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# Optimal PMU Placement Considering Load Loss and Relaying in Distribution Networks

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**ABSTRACT** This paper studies the problem of optimal phasor measurement unit (PMU) placement considering the constraints of full system observability and load loss. The relaying functions of metering devices, such as feeder terminal units (FTUs) and dual-use line relays (DULRs), are taken into account to satisfy the maximum load loss coefficient limit (MLCL) and guarantee the operation of distribution system after the single-branch outage. A mixed-integer linear programming (MILP) is formulated to find the minimal load loss under the given configuration and branch outage with relaying functions of DULRs and pre-existing FTUs. A PMU deployment formulation considering the presence of metering devices and distributed generations (DGs) is solved by the genetic algorithm together with MILP. Various configurations of network and scenarios with different settings of conventional measurements and DGs are simulated in two test cases. Results show that the optimal PMU placement is greatly affected by the pre-existing traditional measurements and DGs as well as MLCL.

**INDEX TERMS** Distribution network, phasor measurement unit, system observability.

## NOMENCLATURE

### A. SETS AND INDICES

| | |
|---|---|
| $\boldsymbol{B}$ | Set of all buses (indexed by $i$ and $j$). |
| $\boldsymbol{L}$ | Set of all lines (indexed by $l$). |
| $\boldsymbol{L_f}$ | Set of lines installed with feeder terminal unit (FTU). |
| $\boldsymbol{E_f}$ | Set of fault lines (indexed by $k$). |
| $\boldsymbol{E_c}$ | Set of all configurations of system. |
| $\boldsymbol{E_{\cdot i}}$ | Set of lines oriented into bus $i$. |
| $\boldsymbol{E_{i\cdot}}$ | Set of lines oriented out of bus $i$. |
| $i_l$ | Head (bus no) of line $l$. |
| $j_l$ | Tail (bus no) of line $l$. |
| $l_{i-j}$ | Line from bus $i$ to bus $j$. |
| $i_c$ | Index of configuration of network. |

### B. PARAMETERS

| | |
|---|---|
| $n_b$ | Number of buses. |
| $n_{al}$ | Number of all lines. |
| $\omega_T$ | Installment cost for a traditional phasor measurement unit (TPMU). |
| $\omega_D$ | Installment cost for a dual-use line relay (DULR). |
| $a_i$ | Number of branches connected to bus $i$. |
| $p_{i,load}$ | Load demand at bus $i$. |
| $p^k_{i,DG}$ | Power generation of distributed generation (DG) at bus $i$ under the fault line $k$. |
| $p^k_{i,loss}$ | Load loss at bus $i$ under the fault line $k$. |
| $P^{max}_{i,DG}$ | The upper power limit of DG at bus $i$. |
| $F_{i,load}$ | Binary parameter that is 1 if bus $i$ is connected with load. |
| $F_{i,DG}$ | Binary parameter that is 1 if bus $i$ is connected with DG. |
| $F_{i,Grid}$ | Binary parameter that is 1 if bus $i$ is connected with substation. |
| $y_l$ | Binary parameter that is 1 if line $l$ is installed with FTU or DULR. |
| $p_{all}$ | Total load demand at all buses. |
| $v^{i_c}_k$ | Load loss coefficient under $i^{th}_c$ configuration of the network under the fault line $k$. |
| $\underline{v_k}$ | The minimal load loss coefficient of all configurations of network under the fault line $k$. |
| $\bar{v}$ | The maximum ratio of load loss that can be cut off from the total load demand. |
| $p^k_l$ | Power flow of line $l$ under the fault line $k$. |
| $X_l$ | Reactive resistance of line $l$. |
| M | Big-M value for power flow. |

$\overline{\theta}$ The upper limit of voltage angle.
$\underline{\theta}$ The lower limit of voltage angle.

### C. VARIABLES

$z$ Vector of state variables.
v Vector of measurement error variables.
$V_i$ Voltage magnitudes of bus $i$ in per unit form.
$\theta_i$ Voltage angle of bus $i$ in per unit form.
$\theta_i^k$ Voltage angle of bus $i$ in per unit form under the fault line $k$.
$h(\cdot)$ Nonlinear measurement function of state variable.
$f_i$ Binary variable that is 1 if bus $i$ is installed with TPMU.
$q_l$ Binary variable that is 1 if line $l$ is installed with DULR.
$x_l^k$ Binary variable that is 1 if line $l$ is switched on under the fault line $k$.
$z_i^k$ Binary variable that is 1 if bus $i$ is the fault bus which cannot be isolated from the fault line by the FTU or DULR under the fault line $k$.

## I. INTRODUCTION

Phasor measurement unit (PMU) is the current most advanced metering device of synchronized measurement technology which can provide real-time voltage and current synchrophasor measurements with high accuracy [1]. PMUs play an important role in state estimation, obtaining full system observability, protection and wide area control in power systems due to its great merits. It is unnecessary to install PMU at each bus in the power system since a PMU can make more than one bus observable. Besides, a full deployment of PMUs in the network is not feasible and realistic because of cost reasons. Optimal PMU placement (OPP) considering the minimal number of required PMUs to make systems full observable has been a specific problem over the past three decades.

