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State Dependent Riccati Equation Based Rotor-Side Converter Control for Doubly Fed Wind Generator

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ABSTRACT This paper proposes a rotor-side converter (RSC) control of doubly fed induction generator (DFIG) through state-dependent Riccati equation (SDRE) approach to enhance the low voltage ride through (LVRT) ability. In order to guarantee the stability of the DFIG-based wind farm under external disturbances, a robust input-to-state stability (ISS)-based controller is presented in cooperation with the SDRE approach. The robust ISS controller is proved to be able to stabilize the wind generation system, while the system dynamics are not guaranteed to be of high performance. By parameterizing the robust ISS controller, the SDRE approach is used to provide guidance for the selection of ISS control parameters. Due to design flexibility, the SDRE control has the ability to enhance dynamic performances of wind generation system through optimizing the state-dependent coefficients. Time-domain simulations are carried out to verify the effectiveness of the proposed control approach. Compared with the conventional PI controller, exact linearization-based nonlinear controller, and robust ISS controller, the proposed SDRE-based RSC control has the ability to optimize the dynamics of the wind generation system during and after faults, and has better LVRT performance.

INDEX TERMS Doubly fed induction generator (DFIG), state-dependent Riccati equation, stability enhancement, low voltage ride through (LVRT).

I. INTRODUCTION

A. MOTIVATION

With shortage of fossil fuels and growing environmental problems, the development of renewable power generation has become a world consensus. As an important renewable energy source, wind generation system has been developed rapidly in recent years. Among various kinds of wind generation systems, doubly fed wind generation systems are widely applied due to the flexibility in power control and low cost [1]. However, the DFIG's characteristics of being sensitive to the change of power grid pose a challenge to the friendly connection of wind power to the grid. The fault ride through ability [2] of wind turbines has become an important index to evaluate the grid-connected wind generation system's friendliness to the power grid.

It should be emphasized that various disturbances could threaten normal operations of the doubly fed wind generation system as well as the power grid. Thus, enhancing the low voltage ride through ability of doubly fed wind generation system has the following requirements: first, the wind generation system should maintain continuous operation during the grid voltage drop to some extent, which requires to limit rotor current, DC-link voltage, and avoid rotor overspeed, etc. Second, reactive power support requirement, which means that the wind generation system can provide dynamic reactive power to support the grid voltage during a grid fault.

In this work, a rotor side converter control approach is proposed to enhance the fault ride through ability of DFIG based wind generation system, based on which stability and dynamic performance enhancement are achieved.

B. RELATED WORKS

In literatures, crowbar circuits are commonly used to absorb the transient energy during fault period [3], [4]. However, the DFIG will act as an induction motor and absorb large amount of reactive power when applying crowbar circuits, which may deteriorate the system dynamics during faults. A transient reconfiguration and coordinating control approach for power converters is proposed in [5]. The grid side converter is reconfigured and connected to the rotor circuit to provide an additional route for the rotor current, while the DC-link voltage is regulated through the energy storage device. However, this method requires additional energy storage devices which will increase the cost. At the same time, as the output current of the converter need to provide part of the demagnetization component to suppress the rotor current, the available output reactive power is smaller. Many research works on low voltage ride through (LVRT) of DFIG use the transient reactive power control to provide transient reactive power support [6]–[8], while additional cost is required. Thus, making full use of the capability of DFIG itself to regulate reactive power is beneficial.

The doubly fed wind generator is commonly controlled through a decoupling approach for active and reactive power, and most of the decoupling control methods are based on conventional PID controllers for the simplicity [9], [10]. However, the PID controller is usually designed based on linearized system model, while the stability may not be guaranteed during system transient, especially under severe fault conditions. Thus, many studies on nonlinear control of doubly fed wind generator have been done to enhance dynamic performance and stability of power system during the transient [2]. In [11], a decentralized nonlinear control for doubly fed wind generator is proposed through differential geometric approach [12]. However, this method achieves exact linearization by regarding the stator currents as constants. In [13], a neural network based controller is proposed to optimize the reference voltage of the rotor-side converter through neural network to enhance the active and reactive power output ability both in steady-state and transient operation. However, this approach does not give full consideration of internal dynamics during transient period. In [14], a sliding mode controller combining the crowbar circuit and the DC unloading circuit is proposed, while the control performance is not good enough.

