IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received March 9, 2018, accepted April 26, 2018, date of publication June 11, 2018, date of current version August 7, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2844022

Empirical Geometrical Bounds on MIMO Antenna Arrays for Optimum Diversity Gain Performance: An Electromagnetic Design Approach

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ABSTRACT We propose a general design methodology for synthesizing surface or volume multi-input-multi-output (MIMO) antenna arrays with optimum cross-correlation diversity gain performance through engineering the array's geometrical shape. The design algorithm is based on approximating arbitrary antenna geometrical configurations by the arrays of infinitesimal (electrically small) dipoles and then using the recently introduced cross-correlation Green's function in order to compute far-field cross correlations without the need to explicitly measure or compute far-zone fields. After directly expressing far-field cross correlation in terms of the geometrical details of the antenna array (position and orientation), the method applies a global optimization strategy (the genetic algorithm) to find optimum positions and orientations of the MIMO antennas' elements within a given geometrical shape, resulting in the statistically best system performance. We provide extensive numerical results, including various array topologies (both fixed and conformal), with investigations of the impact of the array density, positions, and the relative orientations of the composing antenna elements on the attainable diversity gain. This paper also outlines an expansion of the proposed design methodology in order to deal with the important special case when a ground plane is present in the MIMO environment. It is found using the proposed methods that small MIMO receiver terminals can be made to fit any geometrical shape by properly controlling the position and the orientation of each element. All the resulting arrays have dimensions that are smaller than $0.35\lambda \times 0.35\lambda$ with the diversity gain of 80% or greater. It was also found that for each antenna topology, a critical number of antennas per unit area/volume exist, such that no further improvement of the diversity gain is possible. This upper bound is geometrical in nature, but it is obtained through an electromagnetic analysis, clearly demonstrating the impact of relative antenna positioning and orientations. Various 2-D and 3-D antenna array configurations, including disk, ring, spheres, and spherical layers, were investigated and their critical array densities are tabulated. Also, a practical example of conformal arrays mounted on an avionic nose was provided. It was also found that relative orientations alone can be exploited to substantially improve the performance of MIMO arrays by considering different scenarios comparing position-alone, orientation-alone, and position-and-orientation optimization processes with random arrays in terms of diversity gain performance. The method developed in this paper can be expanded to include more complex antenna types, but it is also suitable for scalable computing analysis of continuous large radiating and receiving antenna surfaces and massive MIMO. In particular, for mmWave applications, we expect that the need to optimize large arrays of tiny antennas will increase the demand for accurate and general design algorithms, such as the one proposed in this paper.

INDEX TERMS MIMO, genetic algorithm, infinitesimal dipoles.

I. INTRODUCTION

A. OVERALL VIEW AND MOTIVATIONS

MIMO (Multi-Input, Multi-Output) systems are currently enjoying a resurgence of interest at both the research and industrial levels aiming at developing new infrastructures for communication applications, such as mobile devices, 5G, near-field communications, wireless power transfer, and sensor arrays [1]–[4]. The basic idea in MIMO (see Fig. 1) is

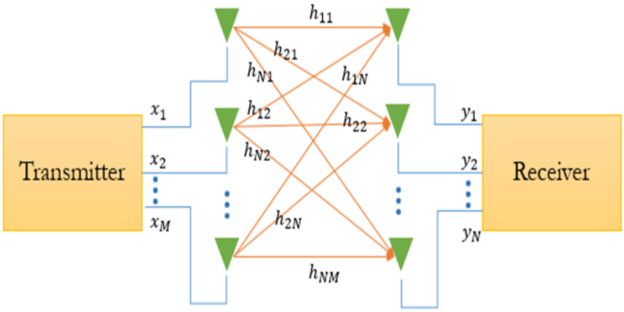

FIGURE 1. **Schematic model of a generic MIMO system. Spatial diversity results from the use of multiple inputs at both the Tx and Rx sides. If *M* and *N* antennas are deployed at the Tx and Rx terminals, respectively, then effectively *MN* independent channels could be established if the stochastic correlation among the antennas themselves, and also among the *MN* Tx-Rx propagation paths can be minimized, resulting in independent spatial links through which information can be transmitted with the same time-frequency resources, leading to improvement in spectral efficiency.**

to exploit the already available rich spatial complexity of most propagation environments in order to attain a positive improvement in the overall performance of the communication link's spectral efficiency [1]. For example, in contrast to the well-attested negative role of fading in classical communications, multiple paths are directly exploited by the designer of the MIMO link in order to increase the channel transmission capacity without burdening the bandwidth. This can happen, for example, if different users are encoded and transmitted via spatially-distinctive beams.

A generic MIMO system contains signal processing parts at the transmitter's (Tx) and receiver's (Rx) terminals, which address coding/decoding and modulation/demodulation aspects; and the electromagnetic part, comprised of antennas plus the channel separating them. Throughout the large and growing MIMO literature, there is a general tendency to separate the analysis of the two parts, which is due to the very different natures of electromagnetic and communication problems. However, it has been repeatedly observed in practice that much of the complexity of the MIMO system goes into the relation between the shape and distribution of the elements of the Tx (transmit) and Rx (receive) antenna arrays and the performance measures of the system. Therefore, the electromagnetic aspects of the system can in fact be taken as variable design parameters and hence profitably incorporated from the beginning into the MIMO development process. The major difficulty of this approach is the notorious complexity of even the minimally necessary electromagnetic analysis needed to describe the simplest MIMO arrays [7]. Indeed, typical full-wave numerical solvers, such as finite element method (FEM) and method of moment, are generally required in order to deal with even simple antennas like half-wavelength wire antennas [8], [9].

Conversely, by characterizing the antenna part of the MIMO system via carefully chosen performance measures relevant to the communication aspect of the system, it is possible to express the MIMO channel matrix H (see Fig. 2), and consequently the channel capacity, in terms of well-defined metrics [3]. Such metrics include the *far-field cross correlation* and the corresponding *diversity gain* [5], [6], which measure the degree of statistical isolation between the Rx antenna ports and are generally considered a fundamental MIMO system metric. The channel information capacity is also another important metric for determining the performance of a MIMO system [1], [2], but due to the complexity of the electromagnetic analysis of the channel propagation behavior, we focus in this paper on free-space ideal environment by considering only the far-field cross-correlation matrix of the TX/Rx antenna systems. In general, it can be shown that the MIMO channel matrix H, and consequently the channel capacity, are functions (among others) of the antenna array cross-correlation matrix R, which in turns determines the diversity gain [1]. However, most published research avoid explicit characterization of how the channel capacity varies (in a functional form) with the correlation matrix R due to the enormous complexity of the problem at the theoretical level.

Since the relationship between the antenna array's characteristics (shape, size, element type, etc) and the corresponding cross-correlation performance is essentially electromagnetic in nature, a new approach to the relevant electromagnetic phenomenon will be helpful if direct analytical solutions do not exist.

Accordingly, in this paper we build on a reexamination of the antenna far-field cross correlation in terms of the cross-correlation Green's function, recently introduced in [12], which allows avoiding working directly with radiated fields by shifting the attention instead toward the radiating/receiving elements themselves, a change of perspective that will be shown in this paper to be quite helpful for the design of optimum MIMO arrays. Moreover, it was shown in [12] that the often popular S-matrix formula for estimating far-field envelope cross-correlation coefficients is not in general valid. Indeed, many examples were given showing that computations of far-field correlations using only port measurements is not adequate. In contrast, the CGF method [12] establishes rigorously that the entire current distribution on the antenna enter into estimating the far-field cross-correlation, a fact that was exploited *computationally* in [40] and the present work in order to design optimum diversity antenna arrays.

Building on this alternative approach to antenna-antenna cross-correlation, and using a powerful global optimization method (the Genetic Algorithm or shortly GA), we employ the newly developed cross-correlation Green's (CGF) function to systematically investigate the impact of the MIMO array's shape on the proper diversity performance metric measured by far-field envelope cross-correlation. The idea is to investigate how positions, orientations, antenna density, and array typologies all interact with each other in order to produce optimum diversity gain multiple-antenna systems. We manage to obtain optimum performance with special shapes by judiciously varying the relative positions and orientations of the individual radiating elements while incorporating full-electromagnetic effects via the use of the cross-correlation Green's function.

The work presented here can also be seen as an optimization of the ''radiate/receive total surface current'' in a given space or volume by noting that an ID array can discretize *continuous* surface current distributions. For example, in massive MIMO, one may think of the entire antenna array as a single combined electrically-large antenna supporting a continuous current distribution. Through such perspective, the optimum design of such antenna system can be interpreted as finding the optimum geometrical shape of this ''big surface antenna'' corresponding to the best diversity gain performance. From the viewpoint of the communication engineer, the present work might then be described as a systematic method allowing the system engineer to modify the antennas' positions and orientations, and/or the overall shape of the Rx MIMO platform, in order to achieve the statistically optimum diversity gain performance while fully taking into account the complete electromagnetic aspects of the process. In recent years, there has been a considerable interest in antenna positioning in mmWave systems [34] and in building surface antenna arrays capable of outperforming massive MIMO [35]. The present work can provide effective algorithmic tools allowing such investigations to be scaled up at the quantitative level. Even though we consider only small number of antennas in the numerical examples to follow, the approach described here based on the CGF is fully scalable and can be applied to various applications involving very-large arrays of tiny antennas, IoT, sensor arrays, antenna positioning, intelligent electromagnetic agents [33], and intelligent antenna surfaces [36].

B. REVIEW OF PREVIOUS WORK

The vast majority of published research papers on optimizing MIMO arrays or, more generally, multiple- antenna diversity systems, assume scalar radiating/receiving antennas where the array element can be adequately modeled by a spherical wave emanating from the source while a plane wave (polarization often ignored) interacts with the scalar recive point. This point-to-point antenna communication model fits nicely with the famialir formalisms of communication theory and signal processing, and hence its extreme popularity. However, it has already been pointed out from the early days of the field that electromagnetic factors like position and orientation of each element play a major role [5]. This has sparked a considerable research at the experimental level that focused on specific antenna types, like dipoles, patch antennas, dielectric resonators, where effects of polarization or orientation are sometimes investigated for those particular devices, though usually for arrays with small number of elements [31], [32], [34], [37]. It appears that a much narrower span of the overall MIMO array research did consider the problem at a somehow more general level, and even less from the electromagnetic viewpoint. In what follows, we present a very brief review of some of these studies that could be reconsidered from the perspective of the present paper. We don't claim that our survey is exhaustive.

