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Joint Energy Management and Interference Coordination With Max-Min Fairness in Ultra-Dense HetNets

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ABSTRACT Interference management and energy management are two important issues in ultra-dense heterogeneous cellular networks (HetNets). However, load balancing investigated for system capacity on interference coordination is not efficient for energy saving in HetNets. Meanwhile, the maximum energy efficiency configuration leads to the serious unfair problem for users associated with larger power node for the ultra-dense scenario. Thus, in this paper, we investigate energy consumption jointly together with interference coordination for ultra-dense HetNets, and formulate max-min energy-efficient enhanced inter-cell interference coordination configuration problem. Due to the non-smooth and mixed programming nature of this formulation, we propose a novel iterative and distributed algorithm to solve the problem by using fractional programming and Lagrangian dual theory. The simulation results verified the effectiveness of our proposed algorithm and fairness achieved for energy efficiency of users, it especially identified a new energy efficiency tradeoff between macro user and small cell user with interference coordination in ultra-dense HetNets.

INDEX TERMS Enhanced inter-cell interference coordination (eICIC), energy efficiency (EE), max-min fairness, load balancing, ultra-dense HetNets.

I. INTRODUCTION

The dense heterogeneous cellular network (HetNet) architecture is a promising solution to meet prolific growth of wireless traffic demand, where macrocells are overlaid with a set of low power cells (e.g. picocell or femtocell) [1]. Since the large power disparities for different cell (e.g. base station (BS)) types in HetNets, macro can cover much larger region than the small cell so that the macro would be overloaded and the amount of users accessed into the small cells is restricted. Offloading UEs into small cell are more important

for improving the capacity of HetNet, which is named load balancing [2]. However, the rate of offloaded UEs in small cells could be dropped significantly by serious interference produced by the higher power macro, which limits the application of cell-specific dense small cells deployment scenario in 5G systems [3].

In order to mitigate this interference for small cell from macro, enhanced inter-cell interference coordination (eICIC) techniques keep macro transmission silent in certain air time, referring to almost blank subframes (ABSs) [4]. There are

two main characteristics in eICIC. First, by determining user association with pico or macro, one can ensure that the macrocell and small cell are not overutilized or underutilized. Second, the macro can mute downlink transmissions in ABS, so that the dense small cell can transmit at a higher throughput with much less interference.

Unfortunately, the only ABS allocation cannot reduce cross-tier interference significantly when the deployment of small cell is dense [5], and the ABS allocation is tightly coupled with user association for the eICIC configuration problem. Some researches [6]–[8] mainly focused on different dynamic ABS configuration schemes with load balance improving the network throughput but neglect the energy efficiency of eICIC configuration. The work [9] revealed that per-tier biasing set for user association is not efficient for energy saving in HetNets. This means that the user association of eICIC for energy saving is quite different from that for load balancing investigated for system capacity on interference coordination. Energy consumption should be considered jointly together with interference coordination in HetNets [10].

Moreover, the maximum energy efficiency of eICIC configuration lead to the serious unfair problem between macrocell user and small cell user in ultra-dense HetNets [11]. There are two main reasons: firstly, the maximum energy efficiency of the entire network only benefits users having the good channel gain, and so the enhancement of network energy efficiency results from the cost of users having the bad channel gain [12]; secondly, for dense small cell in HetNets, most ABSs are allocated to small cell, the user can only be coverage by macrocell which obtains the lower rate [13]. Therefore, how to set the EE-eICIC parameters with fairness, i.e., energy efficiency of joint UE association, air time resources and ABS optimization in fair, is important and left unspecified in ultra-dense HetNets. Our goal is to promote the system capacity and energy efficiency while keeping the energy efficiency of the worst-case user for eICIC configuration in ultra-dense HetNets. In this work, we present an energy-efficient of joint UE association, air time resources and ABS optimization algorithm for ultra-dense HetNets from an max-min optimization-theoretic point of view.

The main contributions of this paper are from both problem formulation and algorithmic aspects.

- Design a novel energy efficient jointly together with interference coordination for ultra-dense HetNets optimization framework: This is a new approach by considering energy efficiency of the worst user maximization, interference coordination, load balancing and eICIC in the design of ultra-dense HetNets optimization framework. So we can promote the capacity and energy efficiency of system while keeping the performance of the larger power node user in ultra-dense HetNets. We formulate the UE association, air time resources and ABSs allocation problem in ultra-dense HetNets as a mixed-integer programming problem.

- Make use of interference mitigation for energy saving in ultra-dense scenario: The interference mitigation, which combines cross-tier interference mitigation and inner-tier interference mitigation, is considered in the energy efficient optimization problem. The cross-tier interference mitigation is adaptive to time domain coordination based on macro-to-pico interference graph in order to ABSs allocation. Moreover, the inner-tier interference mitigation is also taken into consideration in the design of the energy efficient UE association and air time resource allocation optimization for energy saving in ultra-dense small cell networks.
- Develop a max-min energy efficient eICIC configuration algorithm with multiple constraints: Setting one UE associated with macro and pico at the same time and relaxing ABSs constraints, the EE-RELAXED-eICIC optimization problem in fractional form is transformed into subtractive form. We propose an distributed energy efficient eICIC configuration algorithm to solve the transformed optimization problem. The non-convex relaxed optimization problem is then solved in an alternating optimal manner, which is provably within guaranteed convergence and computational complexity. Our rounding algorithm can be assured within a constant gap of the optimum.

The paper is organized as follows. Section II reviews the related works. Section III provides the interference model of ABS protocol in HetNets. Section IV constructs the max-min EE-optimization formulation for user downlink association and ABS allocation. In sections V, we develop an iterative-distributed method to solve the optimization problem. Experimental results are shown in Section VI. We conclude this paper in Section VII.

II. RELATED WORKS

Recently, joint UE association and ABS optimization has attracted considerable interesting in eICIC configuration in HetNets. In this section, we review related work in two categories: optimization model and optimization algorithm for joint UE association and ABS optimization.

A. OPTIMIZATION MODEL OF eICIC CONFIGURATION

Deb *et al.* [4] proposed a maximum weighted proportional fairness of users' rate optimization framework for joint user association and ABS. The user association and ABS ratio issues were jointly formulated as a network-wide logarithmic utility function achieving load balancing and a sort of proportional fairness [14], [15] and as max-min fairness optimization problem of users service rate [16]. In work [17], the adaptive eICIC configuration problem was model as a general optimization formulation with regularization to adjust the bias of small cell range expansion (REB) and the ratio of ABSs with multiple coexisting network services. Liu *et al.* [18] proposed a logarithmic utility of network sum rate framework for both eICIC and FeICIC optimizations in LTE-A HetNets. Reference [19] investigated the network

stability for jointly considering resource allocation and random traffic loads for eICIC in HetNets by formulating a stochastic optimization problem. These works [4], [14], [16]–[18] mainly focused on the capacity of system and rate fairness of each users for eICIC in HetNets, which were not efficient for energy saving in HetNets.

Furthermore, an adaptive energy-efficiency ABS configuration scheme and power control problem was proposed to configure proper ABS ratio according to the practical load [5]. The work [20] formulated the trade off model between the delay and energy consumption of traffic with a distributed energy and delay aware user-BS associations scheme in HetNets. The work [21] established a model of stabilize the network and minimize the delay to optimize jointly user association and spectrum allocation. However, [5], [20], [21] only investigated the energy efficiency of the entire system for ICIC, and were not fit for eICIC configuration and did not take fairness of energy efficiency among users into account [22].

B. OPTIMIZATION ALGORITHM OF eICIC CONFIGURATION

The joint user association and ABS problem is nominally combinatorial. Deb *et al.* [4] developed a novel RELAX-ROUND method to deal with these two coupled configuration jointly. The first step was to handle the relaxed optimization problem, and then to compute the feasible results by rounding the output of the relaxed optimization problem. In work [14], assuming that users can receive resource from two or more base stations (BSs), so the problem became convex, and the solution obtained was upper bounds of the original problem versus a binary association. And the jointly REB and ABSr [17] was solved by the alternating direction method of multipliers, but it needed a general formulation with regularization. With the Lyapunov optimization technique, the work [19] proposed an optimal delay-aware resource allocation and the ratio of ABS configuration method dynamically. Reference [18] proposed a distributed cake-cutting algorithms based on game theory, but it didn't consider the ultra-dense scenarios.

