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Achievable Rates for Full-Duplex Massive MIMO Systems Over Rician Fading Channels

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ABSTRACT We study the uplink and downlink achievable rate of full-duplex large-scale multi-input multi-output (MIMO) systems with a base station (BS) and users over Rician fading channels, based on maximum ratio combining/maximum ratio transmission and zero-forcing reception/zero-forcing transmission processing. Contrary to previous related works over Rayleigh fading channels, this paper assumes that the fast fading MIMO channel matrix follows the Rice distribution which will be more common in future 5G wireless communication systems. We derive approximate expressions of the uplink and downlink achievable rate for perfect channel state information when the number of BS antennas grows large. Based on the theoretical analysis, it is found that when the antennas of the BS are large enough and the power scaling law is applied properly, the impact of multi-user interference, loop interference, inter-user interference, and noise can be suppressed. In addition, the simulation results show that the uplink and downlink achievable rates increase with the number of BS antennas and they will converge to fixed values with the increasing Rician K -factor.

INDEX TERMS Full-duplex, large-scale MIMO, Rician, MRC/MRT, ZFR/ZFT.

I. INTRODUCTION

Nowadays, the fifth generation (5G) mobile communications have gained much attention. Most researchers believe that massive MIMO and full-duplex (FD) technology are the core technologies of 5G. When the number of the base station antennas is large enough, large-scale MIMO systems can effectively overcome small scale fading and additive noise, and the system can gain high spectral and energy efficiency. Moreover, if the loop interference (LI) can be eliminated effectively, the full-duplex system can further improve the spectral efficiency [1]–[5].

Though full-duplex technology can double spectral efficiency, it causes LI which deteriorate the performance of the system. Moreover, since the dynamic range of analog-to-digital converter (ADC) is limited, there is no mature technology to eliminate the LI thoroughly [6]–[8]. However, it is found that when the number of the base station (BS) antennas grows large and the power scaling law is applied properly, the impact of LI can be suppressed. Therefore,

the FD technology still attracts the researchers' attention. In [9], the ergodic capacity of FD relaying systems has been investigated. It has shown that the ergodic capacity degradation in imperfect CSI is higher than degradation in perfect channel state information (CSI). References [10] and [6] analyzed the spectral and energy efficiency of the FD relaying system over Rician fading channels. A multi-user FD massive MIMO systems whose channel is Rayleigh fading channels has been investigated in [11].

However, this paper is difference from [10]. We assume that each user is employed with two antennas (one for transmission and one for reception), while users in [10] are equipped with one antenna. Therefore, the system model of this paper is distinct from that of [10]. Then, since the users in this paper employ two antennas, we study the inter-user interference (IUI) and self-interference in the downlink which doesn't exist in [10]. Contrary to [11], the channel of the system model studied in this paper is Rician fading channel. Moreover, maximum ratio combining/maximum ratio

transmission (MRC/MRT) and zero-forcing reception/zero-forcing transmission (ZFR/ZFT) processing are both considered in this paper. Although partial results had been given in [13], the uplink achievable rate was only analyzed based on MRC/MRT. Section II introduces the FD MU-MIMO system model over Rician fading channels. The uplink and downlink achievable rate for perfect CSI with MRC/MRT and ZFR/ZRT processing are derived in section III and section IV, respectively. Then, we analyze the impact of several interferences/noise including multi-user interference (MUI), LI and noise based on four typical power scaling schemes; The numerical simulation results is analyzed in section V. Conclusions are drawn in Section VI.

The symbols used in this paper are as follows: $(\mathbf{A})^T$, $(\mathbf{A})^H$, $tr\{\mathbf{A}\}$, $\|\mathbf{A}\|$, $\mathbb{E}\{\bullet\}$, represent the matrix transpose, conjugate transpose, matrix trace, the Euclidean norm and expectation respectively; $[\mathbf{A}]_{mn}$ represents the $n \times n$ diagonal entry of matrix \mathbf{A} ; \mathbf{I}_M denotes $M \times M$ identity matrix. $\mathbf{x} \sim \mathcal{CN}(0, \sigma_{ij})$ denotes that \mathbf{x} is a circularly symmetric complex-Gaussian vector whose mean and variance are 0 and σ_{ij} , respectively.

II. SYSTEM MODEL

As shown in Fig. 1, we study a multi-user massive MIMO system, which has a FD BS and N FD users. Assuming that the BS is employed with $2M$ antennas (M antennas for transmission and M antennas for reception), and each user is employed with two antennas (one for transmission and one for reception). We suppose the BS has perfect CSI.

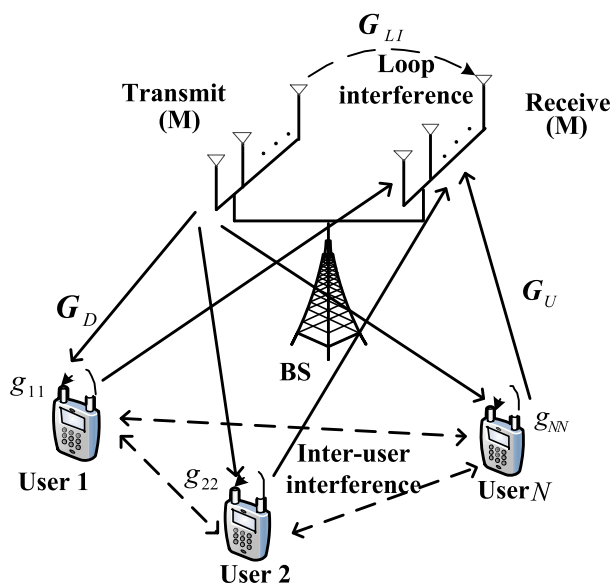


FIGURE 1. System model.

