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# Stabilization of Uncertain Fractional-Order Complex Switched Networks via Impulsive Control and Its Application to Blind Source Separation

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**ABSTRACT** This paper investigates the impulsive stabilization of fractional-order complex switched networks with parametric uncertainty. Using a fractional-order Lyapunov method and matrix inequality techniques, the dynamical characteristics of the controlled impulsive system are well captured, and a novel impulsive stabilizing criterion is derived in terms of algebraic conditions. The stabilization criterion is dependent on system parameters and on the lengths of impulsive intervals. In addition, a simulation example is given to demonstrate the effectiveness of the newly obtained results. Finally, an application of the obtained control pulse is also presented in the blind source separation.

**INDEX TERMS** Complex switched networks, fractional-order dynamic systems, parametric uncertainty, impulsive control.

## I. INTRODUCTION

Complex networks are all around us [1]–[10]. Many natural systems can be modeled by complex networks, as examples, consider aviation networks, power networks, and biological networks. Connected nodes in a complex network may have interactions that change abruptly, leading to a network structure that is irregular and switching dynamically. The structure of such a network is described by a switching topology. For an example, consider community ecology: by its interactions within the community, an individual changes the links between itself and bodies with which it has contact, in real time. This type of network can be modeled as a complex switched network. Other examples of complex switched networks in the real world, include smart grid, bird flock and virus dynamics [1], [3], [7], [9], [10]. Many researchers have become interested in the network evolution and dynamic complexity of complex switched networks [2], [4], [5], [8]. The study of the structure and dynamics in complex switched networks is a rapidly growing field of study, that is challenging because of the complex wiring topology.

To minimize error when constructing these complicated systems, an appropriate mathematical operator must be selected, such as a fractional-order operator. Fractional dynamics is an important research field in nonlocal constitutive systems [11]–[17]. In practice, interconnected control systems are best described by fractional-order dynamic systems. Using an approximation of the fractional derivative, time-domain implementation of analysis and synthesis for fractional-order dynamic systems can be addressed [15]–[17]. However, exploring the deep network structure using this analytical framework is notably difficult. The question of how to develop a framework, which is consistent with the nonstandard approximation for formulating a fractional-order operator and presents a universal viewpoint for fractional dynamics, is significant for successful system construction.

In a variety of engineering disciplines, parametric uncertainty occurs because of incomplete knowledge of mathematical models such as empirical quantities, constitutive laws, etc [11]. For example, biochemical reactions are often modeled by differential/algebraic equations with parametric uncertainty. When analyzing these systems, it is often desirable to assess the robustness of system performance against uncertainty. Even though, as the rapid development of automation technologies, the problem of designing the feedback controller that provides good dynamic performance and robust property for an uncertain nonlinear system is NP-hard in general. The issue of reliable control in the uncertain nonlinear system is still in the early stage and many crucial problems remain to be further solved.

Impulsive control arises naturally in a wide range of applications, such as shaping circuits, high-power laser facility, digital X-ray radiography and accelerator physics. Along with the development of applications, impulsive control has become a powerful tool for complex nonlinear systems over the past several years [18]–[25]. Viewed from the perspective of cybernetics, the impulses are essentially samples of the state variables of the controlled systems at discrete moments. A basic principle for impulsive control is to stabilize a given plant by utilizing only the sampling impulses at discrete moments [18], [22], [25]. Hence, impulsive control can dramatically minimize bandwidth and lessen communication costs.

This paper is concerned with the impulsive stabilization problem of fractional-order complex switched networks with parametric uncertainty. A new framework is introduced, that combines a fractional-order Lyapunov method and fractional-order convergence principle. The newly established framework is consistent with the nonstandard approximation for formulating a fractional-order operator but presents a general viewpoint for fractional dynamics. A constructive procedure for systematic design of the impulsive control is further proposed. One favorable feature of this approach is that the scheme can effectively avoid the difficult problem of solving an impulsive controller. In addition, a concrete application is discussed to show the applicability and quality of the obtained control pulse in the blind source separation.

The remaining section of this paper is arranged below. Section II presents the model description and preliminaries. Impulsive stabilization for the controlled fractional-order complex switched networks is considered in Section III. An illustrative example is formulated in Section IV, the obtained results to be extended into the field of blind source separation are stated in Section V. Finally, Section VI presents the paper's conclusions.

## II. MODEL DESCRIPTION AND PRELIMINARIES

We begin by recalling the definitions of the Caputo derivative and the Mittag-Leffler function.

The Caputo derivative ${}^{C}D_{t_0}^{\alpha}(\cdot)$ of order $\alpha > 0$ of a function $\mathscr{F}(t) \in \mathcal{C}^{n+1}([t_0, +\infty), \Re)$ is defined as

$$ {}^{C}D_{t_0}^{\alpha}\mathscr{F}(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}\frac{\mathscr{F}^{(n)}(s)}{(t-s)^{\alpha-n+1}}\mathrm{d}s, $$

where $t \ge t_0$, $n-1 < \alpha < n$, $n$ is a positive integer, $\Gamma(\cdot)$ is Gamma function.

The Mittag-Leffler function with related one parameter $E_\alpha(\cdot)$ is described as

$$ E_\alpha(z) = \sum_{k=0}^{+\infty}\frac{z^k}{\Gamma(k\alpha+1)}, $$

where $\alpha > 0$, $z$ is a complex number, $\Gamma(\cdot)$ is Gamma function.

Consider a class of fractional-order complex switched networks with parametric uncertainty consisting of $N$ nonidentical coupled nodes

$$ \begin{aligned} {}^{C}D_{t_0}^{\alpha}x_i(t) = \sum_{s=1}^{+\infty}\Bigg[\Big(A_{i\rho} + \Delta A_{i\rho}(t)\Big)x_i(t) + f_{i\rho}(t, x_i(t)) \\ + \sum_{j=1}^{N} b_{ij}^{\rho}(t)\Upsilon(t)x_j(t)\Bigg]\ell_s(t), \\ i = 1, 2, \cdots, N, \quad t \ge t_0 \ge 0, \end{aligned} \tag{1} $$

where the fractional order is $0 < \alpha < 1$, $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in \Re^n$ is the state vector of node $i$, $A_{i\rho}$ is a constant $n \times n$ matrix, $\Delta A_{i\rho}(t)$ is the norm-bounded parametric uncertainty, the nonlinear vector-valued function $f_{i\rho}$ satisfies $f_{i\rho}(t, 0) = 0$, $\mathscr{B}_\rho(t) = \left(b_{ij}^{\rho}(t)\right)_{N\times N}$ is the coupled configuration matrix, when there is a connection between the $i$th node and the $j$th node, $i \neq j$, $b_{ij}^{\rho}(t) \neq 0$, otherwise, $b_{ij}^{\rho}(t) = 0$, and the diagonal element $b_{ii}^{\rho}(t) = -\sum_{j=1, j\neq i}^{N} b_{ij}^{\rho}(t)$, $i = 1, 2, \cdots, N$, $\Upsilon(t) = (r_{ij}(t))_{n\times n}$ is the inner coupled matrix, when two coupled nodes are linked through its $i$th state and $j$th state, $r_{ij}(t) \neq 0$, if not, $r_{ij}(t) = 0$, $i, j \in \{1, 2, \cdots, n\}$, $\ell_s(t)$ is the staircase function, $\ell_s(t) = 1$ for $T_s < t \le T_{s+1}$ with discontinuous switched instants $T_1 < T_2 < \cdots < T_s < \cdots$, and $\lim_{s\to+\infty} T_s = +\infty$, where $T_1 > t_0$, $s \in \{1, 2, \cdots\}$, otherwise, $\ell_s(t) = 0$, the switched signal $\rho \triangleq \rho(t) : [t_0, +\infty) \to \{1, 2, \cdots, m\}$, and $\rho(t) = \varsigma \in \{1, 2, \cdots, m\}$ for $T_s < t \le T_{s+1}$.

