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Adaptive Cooperative Tracking Control of Multi-Agent Systems With Unknown Actuators Hysteresis

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ABSTRACT This paper considers cooperative tracking control of nonlinear multi-agent systems with actuators hysteresis over digraphs. Each agent is modeled by a higher-order nonlinear system in strict-feedback form with the generalized Prandtl–Ishlinskii hysteresis input and unknown time-varying virtual control coefficients. An online adaptive law is introduced for compensating the unknown hysteresis effect. A special Nussbaum-type function is developed to handle unknown time-varying virtual control coefficients. A switching mechanism is utilized to combine the neural networks approximation with an extra robust term, which can take over the authority outside the neural active region. In this sense, the globally uniformly ultimately bounded stability is guaranteed by the proposed distributed adaptive control law. Moreover, all agents ultimately synchronize to the leader node with bounded residual errors. Simulation results justify the proposed algorithm.

INDEX TERMS Adaptive cooperative tracking, multi-agent systems, Nussbaum-type function, Prandtl-Ishlinskii hysteresis.

I. INTRODUCTION

Distributed cooperative control of multi-agent systems have been extensively investigated in the past two decades, mainly due to its ubiquitous applications ranging from the macroscopic area such as formation of satellites, to the micro-/nano-scale field such as diagnosis of nanobots [1]–[3]. The majority of cooperative control problems assume ideal actuators, which means the structure of the controllers do not have hard nonlinearity [4]. The recent years have witnessed a significant interest in cooperative control of multi-agent systems preceded by input nonlinearities, such as input saturation [5], dead-zone [6], hysteresis [7]. It is noteworthy that hysteresis nonlinearities often occur in practice, especially when the actuators are made of smart materials including but not limited to piezoelectric ceramics, or giant magnetostrictive materials [8], [9]. Smart material-based actuators with ultra-high precision and rapid response have broadly potential applications in multi-agent systems, such as micro unmanned aerial vehicles for high-precision cooperative tracking, multiple robots for cooperatively biological exploration, multi-manipulators for micromanipulation and microassembly [10], [11]. However, the actuators hysteresis pose significant challenges in controlling of multi-agent systems.

For control-oriented purpose, hysteresis characteristics, whose input-output relations have memory effect, are generally described by mathematical or phenomenological models, such as the Prandtl-Ishlinskii (P-I) model and the Bouc-Wen model [12]. Since the phenomenon-based P-I model formulates the hysteresis loops by selecting the density function, and hence the P-I model can describe more detailed hysteresis characteristics, which is often used to model hysteresis phenomena within a wide range of real physical systems [12]–[14]. In order to eliminate the effect of hysteresis constraints, one approach is to construct an inverse compensator, which employs an analytical hysteresis inverse function associated to the specific model [13]. Alternatively, adaptive methods with respect to online parameter estimation that do not need an analytical inverse model were developed in [14]–[18]. In [14], a robust adaptive control was developed for eliminating the P-I hysteresis effect without the hysteresis inverse. The adaptive neural networks (NN) control schemes with P-I hysteresis parameter estimation were developed for time-delay and MIMO nonlinear systems in [15]–[17]. Moreover, an adaptive output feedback control using backstepping approach was studied for higher-order nonlinear system with P-I hysteresis input in [18]. Additionally, to describe the more general hysteresis phenomena, a class of generalized P-I hysteresis model that can adjust the density function and hysteresis input function was presented in [19], and an adaptive NN control algorithm with online hysteresis parameter estimation was designed for pure-feedback nonlinear system [20]. An intriguing work [21] proposed a minimal learning parameters-based adaptive NN control algorithm with a concise scheme for nonlinear time-delay system with the generalized P-I hysteresis. It is worth noting that all the aforementioned works only consider a single system instead of multi-agent systems.

On the other hand, cooperative tracking control of higher-order nonlinear multi-agent systems has attracted significant attention for the past few years, and adaptive NN methods are often employed to deal with the agents' unknown dynamics [5]–[7], [22]–[25]. It is well-known that the NN approximation only works on a prescribed compact set called neural active region, which only guarantees semi-globally uniformly ultimately bounded (SGUUB) stability. A switching mechanism was proposed in [26] to combine a NN approximation term and an additional robust term pulling back into the neural active region form outside, which made for a globally uniformly ultimately bounded (GUUB) stability. And this switching mechanism was later modified in [27] for a single system. To avoid the "explosion of complexity" of backstepping technique, dynamic surface control (DSC) or tracking differentiator (TD) were recently applied to eliminate repeated differentiations for higher-order nonlinear multi-agent systems [5], [28], [29]. In addition, works in [29] pointed out that the dynamic surface filter is sensitive to the design constant which may cause system instability or chattering, and inevitably excite set-point jumping during the initial phase, so the second-order TD which has characteristic of the well filtering precision and fast convergence is more suitable for higher-order multi-agent systems. In most existing works on multi-agent systems [5]–[7], [28], [29], the signs of virtual control coefficients with constants were assumed to be known a priori. But most often time, it is difficult to obtain in practice. Fortunately, the Nussbaum-type function can effectively solve the problem of unknown virtual control coefficient for controller design of nonlinear systems [30]. With the seminal work in [31], a distributed adaptive controller with a specific Nussbaum-type function was proposed for low-order multi-agent systems such that leaderless consensus was achieved. It was later studied for leader-following tracking and cooperative regulation problems of higher-order multi-agent systems with unknown constant virtual control coefficients in [32]–[34]. Notice that when there is no a priori knowledge about the signs of unknown time-varying virtual control coefficients, distributed control design of such multi-agent systems becomes much more difficult, and the results in [31]–[34] are no longer applicable. This is another challenging problem to be conducted for the multi-agent systems in a general strict-feedback form with unknown time-varying virtual control coefficients.

This above discussion motivates our present work. In this paper, we investigate cooperative tracking control of nonlinear multi-agent systems in a general strict-feedback form with actuators hysteresis constraints. The generalized P-I model is adopted, which is based on the fact that it can describe a typical hysteresis behavior. The interaction network is expressed by a directed graph. This paper proposes a distributed adaptive control law to achieve cooperative tracking with bounded residual errors, as well as guaranteeing the GUUB stability.

Main contributions of this paper are highlighted as follows.

(i) To the best of our knowledge, cooperative tracking control of nonlinear multi-agent systems with generalized P-I hysteresis inputs has been insufficiently investigated thus far. Compared with the existing results [22]–[25], [28], [29], the agents' dynamics are extended to general higher-order nonlinear systems with generalized P-I hysteresis actuators in the strict-feedback form.

(ii) Unlike the existing works such as [5]–[7] and [31]–[34] that assume the virtual control coefficients to be all ones or unknown constants, this paper considers the rather general multi-agent systems with unknown time-varying virtual control coefficients with unknown signs. With the use of the multiple Nussbaum-type functions, a novel technique and tool is developed for the distributed control design and stability analysis.

(iii) This paper proposes a modified distributed control law with a switching mechanism, such that the constraint of conventional distributed NN control law which only works within the neural active region can be further relaxed, in contrast with the majority of neuro-adaptive cooperative tracking results [5]–[7], [22]–[25], [28], [29]. In this sense, the designed distributed controller can guarantee the GUUB stability.

The rest of this paper is organized as follows. Some preliminaries and problem formulation are introduced in Section II. A distributed adaptive control design and stability analysis are presented in Section III. Simulation example is given in Section IV. Section V concludes this paper.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. GRAPH THEORY

An interaction network of multi-agent systems can be represented by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with the finite nodes set $\mathcal{V} = \{v_1, \ldots, v_N\}$ and the edges set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where $\times$ is the Cartesian product. The node v_j is a neighbor of node v_i if $(v_j, v_i) \in \mathcal{E}$, and the indices set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{j \mid (v_j, v_i) \in \mathcal{E}\}$. Let $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ be an adjacency matrix, where $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. We assume that the graph has no self-loops, i.e., $a_{ii} = 0$. Let $d_i = \sum_{j=1}^{N} a_{ij}$ and $D = \mathrm{diag}(d_1, \ldots, d_N) \in \mathbb{R}^{N\times N}$ be the in-degree of node v_i and in-degree matrix, respectively. Then the graph Laplacian matrix is defined as $L = D - A$. A sequence of edges $(v_i, v_{i_l}), (v_{i_l}, v_{i_m}), \ldots, (v_{i_n}, v_j)$ from node v_i to v_j implies the directions of information flows in a directed graph. A digraph has a spanning tree, if there is a directed path from a root node to any other nodes. Define $B = \mathrm{diag}(b_1, \ldots, b_N) \in \mathbb{R}^N$, where $b_i > 0$ if the node v_i can receive the information from the leader node v_0, otherwise $b_i = 0$. Let $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$ be an augmented graph including the nodes set $\bar{\mathcal{V}} = \{v_0, v_1, \ldots, v_N\}$ and the edges set $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$.

B. GENERALIZED PRANDTL-ISHLINSKII HYSTERESIS

Consider a generalized P-I hysteresis model for describing typical hysteresis behaviors, and its input-output relationship has memory effect [19]. Its output $o_i(t) \in \mathbb{R}$ is determined not just by the current input $u_i(t) \in \mathbb{R}$ but also by the history of input $\{u_i(\tau), \tau \in (0, t]\} \subset \mathbb{R}$. By defining $o_i(t) = H_i\Big(\{u_i(\tau), \tau \in (0, t]\}\Big) := H_i[u_i](t)$, a generalized P-I hysteresis model can be given as

$$o_i(t) := H_i[u_i](t) = \hbar_i\left(u_i(t)\right) - \int_0^{h_0} p_{0i}(r) f_r[u_i](t)dr, \quad (1)$$

where the hysteresis play operator $f_r[u_i](t)$ is inductively defined as

$$f_r[u_i](0) = h_r\big(u_i(0), 0\big),$$
$$f_r[u_i](t) = h_r\Big(u_i(t), f_r[u_i](t_\iota)\Big),$$

and $h_r(h_1, h_2) = \max\Big(h_1 - r, \min(h_1 + r, h_2)\Big)$ with threshold $r \geq 0$ for $t_\iota < t \leq t_{\iota+1}$ and $0 \leq \iota \leq \bar{\iota} - 1$; $0 = t_0 < t_1 < \cdots < t_{\bar{\iota}}$ is a partition of $[0, t_{\bar{\iota}}]$ such that $u_i(t)$ is monotone on each of subintervals $(t_\iota, t_{\iota+1}]$; the density function $p_{0i}(r) \in \mathbb{R}$ vanishes for a large value of constant h_0; $\int_0^{h_0} p_{0i}(r) f_r[u_i](t)dr$ is the Lipschitz continuous operator; and $\hbar_i(u_i) : \mathbb{R} \to \mathbb{R}$ is the hysteresis input function. This model satisfies the following assumptions [20].

Assumption 1: The unknown density function satisfying with $p_{0i}(r) \geq 0$ and $\int_0^{\infty} r p_{0i}(r)dr < \infty$ has an upper bound. There exists a positive constant $\bar{p}_{0i}$ such that $p_{0i}(r) \leq \bar{p}_{0i}$ for all $r \in [0, h_0]$. □

Assumption 2: The locally Lipschitz non-decreasing function $\hbar_i(u_i)$ is smooth and odd, which satisfies $\lim_{u_i \to \infty} \hbar_i(u_i) = \infty$, and there exist positive constants $\underline{\hbar}_i$ and $\bar{\hbar}_i$ such that $\underline{\hbar}_i \leq \hbar_i^{(1)}(u_i) \leq \bar{\hbar}_i$, where $\hbar_i^{(1)}(u_i)$ is the first-order derivative with respect to u_i. □

C. GLOBAL NEURAL NETWORKS DESIGN

According to the universal approximation properties, NN is often utilized to approximate unknown nonlinear functions [35]. A continuous nonlinear function $F_{i,m}(\boldsymbol{Z}_{i,m}) : \mathbb{R}^m \to \mathbb{R}$ on a compact set $\Omega_{\boldsymbol{Z}_{i,m}} \subset \mathbb{R}^m$ can be expressed by

$$F_{i,m}(\boldsymbol{Z}_{i,m}) = \boldsymbol{W}_{i,m}^T \boldsymbol{\varphi}_{i,m}(\boldsymbol{Z}_{i,m}) + \epsilon_{i,m},$$

where $\boldsymbol{Z}_{i,m} = [Z_{i,1}, \ldots, Z_{i,m}]^T \in \mathbb{R}^m$ is the input vector of approximator; $\boldsymbol{\varphi}_{i,m}(\boldsymbol{Z}_{i,m}) : \mathbb{R}^m \to \mathbb{R}^{\upsilon_{i,m}}$ with neurons number $\upsilon_{i,m}$ is a basis functions vector; $\boldsymbol{W}_{i,m} \in \mathbb{R}^{\upsilon_{i,m}}$ is the ideal output weights vector; and $\epsilon_{i,m} \in \mathbb{R}$ is the approximation error. Moreover, suppose that the function $F_{i,m}(\boldsymbol{Z}_{i,m})$ are unknown and bounded by $|F_{i,m}(\boldsymbol{Z}_{i,m})| \leq \bar{F}_{i,m}(\boldsymbol{Z}_{i,m})$, where $\bar{F}_{i,m}(\boldsymbol{Z}_{i,m})$ is a known nonnegative smooth function.

