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Nash Equilibrium-Based Asymptotic Stability Analysis of Multi-Group Asymmetric Evolutionary Games in Typical Scenario of Electricity Market

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ABSTRACT This paper introduces new theoretical insights into Nash equilibrium-based asymptotic stability (NEAS) of two-group and three-group asymmetric evolutionary games in typical scenarios of electricity market (EM). EM competition has become a complex dynamic evolution process accomplished by more complex characteristics of market economy behavior. Replicator dynamics in evolutionary game theory, as well as Lyapunov stability theory, are employed to solve incomplete-information and bounded-rationality game issues in EM, so as to overcome theoretical demerits of classical game theory in solving multi-group games in EM. First, the NEAS of a unilateral two-group asymmetric evolutionary game (AEG) is investigated. Then, this is expanded to a complicated $2 \times 2 \times 2$ trilateral multi-group AEG, and the NEAS of it under different game situations in EM is thoroughly discussed. Finally, a practical case study is conducted for verification. The case illustrates how the factors affect the payoff matrix which will change ultimate evolutionary stable state of the multi-group AEG in EM. One main finding demonstrates the complete dynamics behavior and multi-group evolutionary stable strategy (MESS) of the AEG system in 3-D mixed strategy space. The other one reveals that EM policies formulated by government and other factors can gradually influence the MESS via changing the payoff distribution matrix.

INDEX TERMS Electricity market, asymmetric evolutionary game, Nash equilibrium, multi-group evolutionary stable strategy, asymptotic stability.

I. INTRODUCTION

With continuously increasing energy consumption worldwide, future energy resource utilization will be confronted with a tough challenge. Hence, J. Rifkin first propounded Energy Internet [1], which involves the interconnection and coordination of the power networks over a wide area, the transformation and integration of multiple energy forms, the interconnection and management of a massive distributed power supply, and the use of energy related equipment based on Internet technologies. Under this background, a huge challenge is presented in optimal strategy determination for each decision-making body so as to balance and optimize the interests of all parties.

Moreover, with a very tight coupling between energy Internet and various distributed forms of new energy, the ever-growing electricity market (EM) is gradually opening to participants for which energy trading is becoming increasingly sophisticated. These include the original state grid corporations (SGCs), the power suppliers, the electricity consumer (EC), as well as a large number of emerging stakeholders, e.g., distributed power supply, energy storage, controllable loads, electric vehicle (EV) and new power supply entity (NPSE). In China NPSE is mainly represented by electricity retailers in EM, who produce electricity from various electricity resources and then sell it to customers [2]. Besides, they can provide value-added services such as contract energy management, comprehensive energy conservation and power utilization consultation. NPSEs are numerous in EM and mostly composed of emerging electricity retailer and load aggregator (LA). Here, LA is a new business model and firstly emerged in developed countries as a specialized demand response provider [3]–[6].

From above descriptions, it finds that the development of EM is affected by these stakeholders in varying degrees, which has directly led to a very complex and multifarious electricity price trading behavior in EM.

To address this issue, game theory has been proposed to solve the complex economic behavior issues of different stakeholders [7]. Generally speaking, in the EM, game theory is used to determine the bidding strategies for power suppliers and build EM models. Based on documentary analysis and survey research, the game models in EM are briefly summarized as follows.

Apart from the supply function equilibrium model that is widely used in the competitive EM, the other game models that are widely applied in EM, particularly the generation-side EM, can be divided into three major categories, i.e., the Cournot model [8], [9], the Bertrand model [10], and the Stackelberg model [11], [12], which are all geared to the field of classical game theory. Among these models, the Cournot model, Bertrand model and supply function equilibrium model are three main equilibrium models that are widely used in an oligopolistic EM. Some examples are as follows: Aliabadi *et al.* developed an agent-based simulation game model to simulate GenCos behavior in EM under different market clearing mechanisms [13]; Manbachi *et al.* [14] proposed a Monte-Carlo simulation based dynamic game model in order to achieve generation expansion planning of distributed generation sources in an energy market; an equilibrium game model [15] has been applied in Polish wholesale EM for estimating the impacts on electricity prices and generation levels, and a Stackelberg game model [11] has been developed to investigate the transmission capacities and competition in Western European EM; in addition, the game theory have been employed to investigate carbon emission trading in EM [16], demonstrate usefulness and proficiency of EM participants [17], coordinate generation and transmission expansion planning in EM [18], and reveal multi-agent competitive bidding strategies in EM [19], etc.

However, most of these investigations above are carried out based on classical game theory, thus some theoretical demerits are unavoidable in them, e.g., the perfect rationality is a concept that is not strictly defined; the solution of the Nash equilibrium (which is the key to solving a static game) is a very challenging problem; the formulation of the Nash equilibrium is not given, thus the issues of multiple equilibriums cannot be solved well.

In view of this situation, evolutionary game theory is gradually concerned by scholars for solving incomplete-information and bounded-rationality game issues, which has been employed to simulate EM participant's behavior for determining their bidding strategies and to regulate the market rules [20], or to achieve equilibrium calculations in EM [21].

Evolutionary game theory is originally proposed by Maynard Smith and Price [22]. Subsequently, a well-known RD model [23] is put forward to investigate the evolution of ecologies, which has become an extensively used dynamic equation for mechanism selection. After that, the concept of evolutionary stable strategy (ESS) is proposed [24]. As stated earlier, the players in classical game theory are perfect rational, resulting in the game scenario to be inconsistent with the actual situation and the solving of Nash equilibrium is extremely challenging. To address it, we introduce evolutionary game theory into multi-group games in EM via investigating the Nash equilibrium-based asymptotic stability (NEAS) of unilateral two-group asymmetric evolutionary game (UT-AEG) and trilateral multi-group AEG (TM-AEG) based on the typical scenarios of generation-side EM and demand-side EM, respectively.

We employ evolutionary game theory to discuss the games in typical sentences of the EM. On one hand, this is because evolutionary game theory is very suitable for addressing the incomplete-information and bounded-rationality game issues, which are accordance with actual situations of the EM. On the other hand, the market competition involving multiple interest groups, who belong to the same party/different parties, in EM has become a complex process of dynamic evolution accomplished by more complex characteristics of market economy behavior. Therefore, it is essential to combine the theoretical analysis of multi-group game behavior with its complex dynamic evolution process based on the evolutionary game theory, so that the game behavior of unilateral/multilateral group stakeholders in electricity pricing competition can be investigated in detail.

One of our main findings is that the payoff distribution parameters evidently affect the number of Nash equilibrium states in whatever UT-AEG and TM-AEG in the typical scenarios of generation-side EM and demand-side EM. The other one of our main findings is that the policies for the EM issued by the government can, to some extent, affect the NEAS of the multi-group AEG through changing distribution parameters of the payoff matrix, thus effective interventions implemented by the government can enhance electricity price stability of the EM and promote the energy internetworking to perform a more significant role in resource integration.

The major contribution of this paper can be summarized as follows: the NEAS of the multi-group AEGs in the typical scenarios of an ever-growing and open EM that possesses characteristics of energy Internet are thoroughly investigated based on the novel concept of Replicator Dynamics (RD) in the evolutionary game theory, which can perfectly describe how a population of pure strategies, or replicators, evolve through time [25]–[27], as well as the Lyapunov stability theory (LST), such that the dynamics behaviors and multi-group evolutionary stable strategy (MESS) of the multi-group AEG system are completely revealed and expanded scope from a straightforward two-dimensional plane to an enormously complex three-dimensional strategy combination space.

This paper is structured as follows: we first briefly depict related work in section II, and then we continue with introducing essential evolutionary game theoretic concepts in section III. Subsequently, in section IV, we present the main contributions and we illustrate the strengths of the evolutionary game theory by carrying out a concise two-group evolutionary analysis and a thorough trilateral multi-group evolutionary analysis on typical scenarios of electricity trading in generation-side and demand-side EM, respectively. Moreover, we demonstrate a practical application analysis in order to verify the conclusions drawn in this paper in section V. In section VI, we discuss the implications of EM policy formulated by the government and provide a deeper understanding of the theoretical results. Finally, section VII concludes this paper.

II. RELATED WORK

Evolutionary game theory is originated from biological chemistry and initially employed to reveal the phenomenon of competition in the process of biological evolution [22]. However, few investigations are conducted currently on asymmetric game behavior characteristics of multi-group stakeholders in EM trading, which, as previously noted, is becoming more complex and diverse in the circumstances of energy Internet. We deem that evolutionary game theory is suitable for solving issues of dynamic game in complex networks during the modelling. In net groups, dissemination of information and strategy selecting in evolutionary game can both be seen dynamics behavior that obeys some laws in networks. Hence, how to reveal these dynamics behaviors and find out mechanisms of them have been concerned by scholars [20], [21], [28]–[32].

Currently, in the field of power system, investigations of evolutionary game theory are mostly focused on [33]–[36] behavioral analysis of games in EM, demand side management, electricity pricing and investing, and other electric power economy fields. We also find that some amalgamating complex network theory and evolutionary game can solve some engineering problems that are difficult to be solved via many conventional analytical mathematical methods. However, on the whole, current investigations are focused much more on searching for appropriate evolution rules and topological structure of networks in order to facilitate emergence of collaboration. In addition, the game agents, generally, are simple bilateral game issues. On the contrast, more and more complex agents will emerge, such as trilateral game and even multiple gaming, in EM with dynamic evolution game structure. Hence, their deep evolution game mechanisms, paths and laws are extremely challenging.

The asymptotic stability investigation of this paper is reflected in a deterministic evolutionary game, in which the group game strategy is determined by the properties of an individual, so that the dominant or successful strategy will be spread throughout the population due to a high payoff of it. In contrast with other stochastic processes which are used to describe the update rules of individual strategy, such as the Pairwise Comparison Process [37], Moran Process [38], Fermi Process [39], and Wright-Fisher Process [40], the RD equation has superior mathematical properties for solving the equilibrium strategy of a deterministic evolutionary game. When the evolution of individual strategy occurs in the space, the partial differential equations are used to describe the RD model. Foster and Young [41] introduced randomness into the RD model in 1990, in which a stochastic differential equation is employed to describe the strategy evolution. Sequentially, Cressman and Vickers [42] developed a partial differential equation model in which a one-dimensional spatial variable is adopted to model the ESS of a 2×2 symmetric game. Subsequently, different forms of stochastic noises have been introduced in various stochastic differential equation based dynamic models [43]–[45]. In addition, other forms of dynamics are gradually developed as the differential equation based evolutionary mechanisms, e.g., Smith dynamics [46], Best-response dynamics [47], Logit dynamics [48], and Brown-von Neumann-Nash dynamics [49]. In fact, the theoretical growth of the proportion of players who select each strategy of a group can be calculated via the RD equation. Hence, the RD equation can be used to describe a deterministic evolutionary game, in which the variation of the share of each strategy with time in a population of infinite size without obvious structures can be revealed. That is why the RD equation model is appropriate to be chosen as a mathematical tool for the NEAS analysis of the multi-group AEG system in the typical scenarios of EM in this paper.

Our primary motivation is to enable RD theory in evolutionary game and LST for bilateral asymmetric games. However, we do this just in a two-dimensional strategy plane which is relatively simple when the number of game populations is two. Therefore, when the game populations are expanded in categories in a competitive EM, the corresponding strategy space will be expanded to an extremely complex three-dimensional strategy space as well. The classical game theory cannot perfectly deal with such complex strategy combination issues in evolutionary game situations. Consequently, we consider the original RD equations as well as LST, and derive new strategy space mappings for the complex electricity trading behaviors in the typical scenarios of EM.

III. PRELIMINARIES AND METHODS

In this section we concisely outline evolutionary game theoretic concepts which are necessary to understand the remainder of the paper. We first briefly specify definitions of solution concepts of normal-form game such as Nash equilibrium. Then, we introduce the MESS, RD, and asymptotically stable equilibrium point (ASEP) for UT-AEG and TM-AEG at full length, and moreover, we briefly discuss the concept of evolutionary stable equilibrium (ESE). Finally, we briefly introduce the LST that plays an important role in asymmetrical stability analysis in this paper.

A. NASH EQUILIBRIUM OF NORMAL-FORM GAME

Nash equilibrium is the most important basic concept in game theory. For a normal-form game $\Gamma =< N, S, U >$, composed of three basic elements [7]: a) the game player set

$N = \{1, 2, \cdots, N\}$, b) each player's strategy S_i and strategic space S_i, among which $s_i \in S_i$, $i = 1, 2, \cdots, N$, and c) each player's payoff function or utility function U_i, namely $U_i(s)$: $S_i \to R, i = 1, 2, \cdots, N$. Hence, a normal-form game is also called a strategic-form game, which is generally described via the matrix. According to this, the definition of Nash equilibrium is given as follows. We call a mixed strategy combination, υ_i^*, is a Nash equilibrium, when it meets the following inequality constraints [7]: $U_i(\upsilon_i^*, \upsilon_{-i}^*) \geq U_i(s_i, \upsilon_{-i}^*)$, $\forall s_i \in S_i$, $\forall i$, where $U_i(\upsilon_i^*, \upsilon_{-i}^*)$ is the payoff function of player i when it selects a mixed strategy υ_i^* while others select υ_{-i}^*. When this strategy υ_i^* is a pure strategy, we call it is a pure strategy Nash equilibrium υ^*. Obviously, the mixed strategy Nash equilibrium is a more general definition, thus we can treat the pure strategy as a particular form of mixed strategy. The Nash equilibrium has properties of strategically stable and self-reinforcement. In fact, according to the definition of Nash equilibrium, if each player in the game reaches a Nash equilibrium, then any of the players has no motivation to deviate from this equilibrium, thus one who unilaterally selects any other strategy beyond Nash equilibrium cannot get any extra benefit.

B. MESS

Suppose that the number of groups is n, and the strategies of all the groups constitute a multi-group strategy combination Ω_{group} in a multi-group AEG. Among these the strategy set $X = \{X_1, X_2, \cdots, X_n\} \in \Omega_{\text{group}}$ is assumed as an evolutionary stable strategy combination. Then if, any mutation strategy combination $Y = \{Y_1, Y_2, \cdots, Y_m\} \in \Omega_{\text{group}}$ meets the condition $Y \neq X$, there is an ω where $0 < \omega < 1$, and for any ϖ that meets $0 < \varpi < \omega$, there is an S equal to $S = \varpi Y + (1 - \varpi)X$, then the X is called a MESS if there is a strategy X_i that allows X to meet the inequality criteria in (1) as follows:

$$E(X_i, S^{-i}) > E(Y_i, S^{-i}) \quad i = 1, 2, \cdots, n \tag{1}$$

where S^{-i} is the strategy combination adopted by groups other than group i and which meets $S^{-i} \in S$. Due to the constraints on Y, ω and ϖ, it is obtained that $\forall S^{-i} \neq X$; $E(X_i, S^{-i})$ is the expected payoff for the group i that selects strategy X_i, meanwhile the other groups select S^{-i}, simultaneously, under the same condition, except that the group i selects strategy Y_i, then the expected payoff for the group i is denoted as $E(Y_i, S^{-i})$.

C. RD

Replicator Dynamics (RD) is used to simulate the dynamic adjustment process of the strategies to characterize the response speed of the population in adjusting its size through imitation and learning [25]–[27]. In this process, the effect of selection is highlighted and the evolution law of the number/proportion of the population can be revealed. In other words, when the expected payoff or fitness of a pure strategy X_i, denoted by $E(X_i)$, is higher than the group average payoff or fitness $\bar{E}(X_i)$, even if it is not necessarily a global optimum, the growth rate of the proportion or share (denoted as ρ_i) of the individuals that select strategy X_i in the population will be increased. Hence, this scenario can be described by a dynamic differential equation (i.e., $\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = 0$) of the probability/frequency (ρ_i) of strategy X_i being adopted in population, called an RD equations, where $\frac{\mathrm{d}\rho_i}{\mathrm{d}t}$ is proportional to ρ_i, as well as to the difference between $E(X_i)$ and $\bar{E}(X_i)$, thus the RD equation of the group i is described as

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = \rho_i[E(X_i) - \bar{E}(X_i)] \tag{2}$$

where $i = 1, 2, \ldots, n$, which means the number of the groups in the population. The (2) shows the continuous case. For the discrete case, the RD equation of the group i is described as

$$\rho_i(t+1) = \frac{1 + E(X_i)}{1 + \bar{E}(X_i)}\rho_i(t) \tag{3}$$

D. ASEP

Suppose $\vartheta, \vartheta^* \in \Omega_{\text{group}}$ (a strategy set) are mixed strategies in an evolutionary game, where ϑ^* is an ESS which meets two conditions demonstrated in (4), i.e., i) is an equilibrium condition and ii) is a stability condition, thus the group state $p^* = \vartheta^*$ is called an ASEP. The two conditions are described as

$$\begin{cases} \text{i)}: E(\vartheta, \vartheta^*) \leq E(\vartheta^*, \vartheta^*), & \forall \vartheta \in \Omega_{\text{group}} \\ \text{ii)}: E(\vartheta, \vartheta) < E(\vartheta^*, \vartheta), & \forall \vartheta \neq \vartheta^*, \\ \quad E(\vartheta, \vartheta^*) = E(\vartheta^*, \vartheta^*) \end{cases} \tag{4}$$

where $E(\vartheta, \vartheta)$, $E(\vartheta, \vartheta^*)$, $E(\vartheta^*, \vartheta)$ and $E(\vartheta^*, \vartheta^*)$ are the payoffs or fitness functions under different strategy combinations or game situations.