*Remark 1:* System (1) possesses time-varying topology. If $b_{ij}^{\rho}(t)$ tends to be zero from nonzero, then the link from the $i$th node to the $j$th node, $i \neq j$, may be severed. In contrast, if $b_{ij}^{\rho}(t)$ turns from zero into nonzero, then the link from the $i$th node to the $j$th node, $i \neq j$, may be added.

*Remark 2:* If $b_{ij}^{\rho}(t) \neq b_{ji}^{\rho}(t)$, $i \neq j$, then system (1) is directed. Conversely, if $b_{ij}^{\rho}(t) = b_{ji}^{\rho}(t)$, $i \neq j$, then system (1) is undirected.

For (1), we design the impulsive control as follows:

$$ u_i(t) = \sum_{k=1}^{+\infty} D_k x_i(t)\delta(t - t_k), \quad i = 1, 2, \cdots, N, \tag{2} $$

where $D_k$ is $n \times n$ gain matrix, $\delta(\cdot)$ is the Dirac Delta function, the impulsive time sequence $\{t_k\}_{k=1}^{+\infty}$ satisfies $t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to+\infty} t_k = +\infty$.

Next, we give several basic assumptions for (1).

(A1) There exists a symmetric and positive definite matrix $Q_{i\rho}$ and a continuous function $\psi_{i\rho}(t) \ge 0$, such that

$$ f_{i\rho}^T(t, x_i(t))Q_{i\rho}x_i(t) \le \psi_{i\rho}(t)x_i^T(t)Q_{i\rho}x_i(t), \tag{3} $$

for $i \in \{1, 2, \cdots, N\}$, $t \ge t_0$, $x_i(t) \in \Re^n$.

(A2)

$$ \Delta A_{i\rho}(t) = M_{i\rho}P_{i\rho}(t)N_{i\rho}, \tag{4} $$

where $M_{i\rho}$ and $N_{i\rho}$ are all known constant matrices with matched dimensionality, $P_{i\rho}(t)$ is the unknown time-varying matrix with $P_{i\rho}^T(t)P_{i\rho}(t) \le I$ ($I$ is an identity matrix with matched dimensionality).

(A3) For any two switched instants $T_s$ and $T_{s+1}$, there must be positive integers $\kappa_1$ and $\kappa_2$ to satisfy

$$t_{\kappa_1} = T_s < t_{\kappa_1+1} < \cdots < t_{\kappa_1+\kappa_2} = T_{s+1}, \tag{5}$$

where $t_{\kappa_1}, t_{\kappa_1+1}, \cdots, t_{\kappa_1+\kappa_2}$, are impulsive instants.

*Remark 3:* From (A1), it is easy to see that (3) is a more general condition for $f_{i\rho}$. Obviously, if $f_{i\rho}$ satisfies a Lipschitz condition or block condition on $x_i$, then (3) holds, but not vice versa. Assumption (A2) guarantees the uncertain term to be norm-bounded. By (A3), the switched interval $\tau_{switch} \triangleq T_{s+1} - T_s$ is an integral multiple of the impulsive interval $\tau_{impulse} \triangleq t_{k+1} - t_k$.

Under the impulsive control (2), from (1) and (A3), we obtain the controlled system

$$\begin{aligned} {}^C D_{t_0}^{\alpha} x_i(t) &= \Big(A_{i\rho} + \Delta A_{i\rho}(t)\Big)x_i(t) + f_{i\rho}(t, x_i(t)) \\ &\quad + \sum_{j=1}^{N} b_{ij}^{\rho}(t)\Upsilon(t)x_j(t), \quad t \in (t_{k-1}, t_k], \\ \triangle x_i(t) &= x_i(t_k^+) - x_i(t_k) = D_k x_i(t), \quad t = t_k, \\ x_i(t_0^+) &= x_{i0}, \quad i = 1, 2, \cdots, N, \quad k = 1, 2, \cdots. \end{aligned} \tag{6}$$

In the following, we introduce some of the most common lemmas.

*Lemma 1 [26]:* Let $\mathcal{R}$, $\mathcal{W}$ and $\mathcal{S}(t)$ be real matrices with matched dimensionality, if $\mathcal{S}^T(t)\mathcal{S}(t) \le I$ ($I$ is an identity matrix with matched dimensionality), then

$$\mathcal{R}^T\mathcal{S}^T(t)\mathcal{W}^T + \mathcal{W}\mathcal{S}(t)\mathcal{R} \le \frac{1}{\mathscr{E}}\mathcal{R}^T\mathcal{R} + \mathscr{E}\mathcal{W}\mathcal{W}^T,$$

where $\mathscr{E} > 0$ is a constant.

*Lemma 2:* If $\mathcal{R}$ and $\mathcal{W}$ are real matrices with matched dimensionality, then

$$\mathcal{R}^T\mathcal{W} + \mathcal{W}^T\mathcal{R} \le \mathscr{E}\mathcal{R}^T\mathcal{R} + \frac{1}{\mathscr{E}}\mathcal{W}^T\mathcal{W},$$

where $\mathscr{E} > 0$ is a constant.

Lemma 2 is a direct consequence of Lemma 1: in Lemma 1, let $\mathcal{S}(t) \equiv I$ ($I$ is an identity matrix with matched dimensionality), Lemma 2 can be directly derived from this.

*Lemma 3 [27]:* For positive definite matrix $\mathcal{R}$, symmetric matrix $\mathcal{W}$ and vector $\mathscr{X}$ with matched dimensionality, then

$$\lambda_{min}(\mathcal{R}^{-1}\mathcal{W})\mathscr{X}^T\mathcal{R}\mathscr{X} \le \mathscr{X}^T\mathcal{W}\mathscr{X} \le \lambda_{max}(\mathcal{R}^{-1}\mathcal{W})\mathscr{X}^T\mathcal{R}\mathscr{X},$$

where $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the minimum and maximum eigenvalues, respectively, $\mathcal{R}^{-1}$ represents the inverse matrix of $\mathcal{R}$.

*Lemma 4 [17]:* Let $\mathscr{B}(t)$ be a continuous function defined in $[t_0, +\infty)$, if there exists constant $\mathscr{H}$ such that

$${}^C D_{t_0}^{\alpha}\mathscr{B}(t) \le \mathscr{H}\mathscr{B}(t), \quad t \ge t_0 \ge 0,$$

then

$$\mathscr{B}(t) \le \mathscr{B}(t_0)E_{\alpha}(\mathscr{H}(t-t_0)^{\alpha}), \quad t \ge t_0 \ge 0,$$

where $0 < \alpha < 1$, $E_{\alpha}(\cdot)$ is one-parameter Mittag-Leffler function.

## III. THEORETICAL RESULTS

In this section, the scheme of impulsive control is provided for achieving the stabilization of (1).