Assumption 3: In this paper, to avoid the distraction from the main issues, a linear-in-parameter NN is considered, i.e., only the NN output weights with adjustable parameters are tuned. There exists an ideal weights vector $\boldsymbol{W}_{i,m}$ such that $||\boldsymbol{W}_{i,m}||^2 \leq \bar{w}_{i,m}$ with the constant $\bar{w}_{i,m} > 0$, and it makes a bounded approximation error $|\epsilon_{i,m}| \leq \bar{\epsilon}_{i,m}$ with $\bar{\epsilon}_{i,m} > 0$. □

In the conventional adaptive NN control design, the approximation

$$\hat{F}_{i,m}(\boldsymbol{Z}_{i,m}) = \hat{\boldsymbol{W}}_{i,m}^T \boldsymbol{\varphi}_{i,m}(\boldsymbol{Z}_{i,m}) \quad (2)$$

is often used, where $\hat{\boldsymbol{W}}_{i,m}$ is the estimate of $\boldsymbol{W}_{i,m}$, and it only works on a compact set $\Omega_{\boldsymbol{Z}_{i,m}}$ with semi-global ability. It means that the adaptive NN approximator (2) is disabled when the neural active region is no longer remained. Prior to proceed to obtain global NN design, let constants $\bar{r}_{i,m} > \underline{r}_{i,m} > 0$ be the boundaries of the compact sets $\Omega_{\bar{r}_{i,m}}$ and $\Omega_{\underline{r}_{i,m}}$, respectively. Define the following switching function as

$$S_{i,m}(\boldsymbol{Z}_{i,m}) := \prod_{\ell=1}^{m} s_{i,\ell}(Z_{i,\ell}) \quad (3)$$

with

$$s_{i,\ell}(Z_{i,\ell}) = \begin{cases} 1, & \text{if } |Z_{i,\ell}| < \underline{r}_{i,m} \\ \dfrac{\bar{r}_{i,m}^2 - Z_{i,\ell}^2}{\bar{r}_{i,m}^2 - \underline{r}_{i,m}^2} e^{-\left(\frac{Z_{i,\ell}^2 - \underline{r}_{i,m}^2}{\bar{r}_{i,m}^2 - \underline{r}_{i,m}^2}\right)^2}, & \text{if } \underline{r}_{i,m} \leq |Z_{i,\ell}| \leq \bar{r}_{i,m} \\ 0. & \text{if } |Z_{i,\ell}| > \bar{r}_{i,m} \end{cases}$$

According to [26] and [27], the following global adaptive NN control design can be adopted to replace (2) to obtain the global ability

$$S_{i,m}(\boldsymbol{Z}_{i,m})\phi_{i,m}^{NN} + \Big(1 - S_{i,m}(\boldsymbol{Z}_{i,m})\Big)\phi_{i,m}^{Robust} \quad (4)$$

with

$$\phi_{i,m}^{NN} = \hat{\boldsymbol{W}}_{i,m}^T \boldsymbol{\varphi}_{i,m}(\boldsymbol{Z}_{i,m}),$$
$$\phi_{i,m}^{Robust} = \bar{F}_{i,m}(\boldsymbol{Z}_{i,m}) \tanh\Big(z_{i,m}\bar{F}_{i,m}(\boldsymbol{Z}_{i,m})/\varepsilon_{i,m}\Big),$$

where $\tanh(\cdot)$ is the hyperbolic tangent function; $\varepsilon_{i,m}$ is a positive constant; and the error variable $z_{i,m}$ will be introduced below in (7).

Notice that the switching function $s_{i,\ell}(Z_{i,\ell}) \in [0, 1]$, and $s_{i,\ell}(Z_{i,\ell}) = 1$ within the compact set $\Omega_{\underline{r}_{i,m}}$ and $s_{i,\ell}(Z_{i,\ell}) = 0$ outside $\Omega_{\bar{r}_{i,m}}$. The global adaptive NN design (4) includes the switching function $s_{i,\ell}(Z_{i,\ell})$, a NN approximation term $\phi_{i,m}^{NN}$ and a robust term $\phi_{i,m}^{Robust}$. Define compact sets $\Omega_{\underline{\boldsymbol{r}}_{i,m}} = \Omega_{\underline{r}_{i,1}} \times \cdots \times \Omega_{\underline{r}_{i,m}}$ and $\Omega_{\bar{\boldsymbol{r}}_{i,m}} = \Omega_{\bar{r}_{i,1}} \times \cdots \times \Omega_{\bar{r}_{i,m}}$. When $\Omega_{\boldsymbol{Z}_{i,m}} \subset \Omega_{\underline{\boldsymbol{r}}_{i,m}}$, the NN approximation term $\phi_{i,m}^{NN}$ plays a decisive role and the state falls into the compact set $\Omega_{\underline{\boldsymbol{r}}_{i,m}}$.

Once the state runs outside the $\Omega_{\bar{r}_{i,m}}$, the robust term $\phi_{i,m}^{Robust}$ works to pull the state back to $\Omega_{\bar{r}_{i,m}}$. If $\Omega_{\underline{r}_{i,m}} \subseteq \Omega_{Z_{i,m}} \subseteq \Omega_{\bar{r}_{i,m}}$, both terms (4) work and will drag the state into the compact set $\Omega_{\underline{r}_{i,m}}$. In this sense, the global ability will be obtained. More details can refer to [26] and [27] if needed.

D. NUSSBAUM-TYPE FUNCTION

A special Nussbaum-type function $\mathcal{N}(\xi_{i,m})$ is employed to handle the unknown time-varying virtual control coefficients, in which it has the following properties:

$$\lim_{s\to\infty}\sup \frac{1}{s}\int_0^s \mathcal{N}(\xi_{i,m})d\xi_{i,m} = +\infty,$$
$$\lim_{s\to\infty}\inf \frac{1}{s}\int_0^s \mathcal{N}(\xi_{i,m})d\xi_{i,m} = -\infty.$$

And a crucial technical lemma is developed for the distributed control design and stability analysis.

Lemma 1: For each m, suppose that $g_{i,m}(\boldsymbol{x}_{i,m})$ is an unknown time-varying function with the states vector $\boldsymbol{x}_{i,m} \in \mathbb{R}^m$, which satisfies $g_{i,m}(\boldsymbol{x}_{i,m}) \in [g_m^-, g_m^+]$ with $0 \notin [g_m^-, g_m^+]$, where $g_m^- = \min_{1\le i\le N}\{g_{i,m}\}$ and $g_m^+ = \max_{1\le i\le N}\{g_{i,m}\}$, and all $g_{i,m}(\boldsymbol{x}_{i,m})$ have the identical signs for all $i = 1, \ldots, N$. Let $V_m(t)$ and $\xi_{i,m}(t)$ be smooth functions defined on $[0, t_f)$ with $V_m(t) \ge 0$, $\forall t \in [0, t_f)$. Let $\mathcal{N}(\xi_{i,m})$ be defined by

$$\mathcal{N}(\xi_{i,m}) = \left(e^{\lambda_m\xi_{i,m}} + e^{-\lambda_m\xi_{i,m}}\right)\sin(\xi_{i,m})$$

with $\lambda_m > \frac{1}{\pi}\ln\frac{\bar{g}_m N}{\underline{g}_m} + \beta_m$, where $\underline{g}_m = \min_{1\le i\le N}\{\underline{g}_{i,m}\}$ with $\underline{g}_{i,m}$ being the lower bound of $|g_{i,m}(\boldsymbol{x}_{i,m})|$ and $\bar{g}_m = \max_{1\le i\le N}\{\bar{g}_{i,m}\}$ with $\bar{g}_{i,m}$ being the upper bound of $|g_{i,m}(\boldsymbol{x}_{i,m})|$, and the constant $\beta_m > 0$. If the following inequality holds:

$$V_m(t) \le \sum_{i=1}^N e^{-\beta_m t}\int_0^t g_{i,m}\big(\boldsymbol{x}_{i,m}(\tau)\big)\mathcal{N}\big(\xi_{i,m}(\tau)\big)\dot{\xi}_{i,m}(\tau)e^{\beta_m\tau}d\tau + \sum_{i=1}^N e^{-\beta_m t}\int_0^t \dot{\xi}_{i,m}(\tau)e^{\beta_m\tau}d\tau + \bar{\mu}_m, \quad \forall t \in [0, t_f) \tag{5}$$

where $\bar{\mu}_m$ is a bounded variable, then $V_m(t)$, $\xi_{i,m}(t)$, and $\sum_{i=1}^N e^{-\beta_m t}\int_0^t\left[g_{i,m}\big(\boldsymbol{x}_{i,m}(\tau)\big)\mathcal{N}\big(\xi_{i,m}(\tau)\big) + 1\right]\dot{\xi}_{i,m}(\tau)e^{\beta_m\tau}d\tau$ are bounded on $[0, t_f)$ for all $i = 1, \ldots, N$. □

Proof: See Appendix. ■

E. PROBLEM FORMULATION

Consider a multi-agent system consisting of one leader node and a group of N followers over a digraph. Dynamics of the ith ($i = 1, \ldots, N$) agent in a strict-feedback form with a hysteresis actuator is described as

$$\begin{aligned}\dot{x}_{i,m'} &= f_{i,m'}(\boldsymbol{x}_{i,m'}) + g_{i,m'}(\boldsymbol{x}_{i,m'})x_{i,m'+1} + \zeta_{i,m'},\\ \dot{x}_{i,M} &= f_{i,M}(\boldsymbol{x}_{i,M}) + g_{i,M}(\boldsymbol{x}_{i,M})H_i[u_i](t) + \zeta_{i,M},\\ y_i &= x_{i,1},\end{aligned} \tag{6}$$

where $m' = 1, \ldots, M-1$; $x_{i,m} \in \mathbb{R}$ is the mth state ($m = 1, \ldots, M$); $\boldsymbol{x}_{i,m} = [x_{i,1}, \ldots, x_{i,m}]^T \in \mathbb{R}^m$; $f_{i,m}(\boldsymbol{x}_{i,m})$, $g_{i,m}(\boldsymbol{x}_{i,m}) : \mathbb{R}^m \to \mathbb{R}$ are unknown nonlinear functions assumed to be locally Lipschitz; u_i is the control input; the actuator $H_i[u_i](t)$ is in the generalized P-I hysteresis form (1); $\zeta_{i,m} \in \mathbb{R}$ are unknown external disturbances; and $y_i \in \mathbb{R}$ is the output.

Control Objectives: The reference trajectory of leader node is denoted by $y_0(t)$. In this paper, we aim to design a distributed control law u_i for the multi-agent system (6) such that the following objectives are achieved:

(i) All signals of the closed-loop system are globally uniformly ultimately bounded.
(ii) All followers ultimately synchronize to the leader node with bounded residual errors, i.e., the tracking error $\delta_i = y_i - y_0$ converges to a small neighborhood of the origin for all $i = 1, \ldots, N$.

The following assumptions are made.

Assumption 4: The functions $f_{i,m}(\boldsymbol{x}_{i,m})$ are unknown and bounded by $|f_{i,m}(\boldsymbol{x}_{i,m})| \le \bar{f}_{i,m}(\boldsymbol{x}_{i,m})$, where $\bar{f}_{i,m}(\boldsymbol{x}_{i,m})$ are known nonnegative smooth functions. □

Assumption 5: For each m, the unknown virtual control coefficients $g_{i,m}(\boldsymbol{x}_{i,m})$ for all i are strictly either positive or negative with unknown signs, and there exist known positive constants $\underline{g}_{i,m}$ and $\bar{g}_{i,m}$ such that $\underline{g}_{i,m} \le |g_{i,m}(\boldsymbol{x}_{i,m})| \le \bar{g}_{i,m}$. □

Remark 1: Assumption 5 is reasonable because $g_{i,m}(\boldsymbol{x}_{i,m})$ being away from zero is the controllable condition, and it implies that all $g_{i,m}(\boldsymbol{x}_{i,m})$ have unknown identical control directions, which is made in most control systems [20], [30], [31]. □

Assumption 6: The external disturbances $\zeta_{i,m}$ are bounded by $|\zeta_{i,m}| \le \bar{\zeta}_{i,m}$ for some positive constant $\bar{\zeta}_{i,m}$. □

Assumption 7: The leader's trajectory $y_0 \in \mathbb{R}$ is continuously differentiable, and the available y_0, $\dot{y}_0$ are bounded. □

Assumption 8: The augmented digraph $\bar{\mathcal{G}}$ contains a spanning tree with the unique root node being the leader v_0. □

III. ADAPTIVE CONTROL DESIGN

A. DISTRIBUTED CONTROL LAW DESIGN

In this subsection, a distributed adaptive control law for each agent is proposed. The backstepping incorporating with tracking differentiator (TD) is used to avoid the complexity of repeated differentiation. The design procedure contains M steps. First, the error variables are introduced as follows

$$\begin{aligned} z_{i,1} &= \sum_{j\in\mathcal{N}_i} a_{ij}(y_i - y_j) + b_i(y_i - y_0),\\ z_{i,k'} &= x_{i,k'} - \alpha_{i,k'-1}, \quad (k' = 2, \ldots, M)\end{aligned} \tag{7}$$

where $\alpha_{i,k'-1}$ is the designed intermediate controller, and it will be presented later.