Input-to-State stability (ISS) theory [15] has been widely adopted in the control of nonlinear systems with uncertainties, and is shown to work well in many areas [16], [17]. In [18], an ISS based control of DFIG is proposed, which achieves good active and reactive power support under severe fault conditions. From the results in [18], the ISS based controller design approach is promising for DFIG applications. However, the parameters of the ISS controller are chosen manually and remain fixed during the transient. Although the ISS controller is proved to be inverse optimal and have the ability to stabilize the wind generation system, its dynamic performance is not guaranteed to be of high quality. The lack

of guidance on selecting ISS control parameters will limit the control effect of the ISS controller. Thus, advanced techniques for optimizing ISS control parameters and corresponding comprehensive control process are required to be developed.

State dependent riccati equation (SDRE) [19] has attracted major concern for systematic controller design of nonlinear systems. Through extended linearization, the nonlinear system is transformed into a linear structure with state-dependent coefficient (SDC) matrices [20]. Due to the non-uniqueness of the SDC parameterization, it's free to use SDRE method to enhance controller performance [21]. However, stability is an issue in applying SDRE method [19], and the system is required to conform to the basic structure and conditions in order for the direct application of SDRE method [21].

From above literature reviews, it can be seen that 1) the ISS controller design approach is promising for DFIG applications, while guidance for selecting ISS control parameters is required to guarantee a high quality control performance. 2) the stability issue is required to be addressed in SDRE method to expand application scenario. Thus, by parameterizing the ISS controller, it's free to harness the SDRE method with ISS controller to guarantee the stability and the dynamic performance of the system.

C. MAIN CONTRIBUTIONS

In this paper, a state feedback controller based on the SDRE technique is proposed to enhance the dynamic performance of DFIG based wind turbine. The SDRE technique can fully capture the nonlinearities of the studied system through extended linearization. To directly apply the SDRE method, the controllability and stabilizability of the extended linearized system are guaranteed by applying the ISS theory. The SDRE technique is applied as an auxiliary control to automatically provide guidance and optimize the control parameters, and better transient behavior is achieved. The main contributions of this paper are summarized as follows:

- A state feedback auxiliary controller for RSC is designed through extended linearization via the SDRE technique to enhance the LVRT ability of DFIGs, where the ISS controller is used to guarantee the controllability and stabilizability of the extended linearized system. By parameterizing the ISS controller, the main dynamics of a doubly fed wind generator are clarified, and its corresponding mathematical model is developed.
- Principles on practical SDC parameterization algorithm and the selection of weighting matrices are fully studied in designing the proposed SDRE based state feedback controller.
- The converter's voltage capacity limit as well as a rotor current control mechanism are proposed to update the control signals according to the operational limits. The proposed SDRE based auxiliary controller is embedded in the detailed simulation model of doubly fed

wind generator. The control performance is evaluated through time-domain simulations in Matlab/Simulink.

The advantages of the proposed method can be summarized as follows: first, with parameterized robust ISS controllers, it's free to apply the SDRE method to optimize the ISS control parameters automatically, while stability is guaranteed by the ISS controller. Second, a practical selection of SDC matrices is presented to give an extra freedom to enhance the performance of the SDRE controller. Third, online adjustments can be achieved using the proposed stabilization and performance enhancement approach, which could improve the LVRT ability of doubly fed wind generation system.

D. PAPER ORGANIZATION

This paper is organized as follows: Problem formulation will be introduced in the next section. Section III proposes the theoretical preliminaries of SDRE approach. Section IV gives the detailed description of the controller design approach for doubly fed wind generator. Time-domain simulations are considered in section V as a verification. Section VI concludes the paper.

II. PROBLEM FORMULATION

A. PARAMETERIZED ISS CONTROLLER DESIGN APPROACH

Various definitions of ISS are given in literatures, and the norm-like form is presented in this paper.

Definition 1 [22]: Without loss of generality, consider an affine nonlinear system as follows

$$\dot{x} = f(x) + g(x)d \tag{1}$$

If there exist $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}_\infty$, such that for $\forall x_0, d$ the following holds.

$$|x(t, x_0, d)| \leq \beta(|x_0|, t) + \gamma(\|d\|_\infty) \quad \forall t \geq 0 \tag{2}$$

System (1) is called ISS, and γ is called the asymptotic gain. Where $x \in R^n$, $d \in R^p$, $f : R^n \rightarrow R^n$, and $g : R^n \rightarrow R^{n \times p}$, $|\cdot|$ denotes the Euclidean norm, and $\|\cdot\|_\infty$ represents the (essential) supremum norm. β and γ are comparison functions, the definitions of which are presented in Appendix I.

Definition 2 [23]: Consider the following nonlinear system with control inputs

$$\dot{x} = f(x) + g_1(x)d + g_2(x)u \tag{3}$$

system (3) is called input-to-state stabilizable if the controlled system is ISS with respect to a control law $u = k(x)$.