Many papers only aimed to obtain full observability in the model of PMU placements, and it needs to cover almost a third of the buses in distribution grids. Thus, the cost of placement with sole PMUs can be significantly high due to the great density of PMUs. Therefore, all kinds of available measurements should be incorporated to achieve the observability of distribution networks. Conventional measurements like power flow measurements (PFMs) and injection measurements (IMs) which already exists in the supervisory control and data acquisition (SCADA) system can be used in the OPP model [2]–[7]. Therefore, maximal potential of the pre-existing metering devices could be fully utilized to enhance the power system monitoring performance. Such traditional measurements can be taken into consideration in the optimal placement model to achieve the most economic deployment of PMUs and satisfy full system observability. In [8], a semi-definite programming approach was presented to OPP problem considering the existences of zero injections and conventional measurements. The impact of PMU channel limits was also considered. In [9], both PMUs and PFMs were considered as decision variables simultaneously in the model, and the results showed that the required number of PMUs to make system full observable decreased significantly. However, these traditional measurements are normally fixed before the installments of PMUs. In [10], an MILP model was proposed to obtain full observability with and without conventional measurements respectively. The effects of zero injection buses (ZIBs), IMs and PFMs were also modelled in the formulation by using merging buses.

Besides the leading objective, trying to find the minimal number of PMU devices to obtain full system observability, there are some other pre-defined objectives or realistic constraints in the OPP model. In [11], the novel concept of depth of unobservability was introduced and the placement sites of PMUs could be determined by proposed method based on complete or incomplete observability in power system. In [12], two common contingencies such as single branch outage and single PMU outage were taken into account simultaneously.

Many researches aimed to locate the fault line with the deployment of PMUs [13]–[17]. Ren *et al.* [14] presented a new methodology for locating a fault in distribution systems using high accurate measurements provided by PMUs. In [16], a fault location method was proposed to accurately identify the location of a fault in distribution feeders after the optimal placement of PMUs which ensured the observability of distribution networks.

There is a novel PMU device, dual-use line relay (DULR) which considers the trends in relaying technologies in the future. DULRs are also called branch PMUs which have been discussed in [18] and [19]. DULRs are normally placed on lines, having functions of both measuring and relaying just like FTUs, whereas traditional phasor measurement units (TPMUs) are traditional PMUs which are installed at single bus to collect synchrophasor measurements. With relaying functions, DULRs can improve the reliability of distribution network by removing fault line when branch outage occurs. In [20], DULRs were discussed and modelled elaborately for observability and redundancy of PMU placement, and an MILP method was presented considering realistic costs and practical constraints in OPP model. However, relaying functions of DULRs when outage happens were not taken into consideration in [20].

In this paper, the load loss and reconfiguration after outage are considered with the relaying functions of metering devices in the OPP model. Firstly, both TPMUs and DULRs are used as decision variables in the proposed optimal model to satisfy the observability constraints. Next, observability constraints are defined with the incorporation of traditional measurements consisting of power injections measurements and power flow measurements. These measurements are acquired by pre-existing metering devices in SCADA system such as FTUs. And the presences of DGs could provide more operation options when outages occur and islanding events appear [21], [22].This paper manly focuses on the optimal location of PMUs and DULRs to realize the pre-defined functions. If the location of PMUs does not equipped with CT and VT, then they need to be installed with PMUs. Also, relaying functions of DULRs are taken into account in the constraints of OPP model. The protection actions of DULRs and FTUs can isolate fault line and ensure normal operation of distribution networks when single branch outage occurs. A fault isolation and reconfiguration algorithm is proposed to calculate the minimal load loss coefficient and determine the optimal reconfiguration after the outage. The load loss under any single possible branch outage is restricted within the maximum load loss coefficient limit (MLCL) by seeking the optimal reconfiguration after the outage.

The contribution of this paper can be summarized as follows:

(1) A novel OPP model is proposed with objective function of minimal installment cost and realistic constraints of system observability and MLCL after branch outage. Both TPMUs and DULRs are used as decision variables for full observability in the formulation.

(2) A MILP formulation is proposed to calculate the minimum load loss under the given configuration and branch outage, considering the relaying functions of DULRs and pre-existing FTUs.

(3) DGs are also considered in distribution network which can supply energy in the isolated area formed by DULRs or FTUs, providing more operation options when outage occurs.

This paper is organized as follows. In Section 2, the formulation of mathematical model and fault isolation and reconfiguration algorithm are illustrated. In Section 3, the case study of revised IEEE 33-bus test and revised PG&E 69-bus test and numerical results are presented. The conclusions are noted in Section 4.

## II. MATHEMATICAL FORMULATION OF OPTIMAL PMU PLACEMENT

### A. NUMERICAL METHOD FOR OBSERVABILITY

The primary constraint of OPP model is to satisfy the full observability of power system. In general, there are two main methods for solving observation problem, which are topological method and numerical method. Both of them are utilized in the three-phase balanced distribution systems with the conventional measurements [8]–[10], and this paper also uses numerical method for observability in the three-phase balanced distribution network. Numerical method requires sufficient network messages and great computation, it can guarantee the numerical observability required for successful execution of system state estimation which topological one can't [23]. Numerical method for observability is usually based on the information of weighted least squares (WLS) state estimation.

The measurement model for the WLS state estimation is given by:

$$\mathbf{z} = h(\mathbf{x}) + \mathbf{v} \tag{1}$$

where $\mathbf{x}$ is $(2n_b - 1) \times 1$ state vector, consisting of $n_b$ bus voltage magnitudes $V_i$ and $(n_b - 1)$ bus voltage angles $\theta_i$ with regard to the reference bus; $\mathbf{z}$ is $m \times 1$ measurement vector; $\mathbf{v}$ is $m \times 1$ measurement error vector, and $m$ is the number of measurements.