A direct approach to antenna array design based on electromagnetic theory was developed in [37] and expanded in [38], where the authors attempted to find the optimum current distribution on a given geometrical platform, e.g., apertures or dipoles, producing the maximum diversity gain. This approach, however, is based on working directly with the radiated fields. Since optimal field modes corresponding to maximal diversity gain are known to be mutually orthogonal, the antenna design problem becomes equivalent to searching for those orthogonal field modes corresponding to a given radiating aperture of surface layout. This was formulated as an optimization problems which was solved using approximate methods. While this approach is fully electromagnetic, it also relies on the complete 3D far-fields produced by the arrays, which considerably complicates the optimization process. Moreover, the optimum (orthogonal) radiation modes must be found based on a specific antenna geometry shape (since the vector current basis functions, which determine the antenna shape need to be known in advance before solving for the unknowns, which are those bases' coefficients.) Although Quist and Jensen [37] did attempt to apply the GA algorithm to find optimal dipole locations, the results were still based on prior computation of far-field modes. Additional problems are related to the challenge of finding currents in non-overlapping regions, which was partially resolved in [38]. Overall, that work still did not address in a systematic way how the antenna positions and orientations within a given array topology influence the diversity gain due to the extreme difficulty of dealing with the cross-correlation matrix optimization while working with the full radiated far-field distributions.

The use of infinitesimal dipoles to model antenna-antenna cross-correlation appears to have been attempted in [16]. There, The weed optimization method was used to create a model for PIFA antenna using a number of infinitesimal dipole through the general IDM approach developed before in [28]. Afterward, the S-matrix-based formula for computing the envelope cross-correlation of two PIFA antennas was employed. The idea was to use the weed optimization algorithm to find best position and orientation of the antennas resulting in maximal diversity gain. However, the dipole model was used in [16] only to evaluate the S-matrix of the array using an interaction formula derived in [28]. There are several limitations to this approach. First, the IDM-formula for computing the S-matrix derived in [28] is valid only for weak mutual coupling and does not work for generic electromagnetic coupling scenarios. This difficulty was pointed out and solved within the context of IDM in [29]. Second, the S-matrix formula for estimating envelope cross-correlation is not in general valid. It was already pointed out in several examples in [12] that cross-correlation depends on the current on the entire antenna surface, not only the ports. Therefore, optimizing antenna arrays based on S-matrix formulas is not expected to scale up with generic arrays with variable degrees of mutual coupling. Finally, the work in [16] is difficult to extend to arbitrary antenna arrays with arbitrary typologies because the method there depends essentially on first using the IDM approach to model every specific antennas before starting the optimization process.

Finally, we mention the work presented in [39] where a GA-based method was developed to optimize the positions and orientations of dipole antennas in MIMO communication environments. The authors there rightly noted that the MIMO capacity can be considerably improved by properly positioning and orienting each element in the array. However, while their results agree in general with ours in the sense that overall improvement can be attained with manipulating geometric array data, the scope of the work remains narrower since it focuses on maximizing the channel capacity of a given environment, while in our case we emphasize the organic connection between the general concept of *diversity*, which is broader than channel capacity, and the geometric structure of the array and its antenna density. Indeed, the MIMO capacity that was optimized in [39] depends not only on the nature of the communication channel, e.g., presence of scatters, noise, interference, ergodicity, but also on the structure of the transmitter as well.[1] Moreover, the channel model used in [39] is very limited, since it involves modeling few scattering objects with essentially far-field wave assumption. Although in their methods other more sophisticated channel models could in principle be used, the optimization results will be completely different for different channels and it is not easy to see how the optimum array can be deigned for generic geometric or environmental scenarios. In our approach, we focus directly on diversity as expressed in terms of the cross-correlation matrix and the corresponding diversity gain, which implies that arrays optimized through our approach can be expected to work in more generic scenarios since the optimized positions and orientations are independent of the Tx and the environments. This provides further flexibility since the system engineer can choose to work with different Rx arrays in different complete MIMO system configurations.

C. ORGANIZATION OF THE PRESENT WORK

The paper is organized as follows. In Sec. II, we first start by providing a general view on the main topic of the present work, i.e., finding geometrical bounds on the performance of generic arrays for optimal diversity performance. Afterwards, we begin the formulation of the main method in several steps. First, in Sec. III, the concept of Infinitesimal Dipole MIMO (ID MIMO) is developed mathematically, where each antenna is modeled by an electrically-small (but fully Maxwellian) radiator with position and orientation that will be determined later. In Sec. IV, the far-field envelope cross-correlation is reviewed (Sec. IV-A). Afterwards, the cross-correlation Green's function is integrated into the computation of the envelope correlation by transferring the calculation from the fields to the currents. This will allow easy implementation of position-and-orientation search methods in order to optimize

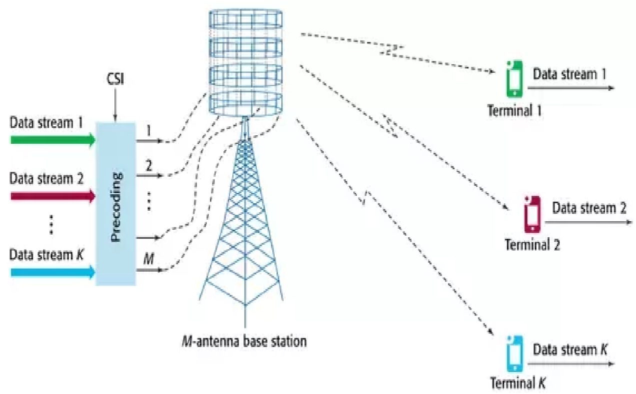

FIGURE 2. Schematic model of a Multi-User MIMO uplink system. For best performance, the Rx antennas must be uncorrelated. For example, Channel State Information (CSI) estimation algorithms are most efficient when the Rx terminals are uncorrelated. This is crucial for massive MU MIMO systems but also for other communication systems exploiting diversity.

for diversity. Before doing so, we need to formulate the diversity gain measure in terms of the entire antenna array (Rx mode only). The general concept is first reviewed in Sec. V-A for general arrays, then specialized into ID MIMO systems in Sec. V-B. In order to stress the importance of the ID MIMO case, we list in Sec. V-C various advantages gained by focusing on this special but fundamental antenna array type. The discussion of the optimization process starts in Sec. VI. We first give a very short view on the Genetic Algorithm in Sec. VI-A and VI-C (excellent detailed accounts exist elsewhere and are mentioned in the references.) After that, various fundamental design examples are introduced in Sec. VII. Following a broad overview in Sec. VII-A, we start by working out basic design examples involving 2D arrays in Sec. VII-B. The all-important concept of critical antenna density is then developed in Sec. VII-C, where we begin to show that an upper bound on the number of antenna elements that can fit a required diversity gain exists. In Sec. VII-D, the results are shown to be consistent also with voulmetric-type arrays where several design examples involving cubic and spherical arrays are given. Excellent diversity gains where obtained for small Rx MIMO terminals, clearly demonstrating how position-and-orientation optimization can considerably improve the diversity gain. We end the basic examples by Sec. VII-E with a more complex problem involving designing a conformal antenna MIMO array to fit an avionic nose (aircraft or missile) where the array is envisioned to mount the curved surface of the support platform. Ground plane effects are fully incorporated into the basic method using an analytical procedure based on image theory. Indeed, in Sec. VIII, it is shown that the presence of ground planes can be taken into account by enlarging the size of the array cross-correlation to include groups of virtual dipoles. The extension is then verified by comparison with full-wave numerical analysis or the half-space Green's function of dipole-over-plane. Finally, because of the fundamental importance of the discovered antenna

[1]Because the MIMO capacity depends directly on the MIMO channel matrix H, which is a strong function of both the Tx and Rx terminals.

density concept, Sec. IX provides additional numerical examples and case studies completely vindicating the impact of the antenna density on the obtainable gain performance. The paper then ends with conclusions and recommendations for future work.

II. GENERIC MIMO ARRAY DESIGN RESEARCH: MOTIVATIONS AND BENEFITS

Design of MIMO arrays requires going simultaneously into numerous (often conflicting) considerations involving both signal processing and electromagnetic aspects, where one can notice that in literature distinct approaches to these two domains have in general evolved into mature and self-contained disciplines each standing on its own. However, since the antenna array remains essentially a physical structure, electromagnetic design parameters, most importantly the array geometry, continues to dominate the final picture. So far, most design approaches to MIMO antenna have focused on working with concrete antenna elements, e.g., microstrip antennas, dielectric resonator antennas, wire antennas, etc, where the MIMO antenna engineer concentrates his effort on perfecting the physical layout of the particular device type he decided to work with. On the other hand, we find that not much is known so far about how *generic* arrays should be designed in order to ensure satisfactory performance. For instance, it is of utmost importance that the MIMO engineer knows roughly the approximate answers to questions such as:

1) How many antennas can I place within a given surface/volume before starting to degrade performance?
2) How roughly should multiple antennas within a given geometric region and positions be oriented?
3) Where roughly should multiple antennas within a given geometric region be placed for best performance?
4) How roughly should multiple antennas within a given geometrical region be simultaneously positioned and oriented?
5) What is the impact of the geometrical shape of the total MIMO array on the overall performance?

The key to understanding the importance of such questions is that the antenna type must be kept as generic as possible in order for the answers to retain their full power of being true rules of thumb or at least reasonable design directives capable of helping the largest possible group of MIMO engineers working on the ground.

The authors believe that one of the most fruitful directions along which investigations of MIMO array design at the generic level can be advanced is discovering certain "empirical upper bounds" on the array performance caused by the interaction of electromagnetics and geometry. By this we mean that regardless to the specifics of the antenna type, how the antennas are excited, or other considerations, it should be possible to have some general knowledge about whether, for example, the number of antennas in massive MIMO can go beyond certain density or not; or whether a cubic antenna will outperform a spherical array for a given size, element density, frequency, etc. These are instances of what we believe can be grouped under the rubric of "fundamental MIMO array limitations" through which it can be approximately decided whether a massive or small MIMO array with such and such geometric and element density can be designed to meet desired overall performance considerations or not. The present paper will work toward attaining some of these goals.

The strategy to be adopted in here can be summarized as follows. We model MIMO arrays by focusing on the least complicated but still fully electromagnetic antenna type, the infinitesimal dipole. The diversity gain of the array is computed in terms of the positions and orientations of each dipole and then a nonlinear optimization problem is formulated in order search for the optimum positions and orientations leading to the best performance. The study is performed for various antenna densities, array shape and sizes, leading to acquisition of general knowledge about some rough empirical upper bounds obeyed by generic MIMO arrays enforced by the electromagnetic nature of the problem.

At this moment, a conflict may arise between the above described desired level of genericity and the hard facts of physics: the antenna type *does*, after all, affect the performance of MIMO arrays. Indeed, arrays of microstrip antennas don't behave exactly like arrays of wires. However, the authors believe that even with this, there is still much that can be gained by projecting the antenna type onto a minimal level of abstraction as above, where each radiating/receiving element is effectively reduced into:

i) Position.
ii) Linear orientation.

In electromagnetic theory, two types of antennas possess this minimal structure: the infinitesimal dipoles of the electric and magnetic types. We propose to focus here mainly on MIMO arrays comprised of infinitesimal electric dipoles. The purpose of the present research is to devise a method allowing us to automatically find

i) Empirical upper bounds on the design parameters of MIMO arrays, with focus laid on the geometry of the system, most prominently positions and orientations of each point dipole relative to others, and
ii) Obtain approximation of continuous large or massive MIMO system with optimum diversity gain when the number of IDs becomes large.

The second goal will require scalable computing platform since the optimization problem even for simple antennas like IDs is expensive, while the first goal can be achieved with small number of dipoles. However, both goals can in principle be attained by one and the same method, the technique to be developed in this paper.