It has been shown in [23] that system (3) is input-to-state stabilizable if and only if there exist an ISS control Lyapunov function (ISS-CLF) V which satisfies the following condition for $\forall x \neq 0$

$$L_{g_2}V(x) = 0 \implies L_fV(x) + |L_{g_1}V(x)|\rho^{-1}(|x|) < 0 \tag{4}$$

where $V(x)$ is a positive definite and radially unbounded function, and $\rho \in K$.

Various kinds of ISS controllers are presented in literatures, such as the Sontag-type control law [15] and the min-norm formula [24]. Although these kinds of formula can stabilize the nonlinear system, the control performance is not guaranteed to be of high quality. To overcome this, a parameterized ISS controller design approach by using the satisficing theory [25] is presented as follows.

Theorem 1 [26]: All robust ISS control laws can be parameterized as follows

$$u = -\beta g_2^T V_x + \xi \tag{5}$$

where $\beta > \max\left(0, 2\frac{\nabla V_f + |\nabla V_{g_1}| \rho^{-1}(|x|)}{\nabla V_{g_2}(\nabla V_{g_2})^T}\right)$, ξ is in the null space of $g_2^T V_x$ and $V_x = (\nabla V)^T$. These ISS controllers have gain margins of $\left(\frac{1}{2}, \infty\right)$.

Remark 1: The parameterized ISS control law gives a large range of freedom in selecting ISS control parameters with respect to certain constraints. It's free to harness other control techniques to enhance the control performance.

B. MODELLING FOR RSC CONTROL OF DFIG WITH ISS CONTROLLER

It has been shown in [27] that, for the transient stability control of doubly fed wind generators, a reduced third-order model can be used. By switching the equilibrium point to the origin, the third order model of DFIG can be formulated in the form of system (3), where corresponding dynamics are represented as follows [18].

$$\begin{cases} x = [\Delta s \ \Delta E'_q \ \Delta E'_d]^T \\ u = [\Delta v_{dr} \ \Delta v_{qr}]^T \\ d = [\Delta V_{ds} \ \Delta V_{qs} \ \Delta P_m]^T \end{cases} \tag{6}$$

$$\begin{cases} f(x) = \begin{bmatrix} \frac{1}{2H} \left(\frac{V_{ds,e}}{X'} \Delta E'_q - \frac{V_{qs,e}}{X'} \Delta E'_d \right) \\ -\omega_s (\Delta s + s_e) \Delta E'_d - \omega_s E'_{de} \Delta s - \frac{1}{T'} \frac{X}{X'} \Delta E'_q \\ \omega_s (\Delta s + s_e) \Delta E'_q - \omega_s E'_{qe} \Delta s - \frac{1}{T'} \frac{X}{X'} \Delta E'_d \end{bmatrix} \\ g_1(x) = \begin{bmatrix} \frac{\Delta E'_q + E'_{qe}}{2HX'} - \frac{\Delta E'_d + E'_{de}}{2HX'} - \frac{1}{2H} \\ 0 \\ \frac{1}{T'} \frac{X - X'}{X'} \\ 0 \\ 0 \end{bmatrix} \\ g_2(x) = \begin{bmatrix} 0 & 0 \\ \omega_s \frac{L_m}{L_{rr}} & 0 \\ 0 & -\omega_s \frac{L_m}{L_{rr}} \end{bmatrix} \end{cases} \tag{7}$$

Where

$$\begin{aligned} X &= \omega_s L_{ss} \\ X' &= \omega_s (L_{ss} - L_m^2 / L_{rr}) \\ T' &= L_{rr} / \omega_s R_r \end{aligned}$$

and the turbine and generator's inertia constant is denoted by H ; the DFIG's active power output and the wind turbine's mechanical power are denoted by P_s and P_m respectively; the rotor slip and the synchronous angle speed are denoted by s and ω_s respectively; the d and q axis stator terminal voltages are denoted by V_{ds} and V_{qs} respectively; the mutual inductance, the rotor self-inductance, and the stator self-inductance are represented by L_m , L_{rr} , and L_{ss} respectively; the rotor circuit time constant is represented by T' ; the stator reactance and transient reactance are denoted by X and X' respectively; R_r represents the rotor resistance; E'_d and E'_q represent the d and q axis voltages behind the transient reactance respectively; the stator currents are denoted by I_{ds} and I_{qs} ; V_{dr} and V_{qr} represent the rotor voltages.