WLS state estimation problem is solved by following iterative equation:

$$\mathbf{G}(\mathbf{x}_u)(\mathbf{x}_{u+1} - \mathbf{x}_u) = \mathbf{H}^T(\mathbf{x}_u)\mathbf{R}^{-1}(\mathbf{z} - \mathbf{h}(\mathbf{x}_u)) \tag{2}$$

where $\mathbf{H}(\mathbf{x}) = \partial\mathbf{h}(\mathbf{x})/\partial\mathbf{x}$ is the $m\times(2n_b-1)$ Jacobian matrix; $\mathbf{R}^{-1}$ is the $m \times m$ weight matrix; $\mathbf{G}(\mathbf{x}) = \mathbf{H}^{\mathrm{T}}(\mathbf{x})\mathbf{R}^{-1}\mathbf{H}(\mathbf{x})$ is the gain matrix and u is the number of iteration.

The nullity of gain matrix $\mathbf{G}$ [24] is equal to:

$$nullity(\mathbf{G}) = (2n_b - 1) - rank(\mathbf{G}) \tag{3}$$

The power system is considered to be numerically observable when the nullity of gain matrix is 0. Network observability can be checked through rank of gain matrix $\mathbf{G}$.

### B. OBJECTIVE FUNCTION AND REALISTIC CONSTRAINTS

The proposed PMU placement scheme uses both TPMUs and DULRs as decision variables in the deployment model. Also, traditional measurements containing power flow measurements obtained by FTUs and power injection measurements are also taken into account. In addition, several network reconfigurations are considered in order to satisfy MLCL when single branch outage occurs.

The basic objective function based on above assumptions is defined as follows:

$$\min(\omega_T \sum_{i=1}^{n_b} a_i f_i + \omega_D \sum_{k=1}^{n_{al}} q_l) \tag{4}$$

The objective function includes the total installment cost of TPMUs and DULRs. The cost of a TPMU depends on its channel numbers which is the number of connected branches.

Realistic constraints are:

$$nullity(\mathbf{G}_{i_c}) = 0 \forall i_c \in \mathbf{E}_c \tag{5}$$

$$q_l = 0 \forall l \in \mathbf{L}_f \tag{6}$$

$$\underline{v_k} = \min_{i_c}\left(v_k^{i_c}\right) \le \bar{v} \forall k \in \mathbf{E}_f, \quad \forall i_c \in \mathbf{E}_c \tag{7}$$

Constraint (5) ensures the full observability of each configuration of the network. Constraint (6) means the lines installed with FTUs are not available for the installation of DULRs. Constraint (7) represents that the minimal load loss coefficient under any single branch outage needs to be less than MLCL, under the optimal reconfiguration and optimal operations of FTUs and DULRs. When the branch outage occurs, FTUs and DULRs should isolate the fault line, and the reconfiguration with the minimum load loss coefficient which is less than the specialized MLCL should be chosen during the given configurations of networks. To be clarified, the optimal PMU placement could guarantee the reconfiguration with minimal load loss coefficient which satisfy the MLCL under N-1 branch outage of the network.

A novel MILP algorithm is proposed to guarantee the constraint (7) under the single branch outage in the following part.

Numerical methodology is applied to ensure the full system observability of distribution grids in different configurations.

### C. FAULT ISOLATION AND RECONFIGURATION ALGORITHM

In this study, we propose a fault isolation and reconfiguration algorithm to isolate the fault line and to reconfigure the network after the outage to ensure the rest of system operate safely with minimal load loss coefficient under the single outage of the network, which is to guarantee the success of constraint (7). FTUs and DULRs take actions to keep the fault line apart when single branch outage occurs.

Two main assumptions are applied in this paper, which are introduced as follows:

*Assumption 1:* FTU or DULR is installed closer to the head of line. If branch outage occurs on line $l_{i-j}$ installed with FTU or DULR, then the bus $i_l$, the head of line $l_{i-j}$, can be isolated from fault with the protection of FTU or DULR.

*Assumption 2:* There are several specific configurations for the distribution network, and the reconfiguration can only happen within the given configurations. The configuration with minimal load loss coefficient among all the given configurations which is less than MLCL would be the optimal reconfiguration network after the fault.

When the branch outage occurs, FTUs and DULRs would try to isolate the fault line, and several reconfigurations would appear with different load loss coefficient considering several given configurations of networks. The one with minimum load loss coefficient which would be chosen as the optimal reconfiguration network after the fault. There would be more operation options for reconfiguration to satisfy the given MLCL under the single branch outage of network if great numbers of different configurations are given.

Here, fault bus is defined as the bus, which cannot be isolated from the fault by FTU or DULR.

If single branch outage occurs on line $k$, then tail of the line $k$, represented by $j_k$ is fault bus. Assumed that bus $t$ is connected with bus $j_k$ through line $l$, bus $t$ would also be fault bus if there is no FTU or DULR on line $l$. With the relaying functions of FTUs or DULRs, branches can be switched on or off to isolate the fault and the whole system could be separated into several sub-areas. The load loss within the formulated sub-areas can be summarized as follows:

1) The area containing the fault buses is denoted as fault sub-area, and the load within the fault area must be cut off.

2) The sub-area without fault bus and without DG or substation cannot provide power supply, so the load gets cut off.

3) The sub-area has no fault bus but has DG or substation, then the load loss needs to be calculated according to the power supply capacity and the network constraint.

Under a given fault line $k (k \in E_f)$ and the given configuration $i_c$, the load loss coefficient can be formulated as the following MILP.