In addition, some works [5], [15], [16] separated the coupled problem to solve the UE association and ABS optimization, respectively. In [5], the association of UE was firstly predetermined with signal to interference plus noise ratio (SINR) between macro and pico in the downlink, and then based on game-theory a utility function of macro's power control can be obtained. In work [16], they solved firstly the optimal ABS ratio problem for given user association, then maximized network utility through user association. By presetting favorable pico operation mode, the optimal UEs association are determined by bias values for each base station [15].

Since the separated method cannot obtain the better solution for the coupled UE association and ABS optimization problem, while these schemes [4], [14], [17], [19] cannot be directly used to solve the max-min energy efficiency optimization problem for eICIC in HetNets. In our approach,

we inspired the relaxed-rounding idea of Deb *et al.* [4], and propose an iterative-relaxed-rounding distribution algorithm to solve the new problem. From what we discussed, few researchers have carried out energy efficiency optimization for eICIC with max-min fair using non-convex distribution optimization in ultra-dense HetNets.

III. SYSTEM MODEL

This paper focuses on a two-layers HetNets system for Time Division Long Term Evolution (TD-LTE), where macrocell is overlapped by small cell in the co-channel deployment scenario. The macrocell and small cell is interference coordination with the subframe and ABS for eICIC configured dynamically. The subframe assignment and user accessing can be carried out in eNodeB for an ABS period. UE denotes the user equipment (e.g. mobile terminal).

User Model: Our network model contains multiple macro and micro BSs in two-layer HetNets. For a UE downlink association, the UE can choose one BS to associate between macro and pico (i.e., only one macro or one pico, but not both). It is assumed that the BS transmits the maximum power and UE accesses with the BS referring to the reference signal received power (RSRP) from all the BSs. The best candidate BS associated of macro and pico are ascertained in terms of RSRP across the overall bandwidth. We consider low speed mobile user, whose channel conditions can be assumed to be slowly changing. Therefore, the eICIC dynamic configuration is valid and effective according to the obtained semi-static channel state information (CSI).

Interference Model: In order to realize the eICIC interference management, it is necessary to divide downlink interference into three types in two-tier HetNets. It is shown in Fig. 1 and described as follow. The macro-to-macro interference is produced by 1:1 frequency reuse in heterogeneous cellular network. For two-layer TD-LTE, macros and picos are deployed in the same bandwidth, called macro-to-pico interference. Picos interfere with each other in the same macrocell, named pico-to-pico interference.

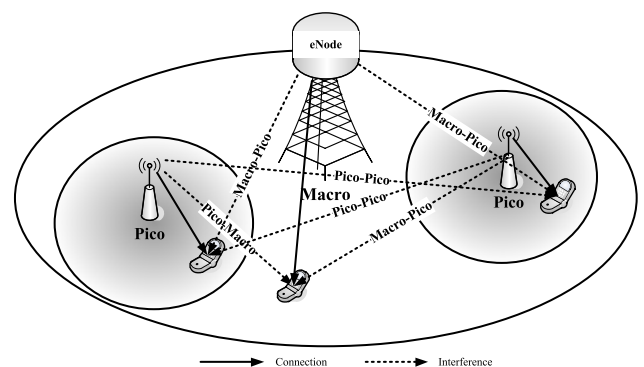


FIGURE 1. Interference model for downlink transmission in HetNets.

eICIC ABS: The domain of time eICIC configuration is employed to avoid inter-layer interference in common channel deployment among macro and pico. The macro makes

downlink silence for ABS to protect the transmission of picos, and only broadcast limited control signals over these specific subframes. In this work, the energy efficient ABS allocation and user scheduling process are performed at each BS which carries out time-domain interference coordination among BSs. And the user scheduling in every ABS-period allocates air time resources in fair among the associated UEs from energy efficiency perspective. Those are presented in Fig. 2.

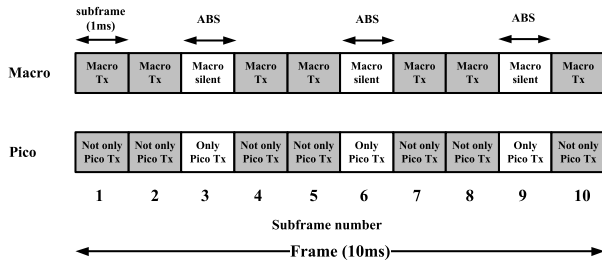


FIGURE 2. The ABS subframe.

SINR Model: In order to obtain the average data rate of the UE, we derive the SINR expression from the interference model. The interfering relationship can be determined by the physical distance or received signal strength between BSs based on whether or not is less than threshold-setting. Due to the user association with macro or pico but not both, we introduce two kinds of user types: macrocell-associated and pico-associated. For a picocell-associated user, the user transmits during ABS subframes and non-ABS subframes. For a macrocell-associated UE, the UE transmits only during non-ABS subframes. During ABSs, the interference is only from the interfering pico since the interference produced by macrocells to picocells keep silent for downlink. During non-ABS sub-frames, the interference to UE is both from all the interfering picocells and from the macrocells.

For the picocell-associated user, the SINR of user is expressed as:

$$SINR_{pico}(u) = \begin{cases} \frac{P_{Rx}(u)}{P_{pico}(u) + N_0} & \text{ABS} \\ \frac{P_{Rx}(u)}{P_{pico}(u) + P_{macro}(u) + N_0} & \text{non-ABS.} \end{cases} \quad (1)$$

For the macrocell-associated user, the SINR of user is expressed as:

$$SINR_{macro}(u) = \frac{P_{Rx}(u)}{P_{pico}(u) + P_{macro}(u) + N_0} \text{non-ABS.} \quad (2)$$

The rate of user depends on the amount of allocated subframes and its experienced SINR. Thus, we can use Shannon capacity formula to get the average rate for user u . The related SINR and rate symbols used in the paper are denoted in Table 1. The important parameters and optimization variables are summarized in the Table 2.

TABLE 1. The list of mathematical notations.

Notation	Description
$P_{Rx}(u)$	The received power of user u for the downlink
$P_{pico}(u)$	The suffered interference in user u from picocells
$P_{macro}(u)$	The suffered interference in user u from macrocells
r_u^{macro}	The rate of user u obtained from macrocell in non-ABS subframes
$r_{u,A}^{pico}$	The rate of user u obtained from picocell in ABS subframes
$r_{u,nA}^{pico}$	The rate of user u obtained from picocell in non-ABS subframes

TABLE 2. Symbol of optimization variables.

Notation	Description
u, K	Acronym of user, number of users, respectively
m, M	Acronym of macrocell, number of macrocells, respectively
p, P	Acronym of pico, number of picos, respectively
m_u	The best accessed candidate macro of user u
p_u	The best accessed candidate pico of user u
N_{sf}	The total number of one ABS period
N_m	The number of subframes transmitted by macro m in non-ABS subframes
A_p	The number of subframes transmitted by pico p in ABS subframes for which the macro m keeps silent
x_u	Air time in non-ABS subframes user u obtains from m_u
$y_{u,A}$	Air time in ABS subframes user u obtains from p_u
$y_{u,nA}$	Air time in non-ABS subframes user u obtains from p_u
p_u^{macro}	The transmission power of macrocell
P_{ref}^{macro}	The reference signals power from macrocell in ABS subframes.
p_u^{pico}	The transmission power of picocell
R_u	The transmission rate of user
P_u	The power consumption of user

IV. OPTIMIZATION PROBLEM FORMULATION

After describing the network model and problem parameters above, we formulate an optimization problem to maximize the energy efficiency of the worst UE with user association, air time subframe allocation for user and ABS allocation between macro and pico.