E. ESE

The ASEP can be considered as an evolutionary equilibrium point of the RD equations shown in (2) or (3), which corresponds to a mixed strategy of a specific population. ESE can be used to describe such a type of strategy, under which, the intrusion of any type of variation will be resisted, i.e., no mutant strategies can invade such a population in which each individual chooses an ESS. Hence, ESE is a dynamic equilibrium under which no individual strategy will be changed unilaterally. It further concludes that ESE must be Nash equilibrium, but the converse is not necessarily true, thus ESE is a type of refinement of Nash equilibrium.

F. LST

LST is used to determine the asymptotic stability of the equilibrium points which are solved via the RD equations. In this paper, we assume that the number of RD equations of an evolutionary game is n_{RD}, i.e., the number of populations involved in the evolutionary game system is n_{RD} ($n_{\text{RD}} = 2$ or 3). We deem that when $n_{\text{RD}} = 2$, the game is a UT-AEG; and $n_{\text{RD}} = 3$ represents a TM-AEG. Then, for each equilibrium point obtained via the RD equation in (2) or (3), the corresponding Jacobian matrix of RD equation can be obtained, which is an n-th-order square matrix, thus we can obtain that the number of eigenvalues for this matrix is no more than n, and they are denoted by λ_i, $i = 1, 2, \cdots, n$. Then, the real part of the eigenvalue λ_i is represented by Re_i, where $i = 1, 2, \cdots, n$. Accordingly, the asymptotic stability of system is determined via analyzing whether the equilibrium point of RD equation is an unstable evolutionary equilibrium point (UEEP) or an ASEP as follows:

$$\begin{cases} \text{an UEEP} & \text{if } \exists i, \text{ making } \mathrm{Re}_i \geq 0 \quad i = 1, 2, \cdots, n \\ \text{an ASEP} & \text{if } \forall i, \text{ making } \mathrm{Re}_i < 0 \quad i = 1, 2, \cdots, n \end{cases} \tag{5}$$

IV. EXPERIMENTAL ILLUSTRATION

In this section, based on the definitions and concepts introduced preciously, a flow chart of the integration process is given as graphically in Figure 1, in order to effectively demonstrate the theoretical framework with NEAS of multi-group AEG, including UT-AEG and TM-AEG, proposed in section III. Based on this integration process, we investigate the asymptotical stability of UT-AEG in a typical scenario of the generation-side EM. Then, we expand the strategy combination space from two-dimensional plane to a complex three-dimensional space, where we investigate the asymptotical stability of TM-AEG in a typical scenario of the demand-side EM.

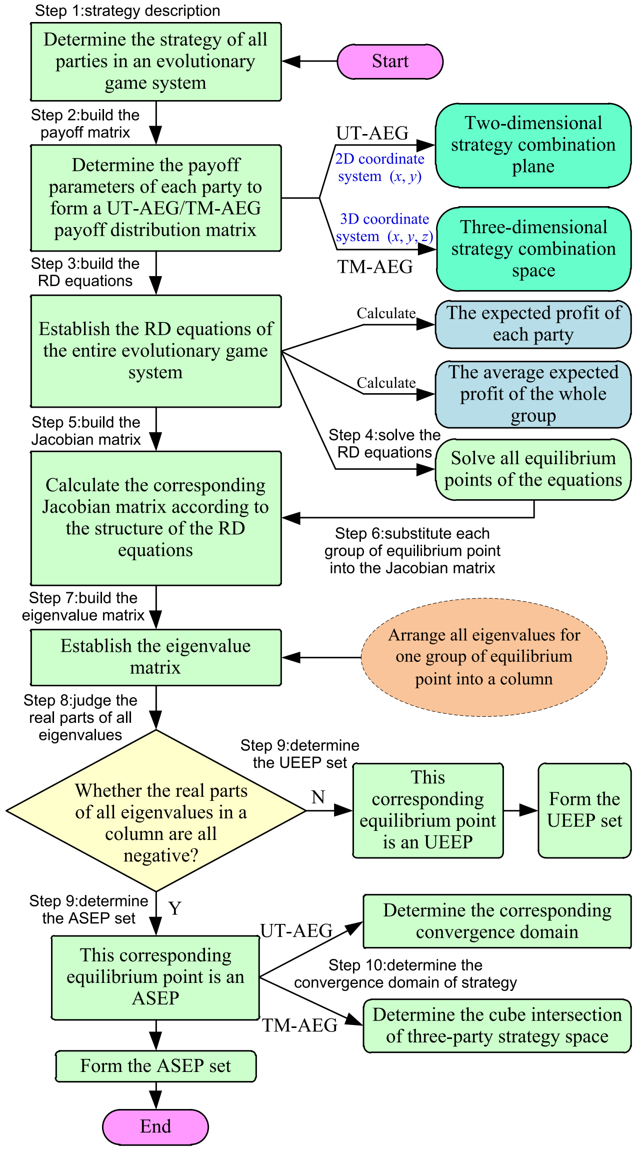

FIGURE 1. A flow chart of the integration process to effectively demonstrate the proposed theoretical framework with NEAS of multi-group AEG, including UT-AEG and TM-AEG.

A. UT-AEG IN THE TYPICAL SCENARIO OF GENERATION-SIDE EM

In the generation-side EM, the profit of each generation corporation (GenCo) is not only determined by their own quotations, but also the quotations from other generation corporations (GenCos), thereby forming a multi-group game issue. Hence, we consider generation-side electricity price bidding as a typical scenario of EM. In this scenario, the feasible pricing set of each GenCo is called a strategy set, and the game can be divided into two categories, i.e., non-cooperative bidding and cooperative bidding. The former means that the quoted prices are offered independently by the GenCos for maximizing their profits. The latter implies that GenCos cooperate with each other to form a pricing alliance, including differential price bidding and unified price bidding.

1) STRATEGY DESCRIPTION

We consider a typical scenario where multiple GenCos participate in electricity price bidding in generation-side EM as a UT-AEG model for NEAS analysis, in which situation two types of GenCo groups are selected as game objects. These are small-sized GenCo groups (SSGC) and large-sized GenCo groups (LSGC). Each type of them is considered to own two bidding strategies that can be implemented, i.e., a high quotation strategy S_{high}, and a basic quotation strategy S_{basic} which is quoted in full accordance with the production costs. In this typical scenario, the payoff distribution of SSGC and LSGC under different strategy combinations is shown in Table 1. These payoff distribution

TABLE 1. The payoff distribution of SSGC and LSGC in the assumed typical scenario of UT-AEG in generation-side EM.

Strategy choice of the two types of GenCos	Strategy S_{high} by LSGC	Strategy S_{basic} by LSGC
Strategy S_{high} by SSGC	(pay_1+pay_4+pay_5, pay_2+pay_3+pay_5)	(pay_1-pay_3, pay_2+pay_3)
Strategy S_{basic} by SSGC	(pay_1+pay_4, pay_2-pay_4)	(pay_1, pay_2)

parameters are chosen taking into consideration some previous asymmetric evolutionary games.

The specific meaning of the payoff distribution parameters shown in Table 1 is briefly interpreted as follows. Assume that the quantity of power generation of individual i (i.e, the power generation enterprise i) in the SSGC and LSGC is q_i. Then its payoff function $u_i(q_i)$ is defined as $u_i(q_i) = q_i \cdot f(Q_{\text{total}}) - C_i(q_i)$, where $f(Q_{\text{total}})$ is the power demand function of users, i.e., the market-clearing price, which can be simplified as a linear demand function, that is, $f(Q_{\text{total}}) = P_{\max} - K_Q \cdot Q_{\text{total}}$. Among this function, $P_{\max}$is the electricity price cap. Hence, if the power demand function is higher than $P_{\max}$, then no users will use the electricity. Here, K_Q is a constant coefficient, and $K_Q = P_{\max}/Q_{\text{total}-\max}$, where Q_{total} and $Q_{\text{total}-\max}$ are the sum of on-grid power generation of all generating companies (i.e., the total quantity of power supply of the market) and the sum of maximum power provided by all power generating companies (i.e., the maximum total power supply of the market).

Therefore, when Q_{total} is reached to $Q_{\text{total}-\max}$, we obtain $f(Q_{\text{total}}) = P_{\max} - (P_{\max}/Q_{\text{total}-\max}) \cdot Q_{\text{total}-\max} = 0$, which indicates that the market-clearing price is zero. In the payoff function, $C_i(q_i)$ is the actual power generation cost function of generating company i, which is usually expressed as a quadratic function of its power generation q_i, i.e., $C_i(q_i) = \alpha_i + \beta_i \cdot q_i + 0.5\chi_i \cdot q_i^2$, where α_i, β_i and χ_i are the no-load operating cost, the intercept of the marginal cost curve, and the slope of the marginal cost curve, respectively. They are all positive constants. For the power generating company i, its marginal cost curve and average cost curve are depicted as $\text{MC}_i(q_i) = \beta_i + \chi_i \cdot q_i$ and $\text{AC}_i(q_i) = \alpha_i/q_i + \beta_i + 0.5\chi_i \cdot q_i$, respectively. The relationship between the two curves is described as $\text{MC}_i(q_i) = \text{AC}_i(q_i) + q_i \cdot \frac{\text{dAC}_i(q_i)}{\text{d}q_i}$. As a result, for a single power generating unit, the minimum value of its average cost curve $\text{MC}_i(q_i)$ can be taken at the point of economic load.

Based on the above specific definition of the payoff distribution parameters in Table 1, when the S_{basic} is selected by SSGC and LSGC, their payoffs are pay_1 and pay_2, respectively; when the S_{high} and S_{basic} are adopted by SSGC and LSGC respectively, LSGC's revenue increased by pay_3, and correspondingly SSGC's revenue decreased by pay_3; in the reverse case, when the S_{high} and S_{basic} are implemented by LSGC and SSGC respectively, SSGC's revenue increased by pay_4, and LSGC's revenue decreased by pay_4; lastly, when they both select S_{high}, they can both earn an additional profit of pay_5. Obviously, when they both chose S_{high}, they can maximize their benefits, as shown in Table 1. However, whether they will both select the high quotation strategy, i.e., whether this strategy combination $\{S_{\text{high}}, S_{\text{high}}\}$ will be involved into a MESS in the actual bidding process needs to be further discussed.

2) RD EQUATIONS

Assume that the S_{high} is selected at ratios of p and q in SSGC and LSGC, respectively, and then the S_{basic} is selected at ratios of $1 - p$ and $1 - q$ in SSGC and LSGC respectively. Here, $0 \leq p \leq 1$ and $0 \leq q \leq 1$, thus the points of (p, q) within the region $[0,1]\times[0,1]$ can be employed to describe the evolutionary dynamics of the game system. Based on (2), we can obtain the RD equations and the corresponding Jacobian matrix $\boldsymbol{J}_{\text{pq}}$ as presented in (6) and (7), respectively,

$$\begin{cases} \text{d}p/\text{d}t = p(1-p)(q \cdot \text{pay}_3 + q \cdot \text{pay}_5 - \text{pay}_3) \\ \text{d}q/\text{d}t = q(1-q)(p \cdot \text{pay}_4 + p \cdot \text{pay}_5 - \text{pay}_4) \end{cases} \tag{6}$$

$$\begin{aligned} \boldsymbol{J}_{\text{pq}} &= \begin{bmatrix} \frac{\partial(\text{d}p/\text{d}t)}{\partial p} & \frac{\partial(\text{d}p/\text{d}t)}{\partial q} \\ \frac{\partial(\text{d}q/\text{d}t)}{\partial p} & \frac{\partial(\text{d}q/\text{d}t)}{\partial q} \end{bmatrix} \\ &= \begin{bmatrix} (1-2p)(q \cdot \text{pay}_3 + q \cdot \text{pay}_5 - \text{pay}_3) & p(1-p)(\text{pay}_3 + \text{pay}_5) \\ q(1-q)(\text{pay}_4 + \text{pay}_5) & (1-2q)(p \cdot \text{pay}_4 + p \cdot \text{pay}_5 - \text{pay}_4) \end{bmatrix} \end{aligned} \tag{7}$$

where $\text{d}p/\text{d}t$ and $\text{d}q/\text{d}t$ are the growth rates of the ratio or frequency of selecting strategy S_{high} in SSGC and LSGC, respectively. Then, make $\text{d}p/\text{d}t = 0$ and $\text{d}q/\text{d}t = 0$ respectively, which obtains $(p, q) = \{(0, \text{pay}_3/(\text{pay}_3+\text{pay}_5)), (1, \text{pay}_3/(\text{pay}_3+\text{pay}_5))\}$ and $(p, q) = \{(\text{pay}_4/(\text{pay}_4+\text{pay}_5), 0), (\text{pay}_4/(\text{pay}_4 + \text{pay}_5), 1)\}$ respectively, representing the ratios of selecting S_{high} in SSGC and LSGC are asymptotically stable, respectively.

Hence, a total of five equilibrium points for RD equations in (6) can be solved, i.e., $(p, q) = \{(\text{pay}_4/(\text{pay}_4 + \text{pay}_5), \text{pay}_3/(\text{pay}_3 + \text{pay}_5)), (0, 0), (1, 0), (0, 1), (1, 1)\}$. As stated earlier, the LST can be used to decide the MESS. Addressed concretely, the five equilibrium points are sequentially substituted into the $\boldsymbol{J}_{\text{pq}}$ shown in (7), such that the real part of the eigenvalues of $\boldsymbol{J}_{\text{pq}}$ can be determined, and the asymptotic stabilities of this UT-AEG system can be achieved, which are shown in Table 2, where $\Delta = (\sqrt{\text{pay}_3 \cdot \text{pay}_4 \cdot \text{pay}_5})/(\sqrt{\text{pay}_3+\text{pay}_5} \cdot \sqrt{\text{pay}_4+\text{pay}_5})$, and $\text{pay}_J > 0$, $J = 1, 2, \cdots, 5$. Table 2 indicates that two ASEPs, two UEEPs, and one saddle point (which is also regarded as an UEEP) are obtained in this UT-AEG system.

3) NEAS ANALYSIS

Based on Table 2, the dynamic adjustment of strategy is shown in Figure 2, where the blue point denotes an UEEP,

TABLE 2. The local asymptotic stabilities of SSGC and LSGC at all equilibrium points.

Local equilibrium point / (p, q)	Eigenvalue distribution of $\boldsymbol{J}_{\text{pq}}$ / (λ_1, λ_2)	Trace of $\boldsymbol{J}_{\text{pq}}$ / $tr(\boldsymbol{J}_{\text{pq}})$	Dose this equilibrium point meet asymptotic stability	MESS
$(\text{pay}_4/(\text{pay}_4+\text{pay}_5), \text{pay}_3/(\text{pay}_3+\text{pay}_5))$	$(\Delta, -\Delta)$	0	No (a saddle point, an UEEP)	Non-existence
(0, 0)	$(-\text{pay}_3, -\text{pay}_4)$	$-(\text{pay}_3+\text{pay}_4)$	Yes (an ASEP)	$\{S_{\text{basic}}, S_{\text{basic}}\}$
(1, 0)	$(\text{pay}_3, \text{pay}_5)$	$\text{pay}_3+\text{pay}_5$	No (an UEEP)	Non-existence
(0, 1)	$(\text{pay}_4, \text{pay}_5)$	$\text{pay}_4+\text{pay}_5$	No (an UEEP)	Non-existence
(1, 1)	$(-\text{pay}_5, -\text{pay}_5)$	-2pay_5	Yes (an ASEP)	$\{S_{\text{high}}, S_{\text{high}}\}$

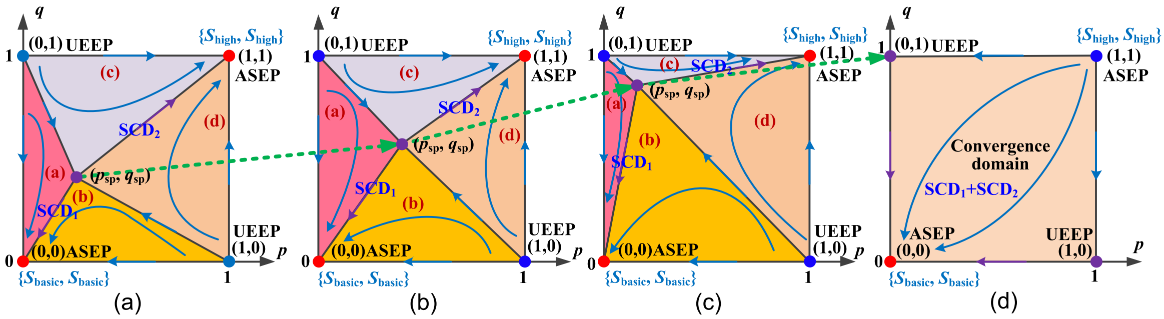

FIGURE 2. Dynamic adjustment process and evolution convergence direction of bidding strategy combination in SSGC and LSGC, where (a), (b) and (c) demonstrate different sizes of domain of convergence SCD_1 and SCD_2 and different coordinates of the saddle point (p_{sp}, q_{sp}), and (d) shows that only one ASEP obtained in (0, 0) and other three UEEPs obtained in (0, 1), (1, 1) and (1, 0).

the red point refers to an ASEP, the purple point represents a saddle point (also an UEEP): $(p_{sp}, q_{sp}) = (\mathrm{pay}_4/(\mathrm{pay}_4+\mathrm{pay}_5), \mathrm{pay}_3/(\mathrm{pay}_3 + \mathrm{pay}_5))$, the solid arrows in blue and purple indicate the dynamic adjustment directions of strategy combination in the convergence domain, i.e., the trajectories of system evolution, and the dashed arrows in green reflect the dynamic adjustment direction of convergence domain.