The downlink and uplink channel matrix can be represented as $\mathbf{G}_D = [\mathbf{g}_{D,1}, \dots, \mathbf{g}_{D,N}] \in \mathbb{C}^{M \times N}$, $\mathbf{G}_U = [\mathbf{g}_{U,1}, \dots, \mathbf{g}_{U,N}] \in \mathbb{C}^{M \times N}$, respectively. For simplicity, we denote the downlink and uplink channel matrix as $\mathbf{G}_a = \mathbf{H}_a \mathbf{D}_a^{1/2}$ ($a \in \{D, U\}$), where \mathbf{H}_a represents the $M \times N$ channel matrix modeling fast fading and \mathbf{D}_a is the $N \times N$ diagonal

matrix. Its n th diagonal entry is $[\mathbf{D}_a]_{nn} = \beta_{a,n}$. There are two components of fast fading matrices, i.e., the deterministic component corresponding to the LOS signal and the Rayleigh distribution of the scattered signal. In this paper, we suppose that every user has different Rician K -factor and K_n is denoted as the n th user's K -factor. Then, the fast fading matrix \mathbf{H}_a can be represented as

$$\mathbf{H} = \bar{\mathbf{H}} \left[\boldsymbol{\Omega} (\boldsymbol{\Omega} + \mathbf{I}_N)^{-1} \right]^{1/2} + \mathbf{H}_w \left[(\boldsymbol{\Omega} + \mathbf{I}_N)^{-1} \right]^{1/2}, \quad (1)$$

where $\boldsymbol{\Omega}$ is $N \times N$ diagonal matrix with $[\boldsymbol{\Omega}]_{nn} = K_n$ is the n th user's K -factor. The random component is \mathbf{H}_w . The entries of \mathbf{H}_w are independent and identically distributed (i.i.d.) Gaussian random variables with zero-mean, variance $1/2$. $\bar{\mathbf{H}}$ denotes the deterministic component, and its elements are given by

$$[\bar{\mathbf{H}}]_{mn} = e^{-j(m-1)(2\pi d/\lambda) \sin(\theta_n)}, \quad (2)$$

where d is the antenna spacing, λ is the wavelength. For simplicity, we consider $d = \lambda/2$ in this paper [9]. θ_n is the arrival angle of the n th user. The LI channel matrix between transmit antennas and receive antennas of the BS can be denoted as $\mathbf{G}_{LI} \in \mathbb{C}^{M \times M}$, and its entries are i.i.d. and follow $\mathcal{CN}(0, \omega_{LI})$ distribution. Let us denote $\mathbf{G}_{IU} = [g_{IU,1}, \dots, g_{IU,N}] \in \mathbb{C}^{N \times N}$ as the inter-user interference (IUI) channel with $[G_{IU}]_{ij}$ representing the channel coefficient from the i th user to the j th user. All the elements in \mathbf{G} are assumed to be i.i.d. with distribution $\mathcal{CN}(0, \mu_{ij})$. Moreover, $[G_{IU}]_{nn} = g_{nn}$ denotes the coefficient of self-interference channel of the n th user. The system model is shown in Fig. 1.

The uplink received signal at the BS \mathbf{y}_U and the received signals at users \mathbf{y}_D can be expressed as

$$\mathbf{y}_U = \sqrt{P_U} \mathbf{F}^H \mathbf{G}_U \mathbf{x}_U + \sqrt{\rho} \sqrt{P_D} \mathbf{F}^H \mathbf{G}_{LI} \mathbf{W} \mathbf{x}_D + \mathbf{F}^H \mathbf{n}_U, \quad (3)$$

$$\mathbf{y}_D = \sqrt{P_D} \mathbf{G}_D^H \mathbf{W} \mathbf{x}_D + \sqrt{P_U} \mathbf{G}_{IU}^H \mathbf{x}_U + \mathbf{n}_D. \quad (4)$$

The uplink beam-forming matrix and the downlink precoding matrix are denoted as $\mathbf{F} = [\mathbf{f}_1, \dots, \mathbf{f}_N] \in \mathbb{C}^{M \times N}$ and $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_N] \in \mathbb{C}^{M \times N}$, respectively; ρ is the factor that depends upon the extent to which the LI is eliminated. The residual LI channel gain is $\rho \omega_{LI}$. The transmit signal vectors of the BS and users are denoted as $\mathbf{x}_D = [x_{D,1}, \dots, x_{D,N}]^T$ and $\mathbf{x}_U = [x_{U,1}, \dots, x_{U,N}]^T$ with $\mathbb{E}\{\|\mathbf{x}_a\|^2\} = 1$, ($a \in \{D, U\}$), respectively. $\mathbf{n}_U \sim (0, \sigma_U \mathbf{I}_M)$ denotes the additive white Gaussian noise (AWGN) vector at the base station and $\mathbf{n}_D \sim (0, \sigma_D \mathbf{I}_M)$ denotes AWGN vector at the users. The transmission powers of the users and BS are P_U and P_D .

With the linear beamforming and precoding matrices, the BS can separate the receive and transmit signals into N streams, i.e., \mathbf{y}_U and \mathbf{y}_D can be divided into N elements. The received signals for the n th user at the BS, i.e., the n th element of \mathbf{y}_U , and the received signal of the n th user,

i.e., the n th element of \mathbf{y}_D are given by

$$y_{U,n} = \sqrt{P_U} \mathbf{f}_n^H \mathbf{g}_{U,n} x_{U,n} + \sum_{i=1, i \neq n}^N \sqrt{P_U} \mathbf{f}_n^H \mathbf{g}_{U,i} x_{U,i} + \sqrt{\rho} \sqrt{P_D} \mathbf{f}_n^H \mathbf{G}_{LI} \mathbf{W} x_D + \mathbf{f}_n^H \mathbf{n}_U \quad (5)$$

$$y_{D,n} = \sqrt{P_D} \mathbf{g}_{D,n}^H \mathbf{w}_n x_{D,n} + \sum_{i=1, i \neq n}^N \sqrt{P_D} \mathbf{g}_{D,n}^H \mathbf{w}_i x_{D,i} + \sqrt{P_U} \mathbf{g}_{U,n}^H \mathbf{x}_U + n_{D,n} \quad (6)$$

The first term of the right side of (5) is the transmit signal of the n th user. The second term corresponds to the signals transmitted by users exclude the n th user, which is MUI for the n th user. The third term denotes LI at the BS. The last term is AWGN at the BS. The first term of the right side of (6) is the downlink signal for n th user. The second term is the MUI which is the downlink signal transmitted for other users. The third term denotes the IUI comes from the signals transmitted by other users. And the last term is AWGN at the n th user.

III. ANALYSIS OF UPLINK AND DOWNLINK ACHIEVABLE RATE WITH MRC/MRT PROCESSING

The beam-forming and precoding matrix of MRC/MRT are written as [6], [11]:

$$\begin{cases} \mathbf{F} = \mathbf{G}_U \\ \mathbf{W} = \mathbf{G}_D (\text{tr}(\mathbf{G}_D^H \mathbf{G}_D))^{-\frac{1}{2}}. \end{cases} \quad (7)$$

We first provide some results that will be used in the following derivations.

