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Fuzzy Dynamic Output Feedback Control for T-S Fuzzy Discrete-Time Systems With Multiple Time-Varying Delays and Unmatched Disturbances

WEI ZHENG[1], HONGBIN WANG[1], HONGRUI WANG[2], SHUHUAN WEN[1], AND ZHI-MING ZHANG[3]

[1]Institute of Electrical Engineering, Yanshan University, Qinhuangdao 066000, China
[2]Institute of Electronic and Information Engineering, Hebei University, Baoding 071000, China
[3]China Machinery Industry Group, China National Heavy Machinery Research Institute, Xi'an 710000, China

Corresponding author: Wei Zheng (xiaoxiong19871110@163.com)

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ABSTRACT This paper addressed the fuzzy dynamic output feedback control problem for a class of nonlinear discrete-time Takagi–Sugeno (T-S) fuzzy systems with multiple time-varying delays and unmatched disturbances. Based on the control input matrix and output matrix, the T-S fuzzy model is employed to approximate the nonlinear discrete-time system. Based on the stochastic system theory and the Bernoulli distribution, the fuzzy dynamic output feedback controller is constructed for the nonlinear discrete-time T-S fuzzy system with multiple time-varying delay and unmatched disturbance. The H_∞ performance analysis is presented, and the cone complementarity linearization algorithm is employed for the stability analysis to deal with the non-convex problem caused by the basis-dependent linear matrix inequalities conditions. Compared with the previous works, the developed controller in this paper is smooth and only uses the system output. The control design conditions are relaxed because of the developed cone complementarity linearization algorithm. The results are further extended to the chemical process case and the mobile robot case. Finally, two simulation examples are performed to show the effectiveness of the proposed methods.

INDEX TERMS T-S fuzzy model, stochastic system theory, fuzzy dynamic output feedback, multiple time-varying delays, unmatched disturbances.

I. INTRODUCTION

Many practical systems are the large nonlinear complex systems and consist of the time-delays and external disturbances in the real world [1], [2]. In these existing literatures, there are two main issues, the time-delays issue and the external disturbances issue, to be solved for the nonlinear uncertain systems [3]. Due to the effect of the time-delays and external disturbances, the systems may instability, and the system control performances of the nonlinear systems are assured hardly [4]. So far, the stability analysis and robust control for the dynamic time-delay systems have attracted a number of researchers over the past years, see [5]–[8] and the references therein. The fuzzy system can be regarded as a fuzzy blending of many local linear models and utilized effectively to approximate nonlinear plants encountered in control engineering. Thus, the method has become an important methodology for the nonlinear system design [9].

On the other hand, it is well known that the fuzzy control theory provides the powerful method to solve the control design issues for the nonlinear systems with the time-delays and external disturbances [10]. Particularly, the nonlinear T–S fuzzy model has attracted lots of attention for the intelligent fuzzy design methods, since it is rigorously effective and conceptually simple for the nonlinear highly complex systems [11]. Based on the dynamic fuzzy back-stepping control theory, an adaptive tracking controller was designed for the nonlinear MIMO system with time-delays [12]. In [13], an adaptive fuzzy-decentralized robust output-feedback control strategy is proposed for a class of large-scale strict-feedback nonlinear systems with unmeasured states and external disturbances. Then, a new adaptive T-S fuzzy method was developed in [14] to improve the haptic feedback fidelity of the nonlinear stochastic system with actuator faults. It should be point out that the above results are based on the parallel distributed compensation design scheme [15], and the fuzzy filter controller needs to share the same premise membership functions. To reduce the conservatism of the filter design methods, the fuzzy-basis-dependent Lyapunov-Krasovskii functional is employed and the stability conditions in the form of LMIs are derived [16]. In order to consider the delay distributions fully, the delay segmentation approach is introduced and the guaranteed cost controller was investigated for a class of interval type-2 T-S fuzzy descriptor system with time-varying delays [17]. However, if the nonlinear system contains the multiple time-delays and external disturbances, the membership functions of T-S fuzzy model will contain the nonlinear uncertainties. Based on the above reasons, the grades of membership for the nonlinear fuzzy system will become uncertain in value, and the non-convex problem caused by the nonlinear system may arise. Therefore, dealing with the trade-off between the less conservative condition and increased design complexity remains an important problem in control system design.

Recently, the adaptive fuzzy and output feedback control theories have been studied extensively in the nonlinear control problems and application problems, see [18], [19], and the references therein. In [20], a fuzzy-model-based static output-feedback method was proposed and the reliable distributed fuzzy controller was constructed. In order to deal with the unknown nonlinear uncertainties caused by the time-varying delays, some adaptive fuzzy and robust output-feedback control strategies [21]–[23] have been developed. In addition, the output-feedback controllers based on the fuzzy theory are often considered as universal effective controllers, and these controller have the better ability to approximate the nonlinear uncertainties. In [24], the dynamic output-feedback control theory was employed for a class of nonlinear industrial systems with unknown disturbances, and the fuzzy network design methodology was presented. Recently, the predictive control and fuzzy output feedback control methods based on the LMIs have been developed for the nonlinear affine/non-affine system [25]–[27]. In [28], the mode-dependent nonrational output feedback control method was proposed for the nonlinear semi-Markovian jump systems with time-varying delays. Then with the help of dynamic output feedback control technique, the two-term approximation theory was investigated in [29] for the Markovian jump systems with time-varying delays and defective mode information. However, most of the above controllers are designed based on the Lyapunov-Krasovskii functional. In addition, it should be mentioned that the unmatched disturbances have not been considered for the nonlinear system with multiple time-varying delays in these literatures. Very few results employed the stochastic system theory and Bernoulli distribution, and consider with the unmatched disturbances for the nonlinear T-S fuzzy system with multiple time-varying delays. The dynamic output feedback technique is more flexible and the required conditions are less conservative. Based on the above discussion, the H_∞ dynamic output-feedback controller for a class of nonlinear T-S fuzzy systems with multiple time-delays and unmatched external disturbances is investigated in this paper.

The objective of this paper is to design the dynamic output-feedback controller for the nonlinear discrete-time T-S fuzzy system, such that the solutions of the closed-loop system converge to an adjustable bounded region. Compared with the previous works, the developed controller in this paper is smooth and only uses the system output. The control design conditions are relaxed because of the developed cone complementarity linearization algorithm. The contributions of this paper are summarized as follows:

(1) The T-S fuzzy model is employed to approximate the nonlinear system based on the control input matrix and output matrix. And the nonlinear uncertainties caused by the multiple time-varying delays and unmatched disturbances can be approximated effectively.

(2) The fuzzy dynamic output feedback controller is constructed based on the stochastic system theory and Bernoulli distribution. The developed controller is smooth and more flexible, and only uses the system output.

(3)By introduced the stochastic system theory and Bernoulli distribution, it can be seen that the solutions of the resultant closed-loop system converge to an adjustable bounded region. The H_∞ performance analysis is presented, and the cone complementarity linearization algorithm is employed for the stability analysis to deal with the non-convex problem caused by the basis-dependent LMIs conditions.

This paper is organized as follows. In Section II, the preliminary knowledge is presented for the nonlinear discrete-time T-S fuzzy systems with multiple time-delays and unmatched disturbances. In Section III, based on the stochastic system theory and Bernoulli distribution, the fuzzy dynamic output feedback controller is constructed. In section IV, the H_∞ stability analysis and cone complementarity linearization algorithm are introduced for the closed-loop system. In Section V, two simulation examples are performed to show the effectiveness of the proposed methods. Finally, Section VI concludes with a summary of the obtained results.

II. SYSTEM DESCRIPTION

In this section, the T-S fuzzy model [30] is employed to approximate the physical plant of the nonlinear discrete-time system with multiple time-varying delays and unmatched disturbances.

Consider a class of nonlinear discrete-time system with multiple time-varying delays and unmatched disturbances as follows

$$\begin{cases} x(k+1) = A_i x(k - \tau_i(k)) + B_i u(k) + E_i d_i(k) \\ z(k) = C_{1i} x(k) + D_i u(k) \\ y(k) = C_{2i} x(k), \quad i \in [1, r] \end{cases} \tag{1}$$

where $x(k) \in R^n$, $y(k) \in R^m$, $z(k) \in R^s$ and $u(k) \in R^q$ are the state variable, measured output, control output and control input of the system, respectively. A_i, B_i, C_{1i}, C_{2i}, D_i, and E_i are the gain matrices with appropriate dimensions. r is the number of IF–THEN rule. $\tau_i(k)$ are the multiple time-varying delays. $d_i(k)$ are the unmatched external disturbances such that [31]

$$|d_i(k_1+1) - d_i(k_2+1)| \le l_i |k_1 - k_2| \tag{2}$$

where l_i is a scalar, k_0 is the initial time for $k_0 < k_1, k_2$, the time-varying delays $d_i(k_1+1)$ and $d_i(k_2+1)$ satisfy the Lipschita conditions [31].