As shown in Figure 2(a), the MESS is obtained at (0, 0) and (1, 1), which means SSGC and LSGC will both select S_{basic} and S_{high}, respectively. In other words, the optimal offers or bids at Nash equilibrium for generators can be achieved at equilibrium points of (0, 0) and (1, 1). Hence, when SSGC and LSGC select S_{high} or S_{basic} simultaneously, they will reach an evolutionary stable state and have Nash equilibrium solutions. Moreover, Figure 2(b) and (c) show that the position of the saddle point will be changed with pay_3, pay_4 and pay_5, however this will always be located in the region $[0,1]\times[0,1]$. In other words, the size of system convergence domains which are constituted by regions (a) and (b), expressed by SCD_1, and regions (c) and (d), expressed by SCD_2, respectively, will be changed. Hence, in a long-term evolution, the probability of this UT-AEG system converging to the two quite different asymptotically stable bidding modes will be changed with the position of the saddle point (p_{sp}, q_{sp}). Among these, one is a reasonable quotation (S_{basic}) that tends to be expected; the other is an unconventional quotation (S_{high}) that tends not to be seen. Nevertheless, the two cases are both evolutionary stable states, which one the system will be eventually converged to depends on the initial conditions of the evolution (i.e., the conditions of the payoff parameters pay_J, $J = 1, 2, \cdots, 5$) is located in SCD_1 or SCD_2.

Therefore, the payoff parameters pay_J, such as pay_3, pay_4 and pay_5 can be adjusted to change the position of the saddle point shown in Figure 2, such that the convergence domains SCD_1 and SCD_2 are manipulated to make the evolutionary trajectory of the system tend to the desired reasonable equilibrium point set. For example, change the parameter pay_5, which means the additional profits of SSGC and LSGC will be decreased when they both select S_{high}, such that the probability of selecting S_{high} by both of them will be reduced significantly; while and that of selecting S_{basic} is increased. This parameter adjustment corresponds to the government supervision of the bidding market, i.e., the government determines the upper and lower limits of the online electricity price for the GenCos based on economic analysis. On one hand to ensure that the GenCos have appropriate profits and development opportunities. On the other hand, the profits of the GenCos (especially the LSGC) are limited when they adopt S_{high}. Therefore, through government supervision, the appropriate adjustments to the payoff parameters pay_J ($J = 1, 2, \cdots, 5$) can be implemented to change the payoff matrix in the UT-AEG (see Table 1), so that the decision-making of price bidding for all groups will be more rational and in line with the actual demand of EM.

In addition, if the government, through its supervision, stipulates that when the GenCos both take the high quotation strategy S_{high} simultaneously, they not only cannot gain additional profits pay_5, but are also given some penalties, that means $\mathrm{pay}_5 < 0$, then the equilibrium point (1, 1) will be transformed from an ASEP into an UEEP, such that an evolutionary stable state cannot be achieved for the game system at this point. Moreover, since $\mathrm{pay}_3 > 0$, $\mathrm{pay}_4 > 0$, when $\mathrm{pay}_4 + \mathrm{pay}_5 < 0$ or $\mathrm{pay}_3 + \mathrm{pay}_5 < 0$, the saddle point ($p_{sp}$, q_{sp}) will disappear. Then the number of system equilibrium points is decreased to 4 and the asymptotic stability can be achieved for the system only if it evolves at (0, 0), as shown in Figure 2(d). From this we draw a conclusion that only when SSGC and LSGC both implement the basic quotation strategy S_{basic}, can an evolutionary stable state be reached and a MESS be formed for this UT-AEG system regardless of the initial states, representing $\{S_{high}, S_{high}\}$ is not a stable strategy combination. Hence, the MESS has a strong feature of expelling intruders, such that unstable strategies will be expelled and eventually eliminated in the process of system evolution. Hence, it can be seen that in the long-term evolution of the system, a more beneficial and rational online electricity price mechanism will tend to be formed for the GenCos through the establishment of reasonable bidding rules that are in line with the actual economic operation of the EM.

B. TM-AEG IN THE TYPICAL SCENARIO OF DEMAND-SIDE EM

Based on the NEAS analysis of the UT-AEG model in a typical scenario of generation-side EM, the convergent domain of the multi-group AEG system is extended from a two-dimensional plane to a three-dimensional space. Concretely speaking, we take the demand-side EM as a typical scenario of gaming, where the aforementioned power grid enterprise (PGE), new power supply entity (NPSE) and electricity consumers (ECS) are treated as three parties of game players participating in the game of time-of-use (TOU) electricity pricing and electricity trading. In this typical scenario, we thoroughly investigate the equilibrium stability issues of a more complex TM-AEG as follows.

1) STRATEGY DESCRIPTION

For convenience, the groups of power grid enterprises, new power supply entities and electricity consumers in the above typical scenario are represented by PGE, NPSE and ECS, respectively. Here, the number of individuals may be very large. Suppose that all three parties have two pure strategies, i.e., the executable strategies of PGE, NPSE and ECS are $\{S_{\text{PG1}}, S_{\text{PG2}}\}$, $\{S_{\text{NP1}}, S_{\text{NP2}}\}$ and $\{S_{\text{EC1}}, S_{\text{EC2}}\}$, respectively.

For the group of PGE, its strategy S_{PG1} chosen at a probability of x means that PGE cooperates with NPSE, and provides a TOU price P_{TOU1} enabling NPSE to be more profitable. Accordingly, S_{PG2} (probability $1-x$) means that PGE cooperates with ECS, and provides a TOU price P_{TOU2} to ECS for their own advantage.

For the group of NPSE, including all kinds of newly-emerging small-scale electricity retailers, LAs, distributed electricity suppliers, emerging production and marketing collectives, and the direct electricity providers of large GenCos, the strategy S_{NP1} chosen at a probability of y indicates cooperation with PGE, under which the electricity sales of NPSE in valley of electricity consumption are reduced and a TOU price P_{TOU3} that benefits PGE more is provided to encourage consumers to purchase more electricity prices of PGE such that the valley can be filled. Correspondingly, S_{NP2} (probability $1-y$) means non-cooperation with PGE, under which some measures will be taken to attract more ECS to purchase electricity at peak times, so that electricity sales will be increased and a TOU price P_{TOU4} that benefits ECS more is provided with a purpose of peak shaving.

For the group of ECS, represented by single-load user groups, the strategy S_{EC1} (chosen at a probability of z) indicates that ECS select TOU prices (P_{TOU1} or P_{TOU2}) provided by PGE to arrange electricity use at different times in order to maximize their interests based on actual demands. S_{EC2} (probability $1-z$) means choosing the TOU prices provided by NPSE (P_{TOU3} or P_{TOU4}) at different times for the purpose of maximizing the benefits.

Therefore, based on the strategy definitions for each stakeholder, we can obtain a trilateral payoff distribution matrix is shown in Table 3, where $a_{\mathcal{X}}$ to $h_{\mathcal{X}}$ in alphabetical order are the payoff distribution parameters, and $\mathcal{X} = 1, 2, 3$. These payoff distribution parameters are chosen partially taking into consideration some previous asymmetric evolutionary games. An example is taken to elaborate the concrete meanings of these parameters as follows.

TABLE 3. The payoff distribution of the TM-AEG for the PGE, NPSE and ECS in the typical scenario of demand-side EM.

Strategy		S_{EC1}	S_{EC2}
S_{PG1}	S_{NP1}	(a_1, a_2, a_3)	(b_1, b_2, b_3)
	S_{NP2}	(c_1, c_2, c_3)	(d_1, d_2, d_3)
S_{PG2}	S_{NP1}	(e_1, e_2, e_3)	(f_1, f_2, f_3)
	S_{NP2}	(g_1, g_2, g_3)	(h_1, h_2, h_3)

For the group of ECS, the payment of them is the sum of the product of the respective electricity consumption of all types of electricity load in all electricity consumption periods and the electricity price of the corresponding time period during a certain electricity cycle (which can be assumed as one day). We assume that the number of ECS is $n_{\text{ec}}(n_{\text{ec}} \geq 1)$, the electricity utilization cycle is one day which is divided into θ periods, thus the time interval is $\Delta t = t_\theta - t_{\theta-1}$, the electricity time set is $T = \{t_1, t_2, \cdots, t_\theta\}$. The total electricity consumption of all types of electricity loads own by the kth consumer during the time period t_θ is denoted as $Q_{\theta,k}$. Here, $k = \{1, 2, \cdots, n_{\text{ec}}\}$. For example, for the consumer numbered n_{ec}, its total electricity consumption is represented as $Q_{\theta,\text{nec}}$. Hence, the electricity consumption distribution matrix of the group of ECS can be obtained, which is denoted by $\boldsymbol{Q}_{\text{users}}$ and it is a $n_{\text{ec}} \times \theta$ matrix as

$$\boldsymbol{Q}_{\text{users}} = \begin{bmatrix} Q_{1,1} & Q_{1,2} & \cdots & Q_{1,\theta} \\ Q_{2,1} & Q_{2,2} & \cdots & Q_{2,\theta} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{n_{\text{ec}},1} & Q_{n_{\text{ec}},2} & \cdots & Q_{n_{\text{ec}},\theta} \end{bmatrix},$$

where $Q_{n_{\text{ec}},\theta}$ refers to the total amount of electricity consumption of the consumer numbered n_{ec} during the θth time period.

In addition, we assume that the electricity price of PGE in each above time period constitutes a price vector as $\boldsymbol{P}_{\text{PG}} = [P_{\text{PG}-1}, P_{\text{PG}-2}, \cdots, P_{\text{PG}-\theta}]$. We also assume that the number of NPSE is $n_{\text{np}}(n_{\text{np}} \geq 1)$. Corresponding to $\boldsymbol{P}_{\text{PG}}$, the electricity price vector of the lth NPSE is $\boldsymbol{P}_{\text{NP}-l} = [P_{l-1}, P_{l-2}, \cdots, P_{l-\theta}]$, for example, for the NPSE numbered n_{np}, its electricity price vector is represented as $\boldsymbol{P}_{\text{NP}-\text{nnp}} = [P_{\text{nnp}-1}, P_{\text{nnp}-2}, \cdots, P_{\text{nnp}-\theta}]$, here $l = \{1, 2, \cdots, n_{\text{np}}\}$. The group of ECS can select the NPSE based on the NPSE's electricity price vector and geographical location. Thereby, we introduce ΔP_l to represent the unit power policy subsidy that is obtained by the group of ECS when they select the lth NPSE. For example, the electricity price selection of the consumer n_{ec} in the θth time period is denoted as $\vartheta_{\text{nec},\theta}$, which is determined based on the level between $P_{\text{PG}-\theta}$ and $P_{\text{nnp}-\theta} - \Delta P_{\text{nnp}}$. If the former is larger, then $\vartheta_{\text{nec},\theta} = P_{\text{nnp}-\theta}$, otherwise $\vartheta_{\text{nec},\theta} = P_{\text{PG}-\theta}$. Here, $P_{\text{PG}-\theta}$

and $P_{\text{nnp}-\theta}$ are the electricity prices provided by the groups of PGE and NPSE during the θth time period, respectively. Therefore, we can obtain the payoff of the consumer n_{ec} in the θth time period, denoted by $\Phi_{\text{nec},\theta}$, which is equal to the selected electricity price of this time period multiplied by the total electricity consumption during that period, namely $\Phi_{\text{nec},\theta} = \vartheta_{\text{nec},\theta} \times Q_{\theta,\text{nec}}$. Moreover, we can obtain the total payoff of this consumer in all time periods $\{t_1, t_2, \cdots, t_\theta\}$ as Φ_{nec}, as well as the user utility U_{nec} in an electricity utilization cycle T. Hence, we can use the payment to subtract the utility to obtain this consumer's total income R_{nec} in that time cycle T, namely $R_{\text{nec}} = U_{\text{nec}} - \Phi_{\text{nec}}$.

Based on the above description, for the group of PGE (maybe we can assume that there is only one PGE), its individual income includes electricity fee charged to the ECS and a certain percentage of network transmission fees charged to the NPSE. Therefore, based on the matrices of quantity distribution and payoff distribution of the ECS described above, we can easily obtain the income distribution matrix of the PGE, $\boldsymbol{y}_{\text{PG}}$, which is an $1 \times \theta$ matrix, namely $\boldsymbol{y}_{\text{PG}} = [y_{\text{PG}-1}, y_{\text{PG}-2}, \cdots, y_{\text{PG}-\theta}]$, thus the total income of the PGE in all time periods is $Y_{\text{PG}} = y_{\text{PG}-1} + y_{\text{PG}-2} + \cdots + y_{\text{PG}-\theta}$. Meanwhile, the cost of PGE, C_{PG}, can be expressed as a quadratic function of its power supply, i.e., $C_{\text{PG}} = a_0 \cdot Q_{\text{PG}}^2 + b_0 \cdot Q_{\text{PG}} + c_0$, where a_0, b_0 and c_0 are cost coefficients of PGE after considering all the cost factors, and they are all non-negative numbers. Hence, the difference between income and cost in a time cycle T can be expressed as the profit of PGE R_{PG}, namely $R_{\text{PG}} = Y_{\text{PG}} - C_{\text{PG}}$.

Likewise, for the group of NPSE, we have assumed that the number of NPSE is n_{np} and their incomes are mainly electricity fees charged to the ECS. Hence, according to the $\boldsymbol{Q}_{\text{users}}$, we can obtain the income distribution matrix of all NPSEs as $\boldsymbol{Y}_{\text{NP}} = [\boldsymbol{Y}_{\text{NP}-1}\ \boldsymbol{Y}_{\text{NP}-2} \cdots \boldsymbol{Y}_{\text{NP}-\text{nnp}}]^{\text{T}}$, which can be determined by using 3 matrix multiplications $\boldsymbol{Y}_{\text{NP}} = \boldsymbol{A}\boldsymbol{Q}_{\text{users}}\boldsymbol{B}$, where $\boldsymbol{A}$ and $\boldsymbol{B}$ are $n_{\text{np}} \times n_{\text{ec}}$ and $\theta \times \theta$ matrix, respectively, and used to calculate the user payoff belonging to NPSE; $\boldsymbol{Y}_{\text{NP}-\text{nnp}}$ is the income distribution vector of the NPSE numbered n_{np}; $\boldsymbol{Y}_{\text{NP}}$ is a $n_{\text{np}} \times \theta$ matrix, and the element in it on row n_{np} and column θ represents the income of electricity fee of the NPSE numbered n_{np} in the θth time period. Therefore, the total income of the NPSE numbered n_{np} in an electricity utilization cycle T is calculated as $y_{\text{NP}-\text{nnp}} = y_{\text{np}-1} + y_{\text{np}-2} + \cdots + y_{\text{np}-\theta}$.

However, the cost of NPSE is different from that of PGE, and analogously, it can also be represented by a quadratic function of its power supply. In other words, we assume that the quantity of power provided by the NPSE numbered n_{np} is Q_{nnp}, such that its cost C_{nnp} can be expressed as $C_{\text{nnp}} = a_{\text{nnp}} \cdot Q_{\text{nnp}}^2 + b_{\text{nnp}} \cdot Q_{\text{nnp}} + c_{\text{nnp}}$, where a_{nnp}, b_{nnp} and c_{nnp} are all non-negative numbers, and they are different from the related parameters of PGE to some degrees. Therefore, we will finally obtain the profit function of the NPSE numbered n_{np}, denoted by R_{nnp}, which can be derived from income minus cost, i.e., $R_{\text{nnp}} = Y_{\text{NP}-\text{nnp}} - C_{\text{nnp}}$.

2) RD EQUATIONS AND ITS JACOBIAN MATRIX

For the groups of PGE, NPSE and ECS in the TM-AEG system, it is assumed that the expected profits are obtained EPG_1, ENP_1 and EEC_1 when they execute the strategy S_{PG1}, S_{NP1} and S_{EC1} respectively. Simultaneously, when they choose S_{PG2}, S_{NP2} and S_{EC2}, the expected profits are expressed as EPG_2, ENP_2 and EEC_2, respectively. Besides, the average expected profits of PGE, NPSE and ECS are achieved as EPG_{av}, ENP_{av} and EEC_{av}, respectively. For example, EPG_{av} here indicates that PGE select EPG_1 at probability of x and EPG_2 at $1 - x$. Then according to previous investigation and based on Table 3, we can obtain these profits described above as

$$\begin{cases} \text{EPG}_1 = y[za_1 + (1-z)b_1] + (1-y)[zc_1 + (1-z)d_1] \\ \text{EPG}_2 = y[ze_1 + (1-z)f_1] + (1-y)[zg_1 + (1-z)h_1] \\ \text{ENP}_1 = z[xa_2 + (1-x)e_2] + (1-z)[xb_2 + (1-x)f_2] \\ \text{ENP}_2 = z[xc_2 + (1-x)g_2] + (1-z)[xd_2 + (1-x)h_2] \\ \text{EEC}_1 = x[ya_3 + (1-y)c_3] + (1-x)[ye_3 + (1-y)g_3] \\ \text{EEC}_2 = x[yb_3 + (1-y)d_3] + (1-x)[yf_3 + (1-y)h_3] \end{cases} \tag{8}$$

$$\begin{cases} \text{EPG}_{\text{av}} = x \cdot \text{EPG}_1 + (1-x) \cdot \text{EPG}_2 \\ \text{ENP}_{\text{av}} = y \cdot \text{ENP}_1 + (1-y) \cdot \text{ENP}_2 \\ \text{EEC}_{\text{av}} = z \cdot \text{EEC}_1 + (1-z) \cdot \text{EEC}_2 \end{cases} \tag{9}$$

The six equations described in (8) demonstrate the expected profits of PGE, NPSE, and ECS when they select the corresponding pure strategy. The three equations in (9) indicate the average expected profits of PGE, NPSE and ECS, namely the average expected profits of all the individuals in the group of PGE, NPSE, and ECS, respectively.