A. PROBLEM FORMULATION

Our objective is to maximize the energy efficiency of the worst UE for eICIC in HetNets. Naturally, it considers the energy efficiency of all users in HetNets. Therefore, it ensures fairness for energy efficiency of eICIC configuration. We jointly optimize these variables $\psi = \{R_u, P_u, x_u, y_{u,A}, y_{u,nA}, A_p, N_m\}$ to obtain the EE-eICIC algorithm with max-min fair. So, the optimization problem (P1) is formulated as:

$$\begin{aligned} & \max_{\psi} \min_u \frac{R_u}{P_u} & (3) \\ & \text{s.t. } C1 : R_u \leq r_u^{macro} \cdot x_u + r_{u,A}^{pico} \cdot y_{u,A} + r_{u,nA}^{pico} \cdot y_{u,nA}, \\ & \quad C2 : P_u \leq p_u^{macro} x_u + (p_u^{pico} + P_{ref}^{macro}) y_{u,A} \\ & \quad \quad \quad + p_u^{pico} y_{u,nA}, \\ & \quad C3 : x_u \cdot (y_{u,A} + y_{u,nA}) = 0, \\ & \quad C4 : A_p + N_m \leq N_{sf}, \quad \forall p, m \in I_{BS}, \\ & \quad C5 : \sum_{u \in U_m} x_u \leq N_m, \quad \forall m \in M, \end{aligned}$$

$$\begin{aligned}
 C6: & \sum_{u \in U_p} y_{u,A} \leq A_p, \quad \forall p \in P, \\
 C7: & \sum_{u \in U_p} y_{u,A} + y_{u,nA} \leq N_{sf}, \quad \forall p \in P, \\
 C8: & x_u \geq 0, y_{u,A} \geq 0, y_{u,nA} \geq 0, \\
 C9: & A_p, N_m \leq N^+, \quad \forall p, m \in I_{BS}, \quad (4)
 \end{aligned}$$

where N^+ represents the set of positive integers.

The $C1$ denotes that the average rate for a user is limited to the available airtime from the associated macro or pico. The formulation $C2$ denotes that the average power consumption for a user is limited to the available airtime from the associated macro or pico. The association $C3$ denotes that the user only can access into one BS, either macrocell or small cell (e.g.picocell), but not both macro and small cell. The constraint $C4$ denotes the ABS subframe required by picos is provided by macros in set I_{BS} . The $C5$ denotes the air time of subframes required by user from a macro is not more than the non-ABS subframes N_m . The $C6$ denotes that air time of ABS subframe required by user from pico is not larger than the ABS subframes A_p . The $C7$ denotes that air time of subframe required by user from pico is less than the period of ABS N_{sf} .

Remark 1: The constraint $C3$ states that one UE can only associates with single macro or pico, P1 is binary programming problem. Moreover, due to the constraint $C8$ involving continuous variables $R_u, P_u, x_u, y_{u,A}, y_{u,nA}$ and the constraint $C9$ containing nonnegative integers variables A_p, N_m , P1 is a mixed binary integer programming problem. It is generally NP-hard and hard to solve [23]. In the paper, we reformulate to solve it with a novel polynomial algorithm.

V. RELAX-ROUNDING SOLUTION

In the section, we develop a two-stage algorithm to solve P1 in polynomial time. Firstly, the integer variables are relaxed into the positive real numbers to make constrain of P1 convex space, which is solvable with an optimization algorithm. In the second stage, the output results of relaxed problem are rounded to obtain a feasible result for the original problem. The solution in detail is as follows.

1) Stage 1: Relaxing. The goal is to solve P2 (i.e.EE-RELAXED-eICIC) with relaxed problem P1. The P2 is formulated by relaxing the constraint $C9$ on N_m and A_p and ignoring the constraint $C3$. After relaxing the constraint $C9$, the N_m and A_p can be taken positive real numbers. The constraints $C3$ removed means that the user could associate with macro and pico at the same time in the downlink. The P2 with optimization variables $\tilde{\psi} = \{\tilde{R}_u, \tilde{P}_u, \tilde{x}_u, \tilde{y}_{u,A}, \tilde{y}_{u,nA}, \tilde{A}_p, \tilde{N}_m\}$ can be formulated as follows:

$$\begin{aligned}
 & \max_{\tilde{\psi}} \min_u \frac{\tilde{R}_u}{\tilde{P}_u} \\
 & \text{s.t. } (C1) - (C2) \text{ and } (C4) - (C8) \\
 & \quad \tilde{A}_p, \tilde{N}_m \in R^+, \forall p, m \in I_{BS}, \quad (5)
 \end{aligned}$$

where R^+ represent the nonnegative real numbers.

2) Stage 2: Rounding. We can obtain the approximative feasible solution of P1 by rounding the solution of P2.

A. PROBLEM TRANSITION

Owing to the constraints $C3$ and $C9$, P1 is a mixed binary integer programming problem. Even if we ignore the constraint $C3$ and relax $C9$ to R^+ , the OP1 is still non-convex problem due to the non-convexity and non-smoothness of the optimization objective (3). However, the structure of (3) is considered as fractional programming in general, which is adopted to design an effective scheme [24].

Without loss of generality, it can be assumed that $R_u > 0$ and $P_u > 0$. For notation simplicity, we define the feasible space of (4) in P2 by $\tilde{\psi}$. So we have

$$\tilde{\eta}_{EE}^{opt} = \max_{\tilde{\psi}^{opt}} \min_u \frac{\tilde{R}_u}{\tilde{P}_u} = \min_u \frac{\tilde{R}_u^{opt}}{\tilde{P}_u^{opt}}, \quad (6)$$

where $\tilde{\psi}^{opt} = \{\tilde{R}_u^{opt}, \tilde{P}_u^{opt}, \tilde{x}_u^{opt}, \tilde{y}_{u,A}^{opt}, \tilde{y}_{u,nA}^{opt}, \tilde{A}_p^{opt}, \tilde{N}_m^{opt}\}$ and $\tilde{\eta}_{EE}^{opt}$ are the optimal solution and result of P2.

In order to solve P2 effectively, we have the proposition as following [25].

Proposition 1: The optimal solution $\tilde{\psi}^{opt}$ can be achieved if and only if

$$\tilde{\eta}_{EE}^{opt} : \max_{\tilde{\psi}^{opt}} \min_u [\tilde{R}_u - \tilde{\eta}_{EE}^{opt} P_u] = \min_u [\tilde{R}_u^{opt} - \tilde{\eta}_{EE}^{opt} \tilde{P}_u^{opt}] = 0, \quad (7)$$

Proof: See Appendix A.

From proposition 1, given $\tilde{\eta}_{EE}^{opt}$, we can solve P2 via its equivalent transform as follow

$$\begin{aligned}
 & \max_{\tilde{\psi}} \min_u [\tilde{R}_u - \tilde{\eta}_{EE}^{opt} \tilde{P}_u] \\
 & \text{s.t. } (C1) - (C2) \text{ and } (C4) - (C8) \\
 & \quad \tilde{A}_p, \tilde{N}_m \in R^+, \forall p, m \in I_{BS}. \quad (8)
 \end{aligned}$$

However, it is impossible to solve (8) for a self-evident task, because $\tilde{\eta}_{EE}^{opt}$ is generally unknown in advance. We use an update parameter $\tilde{\eta}^n$ combines with (8) to obtain the optimal solution of P2 [25]. The specific procedure is described in Algorithm 1.