Objective function:

$$v_k^{i_c} = \min(\sum_{i=1}^{n_b} p_{i,loss}^k)/p_{all} \tag{8}$$

Constraints:

$$z_{j_k}^k = 1 \tag{9}$$

$$x_l^k \geq (1 - y_l) \quad \forall l \in \mathbf{L} \tag{10}$$

$$\begin{cases} z_i^k - z_j^k \leq (1 - x_l^k) \\ z_i^k - z_j^k \geq -(1 - x_l^k) \quad \forall l \in \mathbf{L} \end{cases} \tag{11}$$

$$p_{i,loss}^k \geq z_i^k \times p_{i,load} \quad \forall i \in \mathbf{B} \tag{12}$$

$$F_{i,DG} \times p_{i,DG}^{\max} \geq p_{i,DG}^k \geq F_{i,DG} \times (1 - z_i^k) \times p_{i,DG}^{\max} \quad \forall i \in \mathbf{B} \tag{13}$$

$$-x_l^k \times p_l^{\max} \leq p_l^k \leq x_l^k \times p_l^{\max} \quad \forall l \in \mathbf{L} \tag{14}$$

$$p_{i,loss}^k + p_{i,DG}^k - \sum_{l \in \mathbf{E}_{i\cdot}} p_l^k + \sum_{l \in \mathbf{E}_{\cdot i}} p_l^k = F_{i,Grid} \times p_{i,Grid} + F_{i,load} \times p_{i,load} \quad \forall i \in \mathbf{B} \tag{15}$$

$$\underline{\theta} \leq \theta_i^k \leq \bar{\theta} \quad \forall i \in \mathbf{B} \tag{16}$$

$$p_l^k - M(1 - x_l^k) \leq \frac{\theta_{i_l}^k - \theta_{j_l}^k}{X_l} \leq p_l^k + M(1 - x_l^k) \quad \forall l \in \mathbf{L} \tag{17}$$

where $p_{all} = \sum_{i=1}^{n_b} F_{i,load} \times p_{i,load}$ is the total demand at all buses in the network. Objective function (8) is to determine the minimal load loss coefficient of $i_c^{th}$ configuration under the fault line $k$, with the protections of FTUs and DULRs. After the calculation the load loss coefficients of all the configurations, the optimal reconfiguration after the single outage $k$ could be determined. The proposed method focuses on the active power losses when calculating the load loss coefficient of distribution network. Therefore, DC power flow is used for network constraint in the optimal model in this paper for simplicity. The proposed algorithm is also available for the AC power flow when the losses of active and reactive power are taken into consideration.

Constraints are shown in (9)–(17). Assumed the line $k$ is the fault line, then Constraint (9) represents that the bus located on the tail of fault line is set to be fault bus. Constraint (10) establishes the relationship between the status of line and the installment of FTU or DULR on this line. Constraint (11) establishes the relationship between the status of each line and the type of buses connected to the line. Constraint (12) represents the lower limit of load loss of power at each bus. Constraint (13) ensures the power of DG at each bus is within the capacity limit. Constraint (14) ensures the power flow in each line is within the capacity limit. Constraint (15) ensures the power balance at each bus. Constraint (16) ensures the voltage angle of each bus is within the limit in per unit form. Constraint (17) represents

Kirchhoff's voltage law by using the Big-M method to create linear formulation.

The process of the proposed algorithm is explained through Figure. 1, which shows a simple 20-bus distribution system where bus 1 is connected to the substation, two DGs are connected to bus 16, 20 with the capacity of 300kW and 450kW, 17 loads are connected to the other buses with the same capacity of 100kW.

Assumed $l_{5-6}$ is the fault line as shown in Figure.1, then bus 6 is the fault bus. The type of buses connected with bus 6 can be determined by status of line according to constraint (10) and (11). Since $l_{5-6}$ and $l_{6-11}$ are lines without FTU or DULR, bus 5 and bus 11 is fault bus, so as bus 4. In order to isolate the fault line, $l_{3-4}$, $l_{4-17}$, $l_{6-7}$ and $l_{11-12}$ should be switched off and system would be separated into 5 areas to form the minimal fault area with relaying functions of FTUs and DULRs.

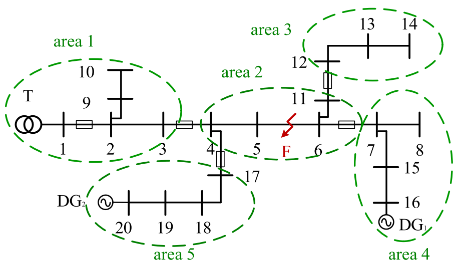


**FIGURE 1.** Example system illustrating the proposed algorithm.

To be specific, area 2 is the fault area, area 1, 4, 5 is supplied by substation, DG1 and DG2 respectively while area 3 is without any supply. The load loss of the whole system is the sum of load loss of these 5 areas. The load loss of Area 2 and 3 is 400kW and 300kW respectively, which is the total loads in the area due to the shortage for supply. Area 1 and 5 have no load loss since the substation or DG can supply enough energy for the area. The load loss of area 4 is 100kW since DG2 could not supply enough energy for other loads in the area. Total load loss of the system is 800 kW and load loss coefficient is approximately 47.1%.

### D. OPTIMIZATION PROCESS

A genetic algorithm is employed to perform the optimization of deployment of PMUs and DULRs since it is too exhaustive for searching all possible placement configurations. The application of GA can reduce the calculation of optimization efficiently and try to find better objective functions results. The genetic algorithm uses evolution of a population through initialization, selection, genetic operators and termination to obtain the optimal solutions. The critical step of the algorithm is the fault isolation and reconfiguration part which uses MILP to find the available PMU deployments, and the most time consumption step is the step using numerical method to make networks full observable. The process of the optimization based on genetic algorithm utilized in this paper is shown in Figure. 2.