Next, according to (2), a set of RD equations of the TM-AEG system can be achieved. Hence, when the groups PGE, NPSE, and ECS select the pure strategy S_{PG1}, S_{NP1} and S_{EC1} at the probability of x, y, and z, respectively, the corresponding rate-of-change of the proportion of individuals who select a pure strategy with the time can be obtained, i.e., the RD equation for the three parties in this TM-AEG system can be obtained as follows:

$$\begin{cases} f_{\text{PG}}(x) = \mathrm{d}x/\mathrm{d}t = x \cdot (\text{EPG}_1 - \text{EPG}_{\text{av}}) \\ f_{\text{NP}}(y) = \mathrm{d}y/\mathrm{d}t = y \cdot (\text{ENP}_1 - \text{ENP}_{\text{av}}) \\ f_{\text{EC}}(z) = \mathrm{d}z/\mathrm{d}t = z \cdot (\text{EEC}_1 - \text{EEC}_{\text{av}}) \end{cases} \tag{10}$$

and then the equations (8) and (9) are substituted into (10), after simplification, a set of new RD equations are obtained as

$$\begin{cases} f_{\text{PG}}(x) = g_{\text{PG1}}(x) \cdot g_{\text{PG2}}(y, z) \\ f_{\text{NP}}(y) = g_{\text{NP1}}(y) \cdot g_{\text{NP2}}(z, x) \\ f_{\text{EC}}(z) = g_{\text{EC1}}(z) \cdot g_{\text{EC2}}(x, y) \end{cases} \tag{11}$$

where $g_{\text{PG1}}(x)$, $g_{\text{NP1}}(y)$ and $g_{\text{EC1}}(z)$ are demonstrated in (12), and $g_{\text{PG2}}(y, z)$, $g_{\text{NP2}}(z, x)$ and $g_{\text{EC2}}(x, y)$ are presented in (13), namely

$$\begin{cases} g_{\mathrm{PG1}}(x) = x(1-x) \\ g_{\mathrm{NP1}}(y) = y(1-y) \\ g_{\mathrm{EC1}}(z) = z(1-z) \end{cases} \tag{12}$$

$$\begin{cases} g_{\mathrm{PG2}}(y,z) = (a_1 - b_1 - c_1 + d_1 - e_1 + f_1 + g_1 - h_1)yz \\ \quad + (b_1 - d_1 - f_1 + h_1)y + (c_1 - d_1 - g_1 + h_1)z + d_1 - h_1 \\ g_{\mathrm{NP2}}(z,x) = (a_2 - b_2 - c_2 + d_2 - e_2 + f_2 + g_2 - h_2)zx \\ \quad + (e_2 - f_2 - g_2 + h_2)z + (b_2 - d_2 - f_2 + h_2)x + f_2 - h_2 \\ g_{\mathrm{EC2}}(x,y) = (a_3 - b_3 - c_3 + d_3 - e_3 + f_3 + g_3 - h_3)xy \\ \quad + (c_3 - d_3 - g_3 + h_3)x + (e_3 - f_3 - g_3 + h_3)y + g_3 - h_3 \end{cases} \tag{13}$$

The Jacobian matrix of the RD equations in (10) is called $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$, which is a 3×3 square matrix where its three rows are the partial derivatives of $f_{\mathrm{PG}}(x)$, $f_{\mathrm{NP}}(y)$ and $f_{\mathrm{EC}}(z)$ for x, y and z, respectively. Obviously, $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$ has no more than three eigenvalues, which are called $\lambda_{\mathcal{X}}$, $\mathcal{X} = 1, 2, 3$. For simplicity, the transformations are implemented as follows: $a_1-b_1-c_1+d_1-e_1+f_1+g_1-h_1 = r_1$, $b_1-d_1-f_1+h_1 = r_2$, $c_1-d_1-g_1+h_1 = r_3$, $d_1-h_1 = r_4$, $a_2-b_2-c_2+d_2-e_2+f_2+g_2-h_2 = s_1$, $e_2-f_2-g_2+h_2 = s_2$, $b_2-d_2-f_2+h_2 = s_3$, $f_2-h_2 = s_4$, $a_3-b_3-c_3+d_3-e_3+f_3+g_3-h_3 = t_1$, $c_3-d_3-g_3+h_3 = t_2$, $e_3-f_3-g_3+h_3 = t_3$, and $g_3-h_3 = t_4$. After that, the Jacobian matrix $\boldsymbol{J}_{\text{PG-NP-EC}}$ is demonstrated as

$$\boldsymbol{J}_{\text{PG-NP-EC}} = \begin{bmatrix} \frac{\partial f_{\mathrm{PG}}(x)}{\partial x} & \frac{\partial f_{\mathrm{PG}}(x)}{\partial y} & \frac{\partial f_{\mathrm{PG}}(x)}{\partial z} \\ \frac{\partial f_{\mathrm{NP}}(y)}{\partial x} & \frac{\partial f_{\mathrm{NP}}(y)}{\partial y} & \frac{\partial f_{\mathrm{NP}}(y)}{\partial z} \\ \frac{\partial f_{\mathrm{EC}}(z)}{\partial x} & \frac{\partial f_{\mathrm{EC}}(z)}{\partial y} & \frac{\partial f_{\mathrm{EC}}(z)}{\partial z} \end{bmatrix} = \begin{bmatrix} (1-2x)\sigma_1 & x(1-x)(r_1 z + r_2) & x(1-x)(r_1 y + r_3) \\ y(1-y)(s_1 z + s_2) & (1-2y)\sigma_2 & y(1-y)(s_1 x + s_3) \\ z(1-z)(t_1 y + t_3) & z(1-z)(t_1 x + t_2) & (1-2z)\sigma_3 \end{bmatrix} \tag{14}$$

where $\sigma_1 = r_1yz + r_2y + r_3z + r_4$, $\sigma_2 = s_1xz + s_2x + s_3z + s_4$, $\sigma_3 = t_1xy + t_2y + t_3x + t_4$.

3) NEAS ANALYSIS

According to the LST, the asymptotic stabilities of this TM-AEG system at all the equilibrium points of the RD equations in (10), which constitute an equilibrium point set (EPS), denoted by Φ_{EPS}, can be found by analyzing the eigenvalues of the Jacobian matrix $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$ in (14). With the aim of solving Φ_{EPS}, a total of four cases are discussed as follows.

Case 1: We make the factors including $g_{\mathrm{PG1}}(x)$, $g_{\mathrm{NP1}}(y)$ and $g_{\mathrm{EC1}}(z)$ in (11) equal to 0, such that eight equilibrium points for the RD equations can be obtained as $\Phi_{\mathrm{EPS0}} = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)\}$. Then each point of Φ_{EPS0} is substituted into $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$, thus the eigenvalues of it for each of the eight equilibrium points can be solved, which are arranged in a column of the eigenvalue matrix $\boldsymbol{J}_{\mathrm{tz}}$. Obviously, it is a 3×8 matrix and described (15), as shown at the bottom of this page.

It can be seen from the eigenvalue matrix $\boldsymbol{J}_{\mathrm{tz}}$ in (15) that the $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$ has three eigenvalues at each equilibrium point of Φ_{EPS0}. Hence, when the real parts $\mathrm{Re}_{\mathcal{X}}$ ($\mathcal{X} = 1, 2, 3$) of all eigenvalues in a column of $\boldsymbol{J}_{\mathrm{tz}}$ are negative, the corresponding equilibrium point is an ASEP, otherwise is an UEEP. At an ASEP, a MESS will be formed for the three parties of PGE, NPSE and ECS, thus the TM-AEG system can reach an evolutionary stable state after a long-term evolution development, so that the Nash equilibrium can be formed. While at an UEEP, MESS cannot be achieved, as well as Nash equilibrium. The asymptotic stabilities of the game system under the eight groups of equilibrium points in Φ_{EPS0} are shown in Table 4.

TABLE 4. The asymptotic stability conditions of the each equilibrium point in Φ_{EPS0} for the PGE, NPSE and ECS in the typical scenario of the demand-side EM.

Equilibrium points in Φ_{EPS0}	Asymptotic stability conditions	Equilibrium points in Φ_{EPS0}	Asymptotic stability conditions
(0, 0, 0)	$d_1<h_1, f_2<h_2, g_3<h_3$	(1, 0, 0)	$h_1<d_1, b_2<d_2, c_3<d_3$
(0, 1, 0)	$b_1<f_1, h_2<f_2, e_3<f_3$	(0, 0, 1)	$c_1<g_1, e_2<g_2, h_3<g_3$
(1, 1, 0)	$f_1<b_1, d_2<b_2, a_3<b_3$	(1, 0, 1)	$g_1<c_1, a_2<c_2, d_3<c_3$
(0, 1, 1)	$a_1<e_1, g_2<e_2, f_3<e_3$	(1, 1, 1)	$e_1<a_1, c_2<a_2, b_3<a_3$

Case 2: We make any two of $g_{\mathrm{PG1}}(x)$, $g_{\mathrm{NP1}}(y)$ and $g_{\mathrm{EC1}}(z)$ equal to zeros and simultaneously any one of $g_{\mathrm{PG2}}(y, z)$, $g_{\mathrm{NP2}}(z, x)$ and $g_{\mathrm{EC2}}(x, y)$ is made to equal to 0. This gives three feasible sets of conditions, namely (i): $g_{\mathrm{PG1}}(x) = 0$, $g_{\mathrm{NP1}}(y) = 0$ and $g_{\mathrm{EC2}}(x, y) = 0$, (ii): $g_{\mathrm{PG1}}(x) = 0$, $g_{\mathrm{EC1}}(z) = 0$ and $g_{\mathrm{NP2}}(z, x) = 0$, and (iii): $g_{\mathrm{NP1}}(y) = 0$, $g_{\mathrm{EC1}}(z) = 0$ and $g_{\mathrm{PG2}}(y, z) = 0$. Now the condition (i) is taken for an example for NEAS analysis. Four groups of solutions under this condition can be obtained, namely $(x, y, z) = \{(0, 0, z_1), (0, 1, z_2), (1, 0, z_3), (1, 1, z_4)\}$, and then which of them is substituted into the $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$ in (14). In turn combining with $g_{\mathrm{EC2}}(x, y) = 0$, it is found that the elements of the third column of $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$ are all zeros, thus the real part of the eigenvalues of $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$ are not

$$\boldsymbol{J}_{\mathrm{tz}} = \begin{array}{c} \begin{matrix} (0,0,0) & (1,0,0) & (0,1,0) & (0,0,1) & (1,1,0) & (1,0,1) & (0,1,1) & (1,1,1) \end{matrix} \\ \begin{bmatrix} d_1 - h_1 & h_1 - d_1 & b_1 - f_1 & c_1 - g_1 & f_1 - b_1 & g_1 - c_1 & a_1 - e_1 & e_1 - a_1 \\ f_2 - h_2 & b_2 - d_2 & h_2 - f_2 & e_2 - g_2 & d_2 - b_2 & a_2 - c_2 & g_2 - e_2 & c_2 - a_2 \\ g_3 - h_3 & c_3 - d_3 & e_3 - f_3 & h_3 - g_3 & a_3 - b_3 & d_3 - c_3 & f_3 - e_3 & b_3 - a_3 \end{bmatrix} \end{array} \tag{15}$$

all negative, which means that $\boldsymbol{J}_{\mathrm{PG-NP-EC}}$ must have a zero eigenvalue, such that the game system is not asymptotically stable and has no ASEPs under this condition. In fact, the four groups of solutions (x, y, z) are substituted into $g_{\mathrm{EC2}}(x, y)$ sequentially, it obtains that $g_{\mathrm{EC2}}(x, y)$ is equal to $g_3 - h_3$, $c_3 - d_3$, $e_3 - f_3$, and $a_3 - b_3$, respectively. Obviously, this is contradictory to $g_{\mathrm{EC2}}(x, y) \equiv 0$, so that the RD equations are unsolvable under condition (i). Similarly, for condition (ii) and (iii), the RD equations are found to be still unsolvable. Consequently, the RD equations have no solutions (x, y, z) in this case, which means that no ASEPs can be obtained for the TM-AEG system.

Case 3: We make any one of $g_{\mathrm{PG1}}(x)$, $g_{\mathrm{NP1}}(y)$ and $g_{\mathrm{EC1}}(z)$ equal to 0 and meanwhile any two of $g_{\mathrm{PG2}}(y, z)$, $g_{\mathrm{NP2}}(z, x)$ and $g_{\mathrm{EC2}}(x, y)$ are made to equal to zeros. Hence three feasible conditions can be obtained: (i): $g_{\mathrm{PG1}}(x) = 0$, $g_{\mathrm{NP2}}(z, x) = 0$ and $g_{\mathrm{EC2}}(x, y) = 0$, (ii): $g_{\mathrm{NP1}}(y) = 0$, $g_{\mathrm{PG2}}(y, z) = 0$ and $g_{\mathrm{EC2}}(x, y) = 0$ and (iii): $g_{\mathrm{EC1}}(z) = 0$, $g_{\mathrm{PG2}}(y, z) = 0$ and $g_{\mathrm{NP2}}(z, x) = 0$. Similarly, the condition (i) is taken as an example for asymptotic stability analysis. Due to $g_{\mathrm{PG1}}(x) = 0$, it obtains $x_0 = 0$ and $x_1 = 1$, under which, the solutions (x, y, z) of the RD equations and its Jacobian matrix $\boldsymbol{J}_{\mathrm{x0x1}}$, and the corresponding eigenvalues and their real parts can be solved, as shown in Table 5, where J_{11}, J_{21}, J_{23}, J_{31} and J_{32} are shown in (16), and J'_{11}, J'_{21}, J'_{23}, J'_{31} and J'_{32} are shown in (17) as follows.

$$\begin{cases} J_{11} = r_4 - (r_3 s_4/s_3) - (r_2 t_4/t_2) + r_1 s_4 t_4/s_3 t_2 \\ J_{21} = -t_4(t_4/t_2 + 1)(s_2 - s_1 s_4/s_3)/t_2 \\ J_{23} = -s_3 t_4(t_4/t_2 + 1)/t_2 \\ J_{31} = -s_4(s_4/s_3 + 1)(t_3 - t_1 t_4/t_2)/s_3 \\ J_{32} = -s_4 t_2(s_4/s_3 + 1)/s_3 \end{cases} \tag{16}$$

where J_{11}, J_{21}, J_{23}, J_{31} and J_{32} shown in (16) are the elements that are not necessarily equal to zero in the Jacobian matrix $\boldsymbol{J}_{\mathrm{x0x1}}$, which is obtained when $x = 0$.

$$\begin{cases} J'_{11} = [\Delta_1 t_1 + \Delta_2 t_2 + \Delta_3 t_3 + \Delta_4 t_4] / [(s_1 + s_3)(t_1+t_2)] \\ J'_{21} = t_3 t_4(s_1 s_4 - s_2 s_3)(t_1 + t_2 + t_3 + t_4) \\ \qquad /[(s_1 + s_3)(t_1 + t_2)^2] \\ J'_{23} = -(s_1 + s_3)(t_3 + t_4)(t_1 + t_2 + t_3 + t_4)/(t_1 t_2)^2 \\ J'_{31} = (s_2 + s_4)(t_1 t_4 - t_2 t_3)(s_1 + s_2 + s_3 + s_4) \\ \qquad /[(s_1 + s_3)^2(t_1 + t_2)] \\ J'_{32} = -(s_2 + s_4)(t_1 + t_2)(s_1 + s_2 + s_3 + s_4)/(s_1 + s_3)^2 \end{cases} \tag{17}$$

where J'_{11}, J'_{21}, J'_{23}, J'_{31} and J'_{32} shown in (17) are the elements that are not necessarily equal to zero in the Jacobian matrix $\boldsymbol{J}_{\mathrm{x0x1}}$, which is obtained when $x = 1$; $\Delta_1 = r_3 s_2 - r_4 s_1 + r_3 s_4 - r_4 s_3$, $\Delta_2 = r_3 s_2 - r_4 s_1 + r_3 s_4 - r_4 s_3$, $\Delta_3 = r_2 s_1 - r_1 s_2 - r_1 s_4 + r_2 s_3$, and $\Delta_4 = r_2 s_1 - r_1 s_4 + r_2 s_3 - r_1 s_2$.