For technical convenience, we denote

$$\begin{aligned} \omega(t) &= \max_{1\le i\le N, 1\le\rho\le m}\Bigg[\lambda_{max}\Bigg(Q_{i\rho}^{-1}\Bigg[Q_{i\rho}A_{i\rho} + A_{i\rho}^T Q_{i\rho} \\ &\quad + \frac{1}{\xi_1}Q_{i\rho}M_{i\rho}M_{i\rho}^T Q_{i\rho} + \xi_1 N_{i\rho}^T N_{i\rho} + 2\psi_{i\rho}(t)Q_{i\rho}\Bigg]\Bigg)\Bigg] \\ &\quad + \max_{1\le i\le N, 1\le\rho\le m}\Bigg[\sum_{j=1}^{N}\Big|b_{ij}^{\rho}(t)\Big|\frac{1}{\xi_2}\lambda_{max}\Big(\Upsilon(t)\Upsilon^T(t)Q_{i\rho}\Big)\Bigg] \\ &\quad + \max_{1\le i\le N, 1\le\rho\le m}\Bigg[\sum_{j=1}^{N}\Big|b_{ji}^{\rho}(t)\Big|\xi_2\lambda_{max}\Big(Q_{i\rho}^{-1}\Big)\Bigg], \end{aligned} \tag{7}$$

$$\sigma = \frac{\max_{1\le i\le N, 1\le\rho\le m}\Big[\lambda_{max}\Big(Q_{i\rho}\Big)\Big]}{\min_{1\le i\le N, 1\le\rho\le m}\Big[\lambda_{min}\Big(Q_{i\rho}\Big)\Big]}, \tag{8}$$

$$\gamma_k = \lambda_{max}\Big([I + D_k]^T[I + D_k]\Big) \ge 0, \quad k = 1, 2, \cdots, \tag{9}$$

where $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ denote the maximum and minimum eigenvalues, respectively, $\xi_1 > 0$ and $\xi_2 > 0$ are some constants, $I$ is an identity matrix with matched dimensionality.

*Theorem 1:* Let (A1)-(A3) hold. If there exists a constant $\hat{\omega} > 0$, such that

$$\omega(t) \le -\hat{\omega} < 0, \quad \text{for } t \ge t_0, \tag{10}$$

$$\gamma_k\sigma E_{\alpha}\Big(-\hat{\omega}(t_k - t_{k-1})^{\alpha}\Big) < 1, \quad k = 1, 2, \cdots, \tag{11}$$

then system (1) is globally asymptotically stable under the impulsive stabilizing control law (2).

*Proof:* Define the Lyapunov function

$$V(t) = \sum_{i=1}^{N} x_i^T(t)Q_{i\rho}x_i(t).$$

Evaluating the Caputo derivative of $V(t)$ along the trajectory of (6) gives

$$\begin{aligned} &{}^C D_{t_0}^{\alpha} V(t) \\ &\le \sum_{i=1}^{N}\Bigg[x_i^T(t)\Big(Q_{i\rho}A_{i\rho} + A_{i\rho}^T Q_{i\rho} \\ &\quad + Q_{i\rho}\Delta A_{i\rho}(t) + \Delta A_{i\rho}^T(t)Q_{i\rho}\Big)x_i(t) \\ &\quad + 2f_{i\rho}^T(t, x_i(t))Q_{i\rho}x_i(t) \\ &\quad + \sum_{j=1}^{N} b_{ij}^{\rho}(t)\Big(x_j^T(t)\Upsilon^T(t)Q_{i\rho}x_i(t) + x_i^T(t)Q_{i\rho}\Upsilon(t)x_j(t)\Big)\Bigg]. \end{aligned}$$

Using the assumption (A1), this becomes

$$
\begin{aligned}
&{}^{C}D_{t_0}^{\alpha}V(t)\\
&\le \sum_{i=1}^{N}\Big[x_i^T(t)\Big(Q_{i\rho}A_{i\rho}+A_{i\rho}^TQ_{i\rho}\\
&\quad +Q_{i\rho}\Delta A_{i\rho}(t)+\Delta A_{i\rho}^T(t)Q_{i\rho}\Big)x_i(t)\\
&\quad +2\psi_{i\rho}(t)x_i^T(t)Q_{i\rho}x_i(t)\\
&\quad +\sum_{j=1}^{N}b_{ij}^{\rho}(t)\Big(x_j^T(t)\Upsilon^T(t)Q_{i\rho}x_i(t)+x_i^T(t)Q_{i\rho}\Upsilon(t)x_j(t)\Big)\Big].
\end{aligned}
$$

Applying (A2), it follows that

$$
\begin{aligned}
&{}^{C}D_{t_0}^{\alpha}V(t)\\
&\le \sum_{i=1}^{N}\Big[x_i^T(t)\Big(Q_{i\rho}A_{i\rho}+A_{i\rho}^TQ_{i\rho}\\
&\quad +Q_{i\rho}M_{i\rho}P_{i\rho}(t)N_{i\rho}+N_{i\rho}^TP_{i\rho}^T(t)M_{i\rho}^TQ_{i\rho}\Big)x_i(t)\\
&\quad +2\psi_{i\rho}(t)x_i^T(t)Q_{i\rho}x_i(t)\\
&\quad +\sum_{j=1}^{N}b_{ij}^{\rho}(t)\Big(x_j^T(t)\Upsilon^T(t)Q_{i\rho}x_i(t)+x_i^T(t)Q_{i\rho}\Upsilon(t)x_j(t)\Big)\Big].
\end{aligned}
\tag{12}
$$

By Lemma 1, there exists a constant $\xi_1 > 0$ such that

$$
\begin{aligned}
&Q_{i\rho}M_{i\rho}P_{i\rho}(t)N_{i\rho}+N_{i\rho}^TP_{i\rho}^T(t)M_{i\rho}^TQ_{i\rho}\\
&\qquad \le \frac{1}{\xi_1}Q_{i\rho}M_{i\rho}M_{i\rho}^TQ_{i\rho}+\xi_1N_{i\rho}^TN_{i\rho}.
\end{aligned}
\tag{13}
$$

According to Lemma 2, there exists a constant $\xi_2 > 0$ such that

$$
\begin{aligned}
&x_j^T(t)\Upsilon^T(t)Q_{i\rho}x_i(t)+x_i^T(t)Q_{i\rho}\Upsilon(t)x_j(t)\\
&\qquad \le \frac{1}{\xi_2}x_i^T(t)Q_{i\rho}\Upsilon(t)\Upsilon^T(t)Q_{i\rho}x_i(t)+\xi_2x_j^T(t)x_j(t).
\end{aligned}
\tag{14}
$$