Step 1: The derivative of $z_{i,1}$ is

$$\dot{z}_{i,1} = (d_i + b_i)f_{i,1}(x_{i,1}) - \sum_{j\in\mathcal{N}_i} a_{ij}\Big(f_{j,1}(x_{j,1}) + g_{j,1}(x_{j,1})x_{j,2}\Big)$$

$$+(d_i+b_i)g_{i,1}(x_{i,1})(z_{i,2}+\alpha_{i,1})+(d_i+b_i)\zeta_{i,1} - \sum_{j\in\mathcal{N}_i} a_{ij}\zeta_{j,1} - b_i\dot{y}_0.$$

Using NN to approximate the unknown function, we obtain

$$\begin{aligned} F_{i,1}(\mathbf{Z}_{i,1}) &= (d_i+b_i)f_{i,1}(x_{i,1}) - \sum_{j\in\mathcal{N}_i} a_{ij}\Big(f_{j,1}(x_{j,1})+g_{j,1}(x_{j,1})x_{j,2}\Big) \\ &= \mathbf{W}_{i,1}^T\boldsymbol{\varphi}_{i,1}(\mathbf{Z}_{i,1})+\epsilon_{i,1}, \end{aligned}$$

where $\mathbf{Z}_{i,1}=[x_{i,1},x_{j,1},\ldots,x_{j,2}]^T$ within a compact set $\Omega_{\mathbf{Z}_{i,1}}$, and there exists a known smooth function $\bar{F}_{i,1}(\mathbf{Z}_{i,1})$ such that $|F_{i,1}(\mathbf{Z}_{i,1})|\le \bar{F}_{i,1}(\mathbf{Z}_{i,1})$ by Assumptions 4 and 5. Considering the switching function $S_{i,1}(\mathbf{Z}_{i,1})$ defined as (3), we obtain

$$\begin{aligned} \dot{z}_{i,1} &= S_{i,1}(\mathbf{Z}_{i,1})\mathbf{W}_{i,1}^T\boldsymbol{\varphi}_{i,1}(\mathbf{Z}_{i,1})+\Big(1-S_{i,1}(\mathbf{Z}_{i,1})\Big)F_{i,1}(\mathbf{Z}_{i,1}) \\ &\quad + (d_i+b_i)g_{i,1}(x_{i,1})(z_{i,2}+\alpha_{i,1})+\rho_{i,1}-b_i\dot{y}_0, \end{aligned} \tag{8}$$

where $\rho_{i,1}=S_{i,1}(\mathbf{Z}_{i,1})\epsilon_{i,1}+(d_i+b_i)\zeta_{i,1}-\sum_{j\in\mathcal{N}_i}a_{ij}\zeta_{j,1}$ is a bounded variable assumed to be bounded by $0<|\rho_{i,1}|\le\bar{\rho}_{i,1}$. For the simplicity, the states vector $\mathbf{Z}_{i,m}$ will be omitted from the corresponding functions $F_{i,m}(\mathbf{Z}_{i,m})$, $\bar{F}_{i,m}(\mathbf{Z}_{i,m})$, $\boldsymbol{\varphi}_{i,m}(\mathbf{Z}_{i,m})$, and $S_{i,m}(\mathbf{Z}_{i,m})$ in the following analysis.

Design the intermediate controller $\alpha_{i,1}$ and adaptive laws as

$$\begin{aligned} \eta_{i,1} &= c_{i,1}z_{i,1}+S_{i,1}z_{i,1}\hat{w}_{i,1}\boldsymbol{\varphi}_{i,1}^T\boldsymbol{\varphi}_{i,1} \\ &\quad +(1-S_{i,1})\bar{F}_{i,1}\tanh(z_{i,1}\bar{F}_{i,1}/\varepsilon_{i,1}) \\ &\quad +\hat{\rho}_{i,1}\tanh(z_{i,1}/\varepsilon_{i,1})-b_i\dot{y}_0, \\ \alpha_{i,1} &= -\mathcal{N}(\xi_{i,1})\eta_{i,1}/(d_i+b_i), \\ \dot{\xi}_{i,1} &= z_{i,1}\eta_{i,1}, \\ \dot{\hat{w}}_{i,1} &= \gamma_{wi,1}(S_{i,1}z_{i,1}^2\boldsymbol{\varphi}_{i,1}^T\boldsymbol{\varphi}_{i,1}-\sigma_{wi,1}\hat{w}_{i,1}), \\ \dot{\hat{\rho}}_{i,1} &= \gamma_{\rho i,1}\Big(z_{i,1}\tanh(z_{i,1}/\varepsilon_{i,1})-\sigma_{\rho i,1}\hat{\rho}_{i,1}\Big), \end{aligned} \tag{9}$$

where $\hat{w}_{i,1}$ is the estimate of $w_{i,1}=||\mathbf{W}_{i,1}||^2$ for reducing the estimate parameter of the NN weights; and $\hat{\rho}_{i,1}$ is the estimate of $\bar{\rho}_{i,1}$; $c_{i,1},\gamma_{wi,1},\gamma_{\rho i,1},\sigma_{wi,1},\sigma_{\rho i,1}$ and $\varepsilon_{i,1}$ are positive design constants.

Choose a Lyapunov function candidate as

$$V_{i,1}=\frac{1}{2}z_{i,1}^2+\frac{1}{2\gamma_{wi,1}}\tilde{w}_{i,1}^2+\frac{1}{2\gamma_{\rho i,1}}\tilde{\rho}_{i,1}^2,$$

where $\tilde{w}_{i,1}=w_{i,1}-\hat{w}_{i,1}$ and $\tilde{\rho}_{i,1}=\bar{\rho}_{i,1}-\hat{\rho}_{i,1}$ are the estimate errors. The time derivative of $V_{i,1}$ along with (8) and (9) is

$$\begin{aligned} \dot{V}_{i,1} &= z_{i,1}\dot{z}_{i,1}-\frac{1}{\gamma_{wi,1}}\tilde{w}_{i,1}\dot{\hat{w}}_{i,1}-\frac{1}{\gamma_{\rho i,1}}\tilde{\rho}_{i,1}\dot{\hat{\rho}}_{i,1} \\ &\le -c_{i,1}z_{i,1}^2+\Big[-g_{i,1}(x_{i,1})\mathcal{N}(\xi_{i,1})+1\Big]\dot{\xi}_{i,1} \\ &\quad +(d_i+b_i)g_{i,1}(x_{i,1})z_{i,1}z_{i,2}+S_{i,1}z_{i,1}\mathbf{W}_{i,1}^T\boldsymbol{\varphi}_{i,1} \\ &\quad +(1-S_{i,1})\Big[|z_{i,1}\bar{F}_{i,1}|-z_{i,1}\bar{F}_{i,1}\tanh(z_{i,1}\bar{F}_{i,1}/\varepsilon_{i,1})\Big] \\ &\quad +\bar{\rho}_{i,1}\Big[|z_{i,1}|-z_{i,1}\tanh(z_{i,1}/\varepsilon_{i,1})\Big]-S_{i,1}z_{i,1}^2w_{i,1}\boldsymbol{\varphi}_{i,1}^T\boldsymbol{\varphi}_{i,1} \\ &\quad +\sigma_{wi,1}\tilde{w}_{i,1}\hat{w}_{i,1}+\sigma_{\rho i,1}\tilde{\rho}_{i,1}\hat{\rho}_{i,1}. \end{aligned} \tag{10}$$

Using Young's inequality and Cauchy-Schwarz inequality [36], we have

$$\begin{aligned} S_{i,1}z_{i,1}\mathbf{W}_{i,1}^T\boldsymbol{\varphi}_{i,1} &\le S_{i,1}(z_{i,1}\mathbf{W}_{i,1}^T\boldsymbol{\varphi}_{i,1})^2+\frac{S_{i,1}}{4} \\ &\le S_{i,1}z_{i,1}^2w_{i,1}\boldsymbol{\varphi}_{i,1}^T\boldsymbol{\varphi}_{i,1}+\frac{1}{4}. \end{aligned} \tag{11}$$

By [37, Lemma 1], we have the following property of the hyperbolic tangent function

$$\begin{aligned} (1-S_{i,1})\Big[|z_{i,1}\bar{F}_{i,1}|-z_{i,1}\bar{F}_{i,1}\tanh\Big(\frac{z_{i,1}\bar{F}_{i,1}}{\varepsilon_{i,1}}\Big)\Big] &\le 0.2785\varepsilon_{i,1}, \\ \bar{\rho}_{i,1}\Big[|z_{i,1}|-z_{i,1}\tanh\Big(\frac{z_{i,1}}{\varepsilon_{i,1}}\Big)\Big] &\le 0.2785\bar{\rho}_{i,1}\varepsilon_{i,1}. \end{aligned} \tag{12}$$

In addition, the following inequalities hold

$$\begin{aligned} (d_i+b_i)g_{i,1}(x_{i,1})z_{i,1}z_{i,2} &\le \frac{d_i+b_i}{2}z_{i,1}^2+\frac{d_i+b_i}{2}\bar{g}_{i,1}^2z_{i,2}^2, \\ \sigma_{wi,1}\tilde{w}_{i,1}\hat{w}_{i,1} &\le -\frac{\sigma_{wi,1}}{2}\tilde{w}_{i,1}^2+\frac{\sigma_{wi,1}}{2}w_{i,1}^2, \\ \sigma_{\rho i,1}\tilde{\rho}_{i,1}\hat{\rho}_{i,1} &\le -\frac{\sigma_{\rho i,1}}{2}\tilde{\rho}_{i,1}^2+\frac{\sigma_{\rho i,1}}{2}\bar{\rho}_{i,1}^2, \end{aligned} \tag{13}$$

by Young's inequality. Substituting inequalities (11), (12) and (13) into (10) yields

$$\begin{aligned} \dot{V}_{i,1} &\le -\Big(c_{i,1}-\frac{d_i}{2}-\frac{b_i}{2}\Big)z_{i,1}^2-\frac{\sigma_{wi,1}}{2}\tilde{w}_{i,1}^2-\frac{\sigma_{\rho i,1}}{2}\tilde{\rho}_{i,1}^2 \\ &\quad +\Big[-g_{i,1}(x_{i,1})\mathcal{N}(\xi_{i,1})+1\Big]\dot{\xi}_{i,1}+0.2785(\bar{\rho}_{i,1}+1)\varepsilon_{i,1} \\ &\quad +\frac{1}{4}+\frac{\sigma_{wi,1}}{2}w_{i,1}^2+\frac{\sigma_{\rho i,1}}{2}\bar{\rho}_{i,1}^2+\frac{d_i+b_i}{2}\bar{g}_{i,1}^2z_{i,2}^2. \end{aligned}$$

Let $V_1=\sum_{i=1}^N V_{i,1}$. It can be obtained that

$$\begin{aligned} \dot{V}_1 &\le -\beta_1 V_1+\sum_{i=1}^N\Big[-g_{i,1}(x_{i,1})\mathcal{N}(\xi_{i,1})+1\Big]\dot{\xi}_{i,1} \\ &\quad +\mu_1+\sum_{i=1}^N\frac{d_i+b_i}{2}\bar{g}_{i,1}^2z_{i,2}^2, \end{aligned} \tag{14}$$

where β_1 and μ_1 are positive constants denoted as

$$\begin{aligned} \beta_1 &= \min_{1\le i\le N}\Big\{2c_{i,1}-d_i-b_i,\ \gamma_{wi,1}\sigma_{wi,1},\ \gamma_{\rho i,1}\sigma_{\rho i,1}\Big\}, \\ \mu_1 &= \sum_{i=1}^N\Big[0.2785(\bar{\rho}_{i,1}+1)\varepsilon_{i,1}+\frac{1}{4}+\frac{\sigma_{wi,1}}{2}w_{i,1}^2+\frac{\sigma_{\rho i,1}}{2}\bar{\rho}_{i,1}^2\Big]. \end{aligned}$$

Multiplying (14) by $e^{\beta_1 t}$ and integrating both sides of (14) over $[0,t]$, we have

$$\begin{aligned} V_1 &\le \frac{\mu_1}{\beta_1}+\Big(V_1(0)-\frac{\mu_1}{\beta_1}\Big)e^{-\beta_1 t} \\ &\quad +\sum_{i=1}^N e^{-\beta_1 t}\int_0^t\Big[-g_{i,1}(x_{i,1})\mathcal{N}(\xi_{i,1})+1\Big]\dot{\xi}_{i,1}e^{\beta_1\tau}d\tau \\ &\quad +\sum_{i=1}^N e^{-\beta_1 t}\int_0^t\frac{d_i+b_i}{2}\bar{g}_{i,1}^2\,z_{i,2}^2e^{\beta_1\tau}d\tau \end{aligned}$$

$$\leq \frac{\mu_1}{\beta_1} + V_1(0) + \sum_{i=1}^{N} \frac{d_i + b_i}{2\beta_1} \bar{g}_{i,1}^2 \sup_{\tau\in[0,t]} \left\{ z_{i,2}^2(\tau) \right\} + \sum_{i=1}^{N} e^{-\beta_1 t} \int_0^t \left[-g_{i,1}(x_{i,1})\mathcal{N}(\xi_{i,1}) + 1 \right] \dot{\xi}_{i,1} e^{\beta_1 \tau} d\tau. \quad (15)$$