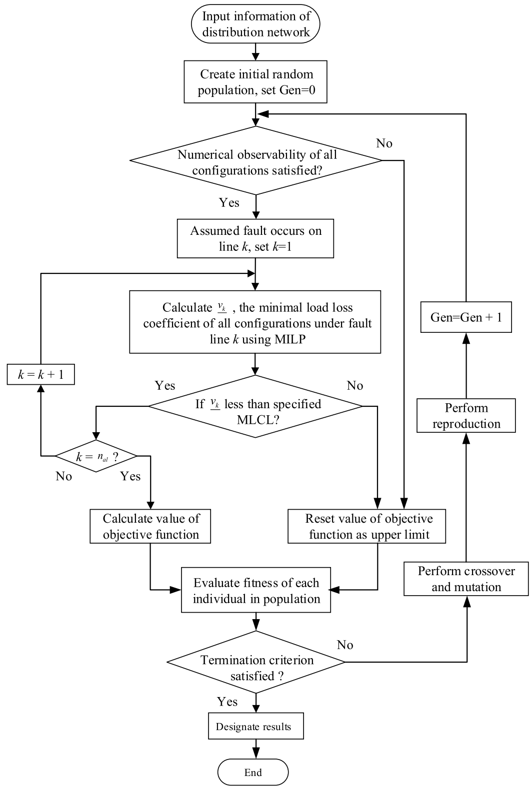


**FIGURE 2.** Block diagram of proposed genetic algorithm.

*Step 1:* Input the information of distribution network, create the initial population and set the maximal number of generation, set generation number as $Gen = 0$;

*Step 2:* Calculate the nullity of gain matrix of each configuration. If all configurations are numerical observable, then turn to Step 3; otherwise, turn to Step 7;

*Step 3:* Assumed a fault occurs on the line k, and set $k = 1$;

*Step 4:* Calculate $\underline{v_k}$, the minimal coefficient of load loss of all configurations using proposed MILP algorithm. If $\underline{v_k}$ is less than the MLCL, then turn to Step 5; otherwise turn to Step 7;

*Step 5:* If the number of assumed fault line, k is less than $n_{al}$, the number of all lines, which means the assumed fault does not cover all the lines, then set $k = k + 1$, turn to Step 4; otherwise, all lines have been assumed to be fault lines, turn to Step 6.

*Step 6:* Calculate the value of objective function, then turn to Step 501; otherwise, turn to Step 7;

*Step 7:* Reset the value of objective function as upper limit, then turn to Step 8;

*Step 8:* Evaluate fitness of each individual in population, if generation number equals the maximal number of generation, then output the results; otherwise perform crossover, mutation and reproduction, set $Gen = Gen + 1$, then turn to Step 2;

## III. CASE STUDIES

Two balanced test distribution systems are applied to evaluate the performance of proposed algorithm in this paper, namely revised IEEE 33-bus test system [25] and revised PG&E 69-bus test system [25], [26]. Balanced distribution systems are under the use of test for simplicity while proposed method in this paper can also be applied in unbalanced distribution systems where PMUs can measure the current of unbalanced lines. Since these two test networks are systems with single power supply, single branch outage which occurs on the lines directly connected to substation are not taken into consideration. These lines are defined as critical lines which could not be set as fault lines in the tests. The rest of lines including tie lines are set of branches which can set with outage individually. However, all lines are available for single branch outage in systems with several power supplies. Also the proposed fault isolation and reconfiguration algorithm is practicable when outage occurs in systems with multi power supplies.

These two distribution test networks are connected with several DGs with different capacities. The impact of DGs is analyzed by considering various capacities and different numbers of DGs. The importance of considering traditional measurements is analyzed by comparing optimal PMU deployment with different numbers and placements of preexisting metering devices such as FTUs. Also, the impact of MLCL is taken into account in the simulation, by setting to be 0.15 and 0.25, respectively. For simplicity, the cost of a DULR is assumed to be equal to the cost of single channel of a TPMU. The cost of a single DULR device is set to be \$2000 according to [20], and the cost of a TPMU depends on its channel capacities which corresponds the number of branches connected to the TPMU.

All programs were solved by MATLAB 2016b, on a Xeon E3-1230 3.30-GHz personal computer with 8G memory.

**TABLE 1.** Settings of scenarios for revised IEEE 33-bus test.

| Scenario | Number of FTUs | Number of IMs | Number of DGs |
|---|---|---|---|
| S1 | 10 | 10 | 2 |
| S2 | 5 | 10 | 2 |
| S3 | 10 | 10 | 5 |
| S4 | 5 | 10 | 5 |

### *A. REVISED IEEE 33-BUS TEST SYSTEM*

The test case is based on a modified version of IEEE 33-bus test network which consists of 32 distributions lines and 5 tie lines. Several possible network operating configurations could be obtained by these lines in the network. Four possible network configurations after branch outage are simulated for revised IEEE 33-bus test network, as follows:

Config.1: open lines 7-20, 8-14, 11-21, 17-32, 24-28;

Config.2: open lines 6-7, 8-9, 13-14, 24-28, 31-32;

Config.3: open lines 6-7, 8-9, 13-14, 27-28, 30-31;

Config.4: open lines 6-7, 8-9, 12-13, 27-28, 17-32;

Specifically, line 1-33 and line 1-2 are the critical lines which could not be fault line because no placement would satisfy the MLCL when branch outage occurs on them.

To analyse the influence of DGs and conventional measurements to the optimal deployment of PMU, four scenarios considering different settings are simulated in the case study as follows:

To be specific, two DGs are connected to Bus 14 and 30 in Scenario 1 and 2 whereas five DGs are connected to Bus 6, 14, 21, 24 and 30 in Scenario 3 and 4. In the case study, all buses are available for TPMUs whereas lines without FTUs are candidate lines for DULRs.

#### 1) IMPACT OF CONVENTIONAL MEASUREMENTS AND DGs

The optimal PMU placements of revised IEEE 33-bus test network under different MLCL in four scenarios are presented in Table 2. Taken Scenario 2 with 25% load loss coefficient limit as an example, it only needs 6 DULRs and 4 TPMUs to make network full observable, costing \$28,000 in total. The optimal locations of DULRs and TPMUs are marked in red in Figure 3. The dashed lines are tie lines depicted in Figure 3.