Then, by employing the T-S fuzzy modeling technique in [30], the system (1) is further rewritten as follows

Plant rule *i*: IF $\theta_1(k)$ is M_{i1}, $\theta_2(k)$ is M_{i2}, ..., and $\theta_j(k)$ is M_{ij} $(j \in [1, r])$, THEN

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} f_i(\theta(k))[A_i x(k-\tau_i(k)) + B_i u(k) + E_i d_i(k)] \\ z(k) = \sum_{i=1}^{r} f_i(\theta(k))[C_{1i}x(k) + D_i u(k)] \\ y(k) = \sum_{i=1}^{r} f_i(\theta(k))[C_{2i}x(k)], \quad i, \in [1, r] \end{cases} \tag{3}$$

where $\theta(k) = [\theta_1(k), \theta_2(k), \ldots, \theta_j(k)]^T$ is the premise variable, M_{ij} is the fuzzy set, $f_i(\theta(k))$ is the fuzzy basis functions of $\theta(k)$ and described as follows

$$f_i(\theta(k)) = \frac{\prod_{j=1}^{r} M_{ij}(\theta_j(k))}{\sum_{i=1}^{r}\prod_{j=1}^{r} M_{ij}(\theta_j(k))} \tag{4}$$

where $M_{ij}(\theta_j(k))$ is the membership function of $\theta_j(k)$ in the fuzzy set M_{ij}.

Assumption 1: For the system (3), if there exist $\prod_{j=1}^{r} M_{ij}(\theta_j(k)) \ge 0$ and $\sum_{i=1}^{r}\prod_{j=1}^{r} M_{ij}(\theta_j(k)) > 0$ for all $i \in [1, r]$ and $j \in [1, r]$, then the following conditions hold

$$\begin{cases} f_i(\theta(k)) \ge 0 \\ \sum_{i=1}^{r} f_i(\theta(k)) = 1, \quad i \in [1, r] \end{cases} \tag{5}$$

Remark 1: The objective of this paper is to design the fuzzy dynamic output feedback controller for the system with *Assumption 1*, such that the solutions of the resultant closed-loop system converge to an adjustable bounded region.

Remark 2: Since the closed-loop system performance can be degraded severely by the time-varying delays and external disturbances, the control design problem for the nonlinear systems presents a tremendous challenge. The adaptive dynamic fuzzy back-stepping control approach is proposed for a class of nonlinear MIMO systems with immeasurable states, without considering the time-delays [12]. The nonrational output feedback control approach is proposed for the nonlinear semi-Markovian jump systems with time-varying delays, without considering the external disturbances [28]. The state estimation problem is addressed for the discrete-time dynamical networks with time-varying delays and bounded disturbances, without considering the unmatched disturbances [32]. Different from the above literatures, the fuzzy dynamic output feedback control problem is addressed in this paper for the nonlinear discrete-time T-S fuzzy system with multiple time-varying delays and unmatched disturbance. With the help of the proposed methods, both the steady-state performance and transient-state performance can be guaranteed. Moreover, by introduced the stochastic system theory and Bernoulli distribution, it can be seen that the solutions of the resultant closed-loop system converge to an adjustable bounded region

Remark 3: For the problem formulated, there are three challenging issues as follows. The first one is how to employ the T-S fuzzy model for the nonlinear system to approximate the nonlinear uncertainties caused by the multiple time-varying delays and unmatched disturbances. The second one is how to introduce the stochastic system theory and Bernoulli distribution to design the fuzzy dynamic output feedback controller, such that the solutions of the resultant closed-loop system converge to an adjustable bounded region. The third one is how to employ the cone complementarity linearization algorithm to deal with the non-convex problem in the LMIs. If the above three issues are solved, the controller will be designed with easy implementation in practical engineering systems.

III. CONTROLLER DESIGN

In this section, based on the stochastic system theory and Bernoulli distribution, the fuzzy dynamic output feedback controller is designed for the system (3). The graphical abstract of the proposed methodology is shown in Fig. 1.

For the system (3), the dynamic output feedback controller is designed as follows.

Controller rule *i*: IF $\theta_1(k)$ is M_{i1}, $\theta_2(k)$ is M_{i2}, ..., and $\theta_j(k)$ is $M_{ij}(j \in [1, r])$, THEN

$$\begin{cases} x_c(k+1) = A_{ci}x_c(k) + B_{ci}y_c(k) \\ u_c(k) = C_{ci}x_c(k) \end{cases} \tag{6}$$

where $x_c(k) \in R^n$, $y_c(k) \in R^m$ and $u_c(k) \in R^q$ are the state vector, control output and control input of the dynamic output feedback controller, respectively. A_{ci}, B_{ci} and C_{ci} are the gain matrices with appropriate dimensions.

Then, for the system (3), the fuzzy dynamic output feedback controller (6) is obtained as follows

$$\begin{cases} x_c(k+1) = \sum_{i=1}^{r} f_i[A_{ci}x_c(k) + B_{ci}y_c(k)] \\ u_c(k) = \sum_{i=1}^{r} f_i[C_{ci}x_c(k)] \end{cases} \tag{7}$$

Remark 4: Due to the existence of the multiple time-varying delays and unmatched disturbances between the system plant and the dynamic output controller (as shown

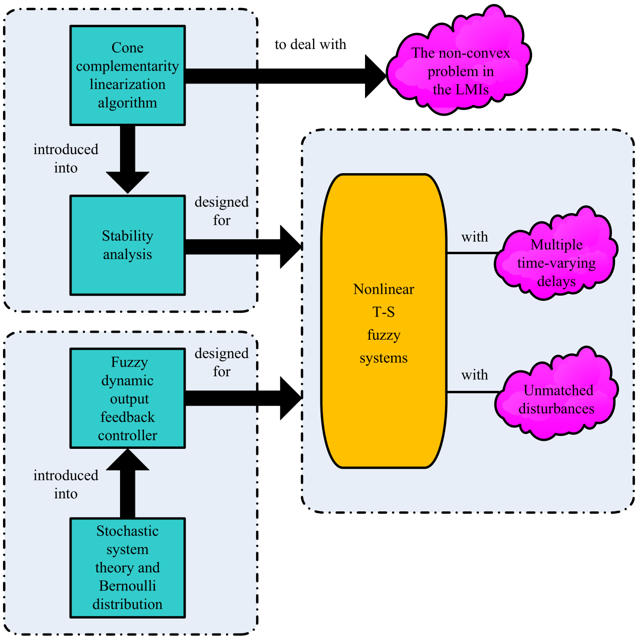

FIGURE 1. The graphical abstract of the proposed methodology.

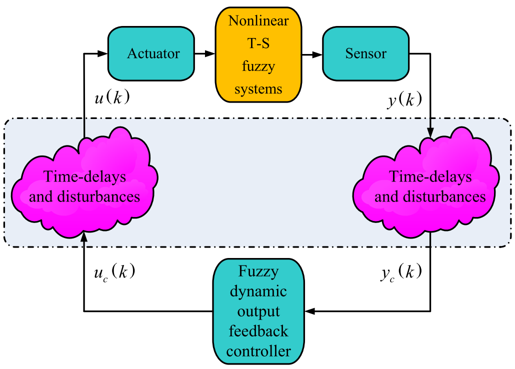

FIGURE 2. Fuzzy dynamic output feedback control for the nonlinear T-S system.

in Fig. 2). The output vector $y(k)$ of the system plant is not equal to the input vector $y_c(k)$ of the controller (i.e. $y(k) \neq y_c(k)$), and the output vector $u_c(k)$ of the controller is not equal to the input vector $u(k)$ of the system plant (i.e. $u_c(k) \neq u(k)$).