Lemma 1: The following results hold [12]:

$$\begin{aligned} E \left\{ \|\mathbf{g}_{a,n}\|^2 \right\} &= \beta_{a,n} M \\ E \left\{ \|\mathbf{g}_{a,n}\|^4 \right\} &= \beta_{a,n}^2 \left(\frac{2MK_n + M}{(K_n + 1)^2} + M^2 \right) \\ E \left\{ \left| \mathbf{g}_{a,n}^H \mathbf{g}_{a,i} \right|^2 \right\} &= \beta_{a,n} \beta_{a,i} \left(\frac{K_n K_i \phi_{ni}^2 + M(K_n + K_i) + M}{(K_n + 1)(K_i + 1)} \right) \end{aligned} \quad (8)$$

A. ANALYSIS OF UPLINK ACHIEVABLE RATE WITH MRC/MRT PROCESSING

Substituting (7) into (5), the received signal of the n th antenna can be rewritten as

$$y_{U,n} = \sqrt{P_U} \mathbf{g}_{U,n}^H \mathbf{g}_{U,n} x_{U,n} + \sum_{i=1, i \neq n}^N \sqrt{P_U} \mathbf{g}_{U,n}^H \mathbf{g}_{U,i} x_{U,i} + \sqrt{\rho} \sqrt{P_D} \sum_{i=1}^N \frac{\mathbf{g}_{U,n}^H \mathbf{G}_{LI}}{\sqrt{\text{tr}(\mathbf{G}_D^H \mathbf{G}_D)}} \mathbf{g}_{D,i} x_{D,i} + \mathbf{g}_{U,n}^H \mathbf{n}_U. \quad (9)$$

Theorem 2: For a multi-user massive MIMO systems with full-duplex over Rician fading channels, the uplink rate of the

n th user for perfect CSI can be derived as

$$R_{U,n}^{MRC} \approx \log_2 \left(1 + \frac{P_U \beta_{U,n} \Delta_0}{P_U \sum_{\substack{i=1, \\ i \neq n}}^N \beta_{U,i} \Delta_1 + \rho \omega_{LI} P_D + \frac{1}{M} \sigma_U} \right) \quad (10)$$

where $\Delta_0 = \left(\frac{2K_n + 1}{(K_n + 1)^2 M} + 1 \right)$,

$\Delta_1 = \left(\frac{K_n K_i \phi_{ni}^2 / M + (K_n + K_i) + 1}{(K_n + 1)(K_i + 1)M} \right)$ and ϕ_{ni} is defined as

$$\phi_{ni} = \frac{\sin\left(\frac{M\pi}{2} [\sin(\theta_n) - \sin(\theta_i)]\right)}{\sin\left(\frac{\pi}{2} [\sin(\theta_n) - \sin(\theta_i)]\right)} \quad (11)$$

Proof: From (9), the uplink achievable rate of the n th user can be expressed as

$$R_{U,n}^{MRC} = \mathbb{E} \left\{ \log_2 \left(1 + \frac{P_U \|\mathbf{g}_{U,n}\|^4}{T_1 + T_2 + \left| \mathbf{g}_{U,n}^H \mathbf{n}_U \right|^2} \right) \right\} \quad (12)$$

where $T_1 = P_U \sum_{\substack{i=1, \\ i \neq n}}^N \left| \mathbf{g}_{U,n}^H \mathbf{g}_{U,i} \right|^2$, $T_2 = \rho P_D \sum_{i=1}^N \frac{\left| \mathbf{g}_{U,n}^H \mathbf{G}_{LI} \mathbf{g}_{D,i} \right|^2}{\text{tr}(\mathbf{G}_D^H \mathbf{G}_D)}$.

When M is large enough, we can approximate (12) by applying [12, Lemma 1] as

$$R_{U,n}^{MRC} \approx \log_2 \left(1 + \frac{P_U \mathbb{E} \left\{ \|\mathbf{g}_{U,n}\|^4 \right\}}{\mathbb{E} \{T_1\} + \mathbb{E} \{T_2\} + \mathbb{E} \left\{ \left| \mathbf{g}_{U,n}^H \mathbf{n}_U \right|^2 \right\}} \right) \quad (13)$$

According to (8), $\mathbb{E} \{T_1\}$ can be derived as

$$\begin{aligned} \mathbb{E} \{T_1\} &= P_U \sum_{i=1, i \neq n}^N \mathbb{E} \left\{ \left| \mathbf{g}_{U,n}^H \mathbf{g}_{U,i} \right|^2 \right\} \\ &= P_U \sum_{i=1, i \neq n}^N \beta_{U,n} \beta_{U,i} \\ &\quad \times \left(\frac{K_n K_i \phi_{ni}^2 + M(K_n + K_i) + M}{(K_n + 1)(K_i + 1)} \right) \end{aligned} \quad (14)$$

Using $\|A\|^2 = \text{tr}(\mathbf{A}\mathbf{A}^H)$ and $\text{tr}(\mathbf{A}\mathbf{B}) = \text{tr}(\mathbf{B}\mathbf{A})$ [6], and [12, Lemma 2], we can derived $\mathbb{E} \{T_2\}$ as

$$\begin{aligned} \mathbb{E} \{T_2\} &= \rho P_D \sum_{i=1}^N \frac{\mathbb{E} \left\{ \left| \mathbf{g}_{U,n}^H \mathbf{G}_{LI} \mathbf{g}_{D,i} \right|^2 \right\}}{\text{tr}(\mathbf{G}_D^H \mathbf{G}_D)} \\ &= \rho P_D \sum_{i=1}^N \frac{\mathbb{E} \left\{ \text{tr} \left(\mathbf{g}_{U,n}^H \mathbf{G}_{LI} \mathbf{g}_{D,i} \mathbf{g}_{D,i}^H \mathbf{G}_{LI}^H \mathbf{g}_{U,n} \right) \right\}}{M \text{tr} \left(\frac{\mathbf{G}_D^H \mathbf{G}_D}{M} \right)} \end{aligned}$$

$$\begin{aligned}
 & \frac{\text{tr} \left(\mathbb{E} \left\{ \mathbf{g}_{U,n}^H \mathbf{g}_{U,n} \right\} \mathbb{E} \left\{ \mathbf{G}_{LI} \mathbf{G}_{LI}^H \right\} \sum_{i=1}^N \mathbb{E} \left\{ \mathbf{g}_{D,i} \mathbf{g}_{D,i}^H \right\} \right)}{\text{tr}(\mathbf{D}_D) M} \\
 &= \rho P_D \frac{\beta_{U,n} M \omega_{LI} M \sum_{i=1}^N \beta_{D,i} M}{\text{tr}(\mathbf{D}_D) M} \\
 &= \rho \omega_{LI} P_D \beta_{U,n} M^2 \tag{15}
 \end{aligned}$$