In this case, when $t_2 \neq 0$, $s_3 \neq 0$, $t_1 + t_2 \neq 0$ and $s_1 + s_3 \neq 0$, we conclude from Table 5 that the RD equations in (10) have the unique solution (x, y, z) for $x = 0$ and $x = 1$, respectively. Addressed concretely, when x equals 0 or 1, the Jacobian matrix's three eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ equals $(J_{11}, \sqrt{J_{23} \cdot J_{32}}, -\sqrt{J_{23} \cdot J_{32}})$ and $(J'_{11}, \sqrt{J'_{23} \cdot J'_{32}}, -\sqrt{J'_{23} \cdot J'_{32}})$, respectively. Hence, $\lambda_2 + \lambda_3 \equiv 0$ and $\mathrm{Re}_2 + \mathrm{Re}_3 \equiv 0$ in condition (i) are met, indicating that the three eigenvalues' real parts cannot be negative simultaneously. Likewise, the same conclusion can be drawn for condition (ii) and (iii). Therefore, the TM-AEG system has no ASEPs in this case.

TABLE 5. The equilibrium points of the TM-AEG system under the conditions of x=0 and x=1.

Item	x=0	x=1
Equilibrium points (x, y, z)	$(0, -t_4/t_2, -s_4/s_3)$	$(1, -(t_3+t_4)/(t_1+t_2), -(s_2+s_4)/(s_1+s_3))$
First to third rows of Jacobian matrix $\boldsymbol{J}_{\mathrm{x0x1}}$	$[J_{11}, 0, 0]$, $[J_{21}, 0, J_{23}]$, $[J_{31}, J_{32}, 0]$	$[J_{11}', 0, 0]$, $[J_{21}', 0, J_{23}']$, $[J_{31}', J_{32}', 0]$
Eigenvalues of $\boldsymbol{J}_{\mathrm{x0x1}}$ $(\lambda_1, \lambda_2, \lambda_3)$	$(J_{11}, \lambda_2, -\lambda_2)$	$(J_{11}', \lambda_2, -\lambda_2)$
Real part of the eigenvalues Re_i	$\mathrm{Re}_2+\mathrm{Re}_3\equiv 0$	$\mathrm{Re}_2+\mathrm{Re}_3\equiv 0$

Case 4: We make $g_{\mathrm{PG2}}(y, z)$, $g_{\mathrm{NP2}}(z, x)$ and $g_{\mathrm{EC2}}(x, y)$ equal to 0, which can make the three RD equations shown in (11) be zero simultaneously, namely

$$\begin{cases} g_{\mathrm{PG2}}(y, z) = r_1 yz + r_2 y + r_3 z + r_4 = 0 \\ g_{\mathrm{NP2}}(z, x) = s_1 zx + s_3 z + s_2 x + s_4 = 0 \\ g_{\mathrm{EC2}}(x, y) = t_1 xy + t_3 x + t_2 y + t_4 = 0 \end{cases} \tag{18}$$

Assume that the solution of (18) is (x_0, y_0, z_0), here z_0 is used to denote x_0 and y_0 because the expression of (x_0, y_0, z_0) is very complicated, thus it is obtained $x_0 = -(s_4+s_3 z_0)/(s_2+s_1 z_0)$, $y_0 = -(r_4+r_3 z_0)/(r_2+r_1 z_0)$. Next, (x_0, y_0, z_0) is substituted into (14) to obtain a new Jacobian matrix, denoted by $\boldsymbol{J}_{\mathrm{x0y0z0}}$. Obviously, the diagonal elements of $\boldsymbol{J}_{\mathrm{x0y0z0}}$ contain $g_{\mathrm{PG2}}(y, z)$, $g_{\mathrm{NP2}}(z, x)$ and $g_{\mathrm{EC2}}(x, y)$, respectively, according to the structure of (14). Then, according to (14) and (18), it obtains that these diagonal elements are equal to 0, thus the Jacobian matrix $\boldsymbol{J}_{\mathrm{x0y0z0}}$ is obtained as

$$\boldsymbol{J}_{x0y0z0} = \begin{bmatrix} 0 & J''_{12} & J''_{13} \\ J''_{21} & 0 & J''_{23} \\ J''_{31} & J''_{32} & 0 \end{bmatrix} \tag{19}$$

where the elements J''_{12}, J''_{13}, J''_{21}, J''_{23}, J''_{31} and J''_{32} are as

$$\begin{cases} J''_{12} = x_0(1 - x_0)(r_1 z_0 + r_2) \\ J''_{13} = x_0(1 - x_0)(r_1 y_0 + r_3) \\ J''_{21} = y_0(1 - y_0)(s_1 z_0 + s_2) \\ J''_{23} = y_0(1 - y_0)(s_1 x_0 + s_3) \\ J''_{31} = z_0(1 - z_0)(t_1 y_0 + t_3) \\ J''_{32} = z_0(1 - z_0)(t_1 x_0 + t_2) \end{cases} \tag{20}$$

Finally, it can be obtained $\lambda_1 + \lambda_2 + \lambda_3 = 0 + 0 + 0 = 0$ owing to the trace of $\boldsymbol{J}_{\mathrm{x0y0z0}}$ is equal to the sum of its eigenvalues, thus their real parts cannot be negative simultaneously, otherwise $\lambda_1 + \lambda_2 + \lambda_3 = 0$ cannot be met. Therefore, no ASEPs can be obtained in this case.

A Summary: We conclude from case 2 to case 4 that ASEPs cannot be obtained in the three cases for this TM-AEG system in the typical scenario of the demand-side EM. In other words, combined equilibrium points cannot be generated in the cube intersection of three-party strategy space, thus Nash equilibriums or evolutionary stable states cannot be achieved in the long-term evolution (i.e., no MESS's are generated), regardless of the strategies adopted by the parties. In addition, in the case 1, we find that, for PGE, NPSE and ECS who participate in electricity trading in the hypothetical typical scenario of EM, only when the payoff distribution parameters presented in Table 3 meet the conditions in Table 4, we can achieve eight evolutionary stable states during the long-term evolution process. That is, the Nash equilibrium can be achieved in this gaming, and the number of ASEPs is 8 and they are $\Phi_{\mathrm{EPS0}} = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)\}$, at each of them the MESS or ESE can be formed. Apart from this, other equilibrium points are revealed as UEEPs. Therefore, we can obtain a complete summary of the NEAS for the four cases in this TM-AEG in a typical scenario of the demand-side EM, as presented in Table 6.

TABLE 6. **A complete summary of the Nash equilibrium based asymptotic stability analysis in the four cases in this TM-AEG.**

Cases	Number of equilibrium points	Eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ at each equilibrium point	Trace of Jacobian matrix at each equilibrium point	Number of ASEPs	Number of UEEPs	Number of MESS's
Case 1	8	Each column of matrix $\boldsymbol{J}_{tz}$ in (16)	Sum of each column of the matrix $\boldsymbol{J}_{tz}$ in (16)	8[a]	0	8[b]
Case 2	0	Non-existent	Non-existent	0	Non-existent	0
Case 3	2	$(J_{11}, \sqrt{J_{23} \cdot J_{32}}, -\sqrt{J_{23} \cdot J_{32}})$ $(J_{11}', \sqrt{J_{23}' \cdot J_{32}'}, -\sqrt{J_{23}' \cdot J_{32}'})$	J_{11} or J_{11}'	0	2	0
Case 4	Depends on payoff parameters	Depends on payoff parameters (number ≤3)	0	0	Equals to number of equilibrium points	0

[a,b]The numbers of ASEPs and MESS's in Case 1 are both 8 on the premise that the parametric conditions in Table IV are met.

Obviously, these eight ASEPs are not necessarily suitable for the requirements of an actual demand-side EM. In fact, we finally discovered that only one ASEP is what we want, which is also commonly adopted by most individuals in each part in the TM-AEG system. This only appropriate ASEP is more in line with the healthy and orderly development of an EM. The only suitable ASEP is $(1, 1, 1)$, which means PGE and NPSE choose to cooperate with each other and provide TOU prices that are more beneficial to them. Meanwhile, ECS regard the two parties as integrated power suppliers and carry out strategy S_{EC1} to maximize their benefits. Hence, the optimal offers or bids at Nash equilibrium for generators and consumers can be achieved at this point. The parameter conditions required to obtain this unique ASEP will be discussed in detail in section VII.

4) NEAS ANALYSIS FOR THE PGE GROUP

As one party of the stakeholders in the TM-AEG system, the group of PGE is considered to be a dominant party. Now, PGE is taken as an objective to discuss the dynamic trend and stable process of ESS for this group in all circumstances, in which complete system dynamics behavior characteristics of PGE will be given.

First, the RD equation of PGE (i.e., the fitness function of PGE) who execute strategy S_{PG1} (proportion is x) is obtained according to (8) $\sim$ (13) as

$$\begin{aligned} f_{\mathrm{PG}}(x) &= \mathrm{d}x/\mathrm{d}t = x \cdot (\mathrm{EPG}_1 - \mathrm{EPG}_{\mathrm{av}}) \\ &= x(1-x)(q_1yz - q_2y - q_3z + q_4) \\ &= g_{\mathrm{PG1}}(x) \cdot g_{\mathrm{PG}}(y, z) \end{aligned} \tag{21}$$

where $g_{\mathrm{PG1}}(x) = x(1-x)$, $g_{\mathrm{PG}}(y, z) = q_1yz - q_2y - q_3z + q_4 = (q_1z - q_2)y - (q_3z - q_4)$, $q_1 = a_1 - b_1 - c_1 + d_1 - e_1 + f_1 + g_1 - h_1$, $q_2 = d_1 + f_1 - h_1 - b_1$, $q_3 = d_1 + g_1 - h_1 - c_1$, and $q_4 = d_1 - h_1$. The RD equation in (21) indicates that the rate of change of the proportion of the individuals who select strategy S_{PG1} in the group of PGE with time is proportional to x, $1 - x$, and $g_{\mathrm{PG}}(y, z)$.

Second, solve the (21) (i.e., $f_{\mathrm{PG}}(x) = 0$) to obtain the equilibrium points of the RD equation, based on which, the dynamic adjustment trajectory of strategy evolution for PGE in all cases can be discussed, such that its complete system dynamics behavior characteristics can be achieved. Besides, according to the structure of $g_{\mathrm{PG1}}(x)$ and $g_{\mathrm{PG}}(y, z)$, on the premise that $(q_1z - q_2) \neq 0$, and based on whether $g_{\mathrm{PG}}(y, z)$ is equal to 0, there are a total of two situations that need to be discussed as follows.

Situation 1: Since $g_{\mathrm{PG}}(y, z) = (q_1z - q_2)y - (q_3z - q_4) \equiv 0$, we obtain $y \equiv (q_3z - q_4)/(q_1z - q_2) \in [0, 1]$, where $z \in [0, 1]$. Hence, $f_{\mathrm{PG}}(x) = \mathrm{d}x/\mathrm{d}t = x(1-x) \cdot g_{\mathrm{PG}}(y, z) \equiv 0$, which indicates that for $\forall x \in (0, 1)$, all the states of PGE will remain stable, regardless of the probability x of selecting S_{PG1} by the individuals in PGE, where the quotation of TOU is provided as P_{TOU1}, together with the probability $(1 - x)$ of selecting S_{PG2}, where the quotation of TOU is provided as P_{TOU2}. Hence, for $\forall x \in (0, 1)$, the expected payoff EPG_1 of the individuals in PGE who select S_{PG1} is exactly equal to the group average payoff $\mathrm{EPG}_{\mathrm{av}}$. This means the proportion of the individuals who choose S_{PG1} or S_{PG2} will remain unchanged when the evolutionary strategy for PGE is asymptotically stable. This dynamic trend of strategy execution is demonstrated in Figure 3.

Situation 2: Since $g_{\mathrm{PG}}(y, z) \neq 0$, we obtain $y \neq (q_3z - q_4)/(q_1zq_2)$, where $y, z \in [0, 1]$. The equilibrium points are obtained by solving $f_{\mathrm{PG}}(x) = 0$, namely $x(1 - x) \cdot g_{\mathrm{PG}}(y, z) = 0$, obviously, it obtains $x_0 = 0$ and $x_1 = 1$ due to

TABLE 7. Complete system dynamics behavioral characteristics analysis for the PGE under the Situation 1 and Situation 2.

y	Case	Constraint	y_{fm}	z	$g_{PG}(y, z)$ $=y_{fm}y$-y_{fz}	pg_0	pg_1	ASEP value	Ultimate trajectory of ESS for PGE	Whether it is a Nash equilibrium (what kind)
$y<y_{fz}/y_{fm}$	(i)	$q_1>0$, $q_2>0$	+	$(q_2/q_1, 1]$	-	-	+	$x=x_0=0$	$\to S_{PG2}$ (ESS)	Yes (an ASEP)
			-	$[0, q_2/q_1)$	+	+	-	$x=x_1=1$	$\to S_{PG1}$ (ESS)	Yes (an ASEP)
	(ii)	$q_1<0$, $q_2<0$	+	$[0, q_2/q_1)$	-	-	+	$x=x_0=0$	$\to S_{PG2}$ (ESS)	Yes (an ASEP)
			-	$(q_2/q_1, 1]$	+	+	-	$x=x_1=1$	$\to S_{PG1}$ (ESS)	Yes (an ASEP)
	(iii)	$q_1<0$, $q_2>0$	-	[0, 1]	+	+	-	$x=x_1=1$	$\to S_{PG1}$ (ESS)	Yes (an ASEP)
	(iv)	$q_1>0$, $q_2<0$	+	[0, 1]	-	-	+	$x=x_0=0$	$\to S_{PG2}$ (ESS)	Yes (an ASEP)
$y>y_{fz}/y_{fm}$	(i)	$q_1>0$, $q_2>0$	+	$(q_2/q_1, 1]$	+	+	-	$x=x_1=1$	$\to S_{PG1}$ (ESS)	Yes (an ASEP)
			-	$[0, q_2/q_1)$	-	-	+	$x=x_0=0$	$\to S_{PG2}$ (ESS)	Yes (an ASEP)
	(ii)	$q_1<0$, $q_2<0$	+	$[0, q_2/q_1)$	+	+	-	$x=x_1=1$	$\to S_{PG1}$ (ESS)	Yes (an ASEP)
			-	$(q_2/q_1, 1]$	-	-	+	$x=x_0=0$	$\to S_{PG2}$ (ESS)	Yes (an ASEP)
	(iii)	$q_1<0$, $q_2>0$	-	[0, 1]	-	-	+	$x=x_0=0$	$\to S_{PG2}$ (ESS)	Yes (an ASEP)
	(iv)	$q_1>0$, $q_2<0$	+	[0, 1]	+	+	-	$x=x_1=1$	$\to S_{PG1}$ (ESS)	Yes (an ASEP)

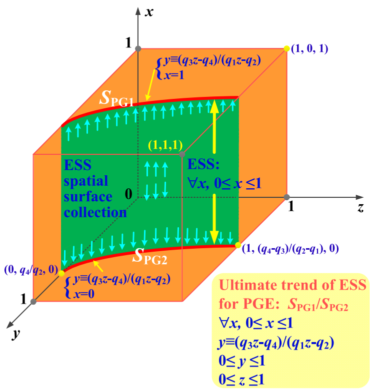

FIGURE 3. Dynamic evolution trend and stabilization process of ESS for PGE in Situation 1.

$g_{PG}(y, z) \neq 0$. This means the ESS for PGE only allows S_{PG1} (when $x = 1$) or S_{PG2} (when $x = 0$), and there are no other mutations that enable the party PGE to reach an evolutionary stable state. At this point, the RD equation $f_{PG}(x)$ in (21) is differentiated in relation to the probability x of selecting S_{PG1}, namely $\mathrm{d}f_{PG}(x)/\mathrm{d}x = (1 - 2x) \cdot g_{PG}(y, z)$. It is calculated that $pg_0 = \{\mathrm{d}f_{PG}(x)/\mathrm{d}x | x = x_0, y \in [0, 1], z \in [0, 1]\} = (1 - 2 \times 0) \cdot g_{PG}(y, z) = g_{PG}(y, z)$, $pg_1 = \{\mathrm{d}f_{PG}(x)/\mathrm{d}x | x = x_1, y \in [0, 1], z \in [0, 1]\} = (1 - 2 \times 1) \cdot g_{PG}(y, z) = -g_{PG}(y, z)$, thus pg_0 and pg_1 are opposites. For convenience, it is denoted that $q_3z - q_4 = y_{fz}$ and $q_1z - q_2 = y_{fm}$, then $g_{PG}(y, z) = y_{fm}y - y_{fz}$ where $z \in [0, 1]$. Owing to $y \neq (q_3z - q_4)/(q_1z - q_2)$, there will be only two possible cases, namely Condition 1: $y < (q_3z - q_4)/(q_1z - q_2)$ and Condition 2: $y > (q_3z - q_4)/(q_1z - q_2)$, i.e., $y < y_{fz}/y_{fm}$ and $y > y_{fz}/y_{fm}$, where $y, z \in [0, 1]$. Since $y_{fm} = q_1z - q_2 \neq 0$ is satisfied under the general constraints, the discussion just needs to be focused on the positive (denoted with '+') and negative (denoted with '−') of y_{fm}. Owing to $z \in [0, 1]$, then the sign of y_{fm} (i.e., + or −) is only related to q_1 and q_2.

Therefore, there are both four cases exist in Condition 1 and Condition 2 above, i.e., (i) $q_1 > 0, q_2 > 0$, (ii) $q_1 < 0, q_2 < 0$, (iii) $q_1 < 0, q_2 > 0$ and (iv) $q_1 > 0, q_2 < 0$. For these cases, based on previous discussions, complete system dynamics behavior of the PGE can be analyzed as demonstrated in Table 7.