Notice that if $z_{i,2}$ is bounded over time interval $[0, t_f)$, then we can obtain the boundedness of $\sum_{i=1}^{N} \frac{d_i+b_i}{2\beta_1} \bar{g}_{i,1}^2 \sup_{\tau\in[0,t]} \left\{ z_{i,2}^2(\tau) \right\}$. Moreover, (15) can be rewritten as

$$V_1 \leq \sum_{i=1}^{N} e^{-\beta_1 t} \int_0^t \left[-g_{i,1}(x_{i,1})\mathcal{N}(\xi_{i,1}) + 1 \right] \dot{\xi}_{i,1} e^{\beta_1 \tau} d\tau + \bar{\mu}_1, \quad (16)$$

where $\bar{\mu}_1 = \frac{\mu_1}{\beta_1} + V_1(0) + \sum_{i=1}^{N} \frac{d_i+b_i}{2\beta_1} \bar{g}_{i,1}^2 \sup_{\tau\in[0,t]} \left\{ z_{i,2}^2(\tau) \right\}$. According to Lemma 1, it can be verified that V_1 and $\xi_{i,1}$ are bounded on $[0, t_f)$. Since [38, Proposition 2] implies $t_f = \infty$, it can be concluded that $z_{i,1}$, $\tilde{w}_{i,1}$ and $\tilde{\rho}_{i,1}$ are uniformly ultimately bounded. The boundedness of $z_{i,2}$ will be guaranteed in the following procedures.

Step k $(k = 2, \ldots, M-1)$*:* In order to avoid the complicated differential computation of the intermediate controller, let $\alpha_{i,k-1}$ pass through a TD to obtain the estimate of $\dot{\alpha}_{i,k-1}$. According to [39, Sec. III], a TD is defined as

$$\begin{aligned} \dot{\omega}_{i,k} &= \omega'_{i,k}, \\ \dot{\omega}'_{i,k} &= -\tau_{i,k} \mathrm{sgn}\left(\omega_{i,k} - \alpha_{i,k-1} + \frac{\omega'_{i,k}|\omega'_{i,k}|}{2\tau_{i,k}} \right), \end{aligned} \quad (17)$$

where $\omega_{i,k}, \omega'_{i,k} \in \mathbb{R}$ are the states of TD, and $\tau_{i,k} > 0$ is a design constant. Let $\varpi_{i,k} = \omega'_{i,k} - \dot{\alpha}_{i,k-1}$ denote the estimate error, which is bounded according to [40, Th. 1.1].

Utilizing NN to approximate the unknown nonlinearity $f_{i,k}(\boldsymbol{x}_{i,k})$, denote $F_{i,k}(\boldsymbol{Z}_{i,k}) = f_{i,k}(\boldsymbol{x}_{i,k}) = \boldsymbol{W}_{i,k}^T \boldsymbol{\varphi}_{i,k}(\boldsymbol{Z}_{i,k}) + \epsilon_{i,k}$, where $\boldsymbol{Z}_{i,k} = \boldsymbol{x}_{i,k}$ within a compact set $\Omega_{\boldsymbol{Z}_{i,k}}$. By Assumption 4, there exists a known smooth function $\bar{F}_{i,k}(\boldsymbol{Z}_{i,k})$ such that $|F_{i,k}(\boldsymbol{Z}_{i,k})| \leq \bar{F}_{i,k}(\boldsymbol{Z}_{i,k})$. Considering (6), (7), (17) and using the switching function $S_{i,k}(\boldsymbol{Z}_{i,k})$ defined as (3), we have

$$\dot{z}_{i,k} = S_{i,k} \boldsymbol{W}_{i,k}^T \boldsymbol{\varphi}_{i,k} + (1 - S_{i,k}) F_{i,k} + g_{i,k}(\boldsymbol{x}_{i,k})(z_{i,k+1} + \alpha_{i,k}) + \rho_{i,k} - \omega'_{i,k},$$

where $\rho_{i,k} = S_{i,k}\epsilon_{i,k} + \zeta_{i,k} + \varpi_{i,k}$ is a bounded variable and is bounded by $0 < |\rho_{i,k}| \leq \bar{\rho}_{i,k}$.

The following intermediate controller $\alpha_{i,k}$ and adaptive laws are designed

$$\begin{aligned} \eta_{i,k} &= c_{i,k} z_{i,k} + S_{i,k} z_{i,k} \hat{w}_{i,k} \boldsymbol{\varphi}_{i,k}^T \boldsymbol{\varphi}_{i,k} + (1 - S_{i,k}) \bar{F}_{i,k} \tanh(z_{i,k} \bar{F}_{i,k}/\varepsilon_{i,k}) + \hat{\rho}_{i,k} \tanh(z_{i,k}/\varepsilon_{i,k}) - \omega'_{i,k}, \\ \alpha_{i,k} &= -\mathcal{N}(\xi_{i,k})\eta_{i,k}, \\ \dot{\xi}_{i,k} &= z_{i,k}\eta_{i,k}, \\ \dot{\hat{w}}_{i,k} &= \gamma_{wi,k}(S_{i,k} z_{i,k}^2 \boldsymbol{\varphi}_{i,k}^T \boldsymbol{\varphi}_{i,k} - \sigma_{wi,k}\hat{w}_{i,k}), \\ \dot{\hat{\rho}}_{i,k} &= \gamma_{\rho i,k}\left(z_{i,k} \tanh(z_{i,k}/\varepsilon_{i,k}) - \sigma_{\rho i,k}\hat{\rho}_{i,k} \right), \end{aligned} \quad (18)$$

where $\hat{w}_{i,k}$ and $\hat{\rho}_{i,k}$ are the estimates of $w_{i,k} = ||\boldsymbol{W}_{i,k}||^2$ and $\bar{\rho}_{i,k}$, respectively; $c_{i,k}$, $\gamma_{wi,k}$, $\gamma_{\rho i,k}$, $\sigma_{wi,k}$, $\sigma_{\rho i,k}$ and $\varepsilon_{i,k}$ are positive design constants.

Select a Lyapunov function candidate as

$$V_{i,k} = \frac{1}{2} z_{i,k}^2 + \frac{1}{2\gamma_{wi,k}} \tilde{w}_{i,k}^2 + \frac{1}{2\gamma_{\rho i,k}} \tilde{\rho}_{i,k}^2,$$

where $\tilde{w}_{i,k} = w_{i,k} - \hat{w}_{i,k}$ and $\tilde{\rho}_{i,k} = \bar{\rho}_{i,k} - \hat{\rho}_{i,k}$ are the estimate errors. Proceeding similarly as in Step 1, and utilizing the aforementioned inequalities properties (11), (12) and (13), the time derivative of $V_{i,k}$ along with (18) results in

$$\dot{V}_{i,k} \leq -\left(c_{i,k} - \frac{1}{2} \right) z_{i,k}^2 - \frac{\sigma_{wi,k}}{2} \tilde{w}_{i,k}^2 - \frac{\sigma_{\rho i,k}}{2} \tilde{\rho}_{i,k}^2 + \left[-g_{i,k}(\boldsymbol{x}_{i,k})\mathcal{N}(\xi_{i,k}) + 1 \right] \dot{\xi}_{i,k} + 0.2785(\bar{\rho}_{i,k} + 1)\varepsilon_{i,k} + \frac{1}{4} + \frac{\sigma_{wi,k}}{2} w_{i,k}^2 + \frac{\sigma_{\rho i,k}}{2} \bar{\rho}_{i,k}^2 + \frac{1}{2} \bar{g}_{i,k}^2 z_{i,k+1}^2.$$

Let $V_k = \sum_{i=1}^{N} V_{i,k}$. We have

$$\dot{V}_k \leq -\beta_k V_k + \sum_{i=1}^{N} \left[-g_{i,k}(\boldsymbol{x}_{i,k})\mathcal{N}(\xi_{i,k}) + 1 \right] \dot{\xi}_{i,k} + \mu_k + \sum_{i=1}^{N} \frac{1}{2} \bar{g}_{i,k}^2 z_{i,k+1}^2, \quad (19)$$

where β_k and μ_k are positive constants denoted as

$$\beta_k = \min_{1\leq i\leq N} \left\{ 2c_{i,k} - 1, \gamma_{wi,k}\sigma_{wi,k}, \gamma_{\rho i,k}\sigma_{\rho i,k} \right\},$$

$$\mu_k = \sum_{i=1}^{N} \left[0.2785(\bar{\rho}_{i,k} + 1)\varepsilon_{i,k} + \frac{1}{4} + \frac{\sigma_{wi,k}}{2} w_{i,k}^2 + \frac{\sigma_{\rho i,k}}{2} \bar{\rho}_{i,k}^2 \right].$$

Multiplying (19) by $e^{\beta_k t}$ and integrating both sides of (19) over $[0, t]$ leads to

$$\begin{aligned} V_k &\leq \frac{\mu_k}{\beta_k} + \left(V_k(0) - \frac{\mu_k}{\beta_k} \right) e^{-\beta_k t} + \sum_{i=1}^{N} e^{-\beta_k t} \int_0^t \left[-g_{i,k}(\boldsymbol{x}_{i,k})\mathcal{N}(\xi_{i,k}) + 1 \right] \dot{\xi}_{i,k} e^{\beta_k \tau} d\tau + \sum_{i=1}^{N} e^{-\beta_k t} \int_0^t \frac{1}{2} \bar{g}_{i,k}^2 z_{i,k+1}^2 e^{\beta_k \tau} d\tau \\ &\leq \frac{\mu_k}{\beta_k} + V_k(0) + \sum_{i=1}^{N} \frac{1}{2\beta_k} \bar{g}_{i,k}^2 \sup_{\tau\in[0,t]} \left\{ z_{i,k+1}^2(\tau) \right\} + \sum_{i=1}^{N} e^{-\beta_k t} \int_0^t \left[-g_{i,k}(\boldsymbol{x}_{i,k})\mathcal{N}(\xi_{i,k}) + 1 \right] \dot{\xi}_{i,k} e^{\beta_k \tau} d\tau. \end{aligned} \quad (20)$$

If $z_{i,k+1}$ is bounded, then (20) can be rewritten as

$$V_k \leq \sum_{i=1}^{N} e^{-\beta_k t} \int_0^t \left[-g_{i,k}(\boldsymbol{x}_{i,k})\mathcal{N}(\xi_{i,k}) + 1 \right] \dot{\xi}_{i,k} e^{\beta_k \tau} d\tau + \bar{\mu}_k, \quad (21)$$

where $\bar{\mu}_k = \frac{\mu_k}{\beta_k} + V_k(0) + \sum_{i=1}^{N} \frac{1}{2\beta_k} \bar{g}_{i,k}^2 \sup_{\tau \in [0,t]} \left\{ z_{i,k+1}^2(\tau) \right\}$. As discussed before, it can be verified that V_k and $\xi_{i,k}$ are bounded by Lemma 1, and $z_{i,k}$, $\tilde{w}_{i,k}$ and $\tilde{\rho}_{i,k}$ are uniformly ultimately bounded. The bounded $z_{i,k+1}$ will be ensured in Step $k+1$.

Step M: At this last step, the actual control law u_i will be designed. According to the Mean Value Theorem, there exists a function $\nu_i(u_i)$ such that

$$\hbar_i(u_i) - \hbar_i(u_i^*) = \hbar_i^{(1)}(u_i)\big|_{u_i=\nu_i(u_i)}(u_i - u_i^*),$$

with the existence of u_i^* satisfying $\hbar_i(u_i^*) = 0$. Denote $h_i(\nu_i) = \hbar_i^{(1)}(u_i)|_{u_i=\nu_i(u_i)}$ and then one has $\hbar_i(u_i) = h_i(\nu_i)u_i - h_i(\nu_i)u_i^*$. By Assumption 2, there exist positive constants $\underline{h}_i$ and $\bar{h}_i$ such that $0 < \underline{h}_i \le h_i(\nu_i) \le \bar{h}_i$, and $h_i(\nu_i)u_i^*$ is also bounded.

Similarly, let $\alpha_{i,M-1}$ pass through a TD by

$$\begin{aligned} \dot{\omega}_{i,M} &= \omega'_{i,M}, \\ \dot{\omega}'_{i,M} &= -\tau_{i,M}\,\mathrm{sgn}\left(\omega_{i,M} - \alpha_{i,M-1} + \frac{\omega'_{i,M}|\omega'_{i,M}|}{2\tau_{i,M}}\right), \end{aligned}$$

where $\omega_{i,M}, \omega'_{i,M} \in \mathbb{R}$ are the states of TD, and $\tau_{i,M} > 0$ is a design constant. Denote the bounded estimate error as $\varpi_{i,M} = \omega'_{i,M} - \dot{\alpha}_{i,M-1}$.