Considering AWGN vector $\mathbf{n}_U \sim (0, \sigma_U \mathbf{I}_M)$, we obtain

$$\mathbb{E} \left\{ \left| \mathbf{g}_{U,n}^H \mathbf{n}_U \right|^2 \right\} = \beta_{U,n} M \sigma_U \tag{16}$$

Substituting (8), (14), (15) and (16) into (13), desired result can be derived.

Several power scaling schemes have been proposed based on the MRC/MRT processing. Then, we can derive the asymptotic uplink sum rate by applying these power scaling schemes.

Proposition 3: Case 1: $P_U = E_U, P_D = E_D$, substituting it into (10), (10) can be expressed as

$$R_{Un,1}^{MRC} \approx \log_2 \left(1 + \frac{E_U \beta_{U,n} \Delta_0}{E_U \sum_{\substack{i=1, \\ i \neq n}}^N \beta_{U,i} \Delta_1 + \rho \omega_{LI} E_D + \frac{\sigma_U}{M}} \right) \tag{17}$$

Proposition 3 shows that when there is no power scaling and $M \rightarrow \infty$, the uplink approximate achievable rate of the n th user tends to infinity. Moreover, we can see from (17) that the MUI, LI and the AWGN at the BS can be wiped out while the desired signal is retained when $M \rightarrow \infty$.

Proposition 4: Case 2: $P_U = E_U/M, P_D = E_D$, substituting it into (10), (10) can be expressed as

$$R_{Un,2}^{MRC} \approx \log_2 \left(1 + \frac{E_U \beta_{U,n} \Delta_0}{E_U \sum_{\substack{i=1, \\ i \neq n}}^N \beta_{U,i} \Delta_1 + \rho \omega_{LI} E_D M + \sigma_U} \right) \tag{18}$$

Proposition 4 shows that the approximate uplink rate of the n th user approaches to a constant when $M \rightarrow \infty$. From power scaling and $M \rightarrow \infty$, the uplink achievable rate of the n th user tends to infinity. Moreover, we can see from (18), we can see that the MUI at the BS tend to 0 while the LI and noise at the BS cannot be eliminated which limit the performance of Case 2.

Proposition 5: Case 3: $P_U = E_U, P_D = E_D/M$, substituting it into (10), (10) can be expressed as

$$R_{Un,3}^{MRC} \approx \log_2 \left(1 + \frac{E_U \beta_{U,n} \Delta_0}{E_U \sum_{\substack{i=1, \\ i \neq n}}^N \beta_{U,i} \Delta_1 + \rho \omega_{LI} \frac{E_D}{M} + \frac{\sigma_U}{M}} \right) \tag{19}$$

Proposition 5 shows that when M grows large, the approximate uplink rate of the n th user tends to infinity. Though, the result of Case 3 is similar with Case 1, the difference between them is that, Case 3 can further restrain the LI by the power scaling at the BS. Therefore, the performance of Case 3 is better than Case 1 in theoretical analysis.

Proposition 6: Case 4: $P_U = E_U/M, P_D = E_D/M$, substituting it into (10), (10) can be expressed as

$$R_{Un,4}^{MRC} \approx \log_2 \left(1 + \frac{E_U \beta_{U,n} \Delta_0}{E_U \sum_{\substack{i=1, \\ i \neq n}}^N \beta_{U,i} \Delta_1 + \rho \omega_{LI} E_D + \sigma_U} \right) \tag{20}$$

Proposition 6 shows that the MUI and LI term can be eliminated with M grows large while the noise term at the BS cannot be wiped out.

B. ANALYSIS OF DOWNLINK ACHIEVABLE RATE WITH MRC/MRT PROCESSING

Substituting (7) into (6), the received signal of the n th user can be rewritten as

$$\begin{aligned}
 y_{D,n} &= \sqrt{P_D} \mathbf{g}_{D,n}^H \frac{\mathbf{g}_{D,n}}{\sqrt{\text{tr}(\mathbf{G}_D^H \mathbf{G}_D)}} x_{D,n} \\
 &+ \sum_{i=1, i \neq n}^N \sqrt{P_D} \mathbf{g}_{D,n}^H \frac{\mathbf{g}_{D,i}}{\sqrt{\text{tr}(\mathbf{G}_D^H \mathbf{G}_D)}} x_{D,i} \\
 &+ \sqrt{P_U} \sum_{i=1}^N g_{in}^* x_{U,i} + n_{D,n}. \tag{21}
 \end{aligned}$$

Theorem 7: For a multi-user massive MIMO systems with full-duplex over Rician fading channels, the approximate downlink achievable rate of the n th user for perfect CSI can be derived as

$$R_{D,n}^{MRC} \approx \log_2 \left(1 + \frac{P_D \beta_{D,n}^2 \Delta_0}{P_D \beta_{D,n} \sum_{i=1, i \neq n}^N \beta_{D,i} \Delta_1 + \frac{\text{tr}(\mathbf{D}_D) \Delta_2}{M}} \right) \tag{22}$$

where $\Delta_2 = \left(P_U \sum_{i=1}^N \mu_{in} + \sigma_D \right)$.

Proof: From (21), we can express the downlink achievable rate of the n th user as

$$R_{D,n}^{MRC} = \mathbb{E} \left\{ \log_2 \left(1 + \frac{P_U \|\mathbf{g}_{D,n}\|^4 (\text{tr}(\mathbf{G}_D^H \mathbf{G}_D))^{-1}}{T_3 + T_4 + |\mathbf{n}_{D,n}|^2} \right) \right\} \quad (23)$$

where $T_3 = P_D \sum_{i=1, i \neq n}^N \frac{|g_{D,n}^H g_{D,i}|^2}{\text{tr}(\mathbf{G}_D^H \mathbf{G}_D)}$, $T_4 = P_U |\mathbf{g}_{IU,n}^H \mathbf{x}|^2$.

