IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received May 1, 2018, accepted May 22, 2018, date of publication June 1, 2018, date of current version July 6, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2842715

An Improved Delay-Dependent Stability Analysis for Markovian Jump Systems With Interval Time-Varying-Delays

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This work was supported by the National Natural Science Foundation of China under Grant 61403278 and Grant 61503280.

ABSTRACT This paper put forward an improved stochastic stability condition for Markovian jump systems with interval time-varying delays. Markov jump parameters are modeled as a continuous-time Markov chain. We choose an improved Lyapunov–Krasovskii functional (LKF) and the linear matrix inequality (LMI) formulation, which can improve stability conditions with delay dependent and more suitable for solving related convex optimization problems. A new inequality processing method and the improved Wirtinger-based inequality with integral are used to deal with the LKF. The new results are showed by the LMI. At the end of this paper, some examples will be given to show that our method is effective and will bring lower conservatism.

INDEX TERMS Markovian jump systems (MJSs), interval time-varying delay, improved Wirtinger-based integral inequality.

I. INTRODUCTION

Markovian jump systems(MJSs) is a stochastic system with multiple modes, the jump transfer between various modes is determined by a set of Markov chains. In the process of practical application, because the equation of state of the system often has a certain randomness, this kind of system can not be described by the linear time invariant system. For example, some industrial systems may change suddenly in different parts of production or a sudden failure of some part of the system. However, this dynamic system can be accurately described by MJSs, which has been widely studied. In 1961, a continuous time model of MJSs has been established by Krasovskii and Lidskii *et al*. With the extensive application of modern computers, the form of discrete time is extended in the subsequent study, and the discrete time model of the MJSs is established. Since then, the analysis and synthesis of MJSs have been widely and deeply studied, and the results of the phased research are reported. For example, In [1], the controllability, observability, stability, detectability and linear two degree control problems of continuous time MJSs have been discussed, and the indispensable and adequate conditions are given. In [2], the problem of stability and control for uncertain MJSs are studied, the necessary and adequate conditions are shown in the form of LMIs. In [3], the H_∞ control problem of the MJSs and provides a design method of the expected controller have been studied. In [4], the H_∞ and H_2 output feedback control problems of MJSs are studied, the adequate conditions for the existence of the expected controller are shown in the form of LMIs. In [5] and [6], a design method for a synovium controller has been provided for a MJSs with uncertainty. In [7]–[9], for the singular systems with Markovian jump parameters, the stability analysis, stabilization, guaranteed cost control and H_∞ control have been discussed, the corresponding analysis and design results are given in the form of LMIs.

However, in practical applications, time delay usually exists in the MJSs, which reduces the normal operation of the system and even affects the instability of the system. Therefore, the research into MJSs with time delay has been focused. Generally speaking, when a time-delay in a MJSs is small enough, compared with the stable condition without time delay, the stability criterion dependent on time delay is not only not conservative, but also can get the upper bound of the maximum time delay that satisfies the stability condition of the system. In the interest of make the stability of the jump system less conservative, researchers have put forward many effective techniques and methods. Such as the convex analysis method, the time delay processing, the model transformation technology, the free weight matrix, the discrete Lyapunov method in [10]–[13], [25], and [26]. These methods have effectively reduced the conservatism of the conditions. However, these methods have some limitations. Either it is not enough to consider the processing of the condition of the function, or the method for the contraction of the LKF can still be improved. In [24], the Jensen's inequality method has been used to deal with inequalities. Then we found that this method was conservative and needed to be improved.

In this paper, we have presented an improved stochastic stability condition for MJSs with interval time-varying delays. The improved Wirtinger-based inequality with double integral and the new inequality processing method are used to handle the LKF. This method has a lower conservativeness compared to the methods mentioned above. At the end of the article, some examples will be given to show that our results are improved.