The time derivative of $z_{i,M}$ is

$$\begin{aligned} \dot{z}_{i,M} = {} & S_{i,M}\boldsymbol{W}_{i,M}^T\boldsymbol{\varphi}_{i,M} + (1-S_{i,M})F_{i,M} + g_{i,M}(\boldsymbol{x}_{i,M})h_i(\nu_i)u_i \\ & + \rho_{i,M} - \int_0^{h_0} g_{i,M}(\boldsymbol{x}_{i,M})p_{0i}(r)f_r[u_i](t)dr - \omega'_{i,M}, \end{aligned}$$

where NN is used to approximate the unknown function $F_{i,M}(\boldsymbol{Z}_{i,M}) = f_{i,M}(\boldsymbol{x}_{i,M}) = \boldsymbol{W}_{i,M}^T\boldsymbol{\varphi}_{i,M}(\boldsymbol{Z}_{i,M}) + \epsilon_{i,M}$ within a compact set $\Omega_{\boldsymbol{Z}_{i,M}}$; and there exists a known smooth function $\bar{F}_{i,M}$ such that $|F_{i,M}| \le \bar{F}_{i,k}$; $S_{i,M}$ is the switching function; and denote $\rho_{i,M} = S_{i,M}\epsilon_{i,M} - g_{i,M}(\boldsymbol{x}_{i,M})h_i(\nu_i)u_i^* + \zeta_{i,M} + \varpi_{i,M}$ which is unknown but bounded by $0 < |\rho_{i,M}| \le \bar{\rho}_{i,M}$; and the hysteresis-related term $g_{i,M}(\boldsymbol{x}_{i,M})p_{0i}(r)f_r[u_i](t)$ can be put as a whole, which is unknown but has an upper bound $\bar{p}_i > 0$ according to Assumptions 1 and 5 and [14].

At this stage, we propose the following actual control law and adaptive updating laws

$$\begin{aligned} \eta_{i,M} &= c_{i,M}z_{i,M} + S_{i,M}z_{i,M}\hat{w}_{i,M}\boldsymbol{\varphi}_{i,M}^T\boldsymbol{\varphi}_{i,M} \\ &\quad + (1-S_{i,M})\bar{F}_{i,M}\tanh\left(\frac{z_{i,M}\bar{F}_{i,M}}{\varepsilon_{i,M}}\right) \\ &\quad + \hat{\rho}_{i,M}\tanh\left(\frac{z_{i,M}}{\varepsilon_{i,M}}\right) \\ &\quad + \mathrm{sgn}(z_{i,M})\int_0^{h_0}\hat{p}_i(t,r)dr - \omega'_{i,M}, \\ u_i &= -\mathcal{N}(\xi_{i,M})\eta_{i,M}, \\ \dot{\xi}_{i,M} &= z_{i,M}\eta_{i,M}, \\ \dot{\hat{w}}_{i,M} &= \gamma_{wi,M}(S_{i,M}z_{i,M}^2\boldsymbol{\varphi}_{i,M}^T\boldsymbol{\varphi}_{i,M} - \sigma_{wi,M}\hat{w}_{i,M}), \\ \dot{\hat{\rho}}_{i,M} &= \gamma_{\rho i,M}\Big(z_{i,M}\tanh(z_{i,M}/\varepsilon_{i,M}) - \sigma_{\rho i,M}\hat{\rho}_{i,M}\Big), \end{aligned}$$

$$\frac{\partial \hat{p}_i(t,r)}{\partial t} = \begin{cases} \gamma_{pi}\Big(|z_{i,M}| - \sigma_{pi}\hat{p}_i(t,r)\Big), & \text{if } 0 \le \hat{p}_i(t,r) < \bar{p}_i \\ -\gamma_{pi}\sigma_{pi}\hat{p}_i(t,r), & \text{if } \hat{p}_i(t,r) \ge \bar{p}_i \end{cases} \tag{22}$$

where $\hat{w}_{i,M}$, $\hat{\rho}_{i,M}$ and $\hat{p}_i(t,r)$ are the estimates of $w_{i,M} = ||\boldsymbol{W}_{i,M}||^2$, $\bar{\rho}_{i,M}$ and $\bar{p}_i$, respectively; $c_{i,M}$, $\gamma_{wi,M}$, $\gamma_{\rho i,M}$, γ_{pi}, $\sigma_{wi,M}$, $\sigma_{\rho i,M}$, σ_{pi} and $\varepsilon_{i,M}$ are positive design constants.

Consider the following Lyapunov function candidate

$$\begin{aligned} V_{i,M} = {} & \frac{1}{2}z_{i,M}^2 + \frac{1}{2\gamma_{wi,M}}\tilde{w}_{i,M}^2 + \frac{1}{2\gamma_{\rho i,M}}\tilde{\rho}_{i,M}^2 \\ & + \frac{1}{2\gamma_{pi}}\int_0^{h_0}\tilde{p}_i^2(t,r)dr, \end{aligned}$$

where $\tilde{w}_{i,M} = w_{i,M} - \hat{w}_{i,M}$, $\tilde{\rho}_{i,M} = \bar{\rho}_{i,M} - \hat{\rho}_{i,M}$ and $\tilde{p}_i(t,r) = \bar{p}_i - \hat{p}_i(t,r)$ are the estimated errors. As previously discussed, it can be induced that the derivative of $V_{i,M}$ is

$$\begin{aligned} \dot{V}_{i,M} \le {} & -c_{i,M}z_{i,M}^2 - \frac{\sigma_{wi,M}}{2}\tilde{w}_{i,M}^2 - \frac{\sigma_{\rho i,M}}{2}\tilde{\rho}_{i,M}^2 \\ & + \Big[-g_{i,M}(\boldsymbol{x}_{i,M})h_i(\nu_i)\mathcal{N}(\xi_{i,M}) + 1\Big]\dot{\xi}_{i,M} \\ & + 0.2785(\bar{\rho}_{i,M}+1)\varepsilon_{i,M} + \frac{1}{4} + \frac{\sigma_{wi,M}}{2}w_{i,M}^2 \\ & + \frac{\sigma_{\rho i,M}}{2}\bar{\rho}_{i,M}^2 + |z_{i,M}|\int_0^{h_0}\Big(\bar{p}_i - \hat{p}_i(t,r)\Big)dr \\ & - \frac{1}{\gamma_{pi}}\int_0^{h_0}\tilde{p}_i(t,r)\frac{\partial}{\partial t}\hat{p}_i(t,r)dr. \end{aligned} \tag{23}$$

Additionally, considering the adaptive law $\frac{\partial \hat{p}_i(t,r)}{\partial t}$ in (22), when the threshold parameter $r \in \Omega_r := \{r \mid 0 \le \hat{p}_i(t,r) < \bar{p}_i\} \subset [0, h_0]$, we have $\frac{\partial \hat{p}_i(t,r)}{\partial t} = \gamma_{pi}\Big(|z_{i,M}| - \sigma_{pi}\hat{p}_i(t,r)\Big)$, the last two terms of (23) can be rewritten as

$$\begin{aligned} & |z_{i,M}|\int_{r\in\Omega_r}\tilde{p}_i(t,r)dr - \frac{1}{\gamma_{pi}}\int_{r\in\Omega_r}\tilde{p}_i(t,r)\gamma_{pi} \\ & \quad \times \Big(|z_{i,M}| - \sigma_{pi}\hat{p}_i(t,r)\Big)dr \le \sigma_{pi}\int_{r\in\Omega_r}\tilde{p}_i(t,r)\hat{p}_i(t,r)dr. \end{aligned}$$

When $r \in \Omega_r^C := \{r \mid \hat{p}_i(t,r) \ge \bar{p}_i\} \subset [0, h_0]$, we have $\frac{\partial \hat{p}_i(t,r)}{\partial t} = -\gamma_{pi}\sigma_{pi}\hat{p}_i(t,r)$. Then, it can be induced that

$$\begin{aligned} & |z_{i,M}|\int_{r\in\Omega_r^C}\Big(\bar{p}_i - \hat{p}_i(t,r)\Big)dr + \frac{1}{\gamma_{pi}}\int_{r\in\Omega_r^C}\tilde{p}_i(t,r) \\ & \quad \times \gamma_{pi}\sigma_{pi}\hat{p}_i(t,r)dr \le \sigma_{pi}\int_{r\in\Omega_r^C}\tilde{p}_i(t,r)\hat{p}_i(t,r)dr. \end{aligned}$$

Combining the above two cases, when all $r \in [0, h_0]$, the last two terms of (23) result in

$$\begin{aligned} & |z_{i,M}|\int_0^{h_0}\Big(\bar{p}_i - \hat{p}_i(t,r)\Big)dr - \frac{1}{\gamma_{pi}}\int_0^{h_0}\tilde{p}_i(t,r)\frac{\partial}{\partial t}\hat{p}_i(t,r)dr \\ & \quad \le \sigma_{pi}\int_0^{h_0}\tilde{p}_i(t,r)\hat{p}_i(t,r)dr. \end{aligned} \tag{24}$$

Moreover, the following inequality holds

$$\sigma_{pi}\int_0^{h_0}\tilde{p}_i(t,r)\hat{p}_i(t,r)dr \le -\frac{\sigma_{pi}}{2}\int_0^{h_0}\tilde{p}_i^2(t,r)dr + \frac{\sigma_{pi}}{2}h_0\bar{p}_i^2, \tag{25}$$

by Young's inequality. Substituting (24) and (25) into (23) yields

$$\begin{aligned}\dot{V}_{i,M} \le& -c_{i,M}z_{i,M}^2 - \frac{\sigma_{wi,M}}{2}\tilde{w}_{i,M}^2 - \frac{\sigma_{\rho i,M}}{2}\tilde{\rho}_{i,M}^2\\ &- \frac{\sigma_{pi}}{2}\int_0^{h_0}\tilde{p}_i^2(t,r)dr\\ &+\Big[-g_{i,M}(\boldsymbol{x}_{i,M})h_i(\nu_i)\mathcal{N}(\xi_{i,M})+1\Big]\dot{\xi}_{i,M}\\ &+0.2785(\bar{\rho}_{i,M}+1)\varepsilon_{i,M}+\frac{1}{4}+\frac{\sigma_{wi,M}}{2}w_{i,M}^2\\ &+\frac{\sigma_{\rho i,M}}{2}\bar{\rho}_{i,M}^2+\frac{\sigma_{pi}}{2}h_0\bar{p}_i^2.\end{aligned}$$

Let $V_M = \sum_{i=1}^N V_{i,M}$. We have

$$\begin{aligned}\dot{V}_M \le& -\beta_M V_M\\ &+\sum_{i=1}^N\Big[-g_{i,M}(\boldsymbol{x}_{i,M})h_i(\nu_i)\mathcal{N}(\xi_{i,M})+1\Big]\dot{\xi}_{i,M}+\mu_M,\end{aligned} \tag{26}$$

where β_M and μ_M are positive constants denoted as

$$\begin{aligned}\beta_M &= \min_{1\le i\le N}\Big\{2c_{i,M},\gamma_{wi,M}\sigma_{wi,M},\gamma_{\rho i,M}\sigma_{\rho i,M},\gamma_{pi}\sigma_{pi}\Big\},\\ \mu_M &= \sum_{i=1}^N\Big[0.2785(\bar{\rho}_{i,M}+1)\varepsilon_{i,M}+\frac{1}{4}+\frac{\sigma_{wi,M}}{2}w_{i,M}^2\\ &\quad+\frac{\sigma_{\rho i,M}}{2}\bar{\rho}_{i,M}^2+\frac{\sigma_{pi}}{2}h_0\bar{p}_i^2\Big].\end{aligned}$$

Multiplying (26) by $e^{\beta_M t}$ and integrating both sides of (26) over $[0, t]$, it becomes

$$\begin{aligned}V_M \le& \sum_{i=1}^N e^{-\beta_M t}\int_0^t\Big[-g_{i,M}(\boldsymbol{x}_{i,M})h_i(\nu_i)\mathcal{N}(\xi_{i,M})+1\Big]\\ &\times\dot{\xi}_{i,M}e^{\beta_M\tau}d\tau+\frac{\mu_M}{\beta_M}+\Big(V_M(0)-\frac{\mu_M}{\beta_M}\Big)e^{-\beta_M t}\\ \le& \sum_{i=1}^N e^{-\beta_M t}\int_0^t\Big[-g_{i,M}(\boldsymbol{x}_{i,M})h_i(\nu_i)\mathcal{N}(\xi_{i,M})+1\Big]\\ &\times\dot{\xi}_{i,M}e^{\beta_M\tau}d\tau+\bar{\mu}_M,\end{aligned} \tag{27}$$

where $\bar{\mu}_M = \frac{\mu_M}{\beta_M} + V_M(0)$. According to Assumptions 2 and 5, [38, Proposition 2], and using Lemma 1, it can be verified that V_M and $\xi_{i,M}$ are bounded, and hence $z_{i,M}$, $\tilde{w}_{i,M}$, $\tilde{\rho}_{i,M}$ and $\tilde{p}_i(t,r)$ are uniformly ultimately bounded. Owing to the boundedness of $z_{i,M}$, it follows that the boundedness of $\sum_{i=1}^N \frac{1}{2\beta_{M-1}}\bar{g}_{i,M-1}^2 \sup_{\tau\in[0,t]}\Big\{z_{i,M}^2(\tau)\Big\}$ is naturally guaranteed at Step $M-1$. Furthermore, employing Lemma 1 for $M-1$ times backward, it can be concluded from the aforementioned recursive design processes that $V_{m'}$, $\xi_{i,m'}$, $z_{i,m'}$, $\tilde{w}_{i,m'}$ and $\tilde{\rho}_{i,m'}$ ($\forall m' = 1,\ldots,M-1$) are uniformly ultimately bounded.