Finally, we mention that the details about the antenna type can also be included using our method as follows. If a specific antenna device is required for a generic MIMO array investigation, then one may approximate the entire device, say a microstrip patch, by a two dimensional sub-array of ID. Other copies of this sub-array will be placed in the vicinity of each other in order to emulate the entire array situation. The technique used in this paper can then be applied to the total array while adding additional constraints to make sure sub-arrays preserve their three-dimensional rigidity throughout optimization, i.e., each su-array used to model a given antenna type will be allowed to change the position of its center of mass and relative orientation only during optimization. This solution however requires also a scalable computing platform and hence lies outside the scope of this paper, which is mainly concerned with the proof of concept of the proposed method.

III. THE INFINITESIMAL DIPOLE MIMO (ID MIMO) ARRAY

IDs enjoys several unique advantages in antenna theory. First, their radiated fields can be expressed in closed-analytical form everywhere in space. Second, their geometry can be fully described by only a spatial position (three variables) and linear orientation (two direction-cosines.) Third, no mutual coupling arises between IDs, regardless to the distances between them. For these reasons, they provide very good initial starting point for addressing fundamental research in the area of MIMO antenna system design and development. In fact, even if the antenna type of interest is more complex device like microstrip antennas, one can usually obtain very good initial knowledge of the expected array performance by examining how ID MIMO arrays would behave in generic situations before moving toward the much more challenging step of computing the full-wave performance of the final MIMO system with the fully-fledged antenna device in use.

However, even when the MIMO array setting is reduced to a point-like antenna type such as the ID, the problem of radiation and reception by antennas comprised of infinitesimal dipoles remains formidable in diversity gain performance studies. The reason is that the process of computing the array cross-correlation between far fields is very complicated, even when the far fields themselves can be obtained analytically from the radiator's data is in the case of IDs. Although the recent introduction of the cross-correlation Green's function has considerably simplified the process of MIMO diversity optimization, it will be shown below that the problem remains extremely complex even when only few IDs are involved. In fact, it does not appear that exact upper bounds on an ID MIMO array can be ever obtained since the corresponding optimization problem is highly nonlinear and is therefore notoriously difficult. In fact, only numerical approaches are possible to generic MIMO antenna research.

For these reasons, the methodology to be introduced in this paper will aim at evolving an intermediate approach that is neither exact or analytical, nor too electromagnetically simplistic like the scalar point source theory popular in MIMO communication-theoretic and array signal processing research. Instead, the problem of ID MIMO arrays will be treated using exact and complete electromagnetic theory by working with the full radiation fields (via the cross-correlation Green's function) as described later.

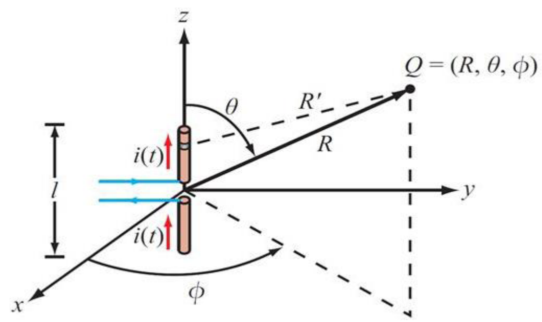

FIGURE 3. An infinitesimal dipole is realized by an electrically-small dipole with $l \ll \lambda$.

Consider an ID MIMO array comprised of N IDs, each located at $\mathbf{r}_n$, and oriented along $\hat{p}_n$, where $n = 1, 2, \ldots, N$ (see Fig. 3). We denote the electric and magnetic fields radiated by each dipole by $\mathbf{E}_n^{\mathrm{id}}(\mathbf{r})$ and $\mathbf{H}_n^{\mathrm{id}}(\mathbf{r})$, respectively, where the excitation amplitude is treated as unity for all dipoles. The ID MIMO array can be represented mathematically by the discrete current:

$$\mathbf{J}_{\mathrm{mimo}}^{\mathrm{id}}(\mathbf{r}) = \sum_{n=1}^{N} \mathbf{p}_n \delta(\mathbf{r} - \mathbf{r}_n), \tag{1}$$

where $\delta(\mathbf{r})$ is the 3-dimensional Dirac delta function.

The geometrical information of the ID MIMO array is then completely captured by the nth ID data

$$G_{\mathrm{MIMO}}^{n} = (\theta_n, \varphi_n, r_n^x, r_n^y, r_n^z)^T, \tag{2}$$

where θ_n and φ_n are the relative orientations of the nth ID. The polarization vector is given then as

$$\hat{p}_n = \hat{x}\cos\phi_d \sin\theta_n + \hat{y}\sin\varphi_n \sin\theta_n + \hat{z}\cos\theta_n. \tag{3}$$

Therefore, each ID data vector consists of five real numbers, three for positions, and two for direction. For the entire MIMO array, the geometrical data vector is given by

$$G_{\mathrm{MIMO}} = \left(G_{\mathrm{MIMO}}^{1} \quad G_{\mathrm{MIMO}}^{2} \quad \ldots\ldots \quad G_{\mathrm{MIMO}}^{N}\right)^T, \tag{4}$$

which is a $5N \times 1$ vector.

Because of the linearity of Maxwell equations, the total field radiated by the entire ID MIMO array is given by

$$\mathbf{E}(\mathbf{r}) = \sum_{n=1}^{N} p_n \mathbf{E}_n^{\mathrm{id}}(\mathbf{r}; \hat{p}_n, \mathbf{r}_n), \tag{5}$$

$$\mathbf{H}(\mathbf{r}) = \sum_{n=1}^{N} p_n \mathbf{H}_n^{\mathrm{id}}(\mathbf{r}; \hat{p}_n, \mathbf{r}_n) \tag{6}$$

where p_n is the (complex) amplitude of the nth vector dipole moment $\mathbf{p}_n = \hat{p}_n p_n$. The electric field radiated by a single dipole with moment $\mathbf{p}_n$ and location $\mathbf{r}_n$ is given by the well-known formula

$$\mathbf{E}_n^{\mathrm{id}}(\mathbf{r}) = \frac{\mu_0 \omega^2}{4\pi} \bar{\mathbf{G}}_e(\mathbf{r} - \mathbf{r}_n) \cdot \hat{p}_n. \tag{7}$$

The electric-field dyadic Green's function $\bar{\mathbf{G}}_e(\mathbf{r}-\mathbf{r}_n)$ can be expressed as [47]

$$\bar{\mathbf{G}}_e(\mathbf{r}-\mathbf{r}_n) = \left(\bar{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right) g(\mathbf{r}-\mathbf{r}_n), \quad (8)$$

where the scalar Green's function is defined as

$$g(\mathbf{r}) := \frac{e^{ikr}}{r}, \quad (9)$$

Here, $k = \omega/c = 2\pi/\lambda$ is the wavenumber in free space, c and λ are the speed of light and the wavelength, respectively, both in free space, and $\bar{\mathbf{I}}$ is the unit dyad. For completeness, we also give the corresponding magnetic field

$$\mathbf{H}_n^{\mathrm{id}}(\mathbf{r}) = \frac{j\omega}{4\pi}\nabla\times\bar{\mathbf{G}}_e(\mathbf{r}-\mathbf{r}_n)\cdot\mathbf{p}_n. \quad (10)$$

Putting up all these results together, the final expression of the electric field radiated by a ID MIMO array is given by

$$\mathbf{E}_{\mathrm{mimo}}^{\mathrm{id}}(\mathbf{r}; G_{\mathrm{MIMO}}) = \frac{\mu_0\omega^2}{4\pi}\sum_{n=1}^{N} p_n\bar{\mathbf{G}}_e(\mathbf{r}-\mathbf{r}_n)\cdot\hat{p}_{\mathrm{n}}. \quad (11)$$

Equation (11) is in fact a closed-form analytical expression of the fields radiated by a MIMO array comprised of arbitrary positioned and oriented Hertzian (infinitesimal) dipoles. The geometrical specifications of the array, which are the main objective of the design process to be developed below, are fully encapsulated within the data structure G_{MIMO} (4).

IV. THE FAR-FIELD CROSS-CORRELATION GREEN'S FUNCTION METHOD

A. ENVELOPE CROSS-CORRELATION COEFFICIENT

The cross-correlation coefficient evaluates the isolation between each port of the receiving system. A low cross-correlation level is a necessary (but not sufficient) condition for a good MIMO system. For example, if we consider a 2×2 MIMO system, the cross-correlation coefficient ρ, can be directly calculated from the radiation patterns of the two receiving antennas. Specifically, let $\mathbf{E}_1(\theta,\varphi)$ and $\mathbf{E}_2(\theta,\varphi)$ be the radiation patterns of the antennas 1 and the antenna 2, respectively. The *envelope cross-correlation coefficient* between the two antennas is defined as follows [5], [6], [8]

$$\rho_e = \frac{\left|\int_{4\pi} d\Omega \mathbf{E}_1(\theta,\varphi)\cdot\mathbf{E}_2^*(\theta,\varphi)\right|}{\sqrt{\int_{4\pi} d\Omega\,|\mathbf{E}_1(\theta,\varphi)|^2}\sqrt{\int_{4\pi} d\Omega\,|\mathbf{E}_2(\theta,\varphi)|^2}}. \quad (12)$$

The derivation of this expression assumes that the far fields $\mathbf{E}_1$ and $\mathbf{E}_2$ impinge on the receive array with a uniform statistical distribution for the direction of arrival. Modifications of (12) to include other types of statistical distributions are straightforward and will not undertaken here.

The envelope cross-correlation coefficient (12) can be evaluated either experimentally or numerically using standard electromagnetic methods in antenna engineering. First, the Lorentz reciprocity theorem of electromagnetics is invoked to use *transmitting* mode antenna radiation patterns instead of the receive mode's data originally instigated in (12). Second, extensive 3D field measurements or expensive numerical computations are often harnessed to perform the integration in (12). An alternative, accurate and less demanding approach is reviewed next.