Substituting (13) and (14) into (12) yields

$$
\begin{aligned}
&{}^{C}D_{t_0}^{\alpha}V(t)\\
&\le \sum_{i=1}^{N}\Big[x_i^T(t)\Big(Q_{i\rho}A_{i\rho}+A_{i\rho}^TQ_{i\rho}\\
&\quad +\frac{1}{\xi_1}Q_{i\rho}M_{i\rho}M_{i\rho}^TQ_{i\rho}+\xi_1N_{i\rho}^TN_{i\rho}\Big)x_i(t)\\
&\quad +2\psi_{i\rho}(t)x_i^T(t)Q_{i\rho}x_i(t)+\sum_{j=1}^{N}\Big|b_{ij}^{\rho}(t)\Big|\\
&\quad \times\Big(\frac{1}{\xi_2}x_i^T(t)Q_{i\rho}\Upsilon(t)\Upsilon^T(t)Q_{i\rho}x_i(t)+\xi_2x_j^T(t)x_j(t)\Big)\Big]\\
&= \sum_{i=1}^{N}\Big[x_i^T(t)\Big(Q_{i\rho}A_{i\rho}+A_{i\rho}^TQ_{i\rho}\\
&\quad +\frac{1}{\xi_1}Q_{i\rho}M_{i\rho}M_{i\rho}^TQ_{i\rho}+\xi_1N_{i\rho}^TN_{i\rho}+2\psi_{i\rho}(t)Q_{i\rho}\Big)x_i(t)\\
&\quad +\sum_{j=1}^{N}\Big|b_{ij}^{\rho}(t)\Big|\frac{1}{\xi_2}\Big(x_i^T(t)Q_{i\rho}\Upsilon(t)\Upsilon^T(t)Q_{i\rho}x_i(t)\Big)\\
&\quad +\sum_{j=1}^{N}\Big|b_{ji}^{\rho}(t)\Big|\xi_2\Big(x_i^T(t)x_i(t)\Big)\Big].
\end{aligned}
\tag{15}
$$

By Lemma 3,

$$
\begin{aligned}
&x_i^T(t)\Big(Q_{i\rho}A_{i\rho}+A_{i\rho}^TQ_{i\rho}\\
&\quad +\frac{1}{\xi_1}Q_{i\rho}M_{i\rho}M_{i\rho}^TQ_{i\rho}+\xi_1N_{i\rho}^TN_{i\rho}+2\psi_{i\rho}(t)Q_{i\rho}\Big)x_i(t)\\
&\le \lambda_{max}\Big(Q_{i\rho}^{-1}\Big[Q_{i\rho}A_{i\rho}+A_{i\rho}^TQ_{i\rho}+\frac{1}{\xi_1}Q_{i\rho}M_{i\rho}M_{i\rho}^TQ_{i\rho}\\
&\quad +\xi_1N_{i\rho}^TN_{i\rho}+2\psi_{i\rho}(t)Q_{i\rho}\Big]\Big)x_i^T(t)Q_{i\rho}x_i(t),
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&x_i^T(t)Q_{i\rho}\Upsilon(t)\Upsilon^T(t)Q_{i\rho}x_i(t)\\
&\le \lambda_{max}\Big(Q_{i\rho}^{-1}\Big[Q_{i\rho}\Upsilon(t)\Upsilon^T(t)Q_{i\rho}\Big]\Big)x_i^T(t)Q_{i\rho}x_i(t),
\end{aligned}
\tag{17}
$$

$$
x_i^T(t)x_i(t)\le \lambda_{max}\Big(Q_{i\rho}^{-1}\Big)x_i^T(t)Q_{i\rho}x_i(t).
\tag{18}
$$

Substituting (16)-(18) into (15),

$$
\begin{aligned}
&{}^{C}D_{t_0}^{\alpha}V(t)\\
&\le \sum_{i=1}^{N}\Big[\lambda_{max}\Big(Q_{i\rho}^{-1}\Big[Q_{i\rho}A_{i\rho}+A_{i\rho}^TQ_{i\rho}+\frac{1}{\xi_1}Q_{i\rho}M_{i\rho}M_{i\rho}^TQ_{i\rho}\\
&\quad +\xi_1N_{i\rho}^TN_{i\rho}+2\psi_{i\rho}(t)Q_{i\rho}\Big]\Big)x_i^T(t)Q_{i\rho}x_i(t)+\sum_{j=1}^{N}\Big|b_{ij}^{\rho}(t)\Big|\\
&\quad \times\frac{1}{\xi_2}\lambda_{max}\Big(Q_{i\rho}^{-1}\Big[Q_{i\rho}\Upsilon(t)\Upsilon^T(t)Q_{i\rho}\Big]\Big)x_i^T(t)Q_{i\rho}x_i(t)\\
&\quad +\sum_{j=1}^{N}\Big|b_{ji}^{\rho}(t)\Big|\xi_2\lambda_{max}\Big(Q_{i\rho}^{-1}\Big)x_i^T(t)Q_{i\rho}x_i(t)\Big]\\
&\le \omega(t)V(t)\\
&\le -\hat{\omega}V(t),\qquad t\in(t_{k-1},t_k].
\end{aligned}
\tag{19}
$$

Applying Lemma 4 to (19),

$$
V(t)\le V(t_{k-1}^{+})E_{\alpha}\Big(-\hat{\omega}(t-t_{k-1})^{\alpha}\Big),\qquad t\in(t_{k-1},t_k],
$$

where $E_{\alpha}(\cdot)$ is one-parameter Mittag-Leffler function.

Note that

$$
V(t)\ge \min_{1\le i\le N,1\le\rho\le m}\Big[\lambda_{min}\Big(Q_{i\rho}\Big)\Big]\sum_{i=1}^{N}x_i^T(t)x_i(t),
$$

$$
V(t_{k-1}^{+})\le \max_{1\le i\le N,1\le\rho\le m}\Big[\lambda_{max}\Big(Q_{i\rho}\Big)\Big]\sum_{i=1}^{N}x_i^T(t_{k-1}^{+})x_i(t_{k-1}^{+}),
$$

hence

$$\sum_{i=1}^{N} x_i^T(t)x_i(t)$$
$$\leq \frac{\max\limits_{1\leq i\leq N, 1\leq\rho\leq m}\left[\lambda_{max}\left(Q_{i\rho}\right)\right]}{\min\limits_{1\leq i\leq N, 1\leq\rho\leq m}\left[\lambda_{min}\left(Q_{i\rho}\right)\right]}\sum_{i=1}^{N} x_i^T(t_{k-1}^+)x_i(t_{k-1}^+)$$
$$\times E_\alpha\Big(-\hat{\omega}(t-t_{k-1})^\alpha\Big)$$
$$= \sigma\sum_{i=1}^{N} x_i^T(t_{k-1}^+)x_i(t_{k-1}^+)E_\alpha\Big(-\hat{\omega}(t-t_{k-1})^\alpha\Big),\ t\in(t_{k-1}, t_k].$$

Let

$$\mu(t) = \sum_{i=1}^{N} x_i^T(t)x_i(t), \tag{20}$$

then we have

$$\mu(t) \leq \sigma\mu(t_{k-1}^+)E_\alpha\Big(-\hat{\omega}(t-t_{k-1})^\alpha\Big), \quad t\in(t_{k-1}, t_k]. \tag{21}$$

On the other hand, when $t = t_k^+$,

$$\mu(t_k^+) = \sum_{i=1}^{N}\Big[(I+D_k)x_i(t_k)\Big]^T\Big[(I+D_k)x_i(t_k)\Big]$$
$$\leq \lambda_{max}\Big([I+D_k]^T[I+D_k]\Big)\sum_{i=1}^{N} x_i^T(t_k)x_i(t_k)$$
$$= \gamma_k\mu(t_k), \quad k = 1, 2, \cdots. \tag{22}$$

By (21) and (22),

$$\mu(t) \leq \sigma\mu(t_0^+)E_\alpha\Big(-\hat{\omega}(t-t_0)^\alpha\Big), \quad \text{for any } t\in(t_0, t_1],$$
$$\mu(t_1^+) \leq \gamma_1\mu(t_1),$$

then

$$\mu(t_1^+) \leq \gamma_1\mu(t_1) \leq \gamma_1\sigma\mu(t_0^+)E_\alpha\Big(-\hat{\omega}(t_1-t_0)^\alpha\Big).$$