In this paper, the symbol "T" represents the matrix transposition, $\mathbb{R}^n$ shows the n-dimensional Euclidean space. 0 indicate the zero matrix. The notation $\{\Omega, \mathcal{F}, \mathcal{P}\}$ indicate the probability space. Ω, $\mathcal{F}$ and $\mathcal{P}$ represent the sample space, σ -algebra of subsets of the sample space and probability measure on $\mathcal{F}$, respectively. $S > 0$ is used to show a symmetric positive-definite matrix. The symbol "$*$" shows a term that is induced by symmetry. When $\kappa(\rho) = i \in S = \{1, \cdots, M\}$, we sign $A_i = A(\kappa(\rho))$.

II. SYSTEM DESCRIPTION AND PROBLEM ANALYSIS

Let us define the following delayed MJSs:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t-d(t)) \\ x(t) = \varphi(t), t \in [-h_2, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ represents the states,r(t) represents having values in a finite-state space $S = \{1, \cdots, M\}$ and a homogenous stationary Markov chain defined on $\{\Omega, \mathcal{F}, \mathcal{P}\}$. The state transition rate matrix $\Xi = (\mu_{ij})_{N\times N}$ is represented by:

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \mu_{ij}\Delta + o(\Delta); \\ if j \neq i, \\ 1 + \mu_{ii}\Delta + o(\Delta); \\ if j = i \end{cases}$$

where $\mu_{ij} \geqslant 0$, if $j \neq i$,$\mu_{ii} = -\sum_{j=1, j\neq i}^{N} \mu_{ij}$.In MJSs (1),d(t) expresses the time vary delay which section is :$h_1 \leqslant d(t) \leqslant h_2$ and $\dot{d}(t) \leqslant \mu$. In MJSs (1), $\varphi(t)$ is defined on interval $[-h_2, 0]$.

Lemma 1 [15]: The given invariant values m and n with $m < n$,for any constant matrix $\mathbb{H} > 0$,and successively differentiable functions $x \in [m, n] \rightarrow \mathbb{R}$,the following inequality shows:

$$\frac{(n-m)^2}{2}\int_m^n\int_\theta^n x^T(u)\mathbb{H}x(u)dud\theta \geqslant \left(\int_m^n\int_\theta^n x(u)dud\theta\right)^T \mathbb{H}\left(\int_m^n\int_\theta^n x(u)dud\theta\right) + 2\Theta_d^T H\Theta_d$$

where

$$\Theta_d = -\int_m^n\int_\theta^n x(u)duds + \frac{3}{s-r}\int_m^n\int_\theta^n\int_u^n x(u)dvduds$$

Definition 1 [24]: The MJSs (1) is stochastically stable, if $r(0) \in S$ and for finite $\varphi(t)$ defined on $[-h_2, 0]$, the following relation holds:

$$\lim_{t\to\infty} E\{\int_0^t x^T(t, \vartheta, r(0))x(t, \vartheta, r(0))d(s)\} < \infty.$$

III. MAIN RESULTS

In this part, we get a improve synchronization method by using aforementioned Lemma.

Theorem 1: For any delay d(t), given arbitrary constant h_1,h_2,and μ. Then, if there exist $n\times n$ matrices $P_i > 0$,$Q_{1i} > 0$,$Q_{2i} > 0$,$Q_{3i} > 0$,$Q_1 > 0$,$Q_2 > 0$,$Q_3 > 0$,$Z_1 > 0$, and $Z_2 > 0$,the MJSs (1) is stochastically stable which the following LMIs keep for any $i = 1, \cdots, N$:

$$\Sigma_i < 0 \tag{2}$$

$$\sum_{j=1}^{N}\mu_{ij}Q_{1j} \leqslant Q_1 \tag{3}$$

$$\sum_{j=1}^{N}\mu_{ij}Q_{2j} \leqslant Q_2 \tag{4}$$

$$\sum_{j=1}^{N}\mu_{ij}Q_{3j} \leqslant Q_3 \tag{5}$$

and (6), shown at the bottom of the next page, with

$$\begin{aligned}\Theta_{1i} &= P_iA_i + (P_iA_i)^T + Q_{1i} + Q_{3i} + h_1Q_1 \\ &\quad + (h_2-h_1)Q_2 + h_2Q_3 - h_1^2Z_1 - \frac{h_1^2}{2}Z_1 \\ &\quad - (h_2-h_1)^2Z_2 - \frac{(h_2-h_1)^2}{2}Z_2 \\ &\quad + A_i^T(\frac{h_1^4}{4}Z_1 + \frac{(h_2^2-h_1^2)^2}{4}Z_2)A_i + \sum_{j=1}^{N}\mu_{ij}P_j \\ \Lambda_{1i} &= P_iA_{di} + A_i^T(\frac{h_1^4}{4}Z_1 + \frac{(h_2^2-h_1^2)^2}{4}Z_2)A_{di} \\ \Theta_{2i} &= -Q_{1i} + Q_{2i} \\ \Theta_{3i} &= -(1-\mu)Q_{2i} \\ &\quad + A_{di}^T(\frac{h_1^4}{4}Z_1 + \frac{(h_2^2-h_1^2)^2}{4}Z_2)A_{di}\end{aligned}$$

Proof: We apply the newly improved inequality method to the Markov structure, a new method $x_t(s) = x(t+s)$,$s \in [-2h_2, 0]$ is defined. And we use an improved LKF: $V(x_t, t, r(t)) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$ where

$$\begin{aligned}V_1(t) &= x^T(t)P(r(t))x(t) \\ V_2(t) &= \int_{t-h_1}^{t} x^T(s)Q_1(r(t))x(s)ds\end{aligned}$$

$$+\int_{t-d(t)}^{t-h_1} x^T(s)Q_2(r(t))x(s)ds$$
$$+\int_{t-h_2}^{t} x^T(s)Q_3(r(t))x(s)ds$$
$$V_3(t)=\frac{h_1^2}{2}\int_{t-h_1}^{t}\int_s^t\int_u^t \dot{x}^T(v)Z_1\dot{x}(v)dvduds$$
$$+\frac{h_2^2-h_1^2}{2}\int_{t-h_2}^{t-h_1}\int_s^t\int_u^t \dot{x}^T(v)Z_2\dot{x}(v)dvduds$$
$$V_4(t)=\int_{-h_1}^{0}\int_{t+\theta}^{t} x^T(s)Q_1x(s)dsd\theta$$
$$+\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} x^T(s)Q_2x(s)dsd\theta$$
$$+\int_{-h_2}^{0}\int_{t+\theta}^{t} x^T(s)Q_3x(s)dsd\theta$$

where P_i, Q_{1i}, Q_{2i}, Q_{3i}, $i=1,2,\cdots,N$, Q_1, Q_2, Q_3, Z_1 and Z_2 are matrices with proper dimensions, which are positive definite, let Υ be the weak infinitesimal generator of the stochastic process x_t, $t\geqslant 0$. Then, for each $r(t)=i$, $i\in S$, the following equation is shown:

$$LV_1(x_t,t,i)=2x^T(t)P_i(A_ix(t)+A_{di}x(t-d(t)))$$
$$+\sum_{j=1}^{N}\mu_{ij}x^T(t)P_jx(t)$$
$$LV_2(x_t,t,i)=x^T(t)Q_{1i}x(t)$$
$$-x^T(t-h_1)Q_{1i}x(t-h_1)$$
$$+x^T(t-h_1)Q_{2i}x(t-h_1)$$
$$-x^T(t-h_2)Q_{3i}x(t-h_2)$$
$$+x^T(t)Q_{3i}x(t)$$
$$-(1-\dot{d}(t))x^T(t-d(t))Q_{2i}x(t-d(t))$$
$$+\int_{t-h_1}^{t} x^T(s)(\sum_{j=1}^{N}\mu_{ij}Q_{1j})x(s)ds$$
$$+\int_{t-d(t)}^{t-h_1} x^T(s)(\sum_{j=1}^{N}\mu_{ij}Q_{2j})x(s)ds$$
$$+\int_{t-h_2}^{t} x^T(s)(\sum_{j=1}^{N}\mu_{ij}Q_{3j})x(s)ds$$
$$LV_3(x_t,t,i)=\frac{h_1^4}{4}(A_ix(t)+A_{di}x(t-d(t)))^TZ_1$$
$$\times(A_ix(t)+A_{di}x(t-d(t)))$$
$$+\frac{(h_2^2-h_1^2)^2}{4}(A_ix(t)+A_{di}x(t-d(t)))^T$$
$$\times Z_2(A_ix(t)+A_{di}x(t-d(t)))$$
$$-\frac{h_1^2}{2}\int_{t-h_1}^{t}\int_s^t \dot{x}^T(s)Z_1\dot{x}(s)dsd\theta$$
$$-\frac{h_2^2-h_1^2}{2}\int_{t-h_2}^{t-h_1}\int_s^t \dot{x}^T(s)Z_2\dot{x}(s)dsd\theta$$
$$LV_4(x_t,t,i)=h_1x^T(t)Q_1x(t)$$
$$+(h_2-h_1)x^T(t)Q_2x(t)$$
$$+h_2x^T(t)Q_3x(t)$$
$$-\int_{t-h_1}^{t} x^T(s)Q_1x(s)ds$$
$$-\int_{t-h_2}^{t-h_1} x^T(s)Q_2x(s)ds$$
$$-\int_{t-h_2}^{t} x^T(s)Q_3x(s)ds$$