B. EXPRESSING THE CROSS-CORRELATION AND DIVERSITY GAIN IN TERMS OF THE ANTENNA CURRENT

In [12], a new approach to envelope cross-correlation coefficient defined by (12) was developed, where the basic motivation is shifting the focus in the computation of antenna-antenna far-field cross-correlation from the far fields to the antennas themselves. To understand how this can be done, consider that antenna 1 and antenna 2 of the Rx array are already connected to their respective sources. The goal is to express the cross-correlation coefficient directly in terms of the *current distributions* $\mathbf{J}_1(\mathbf{r})$ and $\mathbf{J}_2(\mathbf{r})$ on the antennas' respective surfaces instead of the *fields* generated by the two-antenna system, i.e., without the need to deal with full three-dimensional radiation field patterns in the far zone. More specifically, it was shown in [12] and [47] that the numerator of the expression (12) can be replaced by an expression of the form

$$\int_{4\pi} d\Omega\,\mathbf{E}_1(\hat{\mathbf{r}})\cdot\mathbf{E}_2^*(\hat{\mathbf{r}}) = \int_{V1} d^3r' \int_{V2} d^3r''\mathbf{J}_1(\mathbf{r}')\cdot\mathbf{C}(\bar{r'}, r'')\cdot\mathbf{J}_2^*(\mathbf{r}''), \quad (13)$$

while the terms in the denominator of (12) are given by

$$\int_{4\pi} d\Omega|\mathbf{E}_1(\hat{\mathbf{r}})|^2 = \int_{V_1} d^3r' \int_{V_2} d^3r''\mathbf{J}_1(\mathbf{r}')\cdot\mathbf{C}(\bar{r'}, r'')\cdot\mathbf{J}_1^*(\mathbf{r}''), \quad (14)$$

$$\int_{4\pi} d\Omega|\mathbf{E}_2(\hat{\mathbf{r}})|^2 = \int_{V_1} d^3r' \int_{V_2} d^3r''\mathbf{J}_2(\mathbf{r}')\cdot\mathbf{C}(\bar{r'}, r'')\cdot\mathbf{J}_2^*(\mathbf{r}''). \quad (15)$$

Here, $\mathbf{C}(\bar{r'}, r'')$ is called the *cross-correlation Green's function* (CGF) defined by

$$\mathbf{C}(\bar{r'}, r'') := \int_{4\pi} d\Omega\left[\bar{\mathbf{I}} - \hat{r}\hat{r}\right] e^{ik(\mathbf{r}'-\mathbf{r}'')\cdot\hat{r}}, \quad (16)$$

where $\mathbf{r}'$ and $\mathbf{r}''$ are the locations of two point sources within the antennas respective regions V_1 and V_2. Here,

$$\hat{r}(\theta,\varphi) := \frac{\mathbf{r}}{\|\mathbf{r}\|} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta \quad (17)$$

is the standard unit radial position vector.

Details of the derivation and discussion of the physical meaning of the results can be found in [12] and [47]. The CGF has been extended recently to the time domain, where a time-dependent cross-correlation Green's function was developed and integrated to FDTD models for wideband communication applications [13]. Moreover, analytical approximation of the CGF were proposed in [14] while extension to include magnetic dipoles were achieved in [15].

For the purpose of the current paper, we simply use (13)-(16) to compute the cross correlation coefficients (12) between two currents. Therefore, we observe how according to the alternative approach through the CGF (16), generic far-field correlations can be completely reduced to manipulations involving only antenna current excitations integrated across the their physical support regions.

V. DIVERSITY GAIN OF MIMO ARRAYS

A. DIVERSITY GAIN FORMULATION

The diversity gain of an antenna array can be now estimated based on cross-correlation coefficients. If we consider an antenna array with its correlation matrix R (matrix of all the mutual cross-correlation coefficients between every antenna pair in the array), the diversity gain G can be defined as [7], [8]

$$G = \frac{\mathrm{Tr}(\mathrm{R}^2)}{\|\mathrm{R}\|_{\mathrm{Fr}}}, \tag{18}$$

where Tr is the matrix trace (the sum of the diagonal elements), R^2 is element-by-element matrix square, and $\| \|_{\mathrm{Fr}}$ is Frobenius norm square defined as

$$\|A\|_{\mathrm{Fr}} = \sum_{m,n} |a_{m,n}|^2. \tag{19}$$

Here, R is the cross-correlation matrix of pairwise cross-correlation coefficients. It can be explicitly expanded as

$$\mathrm{R} = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{m1} & \rho_{m2} & \cdots & \rho_{mn} \end{pmatrix}. \tag{20}$$

Here, the mnth matrix entry ρ_{mn} is the envelope cross-correlation between the mth and the nth antennas computed by means of (12).

Moreover, we note from (18) that the diversity gain G has a value between 0 and 1. A gain of 1 corresponds to minimum cross-correlation in the system, whereas a diversity gain of 0 indicates a very high degree of statistical coupling between the antennas of the array system. Roughly speaking, for various applications depending on spatial diversity like MIMO and other forms of communication systems, it is crucial to maximize G. Even though a high value of the diversity gain does not necessarily implies that the envelope cross-correlation coefficient between every antenna-antenna pair has been minimized, it remains, for overall performance level evaluation, the diversity measure (18) is considered in many applications, see [5], [8].

B. DIVERSITY GAIN FORMULATION FOR ID MIMO ARRAY SYSTEMS

We now combine the infinitesimal dipole MIMO (ID MIMO) array as introduced in Sec. III with the diversity gain performance measure described in Sec. V-A but using the cross-correlation Green's function (CGF) of Sec. IV instead of original envelope cross-correlation expression (12). The main goal is to directly translate the complicated three-dimensional far-field integration explicit in (12) into simple algebraic expressions directly dependent on the array *geometry* instead of the radiated fields. More specifically, we next present the cross-correlation data in terms of the ID IDM geometrical data (2).

Consider two ID antennas located $\mathbf{r}_m$ and $\mathbf{r}_n$. It is possible to mathematical describe their current distributions by expressions of the form

$$\mathbf{J}_m(\mathbf{r}) = p_m \hat{p}_m \delta(\mathbf{r} - \mathbf{r}_m), \;\; \mathbf{J}_n(\mathbf{r}) = p_n \hat{p}_n \delta(\mathbf{r} - \mathbf{r}_n), \tag{21}$$

where p_m and p_n are complex constants depending on the antenna excitation, while the two unit vectors $\hat{p}_m$ and $\hat{p}_n$ describe the polarization (orientation) of the mth and nth antennas, respectively. By substituting the currents (21) into (13)-(16) then using (12), we arrive at

$$\rho_{mn} = \frac{\hat{p}_m \cdot \bar{\mathbf{C}}(r_1, \mathbf{r}_2) \cdot \hat{p}_n}{\left[\hat{p}_m \cdot \bar{\mathbf{C}}(\mathbf{r}_1, \mathbf{r}_1) \cdot \hat{p}_n\right] \left[\hat{p}_m \cdot \bar{\mathbf{C}}(\mathbf{r}_2, \mathbf{r}_2) \cdot \hat{p}_n\right]}. \tag{22}$$

It is clear that the complex excitations p_m and p_n of each antenna do not contribute to the cross-correlation coefficient because of the normalization factor in (12). However, the *orientation* of each antenna, i.e., $\hat{p}_m$ and $\hat{p}_n$, and the *positions* $\mathbf{r}_m$ and $\mathbf{r}_n$ *are* essential. In other words, we managed to reduce the problem of computing the overall diversity gain of the MIMO system to the *geometrical* data of the array elements, which in the case of the infinitesimal dipole reduces to only five parameters per element as in (2).

In this paper, the expressions in (22) will be deployed in order to estimate the MIMO array diversity gain (18). Design of optimum MIMO arrays will then be translated into finding the position $\mathbf{r}_m$ and orientation $\hat{p}_m$ of every antenna such that the array diversity gain is maximum. We next describe how to formulate this design process using a global optimization algorithm (GA).

C. ADVANTAGES OF USING INFINITESIMAL DIPOLES IN MIMO ARRAY DESIGN RESEARCH

Before proceeding with the algorithm, we mention several main advantages gained by using infinitesimal dipoles as the main radiating elements:

1) The relative excitation of each ID does not enter into the optimization process, leading to full focus on the purely geometrical details of the arrays.
2) Large antennas, i.e., antennas that are not electrically small, can be approximated by groups of infinitesimal dipoles, such as patch or dipole antennas, for example, see the Infinitesimal Dipole Model (IDM) approach [40]. All what is needed is a careful implementation of certain optimization constraint rules placed on the sets of IDs competing against each other through the search process. However, in this case more complicated procedures must be introduced to deal with how each large antenna (modeled by a set

of dipoles) will be excited relative to others. Examples of applications of IDs to model non-electrically-small antennas are numerous, see [28]–[30].

3) In many problems, the most difficult step in designing a novel high-performance antenna system is to know the details of the current *surface distribution* needed. The method presented here can be used to obtain a highly accurate approximation of ideal currents guaranteeing optimum performance. Based on this information, a second stage in the design process may be evolved to realize this ideal current using whatever antenna technology. For example, see [40].
4) Since infinitesimal dipoles are *point sources*, arbitrary array topologies and geometrical shapes can always by approximated by a sufficient number of such dipoles. For instance, it is possible to mimic *continuous* MIMO antenna shapes by clusters of point sources. This is directly relevant, for example, to the now emerging field of *large MIMO systems* [2], [4].
5) The computation of the cross-correlation coefficient using (1) becomes very simple for infinitesimal dipoles [12].
6) Arrays of infinitesimal dipoles can be realized in practice as *electrically-small antennas*. The science and art of designing such antennas is currently in an advanced state, see [21] and [18].
7) Since it is possible to express the fields generated by point sources using *closed-form analytic expressions* [48], use of such antennas in MIMO analysis may provide a deeper insight into the system due to the potential of *combining electromagnetic* analysis with the *statistical* study in a unified analytical formalism.

In sum, clusters of IDs can be deployed either to model each antenna individually (one MIMO antenna represented by one ID), or to model each MIMO antenna by a group of IDs, or to model the entire MIMO array by a continuous antenna shape approximated by the entire groups of IDs. Depending on the application, each of these possible interpretations is possible. For simplicity, we focus in the following examples on small number of IDs. In each case, the mathematical theory, the basic relations, and the results are essentially the same.

VI. DIVERSITY GAIN OPTIMIZATION IN ID MIMO ARRAY SYSTEMS

In this section, we combine the theory contained in Sec. III, IV, and V with a global search algorithm, the Generic Algorithm (GA), in order to find optimal MIMO array antennas' positions and/or orientations capable of maximizing the diversity gain.

A. BACKGROUND AND MOTIVATIONS

In literature, there exists many heuristic global optimization algorithms, such as particle swarm optimization [23], [24], random search algorithms, genetic algorithm [21], [37], ant colony [41], simulated annealing [42]. Such meta-heuristic methods are often characterized by being i) stochastic in contrast to deterministic, and ii) lacking exact or rigorous convergence theory. Algorithms of this type tend to be more effective in handling *general* and *global* search problems than other methods borrowed from functional analysis. While the latter mathematical optimization techniques are known to be very powerful for both *local* optimization problems and some special global problems, they are not always capable of handling generic global optimization scenarios, and hence fail to include within their reach a broad range applications of typically emerging in practice.

Since the optimization problem corresponding to the design of high-diversity gain ID MIMO arrays involve is strongly nonlinear, the process must require the use of a powerful global optimization search algorithm. Moreover, in general there is no prior knowledge about how the various optimization parameters (positions and orientations) should be initialized or restricted, which further necessities the need to use algorithms belonging to the (weak) artificial intelligence algorithms, sometimes known as *soft computing*, which includes evolutionary computing, fuzzy logic, support vector machines, and expert systems. In this paper, we focus on using evolutionary methods. More specifically, we work exclusively with the Genetic Algorithm (GA). Although relatively complex to build, the Genetic Algorithm (GA) is chosen because of its overall stability and consistent performance. Indeed, the GA is known to be robust and powerful when very little information about the optimum solution is available [21]. For detailed information on the GA, many excellent books and papers exist, and consequently we provide here only a very brief review.