Similarly,

$$\mu(t) \leq \sigma\mu(t_1^+)E_\alpha\Big(-\hat{\omega}(t-t_1)^\alpha\Big), \quad \text{for any } t\in(t_1, t_2],$$
$$\mu(t_2^+) \leq \gamma_2\mu(t_2),$$

then

$$\mu(t_2^+) \leq \gamma_2\mu(t_2) \leq \gamma_2\sigma\mu(t_1^+)E_\alpha\Big(-\hat{\omega}(t_2-t_1)^\alpha\Big)$$
$$\leq \gamma_1\gamma_2\sigma^2\mu(t_0^+)E_\alpha\Big(-\hat{\omega}(t_2-t_1)^\alpha\Big)E_\alpha\Big(-\hat{\omega}(t_1-t_0)^\alpha\Big).$$

By a similar procedure, for general $t\in(t_k, t_{k+1}]$,

$$\mu(t) \leq \gamma_1\gamma_2\cdots\gamma_k\sigma^{k+1}\mu(t_0^+)E_\alpha\Big(-\hat{\omega}(t-t_k)^\alpha\Big)$$
$$\times E_\alpha\Big(-\hat{\omega}(t_k-t_{k-1})^\alpha\Big)\cdots E_\alpha\Big(-\hat{\omega}(t_2-t_1)^\alpha\Big)$$
$$\times E_\alpha\Big(-\hat{\omega}(t_1-t_0)^\alpha\Big), \tag{23}$$

that is, for $t\in(t_k, t_{k+1}]$,

$$\sum_{i=1}^{N} x_i^T(t)x_i(t)$$
$$\leq \gamma_1\gamma_2\cdots\gamma_k\sigma^{k+1}\mu(t_0^+)E_\alpha\Big(-\hat{\omega}(t-t_k)^\alpha\Big)$$
$$\times E_\alpha\Big(-\hat{\omega}(t_k-t_{k-1})^\alpha\Big)\cdots E_\alpha\Big(-\hat{\omega}(t_2-t_1)^\alpha\Big)$$
$$\times E_\alpha\Big(-\hat{\omega}(t_1-t_0)^\alpha\Big).$$

Since $\gamma_k$ $(k = 1, 2, \cdots)$ and $\sigma$ are all bounded constants, then by (11), the trivial solution of (6) is globally asymptotically stable. This property also means that system (1) is globally asymptotically stable under the impulsive stabilizing control law (2).

*Remark 4:* Fractional-order complex switched networks (1), which are characterized by discontinuous dynamical systems, have discontinuous motions (motions that are not continuous evoked by switching effect). Our objective is to develop a unified analysis and control design framework for such a hybrid dynamical systems using control system theory and methods. We consider impulsive control law given by the controlled systems of impulsive dynamics, which extends nonimpulsive control systems to impulsive commutative systems.

*Remark 5:* The proposed impulsive-control-based method for transforming nonimpulsive control systems into impulsive commutative systems is highly effective. An important specialty of this architecture is that the hybrid properties of continuous and discontinuous phase could be efficiently integrated and analyzed, whose advantage is considerable flexibility in controlling hybrid systems in which the continuous and discontinuous processes interact. Hence, there is sufficient generality for fractional-order complex switched networks.

*Remark 6:* In the existing research on the impulsive stabilization of complex networks, almost all of the results require $\varrho(I+D_k) < 1$ or $\varrho\Big((I+D_k)^T(I+D_k)\Big) < 1$, where $\varrho(\cdot)$ represents the spectral radius of the matrix. In Theorem 1, this restriction is removed, as can be observed in (9) and (11). Therefore, for some complex networks, even if $\varrho(I+D_k) \geq 1$ or $\varrho\Big((I+D_k)^T(I+D_k)\Big) \geq 1$, the stabilization problem can be solved.

*Remark 7:* Theorem 1 is a sufficient condition under which stabilization control of (1) can be realized with the impulsive strategy (2). The impulsive stabilizing control law (2) requires the information of all state variables at impulse instants. In application, it is more practical to measure certain of the state variables, rather than all state variables. How to design less conservative impulsive control strategies is a challenging problem and an interesting direction for future research.

*Remark 8:* Processing range queries on linear matrix inequality of higher dimensionality is tough question. The curse-of-dimensionality for linear matrix inequality in control design has been widely realized. One obvious remedy to this issue is to achieve some form of dimensionality reduction. The dimensionality of matrix expression in Theorem 1 is equal to the dimensionality of system parameters matrix of (1). This ensures that, even in the increase in the dimensionality of system parameters matrix of (1), the solvability of Theorem 1 should still be available through some numerical algorithms.

In fact, the proposed stabilization scheme can be achieved by numerical programming. A computational procedure for systematic design of the impulsive control of Theorem 1 can be described as follows:

Step I: Choose the gain matrices $D_k$.

Step II: Compute $\gamma_k$ defined in (9). If $\gamma_k \geq 0$, proceed directly to Step III. If $\gamma_k < 0$, return to Step I.

Step III: Select symmetric and positive definite matrices $Q_{i\rho}$ to satisfy (A1) and count $\sigma$ given in (8).

Step IV: Choose constants $\xi_1 > 0$ and $\xi_2 > 0$, and use these to calculate $\omega(t)$ defined in (7).

Step V: Select constant $\hat{\omega} > 0$. If (10) and (11) hold, then according to Theorem 1, the impulsive control (2) can globally asymptotically stabilize (1). If No, return to Step I.

If $\Delta A_{i\rho}(t) = 0$, (1) will degrade into

$$\begin{aligned} {}^{C}D_{t_0}^{\alpha} x_i(t) = \sum_{s=1}^{+\infty} \Bigg[ & A_{i\rho} x_i(t) + f_{i\rho}(t, x_i(t)) \\ & + \sum_{j=1}^{N} b_{ij}^{\rho}(t) \Upsilon(t) x_j(t) \Bigg] \ell_s(t), \\ & i = 1, 2, \cdots, N, \quad t \geq t_0 \geq 0. \end{aligned} \tag{24}$$

Denote

$$\begin{aligned} &\widetilde{\omega}(t) \\ &= \max_{1 \leq i \leq N, 1 \leq \rho \leq m} \left[ \lambda_{max} \left( Q_{i\rho}^{-1} \left[ Q_{i\rho} A_{i\rho} + A_{i\rho}^{T} Q_{i\rho} + 2\psi_{i\rho}(t) Q_{i\rho} \right] \right) \right] \\ &\quad + \max_{1 \leq i \leq N, 1 \leq \rho \leq m} \left[ \sum_{j=1}^{N} \left| b_{ij}^{\rho}(t) \right| \frac{1}{\xi_2} \lambda_{max} \left( \Upsilon(t) \Upsilon^{T}(t) Q_{i\rho} \right) \right] \\ &\quad + \max_{1 \leq i \leq N, 1 \leq \rho \leq m} \left[ \sum_{j=1}^{N} \left| b_{ji}^{\rho}(t) \right| \xi_2 \lambda_{max} \left( Q_{i\rho}^{-1} \right) \right], \end{aligned} \tag{25}$$

where $\lambda_{max}(\cdot)$ denotes the maximum eigenvalue, $\xi_2 > 0$ is some constant.