Using Lemma 1,we have

$$\frac{h_1^2}{2}\int_{t-h_1}^{t}\int_s^t \dot{x}^T(s)Z_1\dot{x}(s)dsd\theta$$

$$\Sigma_i=\begin{bmatrix}\Theta_{1i} & 0 & \Lambda_{1i} & 0 & 0 & 3Z_1 & 0 & 0 & 3Z_2\\ * & \Theta_{2i} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \Theta_{3i} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -Q_{3i} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -3Z_1 & \frac{6}{h_1}Z_1 & 0 & 0 & 0\\ * & * & * & * & * & -\frac{18}{h_1^2}Z_1 & 0 & 0 & 0\\ * & * & * & * & * & * & -3Z_2 & -3Z_2 & \frac{6}{h_2-h_1}Z_2\\ * & * & * & * & * & * & * & -3Z_2 & \frac{6}{h_2-h_1}Z_2\\ * & * & * & * & * & * & * & * & -\frac{18}{(h_2-h_1)^2}Z_2\end{bmatrix} \quad (6)$$

$$\begin{aligned}
&\geqslant [h_1x(t)-\int_{t-h_1}^{t}x(s)ds]^TZ_1[h_1x(t)\\
&-\int_{t-h_1}^{t}x(s)ds]+2[-\frac{h_1}{2}x(t)-\int_{t-h_1}^{t}x(s)ds\\
&+\frac{3}{h_1}\int_{t-h_1}^{t}\int_{s}^{t}x(u)duds]^TZ_1[-\frac{h_1}{2}x(t)\\
&-\int_{t-h_1}^{t}x(s)ds+\frac{3}{h_1}\int_{t-h_1}^{t}\int_{s}^{t}x(u)duds]
\end{aligned}$$

On the other hand,

$$\begin{aligned}
&\frac{h_2^2-h_1^2}{2}\int_{t-h_2}^{t-h_1}\int_{s}^{t}\dot{x}^T(s)Z_2\dot{x}(s)dsd\theta\\
&\geqslant [(h_2-h_1)x(t)-\int_{t-h_2}^{t-h_1}x(s)ds]^TZ_2\\
&[(h_2-h_1)x(t)-\int_{t-h_2}^{t-h_1}x(s)ds]\\
&+2[-\frac{h_2-h_1}{2}x(t)-\int_{t-h_2}^{t-h_1}x(s)ds\\
&+\frac{3}{h_2-h_1}\int_{t-h_2}^{t-h_1}\int_{s}^{t}x(u)duds]^TZ_2\\
&\times[-\frac{h_2-h_1}{2}x(t)-\int_{t-h_2}^{t-h_1}x(s)ds\\
&+\frac{3}{h_2-h_1}\int_{t-h_2}^{t-h_1}\int_{s}^{t}x(u)duds]\\
&=[(h_2-h_1)x(t)-\int_{t-d(t)}^{t-h_1}x(s)ds\\
&-\int_{t-h_2}^{t-d(t)}x(s)ds]^TZ_2[(h_2-h_1)x(t)\\
&-\int_{t-d(t)}^{t-h_1}x(s)ds-\int_{t-h_2}^{t-d(t)}x(s)ds]\\
&+2[-\frac{h_2-h_1}{2}x(t)-\int_{t-d(t)}^{t-h_1}x(s)ds\\
&-\int_{t-h_2}^{t-d(t)}x(s)ds+\frac{3}{h_2-h_1}\int_{t-h_2}^{t-h_1}\int_{s}^{t}x(u)duds]^TZ_2\\
&\times[-\frac{h_2-h_1}{2}x(t)-\int_{t-d(t)}^{t-h_1}x(s)ds-\int_{t-h_2}^{t-d(t)}x(s)ds\\
&+\frac{3}{h_2-h_1}\int_{t-h_2}^{t-h_1}\int_{s}^{t}x(u)duds]
\end{aligned}$$