B. OVERVIEW OF THE GENETIC ALGORITHM

The genetic algorithm is based on an evolution-and-a-selection process of successive ''generations'' of each optimization parameter. Fig. 4 illustrates the main scheme of the genetic algorithm process. At the beginning, each optimization parameter is encoded by a binary function (called *genes*), resulting in a variable's code given by [21]

$$p = \left(\frac{p_{\max} - p_{\min}}{2^{N-1}}\right) \sum_{n=0}^{N-1} 2^n b_n + p_{\min} \tag{23}$$

where p is the decoded value of the parameter; p_{min} and p_{max} are, respectively, the minimum and maximum range bounds of the parameter p; and b_n is the binary function representation of p. A *chromosome* is an array of genes, where each gene corresponds to one parameter. The algorithm starts with a large ''pool'' of random chromosomes. For each chromosome, a cost function is calculated and the chromosomes are ranked from the ''most-fit'' to the ''least-fit'' by an orderly evaluation of the cost function. The least-fit chromosomes are discarded while the most-fit chromosomes become parent, e.g., by swapping them to produce new genes. Next, the cost functions are updated based on the new chromosomes and the process is repeated until the termination criterion is reached.

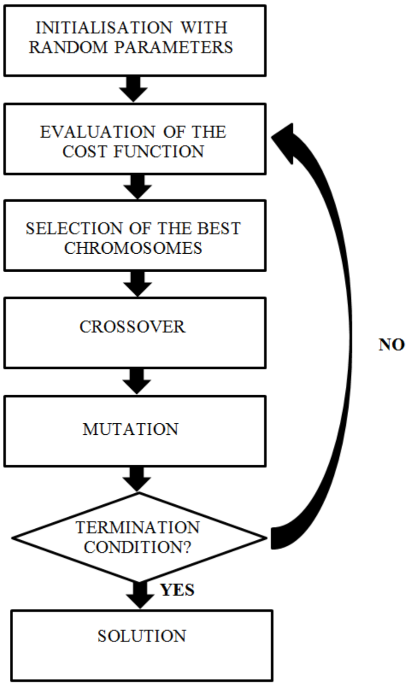

FIGURE 4. Block diagram of the genetic algorithm.

TABLE 1. Main characteristics of the genetic algorithm.

Characteristics	Value
Number of infinitesimal dipoles	Depend on the example
Number of variables	4 (2D shape) and 5 (3D shape)
Number of bits for each variable	5
Number of chromosomes	20
Mutation rate	0.5%
Diversity Gain objective	> 0.8

C. THE GA FORMULATION OF OPTIMUM DIVERSITY-GAIN ID MIMO ARRAY DESIGN

In this paper, our goal is to obtain optimum geometrical dipole arrangements resulting in best diversity gain performance for the overall MIMO system. Consequently, the cost function will be directly related to the diversity gain (18). To avoid a too lengthy optimization time, we choose a termination criterion corresponding to a diversity gain higher than or equal to 0.8 (the maximum possible value is always 1.) The physically relevant optimization parameters are reported in Table 1, where each infinitesimal dipole is associated with a position and an orientation. The position of the dipole is characterized with three Cartesian coordinates $(x_{\mathrm{id}}, y_{\mathrm{id}}, z_{\mathrm{id}})$, while the orientation is captured by two azimuthal and elevation angles θ_{id}, φ_{id}. For each iteration, the diversity gain is computed using the expressions (18)-(22). The cost function is determined by subtracting the desired diversity gain (e.g., the acceptance level of 0.8) from the calculated diversity gain obtained after each iteration (gain error measure). In other words, the optimization cost function at the ith iteration is

$$C_i = |D_{\mathrm{desired}} - D_i|, \tag{24}$$

where D_{desired} is the desired diversity gain (hereafter, we choose $D_{\mathrm{desired}} = 0.8$) and D_i is the calculated diversity at the ith iteration.

The cost function measure (24) is an extremely difficult nonlinear global optimization problem. To see this, note that all the elements of the cross-correlation matrix R as given by (22) are highly nonlinear functions of both the dipole positions $\mathbf{r}_n$ and orientation $\hat{p}_n$. Even worse, the diversity gain expression itself (18) is itself a nonlinear function of the correlation matrix R itself. The final cost (24) appears then to involve a considerably convoluted functional dependence between diversity and the antenna geometry. A very powerful search method like the GA is needed to solve the corresponding optimization problem. *It is interesting to note that even with infinitesimal dipoles, which are the simplest fully-electromagnetic antenna type possible, the problem of optimizing generic multiple-antenna systems for best diversity gain performance is notoriously difficult.* For this reason, the authors believes that gaining knowledge about MIMO arrays in general by starting with ID MIMO scenarios is highly desirable in this case, which is the strategy that will be adopted in what follows.

VII. DESIGN EXAMPLES

A. GENERAL CONSIDERATIONS

In order to simplify the presentation, no additional constraints on the array separation between each two elements were imposed in most examples. Hence, unless the opposite is explicitly stated, no restrictions on the locations of the antennas in the array surface or volume are implicated by the design algorithm. Considering that the main purpose of our paper is to present a general methodology for analyzing the impact of the array geometrical structure on the system's diversity performances, additional geometrical constraints on size, shape, locations can be added into the optimization code whenever a more concrete applications requires doing so, leading to *constrained optimization problems*, a topic well known in evolutionary computing research. The only build-in restrictions added in our basic codes are constraints on the infinitesimal dipole location to ensure that each ID remains within the optimization solution space.

For the dimensions of the different topologies presented throughout the paper, we have chosen maximum dimension around 0.5λ. As many of MIMO systems are embedded into complex and small devices, we try to keep relative small dimensions compared to the wavelength. Obviously, the same study can be performed with other dimensions, for example massive MIMO. Here, a frequency of 10GHz has been chosen for all examples. However as the deign method is frequency independent, the present approach can be performed with any other frequencies. In fact, the main conclusions regarding the geometrical information of the designed arrays are always presented in units normalized with respect to the operating wavelength.

In order to highlight the importance of the MIMO array density, we compute for each example the corresponding density γ, which is defined as the number of antennas per unit area (volume)

$$\gamma = \frac{N}{S} \tag{25}$$

where, N is the number of infinitesimal dipoles and S is the area (or volume) of the considered shape.

B. DISK AND RING MIMO ARRAY TOPOLOGY

For the first design case study, a generic ring MIMO array topology is considered. This shape can be defined by two parameters, R_1 and R_2, the radii of the inner and outer circles, respectively. Since the problem considered in this first example is two dimensional, only four optimization variables will be taken into account, namely $(x_{\text{id}}, y_{\text{id}})$ for the position and $(\theta_{\text{id}}, \phi_{\text{id}})$ for the orientation.

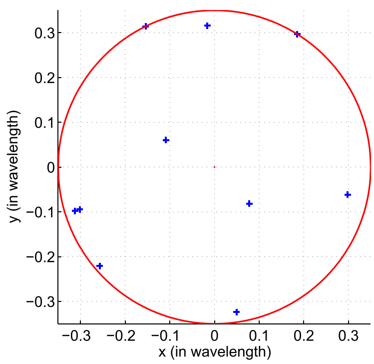

FIGURE 5. Position of the infinitesimal dipoles inside a ring with $R_1 = 0$ and $R_2 = 0.35\lambda$. (Dipole orientations data suppressed.)

In the initial example, a zero inner radius ($R_1 = 0$) is chosen while the outer radius R_2 is set equal to 0.35λ (circular disk) as shown in Fig. 5. The variation of the diversity gain during the optimization process is given in Fig. 6, where it can be seen that a high diversity gain of 0.815 was obtained after 22 iterations.

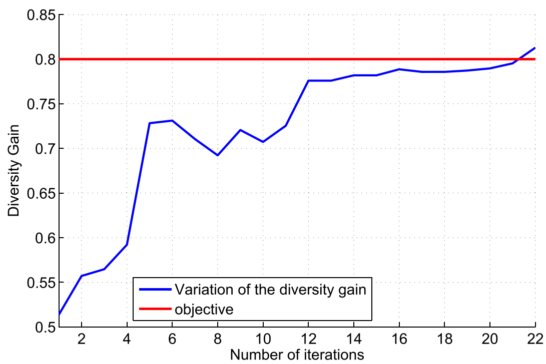

FIGURE 6. Variation of the diversity gain during the optimization process.

Next, in order to investigate the impact of the array density on the optimization behavior, the array surface area is reduced by considering a ring array topology with inner and outer dimensions $R_1 = 0.2\lambda$ and $R_2 = 0.35\lambda$ while keeping the same total number of antennas. Figs. 7 and 8 illustrate the dipole positions obtained by the GA method and the evolution of the diversity gain during optimization process, respectively. It was found that the "critical" number of infinitesimal dipoles (definition to be given shortly) that can be inserted into this shape and area is equal to 10. Table 2 summarizes the design results of the two examples presented in this section.

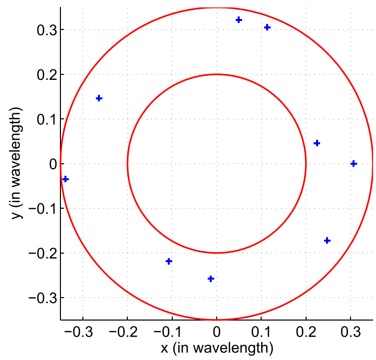

FIGURE 7. Positions of the infinitesimal dipoles inside a ring with $R_1 = 0.2\lambda$ and $R_2 = 0.35\lambda$. (Dipole orientations data suppressed.)

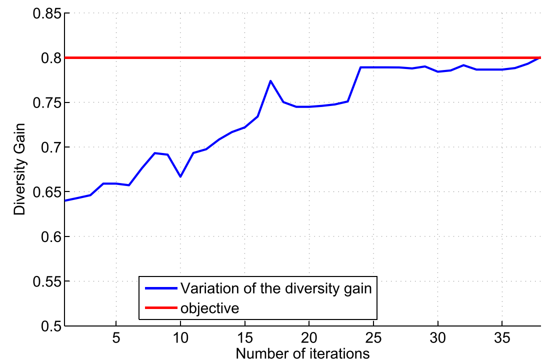

FIGURE 8. Variation of the diversity gain during the optimization process for the problem in Fig. 7 .

TABLE 2. Characteristics of the optimized array for a ring shape.

Ring Geometry	Optimum number of ID	Iterations	Diversity Gain	Critical Density (ID/λ^2)
$R_1 = 0$ $R_2 = 0.35\lambda$	10	22	0.815	26
$R_1 = 0.2$ $R_2 = 0.35\lambda$	10	38	0.8	38

We observe that the number of iterations needed to reach a desired diversity gain (38 iterations) is higher than the example in Fig. 6 (22 iterations). Indeed, the array *density* in the case of circular ring (Fig. 5) is larger than the case of circular disk (Fig. 7), which suggests that the design process of best MIMO with a given geometrical shape arrays becomes more difficult with increasing array density.