For given a $\tilde{\eta}_{EE}^n$, the transform problem (P2) needed be solved in line 3 for Algorithm 1.

$$\begin{aligned}
 & \max_{\tilde{\psi}} \min_u [\tilde{R}_u - \tilde{\eta}^n \tilde{P}_u] \\
 & \text{s.t. } (C1) - (C2) \text{ and } (C4) - (C8) \\
 & \quad \tilde{A}_p, \tilde{N}_m \in R^+, \quad \forall p, m \in I_{BS}, \quad (9)
 \end{aligned}$$

B. PROBLEM REFORMULATION

In order to solve P2, a new variable θ is introduced to re-parameterize the original non-smooth problem P2, because of the non-smoothness of the goal function, into the smooth problem. Therefore, P2 can be reformulated as P3

$$\begin{aligned}
 & \max_{\tilde{\psi}, \theta} \theta \\
 & \text{s.t. } \tilde{R}_u - \tilde{\eta} \tilde{P}_u \geq \theta, \\
 & \quad (C1) - (C2) \text{ and } (C4) - (C8), \\
 & \quad \tilde{A}_p, \tilde{N}_m \in R^+, \quad \forall p, m \in I_{BS}. \quad (10)
 \end{aligned}$$

Algorithm 1 Iterative Energy Efficiency Algorithm for EE-eICIC With max-min Fair

- 1: Initialize: Set the error tolerance $\varepsilon > 0$ and the number of maximum iteration N_{max} , and take the initial energy efficiency $\eta^n = 0$ and iteration number $n = 0$.
- 2: **while** $n \leq N_{max}$ **do**
- 3: Solve this problem P2 for a given $\tilde{\eta}^n$ and get the max-min EE-eICIC subframes allocation policy ψ
- 4: **if** $|\min_u(\tilde{R}_u^n - \tilde{\eta}^n \tilde{P}_u^n)| < \varepsilon$ **then**
- 5: $\tilde{\eta}_{EE}^{opt} = \min_u \frac{\tilde{R}_u^n}{\tilde{P}_u^n}$
- 6: **return** the optimal EE-eICIC subframes allocation policy $\tilde{\psi}^{opt}$ and maximal $\tilde{\eta}_{EE}^{opt}$ with guaranteeing the worse user EE.
- 7: **else**
- 8: set $\tilde{\eta}^{n+1} = \min_u \frac{\tilde{R}_u^n}{\tilde{P}_u^n}$
- 9: $n = n + 1$;
- 10: **end if**
- 11: **end while**

With the property of θ , we obtain the proposition 2 that can be derived directly from Proposition 1.

Proposition 2: For all feasible $\tilde{\psi}$, $\theta \geq 0$ when $0 \leq \tilde{\eta} \leq \tilde{\eta}_{EE}^{opt}$.

It is obvious that the P3 is convex programming, which is easy to solve [26]. However, we design a distributed algorithm to reduce greatly the algorithmic complexity and make the approach amenable to distributed implementation in Ultra-Dense HetNets.

VI. ALGORITHM FOR RELAXED NONLINEAR PROGRAM

In order to solve P3, we exploit the Lagrangian dual decomposition method [4], [26], which is a efficient approach to deal with many networking resource allocation problems for different goals due to its distributed implementation [27]. Next, we state that a dual-based decomposition method for our problem significant declines the complexity of algorithm and also carries out the algorithm in distributed manner.

The Lagrangian of the P3 is presented as:

$$\begin{aligned}
 L(\tilde{x}, \tilde{y}, \tilde{A}, \tilde{N}, \lambda, v, \mu, \rho, \alpha, \beta, \gamma) &= \theta(1 - \sum_u \lambda_u) + \lambda_u \sum_u (R_u - \eta P_u) \\
 &- \sum_u v_u (R_u - r_u^{macro} \cdot x_u - r_{u,A}^{pico} \cdot y_{u,A} - r_{u,nA}^{pico} \cdot y_{u,nA}) \\
 &- \sum_u \rho_u [P_u - p_u^{macro} x_u - (p_{u,A}^{pico} + P_{ref}^{macro}) y_{u,A} \\
 &\quad - p_{u,nA}^{pico} \cdot y_{u,nA}] \\
 &- \sum_{p,m \in I_{BS}} \mu_{p,m} (A_p + N_m - N_{sf}) \\
 &- \sum_m \beta_m (\sum_{u \in U_m} x_u - N_m) - \sum_p \beta_p (\sum_{u \in U_p} y_{u,A} - A_p) \\
 &- \sum_p \gamma_p [\sum_{u \in U_p} (y_{u,A} + y_{u,nA}) - N_{sf}]. \tag{11}
 \end{aligned}$$

Here, the boldface symbol is used to denote vector of variables, such as μ is the vector of variable $\mu_{p,m}$. The variables $\lambda, v, \mu, \rho, \beta, \gamma$ denote dual variables, named Lagrangian multipliers which also is called price factor. Further, we use the vector e to express all the dual variables $e = (\lambda, v, \mu, \rho, \beta, \gamma)$. Also, we can define the variable $\tilde{x}, \tilde{y}, \tilde{A}, \tilde{N}$ as primal variable, and the z is used to express all primal variable vector $z = (\tilde{x}, \tilde{y}, \tilde{A}, \tilde{N})$. So, the Lagrangian function can be expressed as

$$L(z, e) = f(R, P, \theta) - e'g(z). \tag{12}$$

The dual expression of P3 is denoted as

$$\min_{e > 0} \max_z L(z, e). \tag{13}$$

Since the P3 is convex programming, there is no duality gap to obtain P3 optimal allocation by using the Lagrangian dual decomposition [26].

Our algorithm to the P3 problem is to solve it in two steps. Firstly, the primal variables are computed with the feasible space. Then the dual variables are set to the initial value zero and are updated for every iteration.

The primal variables in iteration $(n + 1)$ are updated as

$$z_{n+1} = \arg \max_z L(z, e). \tag{14}$$

The dual variables are set in a subgradient descent manner as

$$e_{n+1} = [e_n + \xi g(e_n)]^+, \tag{15}$$

where e is the vector of dual variable in iteration $n + 1$, ξ is the vector of step sizes, and $[\cdot]^+$ is projection of nonnegative real numbers into the region.

With sufficiently large number of the above steps iterations, we can obtain the optimal solution to P3 by averaging over all iterations as follow.

$$z_{n+1} = \frac{1}{N} \sum_{n=1}^N z_n. \tag{16}$$

A. DECOMPOSITION-BASED APPROACH

With dual decomposition theory, we decompose the primal problem into three problem: UE problem, pico problem and macro problem, which can be carry out in parallel.

For this purpose, we rewrite (11) as follows:

$$\begin{aligned}
 L(z, e) &= \sum_u F_u(e, R_u, P_u, \theta) \\
 &+ \sum_m G_m(e, \{x_u\}_{u \in U_m}, N_m) \\
 &+ \sum_p H_p(e, \{y_u\}_{u \in U_p}, A_p) - N_{sf}, \tag{17}
 \end{aligned}$$

where

$$\begin{aligned}
 F_u(e, R_u, P_u) &= R_u(\lambda_u - v_u) + P_u(-\rho_u - \lambda_u \eta) + (1 - \lambda_u)\theta, \tag{18}
 \end{aligned}$$

$$G_m(\mathbf{e}, \{x_u\}_{u \in m_u}, N_m) = \sum_{u \in U_m} x_u (v_u r_u^{macro} - \rho_u p_u^{macro} - \beta_m) + N_m (\beta_m - \sum_{p,m \in I_{BS}} \mu_{p,m}), \quad (19)$$

$$H_p(\mathbf{e}, \{y_u\}_{u \in p_u}, A_p) = A_p (\beta_p - \sum_{p,m \in I_{BS}} \mu_{p,m}) + \sum_{u \in U_p} y_{u,A} [v_u r_{u,A}^{pico} - \rho_u (p_{u,A}^{pico} + P_{ref}^{macro}) - \beta_p - \gamma_u] + \sum_{u \in U_p} y_{u,nA} (v_u r_{u,nA}^{pico} - \rho_u p_{u,nA}^{pico} - \gamma_u). \quad (20)$$

Thence it follows that

$$\max_z L(\mathbf{z}, \mathbf{e}) = \sum_u \max F_u(\mathbf{e}, R_u, P_u, \theta) + \sum_m \max G_m(\mathbf{e}, \{x_u\}_{u \in m_u}, N_m) + \sum_p \max H_p(\mathbf{e}, \{y_u\}_{u \in p_u}, A_p) - N_{sf}, \quad (21)$$

where the max above involves suitable primal variables for x_u, y_u, N_m, A_p . The above formulations state that primal iteration step is disassemble into the subproblems of each UEs, each picos, and each macros. It is easy to compute each of the subproblems as follows.