By using the robust ISS controller, the doubly fed wind generator model can be formulated as follows.

$$\dot{x} = f(x) + g_1(x)d + g_2(x) \left(-\beta g_2^T V_x + \xi \right) \quad (8)$$

Then the model of DFIG with ISS controller can be formulated in the form of system (3) with a new g_2 function which will be denoted by $g'_2(x)$ in this paper, where $g'_2(x)$ is represented as follows, $V_{g_2}^1$ and $V_{g_2}^2$ are two elements in $g_2^T V_x$, and $\xi = k \left[-V_{g_2}^2 \ V_{g_2}^1 \right]^T$.

$$g'_2(x) = \begin{bmatrix} 0 & 0 \\ \omega_s \frac{L_m}{L_{rr}} \times (-V_{g_2}^1) & -\omega_s \frac{L_m}{L_{rr}} \times V_{g_2}^2 \\ (-\omega_s \frac{L_m}{L_{rr}}) \times (-V_{g_2}^2) & \left(-\omega_s \frac{L_m}{L_{rr}} \right) \times V_{g_2}^1 \end{bmatrix} \quad (9)$$

And the system's new control input can be denoted by $v = [v_1 \ v_2]^T = [\beta \ k]^T$, where the range of β is within the set $\left(\max \left(0, 2 \frac{\nabla V_f + |\nabla V_{g_1}| \rho^{-1}(|x|)}{\nabla V_{g_2}(\nabla V_{g_2})^T} \right), +\infty \right)$. Thus, the problem is to design an auxiliary controller for the new system model to update the control inputs and enhance the dynamic performance of the wind generation system.

III. THEORETICAL PRELIMINARIES OF SDRE METHOD

A. CONTROL OBJECTIVE

The SDRE approach studies the following affine non-linear system.

$$\dot{x} = f(x) + g(x)u \quad (10)$$

where $x \in R^n$, input $u \in R^m$, and $g(x) \neq 0$ for $\forall x$.

In this paper, an infinite horizon performance index is considered as follows.

$$J(x, u) = \frac{1}{2} \int_0^\infty \left[x^T Q(x)x + u^T R(x)u \right] dt \quad (11)$$

where weighting matrices Q and R are assumed to be state dependent and positive semidefinite. The problem is to design a feedback control law $u = k(x)$, such that the performance index (11) is minimized.

B. SDC PARAMETERIZATION AND SDRE CONTROLLER DESIGN

The optimal control problem can be formulated as a problem of solving Hamilton-Jacobi-Bellman (HJB) partial differential equation, which is quite difficult to solve in nonlinear form [28]. The SDC parameterization [29] is proposed to transform a nonlinear system into a linear-like structure, such that the characteristics of linear systems can be obtained in controller design.

The extend linearization of (10) can be formulated as follows.

$$\begin{cases} \dot{f}(x) = A(x)x \\ \dot{g}(x) = B(x) \end{cases} \quad (12)$$

where $A(x)$ is called state-dependent coefficients (SDC), and can be represented as follows [30].

$$A(x) = \int_0^1 \frac{\partial f}{\partial x} \Big|_{x=\lambda x} d\lambda \quad (13)$$

For a one-dimensional system, there exist an unique SDC parameterization denoted by $A(x) = f(x)/x$. While for multivariable systems, there exist infinite number of SDC parameterization.

Remark 2: The non-uniqueness of SDC parameterization for multivariable nonlinear systems gives an extra freedom in designing SDRE controller by choosing different values of SDC.

By using the SDC parameterization, the nonlinear systems is formulated as $\dot{x} = A(x)x + B(x)u$. The nonlinear control problem for the affine nonlinear system (10) can be treated in the similar way as LQR method [31]. In order for the direct application of SDRE approach, the following conditions should be satisfied.

- 1) $f(x)$ is continuous and differentiable on Ω , and $B(x) \in C^0(\Omega)$, where Ω denotes the domain of system states.
- 2) The equilibrium point of system (10) with zero input is 0.
- 3) The weighting matrices should satisfy $Q(x) = Q^T(x) \geq 0$, $R(x) = R^T(x) > 0$, and $Q, R \in C^0(\Omega)$.
- 4) The SDC parameterizations of the nonlinear system (10) should be pointwise stabilizable and detectable for $\forall x$.

Remark 3: The fourth condition is important for the application of SDRE method. A sufficient test for the stabilizability and detectability is to check the ranks of the controllability matrix $M_c(x)$ and the observability matrix $M_o(x)$, where $M_c = [B(x) | A(x)B(x) | \dots | A^{n-1}(x)B(x)]$, $M_o = [C^T(x) | A^T(x)C^T(x) | \dots | A^{n-1}(x)C^T(x)]$, and $C^T(x)C(x) = Q(x)$. Since $Rank(M_c)$ and $Rank(M_o)$ are required to be equal to n for the entire domain of system states, it takes time to check these conditions. By applying the ISS controller as the main control structure as discussed in Section II, the stabilizability and detectability will be guaranteed automatically with no need to check the fourth condition.