C. THE CONCEPT OF CRITICAL ARRAY DENSITY

In our experimental study of the array design algorithm proposed above, three different array shapes have been considered: square, circle and triangle. By comparing the MIMO array densities for these three different geometrical configurations, extensive numerical results suggested that *there exists some critical MIMO array density beyond which it becomes difficult to obtain good diversity gain performance*. Moreover, this critical array density appears to be shape dependent. Indeed, if we consider circular disk array, it is found that ten infinitesimal dipoles is the critical upper limit on the number of antennas that can be inserted into array of an outer radius equal 0.35λ, which corresponds to antenna density of about $26/\lambda^2$. In other words, a circular disk array with this radius cannot accommodate more than ten antennas without severely degrading the diversity gain performance of the MIMO system.

Going back to the array circular disk configuration, if we realize the same total array surface area this time in a *square* shape, the side lengths will be 0.62λ, and applying our design method it was found after repeated optimization processes of this square configuration that the critical number of antennas is twelve infinitesimal dipoles, resulting in a diversity gain higher than the 0.8 obtained with the circular disk configuration. Therefore, *it appears that for a given diversity gain and a fixed total MIMO array surface area, we can insert more antennas inside a square than inside a circle in order to obtain higher diversity gain*.

TABLE 3. **Characteristics of the optimized array of infinitesimal dipoles as a function of shape.**

Shape	Surface	Optimum number of ID	Critical Density (ID/λ^2)
Circle	$0.38\lambda^2$	10	26
Square	$0.38\lambda^2$	12	31
Triangle	$0.38\lambda^2$	13	34

To produce a relevant comparison between three different 2-dimensional topologies (in what follows, the square, the circle and the triangle are chosen as examples of this type), we consider the total same surface area of $0.38\lambda^2$ in all geometrical configurations. For each array type, the optimization algorithm above has been utilized in order to find the critical number of inserted infinitesimal dipoles needed to maintain a diversity gain higher than 0.8. In Table 3, we summed up all the information about the different shapes. *Notice that for the same surface area, a triangular antenna array can accommodate more antennas than square or circular shapes for the same diversity gain performance.*

D. VOLUMETRIC ARRAY TYPE

1) CUBIC MIMO ARRAY TOPOLOGY

For the second type of examples, we move into 3-dimensional-type antenna array systems. Here, two cubes with different dimensions, namely side lengths equal to 0.35λ and 0.5λ, are considered. For a cube side of 0.35λ, the infinitesimal dipole positions obtained through the proposed design algorithm are given in Fig. 9. After 22 iterations, the diversity gain reaches the target value 0.8 by an array of 8 infinitesimal dipoles.

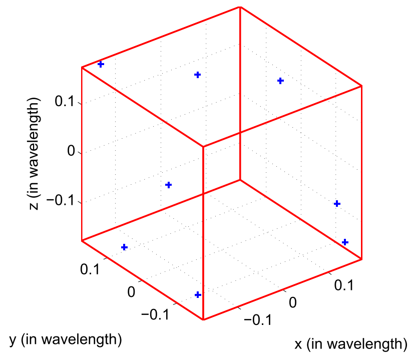

FIGURE 9. **Position of the infinitesimal dipoles inside a cube with a sidelength equals to 0.35λ.**

Next, the performance of the design process of the cubic array above is compared with a second example, also a cubic MIMO system but with a larger sidelength equals 0.5λ. The comparison is reported in Table 4, where it can be noticed here that the critical MIMO array density depends on the cube dimensions. Therefore, while Table 3 shows that in 2-dimensional arrays for the same surface area the critical MIMO array density (needed to obtain good design results) varies with the geometrical shape of the array, *the results of Table 4 suggests that for the same 3-dimensional shape the critical antenna density varies also with the dimension.*

TABLE 4. **Characteristics of the optimized array for a cube.**

Cube Dimensions	Optimum number of ID	Iterations	Diversity Gain	Critical Density (ID/λ^2)
$a = 0.35\lambda$	8	22	0.807	186
$a = 0.5\lambda$	13	75	0.8	104

2) SPHERICAL MIMO ARRAY TOPOLOGY

We continue to work with 3-dimensional arrays and consider a spherical layer with an inner and outer radii R_1 and R_2, respectively. For the first example, the inner radius R_1 is taken to be 0 while the outer radius R_2 is 0.25λ (full sphere.) In Fig. 10, the positions of the optimized infinitesimal dipoles are given together with the problem's geometry. After several experimental tests, the critical number of infinitesimal dipoles that can be inserted into this geometry was found to be 8, corresponding to a diversity gain of 0.8 reached after 42 iterations, with corresponding critical density of 122 dipoles/λ^3. For a cube with the same volume (i.e., cube with a sidelength of 0.4λ), our investigations have demonstrated that the critical number of IDs is 11, giving a density of 172 dipoles/λ^3. Hence, we may conclude that *for the same volume and the same diversity gain, a cubic MIMO array can accommodate more dipoles than a spherical array shape*.

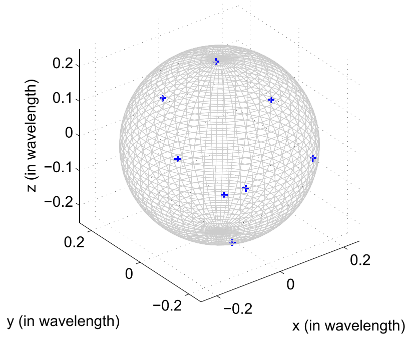

FIGURE 10. Position of the infinitesimal dipoles inside a sphere with $R_1 = 0$ and $R_2 = 0.25\lambda$.

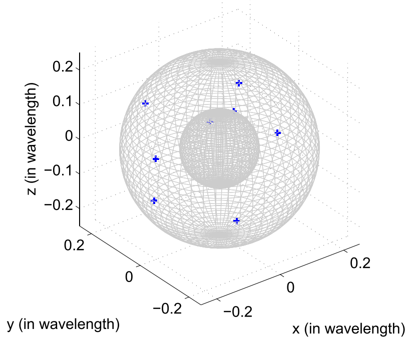

FIGURE 11. Position of the infinitesimal dipoles inside a sphere with $R_1 = 0.1\lambda$ and $R_2 = 0.25\lambda$.

As an another example involving spheres, we consider a spherical layer with $R_1 = 0.1\lambda$ and $R_2 = 0.25\lambda$. Fig. 11 illustrates the geometry and the best positions of the infinitesimal dipoles obtained through optimization. As in the previous example, the critical number of infinitesimal dipoles needed for a diversity gain of at least 0.8 turned out to be also equal to 8, and this is reached after 46 iterations. In table 5, the design data of the last two examples are summarized.

TABLE 5. Characteristics of the optimized array for a sphere.

Dimension of the sphere	Optimum number of ID	Number of iterations	Diversity Gain	Critical Density (ID/λ^2)
$R_1 = 0$ $R_2 = 0.25\lambda$	8	42	0.807	122
$R_1 = 0.1$ $R_2 = 0.25\lambda$	8	46	0.8	130

E. CONFORMAL ARRAYS FOR AVIONIC AND MISSILE SYSTEMS

MIMO systems can be used in many applications, involving wireless communications for cellular networks, or in avionic and missile systems. In the latter type of applications, in order to reduce the volume of the device, the antenna elements have to physically fit into the aircraft or missile support platform upon which they are mounted. In order to demonstrate the usefulness of the design methodology proposed in this paper, we provide here an example involving a more complex geometrical configuration of a MIMO array distribution in space. We choose as *conformal* surface an elliptic paraboloid (Fig. 12), which can be defined as the surface described by the equation

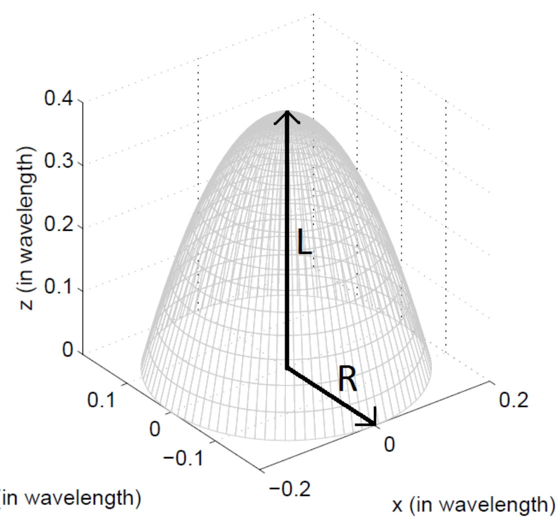

FIGURE 12. Geometry of the elliptic paraboloid used as a conformal array.

$$\frac{X^2}{a^2} + \frac{y^2}{a^2} = \frac{z}{L}, \tag{26}$$

where, a is the radius of the base and L is the overall length. This will serve as a generic geometrical model of a typical aircraft or missile nose. The goal is to design an optimum MIMO array with elements placed anywhere on the surface of the avionic system nose such that the diversity gain of the total array is maximal. Note that this is an example of a constrained optimization problem where while the IDs are free to explore the position space of the problem, each antenna must remain on the allocated surface.

In the first example, the missile or aircraft nose dimensions are chosen as $a = 0.2\lambda$ and $L = 0.4\lambda$. Initially, we consider a fixed orientation for all dipoles (i.e., $\theta_{\text{id}} = \varphi_{\text{id}} = 0$). Using the GA-ID approach, the critical number of infinitesimal dipoles we can place on the surface area turned out to be 4. Fig. 13 presents the conformal shape with the positions of the infinitesimal dipoles. A diversity gain of 0.8 is reached after 43 iterations. Note that because the antennas' directions have been fixed, a relatively large separation of elements was needed in this case to obtain good diversity gain performance.

In the second example, we keep the same geometry but explore a more realistic situation in which the relative antenna orientations are forced to conform to the surface platform of the avionic nose. Indeed, usually in conformal arrays each antenna is tangent to the platform structure, which makes the orientation of the antenna dependent on the local curvature of the physical support structure. In order to take this local orientation of the platform into account, we calculate for each

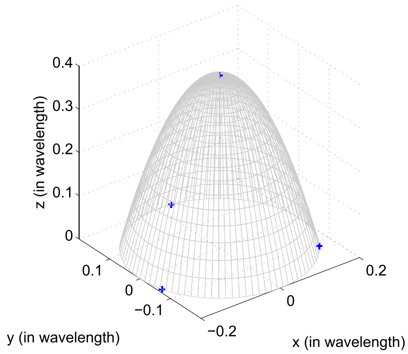

FIGURE 13. Position of the infinitesimal dipoles on the conformal structure ($a = 0.2\lambda$ and $L = 0.4\lambda$) for a fix orientation ($\theta_{ID} = 0$ and $\varphi_{ID} = 0$).