1) Primal variables iteration: At iteration n , the updates of primal variables are the following.

User iteration: For every user, we maximize $F_u(\mathbf{e}, R_u, P_u)$ by calculating as

$$R_u(n+1) = r_u^{macro} \cdot x_u + r_{u,A}^{pico} \cdot y_{u,A} + r_{u,nA}^{pico} \cdot y_{u,nA} |_{\{\lambda_u(n) - v_u(n) > 0\}}, \quad (22)$$

$$P_u(n+1) = p_u^{macro} \cdot x_u + (p_{u,A}^{pico} + P_{ref}^{macro}) \cdot y_{u,A} + p_{u,nA}^{pico} \cdot y_{u,nA} |_{\{-\rho_u(n) - \lambda_u(n) - \eta > 0\}}, \quad (23)$$

$$u^* = \arg \min_u \{1 - \sum_n \lambda_u(n) > 0\},$$

$$u^* = \arg \min_u \{1 - \lambda_u(n) > 0\}, \quad (24)$$

$$\theta(n+1) = \begin{cases} R_u - \eta P_u, & \text{for } u = u^*, \\ 0, & \text{for } u \neq u^*. \end{cases} \quad (25)$$

Macro iteration: for every macro m , we maximize $G_m(\mathbf{e}, \{x_u\}_{u \in m_u}, N_m)$ by calculating as

$$N_m(n+1) = N_{sf} |_{\{\beta_m - \sum_{p,m \in I_{BS}} \mu_{p,m} > 0\}}. \quad (26)$$

To calculate all $\{x_u\}_{u \in U_m}$, every macro chooses the best user u_m^* in iteration n as

$$u_m^* = \arg \max_{u \in U_m} (v_u r_u^{macro} - \rho_u p_u^{macro} - \beta_m > 0). \quad (27)$$

Then the macro m compute $x_u(n+1), u \in U_m$ as

$$x_u(n+1) = \begin{cases} N_{sf}, & \text{for } u = u_m^*, \\ 0, & \text{for } u \neq u_m^*. \end{cases} \quad (28)$$

Pico iteration: At iteration $n+1$, for every pico p , we maximize $H_p(\mathbf{e}, \{y_u\}_{u \in p_u}, A_p)$ by computing

$$A_p = N_{sf} |_{\{\beta_p - \sum_{p,m \in I_{BS}} \mu_{p,m} > 0\}}. \quad (29)$$

To calculate all $\{y_u\}_{u \in U_p}$, every pico p chooses the best user as follows:

$$u_{p,A}^* = \arg \max (v_u r_{u,A}^{pico} - \rho_u (p_{u,A}^{pico} + P_{ref}^{macro}) - \beta_p - \gamma_u > 0),$$

$$u_{p,nA}^* = \arg \max (v_u r_{u,nA}^{pico} - \rho_u p_{u,nA}^{pico} - \gamma_u > 0). \quad (30)$$

Then the pico p computes $y_u(n+1), u \in U_p$.

$$y_{u,A}(n+1) = \begin{cases} N_{sf}, & \text{for } u = u_{p,A}^*, \\ 0, & \text{for } u \neq u_{p,A}^*, \end{cases} \quad (31)$$

$$y_{u,nA}(n+1) = \begin{cases} N_{sf}, & \text{for } u = u_{p,nA}^*, \\ 0, & \text{for } u \neq u_{p,nA}^*. \end{cases} \quad (32)$$

2) Dual variables iteration: Observed that, from (18), (19) and (20), the $\mathbf{e} = (\lambda, v, \mu, \rho, \beta, \gamma)$ is used to allocate ABS subframes for max-min EE optimization. A sub-gradient of \mathbf{e} is given as following. In iteration n , the dual updates are also described user, macro and pico.

For user u , the dual variables are updated as

$$\lambda_u(n+1) = [\lambda_u(n) + \xi(R_u - \eta P_u - \theta)]^+, \quad (33)$$

$$v_u(n+1) = [v_u(n) + \xi(r_u^{macro} \cdot x_u + r_{u,A}^{pico} \cdot y_{u,A} + r_{u,nA}^{pico} \cdot y_{u,nA} - R_u)]^+, \quad (34)$$

$$\rho_u(n+1) = [\rho_u(n) + \xi(p_u^{macro} x_u + (p_{u,A}^{pico} + P_{ref}^{macro}) y_{u,A} + p_{u,nA}^{pico} \cdot y_{u,nA}) - P_u]^+, \quad (35)$$

$$\alpha_u(n+1) = [\alpha_u(n) + \xi(N_{sf} \cdot P_u^{max} - P_u)]^+. \quad (36)$$

For macro m , the dual variables are updated as:

$$\beta_m(n+1) = [\beta_m(n) + \xi(N_m - \sum_{u \in U_m} x_u)]^+, \quad (37)$$

For pico p , the all dual prices for all $\{p, m\} \in I_{BS}$ is updated as:

$$\mu_{p,m}(n+1) = [\mu_{p,m}(n) + \xi(N_{sf} - A_p - N_m)]^+, \quad (38)$$

$$\beta_p(n+1) = [\beta_p(n) + \xi(A_p - \sum_{u \in U_p} y_{u,A})]^+, \quad (39)$$

$$\gamma_u(n+1) = [\gamma_u(n) + \xi(N_{sf} - \sum_{u \in U_p} (y_{u,A} + y_{u,nA}))]^+. \quad (40)$$

Algorithm 2 Optimal Algorithm for Solving P3

- 1: Initialize: set all variables $\mathbf{x}, \mathbf{y}, \mathbf{A}, N, \lambda, \mathbf{v}, \mu, \rho, \alpha, \beta, \gamma$ initial values within feasible space.
- 2: Set initial iteration number $n = 0$ and the number of maximum iteration N .
- 3: **for** $n = 1 : 1 : N$ **do**
- 4: Primal variables update: users update with (22) (23), macros update with (26) and (28), and picos updates with (29), (31) and (32).
- 5: Dual Update: update $\lambda, \mathbf{v}, \mu, \rho, \alpha, \beta, \gamma$ from (33)-(40), respectively.
- 6: $n = n + 1$;
- 7: **end for**
- 8: By averaging the all iterations, we can obtain the optimal solution $\hat{\mathbf{z}}_N = \frac{1}{N} \sum_{n=1}^N \mathbf{z}_n$.

B. ALGORITHM FOR P3

We now describe the algorithm steps to solve (10). Algorithm 2 presents P3 algorithm.

The EE computed by the output of Algorithm 2, which is input of Algorithm 1. The convergence rate of Algorithm 1 is linear [28]. Then we discuss the convergence analysis of Algorithm 2 according to the network parameters.

C. CONVERGENCE ANALYSIS

To estimate the step size and the number of iteration, the convergence property is analyzed for a general method based dual subgradient [13]. It can be seen that the P3 can lead to a simple characterization of the step size and the iteration number with problem parameters. In the subsection, the r_{max} and p_{max} are expressed as the maximum data rate and power consumption of user, respectively.

Proposition 3: Let $\mathbf{z}_n, \hat{\mathbf{z}}_n, \mathbf{z}'(\mathbf{e}_n, \hat{\mathbf{e}}_n, \mathbf{e}')$ be the primal (dual) variables vector for iteration n , the averages for all iterations from 0 to n , and the optimal value, respectively. So we have the proposition as follow:

$$f(\hat{\mathbf{e}}_N) - f(\mathbf{e}') \leq \frac{H}{2\xi N} + \frac{\xi W}{2}, \quad (41)$$

where

$$W = N_{sf}^2 [K(r_{max}^2 + p_{max}^2) + (I + M + 2P)],$$

$$H = K(2 + \eta^2) + N_{sf}^2 (r_{max}^2 + p_{max}^2 \eta^2) (I + M + 2P). \quad (42)$$

Proof: See Appendix B.