When M is large enough, we can approximate (23) by applying [12, Lemma 1] as

$$R_{D,n}^{MRC} \approx \log_2 \left(1 + \frac{(\text{tr}(\mathbf{G}_D^H \mathbf{G}_D))^{-1} P_U \mathbb{E} \left\{ \|\mathbf{g}_{D,n}\|^4 \right\}}{\mathbb{E} \{T_3\} + \mathbb{E} \{T_4\} + \mathbb{E} \left\{ |\mathbf{n}_{D,n}|^2 \right\}} \right) \quad (24)$$

According to (8), $\mathbb{E} \{T_3\}$, $\mathbb{E} \{T_4\}$ can be derived as

$$\begin{aligned} \mathbb{E} \{T_3\} &= P_D \sum_{i=1, i \neq n}^N \frac{\mathbb{E} \left\{ |g_{D,n}^H g_{D,i}|^2 \right\}}{\text{tr}(\mathbf{G}_D^H \mathbf{G}_D)} \\ &= (\text{tr}(\mathbf{G}_D^H \mathbf{G}_D))^{-1} P_D \sum_{i=1, i \neq n}^N \beta_{D,n} \beta_{D,i} \frac{\Delta_1}{M} \end{aligned} \quad (25)$$

$$\begin{aligned} \mathbb{E} \{T_4\} &= P_U \mathbb{E} \left\{ |\mathbf{g}_{IU,n}^H \mathbf{x}|^2 \right\} \\ &= P_U \left| \sum_{i=1}^N g_{in}^* x_{U,i} \right|^2 \\ &= P_U \sum_{i=1}^N \mu_{in} \end{aligned} \quad (26)$$

Considering AWGN vector $\mathbf{n}_D \sim (0, \sigma_D \mathbf{I}_M)$, we obtain

$$\mathbb{E} \left\{ |\mathbf{g}_{D,n}^H \mathbf{n}_D|^2 \right\} = \beta_{D,n} M \sigma_D \quad (27)$$

Substituting (8) and (25) to (27) into (24), the desired result can be derived. In addition, when $K_n = K_i = 0$ (Rayleigh fading channels) and $M \rightarrow \infty$, (22) can be reduced to

$$R_{D,n}^{MRC} \approx \log_2 \left(1 + \frac{P_D \beta_{D,n}^2 M}{P_D \beta_{D,n} \sum_{i=1, i \neq n}^N \beta_{D,i} + \text{tr}(\mathbf{D}_D) \Delta_2} \right) \quad (28)$$

which is consistent with [11, eq. (27)].

Proposition 8: Case 1: $P_U = E_U$, $P_D = E_D$, substituting it into (22), (22) can be re-expressed as

$$R_{Dn,1}^{MRC} \approx \log_2 \left(1 + \frac{E_D \beta_{D,n}^2 \Delta_0}{E_D \beta_{D,n} \sum_{i=1, i \neq n}^N \beta_{D,i} \Delta_1 + \frac{\text{tr}(\mathbf{D}_D) K_1}{M}} \right) \quad (29)$$

where $K_1 = \left(E_U \sum_{i=1}^N \mu_{in} + \sigma_D \right)$.

Proposition 8 shows that when there is no power scaling and $M \rightarrow \infty$, the approximate downlink rate of the n th user tends to infinity. Moreover, we can see from (29) that the MUI, IUI and the AWGN at the users can be wiped out while the desired signal is retained when $M \rightarrow \infty$.

Proposition 9: Case 2: $P_U = E_U/M$, $P_D = E_D$, substituting it into (22), (22) can be re-expressed as

$$R_{Dn,2}^{MRC} \approx \log_2 \left(1 + \frac{E_D \beta_{D,n}^2 \Delta_0}{E_D \beta_{D,n} \sum_{i=1, i \neq n}^N \beta_{D,i} \Delta_1 + \text{tr}(\mathbf{D}_D) K_2} \right) \quad (30)$$

where $K_2 = \frac{E_U}{M^2} \sum_{i=1}^N \mu_{in} + \frac{\sigma_D}{M}$.

Proposition 9 shows that when M grows large, the approximate downlink rate of the n th user tends to infinity. Though, the result of Case 2 is similar with Case 1, the difference between them is that, Case 2 can further restrain the IUI by the power scaling at the users. Therefore, the performance of Case 2 is better than Case 1 in theoretical analysis.

Proposition 10: Case 3: $P_U = E_U$, $P_D = E_D/M$, substituting it into (22), (22) can be re-expressed as

$$R_{Dn,3}^{MRC} \approx \log_2 \left(1 + \frac{E_D \beta_{D,n}^2 \Delta_0}{E_D \beta_{D,n} \sum_{i=1, i \neq n}^N \beta_{D,i} \Delta_1 + \text{tr}(\mathbf{D}_D) K_3} \right) \quad (31)$$

where $K_3 = E_U \sum_{i=1}^N \mu_{in} + \sigma_D$.

Proposition 10 shows that the approximate downlink rate of the n th user approaches to a constant when $M \rightarrow \infty$. From (31), we can see that the MUI at the users tend to 0 while the IUI and the noise cannot be eliminated which limit the performance of Case 3.

Proposition 11: Case 4: $P_U = E_U/M$, $P_D = E_D/M$, substituting it into (22), (22) can be re-expressed as

$$R_{Dn,4}^{MRC} \approx \log_2 \left(1 + \frac{E_D \beta_{D,n}^2 \Delta_0}{E_D \beta_{D,n} \sum_{i=1, i \neq n}^N \beta_{D,i} \Delta_1 + \text{tr}(\mathbf{D}_D) K_4} \right) \quad (32)$$

where $K_4 = \frac{E_U}{M} \sum_{i=1}^N \mu_{in} + \sigma_D$.

Proposition 11 shows that the MUI and IUI term can be eliminated with M grows large while the noise term at the users cannot be wiped out.