Using this and combing, we have

$$\begin{aligned}
&LV(x_t,t,i)\\
&=LV_1(x_t,t,i)+LV_2(x_t,t,i)+LV_3(x_t,t,i)\\
&\quad+LV_4(x_t,t,i)\\
&\leqslant 2x^T(t)P_i(A_ix(t)+A_{di}x(t-d(t)))\\
&\quad+\sum_{j=1}^{N}\mu_{ij}x^T(t)P_jx(t)+x^T(t)Q_{1i}x(t)\\
&\quad-x^T(t-h_1)Q_{1i}x(t-h_1)+x^T(t-h_1)Q_{2i}x(t-h_1)\\
&\quad-x^T(t-h_2)Q_{3i}x(t-h_2)+x^T(t)Q_{3i}x(t)\\
&\quad-(1-\dot{d}(t))x^T(t-d(t))Q_{2i}x(t-d(t))\\
&\quad+\frac{h_1^4}{4}(A_ix(t)+A_{di}x(t-d(t)))^TZ_2\\
&\quad\times(A_ix(t)+A_{di}x(t-d(t)))\\
&\quad+\frac{(h_2^2-h_1^2)^2}{4}(A_ix(t)+A_{di}x(t-d(t)))^TZ_2\\
&\quad\times(A_ix(t)+A_{di}x(t-d(t)))\\
&\quad-[h_1x(t)-\int_{t-h_1}^{t}x(s)ds]^TZ_1[h_1x(t)-\int_{t-h_1}^{t}x(s)ds]\\
&\quad-2[-\frac{h_1}{2}x(t)-\int_{t-h_1}^{t}x(s)ds+\frac{3}{h_1}\int_{t-h_1}^{t}\int_{s}^{t}x(u)duds]^TZ_1\\
&\quad\times[-\frac{h_1}{2}x(t)-\int_{t-h_1}^{t}x(s)ds+\frac{3}{h_1}\int_{t-h_1}^{t}\int_{s}^{t}x(u)duds]\\
&\quad-[(h_2-h_1)x(t)-\int_{t-d(t)}^{t-h_1}x(s)ds-\int_{t-h_2}^{t-d(t)}x(s)ds]^TZ_2\\
&\quad\times[(h_2-h_1)x(t)-\int_{t-d(t)}^{t-h_1}x(s)ds-\int_{t-h_2}^{t-d(t)}x(s)ds]\\
&\quad-2[-\frac{h_2-h_1}{2}x(t)-\int_{t-d(t)}^{t-h_1}x(s)ds-\int_{t-h_2}^{t-d(t)}x(s)ds\\
&\quad+\frac{3}{h_2-h_1}\int_{t-h_2}^{t-h_1}\int_{s}^{t}x(u)duds]^TZ_2\\
&\quad\times[-\frac{h_2-h_1}{2}x(t)-\int_{t-d(t)}^{t-h_1}x(s)ds-\int_{t-h_2}^{t-d(t)}x(s)ds\\
&\quad+\frac{3}{h_2-h_1}\int_{t-h_2}^{t-h_1}\int_{s}^{t}x(u)duds]+h_1x^T(t)Q_1x(t)\\
&\quad+(h_2-h_1)x^T(t)Q_2x(t)+h_2x^T(t)Q_3x(t)
\end{aligned}$$

