aligned}$$

Note that the interpretation of the constraints above, excluding the third constraint, is the same as that in Section IV.D.2). The third constraint ( $\sum_{i=1}^n h_j = 1 \quad \forall i \in n, j \in m;$ ) discloses that only one bidder can win an object.

D. SIMULATION RESULTS

On the condition of three bidders, first, second, third, and three heterogeneous objects,  $a, b$  and  $c$ , a bidder is permitted to sequentially submit its bid without any combination. As is the case in a first-price heterogeneous multi-object CA, every bidder obtains a ladder quantity from its vendor (the same parameter value as in Table 11). In other words, two different raw material costs are given to the bidder based on the ladder quantity. The actual raw material cost  $C_1$  can be calculated by the weighted average cost. Given the other special cost, the actual cost can be reached, and by optimizing the bid price, the winning share can be disclosed. Taking the second bidder and object  $b$  as an example,  $Q_1$  for the second bidder for object  $b$  is equal to 15, and as a result, the weighted average can be reached. The second bidder obtains the actual cost equal to \$1.15, which optimizes the bid price at \$1.35 and finally obtains the winning share of 1, with the corresponding profit equaling \$0.2 per unit.

TABLE 16. Simulation results(Q = 50) of the Petri net in Fig. 7.

1st Bidder										
object	$C_{1-1}$	$C_{1-2}$	$C_2$	$C_3$	$G$	$Q_1$	$C_0$	$X$	$H$	profit
$a$	2.3	2.0	1.6	5.0	30	10	2.792	3.19	0	0.32
$b$	0.7	0.5	1.6	5.0	30	10	1.272	1.47	0	0.15
$c$	1.5	1.2	1.6	5.0	30	10	1.992	2.19	0	0.16
2nd Bidder										
object	$C_{1-1}$	$C_{1-2}$	$C_2$	$C_3$	$G$	$Q_1$	$C_0$	$X$	$H$	profit
$a$	2.2	1.9	1.5	3.0	25	15	2.580	2.98	1	0.37
$b$	0.7	0.5	1.5	3.0	25	15	1.150	1.35	1	0.20
$c$	1.3	1.1	1.5	3.0	25	15	1.750	1.95	1	0.20
3rd bidder										
object	$C_{1-1}$	$C_{1-2}$	$C_2$	$C_3$	$G$	$Q_1$	$C_0$	$X$	$H$	profit
$a$	2.5	2.1	1.8	5.2	45	30	3.380	3.98	0	0.56
$b$	0.9	0.6	1.8	5.2	45	30	1.820	2.02	0	0.19
$c$	1.6	1.2	1.8	5.2	45	30	2.480	2.88	0	0.33

In Table 16, we obtain several simulation results, and the corresponding doubts lead to further analysis:

(1) The same winner has been selected in a first-price and sealed-bid heterogeneous multi-object sequential auction in Table 16 and CA in Table 11. However, our first question is whether the type of auction mechanism affects who the winner will be.

(2) The winner price in the CA is generally lower than that in the sequential auction. The question is whether the type of auction mechanism affects the value of the winner's price.

(3) Based on the answers of the two questions above, can we find a type of auction mechanism that benefits supply chain coordination, and if so, which one is better?

VI. COMPARISON OF THE TWO TYPES OF AUCTION

With several simulation results and the three questions at the end of Section 5.4, this work compares the two auction mechanism types. The performance for comparison is the



total supply chain cost based on supply chain coordination theory [21]. The total cost of a supply chain can be defined as the sum of the auctioneer’s purchasing cost, the auctioneer’s object management cost, and the bidder’s actual cost. The auctioneer’s purchasing cost is equal to the sum of the winner’s bid of every object, while the amount of the auctioneer’s object management cost is the sum of all individual object’s management costs or bundled management costs. Accordingly, a hierarchical Petri net model can be made to compare these two different mechanisms in Fig. 8.