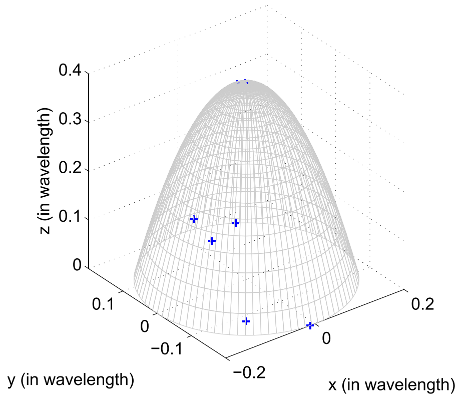

FIGURE 15. Position of the infinitesimal dipoles on the conformal structure ($R = 0.2\lambda$ and $L = 0.4\lambda$) for an optimized orientation.

antenna position the tangent at the surface on this point and orient the infinitesimal dipole along this tangent. In other words, we add orientation constraints to the optimization process in addition to the position constraints mentioned in the previous paragraph. It is also possible to choose the antenna to be normal to the surface or tilted with any angle; the horizontal direction has been chosen here for convenience. Fig. 14 shows the optimized positions of the dipoles where it is found that the critical number of antennas needed in this case to obtain good diversity gain performance is 5. We note that relative to position-only conformal optimization, conformal position-and-orientation optimization did not lead to a significant reduction of the MIMO array size or to a reduction in the critical antenna density.

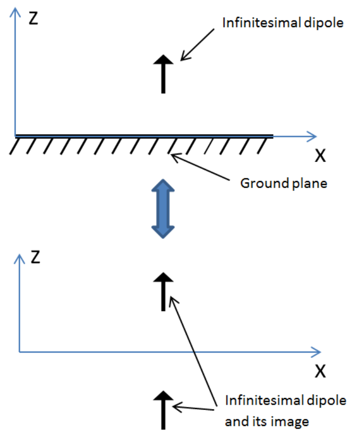

FIGURE 16. Schematic representation of image theory.

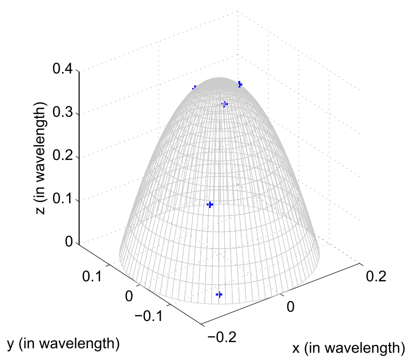

FIGURE 14. Position of the infinitesimal dipoles on the conformal structure ($a = 0.2\lambda$ and $L = 0.4\lambda$) for a fix orientation (conformal orientation).

Finally, if we consider a completely unrestricted orientation for each MIMO antenna, Fig. 15 shows that the critical number of conformal infinitesimal dipoles that can be placed on the platform is equal to 8. This example demonstrates again that the polarization of the MIMO antennas (relative orientations with respect to other antennas) is an important parameter in the design of MIMO system. Indeed, *with a proper antenna orientations optimization, we can significantly increase the density of the array compared with the standard examples in literature where isotropic (scalar) antennas are usually assumed. We also note how the inclusion of the dipole orientations in the design process helped bringing down the total size of the MIMO array.*

VIII. GROUND PLANE CONSIDERATION

A. MOTIVATIONS

In practice, many antennas are frequently located over a ground plane, which introduces significant modification in the electromagnetic aspects of the system such as radiation pattern, scattering parameters, gain, and so on. For our application under consideration, changes in the array's radiation pattern caused by the presence of ground plane are fundamental and cannot be ignored. In order to make the algorithm proposed in this paper applicable to a wide range of practical MIMO system, we here describe how to explicitly take into account the impact of such ground plane on the diversity gain and antenna array density. First, the necessary electromagnetic theory allowing us to incorporate ground plane effects into the calculation of the diversity gain will be presented and validated with a simple example. In the second part, we apply this theory to the optimization of a planar antenna array over an infinite ground plane.

To take into account the electromagnetic ground plane effect, we employ *image theory*, which states that for each

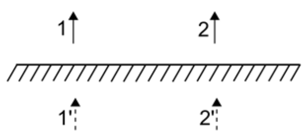

FIGURE 17. Schematic representation of two infinitesimal dipoles over a ground plane.

antenna over a ground plane the equivalent configuration is a two-antenna array this time radiating in free space, where the second antenna is the exact mirror of the first one [8], [9]. As can be seen from Fig. 16, when there is more than one antenna above the ground plane, it is necessary to pair each original antenna together with its "image" (beneath the ground plane) and treat the whole as a "single antenna system." Therefore, antenna pairs will be taken as the basic elements of the new array obtained after applying image theory. Because of this modification, the correlation matrix R of the new system, i.e., the original array plus the ground plane, will corresponds to an *enlarged* or *augmented* array comprised of the original one with its mirror image stacked together in *free space*. The advantage of using image theory is that we can still use exactly the same theory developed in Sec. III, in which each infinitesimal dipole radiates in infinite and homogeneous free space environment, allowing us then to continue to use the exact analytical expressions of the dipole radiation field and hence the cross-correlation Green's function.

B. THE CORRELATION IMAGED SUB-MATRIX METHOD

Each coefficient ρ_{mn} of the cross-correlation matrix R (20) will be modified by the presence of the ground plane. Careful analysis shows that the change of the numerical values of ρ_{mn} can be taken care of by the incorporation of a "sub-cross-correlation matrix" including each antenna and its images. The calculation of each coefficient of the *modified* cross-correlation matrix ρ'_{mn} is accomplished with the help of the following expression

$$\rho'_{i,j} = \sum_{m=i}^{i'} \sum_{n=j}^{j'} \rho_{mn}, \tag{27}$$

where i' the index of the the image of the dipole i, and, similarly, j' is the index of the image of the dipole j. For example, for the case depicted in Fig. 17, where we consider two infinitesimal dipoles over an infinite ground plane, the cross-correlation matrix R will be expressed as follow

$$\mathrm{R}' = \begin{pmatrix} \rho'_{11} & \rho'_{12} \\ \rho'_{21} & \rho'_{22} \end{pmatrix}, \tag{28}$$

with

$$\begin{aligned} \rho'_{11} &= \rho_{11} + \rho_{11'} + \rho_{1'1} + \rho_{1'1'}, \\ \rho'_{12} &= \rho_{12} + \rho_{12'} + \rho_{1'2} + \rho_{1'2'}, \\ \rho'_{21} &= \rho_{21} + \rho_{21'} + \rho_{2'1} + \rho_{2'1'}, \\ \rho'_{22} &= \rho_{22} + \rho_{22'} + \rho_{2'2} + \rho_{2'2'}, \end{aligned} \tag{29}$$

where $1'$ and $2'$ are, respectively, the images of the dipole 1 and 2 as shown in Fig. 17.

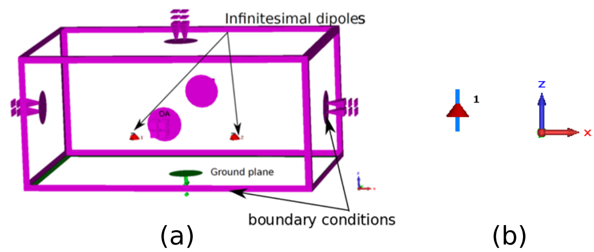

FIGURE 18. Simulation setup of two Infinitesimal Dipoles over an infinite ground plane (a) - Close-up view of an Infinitesimal Dipole on CST (b).

C. VALIDATION AND A MIMO ARRAY DESIGN EXAMPLES WITH GROUND PLANE

To verify the previous formulation, we consider two infinitesimal dipole antennas over a ground plane. The analytical model based on (22) and 28 are used to compute the correlation matrix R and the results are compared with those obtained using the full-wave FDTD electromagnetic solver CST Microwave Studio. Fig. 18 shows the CST model used for the FDTD simulation. Since FDTD is a differential equation solver, the infinite half space over the ground plane has to be truncated by an an electromagnetic radiation absorbing boundary condition (radiation box), the larger the box, the more accurate the results but slower the solution speed. The figure also shows the perfect electric conductor (PEC) boundary conditions of the electric wall (in Green's) representing the infinite ground plane and an open space (in purple) in all other directions. The Infinitesimal dipole itself is simply represented on CST by a lumped port with small dimensions (around 0.01λ).

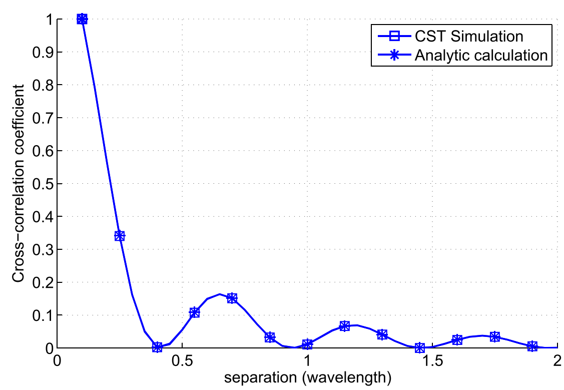

FIGURE 19. Variation of the cross-correlation coefficient in function of the separation between two infinitesimal dipoles in simulation and with an analytic calculation for ($\theta_{\mathrm{ID}_1} = 0, \varphi_{\mathrm{ID}_1} = 0$) and ($\theta_{\mathrm{ID}_2} = 0, \varphi_{\mathrm{ID}_2} = 0$).

In Fig. 19, the FDTD simulation's estimation of the cross correlation coefficient (12) and the analytic calculation using the formula (28) are compared as functions of the separation between the antennas. The two dipoles have the same orientation ($\theta_{\mathrm{id}} = 0$ and $\varphi_{\mathrm{id}} = 0$) and are located over a ground

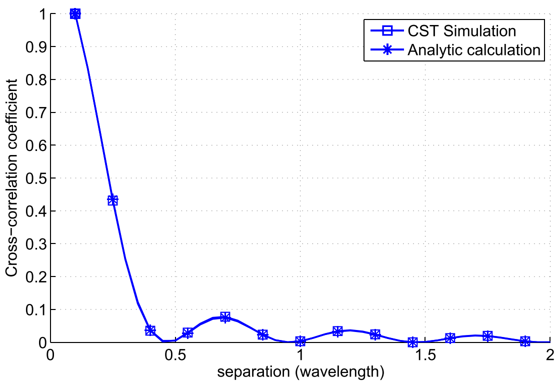

FIGURE 20. **Variation of the cross-correlation coefficient in function of the separation between two infinitesimal dipoles in simulation and with an analytic calculation for ($\theta_{\mathrm{ID}_1} = \pi/8$ and $\varphi_{\mathrm{ID}_1} = \pi/8$)) and ($\theta_{\mathrm{ID}_2} = \pi/4$ and $\varphi_{\mathrm{ID}_2} = \pi/4$).**

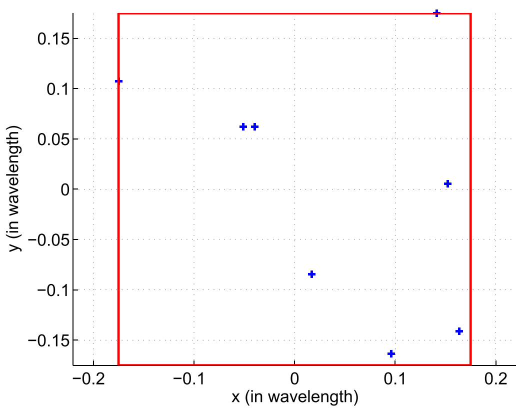

FIGURE 21. **Positions of the infinitesimal dipoles inside a square with a side of 0.35λ over an infinite ground plane, where each dipole is positioned at a distance 0.1λ from the ground plane. (Dipole orientations data are suppressed.)**

plane at a distance of 0.2λ. The theoretical calculation of the cross-correlation agrees perfectly with simulation result.