Then we can get

$$\Upsilon V(x_t,t,i)\leqslant \eta^T(t)\Sigma_i\eta(t)$$

where $\eta(t)=[x^T(t)\quad x^T(t-h_1)\quad x^T(t-d(t))\quad x^T(t-h_2)\quad \int_{t-h_1}^{t}x(s)ds\quad \int_{t-h_1}^{t}\int_{s}^{t}x(u)duds\quad \int_{t-d(t)}^{t-h_1}x(s)ds\quad \int_{t-h_2}^{t-d(t)}x(s)ds\quad \int_{t-h_2}^{t-h_1}\int_{s}^{t}x(u)duds]^T$.

We can get the above inequalities are equivalent to $\Upsilon V(x_t,t,i)<0$. After that, we use Definition 1, the stochastic stability is established.

Remark 1: It can be seen from the derivation results of previous papers that their results are conservative because conservative scaling methods have been used in their derivation. Such as model transformation in delay system, Jensen's inequality method, and free weighting matrix method. In [24], we can see that dealing with $\int_{t-h_1}^{t}h_1\dot{x}^T(s)Z_1\dot{x}(s)ds$ and $\int_{t-h_2}^{t-h_1}(h_2-h_1)\dot{x}^T(s)Z_2\dot{x}(s)ds$ in LKF by Jensen's inequality method can make the result more conservative. In this paper, we use a new $V_3(t)$ in LKF to proof the Theorem 1, which is diffierent from [24], and the improved Wirtinger-based integral inequality is used to handle $V_3(t)$. These methods will bring lower conservatism. Therefore, it is more reasonable to study the comprehensive problem of MJSs with time-varying delay.

When μ (time derivative of delay) is unknown, by modifying, we can deduce Corollary 1 from Theorem 1.

Corollary 1: For any delay d(t), given arbitrary constant h_1 and h_2, if there exist $n \times n$ matrices $G_i > 0, F_{1i} > 0, F_{3i} > 0, F_1 > 0, F_3 > 0, D_1 > 0$ and $D_2 > 0$, the MJSs (1) is stochastically stable which the following LMIs keep for any $i = 1, \cdots, N$:

$$\Sigma_i^{\angle} < 0 \tag{7}$$

$$\sum_{j=1}^{N} \mu_{ij} F_{1j} \leqslant F_1 \tag{8}$$

$$\sum_{j=1}^{N} \mu_{ij} F_{3j} \leqslant F_3 \tag{9}$$

and (10), shown at the bottom of the next page, with

$$\begin{aligned}
\Theta_{1i}^{\angle} &= G_iA_i + F_{1i} + h_2F_3 + F_{3i} + (G_iA_i)^T + h_1F_1 \\
&\quad - h_1^2D_1 + \sum_{j=1}^{N} \mu_{ij}G_j - \frac{h_1^2}{2}D_1 \\
&\quad - (h_2 - h_1)^2D_2 - \frac{(h_2 - h_1)^2}{2}D_2 \\
&\quad + A_i^T(\frac{h_1^4}{4}D_1 + \frac{(h_2^2 - h_1^2)^2}{4}D_2)A_i \\
\Theta_{2i}^{\angle} &= -F_{1i} \\
\Theta_{3i}^{\angle} &= A_{di}^T(\frac{h_1^4}{4}D_1 + \frac{(h_2^2 - h_1^2)^2}{4}D_2)A_{di}
\end{aligned}$$

When $h_1 = 0$, we can get Corollary 2 from Theorem 1.