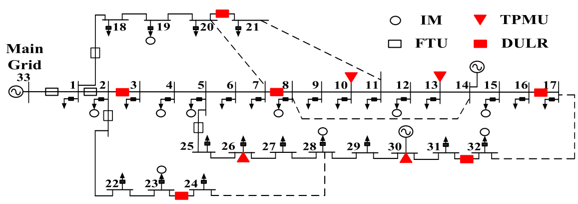


**FIGURE 3.** Location of PMUs with 15% load loss coefficient limit in Scenario 2 for revised IEEE 33-bus test.

Under 15% load loss coefficient limit, there is no feasible placement in Scenario 1 and 2 while placement could be obtained in Scenario 3 and 4 as the number of DGs increases from 2 to 5. The reason for no solution in Scenario 1 and 2 under 15% load loss coefficient limit is that the minimum load loss coefficient of power system is 23.26%, 19.64% and 16.54% in Configuration 3 and 4 when single branch outage occurs on line 2-3, 3-4 and 4-5 respectively. With the addition of DGs installed in Bus 6, 21 and 24, the MLCL of power system can be satisfied and the locations of TPMUs and DULRs can be found in Scenario 3 and 4.

As shown in Table 2, compared results in Scenario 3 with results in Scenario 4 under 25% load loss coefficient limit, the required number of TPMUs decreases from 4 to 3 and the total installment cost declines by \$4,000, from \$26,000 to \$22,000 with three more DGs in Scenario 4. The results prove that the increased number of DGs can reduce the required number of TPMUs and DULRs.

Compared results in Scenario 2 with results in Scenario 1 and results in Scenario 4 with results in Scenario 3 respectively, the number of TPMUs and DULRs and installment cost decrease efficiently with the same number of IMs and DGs as the increased number of FTUs. For example, compared results in Scenario 4 with results in Scenario 3 under 15% load loss coefficient limit and 3, the number of both DULRs and TPMUs decreases by 1 and the cost declines from \$30,000 to \$24,000. So it is quite critical to consider

**TABLE 2. Optimal PMU placement under different MLCLs and scenarios for revised IEEE 33-Bus test.**

| Scenario | 15% load loss coefficient limit | | 25% load loss coefficient limit | |
|---|---|---|---|---|
| | Location of PMUs | Installment Cost (in USD) | Location of PMUs | Installment Cost (in USD) |
| S1 | No Placement | N/A | 3TPMUs:13,26,30<br>6DULRs:2-3,6-7,9-10,16-17,20-21,23-24 | 24,000 |
| S2 | No Placement | N/A | 4TPMUs: 10,13,26,30<br>6DULRs:2-3,7-8,16-17,20-21,23-24,31-32 | 28,000 |
| S3 | 3TPMUs:13,26,30<br>6DULRs: 2-3,7-8,9-10,16-17,20-21,22-23 | 24,000 | 3TPMUs:13,26,30<br>5DULRs:7-8,9-10,16-17,20-21,22-23 | 22,000 |
| S4 | 4TPMUs:10,13,26,31<br>7DULRs:2-3,7-8，8-14,16-17,20-21,22-23,29-30 | 30,000 | 4TPMUs:13,17,21,30<br>5DULRs: 4-5,7-8,9-10,23-24,26-27 | 26,000 |

the presence of conventional measurements in the problem of PMU deployment which enhances observability of system and decrease the required number of PMUs.

### 2) IMPACT OF MAXIMUM LOAD LOSS COEFFICIENT LIMIT

To analyse the impact of MLCL, tests under two MLCLs (15% and 25%) are simulated individually. As shown in Table 3, it can be seen that as the increase of MLCL, the required number of DULRs and TPMUs are reduced. Particularly in Scenario 1 and 2 in Table 2, the results show that there is no available PMU placement to satisfy 15% load loss coefficient limit whereas optimal locations of DULR and TPMU can be found under 25% load loss coefficient limit. The minimal load loss coefficients under fault line 2-3, 3-4 and 4-5 are 23.26%, 19.64% and 16.54% under Configuration 3 and 4 individually, resulting in no feasible placement under 15% load loss coefficient limit in Scenario 1 and 2. As MLCL increases up to 25%, optimal PMU placement can be found in all scenarios shown in Table 2 and the installment cost declines efficiently. For example, the installment cost of PMUs decreases from 15 to 13 in Scenario 4 compared results under 25% load loss coefficient limit with one of 15% load loss coefficient limit. So more TPMUs and DULRs are required when the system has restrict MLCL.

**TABLE 3. Settings of scenarios for revised PG&E 69-bus test.**

| Scenario | Number of FTUs | Number of IMs | Number of DGs |
|---|---|---|---|
| S1 | 20 | 20 | 2 |
| S2 | 10 | 20 | 2 |
| S3 | 20 | 20 | 6 |
| S4 | 10 | 20 | 6 |

## B. REVISED PG&E 69 TEST SYSTEM

This test case is based on a revised PG&E 69-bus test network which consists of 68 distributions lines and 5 tie lines. Similar to revised IEEE 33-bus test network, four possible network configurations after branch outage are simulated as follows:

Config.1: open lines 10-65, 12-19, 14-68, 26-53, 38-47;
Config.2: open lines 13-14, 42-43, 49-50, 10-65, 12-19;
Config.3: open lines 5-6, 12-19, 13-14, 18-19, 43-44;
Config.4: open lines 9-10, 8-41, 16-17, 10-65, 22-23;

The critical lines in this case study are line 1-2 and line 1-69 which could not set as fault lines. Four scenarios considering different settings of DGs and conventional measurements are shown in Table 3.