IV. ANALYSIS OF UPLINK AND DOWNLINK ACHIEVABLE RATE WITH ZFR/ZFT PROCESSING

The beam-forming and precoding matrix of ZFR/ZFT are written as

$$\begin{cases} \mathbf{F} = \mathbf{G}_U (\mathbf{G}_U^H \mathbf{G}_U)^{-1} \\ \mathbf{W} = \mathbf{G}_D (\mathbf{G}_D^H \mathbf{G}_D)^{-1} \left(\text{tr} \left((\mathbf{G}_D^H \mathbf{G}_D)^{-1} \right) \right)^{-\frac{1}{2}} \end{cases} \quad (33)$$

A. ANALYSIS OF UPLINK ACHIEVABLE RATE WITH ZFR/ZFT PROCESSING

For ZF processing, we have $\mathbf{F}^H \mathbf{G}_U = \mathbf{I}_N$ and $\mathbf{G}_D^H \mathbf{W} = \left(\text{tr} \left((\mathbf{G}_D^H \mathbf{G}_D)^{-1} \right) \right)^{-\frac{1}{2}} \mathbf{I}_M$, so that [6], [7]

$$\mathbf{f}_n^H \mathbf{g}_{U,i} = \delta_{ni} \quad (34)$$

$$\mathbf{g}_{D,n}^H \mathbf{w}_i = \left(\text{tr} \left((\mathbf{G}_D^H \mathbf{G}_D)^{-1} \right) \right)^{-\frac{1}{2}} \delta_{in} \quad (35)$$

where $\delta_{ni} = 1$ or 0 when $n = i$ or $n \neq i$. Substituting (34) into (5), the received signal for the n th user of BS is re-written as

$$y_{U,n} = \sqrt{P_U} x_{U,n} + \sqrt{\rho} \sqrt{P_D} \mathbf{f}_n^H \mathbf{G}_{LI} \mathbf{W} x_D + \mathbf{f}_n^H \mathbf{n}_U \quad (36)$$

From (36), the uplink rate for the n th user of BS can be expressed as

$$R_{U,n}^{ZF} = \mathbb{E} \left\{ \log_2 \left(1 + \frac{P_U}{\Delta_{LI} + \left[(\mathbf{G}_U^H \mathbf{G}_U)^{-1} \right]_{nn} \sigma_U} \right) \right\} \quad (37)$$

where $\Delta_{LI} = \rho P_D |\mathbf{f}_n^H \mathbf{G}_{LI} \mathbf{W}|^2$.

With the help of [12, Lemma 1], we can obtain

$$R_{U,n}^{ZF} \approx \log_2 \left(1 + \frac{P_U}{\mathbb{E} \{ \Delta_{LI} \} + \left[(\mathbf{G}_U^H \mathbf{G}_U)^{-1} \right]_{nn} \sigma_U} \right) \quad (38)$$

Compute $E \{ \Delta_{LI} \}$:

$$\begin{aligned} \mathbb{E} \{ \Delta_{LI} \} &= \rho P_D \mathbb{E} \left\{ \left| \mathbf{f}_n^H \mathbf{G}_{LI} \mathbf{W} \right|^2 \right\} \\ &= \rho P_D \mathbb{E} \left\{ \text{tr} \left(\mathbf{f}_n^H \mathbf{G}_{LI} \mathbf{W} \mathbf{W}^H \mathbf{G}_{LI}^H \mathbf{f}_n \right) \right\} \\ &= \rho P_D \text{tr} \left\{ \mathbb{E} \left\{ \mathbf{f}_n^H \mathbf{f}_n \right\} \mathbb{E} \left\{ \mathbf{G}_{LI} \mathbf{G}_{LI}^H \right\} E \left\{ \mathbf{W} \mathbf{W}^H \right\} \right\} \\ &= \frac{1}{M} \rho P_D \omega_{LI} M \left[\left(\frac{\mathbf{G}_U^H \mathbf{G}_U}{M} \right)^{-1} \right]_{nn} \\ &= \rho P_D \omega_{LI} \beta_{U,n}^{-1} \end{aligned} \quad (39)$$

Substituting (39) into (38), we can obtain

$$\begin{aligned} R_{U,n}^{ZF} &\approx \log_2 \left(1 + \frac{P_U}{M^{-1} \rho P_D \omega_{LI} \beta_{U,n}^{-1} + M^{-1} \beta_{U,n}^{-1} \sigma_U} \right) \\ &= \log_2 \left(1 + \frac{P_U M \beta_{U,n}}{\rho P_D \omega_{LI} M + \sigma_U} \right) \end{aligned} \quad (40)$$

B. ANALYSIS OF DOWNLINK ACHIEVABLE RATE WITH ZFR/ZFT PROCESSING

Substituting (35) into (6), the received signal for the n th user is re-written as

$$y_{D,n} = \sqrt{P_D} \left(\text{tr} \left((\mathbf{G}_D^H \mathbf{G}_D)^{-1} \right) \right)^{-\frac{1}{2}} x_{D,n} + \sqrt{P_U} \mathbf{g}_{DU,n}^H \mathbf{x}_U + n_{D,n} \quad (41)$$

From (41), the downlink rate for the n th user can be expressed as

$$R_{D,n}^{ZF} \approx \log_2 \left(1 + \frac{P_D \left(\text{tr} \left((\mathbf{G}_D^H \mathbf{G}_D)^{-1} \right) \right)^{-1}}{P_U \sum_{i=1}^N \mu_{in} + \sigma_D} \right) \quad (42)$$

With the aid of [12, Lemma 2], we can get $\frac{1}{M} \mathbf{G}_D^H \mathbf{G}_D \rightarrow \mathbf{D}_D$. Then, we can have

$$R_{D,n}^{ZF} \approx \log_2 \left(1 + \frac{M P_D}{\text{tr} \left((\mathbf{D}_D)^{-1} \right) \left(P_U \sum_{i=1}^N \mu_{in} + \sigma_D \right)} \right) \quad (43)$$

The four power scaling schemes which are applied in this section is similar to the MRC/MRT cases. In order to avoid duplication, we will not go into details here. Based on the above theoretical analysis, we find that MUI cannot be eliminated by MRC/MRT processing, but ZFT/ZFR processing can do that. Therefore, the performance of ZFT/ZFR is better than that of MRC/MRT.