*Corollary 1:* Let (A1) and (A3) hold. System (24) is globally asymptotically stable under the impulsive stabilizing control law (2), if there exists a constant $\hat{\omega} > 0$, such that

$$\begin{aligned} &\widetilde{\omega}(t) \leq -\hat{\omega} < 0, \quad \text{for } t \geq t_0, \\ &\gamma_k \sigma E_\alpha \Big( -\hat{\omega}(t_k - t_{k-1})^{\alpha} \Big) < 1, \quad k = 1, 2, \cdots. \end{aligned}$$

If (1) does not produce mode switching, (1) will degrade into

$$\begin{aligned} {}^{C}D_{t_0}^{\alpha} x_i(t) = & \Big( A_i + \Delta A_i(t) \Big) x_i(t) + f_i(t, x_i(t)) \\ & + \sum_{j=1}^{N} b_{ij}(t) \Upsilon(t) x_j(t), \\ & i = 1, 2, \cdots, N, \quad t \geq t_0 \geq 0. \end{aligned} \tag{26}$$

In this way, assumptions (A1) and (A2) can be rewritten as follows.

(A4) There exist a symmetric and positive definite matrix $Q_i$ and a continuous function $\psi_i(t) \geq 0$, such that

$$f_i^{T}(t, x_i(t)) Q_i x_i(t) \leq \psi_i(t) x_i^{T}(t) Q_i x_i(t),$$

for $i \in \{1, 2, \cdots, N\}$, $t \geq t_0$, $x_i(t) \in \Re^n$.

(A5)

$$\Delta A_i(t) = M_i P_i(t) N_i,$$

where $M_i$ and $N_i$ are all known constant matrices with matched dimensionality, $P_i(t)$ is the unknown time-varying matrix with $P_i^{T}(t) P_i(t) \leq I$ ($I$ is an identity matrix with matched dimensionality).

We also denote

$$\begin{aligned} \widehat{\omega}(t) = & \max_{1 \leq i \leq N} \left[ \lambda_{max} \left( Q_i^{-1} \left[ Q_i A_i + A_i^{T} Q_i + \frac{1}{\xi_1} Q_i M_i M_i^{T} Q_i \right.\right.\right. \\ & \left.\left.\left. + \xi_1 N_i^{T} N_i + 2\psi_i(t) Q_i \right] \right) \right] \\ & + \max_{1 \leq i \leq N} \left[ \sum_{j=1}^{N} \left| b_{ij}(t) \right| \frac{1}{\xi_2} \lambda_{max} \left( \Upsilon(t) \Upsilon^{T}(t) Q_i \right) \right] \\ & + \max_{1 \leq i \leq N} \left[ \sum_{j=1}^{N} \left| b_{ji}(t) \right| \xi_2 \lambda_{max} \left( Q_i^{-1} \right) \right], \end{aligned} \tag{27}$$

$$\widehat{\sigma} = \frac{\max_{1 \leq i \leq N} \left[ \lambda_{max} \left( Q_i \right) \right]}{\min_{1 \leq i \leq N} \left[ \lambda_{min} \left( Q_i \right) \right]}, \tag{28}$$

where $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ denote the maximum and minimum eigenvalues, respectively, $\xi_1 > 0$ and $\xi_2 > 0$ are some constants.

*Corollary 2:* Let (A4) and (A5) hold. System (26) is globally asymptotically stable under the impulsive stabilizing control law (2), if there exists a constant $\hat{\omega} > 0$, such that

$$\begin{aligned} &\widehat{\omega}(t) \leq -\hat{\omega} < 0, \quad \text{for } t \geq t_0, \\ &\gamma_k \widehat{\sigma} E_\alpha \Big( -\hat{\omega}(t_k - t_{k-1})^{\alpha} \Big) < 1, \quad k = 1, 2, \cdots. \end{aligned}$$

## IV. A NUMERICAL EXAMPLE

We consider a nearest-neighbor fractional-order coupled complex switched network with five nodes. Assume that the switched signal $\rho \triangleq \rho(t) : [t_0, +\infty) \rightarrow \{1, 2\}$ and the

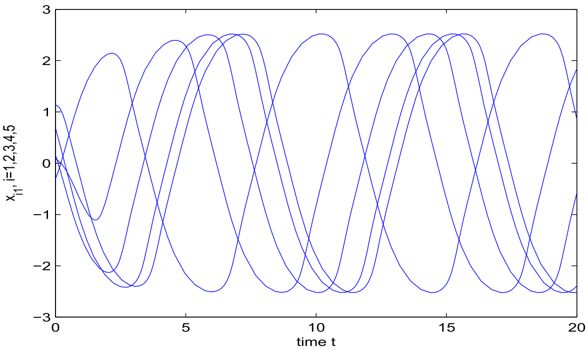


**FIGURE 1.** Time responses of states $x_{i1}$ ($i = 1, 2, \cdots, 5$) without impulsive control.

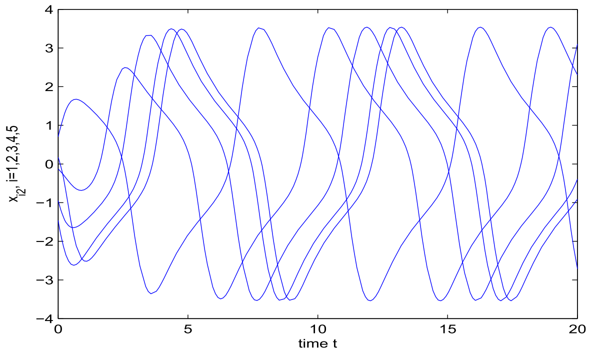


**FIGURE 2.** Time responses of states $x_{i2}$ ($i = 1, 2, \cdots, 5$) without impulsive control.

switched interval $T_{s+1} - T_s = \tau_{switch} = 0.02$. Next, the network model is described by $x_i = (x_{i1}, x_{i2}, x_{i3})^T$,

$$A_{i\rho} = \begin{bmatrix} -25a_{i\rho} & 0 & 0 \\ 0 & -29a_{i\rho} & 0 \\ 0 & 0 & -30a_{i\rho} \end{bmatrix},$$

$$\Delta A_{i\rho}(t) = \sin(a_{i\rho}t) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

$$f_{i\rho}(t, x_i(t)) = \Big(0, -x_{i1}x_{i3}, x_{i1}x_{i2}\Big)^T,$$

$$\mathscr{B}_1(t) = \Big(b^1_{ij}(t)\Big)_{5\times5} = \begin{bmatrix} -\sin(t) & 0 & 0 & 0 & \sin(t) \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix},$$

$$\mathscr{B}_2(t) = \Big(b^2_{ij}(t)\Big)_{5\times5} = 0.5 \begin{bmatrix} -\sin(t) & 0 & 0 & 0 & \sin(t) \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix},$$

$$\Upsilon(t) = I \ \ (I \text{ is an identity matrix}),$$

in addition, $\alpha = 0.5$, $a_{11} = a_{21} = a_{31} = a_{41} = a_{51} = 0.2$, $a_{12} = a_{22} = a_{32} = a_{42} = a_{52} = 0.3$.

The corresponding time response curves of states $x_{i1}$, $x_{i2}$, and $x_{i3}$ ($i = 1, 2, \cdots, 5$) without impulsive control are depicted in Figures 1-3. It is seen that the uncontrolled network is disorganized. One limitation of the disorganized nearest-neighbor complex network may be that the network nodes do not naturally lie on a metric space, thus restricting the community detection.