By formulating the nonlinear system (10) in SDC form (12), the following optimal control law can be obtained

with the similar structure of LQR formulation.

$$u = -R^{-1}(x)B^T(x)P(x)x \quad (14)$$

where $P(x)$ is obtained through solving the following State-Dependent Riccati Equation

$$P(x)A(x) + A^T(x)P(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \quad (15)$$

From the above discussions, the SDRE approach can take full consideration of the system's nonlinearities and capture the characteristics of linear systems by performing SDC parameterization, which results in a computationally simple and effective control design for online applications. The detailed SDRE controller design for the DFIG system will be discussed in the next section.

IV. SDRE BASED AUXILIARY CONTROL FOR DFIG

In this section, an SDRE based state feedback auxiliary controller is proposed to enhance the LVRT ability of DFIGs and its structure diagram is depicted in Fig 1. The ISS controller is used to guarantee the stability of the wind generation system under external disturbances, and the SDRE controller acts as an auxiliary control for online tuning the ISS control parameters. The detailed description on the selection of SDC parameterization, the selection of weighting matrices, protection circuits, and the flowchart of the proposed control design approach are presented in the following paragraphs.

A. SDC PARAMETERIZATION

As discussed in Section III-B, multivariable systems have infinite number of SDC parameterization, and can be represented as follows.

$$A(\alpha, x) = \left[\prod_{j=1}^{n-1} (1 - \alpha_j) \right] A_1(x) + \sum_{i=2}^{n-1} \alpha_{i-1} \left[\prod_{j=i}^{n-1} (1 - \alpha_j) \right] A_i(x) + \alpha_{n-1} A_n(x) \quad (16)$$

where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_{n-1})^T$ is the weighting coefficient vector with $\alpha_i \in \Omega_\alpha \subseteq R$, and $A_1(x), \dots, A_n(x)$ are n distinct SDC matrices.

For example, assume one nonlinear element of a system is denoted by x_1x_2 . The SDC parameterization can be obtained by factoring x_1 as $(1 - \alpha)x_1 + \alpha x_1$, such that $x_1x_2 = (1 - \alpha)x_1x_2 + \alpha x_1x_2 = A'(\alpha, x)x$, where $x = [x_1, x_2]^T$, $A'(\alpha, x) = \{[a_1, a_2]\}$ with $a_1 = (1 - \alpha)x_2$, $a_2 = \alpha x_1$. Thus, a practical way to realize SDC parameterization is to parameterize and apportion every nonlinear elements in $f(x)$ [21].

By using the above practical SDC parameterization, the SDC matrix $A(\alpha, x)$ of DFIG with ISS controller can be

obtained as follows.

$$A(\alpha, x) = \begin{bmatrix} 0 & \frac{1}{2H} \cdot \frac{V_{dse}}{X'} & -\frac{1}{2H} \cdot \frac{V_{qse}}{X'} \\ -E'_{de} - (1 - \alpha_1)\Delta E'_d & -\frac{1}{T'} \cdot \frac{X'}{X'} & -s_e - \alpha_1 \Delta s \\ -E'_{qe} + (1 - \alpha_2)\Delta E'_q & s_e + \alpha_2 \Delta s & -\frac{1}{T'} \cdot \frac{X'}{X'} \end{bmatrix} \quad (17)$$

B. SELECTION OF COEFFICIENTS

The SDC coefficient α in (17) will be chosen in the range $[0, 1]$. The control effort is directly linked to the pointwise controllability of the SDC parameterization. Thus, the value of SDC coefficient α is selected to maximize the pointwise controllability. The controllability matrix is denoted by $M_c(x)$ as discussed in III-B, then the choice of α which gives the largest value of $|\det(M_c M_c^T)|$ at each iteration throughout the entire domain will be used in the controller design.

In order for a desired control performance, the matrices $Q(x)$ and $R(x)$ can be either constant or functions of system states. For example, $Q(x)$ can be chosen as an increasing function and $R(x)$ can be chosen as a decreasing function with respect to x , such that the control effort at the origin is saved. To realize the minimization of control objective (11), the corresponding Hamiltonian function is required to be convex. The following theorem gives a sufficient condition for the convexity of (11).

Theorem 2 [32]: The function $l(x) = x^T Q(x)x$ is globally convex with respect to x by selecting $Q(x) = Q_0 + Q_1(x)$, where $Q_0 = \text{diag}\{c_{11}, \dots, c_{nn}\}$ with $c_{ii} > 0$, and $Q_1(x) = \text{diag}\{q_1(x_1), \dots, q_n(x_n)\}$ with $q_i(x_i) = \sum_{j=1}^{p_i} a_{ij}x_i^{2j}$, and $a_{ij} \geq 0$.