Fig. 20 presents the case when the two dipoles over the ground plane have been rotated. According to image theory, the image of an antenna rotated with the angle $(\theta_{\mathrm{id}}, \varphi_{\mathrm{id}} = 0)$ will have an orientation equals to $\theta_{\mathrm{ID}'_1} = -\theta_{\mathrm{ID}_1}$ and $\varphi_{ID_1} = \varphi_{\mathrm{ID}_1}$. The first dipole is rotated with an angle $\theta_{\mathrm{ID}_1} = \pi/8$ and $\varphi_{\mathrm{ID}_1} = \pi/8$ and the second dipole is rotated by $\theta_{\mathrm{ID}_2} = \pi/4$ and $\varphi_{\mathrm{ID}_2} = \pi/4$. Excellent agreement between theory and simulation is again observed, demonstrating that the sub-matrix method (28) can be used to describe arbitrary ID MIMO arrays with generic positions and orientations above the ground plane.

After validating imaged sub-matrix method, we apply the augmented ID MIMO formulation to the optimization of a planar antenna array over an infinite ground plane. Each infinitesimal dipole is located at 0.1λ from the ground, while the array shape considered here is square with side lengths of 0.35λ. In Fig. 21, the optimum positions of the infinitesimal dipoles over the infinite ground plane resulting in a diversity gain higher than 0.8 are given. In the design process, the antennas' relative polarizations are allowed to change. However, while the GA randomly modifies the orientation of each dipole, the design code automatically changes the direction of its associated imaged dipole internally and without assigning additional optimization parameters. Therefore, the numerical cost of optimization arrays while still slightly higher than the case of infinite space, it is still very efficient compared with full-wave analysis since the entire process of obtaining the radiation fields remains analytical.

IX. IMPACT OF MIMO ARRAY DENSITY ON DIVERSITY GAIN PERFORMANCE

Throughout the various design examples encountered so far, and especially in the case of conformal arrays, it could be noticed that the relative orientations of the MIMO antennas within a given geometrical array configuration may greatly impact the obtainable critical antenna density of the optimal array compatible with a satisfactory diversity gain performance. Because of the importance of this observation for the design and development of optimum MIMO systems, we further investigate in this section the influence of the different parameters of MIMO antenna arrays on the diversity gain.

The influence of the polarization on the diversity gain has already been observed for many years [5]. For example, [43] investigated the influence of antenna polarization on the diversity gain of a MIMO system in handset devices, where they compared this 'true polarization diversity' (TPD) to the conventional orthogonal polarization diversity (OPD). They showed that a proper polarization and position can double the diversity gain for the same volume. With the same idea, [44] and [46] found that design of a MIMO system with different radiation pattern or polarization for each antenna can significantly improve the diversity gain. However, to our best knowledge all such studies of the impact of antenna polarization on diversity gain published so far have been conducted while attention is focused on very specific and concrete applications. In this section, and continuing the overall spirit of the present paper, we attempt to provide a more generic and generalizable conclusions regarding this important design issue by applying the special method developed here, namely, the ID-CGF technique, to MIMO array design.

For brevity, we consider here a square shape in which the number of dipoles fitted into this region is increased in progressive fashion. The relative orientations of the composing ID antennas will be taken from the following four schemes:

- CASE I: A random positions for infinitesimal dipoles but with fixed orientation ($\theta_{\mathrm{id}} = 0$, $\varphi_{\mathrm{id}} = 0$).
- CASE II: Optimized positions for the infinitesimal dipoles but with no orientation variation ($\theta_{\mathrm{id}} = 0$, $\varphi_{\mathrm{id}} = 0$).
- CASE III: A random distribution (same distribution as the CASE I but now with an optimized orientations.)
- CASE IV: Optimized positions and orientations for the array of infinitesimal dipoles.

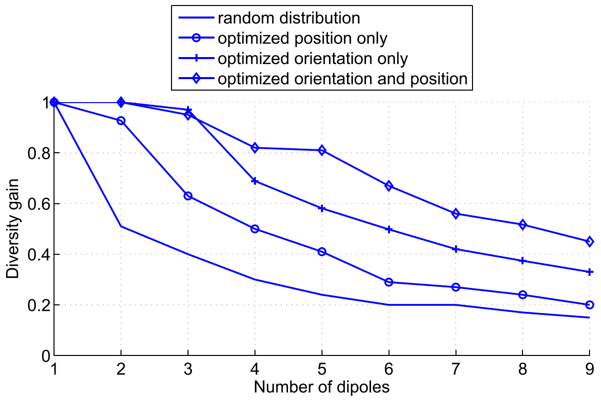

FIGURE 22. Variation of the diversity gain as function of the number of dipoles inside a square MIMO array with a sidelength 0.25λ.

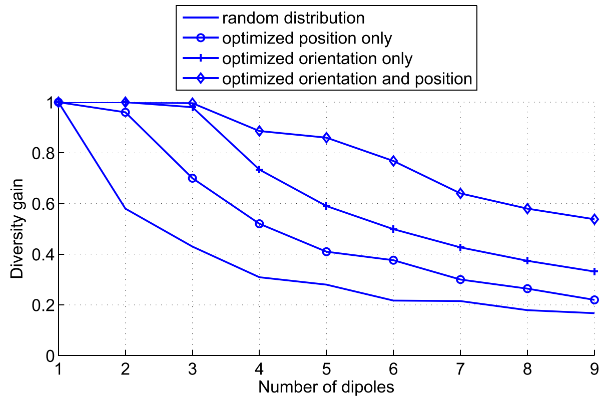

FIGURE 23. Variation of the diversity gain as function of the number of dipoles inside a cube MIMO array with a sidelength 0.25λ.

For the first example, a 2D square shape array with a sidelength of 0.25λ is considered, where the number of infinitesimal dipoles is increaseed progressively. Fig. 22 presents the variation of the diversity gain as function of the number of dipoles for the four different cases described above.[2] For a target diversity gain of 0.8, the maximum number of dipoles that can be inserted into array without degrading the diversity gain below the desired target is equal to 1 for CASE I, 2 for CASE II, 3 for CASE III, and 5 for CASE IV. *These results strongly suggest that the choice of antenna relative orientations (polarization) is the parameter which has the largest influence on the possibility of reaching a high diversity gain in the design of generic MIMO arrays.* From the same data, we can clearly see that in general increasing the number of elements inside an array of fixed size tend to degrade the diversity gain. This effect can be easily explained by the geometrical considerations of our study. Indeed in a classical antenna array with *fixed* inter-element spacings, the diversity gain will obviously increase when we increase the number of elements. However, in the present manuscript, such condition of fixed interspacing distance between the MIMO elements is no longer imposed since instead we are more interested in investigating antenna *density* as a design parameters. Furthermore, since for antenna engineers "array's size matters," it is of utmost importance to keep the overall surface or volume of the MIMO system at the smallest level possible. In this sense, the more elements we insert into a fixed volume or area, interspacing distances between the antennas will inevitably tend to decrease, leading to the onset of the well-known field-field correlations and hence immediately degrading the diversity gain. However, when considering another design parameter such as the antenna orientation, more spatial diversity in the antenna array can be achieved by exploiting such additional *spatial* electromagnetic degrees of freedom, and therefore it becomes possible to increase the array antenna density while keeping a good diversity gain.

[2]Notice that though a diversity gain for one-element array has no physical meaning, we have decided to arbitrarily and nominally set a diversity gain equal to one in this specific case in order to have a better estimation of the diversity gain variation in this section.

For completeness, we also carry out the same previous invistigation but for a volumetric array configuration, where the design a cube arrays with sidelength 0.25λ is investigated. Fig. 23 presents the variation of the diversity gain as function of the number of dipoles. As in the previous 2D case, we progressively increase the number of dipoles inside the array volume and find the maximum diversity gain obtained after 100 iterations. For the 3D case the same conclusions reached in the square array data of Fig. 22 where also found here. That is, the larger the number of optimization variables in the search algorithm, the larger the critical number of dipole that can be inserted into the array's volume order to reach a diversity gain of 0.8. In fact, in most of our extensive design examples, *it appears that the relative orientations of the infinitesimal dipoles appear to be more important than positions*.

Finally, we mention that if a very large number of IDs is considered, the optimization of the ID distribution (position and orientation) can also be seen as a way to optimize *continuous* surface current distributions, the ID array being treated this case as a "discretized approximating" of this continuous current [40]. Consequently, *the ability to exploit each dipole orientation in optimizing diversity gain already established above can be of considerable help if we want to implement continuous currents on specially-engineered printed antennas to meet very high-diversity-gain performance measures.* Such approach is beyond the scope of the present paper but seems to present a natural next step in continuing the research results of the current paper.

X. CONCLUSION

We presented a design algorithm to implement a general methodology capable of synthesizing surface or volume MIMO antenna arrays with optimum cross-correlation diversity gain performance. In particular, the method can deal with arbitrarily-shaped MIMO arrays, including conformal arrays. The proposed design methodology is based on a new formulation of the cross-correlation problem in which far-field cross correlation's calculation is transferred from the far-zone to the antennas themselves using the concept of cross-correlation Green's function. Based on this idea, a global optimization technique, here the Genetic Algorithm (GA), was applied to directly modify the inter-spacing distances and relative orientations of the array elements. The paper provided extensive numerical examples to validate the method, including circular, cubic, and conformal array shapes. Moreover, a special routine using image theory was proposed and validated in order to generalize the design method to the practical case when ground plane is present in the MIMO array environment.

Some of the findings include the identification of the importance played by the MIMO antenna density in the design process. Indeed, one of the main goals of the paper was discovering empirical bounds on how far the diversity gain of generic MIMO antenna arrays can be improved by inserting more antennas into a given area/volume and for a fixed topology. It was found that different MIMO arrays with different typologies correspond to different critical antenna densities. Moreover, the relative orientations of the MIMO array antennas were found to be important. In particular, design rules and observations regarding how these two factors (and others) affect the practical attainability of optimum MIMO systems had been thoroughly documented and discussed throughout the paper. Currently, implementation of new MIMO antennas inspired by configurations obtained by the method of this paper, e.g., using slots on a waveguide or 3D printing and printed-circuit antenna technology are being explored. Moreover, the design algorithm itself can be extended in a straightforward manner to antennas with arbitrary size by merely increasing the computational cost but without modification in the basic concept as such.

In the main, the present work has amply demonstrated the fact that taking into account spatial aspects in the electromagnetic problem, here the interrelation between the far-field correlation and the antenna geometrical/polarization structure, can enhance the performance of communication systems without the need to introduce additional signal processing at the Tx/Rx terminals. It also showed that steady improvement of diversity gain will always run into fundamental upper bounds or limits set by the geometrical configuration of the antenna array.

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