Obviously, we can get

$$M_{i\rho} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$
$$N_{i\rho} = [1 \ \ 1 \ \ 1],$$
$$P_{i\rho}(t) = \sin(a_{i\rho}t).$$

Next, we perform a specific calculation.

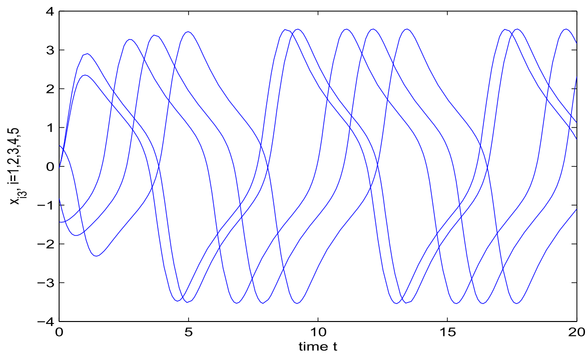


**FIGURE 3.** Time responses of states $x_{i3}$ ($i = 1, 2, \cdots, 5$) without impulsive control.

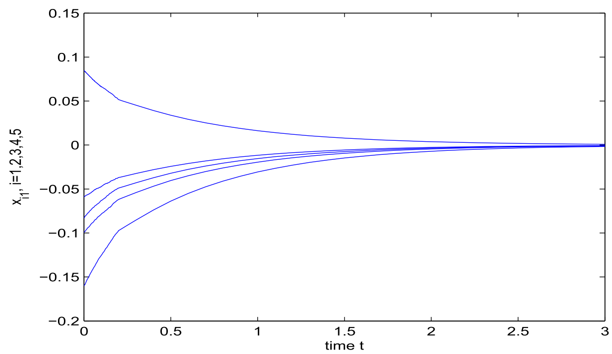


**FIGURE 4.** The states $x_{i1}$ ($i = 1, 2, \cdots, 5$) of the controlled network under the designed impulsive control law with $\mathscr{N} = 10$.

Step I: We choose the gain matrices $D_k = \text{diag}(-0.7, -0.7, -0.7)$.

Step II: We compute $\gamma_k$ as defined in (9), and obtain $\gamma_k = 0.09$.

Step III: For the symmetric and positive definite matrices $Q_{i\rho}$, we select $Q_{i\rho} = I$ ($I$ is an identity matrix) to satisfy (A1), and then determine using (8) that $\sigma = 1$.

Step IV: We choose constants $\xi_1 = 1$ and $\xi_2 = 1$, and then calculate $\omega(t) = -2.0667$ using (7).

Step V: We Select the constant $\hat{\omega} = 2 > 0$, and observe that (10) and (11) hold.

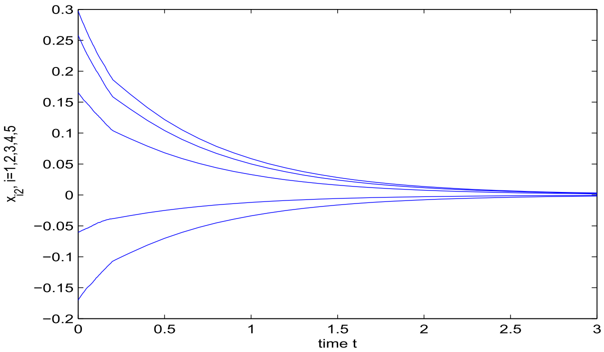


**FIGURE 5.** The states $x_{i2}$ ($i = 1, 2, \cdots, 5$) of the controlled network under the designed impulsive control law with $\mathscr{N} = 10$.

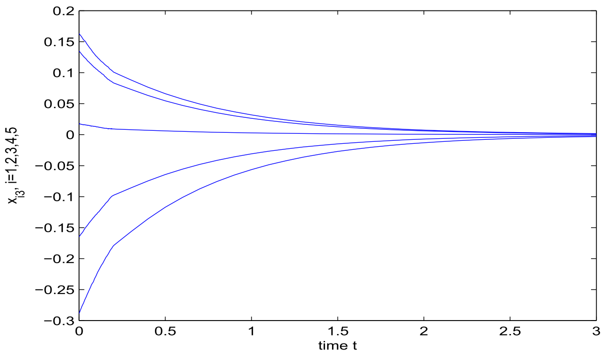


**FIGURE 6.** The states $x_{i3}$ ($i = 1, 2, \cdots, 5$) of the controlled network under the designed impulsive control law with $\mathscr{N} = 10$.

Let $t_{k+1} - t_k = \tau_{impulse}$. If there is some integer $\mathscr{N} > 0$, such that $\mathscr{N}\tau_{impulse} = \tau_{switch}$, then (A3) holds.

According to Theorem 1, the impulsive control (2) can globally asymptotically stabilize (1).

In the first simulation example below, we select $t_{k+1} - t_k = 0.002$, i.e., $\mathscr{N} = 10$. The controlled network states under the designed impulsive control law are depicted in Figures 4-6. In the second simulation example below, we select $t_{k+1} - t_k = 0.004$, i.e., $\mathscr{N} = 5$. The controlled network states under the designed impulsive control law are depicted in Figures 7-9. The simulation results indicate that the controlled network is globally asymptotically stable. This suggests that, to some extent, the proposed methods have good dynamic property and negligible vibration, and efficiently stabilize the controlled network.

## V. APPLICATION TO BLIND SOURCE SEPARATION

In this section, we formulate the obtained results to be applied in the field of blind source separation.

Source signals are "tiger", "tree", "sky", which are described in Figure 10.

After vectorization, each component in vectors for source signals is assumed to be independent of each other. Let the impulse signal in Section IV "A Numerical Example" as the unknown sources to be mixed.

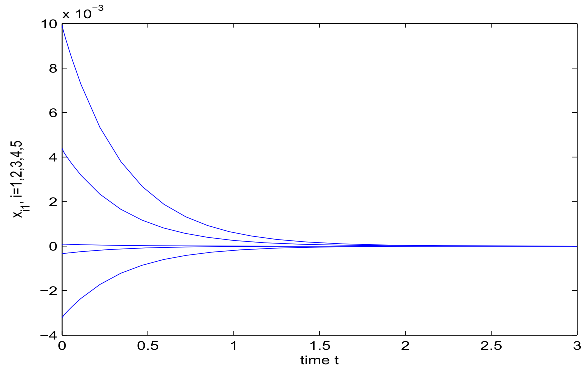


**FIGURE 7.** The states $x_{i1}$ ($i = 1, 2, \cdots, 5$) of the controlled network under the designed impulsive control law with $\mathscr{N} = 5$.

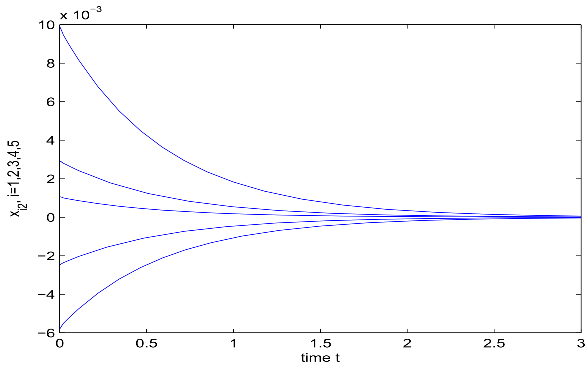


**FIGURE 8.** The states $x_{i2}$ ($i = 1, 2, \cdots, 5$) of the controlled network under the designed impulsive control law with $\mathscr{N} = 5$.