Base on the above reasons as show in *Remark 4*, the stochastic system theory in [33] is employed to describe the phenomenon as follows

$$\begin{cases} y_c(k) = \alpha(k)\, y(k) \\ u(k) = \beta(k)\, u_c(k) \end{cases} \tag{8}$$

$$\begin{cases} Prob\{\alpha(k) = 1\} = E\{\alpha(k)\} = \bar{\alpha} \\ Prob\{\alpha(k) = 0\} = 1 - \bar{\alpha} \\ Prob\{\beta(k) = 1\} = E\{\beta(k)\} = \bar{\beta} \\ Prob\{\beta(k) = 0\} = 1 - \bar{\beta} \end{cases} \tag{9}$$

with

$$\alpha(k) = \begin{cases} 1, & y_c(k) = y(k)\ (e.t. y_c(k) \text{ is available}) \\ 0, & y_c(k) \neq y(k)\ (e.t. y_c(k) \text{ is unavailable}) \end{cases} \tag{10}$$

$$\beta(k) = \begin{cases} 1, & u_c(k) = u(k)\ (e.t. u(k) \text{ is available}) \\ 0, & u_c(k) \neq u(k)\ (e.t. u(k) \text{ is unavailable}) \end{cases} \tag{11}$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the positive scalars. $\alpha(k)$ and $\beta(k)$ are the random variables representing the measurements probabilistic-density functions. If $\alpha(k) = 1$ $(\beta(k) = 1)$ representing successful transmission. If $\alpha(k) = 0$ $(\beta(k) = 0)$ representing failed transmission. $\{\alpha(k)\}$ and $\{\beta(k)\}$ obey the Bernoulli distribution and representing the missing measurements nature from the sensors to the controller due to the time-delays, external disturbances, suddenly breakdown or limited capacity in information transmission (as shown in Fig.2). $Prob\{\alpha(k)\}$ and $Prob\{\beta(k)\}$ are the probability of the random variables $\alpha(k)$ and $\beta(k)$, respectively. $E\{\alpha(k)\}$ and $E\{\beta(k)\}$ are the mathematical expectation values of the random variables $\alpha(k)$ and $\beta(k)$, respectively.

Based on the stochastic equation (8), the fuzzy dynamic output feedback controller (7) is further rewritten as follows

$$\begin{cases} x_c(k+1) = \sum_{i=1}^{r} f_i \left[A_{ci} x_c(k) + B_{ci} \alpha(k)\, y(k)\right] \\ u(k) = \beta(k) \sum_{i=1}^{r} f_i \left[C_{ci} x_c(k)\right] \end{cases} \tag{12}$$

By employing the controller (12) for the system (3), the nonlinear discrete-time T-S fuzzy closed-loop system is obtained as follows

$$\begin{cases} \bar{x}(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} f_i f_j \left[\left(A_{1ij} + A_{2ij}\right)\bar{x}(k) + \bar{E}_i d_i(k)\right] \\ z(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} f_i f_j \left[\left(C_{1ij} + C_{2ij}\right)\bar{x}(k)\right] \end{cases} \tag{13}$$

where

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}, \quad A_{1ij} = \begin{bmatrix} A_i & \bar{\beta} B_i C_{ci} \\ \bar{\alpha} B_{ci} C_{2i} & A_{ci} \end{bmatrix},$$
$$A_{2ij} = \begin{bmatrix} 0 & \tilde{\beta}(k)\, B_i C_{ci} \\ \tilde{\alpha}(k)\, B_{ci} C_{2i} & 0 \end{bmatrix}, \quad \bar{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix},$$
$$C_{1ij} = \begin{bmatrix} C_{1i} & \bar{\beta} D_i C_{ci} \end{bmatrix}, \quad C_{2ij} = \begin{bmatrix} 0 & \tilde{\beta}(k)\, D_i C_{ci} \end{bmatrix},$$

in which

$$\begin{cases} \tilde{\alpha}(k) = \alpha(k) - \bar{\alpha} \\ \tilde{\beta}(k) = \beta(k) - \bar{\beta} \end{cases} \tag{14}$$

Based on the definition of the stochastic variables, with (8)-(11), one has

$$\begin{cases} E\{\tilde{\alpha}(k)\} = E\{\alpha(k) - \bar{\alpha}\} = 0 \\ E\left\{\tilde{\beta}(k)\right\} = E\left\{\beta(k) - \bar{\beta}\right\} = 0 \\ E\left\{\tilde{\alpha}^2(k)\right\} = E\{\alpha(k) - \bar{\alpha}\} = \bar{\alpha}(1 - \bar{\alpha}) = a^2 \\ E\left\{\tilde{\beta}^2(k)\right\} = E\left\{\beta(k) - \bar{\beta}\right\} = \bar{\beta}\left(1 - \bar{\beta}\right) = b^2 \end{cases} \tag{15}$$

where a and b are the scalars.

For the problem formulated, there exist the *Lemmas 1* and **2** as follows.

Lemma 1: For the system (13), if there exist the positive scalar χ, positive matrix G such that $\chi > 0$ and $G > 0$ for the initial conditions $\bar{x}(0) \equiv 0$ and $d(k) \equiv 0$, the following inequalities holds

$$E\left\{\sum_{k=0}^{\infty} |\bar{x}(k)|^2\right\} < x_0^T G x_0 \tag{16}$$

$$E\left\{\left(\sum_{k=0}^{\infty} |z(k)|^2\right)^{1/2}\right\} \leq \chi \left\|d_i(k)\right\|_2 \tag{17}$$

where x_0 is the initial state of the nonlinear system. Then the closed-loop system is stable.

Lemma 2: For the system (13), if there exist the negative matrices $\mathcal{M}_{il}$ such that

$$\begin{cases} \mathcal{M}_{il(i=l)} < 0 \\ \dfrac{\mathcal{M}_{il(i=l)}}{r-1} + \dfrac{\left(\mathcal{M}_{il(i\neq l)} + \mathcal{M}_{li(i\neq l)}\right)}{2} < 0, i, \quad l \in [1, r] \end{cases} \tag{18}$$

Then, the following inequality holds

$$\sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l \mathcal{M}_{il} < 0 \tag{19}$$

IV. STABILITY ANALYSIS

In this section, the H_∞ performance analysis is presented at first, then the cone complementarity linearization algorithm is employed for the system (13) to deal with the non-convex problem caused by the LMIs conditions.

Remark 5: First, the H_∞ stability analysis is presented for the system (13). The fuzzy discrete Lyapounov-Krasovskii functional (24) is introduced, and a non-convex condition will be produced when obtaining the LMIs conditions for the controller design. Secondly, in order to deal with the non-convex problem, the cone complementarity linearization algorithm is employed for the fuzzy dynamic output feedback controller design.

A. H_∞ STABILITY ANALYSIS

In this section, the H_∞ stability analysis is presented for the system (13).

Theorem 1: For the system (13), if there exist the positive scalar $\chi > 0$, identity matrix I, positive matrix $Q_i > 0$, and nonsingular positive matrix $Q_j > 0$ such that

$$\begin{bmatrix} \Upsilon_j & \Theta_{il} \\ * & \bar{Q}_i \end{bmatrix} < 0 \tag{20}$$

where

$$\bar{Q}_i = diag\left\{-Q_i, -\chi I\right\} \tag{21}$$

$$\Upsilon_j = diag\left\{-Q_j^{-1}, -Q_j^{-1}, -I, -I\right\} \tag{22}$$

$$\Theta_{il} = \begin{bmatrix} A_{1il} & \bar{E}_i \\ \bar{A}_{2il} & 0 \\ C_{1il} & 0 \\ \bar{C}_{2il} & 0 \end{bmatrix} \tag{23}$$

in which

$$A_{1il} = \begin{bmatrix} 0 & bB_iC_{ci} \\ aB_{ci}C_{1i} & 0 \end{bmatrix}, \quad \bar{A}_{2il} = \begin{bmatrix} 0 & bB_iC_{ci} \\ aB_{ci}C_{2i} & 0 \end{bmatrix},$$

$$C_{1il} = \bar{C}_{2il} = \begin{bmatrix} 0 & bD_iC_{ci} \end{bmatrix}, \quad \bar{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix}.$$

Then, it can be seen that the solutions of the closed-loop system (13) converge to an adjustable bounded region.

Proof: For the system (13), choose a fuzzy discrete Lyapunov-Krasovskii functional $f_i(\cdot)$ as follows

$$V(k) = \bar{x}^T(k)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(k) \tag{24}$$

where $f_i(\cdot)$ is the fuzzy basis functions, and $Q_i > 0$ is a positive matrix.

With the *Lemma 1*, the mathematical expectation value of $E\{V(k+1) - V(k)\}$ satisfies

$$\begin{aligned} &E\{V(k+1) - V(k)\} \\ &= E\{(\bar{x}^T(k)\sum_{j=1}^{r} f_j^{+}(\sum_{i=1}^{r}\sum_{l=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} f_i f_l f_s f_t (A_{1il} + A_{2il})^T \\ &\quad \times Q_j (A_{1il} + A_{2il}))\bar{x}(k))\} - \bar{x}^T(k)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(k) \\ &\leq \bar{x}^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l \left(A_{1il}^T Q_j A_{1il} + \bar{A}_{2il}^T Q_j \bar{A}_{2il}\right)\bar{x}(k) \\ &\quad - \bar{x}^T(k)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(k) \\ &= \bar{x}^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l \left[A_{1il}^T Q_j A_{1il} + \bar{A}_{2il}^T Q_j \bar{A}_{2il} - Q_i\right]\bar{x}(k) \\ &= \bar{x}^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l \left(\Theta_{il}^T \hat{Q}_j \Theta_{il} - Q_i\right)\bar{x}(k) \end{aligned} \tag{25}$$

where $f_j^{+} = f_j(\theta(k+1))$ and $\hat{Q}_j = diag\left\{Q_j, Q_j\right\}$.