B. STABILITY ANALYSIS

We are now ready to present our main results which show the stability and the tracking performance.

Theorem 1: Consider the nonlinear multi-agent system (6) preceded by hysteresis actuator (1). Let Assumptions 1-8 hold. Design the distributed control law and adaptive updating laws as (9), (18) and (22). Then the following objectives can be achieved for any bounded initial conditions.

(i) All signals of the closed-loop system remain globally uniformly ultimately bounded.

(ii) All outputs of the agents ultimately synchronize to the leader's trajectory with bounded residual errors, i.e., the tracking error $\boldsymbol{\delta} = \boldsymbol{y} - \boldsymbol{y}_0$ remains on the compact set $\Omega_{\boldsymbol{\delta}}$ specified as

$$\Omega_{\boldsymbol{\delta}} := \Bigg\{\boldsymbol{\delta}\ \Bigg|\ \|\boldsymbol{\delta}\| \le \frac{\sqrt{2\vartheta_1^*}}{\underline{\sigma}(L+B)}\Bigg\},$$

whose size $\vartheta_1^* > 0$ can be adjusted by the choice of the design parameters, where $\boldsymbol{y} = [y_1,\ldots,y_N]^T \in \mathbb{R}^N$, $\boldsymbol{y}_0 = [y_0,\ldots,y_0]^T \in \mathbb{R}^N$, and $\underline{\sigma}(L+B)$ is the minimum singular value of matrix $L+B$. □

Proof: For all $m = 1,\ldots,M$, denote $\boldsymbol{z}_m = [z_{1,m},\ldots,z_{N,m}]^T$, $\tilde{\boldsymbol{w}}_m = [\tilde{w}_{1,m},\ldots,\tilde{w}_{N,m}]^T$, $\tilde{\boldsymbol{\rho}}_m = [\tilde{\rho}_{1,m},\ldots,\tilde{\rho}_{N,m}]^T$ and $\tilde{\boldsymbol{p}} = [\tilde{p}_1(t,r),\ldots,\tilde{p}_N(t,r)]^T$, and let $\Gamma_{wm} = \mathrm{diag}(\gamma_{w1,m},\ldots,\gamma_{wN,m})$, $\Gamma_{\rho m} = \mathrm{diag}(\gamma_{\rho 1,m},\ldots,\gamma_{\rho N,m})$, $\Gamma_p = \mathrm{diag}(\gamma_{p1},\ldots,\gamma_{pN})$ be the constant matrices. In addition, let ς_M and $\varsigma_{m'}$ be the upper bounds of the terms

$$\sum_{i=1}^N e^{-\beta_M t}\int_0^t\Big[-g_{i,M}(\boldsymbol{x}_{i,M})h_i(\nu_i)\mathcal{N}(\xi_{i,M})\dot{\xi}_{i,M}+\dot{\xi}_{i,M}\Big]e^{\beta_M\tau}d\tau$$

and

$$\begin{aligned}&\sum_{i=1}^N e^{-\beta_{m'}t}\int_0^t\Big[-g_{i,m'}(\boldsymbol{x}_{i,m'})\mathcal{N}(\xi_{i,m'})\dot{\xi}_{i,m'}+\dot{\xi}_{i,m'}\\ &\qquad\qquad+\frac{1}{2}\bar{g}_{i,m'}^2\,z_{i,m'+1}^2\Big]e^{\beta'_m\tau}d\tau,\end{aligned}$$

respectively.

From the previous result (27), one has

$$\begin{aligned}V_M &= \frac{1}{2}\boldsymbol{z}_M^2+\frac{1}{2}\tilde{\boldsymbol{w}}_M^T\Gamma_{wM}^{-1}\tilde{\boldsymbol{w}}_M+\frac{1}{2}\tilde{\boldsymbol{\rho}}_M^T\Gamma_{\rho M}^{-1}\tilde{\boldsymbol{\rho}}_M\\ &\quad+\frac{1}{2}\int_0^{h_0}\tilde{\boldsymbol{p}}^T\Gamma_p^{-1}\tilde{\boldsymbol{p}}dr\\ &\le \varsigma_M+\frac{\mu_M}{\beta_M}+V_M(0).\end{aligned}$$

Furthermore, according to the recursive design results (21) and (16) from Step $M-1$ to Step 1, one has

$$\begin{aligned}V_{m'} &= \frac{1}{2}\boldsymbol{z}_{m'}^2+\frac{1}{2}\tilde{\boldsymbol{w}}_{m'}^T\Gamma_{wm'}^{-1}\tilde{\boldsymbol{w}}_{m'}+\frac{1}{2}\tilde{\boldsymbol{\rho}}_{m'}^T\Gamma_{\rho m'}^{-1}\tilde{\boldsymbol{\rho}}_{m'}\\ &\le \varsigma_{m'}+\frac{\mu_{m'}}{\beta_{m'}}+V_{m'}(0).\end{aligned}$$

Consider the global Lyapunov function candidate $V = \sum_{m=1}^M V_m$, and denote $\vartheta_m = \varsigma_m + \frac{\mu_m}{\beta_m} + V_m(0)$ for each m.

It can be guaranteed that all signals of the closed-loop system remain on the compact set $\Omega = \Omega_1 \times \cdots \times \Omega_M$, where

$$\Omega_{m'} := \Big\{[z_{m'}^T, \tilde{w}_{m'}^T, \tilde{\rho}_{m'}^T]^T \mid z_{m'}^2 + \tilde{w}_{m'}^T\Gamma_{wm'}^{-1}\tilde{w}_{m'} + \tilde{\rho}_{m'}^T\Gamma_{\rho m'}^{-1}\tilde{\rho}_{m'} \le 2\vartheta_{m'}\Big\},$$
$$\Omega_M := \Big\{[z_M^T, \tilde{w}_M^T, \tilde{\rho}_M^T, \tilde{p}^T]^T \mid z_M^2 + \tilde{w}_M^T\Gamma_{wM}^{-1}\tilde{w}_M + \tilde{\rho}_M^T\Gamma_{\rho M}^{-1}\tilde{\rho}_M + \int_0^{h_0}\tilde{p}^T\Gamma_p^{-1}\tilde{p}dr \le 2\vartheta_M\Big\}.$$

As time $t \to \infty$, according to (16), (21) and (27), it can be shown that Ω eventually converges to the compact set $\Omega' = \Omega_1' \times \cdots \times \Omega_M'$, where

$$\Omega_{m'}' := \Big\{[z_{m'}^T, \tilde{w}_{m'}^T, \tilde{\rho}_{m'}^T]^T \mid z_{m'}^2 + \tilde{w}_{m'}^T\Gamma_{wm'}^{-1}\tilde{w}_{m'} + \tilde{\rho}_{m'}^T\Gamma_{\rho m'}^{-1}\tilde{\rho}_{m'} \le 2\vartheta_{m'}^*\Big\},$$
$$\Omega_M' := \Big\{[z_M^T, \tilde{w}_M^T, \tilde{\rho}_M^T, \tilde{p}^T]^T \mid z_M^2 + \tilde{w}_M^T\Gamma_{wM}^{-1}\tilde{w}_M + \tilde{\rho}_M^T\Gamma_{\rho M}^{-1}\tilde{\rho}_M + \int_0^{h_0}\tilde{p}^T\Gamma_p^{-1}\tilde{p}dr \le 2\vartheta_M^*\Big\}, \quad (28)$$

with $\vartheta_m^* = \varsigma_m + \frac{\mu_m}{\beta_m}$.

As a consequence, for the closed-loop system, given any initial compact set Ω_0, as long as the initial conditions start in Ω_0, the proposed distributed control law will guarantee that all signals remain on the compact set Ω and eventually converge to the compact set Ω', regardless of whether they are subsets of the NN active region Ω_{NN} or not. In this sense, our stability result is global, and hence all signals of the closed-loop system are GUUB.

Additionally, it can be shown that the size of ultimately compact set Ω' depends on the choice of design parameters $c_{i,m}$, $\gamma_{wi,m}$, $\gamma_{\rho i,m}$ and γ_{pi}. In particular, we can tune these design parameters $c_{i,m}$, $\gamma_{wi,m}$, $\gamma_{\rho i,m}$ and γ_{pi} to make β_m increase, which together with (28) implies that the size of Ω' can be made arbitrarily small.

It can be verified that (28) implies $||z_1|| \le \sqrt{2\vartheta_1^*}$. [23, Lemma 2] implies that as long as $||z_1||$ is bounded, $||y - y_0|| \le ||z_1||/\underline{\sigma}(L + B)$, i.e., $||\delta|| \le \sqrt{2\vartheta_1^*}/\underline{\sigma}(L + B)$ guarantees the boundedness of tracking errors. So all outputs of the agents ultimately synchronize to the leader's trajectory with bounded residual errors. This completes the proof. ■

IV. SIMULATION EXAMPLE

Consider a multi-agent system consisting of five following agents and one leader as shown in Figure 1. Each follower node is a single-link manipulator with actuator hysteresis [18], which can be modeled as the Lagrangian dynamics

$$J_i\ddot{q}_i + B_i\dot{q}_i + M_ig\ell_i\sin(q_i) = H_i[u_i](t) + \zeta_i, \quad (29)$$

where $i = 1, \ldots, 5$; q_i, $\dot{q}_i$ and $\ddot{q}_i$ are the angle position, angular velocity and angular acceleration, respectively; J_i is the total rotational inertia; B_i is the overall damping coefficient; M_i is the total mass of the link; g is the gravitational acceleration; ℓ_i is the distance from the joint axis to the link center of mass; ζ_i is the external disturbance; $H_i[u_i](t)$ is the actuator signal with the generalized P-I hysteresis; and denote $y_i = q_i$ as the output signal.

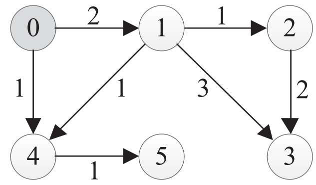

FIGURE 1. Topology of the digraph $\bar{\mathcal{G}}$.

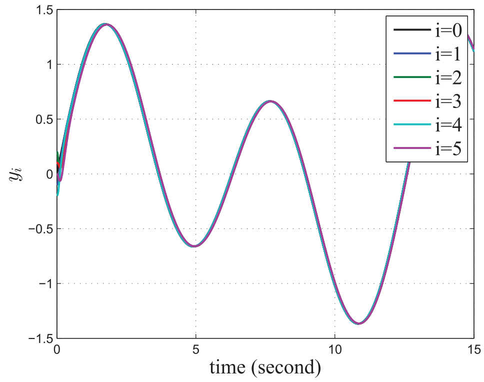

FIGURE 2. Evolutions of the output trajectories y_i.

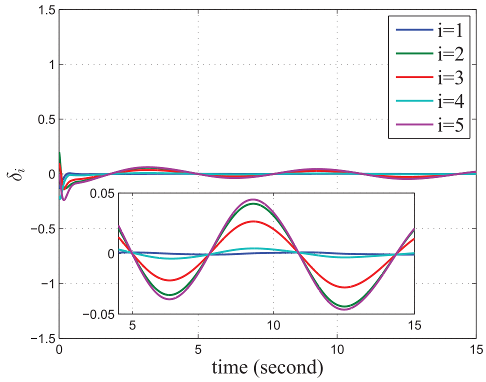

FIGURE 3. Evolutions of the tracking errors δ_i.

The leader's trajectory is given as $y_0 = \sin(t) + 0.5\sin(0.5t)$. In the simulation, the system parameters J_i, B_i, M_i, ℓ_i and the hysteresis input function $\hbar_i(u_i)(t) = 0.2\big(\tanh(u_i) + 9u_i\big)$ and density function $p_{0i}(r) = 0.08e^{-0.067(r-1)^2}$ are unknown to the distributed controller design. The unknown disturbance ζ_i is random but bounded.