TABLE 8. Six representative game situations.

The sequence number of game situations shown in Figure 4	The constraints of six representative game situations for PGE				
	x	y	z	q_1	q_2
(a)	1	$[0, 1] \cap [0, (q_3z\text{-}q_4)/(q_1z\text{-}q_2))$	[0, 1]	$(-\infty, 0)$	$(0, +\infty)$
(b)	0	$[0, 1] \cap [0, (q_3z\text{-}q_4)/(q_1z\text{-}q_2))$	[0, 1]	$(0, +\infty)$	$(-\infty, 0)$
(c)	0	$[0, 1] \cap [0, (q_3z\text{-}q_4)/(q_1z\text{-}q_2))$	$[0, 1] \cap (q_2/q_1, 1]$	$(0, +\infty)$	$(0, +\infty)$
(d)	1	$[0, 1] \cap [0, q_4/q_2] \cap [0, (q_3z\text{-}q_4)/(q_1z\text{-}q_2))$	$[0, 1] \cap [0, q_2/q_1)$	$(0, +\infty)$	$(0, +\infty)$
(e)	0	$[0, 1] \cap [0, q_4/q_2] \cap [0, (q_3z\text{-}q_4)/(q_1z\text{-}q_2))$	$[0, 1] \cap [0, q_2/q_1)$	$(-\infty, 0)$	$(-\infty, 0)$
(f)	1	$[0, 1] \cap [0, (q_3z\text{-}q_4)/(q_1z\text{-}q_2))$	$[0, 1] \cap (q_2/q_1, 1]$	$(-\infty, 0)$	$(-\infty, 0)$

Accordingly, Table 8 shows six representative game situations that are chosen from Table 7. Under these game situations, the dynamic trajectories of ESS in a long period of systematic evolution and development for PGE are illustrated in Figure 4, where the green surface is a spatial surface collection of ESS that is obtained in Situation 1. Equally, for the other two stakeholders, NPSE and ECS, similar phase trajectory charts about the stabilization processes of ESS can be achieved.

A Summary for All Situations: We find out from Table 7 that 12 cases can be obtained for the PGE to achieve an ESS in Situation 2 on the premise that the payoff parameters meet the corresponding conditions in this table. Among these cases, the selection of the strategy S_{PG1} and S_{PG2} accounts for half of each. When Situation 1 is counted, there will be a total of 13 cases where an ASEP for PGE can be obtained. At each ASEP, an ESS can be formed and an evolutionary stable state can be achieved for PGE after a long-term evolution. Moreover, Figure 4 shows that at these ASEPs a strong resistance to incursion of mutant strategies into PGE will be formed and these ESS's will lead to dynamic equilibriums

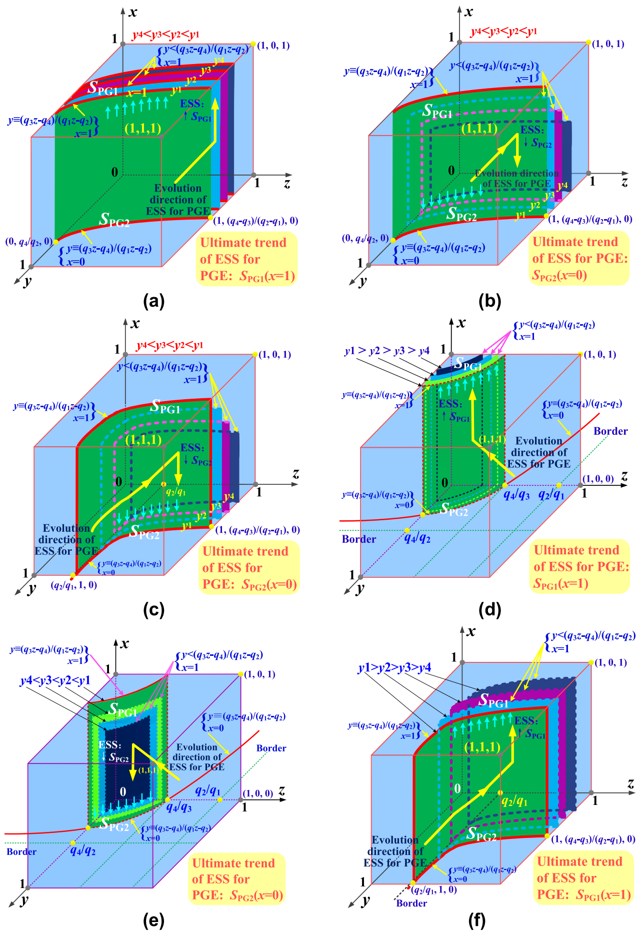

FIGURE 4. Dynamic evolution trend and stabilization process of ESS for PGE under six representative scenarios presented in Table 8 in Situation 2, where (a) shows the ultimate trend of ESS for PGE is strategy S_{PG1} (x=1), (b) shows the ultimate trend of ESS for PGE is strategy S_{PG2} (x=0), (c) shows the ultimate trend of ESS for PGE is strategy S_{PG2} (x=0), (d) shows the ultimate trend of ESS for PGE is strategy S_{PG1} (x=1), (e) shows the ultimate trend of ESS for PGE is strategy S_{PG2} (x=0), and (f) shows the ultimate trend of ESS for PGE is strategy S_{PG1} (x=1).

in system. In these equilibrium states, no individuals will be willing to change their strategies unilaterally. Hence, they are absolutely evolutionary equilibriums.

V. AN ACTUAL CASE FOR VERIFICATION

In this section, taking new energy accommodation for an example, we demonstrate a practical application analysis based on a TM-AEG in a typical scenario of the generation-side EM, in order to verify the main findings we have achieved in previous sections. In particular, we want to verify a finding that no more than 8 ASEPs can be achieved in the typical scenario of TM-AEG and the MESS obtained in the gaming shows strong properties of expelling invaders and resistance to any variation, and besides, the evolutionary stable equilibriums obtained during the process of system evolution are both strict Nash equilibriums.

In this case study, the game agents are three-party enterprises who participate in new energy accommodating in an ever-growing, opened and competitive generation-side EM, and they are new energy generation enterprises (mainly wind and photovoltaic power GenCos), traditional fossil energy generation enterprises (mainly thermal power GenCos) and the power grid enterprises. Here, they are denoted by NEGE, TEGE and PGES respectively. On one hand the interests of TEGE will be reduced when wind/photovoltaic power (i.e. NEGE) integrated. On the other hand the safe and stable operation of power system (i.e. PGES) will be influenced by wind power due to its intermittency and randomness in generation. As a result, PGES are not very positive to participate in wind power accommodating, thus a conflict of interest will be formed eventually between NEGE, TEGE and PGES on the development and utilization of new energy sources. Obviously, this is a representative multi-group AEG issue, or called a wind/photovoltaic-thermal-gird multi-gaming issue in detail, when the three parties are seen as different stakeholders participating in exploration and exploitation of new energy under which integrated.

A. STRATEGY DESCRIPTION

In this trilateral evolutionary game, two pure strategies are formed in actual operation for each group as follows: NEGE choose strategy S_{NEGE1} at a proportion/probability u and S_{NEGE2} at $1-u$, implying cooperation and non-cooperation with TEGE, respectively, and correspondingly, the new energy generation outputs (mainly photovoltaic and wind power) are W_{NEGE1} and W_{NEGE2}, respectively. TEGE select strategy S_{TEGE1} at a probability v and S_{TEGE2} at $1-v$, implying cooperation and non-cooperation with NEGE, respectively, and accordingly, the traditional fossil energy generation outputs (mainly thermal power) are W_{TEGE1} and W_{TEGE2}, respectively. PGES implement strategy S_{PGES1} at a probability w and S_{PGES2} at $1-w$, implying PGES positively and negatively accommodate new energy generation, respectively, and the accommodations are W_{PGES1} and W_{PGES2}, respectively. Here, u, v, and $w \in [0, 1]$.

Hence, similar to Table 3, we can obtain the payoff distribution matrix of this wind/photovoltaic-thermal-gird game (a TM-AEG) system as

$$
\begin{array}{ccc}
 & & \begin{array}{cc} S_{\mathrm{NEGE1}}(u) & S_{\mathrm{NEGE2}}(1-u) \end{array} \\
S_{\mathrm{PGES1}}(w) & \left\{\begin{array}{l} S_{\mathrm{TEGE1}}(v) \\ S_{\mathrm{TEGE2}}(1-v) \end{array}\right. & \multirow{2}{*}{$\begin{bmatrix} (A_1,A_2,A_3) & (B_1,B_2,B_3) \\ (C_1,C_2,C_3) & (D_1,D_2,D_3) \\ (E_1,E_2,E_3) & (F_1,F_2,F_3) \\ (G_1,G_2,G_3) & (H_1,H_2,H_3) \end{bmatrix}$} \\
S_{\mathrm{PGES2}}(1-w) & \left\{\begin{array}{l} S_{\mathrm{TEGE1}}(v) \\ S_{\mathrm{TEGE2}}(1-v) \end{array}\right. &
\end{array}
\tag{22}
$$

where A_Υ, B_Υ, C_Υ, D_Υ, E_Υ, F_Υ, G_Υ and H_Υ are the payoff parameters of the payoff matrix obtained in this TM-AEG system, and $\Upsilon = 1, 2, 3$. The (22) demonstrates a payoff distribution matrix of this TM-AEG system, in which the group of PGES implements strategy S_{PGES1} at a probability of w and S_{PGES2} at $1-w$, the group of TEGE selects strategy S_{TEGE1} at a probability of v and S_{TEGE2} at $1-v$, and the group of NEGE chooses strategy S_{NEGE1} at a probability of u and S_{NEGE2} at $1-u$. Therefore, there are a total of 8 strategy combinations, and in each one the corresponding payoff distribution parameters of the three parties are given.

Based on (22) and according to (8)∼(14), the RD equations of this TM-AEG system can be obtained as in (23), as shown at the bottom of this page, and the corresponding Jacobian matrix can also be calculated as in (24), which is denoted by $\boldsymbol{J}_{NE-TE-PG}$, as shown at the bottom of this page, where $\sigma_4 = R_1vw + R_2v + R_3w + R_4$, $\sigma_5 = Q_1wu + Q_2w + Q_3u + Q_4$, $\sigma_6 = T_1uv + T_2u + T_3v + T_4$; and R_M, Q_M and T_M, (M = 1, 2, 3, 4) are presented as

$$\begin{bmatrix} R_1 & R_2 & R_3 & R_4 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ T_1 & T_2 & T_3 & T_4 \end{bmatrix} = \begin{bmatrix} A_1-B_1-C_1+D_1-E_1+F_1+G_1-H_1 & B_1-D_1-F_1+H_1 & C_1-D_1-G_1+H_1 & D_1-H_1 \\ A_2-B_2-C_2+D_2-E_2+F_2+G_2-H_2 & E_2-F_2-G_2+H_2 & B_2-D_2-F_2+H_2 & F_2-H_2 \\ A_3-B_3-C_3+D_3-E_3+F_3+G_3-H_3 & C_3-D_3-G_3+H_3 & E_3-F_3-G_3+H_3 & G_3-H_3 \end{bmatrix} \tag{25}$$

Analogously, we can conclude that the RD equations presented in (23) only have 8 possible asymptotical stable equilibrium points (ASEPs): $(u, v, w) = \{(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 0), (1, 0, 1), (0, 1, 1), (1, 1, 1)\}$. When these equilibrium points meet the conditions similar to Table 4, they will make this TM-AEG system evolve into an evolutionary stable state and achieve a Nash equilibrium.

B. SCENARIO DISCUSSION

Here, we consider two scenarios as follows: one is no policy intervention on this gaming, and the other one is policies of EM issued by government to intervene this gaming, demonstrated as follows.

Scenario I): No policy intervention on this game. Obviously, PGES can execute S_{PGES1} or S_{PGES2} due to no policy intervention, under which TEGE can adopt S_{TEGE1} or S_{TEGE2}, at this moment NEGE will always select S_{NEGE2} for gaining more advantage while the payoffs of TEGE are decreased. This demonstrates that NEGE choose not to cooperate with TEGE no matter what strategy they adopt, from which it obtains $B_3 > A_3$, $A_2 > B_2$, $D_3 > C_3$ and $C_2 > D_2$. Besides, analogous conclusions can be drawn from other cases. Hence, the inequality constraints of the payoff parameters can be obtained as shown by the right-hand big blue oval in Figure 5.

Moreover, owing to new energy integration, an additional cost is needed for PGES to maintain output, thus the payoff will be increased when they choose strategy S_{PGES2}, compared to strategy S_{PGES1}, from which it is obtained that $E_1 > A_1$, $G_1 > C_1$, $F_1 > B_1$, and $H_1 > D_1$, as shown in the left-hand small blue oval in Figure 5.

On the whole, when no policies are introduced, we find that the payoff matrix will not be affected. Then, the MESS that is beneficial to all parties will be achieved in such a scenario where NEGE and TEGE are always uncooperative, while PGES perform renewable energy accommodation in a negative way, as illustrated in Figure 5, where the up arrow (↑) means the payoff is increased after choosing the corresponding strategy, conversely, the down arrow (↓) implies the payoff is decreased. In this scenario, the only ASEP is obtained at (0, 0, 0), and the Nash equilibrium under the MESS $\{S_{NEGE2}, S_{TEGE2}, S_{PGES2}\}$ will be achieved.

Scenario II): Government intervention involved in the game. In this scenario, the payoff parameters can be regulated, thereby affecting the evolution stability of system, i.e., the selection of ASEP and MESS. Here, the policy interventions are conducted to promote the new energy enterprises to participate in trading. Under such circumstances, the policy can affect the initial state of system, and conversely, which will determine the eventual evolutionary stable state of this gaming system. Therefore, under policy interventions, in order to promote the development and utilization of new energy resources and improve the grid-connection certainty, this TM-AEG system is expected to achieve a unique MESS at (1, 1, 1) finally, which means that NEGE cooperate with TEGE to output generation of W_{NEGE1} and W_{TEGE1}, respectively, together with PGES actively accommodate new energy of W_{PGES1}. At this moment the parameters in (22) will be changed into $A'_{\mathcal{X}}$ to $H'_{\mathcal{X}}$ alphabetically, where $\mathcal{X} = 1, 2, 3$.

$$\begin{cases} f_{NEGE}(u) = u(1-u)(R_1vw + R_2v + R_3w + R_4) \\ f_{TEGE}(v) = v(1-v)(Q_1wu + Q_2w + Q_3u + Q_4) \\ f_{PGES}(w) = w(1-w)(T_1uv + T_2u + T_3v + T_4) \end{cases} \tag{23}$$

$$\boldsymbol{J}_{\text{NE-TE-PG}} = \begin{bmatrix} \frac{\partial f_{NEGE}(u)}{\partial u} & \frac{\partial f_{NEGE}(u)}{\partial v} & \frac{\partial f_{NEGE}(u)}{\partial w} \\ \frac{\partial f_{TEGE}(v)}{\partial u} & \frac{\partial f_{TEGE}(v)}{\partial v} & \frac{\partial f_{TEGE}(v)}{\partial w} \\ \frac{\partial f_{PGES}(w)}{\partial u} & \frac{\partial f_{PGES}(w)}{\partial v} & \frac{\partial f_{PGES}(w)}{\partial w} \end{bmatrix} = \begin{bmatrix} (1-2u)\sigma_4 & u(1-u)(R_1w+R_2) & u(1-u)(R_1v+R_3) \\ v(1-v)(Q_1w+Q_3) & (1-2v)\sigma_5 & v(1-v)(Q_1u+Q_2) \\ w(1-w)(T_1v+T_2) & w(1-w)(T_1u+T_3) & (1-2w)\sigma_6 \end{bmatrix} \tag{24}$$

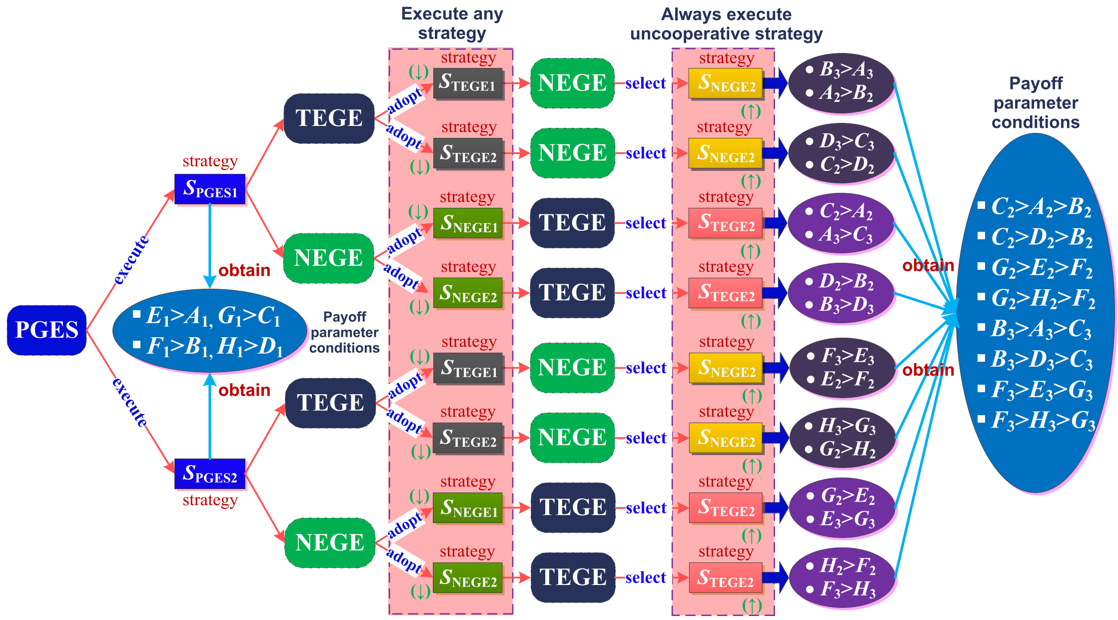

FIGURE 5. Strategy selection process for the NEGE, TEGE and PGES considering no government interventions implemented in the game system.