By employing the Schur complement, one knows that the inequality (20) is equivalent to the inequality (26) as follows

$$\Theta_{il}^T \hat{Q}_j \Theta_{il} - Q_i < 0, \quad i, j, l \in [1, r] \tag{26}$$

For the system (13), defining the parameter Ξ such that

$$\Xi = \sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l \left(\Theta_{il}^T \hat{Q}_j \Theta_{il} - Q_i\right) < 0 \tag{27}$$

With (24) and (27), one has

$$\begin{aligned} E\left\{\bar{x}^T(k+1)\left(\sum_{j=1}^{r} f_j^{+} Q_j\right)\bar{x}(k+1)\right\} - x^T(k)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(k) \\ \leq -\mu(-\Xi)\bar{x}^T(k)\, G x^T(k) \end{aligned} \tag{28}$$

where μ is a positive scalar.

Based on the *Lemma 1*, taking the mathematical expectation value for right side of (28), one has

$$E\left\{x^T(\varepsilon+1)\left(\sum_{j=1}^{r} f_j Q_j\right)\bar{x}(\varepsilon+1)\right\}-\bar{x}^T(0)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(0) \leq -\mu(-\Xi)\,E\left\{\sum_{k=0}^{\varepsilon}|\bar{x}(k)|^2\right\} \tag{29}$$

where $k=0,1,\ldots,\varepsilon$ with $\varepsilon \geq 1$ is a positive integer scalar.

With the *Lemma 1* and (29), one has

$$\begin{aligned} E\left\{\sum_{k=0}^{\varepsilon}|x(k)|^2\right\} &\leq (\mu(-\Xi))^{-1}\times\left\{\bar{x}^T(0)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(0)\right. \\ &\quad \left. - E\left\{\bar{x}^T(\varepsilon+1)\left(\sum_{j=1}^{r} f_j^{+} Q_j\right)\bar{x}(\varepsilon+1)\right\}\right\} \\ &\leq (\mu(-\Xi))^{-1}\bar{x}^T(0)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(0) \end{aligned} \tag{30}$$

where $\bar{x}(0)$ is the initial condition.

Assumption 2: For the system (13), if there exist the positive scalar $\varepsilon \geq 1$ and the nonsingular positive matrix $Q_j > 0$ such that

$$E\left\{\bar{x}^T(\varepsilon)\left(\sum_{j=1}^{r} f_j^{+} Q_j\right)\bar{x}(\varepsilon)\right\} \geq 0 \tag{31}$$

then based on the *Lemma 1,* the following inequality holds

$$\begin{aligned} E\left\{\sum_{k=0}^{\varepsilon}|\bar{x}(k)|^2\right\} &\leq (\mu(-\Xi))^{-1}\bar{x}^T(0)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(0) \\ &= \bar{x}^T(0)\left((\mu(-\Xi))^{-1}\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(0) \\ &= \bar{x}^T(0)\,G\bar{x}(0) \end{aligned} \tag{32}$$

where

$$G = (\mu(-\Xi))^{-1}\sum_{i=1}^{r} f_i Q_i \tag{33}$$

With (27) and (33), one can obtain $\Xi < 0$ and $G > 0$. Thus, based on the above analysis and *Lemma 1*, it can be seen that the nonlinear T-S fuzzy closed-loop system (13) is stable.

Then, for the system (13), the H_∞ performance index J is introduced as follows

$$J = E\left\{z^T(k)\,z(k)\right\}-\chi^2 d_i^T(k)\,d_i(k)+E\left\{V(k+1)-V(k)\right\} \tag{34}$$

For (34), defining the parameter $\xi_i(k)$ as follows

$$\xi_i(k) = \begin{bmatrix} \bar{x}(k) \\ d_i(k) \end{bmatrix} \tag{35}$$

With (34) and (35), one has

$$\begin{aligned} J &= E\left\{z^T(k)\,z(k)\right\}-\chi^2 d_i^T(k)\,d_i(k)+E\left\{V(k+1)-V(k)\right\} \\ &= E\left\{\xi_i^T(k)\sum_{i=1}^{r}\sum_{l=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} f_i f_l f_s f_t\left(\begin{bmatrix} C_{1il}+C_{2il} & 0\end{bmatrix}^T\right.\right. \\ &\quad \left.\left.\times\begin{bmatrix} C_{1il}+C_{2il} & 0\end{bmatrix}\right)\xi_i(k)\right\} \\ &\quad + E\left\{\xi_i^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} f_i f_l f_s f_t\right. \\ &\quad \left.\times\left(\begin{bmatrix} A_{1il}+A_{2il} & \bar{E}_i\end{bmatrix}^T Q_j\begin{bmatrix} A_{1il}+A_{2il} & \bar{E}_i\end{bmatrix}\right)\xi_i(k)\right\} \\ &\quad -\chi^2 d_i^T(k)\,d_i(k)-\bar{x}^T(k)\left(\sum_{i=1}^{r} f_i Q_i\right)\bar{x}(k) \\ &= \xi_i^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} f_i f_l f_s f_t \\ &\quad \times\begin{bmatrix} C_{1il}^T C_{1st}+\bar{C}_{2il}^T\bar{C}_{2st} & 0 \\ 0 & 0\end{bmatrix}\xi_i(k) \\ &\quad +\xi_i^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} f_i f_l f_s f_t \\ &\quad \times\begin{bmatrix} A_{1il}^T Q_j A_{1st}+\bar{A}_{2il}^T Q_j\bar{A}_{2st} & A_{1il}^T Q_j\bar{E}_s \\ \bar{E}_i^T Q_j A_{1st} & \bar{E}_i^T Q_j\bar{E}_s\end{bmatrix}\xi_i(k) \\ &\quad -\xi_i^T(k)\sum_{i=1}^{r} f_i\begin{bmatrix} Q_i & 0 \\ 0 & \gamma^2 I\end{bmatrix}\xi_i(k) \end{aligned} \tag{36}$$

where

$$\begin{aligned} J &\leq \xi_i^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l\begin{bmatrix} C_{1il}^T C_{1il}+\bar{C}_{2il}^T\bar{C}_{2il} & 0 \\ * & 0\end{bmatrix}\xi_i(k) \\ &\quad +\xi_i^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l \\ &\quad \times\begin{bmatrix} A_{1il}^T Q_j A_{1il}+\bar{A}_{2il}^T Q_j\bar{A}_{2il} & A_{1il}^T Q_j\bar{E}_i \\ \bar{E}_i^T Q_j A_{1il} & \bar{E}_i^T Q_j\bar{E}_i\end{bmatrix}\xi_i(k) \\ &\quad -\xi_i^T(k)\sum_{i=1}^{r} f_i\begin{bmatrix} Q_i & 0 \\ 0 & \chi^2 I\end{bmatrix}\xi_i(k) \\ &= \xi_i^T(k)\sum_{j=1}^{r} f_j^{+}\sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l\left\{\left[\begin{bmatrix} C_{1il}^T C_{1il}+\bar{C}_{2il}^T\bar{C}_{2il} & 0 \\ * & 0\end{bmatrix}\right.\right. \\ &\quad \left.\left.+\begin{bmatrix} A_{1il}^T Q_j A_{1il}+\bar{A}_{2il}^T Q_j\bar{A}_{2il} & A_{1il}^T Q_j\bar{E}_i \\ * & \bar{E}_i^T Q_j\bar{E}_i\end{bmatrix}-\begin{bmatrix} Q_i & 0 \\ 0 & \gamma^2 I\end{bmatrix}\right]\right\} \end{aligned}$$

$$\times \xi_i(k)$$
$$= \xi_i^T(k)\sum_{j=1}^{r} f_j^+ \sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l \left\{ \begin{bmatrix} A_{1il} & \bar{E}_i \\ \bar{A}_{2il} & 0 \end{bmatrix}^T \begin{bmatrix} Q_i & 0 \\ 0 & Q_j \end{bmatrix} \right.$$
$$\times \begin{bmatrix} A_{1il} & \bar{E}_i \\ \bar{A}_{2il} & 0 \end{bmatrix}$$
$$\left. + \begin{bmatrix} C_{1il} & 0 \\ \bar{C}_{2il} & 0 \end{bmatrix}^T \begin{bmatrix} C_{1il} & 0 \\ \bar{C}_{2il} & 0 \end{bmatrix} - \begin{bmatrix} Q_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} \right\} \xi_i(k)$$

By employing the Schur complement and considering the inequality (20), one has

$$J \le E\left\{ z^T(k)\, z(k) \right\} - \chi^2 d_i^T(k)\, d_i(k) + \Delta V(k) \le 0 \quad (37)$$

The proof for the *Theorem 1* is completed.