Remark 2: For convex programming, the property of convergence is on the basis of a general slater vector [26]. In the proof of Proposition 2 (Appendix B), we have shown that the optimal dual price, similar to convergence analysis in [29], have the upper bound depending on the network parameters. The upper bound models the property of convergence for P3.

Proposition 4 (The Step Size and the Iterations Number): Assume that the gap between the objective of every user and the optimal value is no more than σ , so the step size ξ and the

number of iterations N are set to as follows:

$$\frac{\xi W}{2} \leq \frac{K\sigma}{2} \text{ and } \frac{H}{2\xi N} \leq \frac{K\sigma}{2}. \quad (43)$$

The above inequality implies,

$$\xi = \frac{K\sigma}{W} \text{ and } N = \frac{HW}{(K\sigma)^2}. \quad (44)$$

Further, a moments reflection states that $\xi = O(\sigma/N_{sf}^2(r_{max}^2 + p_{max}^2))$ and $N = O(N_{sf}^2(r_{max}^2 + p_{max}^2)(2 + \eta^2)/\sigma^2)$. We can decompose the macro-pico interference map into several disjoint components, so that run the max-min P3 algorithm can be carried out for each component independently in parallel. So the implement time of this algorithm can be significantly reduced.

For the convex P3, Algorithm 1 with Algorithm 2 can obtain the optimal solution. This is because of a requirement of Dinkelbach's generalized algorithm (Algorithm 1) is to globally solve P2 for any value of the parameter η , which is optimal achieved by Algorithm 2. However, the results obtained by Algorithm 1 with Algorithm 2 is not feasible for original problem (P1), we use the following method to deal with.

VII. INTEGER ROUNDING ALGORITHM

Unlike the relaxed problem, each user can only receive resource from macrocell or picocell but not both in practice. Another, the solution of P2 may violate feasibility of original problem. By the rounding way as follow, the feasible results can be obtained for the original problem:

$$Round(x) = \begin{cases} floor(x) & x < \frac{N_{sf}}{2}, \\ ceil(x) & x \geq \frac{N_{sf}}{2}. \end{cases} \quad (45)$$

In the next, the feasible solutions are computed approximately with rounding the result of the Algorithm 1 with Algorithm 2. The detail describes are shown in Algorithm 3.

Algorithm 3 can be divided into three aspects. Firstly, we make N_m and A_p into integer values with the rounding function (45). Secondly, we compare the EE of user obtained from macro and pico to determine the UE association. Thirdly, each user available average time ratio are obtained by filling up the available subframes, the average time ratio is computed. And we can obtain the rate and power consumption of the UE. Finally, the EE of every user is calculated as $\eta_u^* = \frac{R_u^*}{P_u^*}$.

Performance analysis: We presents the lower bound performance of our proposed algorithm by theoretical analysis. Proposition 1 states that Algorithm 1 with Algorithm 2 can achieve the optimal solution of P2, if we assume that there is a sufficiently large but polynomial number of iterations and suitable step size for Algorithm 1 and Algorithm 2. So the performance of our proposed algorithm depend on the integer rounding the output of Algorithm 3.

Proposition 5: Let η^{opt} be the optimal results of original problem and let η^* be the output of Algorithm 3. It is assumed

Algorithm 3 The Input Is the Result Obtained By Algorithm 1 With Algorithm 2

1: *Max-Min EE-ABS allocation*: To make N_m^* and A_p^* to integer number.

$$N_m^* = \text{Round}(\tilde{N}_m) \quad \text{and} \quad A_p^* = \text{Round}(\tilde{A}_p), \quad (46)$$

where \tilde{N}_m and \tilde{A}_p are output of Algorithm 1 with Algorithm 2.

2: *User EE-Association with fair*:

$$R_u^{\text{macro}} = r_{u,A}^{\text{macro}} \cdot \tilde{x}_u, \quad P_u^{\text{macro}} = p_{u,A}^{\text{macro}} \cdot \tilde{x}_u, \quad (47)$$

$$\begin{aligned} R_u^{\text{pico}} &= r_{u,A}^{\text{pico}} \cdot \tilde{y}_{u,A} + r_{u,nA}^{\text{pico}} \cdot \tilde{y}_{u,nA}, \\ P_u^{\text{pico}} &= (P_{u,A}^{\text{pico}} + P_{\text{ref}}^{\text{macro}}) \cdot \tilde{y}_{u,A} + P_{u,nA}^{\text{pico}} \cdot \tilde{y}_{u,nA}, \end{aligned} \quad (48)$$

where $\tilde{x}_u, \tilde{y}_{u,A}, \tilde{y}_{u,nA}$ is output of Algorithm 2.

So, we can obtain $\eta_u^{\text{pico}} = \frac{R_u^{\text{pico}}}{P_u^{\text{pico}}}, \eta_u^{\text{macro}} = \frac{R_u^{\text{macro}}}{P_u^{\text{macro}}}$.

If $\eta_u^{\text{macro}} > \eta_u^{\text{pico}}$, the UE associates with macro, otherwise it associates with pico.

3: *Energy Efficiency Computation*:

Firstly, calculate the time ratio of non-EE-ABS and EE-ABS.

$$X_m = \sum_{u \in U_m^*} \tilde{x}_u, \quad (49)$$

$$Y_{p,A} = \sum_{u \in U_p^*} \tilde{y}_{u,A}, \quad Y_{p,nA} = \sum_{u \in U_p^*} \tilde{y}_{u,nA}, \quad (50)$$

the X_m and $X_{m,A}$ denote the non-EE-ABS and EE-ABS usage for macro, and $Y_{p,nA}$ and $Y_{p,A}$ denote the non-EE-ABS and EE-ABS usage for pico, U_m^* denote the set of UE accessed into macro, and so are U_p^* for pico.

Secondly, calculate the air time of subframe for every user

$$x_u^* = \frac{\tilde{x}_u \cdot N_m^*}{X_m}, \quad (51)$$

$$y_{u,A}^* = \frac{\tilde{y}_{u,A} \cdot A_p^*}{Y_{p,A}}, \quad (52)$$

$$y_{u,nA}^* = \frac{\tilde{y}_{u,nA} \cdot (N_{sf} - A_p^*)}{Y_{p,nA}}, \quad (53)$$

Finally, the rate and power consumption of user is obtained in macro and pico.

For $u \in U_m^*, R_u^* = r_{u,A}^{\text{macro}} \cdot x_u^*, P_u^* = p_{u,A}^{\text{macro}} \cdot x_u^*$.

For $u \in U_p^*, R_u^* = r_{u,A}^{\text{pico}} \cdot y_{u,A}^* + r_{u,nA}^{\text{pico}} \cdot y_{u,nA}^*, P_u^* = (P_{u,A}^{\text{pico}} + P_{\text{ref}}^{\text{macro}}) \cdot y_{u,A}^* + P_{u,nA}^{\text{pico}} \cdot y_{u,nA}^*$.

Hence, the energy efficiency of user: $\eta_u^* = \frac{R_u^*}{P_u^*}$.

that there is a sufficiently large number of iterations and suitable step size for Algorithm 1 and Algorithm 2. For any given $\xi > 0$, then

$$\eta^{opt} \leq 2(1 + \xi)\eta^*. \quad (54)$$

Proof: In Appendix C.

Proposition 5 states that the lower bound of our proposed algorithm is approximate to 2 for polynomial time, which is important for this NP-hard problem.

VIII. NUMERICAL RESULTS

In the section, some numerical results are shown to validate the performance of our proposed algorithm. The parameters in the simulation are summarized in Table 3. In simulation scene, a typical 4G/5G macrocell with a transmission radius of 500 m is considered for a two-layer HetNets. We locate the BS of macrocell at the center and set the density of macros $\frac{1}{500^2}$. It will be specified that the density of 100 picocells and 250 UEs are randomly distributed within the macrocell. Note that each point in simulation figures is obtained by averaging 500 independent runs.

TABLE 3. Simulation parameters.