From the above discussion, the state-dependent weighting matrices $Q(x)$ and $R(x)$ are selected as follows.

$$\begin{cases} Q(x) = \text{diag}(50, 100, 100) + Q_1(x) \\ Q_1(x) = 0.1 \text{diag}(q_1(x_1), q_2(x_2), q_3(x_3)) \\ q_i(x_i) = (x_i^2 + x_i^4), i = 1, 2, 3 \\ R(x) = \text{diag}(1, 1) \end{cases} \quad (18)$$

C. SELECTION OF OPERATIONAL LIMITS AND PROTECTION CIRCUITS

The rotor voltage V_{dr} and V_{qr} may exceed the feedback converter's voltage capacity limit under external perturbations. Thus, it is necessary to regulate V_{dr} and V_{qr} to meet the converter's capacity limit. In this paper, the maximum rotor voltage limit is denoted by V_{r_max} . Considers the d/q-axis output rotor voltages of the proposed controller is denoted by V_{dr}^* and V_{qr}^* , V_{dr} and V_{qr} are limited as $V_{dr}^* V_{r_max} / V_r^*$ and $V_{qr}^* V_{r_max} / V_r^*$ respectively once the rotor voltage exceeds its maximum limit, where $V_r^* = \sqrt{V_{dr}^{*2} + V_{qr}^{*2}}$.

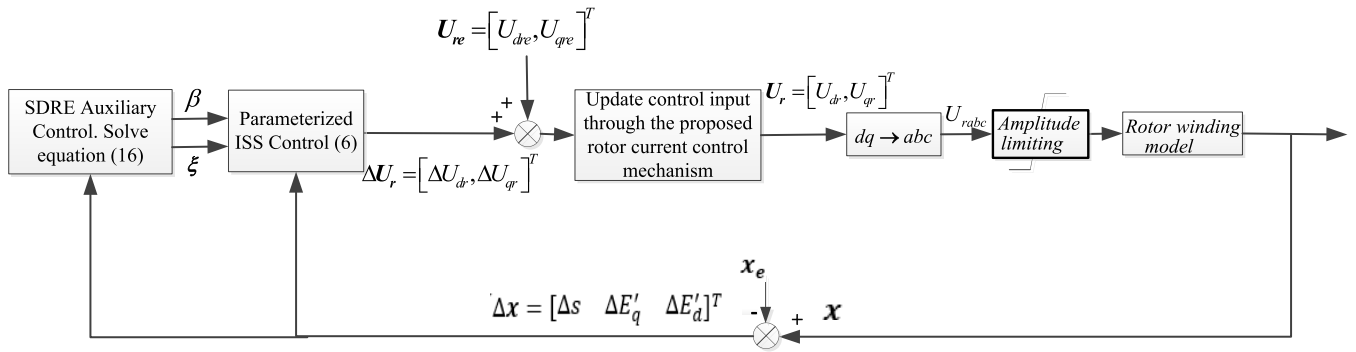


FIGURE 1. The structure diagram of the proposed SDRE based control.

When grid fault occurs, the voltage dip will cause over-current in the rotor windings, especially under severe fault conditions. Thus, crowbar circuit should be considered to protect the DFIG from being damaged. Since the large overcurrent stimulated by the stator flux linkage usually occurs at the initial time of the fault. An active crowbar circuit [33] is applied with the proposed auxiliary control strategy. By doing this, the proposed SDRE based auxiliary control approach can be regained during the transient to provide reactive power support and enhance the dynamic performance of the system. In this paper, the active crowbar circuit is activated when the rotor current exceeds its maximum limit, and quits after about 1.5 system cycles (0.025s).

In order to guarantee the rotor current to be within its maximum limit during all fault period, the following rotor current control mechanism is applied during the transient when the rotor current exceeds a certain threshold value. Based on the DFIG model in Section II, the rotor current can be estimated as follows.

$$I_{r_estimate} = \sqrt{\left(\frac{V_{dr}}{s} - V_{ds} - X_s I_{qs}\right)^2 + \left(\frac{V_{qr}}{s} - V_{qs} + X_s I_{ds}\right)^2} / X_r \quad (19)$$

where R_r is omitted for brevity.