Corollary 2: For any delay d(t), given arbitrary constant h_2 and μ, if there exist $n \times n$ matrices $G_i > 0, F_{2i} > 0, F_{3i} > 0, F_2 > 0, F_3 > 0$ and $D_2 > 0$, the MJSs (1) is stochastically stable which the following LMIs keep for any $i = 1, \cdots, N$:

$$\Sigma_i^{\S} < 0 \tag{11}$$

$$\sum_{j=1}^{N} \mu_{ij} F_{2j} \leqslant F_2 \tag{12}$$

$$\sum_{j=1}^{N} \mu_{ij} F_{3j} \leqslant F_3 \tag{13}$$

where

$$\Sigma_i^{\S} = \begin{bmatrix} \Theta_{1i}^{\S} & \Lambda_{1i}^{\S} & 0 & 0 & 3D_2 \\ * & \Theta_{2i}^{\S} & 0 & 0 & 0 \\ * & * & -F_{3i} & 0 & 0 \\ * & * & * & -3D_2 & \frac{6}{h_2}D_2 \\ * & * & * & * & -\frac{18}{h_2^2}D_2 \end{bmatrix} \tag{14}$$

with

$$\begin{aligned}
\Theta_{1i}^{\S} &= G_iA_i + (G_iA_i)^T + F_{2i} + F_{3i} + h_2F_2 \\
&\quad + h_2F_3 - h_2^2D_2 - \frac{h_2^2}{2}D_2 \\
&\quad + A_i^T(\frac{h_2^4}{4}D_2)A_i + \sum_{j=1}^{N} \mu_{ij}G_j \\
\Lambda_{1i}^{\S} &= G_iA_{di} + A_i^T(\frac{h_2^4}{4}D_2)A_{di} \\
\Theta_{2i}^{\S} &= (\mu - 1)F_{2i} + A_{di}^T(\frac{h_2^4}{4}D_2)A_{di}
\end{aligned}$$

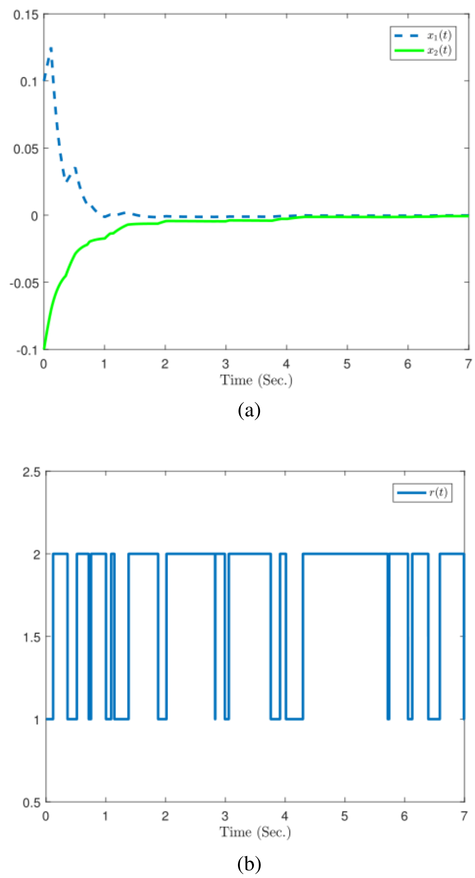

FIGURE 1. (a) Time response of $x_1(t), x_2(t)$. (b) Time response of $r(t)$.

IV. NUMBERICAL EXAMPLES

In this part, three examples have been given to show the effectiveness of our method.

When the time delay are time-invariant, in order to show the Theorem 1 have lower conservatism in MJSs with time delay, we provide the following example.

Example 1: Let us choose a MJSs in (1) with two modes and the matrix *parameters* [18]:

$$A_1 = \begin{bmatrix} 0.5 & -1 \\ 0 & -3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -5 & 1 \\ 1 & 0.2 \end{bmatrix}$$

TABLE 1. Different results to upper bound allowed h_2 for example 1.