Notation	Description	Value
	Transmit power of Macrocells	46dBm
	Transmit power of Small cells	30dBm
	Reference signal power from macro in ABS	20dBm
	Noise power spectrum density	-174dBm/Hz
	Path-loss of Macrocell	$28.3+22.0\log_{10}l, lkm$
	Path-loss of Small cell	$30.5+36.7\log_{10}l, lkm$
	N_{sf} frame	40
	Bandwidth	20MHz

A. PERFORMANCE ANALYSIS OF MAX-MIN EE-eICIC

To analyze of performance of max-min EE-eICIC, we compare the proposed scheme with other schemes described as follows.

1) Max log rate with eICIC (MaxSUMlogRate) [4], [16], [17]: The function $\ln(R_u)$ is to strike a throughput of system with the fairness of user throughput. This is to say, the Max $\ln(R_u)$ for eICIC is to maximize the network rate with each UE rate fairness for load balancing between macrocell and small cell.

2) Max EE with eICIC (MaxEE) [5], [20], [21]: For a fair comparison, we modified the MaxEE [5], [20], [21] to maximum the EE of system with EE-ABS allocation and UEs EE-association between picocell and macrocell, but it don't consider the fairness of users.

3) The proposed method max-min EE with eICIC (MaxMinEE): The proposed method with max-min EE-eICIC is presented for EE-ABS allocation and UEs EE-association between macrocell and picocell with fairness assurance among users. For this purpose, the energy efficiency of the worst UE is maximized for eICIC in HetNets.

In Fig.3 and Fig.4, we compare the EE and capacity performance among these schemes from three aspects: the best-case user, the worst-case user and the network. In Fig.3, it can be observed that the energy efficiency is fairly large between the best user and the worst user for the MaxEE, nevertheless the energy efficiency of each user for the MaxMinEE can be balanced well but it is a little loss in energy efficiency of network than MaxEE. The energy efficiency of the worst-case

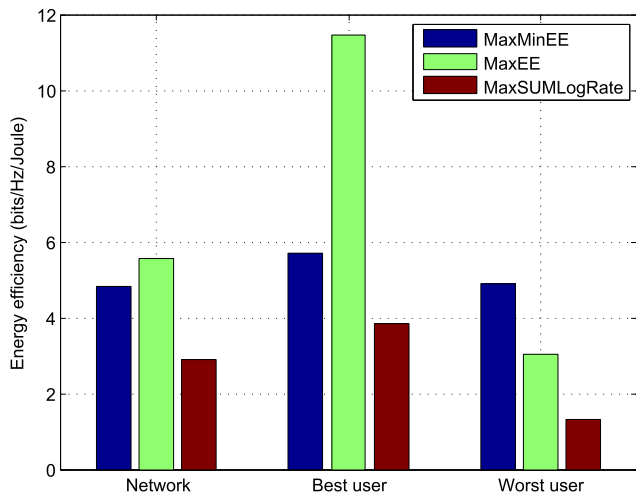


FIGURE 3. Energy efficiency of the network, the best UE, and the worst UE.

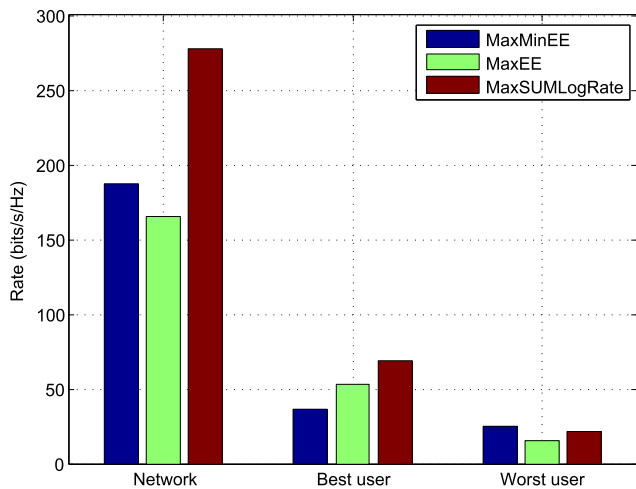


FIGURE 4. Capacity of the network, the best UE, and the worst UE.

user is at the cost of the total network energy efficiency for MaxMinEE as compared with the MaxEE. This the balance of energy efficiency between network and individual user is analogous to the tradeoff between the throughput of network and user rate [22].

Furthermore, comparing with the other algorithms, the capacity performance of MaxMinEE is shown in Fig. 4. As is shown in Fig.3 and Fig.4, the energy efficiency of the MaxMinEE and the MaxEE improves at the cost of the total sum rates comparing with MaxSUMlogRate. Furthermore, MaxMinEE assures the fairness of rate among user as compared with the other two strategies, and can achieve almost the same rate for the best and worst user. This is because of MaxMinEE augmenting the transmit data rate of the worst user to guarantee fairness for the energy efficiency of user. For the MaxEE, since the different of energy efficiency among the best and worst user increases, the different of their data rate augments as well.

Fig. 5 compares the load of the MaxMinEE, MaxEE and MaxSUMlogRate in 250-user per macrocell. We can see that more users are shift to associate with pico so that the load of macro decreases with the increasing of picocell. The proposed scheme with MaxMinEE leads to further load balancing than the MaxEE and MaxSUMlogRate. The is because the MaxMinEE makes full use of picocells to augment their air time, while avoids the picocells overload.

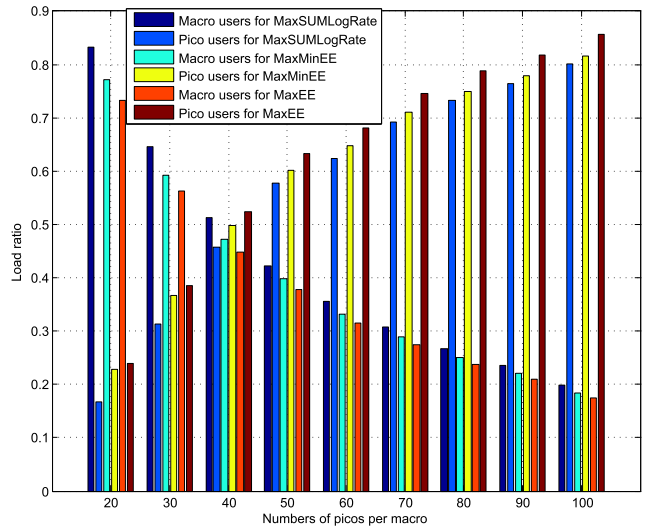


FIGURE 5. Load vs. number of picos.

Fig. 6 reveals the effect of different densities of picocell on the ABS ratio in the 250 users system for the MaxMinEE, MaxEE and MaxSUMlogRate in HetNets. It can be found that the ABS ratio of the MaxMinEE, MaxEE and MaxSUMlogRate increases when pico become denser, but the MaxEE has the largest ratio of ABS. This is due to the fact that the picocell is more energy efficiency and spectral efficiency. For MaxSUMlogRate, the macro-associated users have large SINR to obtain the larger rate, so the macrocells have been allocated

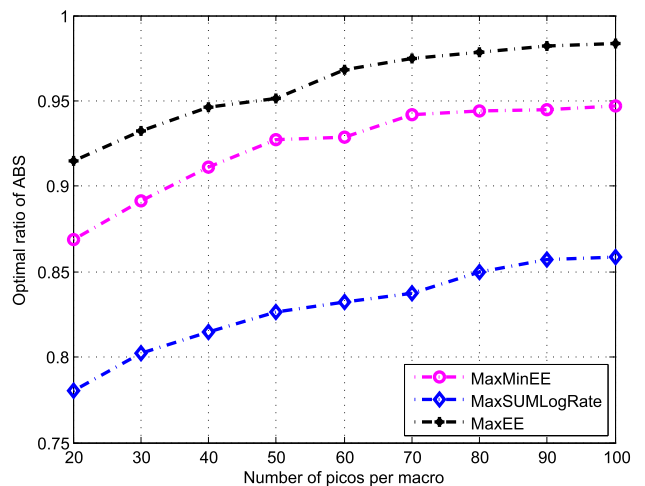


FIGURE 6. Optimal ratio of ABS vs. number of picos.

much more transmission time. The MaxEE augments the air time for picocell and the ABS becomes large. With the density of pico increases, much more users access into picocell for MaxEE. The MaxMinEE need to consider the worst-case user energy efficiency so that the more ABS is allocated to the macrocell, which solves the serious unfair problem between macrocell user and small cell for the maximum energy efficiency eICIC configuration.