Let I_{r_mea} denotes the measured rotor current, I_{r_thr} denotes the threshold value of rotor current, and I_{r_max} denotes the maximum limit of rotor current. In this paper, the I_{r_thr} is selected as $(1 - a\%) I_{r_max}$, such that a security margin is guaranteed to avoid rotor overcurrent during the transient. The main point of the proposed rotor current control mechanism is regulating $I_{r_estimate}$ around I_{r_thr} if $I_{r_mea} > I_{r_thr}$. Thus, V_{dr} and V_{qr} are tuned proportionally as follows when $I_{r_mea} > I_{r_thr}$: $V_{dr} = V_{dr}^*/k + s(1 - 1/k)V_{ds} + sX(1 - 1/k)I_{qs}$, and $V_{qr} = V_{qr}^*/k + s(1 - 1/k)V_{qs} + sX(1 - 1/k)I_{ds}$, where V_{dr}^* and V_{qr}^* represent the original control output, and k represents the ratio of I_{r_mea} to I_{r_thr} .

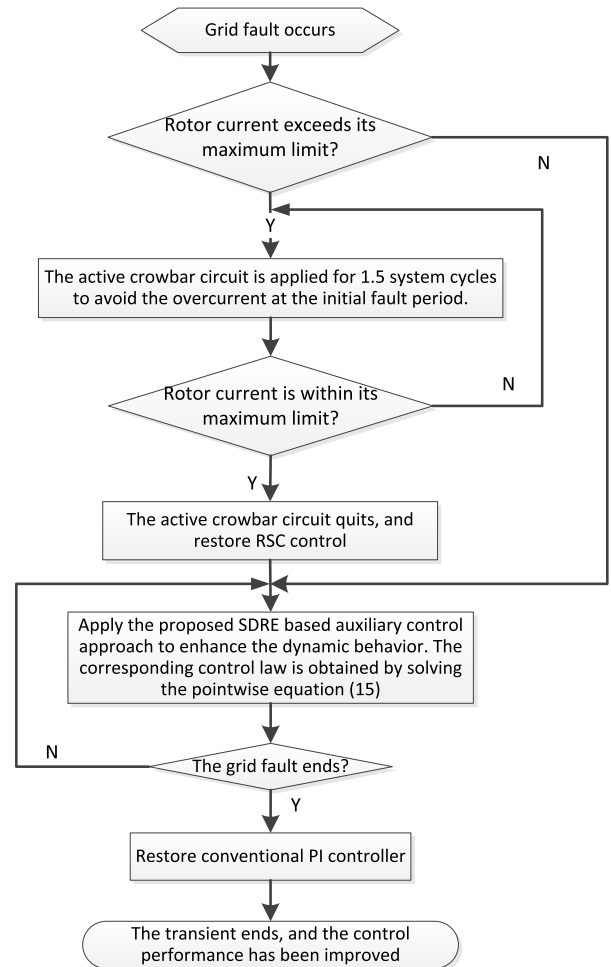


FIGURE 2. Flowchart of the proposed control scheme.

D. FLOWCHART OF THE PROPOSED SDRE BASED CONTROL

The flowchart of the proposed SDRE based auxiliary control is depicted in Fig. 2. The active crowbar circuit is applied to avoid the overcurrent at the initial fault period, while the rotor current control mechanism proposed in IV-C is used to avoid rotor overcurrent and guarantee a certain security margin

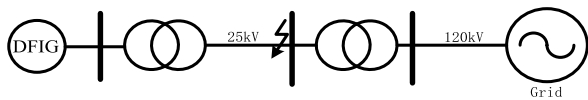


FIGURE 3. The structure of SMIB system with DFIG based wind farm.

during the transient. The proposed SDRE based auxiliary control is applied during the fault period to enhance the dynamic behavior and the LVRT ability of the DFIG based wind generation system. At each time instant, the SDC matrices are regarded as constant, and a control law is calculated by solving the equation (15).

V. CASE STUDY

In this section, the proposed SDRE based auxiliary controller design approach is verified through time domain simulations. Tests on a Single Machine Infinite Bus (SMIB) system and a 4-machine-2-area system are considered under fault conditions. The proposed SDRE based auxiliary controller is implemented in Matlab/Simulink and the simulations are performed in Matlab R2016a.

In this paper, the maximum limit of rotor current is selected as $2 p.u.$ [33]. That is, I_{r_max} is selected as $2 p.u.$ in this paper, while I_{r_thr} is selected as $1.75 p.u.$ to guarantee a 12.5% security margin.

A. TESTS ON A SINGLE MACHINE INFINITE BUS SYSTEM

The structure of the test system is depicted in Fig.3 with a 9 MW wind farm.

The PI controller with active crowbar, and the original ISS controller are simulated respectively. The control parameters of the original ISS controller are selected manually as follows.

$$\beta = 2 \cdot \max \left(0, 2 \frac{V_x^T f + |V_x^T g_1| \rho^{-1}(|x|)}{V_x^T g_2 g_2^T V_x} \right) + 8$$

$$\xi = 9.5 [V_{g_2}^2 \quad -V_{g_2}^1]^T \tag{20}$$

The conventional PI control parameters are selected based on General Electric Company’s 1.5 MW wind generator in MATLAB/Simulink R2016a. Case studies on a severe and a light fault conditions are considered and discussed in the following paragraphs.