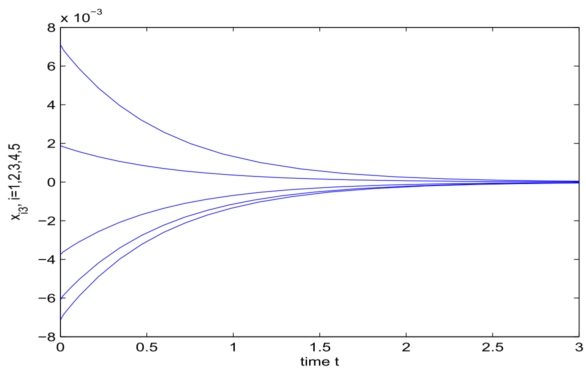


**FIGURE 9.** The states $x_{i3}$ ($i = 1, 2, \cdots, 5$) of the controlled network under the designed impulsive control law with $\mathscr{N} = 5$.

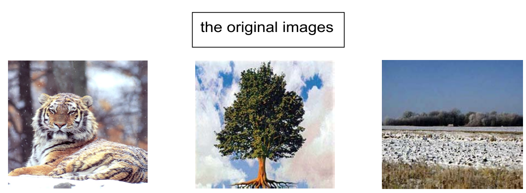


**FIGURE 10.** The original images.

Here, the aim of blind source separation is to look for a filter, such that the components of reconstructed signals are mutually independent.

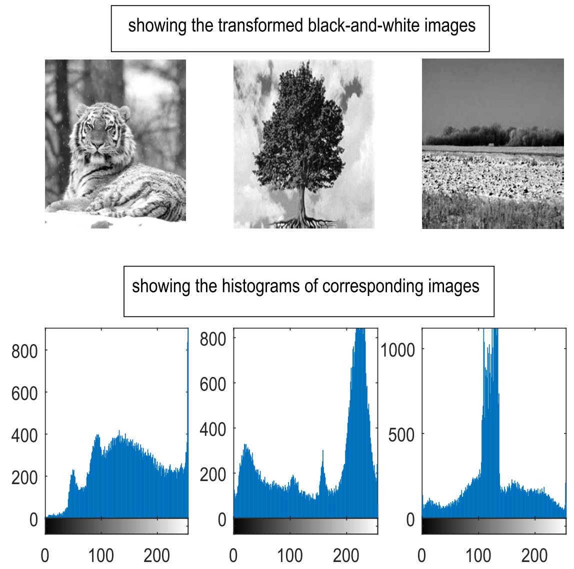


**FIGURE 11.** The transformed black-and-white images and the related histograms.

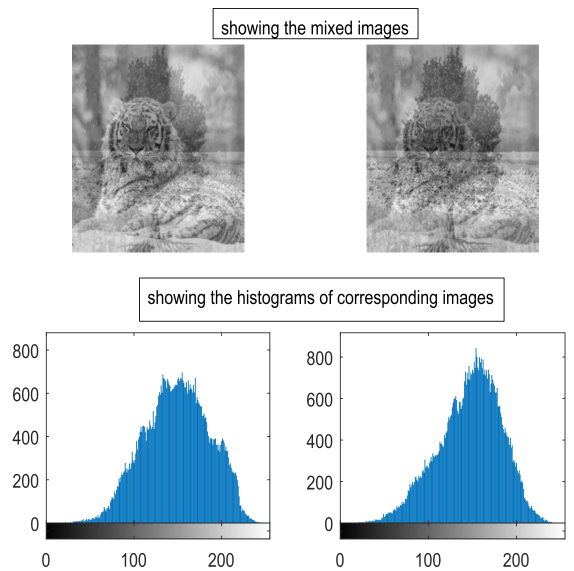


**FIGURE 12.** The mixed images effected by the impulse signal in Section IV "A Numerical Example" and the related histograms.

Applying the learning algorithm as Amari *et al.* [28], Figure 11 gives the transformed black-and-white images and the related histograms. Figure 12 shows the mixed images effected by the impulse signal in Section IV "A Numerical Example" and the related histograms.

In order to describe the characteristics of blind source separation more intuitively, the full process from the original images to the separated images is illustrated in Figure 13.

In order to analyze conveniently, besides qualitative analysis, and sometimes we also need the data of quantitative analysis. A graphical illustration of the histograms of original images and separated images can be seen in Figure 14.

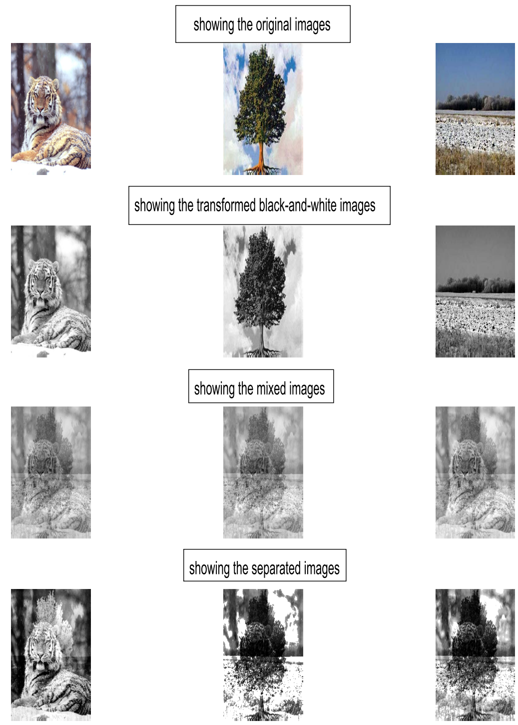


**FIGURE 13.** The full process from the original images to the separated images.

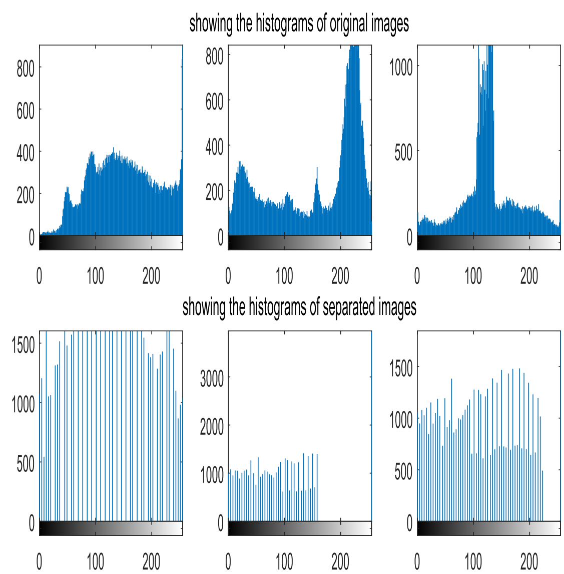


**FIGURE 14.** The histograms of original images and separated images.

## VI. CONCLUSIONS

Looked through the paper, we investigate thoroughly the impulsive stabilizing control law for a class of uncertain fractional-order complex switched networks. By virtue of fractional-order Lyapunov method and impulsive control law, a less conservative impulsive stabilizing criterion is obtained. Several tests about blind source separation using the derived control pulse are carried out to demonstrate that our control algorithm is with a high-performance level. Further investigations may aim to design the pinning impulsive strategy for fractional-order complex switched networks and the delay-dependent impulsive control for fractional-order time-delay complex switched networks.

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