Remark 6: For the system (13), due to introducing the fuzzy discrete Lyapounov-Krasovskii functional (24), the non-convex condition will be produced when obtaining the LMIs conditions for the controller design. Thus, it will be more difficult to solve the parameter matrices A_{ci}, B_{ci} and C_{ci} for the proposed controller. In order to deal with the non-convex problem, the cone complementarity linearization algorithm is employed for the controller design in Section *B*.

B. CONE COMPLEMENTARITY LINEARIZATION ALGORITHM

In this section, the cone complementarity linearization algorithm is employed for the fuzzy dynamic output feedback controller design. And the control design conditions are relaxed because of the developed cone complementarity linearization algorithm.

Remark 7: The dynamic output feedback technique is more flexible and the required conditions on the nonlinear systems are less conservative. The proposed controller is smooth and the precise time delays are not required for the control implementation. But the control design conditions for the nonlinear systems are more conservative because of the non-convex problem. Thus, based on the *Lemma 2*, the more flexible conditions will be presented in the *Theorem 2* as follows.

Theorem 2: For the system (13), if there exist the positive scalar $\chi > 0$, identity matrix I, positive matrix $Q_i > 0$, and nonsingular positive matrix $L_i > 0$ such that

$$\begin{bmatrix} \hat{L}_i & \hat{\Theta}_{il} \\ * & \bar{Q}_i \end{bmatrix} < 0 \quad (38)$$

where

$$\bar{Q}_i = diag\{-Q_i, -\chi I\} \quad (39)$$
$$\hat{L}_i = diag\{-L_i, -L_i, -I, -I\} \quad (40)$$
$$\hat{\Theta}_{il} = \begin{bmatrix} \hat{A}_{1il} & \bar{E}_i \\ \hat{\hat{A}}_{2il} & 0 \\ \hat{C}_{1il} & 0 \\ \hat{\hat{C}}_{2il} & 0 \end{bmatrix} \quad (41)$$

in which $\hat{A}_{1il}$, $\hat{\hat{A}}_{2il}$, $\hat{C}_{1il}$ and $\hat{\hat{C}}_{2il}$ are the system parameter matrices with appropriate dimensions, then the following conditions hold

$$\bar{\mathcal{N}}_{ilj(i=l)} < 0, \quad i, l, j \in [1, r] \quad (42)$$
$$\frac{\bar{\mathcal{N}}_{ilj(i\neq l)}}{r-1} + \frac{\left(\bar{\mathcal{N}}_{ilj(i\neq l)} + \bar{\mathcal{N}}_{lij(i\neq l)}\right)}{2} < 0 \quad (43)$$
$$Q_i L_i = I \quad (44)$$

where $\bar{\mathcal{N}}_{ilj} = \begin{bmatrix} \hat{L}_i & \hat{\Theta}_{il} \\ * & \bar{Q}_i \end{bmatrix} < 0$ is a negative matrix

Then, it can be seen that the solutions of the resultant closed-loop system (13) converge to an adjustable bounded region.

Proof: For the matrix $\hat{\Theta}_{il}$ in (41), the parameter matrices $\hat{A}_{1il}, \hat{\hat{A}}_{2il}, \hat{C}_{1il}$ and $\hat{\hat{C}}_{2il}$ are defined as follows

$$\begin{cases} \hat{A}_{1il} = \Lambda_i + EK_i\Phi_i + \Omega_i\bar{C}_{ci} \\ \hat{\hat{A}}_{2il} = EK_iR_i + S_i\bar{C}_{ci} \\ \hat{C}_{1il} = \Psi_i + \bar{\beta}D_i\bar{C}_{ci} \\ \hat{\hat{C}}_{2il} = bD_i\bar{C}_{ci} \end{cases} \quad (45)$$

where

$$\Lambda_i = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad K_i = \begin{bmatrix} A_{ci} & B_{ci} \end{bmatrix},$$
$$\Phi_i = \begin{bmatrix} 0 & I \\ \bar{\alpha}C_{2i} & 0 \end{bmatrix}, \quad \Omega_i = \begin{bmatrix} \bar{\beta}B_i \\ 0 \end{bmatrix}, \quad \bar{C}_{ci} = \begin{bmatrix} 0 & C_{ci} \end{bmatrix},$$
$$R_i = \begin{bmatrix} 0 & 0 \\ aC_{2i} & 0 \end{bmatrix}, \quad S_i = \begin{bmatrix} bB_i \\ 0 \end{bmatrix}, \quad \Psi_i = \begin{bmatrix} C_{1i} & 0 \end{bmatrix}.$$

With (45), the inequality (20) can be rewritten as follows

$$\begin{bmatrix} \Upsilon_j & \hat{\Theta}_{il} \\ * & \bar{Q}_i \end{bmatrix} < 0 \quad (46)$$

Let $\Upsilon_j = \hat{L}_i$, one has

$$\begin{bmatrix} \hat{L}_i & \hat{\Theta}_{il} \\ * & \bar{Q}_i \end{bmatrix} < 0 \quad (47)$$

With the *Lemma 2* and (47), the following inequality holds

$$\sum_{j=1}^{r} f_j^+ \sum_{i=1}^{r}\sum_{l=1}^{r} f_i f_l \bar{\mathcal{N}}_{ilj} < 0 \quad (48)$$

With the above analysis, it can be seen that if Q_i and L_i are the feasible solutions of (42)-(44), the gain matrices of the controller A_{ci}, B_{ci} and C_{ci} can be solved in the *Theorem 2.* The proof for the *Theorem 2* is completed.

Remark 8: From the *Theorem 2,* it can be seen that the condition (44) is a non-convex set. In order to deal with the non-convex problem, the cone complementarity linearization algorithm is introduced as follows.

Lemma 3 [34]: The basic idea of the cone complementarity linearization algorithm is that if there exist the positive matrices $Q_i \in R^{n\times n}$ and $L_i \in R^{n\times n}$ such that $Q_i > 0$, $L_i > 0$ and

$\begin{bmatrix} Q_i & I \\ I & L_i \end{bmatrix} \geq 0$ for $i \in [1, r]$, then the following conditions hold

$$\begin{cases} Tr\,(Q_iL_i) > n \\ Tr\,(Q_iL_i) = n, Q_i = L_i = I \end{cases} \tag{49}$$

where $Tr\,(Q_iL_i)$ is the trace of the matrix Q_iL_i.

Note, the LMIs can be solved by the LMIs toolbox, and the basic feasible solutions A_{ci}, B_{ci}, C_{ci}, Q_i and L_i can be obtained. With the above analysis, it can be seen that the nonlinear minimization problem can be instead of the non-convex feasibility problem in the *Theorem 2*. Thus, the LMIs conditions for the nonlinear minimization problem are given as follows

$$minTr\left(\sum_{i=1}^{r} Q_iL_i\right) \quad s.t.\,(19),\ (20)\ and \begin{bmatrix} Q_i & I \\ I & L_i \end{bmatrix} \geq 0 \tag{50}$$

where $minTr\,(\cdot)$ is the minimum value for the trace of matrix $\sum_{i=1}^{r} Q_iL_i$.

For the nonlinear minimization problem, if there exist the feasible solutions rn such that the following conditions hold

$$\begin{cases} Tr\left(\sum_{i=1}^{r} Q_iL_i\right) > rn \\ Tr\left(\sum_{i=1}^{r} Q_iL_i\right) = rn, \quad Q_i = L_i = I \end{cases} \tag{51}$$

then the LMIs conditions are solvable in the *Theorem 2*.

For the system (13), the cone complementarity linearization algorithm is designed as follows (Step 1-5) [34].

Step 1. Select the membership functions and construct the fuzzy rules for the system (3). Giving the H_∞ performance index J for the closed-loop system (13). Go to Step 2.

Step 2. Solve the LMIs (42), (43), and $\begin{bmatrix} Q_i & I \\ I & L_i \end{bmatrix} \geq 0$ in the *Lemma 3* to obtain the initial feasible solutions (A^0_{ci}, B^0_{ci}, C^0_{ci}, Q^0_i and L^0_i). Then, set $k = 0$, where k is the numbers of the iterations. Go to Step 3.

Step 3. Solve the following LMIs for the parameter matrices A_{ci}, B_{ci}, C_{ci}, Q_i and L_i such that

$$minTr\left(\sum_{i=1}^{r} (Q_iL_i)\right), \quad s.t.\ (19),\ (20)\ and \begin{bmatrix} Q_i & I \\ I & L_i \end{bmatrix} \geq 0 \tag{52}$$

Set $Q^{k+1}_i = Q_i$ and $L^{k+1}_i = L_i$. Go to Step 4.