IX. CONCLUSION

In this work, we consider energy efficient jointly together with interference coordination in HetNets, and proposed max-min energy-efficient eICIC configuration algorithm. The proposed algorithm maintains a good load balancing with energy saving in ultra-dense HetNets. The results suggested that max-min energy-efficient eICIC configuration brings about further load balancing over MaxEE and MaxSUMlogRate methods, can obtain efficient ABSs allocation between macro and pico to keep the performance of the macrocell user in ultra-dense HetNets.

**APPENDIX A
PROOF OF PROPOSITION 1**

We follow the similar skill in [25] to prove Proposition 1. We prove it from the sufficiency and the necessity. We assumed that the optimal subframe allocation scheme of P1 is ψ^* , and for any feasible subframe allocation $\psi \in (C1) - (C9)$, there are

$$\begin{aligned} \min_u [R_u(\psi) - \eta_{EE}^{opt} P_u(\psi)] &\leq 0, \\ \min_u [R_u(\psi^*) - \eta_{EE}^{opt} P_u(\psi^*)] &= 0. \end{aligned} \tag{55}$$

Rearranging (55), we can obtain

$$\begin{aligned} \min_u \frac{R_u(\psi)}{P_u(\psi)} &\leq \eta_{EE}^{opt}, \\ \min_u \frac{R_u(\psi^*)}{P_u(\psi^*)} &= \eta_{EE}^{opt}. \end{aligned} \tag{56}$$

Therefore, ψ^* is also the optimal solution of P1. This completes the sufficiency.

Then we give the necessity of proof. For any feasible space $\psi \in (C1) - (C9)$, there are

$$\begin{aligned} \min_u \frac{R_u(\psi)}{P_u(\psi)} &\leq \eta_{EE}^{opt}, \\ \min_u \frac{R_u(\psi^{opt})}{P_u(\psi^{opt})} &= \eta_{EE}^{opt}. \end{aligned} \tag{57}$$

Rearranging (57) yields

$$\begin{aligned} \min_u [R_u(\psi) - \eta_{EE}^{opt} P_u(\psi)] &\leq 0, \\ \min_u [R_u(\psi^{opt}) - \eta_{EE}^{opt} P_u(\psi^{opt})] &= 0. \end{aligned} \tag{58}$$

Hence, we can see that the minimum of (8) is 0 and can be achieved by ψ^{opt} . The ψ^{opt} is also the optimal solution of P1. We obtain the necessity of proof.

**APPENDIX B
PROOF OF PROPOSITION 3**

For the convenience of proof, we have made the technical assumption that there is at least one user in every macrocell (picocell) that can not be covered by any picocell (macrocell).

Assumption 1: For each macrocell (picocell), there is one user at least that obtains nonzero rate only from macro (pico).

We will simply provide a proof sketch. As is in [4] and [29], the dual update iteration $e_{n+1} = [e_n + \xi g(e_n)]^+$ can be used to as $f(\tilde{e}_N) - f(e^*) \leq \frac{\|e^*\|^2 - \|e_0\|^2}{2\xi N} + \frac{\xi \|g(z)\|^2}{2}$.

In our proof, a moments reflection states that $\|g(z)\|^2 \leq KR_{max}^2 + KP_{max}^2 + IN_{sf}^2 + MN_{sf}^2 + 2PN_{sf}^2 \leq K(R_{max}^2 + P_{max}^2) + N_{sf}^2(I + M + 2P)$, where R_{max} is the maximum rate a user can obtain in all iterations.

And we set $R_{max} = N_{sf} r_{max}$ and $P_{max} = N_{sf} p_{max}$ as a bound. At the beginning the dual variables can be set with arbitrarily small initial value, then we have upon substituting the bound $\|g(z)\|^2 \leq K(R_{max}^2 + P_{max}^2) + N_{sf}^2(I + M + 2P) \leq N_{sf}^2 [K(r_{max}^2 + p_{max}^2) + (I + M + 2P)]$.

Therefore,

$$\begin{aligned} f(\tilde{e}_N) - f(e^*) &\leq \frac{\|e^*\|^2}{2\xi N} + \frac{\xi N_{sf}^2}{2} [K(r_{max}^2 + p_{max}^2) \\ &\quad + (I + M + 2P)]. \end{aligned} \tag{59}$$

All that remains will be bounded by $\|e^*\|^2$.

Lemma B.1: Under assumption 1, we have the following bound:

$$\|e^*\|^2 \leq K(2 + \eta^2) + N_{sf}^2(r_{max}^2 + p_{max}^2 \eta^2)(I + M + 2P). \tag{60}$$

Proof: We roughly sketch how to bound the quantity. To bound the variables, we note that from the duality theory:

$$z^* = \arg \max_z L(z, e). \tag{61}$$

And there is no duality-gap.

To bound λ_u^* , consider the term $\lambda_u(k) - v_u(k) > 0$. With assumption 1, for (61) to hold, it must be that $1 - \lambda_u^* \geq 0$ from which it follows that $\sum_u \lambda_u^{*2} \leq K$.

Similarly, it can be shown that $\sum_u v_u^{*2} \leq K$ and $\sum_u \rho_u^{*2} < K\eta^2$.

To bound β_m^* , consider the term $\sum_{u \in U_m} x_u(v_u r_u^{macro} - \rho_u p_u^{macro} - \beta_m)$ in the expansion of $L(z, e^*)$. By Assumption 1, for (61) to hold, it must be that $v_u^* r_u^{macro} - \rho_u^* p_u^{macro} - \beta_m^* \geq 0$ from which it follows that $\beta_m^{*2} \leq \max_u (v_u^{*2} r_u^2 + \rho_u^{*2} p_u^2) \leq \max_u (v_u^{*2} R_u^2 + \rho_u^{*2} P_u^2) \leq N_{sf}^2 (r_{max}^2 + \eta^2 P_{max}^2)$.

Similarly, it can be obtained that, $\beta_p^{*2} \leq N_{sf}^2 (r_{max}^2 + \eta^2 P_{max}^2)$, $\mu_{p,m}^* \leq N_{sf}^2 (r_{max}^2 + \eta^2 P_{max}^2)$. Also γ^* satisfies the same bound as β_p^* . So, we can get the claim (60). By substituting (60) into (59), we can obtain the result in Proposition 2.

**APPENDIX C
PROOF OF PROPOSITION 5**

Assumption 1: Assume that the error of solution is less than ξ' by Algorithm 1 and Algorithm 2 on the conditions that there

is a sufficiently large but polynomial number of iterations and suitable step size for Algorithm 1 and Algorithm 2.

For any macrocell m , we can have $x_u^* = \frac{\tilde{x}_u \cdot N_m}{X_m} \geq \frac{\tilde{x}_u \cdot \tilde{N}_m}{X_m} (1 - \frac{2}{N_{sf}}) \geq \frac{\tilde{x}_u \cdot \tilde{N}_m}{N_m + \xi'} (1 - \frac{2}{N_{sf}}) \geq \frac{\tilde{x}_u}{1 + \xi'} (1 - \frac{2}{N_{sf}})$, for any user associated with macrocell. Since the rounding function $Round(x)$ is defined in (45), there is $z(1 - 2/N_{sf}) \leq Round(z) \leq z(1 + 2/N_{sf})$ [4].

Suppose ξ is defined as a function of ξ' , the above formulation can be written as $x_u^* \geq \frac{\tilde{x}_u}{1 + \xi}$.

In part 2 of Algorithm 3, user associates with macrocell when the energy efficiency of user gets from the macrocell is higher, so we have $\tilde{\eta}_u \leq 2(1 + \xi)\eta_u^*$.

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