V. SIMULATION RESULTS

In this section, we analyze the simulation results of uplink and downlink sum achievable rate of the system model proposed in this paper by building a simulation environment where N users uniformly distributed in a cell with a radius of 1000m. Moreover, the sum rate of this system are defined as $C_{sum} = C_U + C_D$, where C_U and C_D are the uplink and downlink sum achievable rate of the system, respectively. For simplicity, we suppose that all users have same Rician K -factor, which is defined as K . Assuming the minimum distance from the user to the BS is $r_{min} = 100m$, and r_n is the distance from the n th user to the BS. a_n is denoted as a log-normal random variable with standard deviation $\sigma = 8dB$. $\nu = 3.8$ is the path loss exponent. The large-scale channel fading can be modelled as $\beta_n = a_n (r_n/r_{min})^{-\nu}$ [7]. In this simulation, we assume that the number of users is $N = 10$, the variances of the noise are $\sigma_D = \sigma_U = 1$, the residual LI power is $\rho = \beta_{LI} = 0dB$, and $E_U = 10dB, E_D = N E_U$.

Fig. 2 and Fig. 3 depict the uplink and downlink sum achievable rate of Case 1 to Case 4 which are considered in above section versus different number of BS antennas. In this simulation, we choose Rician K -factor $K = 6dB$. In Fig. 2, it can be seen that the uplink sum rates of Case 1, Case 3 and Case4 increase with the number of BS antenna. Since the

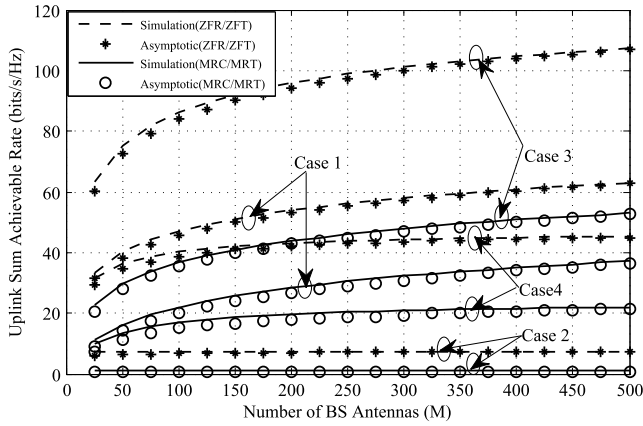


FIGURE 2. Uplink sum rate of full-duplex multi-user MIMO system over Rician fading channels versus the number of BS antennas for $K = 6dB$, $\rho = \beta_{LI} = 0dB$.

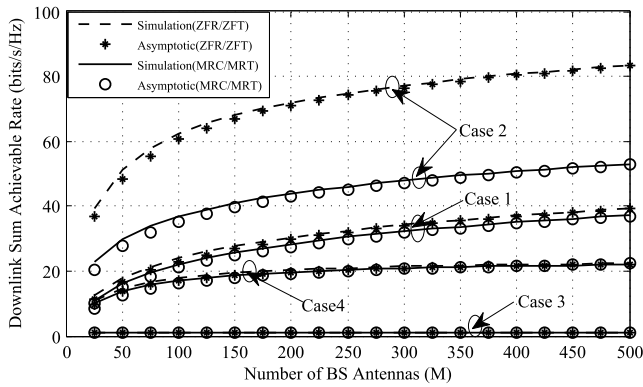


FIGURE 3. Downlink sum rate of full-duplex multi-user MIMO system over Rician fading channels versus the number of BS antennas for $K = 6dB$, $\rho = \beta_{LI} = 0dB$.

MUI, LI term and the AWGN at the BS of Case 1 and Case 3 tend to 0 when M grows large, their growth trend is more obvious than Case4, which cannot wipe out the AWGN at the BS. Moreover, Case 3 can further restrain the LI by the power scaling at the BS. As shown in Fig.2, the performance of Case 3 is best. We also can see that the uplink sum rate of Case 2 approaches to a constant. Since the LI and the AWGN at the BS cannot be eliminated. Therefore, the performance of Case 2 is worst. Fig. 3 shows that the downlink sum rates of Case 1, Case 2 and Case4 increase with the number of BS antenna. Since the MUI, IUI term and the AWGN at the BS of Case 1 and Case 2 tend to 0 when M tends to infinity, their growth trend is more obvious than Case4, which cannot wipe out the AWGN at the users. Moreover, Case 2 can further restrain the IUI by the power scaling at the users. As shown in Fig. 3, the performance of Case 2 is best. We also can see that the downlink sum rate of Case 3 approaches to a constant. Since the IUI and the AWGN at the users cannot be eliminated. Therefore, the performance of Case 3 is worst. Fig. 2 shows that Case 3 has the best performance when ZFR/ZFT processing is adopted. However, when MRC/MRT

processing is applied, we can see clearly the performance of Case 2 is best. In general, the system performance after ZFR/ZFT approach is better than MRC/MRT processing.

As shown in Fig. 2, Case 4 performance with ZFR/ZFT is better than Case 1 performance with MRC/MRT. However, as the number of antennas increases, the performance gap between two gradually decreases. From Fig. 3, Case 1 performance with MRC/MRT is better than Case 4 performance with ZFR/ZFT, and the performance gap gradually increases with the the number of antennas M . Fig. 4 depicts the FD sum rate of Case 1 to Case 4 which are considered in above section versus different number of BS antennas. Therefore, it is reasonable to have a cross point between the curve of Case 1 with MRC/MRT and Case 4 with ZFR/ZFT in Fig. 4.

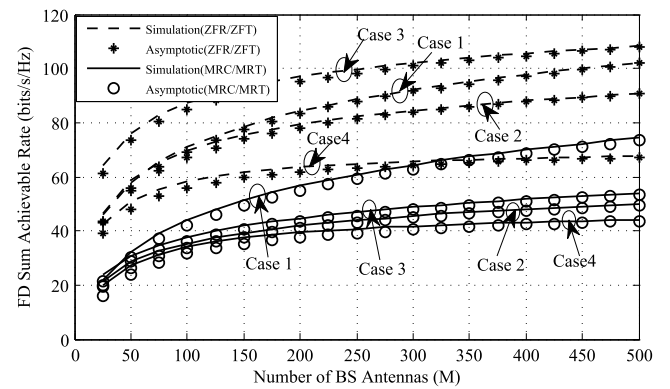


FIGURE 4. FD sum rate of full-duplex multi-user MIMO system over Rician fading channels versus the number of BS antennas for $K = 6dB$, $\rho = \beta_{LI} = 0dB$.