Step 4. If the LMIs (42), (43) and $\begin{bmatrix} \hat{L}_i & \hat{\Theta}_{il} \\ * & \bar{Q}_i \end{bmatrix} < 0$ are feasible for the parameter matrices A_{ci}, B_{ci}, C_{ci}, Q_i and L_i obtained in Step 3, then go to Step 5. If the LMIs (42), (43) and $\begin{bmatrix} \hat{L}_i & \hat{\Theta}_{il} \\ * & \bar{Q}_i \end{bmatrix} < 0$ are unfeasible with $k < N$, where N is the maximum number of iterations allowed, then set $k = k + 1$ and return to Step 3.

Step 5. Output the feasible solutions A^k_{ci}, B^k_{ci}, C^k_{ci}, Q^k_i and L^k_i. Then let $A^k_{ci} = A_{ci}$, $B^k_{ci} = B_{ci}$, $C^k_{ci} = C_{ci}$, $Q^k_i = Q_i$ and $L^k_i = L_i$. STOP.

Remark 9: The static output feedback control problem has been studied extensively for the nonlinear uncertain system with time-delays [20], [25]. It is well known that the dynamic controller provides more freedom and results in less conservativeness than the static controller. Compared with the design conditions for the static output feedback control, the design conditions for the fuzzy dynamic output feedback control are relaxed in this paper. In addition, with the help of the cone complementarity linearization algorithm, the non-convex problem is solved effectively.

V. SIMULATION RESULTS

In this section, two simulation examples are performed to show the effectiveness of the proposed methods.

A. EXAMPLE 1

Consider a chemical stirred tank reactor system as follows [35]

$$\begin{cases} \dot{C}_A = qV^{-1}\left(C_{Af_{si}k} - C_A\right) - a_0 \exp\left(-\frac{E}{RT}\right) C_A \\ \dot{T} = qV^{-1}\left(T_{f_{si}k} - T\right) - a_1 \exp\left(-\frac{E}{RT}\right) C_A + a_2\left(T_C - T\right) \end{cases} \tag{53}$$

where $\kappa = 1, 2$ is the mode index, f_{si} is the feed stream index, and the use details of parameters κ and f_{si} were shown in [35]. C_A, T and T_C are the reactant concentration, reactor temperature and coolant temperature, respectively (as shown in Fig. 3).

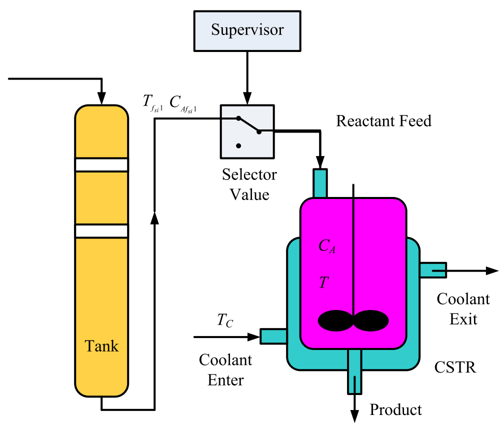

FIGURE 3. The schematic diagram of chemical stirred tank reactor.

In this paper, the mode index κ is chosen as $\kappa = 1$ [35], then the system (53) is rewritten as follows

$$\begin{cases} \dot{C}_A = qV^{-1}\left(C_{Af_{si}1} - C_A\right) - a_0 \exp\left(-\frac{E}{RT}\right) C_A \\ \dot{T} = qV^{-1}\left(T_{f_{si}1} - T\right) - a_1 \exp\left(-\frac{E}{RT}\right) C_A + a_2\left(T_C - T\right) \end{cases} \tag{54}$$

In addition, the using details of the parameters in (54) are shown in [35].

For the chemical stirred tank reactor system (54), the desired nominal operating values are $C_A^* = 0.5mol/L$, $T^* = 350K$ and $T_C^* = 280K$. In this case, $x_1 = C_A - C_A^*$ and $x_2 = T - T^*$ are the state variables. With [35, Table 1], the model index κ is given as $\kappa = 1$, then the system (54) is rewritten as follows

$$(\kappa = 1): \begin{cases} x_1(k+1) = x_2(k) + 0.5(1.5 - x_1(k)) \\ \quad -a_0 x_1(k) e^{-8750/(x_2(k)+350)} - x_2(k) \\ x_2(k+1) = a_2 u(k) - 2.592 x_2(k) \\ \quad -a_1 x_1(k) e^{-8750/(x_2(k)+350)} - 104.6 \\ y = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \end{cases} \quad (55)$$

Consider the chemical stirred tank reactor system (55) with the multiple time-varying delays and unmatched disturbances, the system (55) is further rewritten as follows

$$(\kappa=1): \begin{cases} x_1(k+1)=x_2(k-\tau_2(k))+0.5(1.5-x_1(k-\tau_1(k))) \\ \quad -a_0 x_1(k-\tau_1(k)) e^{-8750/(x_2(k-\tau_2(k))+350)} \\ \quad -x_2(k-\tau_2(k)) + d_1(k) \\ x_2(k+1) = a_2 u(k) - 2.592 x_2(k-\tau_2(k)) \\ \quad -a_1 x_1(k-\tau_1(k)) e^{-8750/(x_2(k-\tau_2(k))+350)} \\ \quad -104.6 + d_2(k) \\ y = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \end{cases} \quad (56)$$

For the problem formulated, the system (56) is transformed into the form of system (1) as follows

$$\begin{cases} x(k+1) = A_i x(k-\tau_i(k)) + B_i u(k) + E_i d_i(k) \\ z(k) = C_{1i} x(k) + D_i u(k) \\ y(k) = C_{2i} x(k), i \in [1, r] \end{cases} \quad (57)$$

where $r = 2$.

By employing the T-S fuzzy model technique in [30], the system (57) is rewritten as follows

$$\begin{cases} x(k+1)=\sum_{i=1}^{2} f_i(\theta(k))[A_i x(k-\tau_i(k))+B_i u(k)+E_i d_i(k)] \\ z(k) = \sum_{i=1}^{2} f_i(\theta(k))[C_{1i} x(k) + D_i u(k)] \\ y(k) = \sum_{i=1}^{2} f_i(\theta(k))[C_{2i} x(k)], i \in [1, 2] \end{cases} \quad (58)$$

where $x_1 = C_A - C_A^*$ and $x_2 = T - T^*$ are the state variables. $y(k)$, $z(k)$ and $u(k)$ are the measured output, control output and control input of the system, respectively. $d_1(k)$ and $d_2(k)$ are the unmatched disturbances. $A_1, A_2, B_1, B_2, C_{11}, C_{12}, C_{21}, C_{22}, D_1, D_2, E_1$ and E_2 are the gain matrices with appropriate dimensions and given as follows

$$A_1 = \begin{bmatrix} 4.4800 & 210.2019 \\ -0.0468 & -1.899 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.9868 & 30.1905 \\ -0.2328 & -1.2400 \end{bmatrix},$$

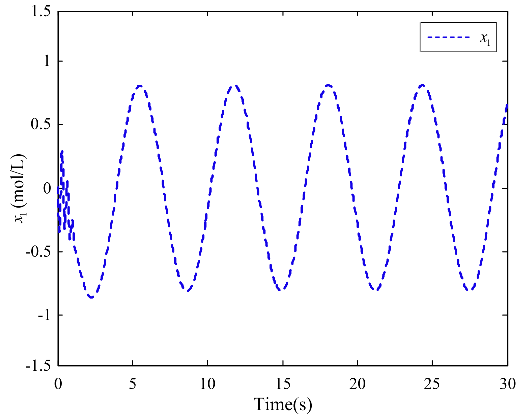

FIGURE 4. The response of system state variable x_1.

$$B_1 = \begin{bmatrix} 2.0921 \\ 0.2305 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.4681 \\ 0.3549 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 1 \\ 0.7 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \quad C_{11} = C_{12} = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix},$$

$$C_{21} = C_{22} = \begin{bmatrix} 0.02 & 0.05 \end{bmatrix}, \quad D_1 = D_2 = 0.01.$$

The membership functions for the fuzzy rules 1 and 2 are given as follows

$$\begin{cases} f_1(\theta(k)) = \dfrac{2 - \sin\theta(k)}{2}, & 0 \le \theta(k) \le \pi \\ f_2(\theta(k)) = 1 - f_1(\theta(k)) \end{cases} \quad (59)$$