	By[17]	By[18]	By[19]	By[20]	By[24]	By Theorem 1
h_2	0.84	1.23	0.40	0.73	1.33	1.87

$$A_{d1} = \begin{bmatrix} 0.5 & -0.2 \\ 0.2 & 0.3 \end{bmatrix}$$
$$A_{d2} = \begin{bmatrix} -0.3 & 0.5 \\ 0.4 & -0.5 \end{bmatrix}$$

with transition rates matrix

$$\Xi = \begin{bmatrix} -7 & 7 \\ 3 & -3 \end{bmatrix}$$

We now use Theorem 1 to system (1) by chooseing $\mu = 0$ and $h_1 = 0$. Then we choose LMIs (6) to get our results. In addition, the upper bound h_2 represents the time-varying delay, which obtained for values of μ and h_1(see Table 1). It is clear that our results are better conserved than those in [17]–[20] and [24].

When the time delays are time-variant, in order to show the Theorem 1 have lower conservatism, we consider the following example.

Example 2: We now use a MJSs in (1) with two modes and the matrix *parameters* [21]:

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix}$$
$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix}$$
$$A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix}$$

TABLE 2. Different results to upper bound allowed h_2 for example 2.

	$\mu = 0.6$	$\mu = 0.8$	$\mu = 1.6$
h_2 by Theorem 1 of [22]	0.4428	0.3795	0.3469
h_2 by Theorem 2 of [22]	0.4492	0.4341	0.4314
h_2 by [21]	0.4927	0.4261	0.3860
h_2 by Theorem 1 of [24]	0.5159	0.4814	0.4789
h_2 by Theorem 1 of this paper	0.5342	0.5147	0.5029

with transition rates matrix

$$\Xi = \begin{bmatrix} -0.1 & 0.1 \\ 0.8 & -0.8 \end{bmatrix}$$

We also use Theorem 1 to system (1) by chooseing $h_1 = 0$. Then we choose LMIs (6) to get our results. In addition, the upper bound h_2 represents the time delay,which obtained for different values of μ. The comparison results are given in Table 2, which indicates that Theorem 1 is better than those in [21], [22], and [24]. This means that our results have less conservative, and are more suitable for solving related convex optimization problems.

When the lower bound h_1 is biger than zero, the next example shows that Theorem 1 is less conserved.

Example 3: We also use Theorem 1 to a MJSs in (1) by chooseing $\mu = 0, h_1 = 0.5, and \mu_{11} = -7$, and the MJSs parameters are given as in Exmple 1. Then we choose LMIs (6) to get our results. In addition, the upper bound h_2 represents the time delay, which obtained for different values

$$\Sigma_i = \begin{bmatrix} \Theta_{1i}^{\angle} & 0 & \Lambda_{1i} & 0 & 0 & 3D_1 & 0 & 0 & 3D_2 \\ * & \Theta_{2i}^{\angle} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Theta_{3i}^{\angle} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -F_{3i} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -3D_1 & \frac{6}{h_1}D_1 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{18}{h_1^2}D_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -3D_2 & -3D_2 & \frac{6}{h_2-h_1}D_2 \\ * & * & * & * & * & * & * & -3D_2 & \frac{6}{h_2-h_1}D_2 \\ * & * & * & * & * & * & * & * & -\frac{18}{(h_2-h_1)^2}D_2 \end{bmatrix} \quad (10)$$

TABLE 3. Different results to upper bound allowed h_2 for example 3.

	$\mu=-1$	$\mu=-2$	$\mu=-3$
h_2 by [23]	0.6898	1.1077	1.2455
h_2 by Theorem 1 of [24]	0.6976	1.1384	1.5091
h_2 by Theorem 1 of this paper	0.9324	1.2508	1.7531

of μ_{22} (see Table 3). It can be seen through comparison that our results are less conservative than those in [23] and [24] when $h_1 > 0$.

V. CONCLUSIONS

In this paper, we have been used a new LKF and an improved Wirtinger-based double integral inequality for MJSs with interval time-varying-delays, which are shown in LMIs. Using a new inequality approximation method, our results have lower conservativeness than published papers with Jensen's inequality method. At the end of the article, three examples have been given to show the effectiveness of our method.

In the next study, we will discuss MJSs of the filter design and controller design, and consider the system design in the finite frequency domain.

ACKNOWLEDGMENT

The authors are very indebted to the Editor and the anonymous reviewers for their insightful comments and valuable suggestions that have helped improve the academic research.

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