To be specific, two DGs are connected to Bus 47 and 63 in Scenario 1 and 2 and six DGs are connected to Bus 6, 20, 31, 47, 63 and 67 in Scenario 3 and 4 respectively. Among the four scenarios of the case study, all buses are available for TPMUs whereas lines without FTUs are candidate lines for DULRs.

### 1) IMPACT OF CONVENTIONAL MEASUREMENTS AND DGs

Similar with the case study of the revised IEEE 33-bus test system, various scenarios consisting of different numbers of conventional measurements and DGs are implemented. Optimal PMU deployments are listed out in Table 4. Taken Scenario 1 with 25% load loss coefficient limit as an example, it only needs 10 DULRs and 13 TPMUs to obtain the full observability, costing $62,000 totally. The optimal locations of DULRs and TPMUs are marked in red in Figure. 4.

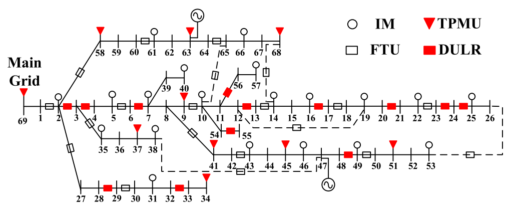


**FIGURE 4. Location of PMUs with 25% load loss coefficient limit in Scenario 1 for revised PG&G 69-bus test.**

By comparison results in Scenario 2 with results in Scenario 1 and results in Scenario 4 with results in Scenario 3 respectively, as the increase of number of conventional measurements, the corresponding installment cost is decreased correspondingly. With four more DGs connected in the network under 25% load loss coefficient limit, the installment cost decreases by $4,000 efficiently. The results prove that the addition of DGs also helps to decline the total cost of PMUs.

**TABLE 4.** Optimal PMU placement under different MLCLs and scenarios for revised PG&E 69-Bus test.

| Scenario | 15% load loss coefficient limit | | 25% load loss coefficient limit | |
|---|---|---|---|---|
| | Location of PMUs | Installment Cost (in USD) | Location of PMUs | Installment Cost (in USD) |
| S1 | No placement | N/A | 10 TPMUs: 9, 34, 37, 41, 45, 51, 58, 63, 68, 69<br>13 DULRs: 2-3, 3-4, 6-7, 11-56, 12-13, 16-17, 20-21, 23-24, 24-25, 28-29, 32-33, 48-49, 54-55 | 62,000 |
| S2 | No placement | N/A | 10 TPMUs: 1, 18, 24, 42, 44, 51, 55, 57, 63, 67<br>14 DULRs: 2-3,4-5,6-7,8-9,10-11,12-13,14-15,20-21,28-29,30-31,33-34,37-38,47-48,59-60 | 64,000 |
| S3 | 8 TPMUs: 20, 23, 29, 34, 45, 55, 56, 60<br>16 DULRs: 1-69,2-3,4-5,7-39,8-9,12-13,15-16,24-25,32-33,36-37,41-42,47-48,50-51,52-53,63-64,67-68 | 60,000 | 8 TPMUs: 8,24,28,33,40,50,55,69<br>15 DULRs: 2-3,3-4,11-56,12-13,15-16,20-21,37-38,41-42,44-45,45-46,48-49,52-53,59-60,63-64,67-68 | 58,000 |
| S4 | 8 TPMUs: 17,20,33,42,44,55,64,69<br>17 DULRs: 2-3,4-5,7-39,8-9,11-56,12-13,23-24,25-26,28-29,30-31,37-38,47-48,50-51,51-52,58-59,61-62,67-68 | 62,000 | 10 TPMUs: 1,8,24,34,37,40,51,55,57,60<br>13 DULRs: 2-3,3-4,11-12,12-13,16-17,20-21,27-28,30-31,41-42,44-45,47-48,63-64,67-68 | 60,000 |

### 2) IMPACT OF MAXIMUM LOAD LOSS COEFFICIENT LIMIT

The optimal placements of PMUs under two MLCLs are presented in Table 4. Similar with the results in the case study of revised IEEE 33-bus system, there is no placement under 15% load loss coefficient limit in Scenario 1 and 2 due to excessive load loss on fault line 2-3, 2-58 and 58-59.

The minimum coefficient of load loss under these fault lines are 23.13%, 19.87% and 18.37% under Configuration 3 and 4 respectively. With the increase of MLCL, the optimal placement can be found in Scenario 1 and 2 under 25% load loss coefficient limit and the total cost is decreased remarkably as shown in Table 4.

## IV. CONCLUSIONS

In this paper, we studied the problem of optimal PMU placement with both DULRs and TPMUs, considering constraints of full observability of distribution system and the MLCL after single branch outage. The relaying functions of both DULRs and FTUs are also taken into account when single branch outage occurs. The minimum load loss under the given reconfiguration and specific branch outage was formulated as a MILP problem, considering the relaying function of existed FTUs and applied DULRs.

Results of two case studies show that the optimal PMU placement is highly affected by presence of conventional measurements especially the number of FTUs. The investment cost of PMU placement can be reduced efficiently considering the traditional measurements obtained by pre-existing metering devices. In addition, MLCL influences the deployment of PMUs. Hence, the proposed approach provides an optimal deployment considering the relaying of DULRs and FTUs to satisfy the MLCL to the OPP problem.

However, zero injection buses are not considered in the OPP model in this paper. In future, we will focus on the effects of zero injection buses in the DSSE to reduce the number of PMUs and the cost of installment.