Base on the *Lemma 1*, with (8) and (9), the parameters $\bar{\alpha}$, $\bar{\beta}\chi$ and G are given as $\bar{\alpha} = 0.7$, $\bar{\beta} = 0.7$, $\chi = 1$ and $G = [1.50]$. Then, based on the stochastic system theory and Bernoulli distribution, the fuzzy dynamic output feedback controller is designed as follows

$$\begin{cases} x_c(k+1) = \sum_{i=1}^{2} f_i[A_{ci} x_c(k) + B_{ci} y_c(k)] \\ u_c(k) = \sum_{i=1}^{2} f_i[C_{ci} x_c(k)], \quad i \in [1, 2] \end{cases} \quad (60)$$

where

$$A_{c1} = \begin{bmatrix} 0.3948 & -0.1703 \\ -0.5537 & 0.3445 \end{bmatrix}, \quad A_{c2} = \begin{bmatrix} -0.0360 & 0.2072 \\ -0.0333 & -0.2645 \end{bmatrix},$$

$$B_{c1} = \begin{bmatrix} 0.0577 \\ 0.5280 \end{bmatrix}, \quad B_{c2} = \begin{bmatrix} -0.0455 \\ -0.4962 \end{bmatrix},$$

$$C_{c1} = \begin{bmatrix} -5.3759 & -1.6552 \end{bmatrix}, \quad C_{c2} = \begin{bmatrix} 12.6825 & 8.7557 \end{bmatrix}.$$

For the simulation, the initial states are given as$x(0) = \begin{bmatrix} -0.36 & 52 \end{bmatrix}^T$. The multiple time-varying delays are given as $\tau_1 = 0.25(1 + \sin k)$ and $\tau_2 = 0.1(1 + \sin k)$. The unmatched disturbances are given as $d_1(k) = \sin(0.1k)$ and $d_2(k) = 0.8\sin(0.2k)$. The responses of the system state variables x_1 and x_2 are shown in Figs. 4 and 5. The control input is shown in Fig. 6. From the three figures, it can be seen that the proposed method is effective and can stabilize the chemical stirred tank reactor system quickly.

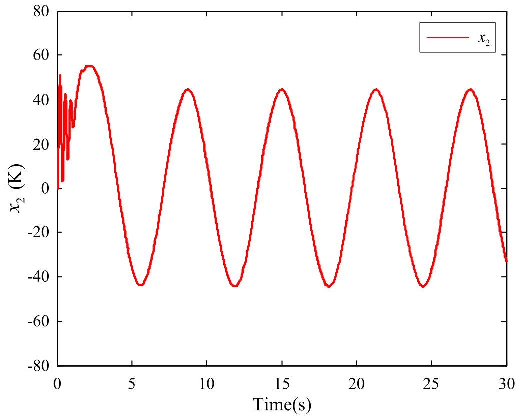

FIGURE 5. The response of system state variable x_2.

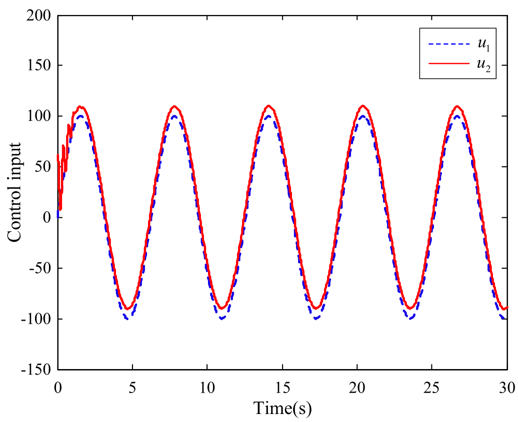

FIGURE 6. The response of system control input.

B. EXAMPLE 2

Consider a nonlinear mobile robot system as follows [30]

$$\begin{aligned} x_1(k+1) &= x_1(k) + \frac{v_r t_r}{l_r} u(k) \\ x_2(k+1) &= x_2(k) + \sin(x_1(k)) \\ x_3(k+1) &= x_3(k) + \cos(x_1(k)) \\ y(k) &= 1.5x_2(k) + 1.5x_3(k) \end{aligned} \tag{61}$$

where $x_1(k)$ is the direction angle of the mobile robot, $x_2(k)$ and $x_3(k)$ are the mobile robot position coordinates in the global coordinate system. $u(k)$ is the control input. $v_r = 0.9m/s$ is the linear velocity of robot, $l_r = 2.5m$ is the length of robot and $t_r = 1.0s$ is the sampling time.

Consider the mobile robot system (61) with the multiple time-varying delays and unmatched disturbances, the system (61) is further rewritten as follows

$$\begin{aligned} x_1(k+1) &= x_1(k-\tau_1(k)) + \frac{v_r t_r}{l_r} u(k) + 0.01 d_1(k) \\ x_2(k+1) &= x_2(k-\tau_2(k)) + \sin(x_1(k)-\tau_1(k)) + 0.02 d_2(k) \\ x_3(k+1) &= x_3(k-\tau_3(k)) + \cos(x_1(k-\tau_1(k))) + 0.03 d_3(k) \\ y(k) &= 1.5x_2(k-\tau_2(k)) + 1.5x_3(k-\tau_3(k)) \end{aligned} \tag{62}$$

For the problem formulated, the system (62) is transformed into the form of system (1) as follows

$$\begin{cases} x(k+1) = A_i x(k-\tau_i(k)) + B_i u(k) + E_i d_i(k) \\ z(k) = C_{1i} x(k) + D_i u(k) \\ y(k) = C_{2i} x(k), \quad i \in [1, r] \end{cases} \tag{63}$$

where $r = 3$.

By employing the T–S fuzzy model technique in [30], the system (63) is rewritten as follows

$$\begin{cases} x(k+1) = \sum_{i=1}^{3} f_i(\theta(k))[A_i x(k-\tau_i(k)) + B_i u(k) + E_i d_i(k)] \\ z(k) = \sum_{i=1}^{3} f_i(\theta(k))[C_{1i} x(k) + D_i u(k)] \\ y(k) = \sum_{i=1}^{3} f_i(\theta(k))[C_{2i} x(k)], \quad i \in [1, 3] \end{cases} \tag{64}$$

where $x_1(k)$, $x_2(k)$ and $x_3(k)$ are the state variables of the mobile robot system. $y(k)$, $z(k)$ and $u(k) = \begin{bmatrix} v(k) & \omega(k) \end{bmatrix}^T$ are the measured output, control output and control input of the system, respectively. $v_p(k)$ and $\omega_p(k)$ are the linear velocity and angular velocity of the mobile robot, respectively. $d_1(k)$, $d_2(k)$ and $d_3(k)$ are the unmatched disturbances. A_1, A_2, A_3, B_1, B_2, B_3, C_{11}, C_{12}, C_{13}, C_{21}, C_{22}, C_{23}, D_1, D_2, D_3, E_1, E_2 and E_3 are the gain matrices with appropriate dimensions and given as follows

$$\begin{aligned} &A_1 = \begin{bmatrix} 1 & 2 & 0.8 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ &B_1 = \begin{bmatrix} 0.03 \\ 0.01 \\ 0.02 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.04 \\ 0.03 \\ 0.05 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.05 \end{bmatrix}, \\ &E_1 = \begin{bmatrix} -0.01 \\ 0.07 \\ 0.02 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.02 \\ 0.05 \\ 0.03 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0.05 \\ 0.02 \\ -0.06 \end{bmatrix}, \\ &C_{11} = C_{12} = C_{13} = \begin{bmatrix} 0.14 & -0.07 & 0.06 \end{bmatrix}, \\ &C_{21} = C_{22} = C_{23} = \begin{bmatrix} 0.02 & -0.05 & 0.04 \end{bmatrix}, \\ &D_1 = D_2 = D_3 = 0.01. \end{aligned}$$

The membership functions for the fuzzy rules 1, 2 and 3 are given as follows

$$\begin{cases} f_1(\theta(k)) = 1 - \frac{\sin\theta(k)}{2}, \quad 0 \le \theta(k) \le \pi \\ f_2(\theta(k)) = 1 - \frac{\sin\theta(k)}{2} \\ f_3(\theta(k)) = 1 - f_1(\theta(k)) - f_2(\theta(k)) \end{cases} \tag{65}$$

Based on the *Lemma 1*, with (8) and (9), the parameters $\bar{\alpha}$, $\bar{\beta}\chi$ and G are given as $\bar{\alpha} = 0.9$, $\bar{\beta} = 0.8$, $\chi = 1$ and $G = [1.50]$. Then, based on the stochastic system theory and Bernoulli distribution, the fuzzy dynamic output feedback controller is

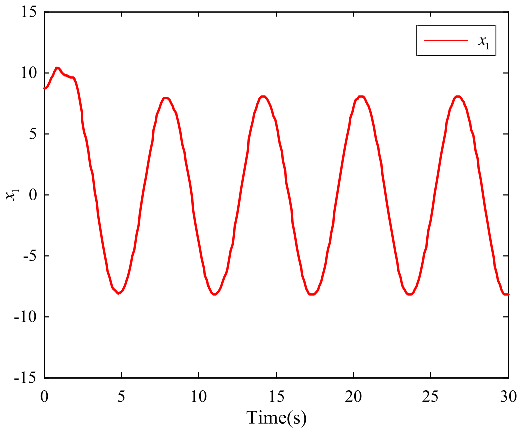

FIGURE 7. The response of system state variable x_1.