Therefore, we find that only these changed payoff parameters meet five conditions simultaneously as follows, can the above wish be achieved. They are presented as

i) $E'_1 < A'_1$, $C'_2 < A'_2$ and $B'_3 < A'_3$;
ii) $C'_1 > G'_1$ or $E'_2 > G'_2$ or $H'_3 > G'_3$;
iii) $B'_1 > F'_1$ or $H'_2 > F'_2$ or $E'_3 > F'_3$;
iv) $H'_1 > D'_1$ or $B'_2 > D'_2$ or $C'_3 > D'_3$;
v) $D'_1 > H'_1$ or $F'_2 > H'_2$ or $G'_3 > H'_3$.

Therefore, when meeting the above five conditions, an only one MESS, namely $\{S_{NEGE1}, S_{TEGE1}, S_{PGES1}\}$, will be formed via policy interventions in the gaming system, and which will achieve an evolutionary stable state after a long-term evolution and development. According to this MESS, the government will be capable of formulating grid-connection policies with lower costs for new energy enterprises to implement market intervention in EM, thus not only the new energy generation enterprises (the NEGE) will be motivated to participate in EM transaction in order to promote their development, but also the capability of accommodating their power generation will be improved, especially for the photovoltaic and wind power. This also further verifies the conclusion drawn from the analysis of multi-group AEG in the typical scenarios of EM.

In this section, we select a representative three-party game of photovoltaic/wind-thermal-grid as an actual case to conduct a brief scenario discussion for verifying the main findings made in this paper. From this case study we conclude that the policy intervention on EM by government can regulate the payoff parameters of the gaming in this system, which will finally affect the selection of ASEP and MESS, together with the formation of Nash equilibrium and evolutionary stable state of the AEG system in the process of long-term evolution and development.

VI. DISCUSSION

In this section, we discuss the factors that affect the asymptotical stability of the evolution gaming between PGE, NPSE and ECS. In addition, we briefly discuss the Nash equilibrium decomposition method when solving the Nash equilibrium solution of the three-party game system. From the discussion we can find that multiple factors can, to some extent, influence the ultimate evolutionary stable states, and the energy Internet can play a better role in resource integration of the EM. Moreover, we can obtain some relevant policy implications from the discussion, which can provide reference for the government to make policies for the EM with characteristics of energy Internet.

Firstly, we discuss the factors that affect the asymptotic stability. In the TM-AEG system, PGE, NPSE and ECS try to maximize their interests in electricity trading. For PGE, its payoffs involve the electricity charged to NPSE when providing electricity and a transaction fee charged to ECS. For NPSE, the main means of competition is their price of electricity changing along with the TOU prices announced by PGE. Their income consists essentially of the electricity charged to ECS and is constrained by its maximum power supply. In addition, the total cost is relatively low. For ECS, its benefits are user utility minus user spending. The former is the utility of various loads, and the latter is the payment for electricity. At ASEPs, for PGE and NPSE, the main game strategy is the TOU prices. For ECS, it is the optimal choice of TOU prices provided by PGE and NPSE based on the electricity demands of each type of load. Hence, in an

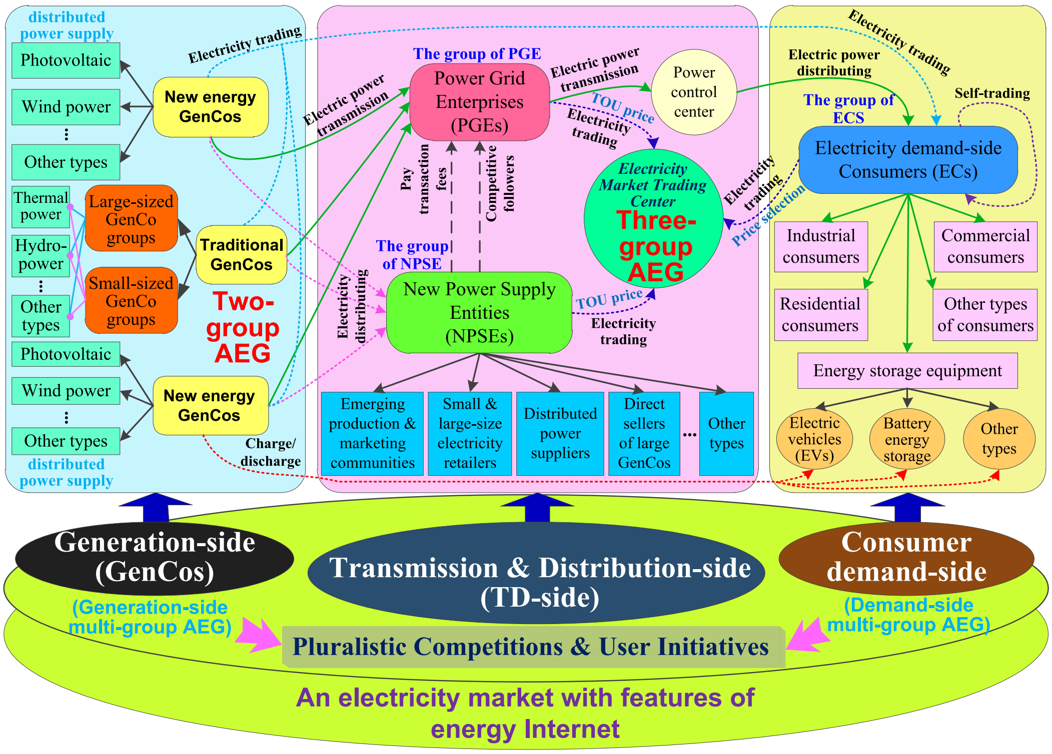

FIGURE 6. Electricity trading model of an opened and competitive EM with features of energy Internet for the three groups of PGE, NPSE and ECS.

EM with features of energy Internet, the factors that may affect the stable equilibrium states of the game system can be summarized, including: the level of the transaction fee between PGE and NPSE, the number of NPSEs, the capacity limit and cost factor of each NPSE, the way to pay the fee, the electricity trading rules, the possession of information on transactions and the degree of user participation, etc. This indicates the complexity and diversity of the marketization of electricity trading under the background of energy Internet, as shown in Figure 6. We find that these factors above can change the payoff parameters, thus changing the ASEPs and the convergence domains. Here, we give two examples as follows.

When the installed capacity of PGE is sufficient, the valley time price under the energy Internet usually arises near the peak load, thus at peak times, PGE will be inclined to choose strategy S_{PG2} so as not to cooperate with NPSE to provide a TOU price which can make ECS more profitable. Meanwhile, NPSE select S_{NP2}, which is uncooperative with PGE, to provide a TOU price that enables ECS to gain more benefits. At the same time ECS execute strategy S_{EC1} or S_{EC2} according to the characteristics of their own electricity loads. Hence, during the peak time, PGE and NPSE form an asymmetric non-cooperative game relationship in which they both try to attract more ECs to use electricity via competition with the purpose of peak shaving. However, in an electricity trough, PGE and NPSE are less involved in price competition, thus PGE and NPSE collaborate with each other and execute strategy S_{PG1} and S_{NP1} respectively to provide TOU prices that benefit both parties. Meanwhile, ECS still implement strategy S_{EC1} or S_{EC2} with the purpose of profit maximization, so that the goal of valley filling may be accomplished.

When the number of NPSEs increases, obviously, the impacts of the electricity price on peak shaving and valley filling will be weakened. As a result, the fluctuations in electricity price will be flattened. Hence, with increasing numbers of ECS individuals, the electricity price provided by each competitor at each time-period will be fairer and more reasonable for consumers, as well as being more stable (i.e., more closer to the rational price of this period). Furthermore, with the increasingly opening up of EM, the individual number and category of NPSE, especially the addition of incremental distribution networks, will be gradually increased. Consequently, within the larger energy Internet, complementarity of resources and time will lead to a more dramatic decline in electricity prices, which will also become flatter in fluctuations. In other words, the energy Internet will play a promising role in resource allocation of new EM.

Secondly, we discuss the impact of policy interventions on the TM-AEG. In order to guide the three parties, namely PGE, NPSE, and ECS, to form an active and order participation in the EM game, with the purpose of creating a more stable electricity price, the PGE can be treated as one party of power supply entities. Moreover, only one MESS, i.e., $\{S_{PG1}, S_{NP1}, S_{EC1}\}$, is expected to be achieved finally, under which PGE and NPSE choose to cooperate with each other and provide TOU prices that are more beneficial to them. Meanwhile, ECS regard the two parties as integrated power suppliers and carry out strategy S_{EC1} to maximize their benefits.

Therefore, an only one ASEP {1, 1, 1} is required to be obtained, while other ASEPs will disappear and eventually be transformed into UEEPs during the system evolution. This can be achieved via policy intervention implemented by the government. Assume that the payoff parameters of the TM-AEG in Table 3 will be changed into $a'_{\mathcal{X}}$, $b'_{\mathcal{X}}$, $c'_{\mathcal{X}}$, $d'_{\mathcal{X}}$, $e'_{\mathcal{X}}$, $f'_{\mathcal{X}}$, $g'_{\mathcal{X}}$ and $h'_{\mathcal{X}}$, due to the government supervision and intervention, where $\mathcal{X} = 1, 2, 3$. Hence, the new eigenvalue matrix $\boldsymbol{J}'_{\mathrm{tz}}$ is obtained as (26), as shown at the bottom of this page, where each column represents the whole eigenvalues of an ASEP.

TABLE 9. The conditions for the elements to be satisfied in the new system eigenvalue matrix $\boldsymbol{J}'_{\mathrm{tz}}$.

Condition number	Distribution parameters condition	Retained ASEP	Disappeared ASEP	Condition description
Condition 1	$e_1'<a_1'$, $c_2'<a_2'$ and $b_3'<a_3'$	(1, 1, 1)	(0, 1, 1), (1, 0, 1), (1, 1, 0)	(1, 1, 1) is still an ASEP, while (0, 1, 1), (1, 0, 1) and (1, 1, 0) are transformed into UEEPs
Condition 2	$c_1'>g_1'$ or $e_2'>g_2'$ or $h_3'>g_3'$	None	(0, 0, 1)	(0, 0, 1) is transformed into an UEEP
Condition 3	$b_1'>f_1'$ or $h_2'>f_2'$ or $e_3'>f_3'$	None	(0, 1, 0)	(0, 1, 0) is transformed into an UEEP
Condition 4	$h_1'>d_1'$ or $b_2'>d_2'$ or $c_3'>d_3'$	None	(1, 0, 0)	(1, 0, 0) is transformed into an UEEP
Condition 5	$d_1'>h_1'$ or $f_2'>h_2'$ or $g_3'>h_3'$	None	(0, 0, 0)	(0, 0, 0) is transformed into an UEEP

Since $\{S_{\mathrm{PG1}}, S_{\mathrm{NP1}}, S_{\mathrm{EC1}}\}$ as the only MESS to be achieved at (1, 1, 1), the eigenvalues in (26), will be required to meet five conditions simultaneously, as shown in Table 9, thus a trilateral Nash equilibrium can be achieved after a long system evolution, and the pricing strategy of each party will tend to be more reasonable. Moreover, EM policies issued by the government will reduce electricity price fluctuation and further promote market stability, and besides, the competitions can be regulated more orderly and rational to all parties. This indicates that effective supervision and control via policy intervention contributes to form stable electricity pricing mechanisms in EM trading, meanwhile the promotion of peak shaving and valley filling for electric network and resource allocation of EM can be achieved.

Lastly, we discuss a decomposition method for solution of Nash equilibrium in practical cases. We have found that the convergence domain of the TM-AEG in is a highly complex three-dimensional space, which makes the Nash equilibrium solutions are very difficult to be obtained directly. Moreover, the electricity behaviors of ECS (e.g., the selection behavior of TOU prices and distribution of electricity consumption) are generally hard to obtain, and sometimes even completely unknown. Hence, machine learning approaches, such as emotional learning [50] and reinforcement learning [51], can be introduced into the grid-retailer-user game to simulate their trading behaviors (i.e., the multi-agent dynamic game interactions) in order to achieve the final optimal evolutionary stable equilibrium, also as Nash equilibrium, during the evolutionary process of multi-group dynamic game in a competitive EM. This involves game input/output, thus this grid-retailer-user game can be decomposed into a two-layer bilateral interactive game, i.e., PGE-NPSE game and NPSE-ECS game, as shown in Figure 7, where the machine learning approaches, such as the multi-agent reinforcement learning algorithm, Q-learning, affective learning, belief-based learning, and deep learning, can be used to search the optimal Nash equilibrium, which is described in detail as follows.

As illustrated in Figure 7, the groups of PGE, NPSE, and ECS as three parties (i.e., Party-1, Party-2, and Party-3) constitute a trilateral multi-group AEG, called grid-retailer-user AEG, in which the system evolution stable equilibrium can be achieved as the refinement of Nash equilibrium. As stated earlier, it is very difficult to directly address the issue of Nash equilibrium solving of this three-party evolutionary game system. In Figure 7, we deem that the reinforcement learning combining with depth perception and psychology can be employed to predict the user's electricity utilization behavior. This will facilitate the demand side response. In addition, for an intelligent community or demonstration area, since the user's electricity behavior is totally unknown, the machine learning methods such as emotional learning and reinforcement learning can be employed to simulate the electricity transaction behavior of three parties, including grid, user, and retailer. This behavioral process is a multi-party dynamic game interaction process in the competitive circumstance of EM. Therefore, we need to treat these three parties as different agents and use the multi-agent reinforcement learning algorithm to solve the optimal Nash equilibrium solution for the game issue.

In Figure 7, we propose a scheme to decompose this grid-retailer-user AEG into a two-layer two-two or bilateral interactive game in our next investigation plan. This two-layer game can also be regarded as a Stackelberg game or a master-slave game.

In particular, the PGE-NPSE game can be seen as a master game or upper game, where the group of PGE is

$$\boldsymbol{J}'_{\mathrm{tz}} = \begin{bmatrix} d'_1 - h'_1 & h'_1 - d'_1 & b'_1 - f'_1 & c'_1 - g'_1 & f'_1 - b'_1 & g'_1 - c'_1 & a'_1 - e'_1 & e'_1 - a'_1 \\ f'_2 - h'_2 & b'_2 - d'_2 & h'_2 - f'_2 & e'_2 - g'_2 & d'_2 - b'_2 & a'_2 - c'_2 & g'_2 - e'_2 & c'_2 - a'_2 \\ g'_3 - h'_3 & c'_3 - d'_3 & e'_3 - f'_3 & h'_3 - g'_3 & a'_3 - b'_3 & d'_3 - c'_3 & f'_3 - e'_3 & b'_3 - a'_3 \end{bmatrix} \tag{26}$$

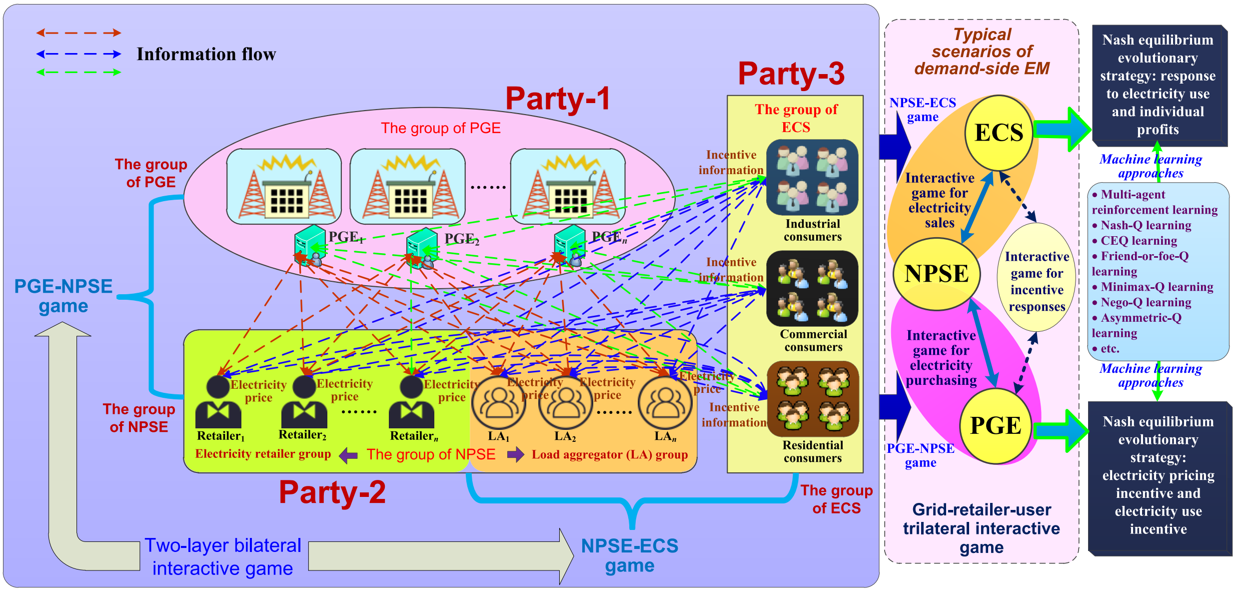

FIGURE 7. A Nash equilibrium decomposition scheme for the TM-AEG involving the PGE, NPSE and ECS who participate in the electricity trading of EM.

seen as a special type of electricity retailer, thus the groups of PGE and NPSE constitute the electricity suppliers and service operators. In addition, the NPSE-ECS game can be treated as a slave game or lower game, in which the electricity consumption behavior of users, including industrial/commercial/residential consumers, can be simulated via an evolutionary game model containing personal privacy. In the upper game, the price competition among the electricity suppliers can be simulated through a non-cooperative game model.