During the simulation, the initial conditions are given as $x_1(0) = [-0.1; 0.1]$, $x_2(0) = [0.2; 0.1]$, $x_3(0) = [0.1; 0.1]$, $x_4(0) = [-0.2; 0.1]$, $x_5(0) = [0; 0]$, $\xi_{i,1}(0) = \xi_{i,2}(0) = 0$, $\hat{w}_{i,2}(0) = 0$, $\hat{\rho}_{i,2}(0) = 0$, $\hat{p}_i(0, r) = 0$, and $\omega_{i,2}(0) =$

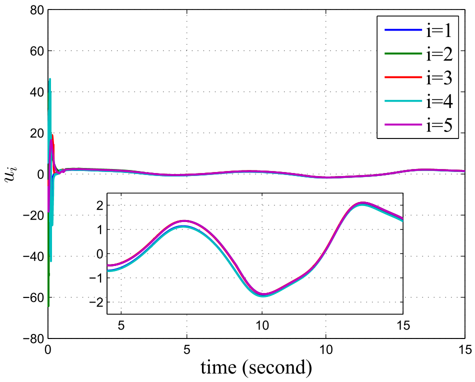

FIGURE 4. Evolutions of the control signals u_i.

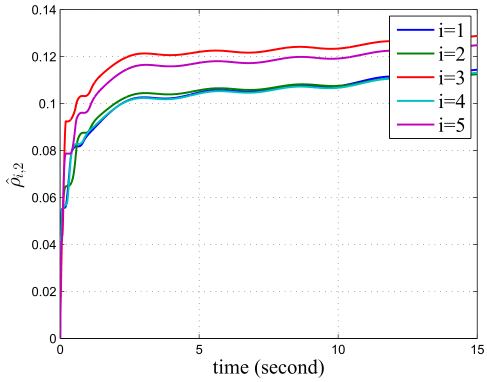

FIGURE 7. Evolutions of the estimated signals $\hat{\rho}_{i,2}$.

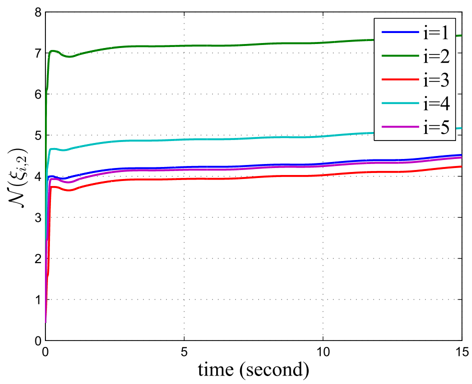

FIGURE 5. Evolutions of the Nussbaum-type function signals $\mathcal{N}(\xi_{i,2})$.

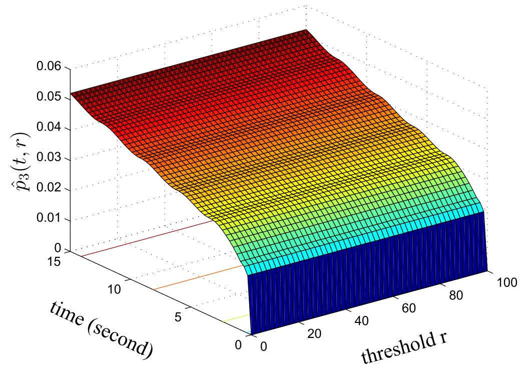

FIGURE 8. The convergent behavior of the estimate values $\hat{p}_3(t, r)$.

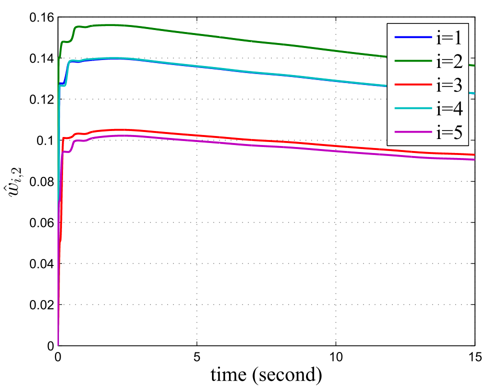

FIGURE 6. Evolutions of the estimated signals $\hat{w}_{i,2}$.

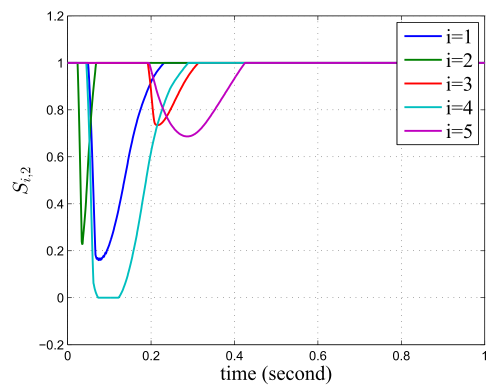

FIGURE 9. Evolutions of the switching function signals $S_{i,2}$.

$\omega'_{i,2}(0) = 0$. According to the structure of Lagrangian system (29), the boundaries $\bar{g}_1 = \max_{1\le i\le 5}\{\bar{g}_{i,1}\} = 1.2$, $\bar{g}_2 = \max_{1\le i\le 5}\{\bar{g}_{i,2}\} = 1.5$, $\underline{g}_1 = \max_{1\le i\le 5}\{\underline{g}_{i,1}\} = 0.8$, $\underline{g}_2 = \max_{1\le i\le 5}\{\underline{g}_{i,2}\} = 0.5$, and $\bar{F}_{i,2} = 4 + \dot{q}_i^2$, and the hysteresis-related parameter $r \in [0, 100]$ and $\bar{p}_i = 5$ can be known a priori. In the distributed control scheme (9), (18) and (22), the design parameters are chosen as $c_{i,1} = 16$, $c_{i,2} = 10$, $\tau_{i,2} = 10$, $\varepsilon_{i,2} = 0.05$, $\underline{r}_{i,2} = 1.5$, $\bar{r}_{i,2} = 2.5$, $\gamma_{wi,2} = 1.2$, $\gamma_{\rho i,2} = 0.98$, $\gamma_{pi} = 0.2$, $\sigma_{wi,2} = \sigma_{\rho i,2} = \sigma_{pi} = 0.01$ for all $i = 1, \ldots, 5$, and $\lambda_1 = \lambda_2 = 2$. In this example, six neurons are used for each NN, and the Sigmoid basis functions are the NN activation functions.

The simulation results are shown in Figures 2-9. The trajectories of the angle positions and reference signal are depicted as Figure 2. Figure 3 graphically shows that the tracking errors converge to a neighborhood of the origin. The control signals are given in Figure 4. In addition, the Nussbaum-type function signals $\mathcal{N}(\xi_{i,2})$ and the estimated signals $\hat{w}_{i,2}$ and $\hat{\rho}_{i,2}$ are illustrated through Figures 5-7, respectively. The convergent behavior of estimate values $\hat{p}_3(t, r)$ is shown in Figure 8. The switching function signals $S_{i,2}$ are shown in Figure 9.

V. CONCLUSION

In this paper, we studied a cooperative tracking control problem of nonlinear multi-agent systems in strict-feedback form with the generalized P-I hysteresis inputs over digraphs. The use of the multiple Nussbaum-type functions is developed to handle the problem of unknown time-varying virtual control coefficients. By using backstepping incorporating with TD technique, a distributed adaptive control law with a switching mechanism is proposed to guarantee the GUUB stability. Moreover, the leader-following tracking can be achieved with bounded residual errors.

APPENDIX

PROOF OF LEMMA 1

Proof: For each $m = 1, \ldots, M$ and all $i = 1, \ldots, N$, construct the following Nussbaum-type function

$$\mathcal{N}(\xi_{i,m}) = \left(e^{\lambda_m \xi_{i,m}} + e^{-\lambda_m \xi_{i,m}}\right)\sin(\xi_{i,m}) \tag{A1}$$

where $\lambda_m > 0$ is a design constant. Integrating both sides of (A1) over $[0, \xi_{i,m}]$ gives

$$\begin{aligned}\mathcal{M}(\xi_{i,m}) &= \int_0^{\xi_{i,m}} \mathcal{N}(\sigma)d\sigma \\ &= \frac{1}{1+\lambda_m^2}\Big[\lambda_m\left(e^{\lambda_m\xi_{i,m}} - e^{-\lambda_m\xi_{i,m}}\right)\sin(\xi_{i,m}) \\ &\quad - \left(e^{\lambda_m\xi_{i,m}} + e^{-\lambda_m\xi_{i,m}}\right)\cos(\xi_{i,m}) + 2\Big].\end{aligned}$$

Then it is straightforward that the following properties hold.

(i) $\mathcal{N}(\xi_{i,m})$ is an odd function, and is nonnegative for $\xi_{i,m} \in [2n\pi - 2\pi, 2n\pi - \pi]$, and is nonpositive for $\xi_{i,m} \in [2n\pi - \pi, 2n\pi]$ with n being a positive integer.

(ii) $\mathcal{M}(\xi_{i,m})$ is an even function. Over the interval $\xi_{i,m} \in [0, 2n\pi]$, its minimal value can be taken at $\xi_{i,m} = 2n\pi$ as

$$\mathcal{M}(2n\pi) = \frac{1}{1+\lambda_m^2}\Big[-\left(e^{\lambda_m 2n\pi} + e^{-\lambda_m 2n\pi}\right) + 2\Big],$$

and its maximum value can be taken at $\xi_{i,m} = 2n\pi - \pi$ as

$$\begin{aligned}&\mathcal{M}(2n\pi - \pi) \\ &\quad = \frac{1}{1+\lambda_m^2}\Big[e^{\lambda_m(2n\pi-\pi)} + e^{-\lambda_m(2n\pi-\pi)} + 2\Big].\end{aligned}$$

We will prove the boundedness of $\xi_{i,m}$ $(i = 1, \ldots, N)$ by seeking a contradiction. Therefore, suppose that there exist some $\xi_{i,m}$ which are unbounded. Without loss of generality, we assume that $\xi_{1,m}, \xi_{2,m}, \ldots, \xi_{q,m}$ are unbounded, while $\xi_{q+1,m}, \xi_{q+2,m}, \ldots, \xi_{N,m}$ are bounded with $1 \le q \le N$.

Let the related functions defined on $[0, t_f)$ be smooth. From the smoothness of $\xi_{i,m}$, there exists a monotonic increasing sequence $\{t_s\}$ $(s = 0, 1, 2\ldots)$ on the time interval $[0, t_f)$, such that

$$t_s = \begin{cases}\min\limits_{1\le i\le q}\Big\{t : \xi_{i,m}(t) = 2n\pi\Big\}, & \text{if all } g_{i,m}(\boldsymbol{x}_{i,m}) > 0 \\ \min\limits_{1\le i\le q}\Big\{t : \xi_{i,m}(t) = 2n\pi - \pi\Big\}, & \text{if all } g_{i,m}(\boldsymbol{x}_{i,m}) < 0\end{cases}$$

with $\lim_{s\to\infty} t_s = t_f$.

According to Assumption 5, for each m, all unknown time-varying functions $g_{i,m}(\boldsymbol{x}_{i,m})$ possess the same signs which are strictly either positive or negative. Hence, the following analysis can be divided into two cases.

Case 1 (When all $g_{i,m}(\boldsymbol{x}_{i,m}) > 0$): In this case, similarly, we define the monotonic increasing sequence $\{t_s^1\}$ and $\{t_s^2\}$ as

$$\begin{aligned}t_s^1 &= \Big\{t : \xi_{i,m}(t) = 2n\pi - 2\pi\Big\}, \quad \text{for } 1 \le s \le q \\ t_s^2 &= \Big\{t : \xi_{i,m}(t) = 2n\pi - \pi\Big\}, \quad \text{for } 1 \le s \le q.\end{aligned}$$

With the definition of t_s, there exists a p with $1 \le p \le q$ such that $\xi_{p,m}(t_s) = 2n\pi$. It is noted that $\xi_{i,m}(t) \le 2n\pi$ when $i \ne p$ for all $t \in [0, t_f)$.

Consider $V_m(t)$ over the time interval $[0, t_s]$, the inequality (5) can be rewritten as

$$\begin{aligned}&V_m(t_s) \\ &\le \sum_{i=1, i\ne p}^{q} e^{-\beta_m t_s}\int_0^{t_s} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{\beta_m\tau}d\tau \\ &\quad + e^{-\beta_m t_s}\int_0^{t_s} g_{p,m}(\boldsymbol{x}_{p,m})\mathcal{N}(\xi_{p,m})\dot{\xi}_{p,m}e^{\beta_m\tau}d\tau \\ &\quad + \sum_{i=1}^{q} e^{-\beta_m t_s}\int_0^{t_s}\dot{\xi}_{i,m}e^{\beta_m\tau}d\tau \\ &\quad + \sum_{i=q+1}^{N} e^{-\beta_m t_s}\int_0^{t_s}\Big[g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m}) + 1\Big]\dot{\xi}_{i,m}e^{\beta_m\tau}d\tau \\ &\quad + \bar{\mu}_m.\end{aligned} \tag{A2}$$

Noting that $0 < e^{-\beta_m(t_s-\tau)} \le 1$ for $\tau \in [0, t_s]$, and $\xi_{i,m}(t_s) \le 2n\pi$, one has

$$\begin{aligned}\sum_{i=1}^{q} e^{-\beta_m t_s}\int_0^{t_s}\dot{\xi}_{i,m}e^{\beta_m\tau}d\tau &\le \sum_{i=1}^{q}\int_0^{t_s}\dot{\xi}_{i,m}e^{-\beta_m(t_s-\tau)}d\tau \\ &\le q2n\pi - \sum_{i=1}^{q}\xi_{i,m}(0).\end{aligned}$$

Then (A2) can be transformed into

$$\begin{aligned}V_m(t_s) &\le \sum_{i=1, i\ne p}^{q}\int_0^{t_s} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s-\tau)}d\tau \\ &\quad + \int_0^{t_s} g_{p,m}(\boldsymbol{x}_{p,m})\mathcal{N}(\xi_{p,m})\dot{\xi}_{p,m}e^{-\beta_m(t_s-\tau)}d\tau \\ &\quad + q2n\pi + \mu_{m1},\end{aligned} \tag{A3}$$

where $\mu_{m1} = \sum_{i=q+1}^{N}\int_0^{t_s}[g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m}) + 1]\dot{\xi}_{i,m}e^{-\beta_m(t_s-\tau)}d\tau + \bar{\mu}_m - \sum_{i=1}^{q}\xi_{i,m}(0)$. Owing to the boundedness of $g_{i,m}(\boldsymbol{x}_{i,m})$ and $\xi_{i,m}$ for $q+1 \le i \le N$, μ_{m1} is bounded.