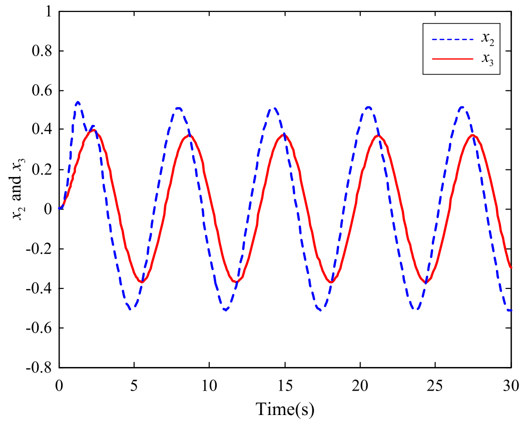

FIGURE 8. The response of system state variables x_2 and x_3.

designed as follows

$$\begin{cases} x_c(k+1) = \sum_{i=1}^{3} f_i\left[A_{ci}x_c(k) + B_{ci}y_c(k)\right] \\ u_c(k) = \sum_{i=1}^{3} f_i\left[C_{ci}x_c(k)\right], \quad i \in [1,3] \end{cases} \tag{66}$$

where

$$A_{c1} = \begin{bmatrix} 0.4079 & -0.2013 & 0.8527 \\ -0.6860 & 0.4778 & 0.9173 \\ 0.6981 & -0.6149 & -0.2846 \end{bmatrix},$$
$$A_{c2} = \begin{bmatrix} -0.2493 & 0.3103 & 0.8255 \\ -0.2446 & -0.5726 & -0.6842 \\ 0.4413 & 0.6915 & -0.4891 \end{bmatrix},$$
$$A_{c3} = \begin{bmatrix} 0.1236 & -0.2583 & -0.3719 \\ 0.1206 & 0.5561 & 0.8619 \\ 0.1201 & -0.1946 & 0.0925 \end{bmatrix},$$
$$B_{c1} = \begin{bmatrix} 0.1468 \\ 0.4190 \\ -0.5558 \end{bmatrix}, \quad B_{c2} = \begin{bmatrix} -0.0544 \\ -0.5843 \\ 0.5387 \end{bmatrix},$$
$$B_{c3} = \begin{bmatrix} 0.1002 \\ 0.1118 \\ -0.0619 \end{bmatrix}, \quad C_{c1} = \begin{bmatrix} -4.4874 & -1.4798 & -0.5852 \end{bmatrix},$$
$$C_{c2} = \begin{bmatrix} 22.5714 & 7.8688 & 0.3159 \end{bmatrix},$$
$$C_{c3} = \begin{bmatrix} 0.2336 & -0.2689 & -0.4915 \end{bmatrix}.$$

For the simulation, the initial states are given as $x(0) = \begin{bmatrix} 8.8 & 0 & 0 \end{bmatrix}^T$. The multiple time-varying delays are given as $\tau_1 = 0.25(1 + \sin k)$, $\tau_2 = 0.1(1 + \sin k)$ and $\tau_3 = 0.2(1 + \sin k)$. The unmatched disturbances are given as $d_1(k) = \sin(0.1k)$, $d_2(k) = 0.5\sin(0.3k)$ and $d_3(k) = 0.9\sin(0.25k)$. The responses of the system state variable x_1 is shown in Fig. 7, the responses of the system state variables x_2 and x_3 are shown in Fig. 8. The control input is shown in Fig. 9. From the three figures, it can be seen that the proposed method is effective and can stabilize the mobile robot system quickly.

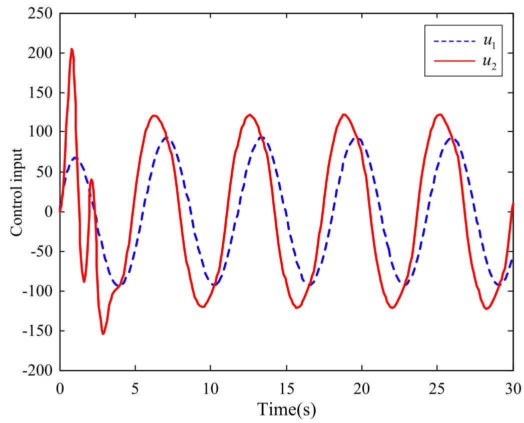

FIGURE 9. The response of system control input.

VI. CONCLUSIONS

This paper addressed the fuzzy dynamic output feedback control problem for a class of nonlinear discrete-time T-S fuzzy systems with multiple time-varying delays and unmatched disturbances. The T-S fuzzy model is employed for the nonlinear system, and the nonlinear uncertainties caused by the multiple time-delays and unmatched disturbances can be approximated effectively. The proposed fuzzy dynamic output feedback controller is smooth and flexible, constructed based on the stochastic system theory and Bernoulli distribution, and only uses the system output. The cone complementarity linearization algorithm is employed for the stability analysis, and the control design conditions are relaxed. By introducing the stochastic system theory and Bernoulli distribution for the controller design, it can be seen that the designed controller renders that the closed-loop system has better transient state performance and better steady state performance. Finally, two simulation examples are performed to show the effectiveness of the proposed methods. In the future work, the finite/fixed-time stabilization for the stochastic system with asymmetrical time-varying delays will be considered.

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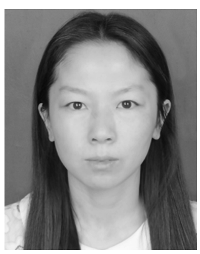

WEI ZHENG was born in Baoding, Hebei, China. She received the B.S. degree from Yanshan University, China, and the M.S. degree from the Institute of Electrical Engineering, Yanshan University, in 2011, where she is currently pursuing the Ph.D. degree in control theory and control engineering. She has co-authored about seven journal papers in mathematical and technical journals. Her research interests cover process automation, robot control technology, variable structure control system, robust control, and nonlinear system.

HONGBIN WANG was born in Harbin, Heilongjiang, China, in 1966. He received the B.S. and M.S. degrees in electrical engineering and automation from Northeast National Heavy Machinery University, Qiqihar, in 1988 and 1993, respectively, and the Ph.D. degree from the Institute of Electrical Engineering, Yanshan University, Qinhuangdao, China, in 2005. He was the Vice-dean of the Institute of Electrical Engineering in 2008 and the Dean of the Liren Institute, Yanshan University, in 2016. He is currently a Professor with the Institute of Electrical Engineering, Yanshan University. He is the (co)-author of many books and about 60 papers. He holds 10 national invention patents. As (a)an (co)-investigator, he was finished and carrying out more than 31 projects, supported by the Key Construction Program of the National 985 Project, the National Natural Science Foundation of China, the Natural Science Foundation of Hebei Province of China, and other important engineering project foundations where applied practically.

HONGRUI WANG was born in Harbin, Heilongjiang, China, in 1956. He received the B.S. and M.S. degrees in electrical engineering and automation from Northeast National Heavy Machinery University, Qiqihar, in 1979 and 1981, respectively, and the Ph.D. degree from the Institute of Electrical Engineering, Yanshan University, Qinhuangdao, in 2002. He was the Vice-President of Yanshan University from 1997 to 1999. He was the Headmaster of Hebei University from 1999 to 2012, where he has been the Party Secretary since 2012. He is currently a Professor with the Institute of Electrical Engineering, Yanshan University. He is the (co)-author of many books and about 158 papers. He is a member of the Chinese Association of Automation, an Executive Director of the China Higher Education Management Research Association, the Director of the Fifth Council of China Association for International Education, and an Executive Vice President of the Hebei Higher Education Association.

SHUHUAN WEN was born in Qiqihar, Heilongjiang, China, in 1972. She received the Ph.D. degree in control theory and control engineering from Yanshan University, Qinhuangdao, China, in 2005. She was a Visiting Scholar with Ottawa University, Carleton University, and Simon Fraser University, Canada, from 2011 to 2013. She is currently a Professor of automatic control with the Department of Electric Engineering, Yanshan University. She has co-authored one book and about 40 papers. Her research interests include humanoid robot control, force/motion control of parallel robot, fuzzy control, and 3-D object recognition and reconstruction.

ZHI-MING ZHANG was born in Chengde, Hebei, China. He received the B.S. degree from Shandong University, China, and the M.S. degree from the Institute of Electrical Engineering, Yanshan University, in 2008, where he is currently pursuing the Ph.D. degree in control theory and control engineering. His research interests include humanoid robot control, force/motion control of parallel robot, fuzzy control, and 3-D object recognition and reconstruction.