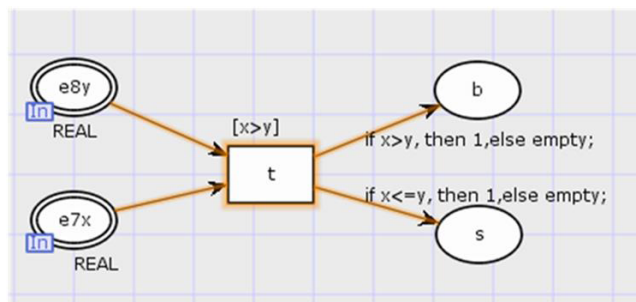


FIGURE 8. HPN for comparing two mechanisms.

Note that place  $e8y$  is the ending place of Fig. 7 and place  $e7x$  is the ending place of Fig. 5. Transition  $t$  has a guard of  $[x > y]$ . If  $x > y$ , transition  $t$  fires a token to place  $b$ . If  $x \leq y$ , transition  $t$  fires a token to place  $s$ . The aim of this model is to find a relatively better auction mechanism for supply chain coordination and provide the answer to the questions at the end of Section 5.4.

A CPN tool [62] for timed and colored Petri net models is employed for the modeling, analysis, control, and performance evaluation of a FpSbCA and a sequential auction. With 1000 repeated simulation results, this study answers the three questions appearing at the end of Section V.D.

TABLE 17. An Example of a comparison.

object	Q	sequential auction	combinatorial auction
a	30	2.98	
b	30	1.35	
c	30	1.95	
Total:		6.28	5.898

In Table 17, we find that the choice of an auction mechanism leads to different bidding results that manifest in different bid prices and different winners. The winner’s bid in a sequential auction is “\$2.98, \$1.35 and \$1.95” for object a, b and c, respectively with total value of “\$6.28”, while in a combinatorial auction, the winner’s bid is “\$5.898” for the object package of a&b&c. The procurement price of the sequential auction is 6.1% higher than that of the combinatorial one. In a FpSbCA, a winner definitely submits the lowest bundled subset bid price; however, not every individual object’s bid

in the winner’s bundled subset is definitely the lowest one. A bidder usually wins the CA by submitting the lowest bid for the principal bidding object that captures the majority of the bidding money. However, in a first-price sealed-bid heterogeneous multi-object sequential auction, an individual bidding object’s winner is definitely the lowest bid owner.

Regarding the third question, a FpSbCA generally dominates a sequential model in terms of supply chain coordination performance. If there are multitudinous bidding objects in an auction with higher management costs, a CA usually achieves better supply chain coordination with a simpler supply chain structure. Conversely, if a few bidding objects are involved in an auction with a lower management cost, performance in supply chain coordination usually depends on the difference in the winner’s bid price between a CA and a sequential auction. All these findings in the simulation results demonstrate that a CA is not a better choice if a few bidding objects are involved with a lower management cost and no more cost-down space for the object. The CA is more suitable for an auction with multitudinous bidding objects, more cost-down space for the bidding object, and a complex supply chain structure to be simplified.

VII. CONCLUSIONS

This work is the first attempt to illustrate every process using colored timed tokens as bidding data and transition rules embedded in every sub-process of bidding algorithms. It investigates two types of auction mechanisms, namely first-price sealed-bid heterogeneous multi-object combinatorial and sequential auction mechanisms based on Petri nets, and compares the supply chain coordination performance based on the two models. A large number of simulation results demonstrate that a combinatorial auction dominates a sequential auction in supply chain coordination based on the given assumptions.

In this study, however, the combinatorial auction Petri net depends on a case-based context algorithm, which may not be suitable for every physical multi-object combinatorial auction, much less so for abstract multi-objects. Hence, it is important to focus on the application of auction Petri nets in abstract multi-objects in the future.

Our future work will focus on the following direction: Modeling combinatorial auctions with new mechanism design that relaxes constraints such as item choice, ladder quantities, and ladder price. If weighted inhibitor arcs are added to a colored timed Petri net, bidding supervisors can be established based on the modeling of bad tendering such as collusion, betray the bidding, and leakage of tender. The future work will also consider the distributed auctions based on reconfigurable wireless sensor networks [76].

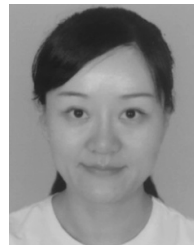
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