Several laboratories such as Power Standards Lab have devoted to developing a novel powerful micro-phasor measurement unit which has diagnostic and control applications that appear promising in the future. With the development of PMU, the price of PMU and its communication devices will decline and it is expected to apply PMUs in distribution level with acceptable cost.

## REFERENCES


[1] J. De La Ree, V. Centeno, J. S. Thorp, and A. G. Phadke, "Synchronized phasor measurement applications in power systems," *IEEE Trans. Smart Grid*, vol. 1, no. 1, pp. 20–27, Jun. 2010.

[2] S. Chakrabarti, E. Kyriakides, and D. G. Eliades, "Placement of synchronized measurements for power system observability," *IEEE Trans. Power Del.*, vol. 24, no. 1, pp. 12–19, Jan. 2009.

[3] N. H. Abbasy and H. M. Ismail, "A unified approach for the optimal PMU location for power system state estimation," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 806–813, May 2009.

[4] F. Aminifar, A. Khodaei, M. Fotuhi-Firuzabad, and M. Shahidehpour, "Contingency-constrained PMU placement in power networks," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 516–523, Feb. 2010.

[5] S. Azizi, G. B. Gharehpetian, and A. S. Dobakhshari, "Optimal integration of phasor measurement units in power systems considering conventional measurements," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 1113–1121, Jun. 2013.

[6] M. Esmaili, K. Gharani, and H. A. Shayanfar, "Redundant observability PMU placement in the presence of flow measurements considering contingencies," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3765–3773, Nov. 2013.

[7] A. Pal, C. Mishra, A. K. S. Vullikanti, and S. S. Ravi, "General optimal substation coverage algorithm for phasor measurement unit placement in practical systems," *IET Gener. Transmiss. Distrib.*, vol. 11, no. 2, pp. 347–353, Jan. 2017.

[8] N. M. Manousakis and G. N. Korres, "Optimal PMU placement for numerical observability considering fixed channel capacity—A semidefinite programming approach," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 3328–3329, Jul. 2016.

[9] R. Kavasseri and S. K. Srinivasan, "Joint placement of phasor and conventional power flow measurements for fault observability of power systems," *IET Gener. Transmiss. Distrib.*, vol. 5, no. 10, pp. 1019–1024, Oct. 2011.

[10] B. Gou, "Optimal placement of PMUs by integer linear programming," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1525–1526, Aug. 2008.
[11] R. F. Nuqui and A. G. Phadke, "Phasor measurement unit placement techniques for complete and incomplete observability," *IEEE Trans. Power Del.*, vol. 20, no. 4, pp. 2381–2388, Oct. 2005.
[12] K. G. Khajeh, E. Bashar, A. M. Rad, and G. B. Gharehpetian, "Integrated model considering effects of zero injection buses and conventional measurements on optimal PMU placement," *IEEE Trans. Smart Grid*, vol. 8, no. 2, pp. 1006–1013, Mar. 2017.
[13] S. P. Pokharel and S. Brahma, "Optimal PMU placement for fault location in a power system," in *Proc. North Amer. Power Symp.*, Oct. 2009, pp. 1–5.
[14] J. Ren, S. S. Venkata, and E. Sortomme, "An accurate synchrophasor based fault location method for emerging distribution systems," *IEEE Trans. Power Del.*, vol. 29, no. 1, pp. 297–298, Feb. 2014.
[15] Q. Jiang, X. Li, B. Wang, and H. Wang, "PMU-based fault location using voltage measurements in large transmission networks," *IEEE Trans. Power Del.*, vol. 27, no. 3, pp. 1644–1652, Jul. 2012.
[16] A. Rajeev, T. S. Angel, and F. Z. Khan, "Fault location in distribution feeders with optimally placed PMU's," in *Proc. Int. Conf. Advancements Power Energy*, Jun. 2015, pp. 438–442.
[17] S. S. Geramian, H. A. Abyane, and K. Mazlumi, "Determination of optimal PMU placement for fault location using genetic algorithm," in *Proc. Int. Conf. Harmon. Quality Power*, Sep./Oct. 2008, pp. 1–5.
[18] O. Gómez, C. Portilla, and M. A. Ríos, "Reliability analysis of substation monitoring systems based on branch PMUs," *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 962–969, Mar. 2015.
[19] R. Emami and A. Abur, "Robust measurement design by placing synchronized phasor measurements on network branches," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 38–43, Feb. 2010.
[20] A. Pal, A. K. S. Vullikanti, and S. S. Ravi, "A PMU placement scheme considering realistic costs and modern trends in relaying," *IEEE Trans. Power Syst.*, vol. 32, no. 1, pp. 552–561, Jan. 2017.
[21] K. Narayanan, S. A. Siddiqui, and M. Fozdar, "Identification and reduction of impact of islanding using hybrid method with distributed generation," in *Proc. Power Energy Soc. Gen. Meeting*, Jul. 2015, pp. 1–5.
[22] K. Narayanan, S. A. Siddiqui, and M. Fozdar, "Hybrid islanding detection method and priority-based load shedding for distribution networks in the presence of DG units," *IET Gener. Transmiss. Distrib.*, vol. 11, no. 3, pp. 586–595, Feb. 2017.
[23] N. M. Manousakis and G. N. Korres, "An advanced measurement placement method for power system observability using semidefinite programming," *IEEE Syst. J.*, to be published, doi: 10.1109/JSYST.2017.2688470.
[24] J. G. Mcdonald, *Introduction to Matrix Computations*. San Francisco, CA, USA: Academic, 1973.
[25] M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Trans. Power Del.*, vol. 4, no. 2, pp. 1401–1407, Apr. 1989.
[26] M. E. Baran and F. F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Trans. Power Del.*, vol. 4, no. 1, pp. 725–734, Jan. 1989.

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