Taking the intelligent community for an example, under the premise of considering the consumer's power consumption privacy behavior and price competition among suppliers, we can establish a multi-supplier and multi-home user based master-slave game model from the perspective of the electricity retail market to investigate the issues of electricity demand response in the future. In this example, the demand-responsive behavior based on price competition between suppliers and users is conducive to achieving a balance between power supply and demand, so as to maintain the safe operation of the power systems.

In the future, we can investigate the interactive game between multiple users and suppliers in different game situations. Thereby, two situations are considered as follows.

Situation 1: We do not consider the participation of electricity sellers. At this time, there is only a simple interaction game between the grid side and the user side. At this point, we need to investigate the inputs of the game issue, that is, the value of the objective function of the grid side and user side under the combination of incentive and electricity consumption plans on the obtained Pareto frontier. We also need to investigate the outputs of the game issue, that is, the optimal incentive pricing strategy and the electricity consumption planning strategy in the situation where the grid side and the user side meet the maximization of their expected benefits.

Situation 2: We consider the participation of electricity sellers, i.e., the new power supply entities (NPSEs). As shown in Figure 7, NPSEs mainly include electricity retailer group and load aggregator (LA) group. At this point, the TM-AEG is divided into a two-layer bilateral interactive game, as illustrated in Figure 7. This two-layer bilateral interactive game contains a PGE-NPSE game and a NPSE-ECS game, which are introduced as follows.

- PGE-NPSE gaming: the game inputs are Pareto frontier of objectives such as peak shaving and load leveling on PGE side and electricity purchasing cost of NPSE, together with their objective function value under different action strategy combinations: $\{S_{\mathrm{PG}i}, S_{\mathrm{NP}i}\}$; and the game outputs are optimal Nash equilibrium solutions of pure strategy, including optimal electricity purchasing price strategy and optimal electricity purchasing plan strategy for NPSE.
- NPSE-ECS gaming: the game inputs are Pareto frontier of objectives such as profit of electricity selling on NPSE side, as well as electricity gains (i.e., the utility of electricity minus the payment of electricity) and electricity use comfort (for residential users only) on ECS side, together with their objective function value under different action strategy combinations: $\{S_{\mathrm{NP}i}, S_{\mathrm{EC}i}\}$; and the game outputs are optimal Nash equilibrium solutions of pure strategy, including optimal electricity selling strategy for NPSE and electricity use plan strategy for ECS.

VII. CONCLUSION

In this paper, we investigated the Nash-equilibrium based asymptotic stability of multi-group AEG, including UT-AEG and TM-AEG, based on Replicator Dynamics theory and LST for some typical scenarios in an ever-growing and opened EM with features of energy Internet. In addition, we discussed the impacts of EM policies introduced by the government on the asymptotical stability of multi-group AEG in typical scenario of the EM. The main contributions are summarized as follows:

1) A bilateral 2×2 AEG model (i.e., the UT-AEG model) for electricity price bidding of generation-side EM is established, as well as a trilateral 2×2×2 AEG model (i.e., the TM-AEG model) for electricity trading of demand-side EM, thus the phase trajectory of AEG has been extended from a two-dimensional surface to a three-dimensional space, which reveals more complex dynamic processes of strategy adjustment and more diversified MESS selection behavior of different stakeholders.

2) The TM-AEG in the typical scenarios of EM shows that the number of possible equilibrium states is only eight and they are asymptotically stable when the payoff parameters meet certain conditions. In these states, the MESS shows strong properties of expelling invaders and resistance to any variation, besides, the evolutionary stable equilibriums are both strict Nash equilibriums.

3) EM policies issued by government can affect the NEAS of multi-group AEG via changing the distribution parameters of payoff matrix, thus effective interventions will improve the stability of electricity prices and promote the energy Internet to play a more significant role in resource allocation, which means the complementarities between resources and time flexibility will lead to a lower and stable electricity price.

Hence, we give the relevant policy implications as follows, which may be helpful for EM policy formulating by the relevant government departments.

- First, the policymakers should respect the interests of all sides in the game, try to establish a reasonable profit distribution mechanism, guide them to cooperate, and jointly promote the healthy and orderly development of the open and ever-growing EM.
- Second, in order to reduce the high profits from high pricing by the GenCos, the government must take measures to make reasonable bidding rules for the on-grid competitive bidding of power GenCos. The goal of government regulation is to make the power generation bid as close to its marginal cost as possible in order to harmonize the benefits for power GenCos with the social benefits, and finally form an efficient EM. This principle should be reflected in the established bidding rules.
- Third, in the TM-AEG involving the NEGE, TEGE and PGES, especially the representative wind/photovoltaic-thermal-gird game when considering new energy integrated, if the government does not carry out policy guidance to all parties involved in grid connection of wind power/photovoltaic, then they cannot develop healthfully due to their lack of market competitiveness. However, the government could introduce relevant policies for new energy (power, photovoltaic, etc.) integration, so as to change the trilateral payoffs, such that the new energy industry can be guided to develop healthily, but the policies need to meet certain conditions that have been discussed in this paper, which is of theoretical significance and practical value for ensuring the rational development of new energy.

Finally, we have to recognize that the discussions in this paper on Nash-equilibrium based asymptotic stability of multi-group AEG in typical scenarios of generation-side and demand-side EM are not very rigorous, and moreover, it is still a huge challenge to apply evolutionary game theory to the evolution study of practical engineering systems, especially the establishment of simulation systems for analysis of actual evolution processes, which will be next research direction of the authors. Specifically, build an engineering feasible simulation system for studying the characteristics and regularities of equilibrium stability in a long-term evolution and development of system.

APPENDIX

$\Gamma = < N, S, U >$	a normal-form game
S_i	strategic space of paly i in a game
i	the number of the players/groups/ strategies/individuals in a game system or a population
R	the real number field
Ω_{group}	multi-group strategy combination
X, Y	strategy set
ω, ϖ	parameters and belong to (0, 1)
S^{-i}	strategy combination adopted by groups other than group i
$E(X_i, S^{-i})$	expected payoff for the group i that selects strategy X_i, meanwhile the other groups select S^{-i}
$E(Y_i, S^{-i})$	expected payoff for the group i that selects strategy Y_i, meanwhile the other groups select S^{-i}
$E(X_i)$	expected payoff or fitness of a pure strategy X_i
$\bar{E}(X_i)$	group average payoff or fitness
ρ_i	growth rate of the proportion or share of the individuals that select strategy X_i in a population
ϑ, ϑ^*	mixed strategies in an evolutionary game
$E(\vartheta, \vartheta)$, $E(\vartheta, \vartheta^*)$, $E(\vartheta^*, \vartheta)$, $E(\vartheta^*, \vartheta^*)$	payoffs or fitness functions under different strategy combinations or game situations
n_{RD}	number of populations involved in an evolutionary game system

S_{high}	a high quotation strategy
S_{basic}	a basic quotation strategy
pay_J	payoffs of SSGC and LSGC, $J = 1, 2, 3, 4, 5$
$u_i(q_i)$	payoff function of the power generation enterprise i
$f(Q_{total})$	power demand function of users or the market-clearing price
P_{max}	electricity price cap
K_Q	a constant coefficient
Q_{total}	the sum of on-grid power generation of all generating companies
$Q_{total-max}$	the sum of maximum power provided by all power generating companies
$C_i(q_i)$	actual power generation cost function of generating company i
$\alpha_i, \beta_i, \chi_i$	the no-load operating cost, the intercept of the marginal cost curve, and the slope of the marginal cost curve, respectively
$\text{MC}_i(q_i)$	marginal cost curve of the power generating company i
$\text{AC}_i(q_i)$	average cost curve of the power generating company i
$tr(\boldsymbol{J}_{pq})$	trace of $\boldsymbol{J}_{pq}$
$\text{SCD}_1, \text{SCD}_2$	size of system convergence domains
(p_{sp}, q_{sp})	a saddle point of the UT-AEG system
Υ	the number of the payoff distribution parameters or eigenvalues or real parts of an eigenvalue in a TM-AEG, and $\Upsilon = 1, 2, 3$
$a_\Upsilon, b_\Upsilon, c_\Upsilon, d_\Upsilon, e_\Upsilon, f_\Upsilon, g_\Upsilon, h_\Upsilon, A_\Upsilon, B_\Upsilon, C_\Upsilon, D_\Upsilon, E_\Upsilon, F_\Upsilon, G_\Upsilon, H_\Upsilon$	payoff parameters of the payoff matrix obtained in a TM-AEG system
n_{ec}	number of ECS
θ	number of time periods in an electricity utilization cycle
Δt	time interval
t_θ	time period
T	electricity time set
$Q_{\theta,k}$	total quantity of electricity consumption of the kth consumer's all types of electricity loads during the time period t
$\boldsymbol{Q}_{users}$	electricity consumption distribution matrix of the group of ECS
$Q_{\theta,nec}$	total electricity consumption of the consumer numbered n_{ec}
$Q_{n_{ec},\theta}$	total amount of electricity consumed by the user n_{ec} in the θth time period
$\boldsymbol{P}_{PG}$	price vector of the group of PGE
n_{np}	number of the group of NPSE
$\boldsymbol{P}_{NP-l}$	electricity price vector of the lth NPSE
$\boldsymbol{P}_{NP-nnp}$	electricity price vector of the group of NPSE
l	the number set of the group of NPSE
ΔP_l	unit power policy subsidy obtained by the ECS when selecting the lth NPSE
$\vartheta_{nec,\theta}$	electricity price selection of the consumer n_{ec} in the θth time period
$P_{PG-\theta}, P_{nnp-\theta}$	electricity price provided by the PGE and NPSE in the θth time period, respectively
$\Phi_{nec,\theta}$	payoff of the consumer n_{ec} in the θth time period
Φ_{nec}	total payoff of this consumer in all time periods
U_{nec}	user utility
R_{nec}	consumer's total income
$\boldsymbol{y}_{PG}$	income distribution matrix of the PGE
Y_{PG}	total income of the PGE in all time periods
C_{PG}	cost of PGE in a time cycle T
a_0, b_0, c_0	cost coefficients of PGE after considering all the cost factors
R_{PG}	profit of PGE in a time cycle T
$\boldsymbol{Y}_{NP}$	income distribution matrix of all NPSEs
$\boldsymbol{A}, \boldsymbol{B}$	$n_{np} \times n_{ec}$ and $\theta \times \theta$ matrix, respectively, used to calculate user payoff belonging to the group of NPSE
$\boldsymbol{Y}_{NP\text{-}nnp}$	income distribution vector of the NPSE numbered n_{np}
y_{NP-nnp}	total income of the NPSE numbered n_{np} in a time cycle T
Q_{nnp}	quantity of power provided by the NPSE numbered n_{np}
C_{nnp}	cost of the NPSE numbered n_{np}
$a_{nnp}, b_{nnp}, c_{nnp}$	non-negative numbers
R_{nnp}	profit function of the NPSE numbered n_{np}
Φ_{EPS}	equilibrium point set
Φ_{EPS0}	equilibrium point set obtained in Case 1
$\boldsymbol{J}_{tz}, \boldsymbol{J}'_{tz}$	eigenvalue matrix
p, q, x, y, z, u, v, w	ratio or probability of choosing a strategy
$\boldsymbol{J}_{pq}, \boldsymbol{J}_{PG-NP-EC}, \boldsymbol{J}_{x0x1}, \boldsymbol{J}_{x0y0z0}, \boldsymbol{J}_{NE-TE-PG}$	Jacobian matrices

S_{PG1}, S_{PG2}	the executable strategies of PG
S_{NP1}, S_{NP2}	the executable strategies of NP
S_{EC1}, S_{EC2}	the executable strategies of EC
EPG_1, ENP_1, EEC_1 EPG_2, ENP_2, EEC_2	expected profits
EPG_{av}, ENP_{av}, EEC_{av}	average expected profits
$f_{PG}(x)$, $f_{NP}(y)$, $f_{EC}(z)$	rate-of-change of the proportion of individuals who select a pure strategy in the group of PGE, NPSE, and ECS with the time
$g_{PG1}(x)$, $g_{NP1}(y)$, $g_{EC1}(z)$, $g_{PG2}(y,z)$, $g_{NP2}(z,x)$, $g_{EC2}(x,y)$	multiplicative factor functions in the RD equations of the TM-AEG system
$\lambda_{\mathcal{X}}$	the eigenvalue numbered $\mathcal{X}$
$\mathrm{Re}_{\mathcal{X}}$	the real part of the eigenvalue numbered $\mathcal{X}$
r_1, r_2, r_3, r_4, s_1, s_2, s_3, s_4, t_1, t_2, t_3, t_4	intermediate parameters used for simplicity
J_{11}, J_{21}, J_{23}, J_{31}, J_{32}	elements in the Jacobian matrix $\boldsymbol{J}_{x0x1}$ which is obtained when $x=0$
J'_{11}, J'_{21}, J'_{23}, J'_{31}, J'_{32}	elements in the Jacobian matrix $\boldsymbol{J}_{x0x1}$ which is obtained when $x=1$
Δ_1, Δ_2, Δ_3, Δ_4	parameters used for a concise description
J''_{12}, J''_{13}, J''_{21}, J''_{23}, J''_{31}, J''_{32}	elements in the Jacobian matrix $\boldsymbol{J}_{x0y0z0}$
$g_{PG1}(x)$, $g_{PG}(y,z)$	multiplicative factor functions in the RD equation of the group of PGE
pg_0, pg_1, y_{fz}, y_{fm}	parameters calculated in situations for the group of PGE
S_{NEGE1}, S_{NEGE2}	the executable strategies of NEGE
S_{TEGE1}, S_{TEGE2}	the executable strategies of TEGE
S_{PGES1}, S_{PGES2}	the executable strategies of PGES
W_{NEGE1}, W_{NEGE2}	new energy generation outputs (mainly photovoltaic and wind power)
W_{TEGE1}, W_{TEGE2}	traditional fossil energy generation outputs (mainly thermal power)
W_{PGES1}, W_{PGES2}	accommodations of new energy generation by the power grid enterprises
$f_{NEGE}(u)$, $f_{TEGE}(v)$, $f_{PGES}(w)$	rate-of-change of the proportion of individuals who select a pure strategy in the group of NEGE, TEGE, and PGES with the time
σ_4, σ_5, σ_6, R_M, Q_M, T_M	intermediate parameters used for simplicity
$a'_{\mathcal{X}}$, $b'_{\mathcal{X}}$, $c'_{\mathcal{X}}$, $d'_{\mathcal{X}}$, $e'_{\mathcal{X}}$, $f'_{\mathcal{X}}$, $g'_{\mathcal{X}}$, $h'_{\mathcal{X}}$, $A'_{\mathcal{X}}$, $B'_{\mathcal{X}}$, $C'_{\mathcal{X}}$, $D'_{\mathcal{X}}$, $E'_{\mathcal{X}}$, $F'_{\mathcal{X}}$, $G'_{\mathcal{X}}$, $H'_{\mathcal{X}}$	new payoff parameters of the payoff matrix obtained in a TM-AEG system
NEAS	Nash equilibrium-based asymptotic stability
AEGs	asymmetric evolutionary games
EM	electricity market
MESS	multi-group evolutionary stable strategy
SGCs	state grid corporations
EC	electricity consumer
EV	electric vehicle
NPSE	new power supply entity
PGE	power grid enterprise
ECS	electricity consumers
LA	load aggregator
UT-AEG	unilateral two-group asymmetric evolutionary game
TM-AEG	trilateral multi-group asymmetric evolutionary game
RD	replicator dynamics
LST	Lyapunov stability theory
ASEP	asymptotically stable equilibrium point
ESE	evolutionary stable equilibrium
UEEP	unstable evolutionary equilibrium point
GenCo	generation corporation
GenCos	generation corporations
SSGC	small-sized GenCo groups
LSGC	large-sized GenCo groups
NEGE	individual sets of new energy generation enterprises
TEGE	the individual sets of traditional fossil energy generation enterprises
PGES	the individual sets of power grid enterprises
ESS	evolutionary stable strategy
TOU	time-of-use

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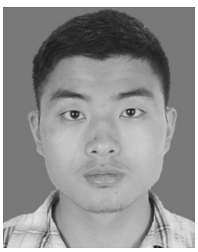

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