It is noted that the maximum value of the first term at the right-hand side of (A3) is obtained at $\xi_{i,m} = 2n\pi - \pi$, and $\mathcal{N}(\xi_{i,m})$ is nonnegative for $\xi_{i,m} \in [2n\pi - 2\pi, 2n\pi - \pi]$, and thus the time interval $[0, t_s]$ is decomposed into $[0, t_s^1]\bigcup[t_s^1, t_s^2]\bigcup[t_s^2, t_s]$. Utilizing the integral property and noting that $0 < e^{-\beta_m(t_s^2-\tau)} \leq e^{-\beta_m(t_s-\tau)} \leq 1$ for $\tau \in [0, t_s^1]$, we have

$$\begin{aligned}
&\sum_{i=1,i\neq p}^{q}\int_0^{t_s} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s-\tau)}d\tau\\
&\quad\leq \sum_{i=1,i\neq p}^{q}\int_0^{t_s^1} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s^1-\tau)}d\tau\\
&\qquad+\sum_{i=1,i\neq p}^{q}\sup_{\substack{\xi_{i,m}\in[2n\pi-2\pi,\\2n\pi-\pi]}}\left\{\int_{t_s^1}^{t_s^2} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\right.\\
&\qquad\qquad\left.\times\,\dot{\xi}_{i,m}e^{-\beta_m(t_s^2-\tau)}d\tau\right\}\\
&\quad\leq \sum_{i=1,i\neq p}^{q}\int_0^{t_s^1} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s^1-\tau)}d\tau\\
&\qquad+\bar{g}_m(q-1)\int_{2n\pi-2\pi}^{2n\pi-\pi}\mathcal{N}(\sigma)d\sigma,
\end{aligned}\tag{A4}$$

where $\bar{g}_m = \max_{1\leq i\leq N}\{\bar{g}_{i,m}\}$ with constants $\bar{g}_{i,m}$ being the upper bound of $|g_{i,m}(\boldsymbol{x}_{i,m})|$. To continue, when $i = p$, using the integral property and noting that $e^{-\beta_m(t_s-\tau)} \geq e^{-\beta_m(t_s-t_s^2)} = e^{-\beta_m\pi} > 0$ for $\tau \in [t_s^2, t_s]$, we have

$$\begin{aligned}
&\int_0^{t_s} g_{p,m}(\boldsymbol{x}_{p,m})\mathcal{N}(\xi_{p,m})\dot{\xi}_{p,m}e^{-\beta_m(t_s-\tau)}d\tau\\
&\leq \int_0^{t_s^1} g_{p,m}(\boldsymbol{x}_{p,m})\mathcal{N}(\xi_{p,m})\dot{\xi}_{p,m}e^{-\beta_m(t_s^1-\tau)}d\tau\\
&\quad+\sup_{\substack{\xi_{p,m}\in[2n\pi-2\pi,\\2n\pi-\pi]}}\left\{\int_{t_s^1}^{t_s^2} g_{p,m}(\boldsymbol{x}_{p,m})\mathcal{N}(\xi_{p,m})\dot{\xi}_{p,m}e^{-\beta_m(t_s^2-\tau)}d\tau\right\}\\
&\quad+\inf_{\substack{\xi_{p,m}\in[2n\pi-\pi,\\2n\pi]}}\left\{\int_{t_s^2}^{t_s} g_{p,m}(\boldsymbol{x}_{p,m})\mathcal{N}(\xi_{p,m})\dot{\xi}_{p,m}e^{-\beta_m(t_s-\tau)}d\tau\right\}\\
&\leq \int_0^{t_s^1} g_{p,m}(\boldsymbol{x}_{p,m})\mathcal{N}(\xi_{p,m})\dot{\xi}_{p,m}e^{-\beta_m(t_s^1-\tau)}d\tau\\
&\quad+\bar{g}_m\int_{2n\pi-2\pi}^{2n\pi-\pi}\mathcal{N}(\sigma)d\sigma+\underline{g}_m e^{-\beta_m\pi}\int_{2n\pi-\pi}^{2n\pi}\mathcal{N}(\sigma)d\sigma,
\end{aligned}\tag{A5}$$

where $\underline{g}_m = \min_{1\leq i\leq N}\{\underline{g}_{i,m}\}$ with constants $\underline{g}_{i,m}$ being the lower bound of $|g_{i,m}(\boldsymbol{x}_{i,m})|$. Then, substituting (A4) and (A5) into (A3) yields

$$\begin{aligned}
V_m(t_s) &\leq \sum_{i=1}^{q}\int_0^{t_s^1} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s^1-\tau)}d\tau\\
&\quad+\bar{g}_m q\int_{2n\pi-2\pi}^{2n\pi-\pi}\mathcal{N}(\sigma)d\sigma+\underline{g}_m e^{-\beta_m\pi}\\
&\quad\times\int_{2n\pi-\pi}^{2n\pi}\mathcal{N}(\sigma)d\sigma+q2n\pi+\mu_{m1},\\
&= \sum_{i=1}^{q}\int_0^{t_s^1} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s^1-\tau)}d\tau\\
&\quad+\bar{g}_m q\mathcal{M}(2n\pi-\pi)+\underline{g}_m e^{-\beta_m\pi}\mathcal{M}(2n\pi)\\
&\quad+q2n\pi+\mu_{m1}\\
&\quad-\bar{g}_m q\mathcal{M}(2n\pi-2\pi)-\underline{g}_m e^{-\beta_m\pi}\mathcal{M}(2n\pi-\pi).
\end{aligned}\tag{A6}$$

Recalling the properties of $\mathcal{M}(\xi_{i,m})$, one has

$$\begin{aligned}
&\bar{g}_m q\mathcal{M}(2n\pi-\pi)+\underline{g}_m e^{-\beta_m\pi}\mathcal{M}(2n\pi)+q2n\pi+\mu_{m1}\\
&\quad=\frac{e^{\lambda_m 2n\pi}}{1+\lambda_m^2}\left[\bar{g}_m q e^{-\lambda_m\pi}-\underline{g}_m e^{-\beta_m\pi}+\frac{q2n\pi(1+\lambda_m^2)}{e^{\lambda_m 2n\pi}}\right]\\
&\qquad+\frac{e^{-\lambda_m 2n\pi}}{1+\lambda_m^2}\left(\bar{g}_m q e^{\lambda_m\pi}-\underline{g}_m e^{-\beta_m\pi}\right)+\mu_{m2},
\end{aligned}\tag{A7}$$

where $\mu_{m2} = \frac{2\bar{g}_m q}{1+\lambda_m^2}+\frac{2\underline{g}_m e^{-\beta_m\pi}}{1+\lambda_m^2}+\mu_{m1}$ is bounded. Since λ_m is designed as $\lambda_m > \frac{1}{\pi}\ln\frac{\bar{g}_m N}{\underline{g}_m}+\beta_m$, it results in $\bar{g}_m q e^{-\lambda_m\pi} - \underline{g}_m e^{-\beta_m\pi} < 0$. Moreover, it is noted that

$$\lim_{n\to\infty}\frac{q2n\pi(1+\lambda_m^2)}{e^{\lambda_m 2\,n\pi}} = 0 \text{ and } \lim_{n\to\infty}\frac{e^{-\lambda_m 2\,n\pi}}{1+\lambda_m^2} = 0.$$

Thus, it can be guaranteed that as $n\to\infty$, (A7) $\to -\infty$.

Consider the last two terms in (A6) as

$$\begin{aligned}
&-\bar{g}_m q\mathcal{M}(2n\pi-2\pi)-\underline{g}_m e^{-\beta_m\pi}\mathcal{M}(2n\pi-\pi)\\
&\quad=\frac{e^{\lambda_m(2n\pi-\pi)}}{1+\lambda_m^2}\left(\bar{g}_m q e^{-\lambda_m\pi}-\underline{g}_m e^{-\beta_m\pi}\right)\\
&\qquad+\frac{e^{-\lambda_m(2n\pi-\pi)}}{1+\lambda_m^2}\left(\bar{g}_m q e^{\lambda_m\pi}-\underline{g}_m e^{-\beta_m\pi}\right)-\frac{2\bar{g}_m q}{1+\lambda_m^2}\\
&\qquad-\frac{2\underline{g}_m e^{-\beta_m\pi}}{1+\lambda_m^2}.
\end{aligned}\tag{A8}$$

Similarly, it can be verified that as $n\to\infty$, (A8) $\to -\infty$.

To proceed, the following considered term over the time interval $[0, t_s^1]$ is divided into finite subintervals as

$$\begin{aligned}
&\sum_{i=1}^{q}\int_0^{t_s^1} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s^1-\tau)}d\tau\\
&=\sum_{i=1}^{q}\int_0^{t_s^{2\pi}} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s^{2\pi}-\tau)}d\tau\\
&\quad+\sum_{i=1}^{q}\int_{t_s^{2\pi}}^{t_s^{4\pi}} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s^{4\pi}-\tau)}d\tau\\
&\quad+\cdots+\sum_{i=1}^{q}\int_{t_s^{2n\pi-4\pi}}^{t_s^1} g_{i,m}(\boldsymbol{x}_{i,m})\mathcal{N}(\xi_{i,m})\dot{\xi}_{i,m}e^{-\beta_m(t_s^1-\tau)}d\tau,
\end{aligned}\tag{A9}$$

where the monotonic increasing sequences are defined as $t_s^{2\pi} = \{t : \xi_{i,m}(t) = 2\pi\}$, $t_s^{4\pi} = \{t : \xi_{i,m}(t) = 4\pi\}$, ..., $t_s^{2n\pi-4\pi} = \{t : \xi_{i,m}(t) = 2n\pi - 4\pi\}$, respectively. Considering the general term of (A9) and decomposing the interval $[t_s^{2n\pi-4\pi}, t_s^1]$ into

$[t_s^{2n\pi-4\pi}, t_s^{2n\pi-3\pi}] \bigcup [t_s^{2n\pi-3\pi}, t_s^1]$, we have

$$\sum_{i=1}^{q} \int_{t_s^{2n\pi-4\pi}}^{t_s^1} g_{i,m}(\boldsymbol{x}_{i,m}) \mathcal{N}(\xi_{i,m}) \dot{\xi}_{i,m} e^{-\beta_m(t_s^1-\tau)} d\tau \\ \leq \bar{g}_m q \int_{2n\pi-4\pi}^{2n\pi-3\pi} \mathcal{N}(\sigma) d\sigma + \underline{g}_m q e^{-\beta_m \pi} \int_{2n\pi-3\pi}^{2n\pi-2\pi} \mathcal{N}(\sigma) d\sigma. \tag{A10}$$

Then using the similar technique as proved previously, it is derived that as $n \rightarrow \infty$, (A10) $\rightarrow -\infty$, when $\lambda_m > \frac{1}{\pi} \ln \frac{\bar{g}_m N}{\underline{g}_m} + \beta_m$.

Combining all aforementioned results, it can be obtained that $V_m(t_s) \rightarrow -\infty$ from (A3) as $n \rightarrow \infty$. Obviously, it contradicts with $V_m(t) \geq 0$. Therefore, all $\xi_{i,m}$ $(i = 1, \ldots, N)$ must be bounded on $[0, t_f)$. As a consequence, it can be further induced that V_m, $\xi_{i,m}$, and $\sum_{i=1}^{N} e^{-\beta_m t} \int_0^t \left[g_{i,m}\left(\boldsymbol{x}_{i,m}(\tau)\right) \mathcal{N}\left(\xi_{i,m}(\tau)\right) + 1\right] \dot{\xi}_{i,m}(\tau) e^{\beta_m \tau} d\tau$ are bounded on $[0, t_f)$.

Case 2 (When all $g_{i,m}(\boldsymbol{x}_{i,m}) < 0$): The proof is similar to that of Case 1. Hence, it is omitted. ■

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