Fig. 5 depicts the uplink sum achievable rate of Case 1 to Case 4 which are considered in above section versus different Rician K -factor. In this simulation, the number of BS antennas is set to $M = 200$. Fig. 5 shows that the uplink sum rates of Case 1, Case 3 and Case4 increase with Rician

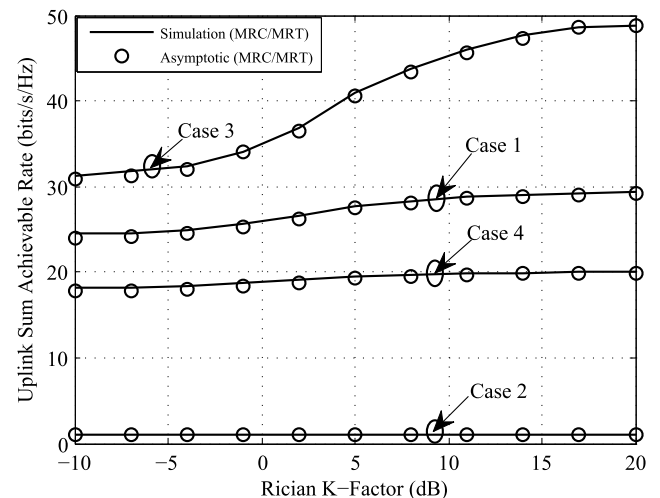


FIGURE 5. Uplink sum rate of full-duplex multi-user MIMO system over Rician fading channels versus Rician K -Factor for $M = 200$, $\rho = \beta_{LI} = 0dB$.

K -factor. Since the number of BS antennas in this simulation is large enough, the impact of the MUI, LI and the AWGN at the BS of Case 1 and Case 3 can be limited. Moreover, Case 3 can further restrain the LI by the power scaling at the BS. The performance of Case 3 is better than Case 1. Owing to Case 4 cannot wipe out the AWGN, the growth trend of Case 4 is not obvious. From Fig. 5, we also can see that as K increases, the uplink sum rates trend to be constant. Since the impact of the LI and the AWGN cannot be eliminated, Case 2 has the worst performance.

As shown in Fig. 5, Case 3 has the best performance. In order to further analyze the performance of Case 3, we study the uplink sum rate of Case 3 versus Rician K -Factor for different number of BS antennas. The following four cases are considered in Fig. 6: 1) $M = 50$; 2) $M = 150$; 3) $M = 250$; 4) $M = 350$. From Fig. 6, we can see that the uplink sum rate of four cases approach fixed values as K tends to infinity. Moreover, the gap between these curves decreases as the number of BS antennas grows.

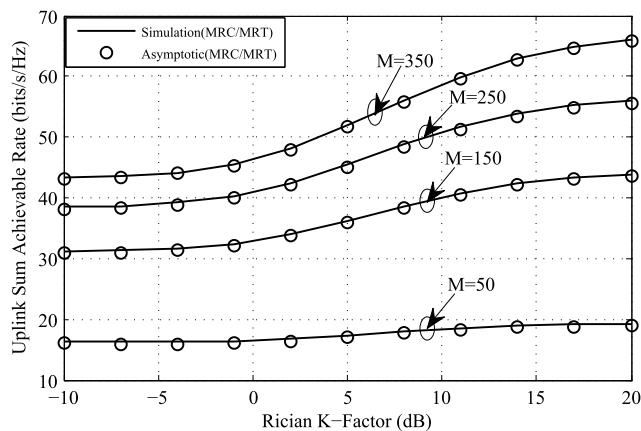


FIGURE 6. Uplink sum rate of Case 3 versus Rician K -Factor under different number of BS antennas for $N = 10$, $\rho = \beta_{LI} = 0dB$.

Fig. 7 depicts the downlink sum achievable rate of cases 1 to case 4 which are considered in above section versus different Rician K -factor. In this simulation, the number of

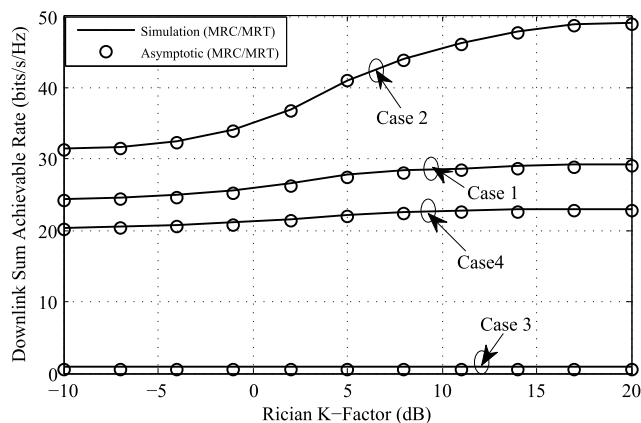


FIGURE 7. Downlink sum rate of full-duplex multi-user MIMO system over Rician fading channels versus Rician K -Factor for $M = 200$, $\rho = \beta_{LI} = 0dB$.

BS antennas is set to $M = 200$. Fig. 7 shows that the uplink sum rates of Case 1, Case 2 and Case4 increase with Rician K -factor. Since the number of BS antennas in this simulation is large enough, the impact of the MUI, LI and the AWGN at the BS of Case 1 and Case 2 can be limited. Moreover, Case 2 can further restrain the IUI by the power scaling at the BS. The performance of Case 3 is better than Case 1. Owing to Case 4 cannot wipe out the AWGN, the growth trend of Case 4 is not obvious. From Fig. 7, we also can see that as K increases, the downlink sum rates trend to be constant. Since the impact of the IUI and the AWGN cannot be eliminated, Case 3 has the worst performance.

VI. CONCLUSION

We study the uplink and downlink sum rates of multi-user FD large-scale MIMO systems over Rician fading channels in the paper. The uplink and downlink achievable rates under perfect CSI is derived based on MRC/MRT and ZFR/ZFT processing. The simulation results show that the detrimental effect of MUI, LI and the AWGN in the uplink can be eliminated by increasing the received antennas of BS and applying the power scaling law in the BS. Similarly, in the downlink, the adverse influence of MUI, IUI and the AWGN can be eliminated by increasing the transmitting antennas of BS and applying the power scaling law in the users. In addition, the simulation results indicates that the uplink and downlink sum achievable rates increase with the number of BS antennas and they will converge to fixed values with the increasing Rician K -factor.

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