Case 1: A three phase to ground fault is applied from $t = 0s$ to $0.1s$, and the corresponding grounding resistance is 0.5Ω .

The proposed SDRE based controller is simulated and compared with the conventional PI controller as well as the original ISS controller. The dynamics of the wind farm’s rotor current, active power, reactive power, terminal voltage, and rotor speed are depicted in Fig. 4-8 respectively.

The two horizontal dotted lines in Fig. 4 denote the maximum limit I_{r_max} and the threshold value I_{r_thr} respectively. It can be seen from Fig. 4 that the rotor current exceeds its maximum limit at the initial fault period, thus the active crowbar circuit is applied to reduce the rotor overcurrent.

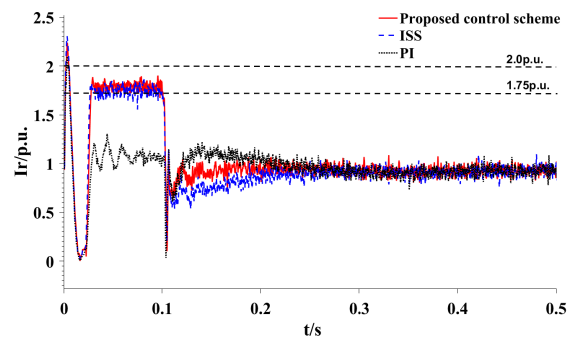


FIGURE 4. The rotor current of DFIG.

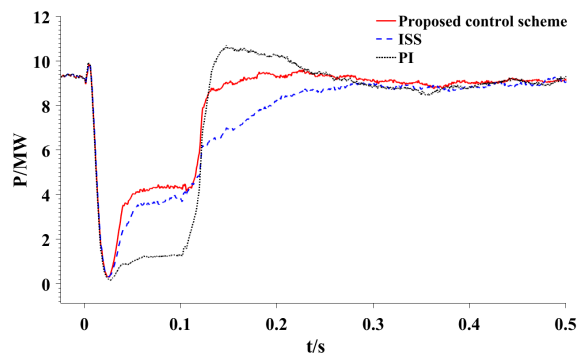


FIGURE 5. The output active power of wind farm.

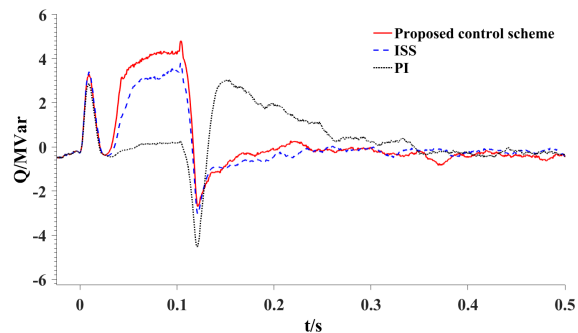


FIGURE 6. The output reactive power of wind farm.

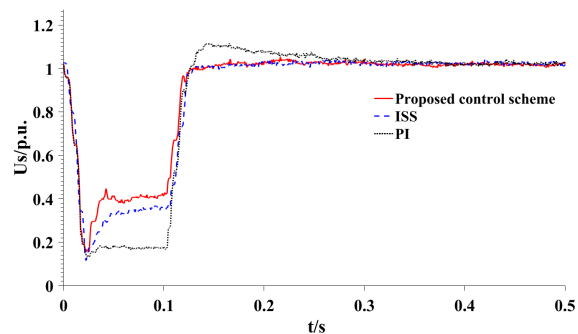


FIGURE 7. The terminal voltage of wind farm.

After 0.025s, the active crowbar circuit quits to regain the converter control. Based on the rotor current control mechanism, the rotor current is controlled around the threshold value during the transient period.

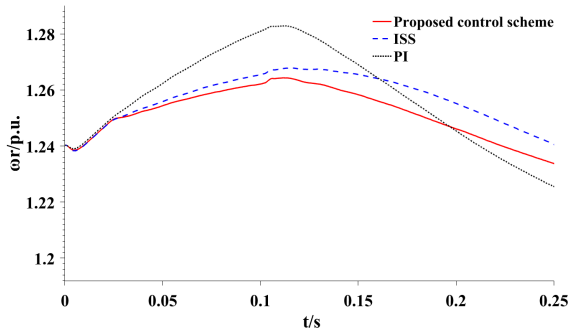


FIGURE 8. The rotor speed of wind turbine.

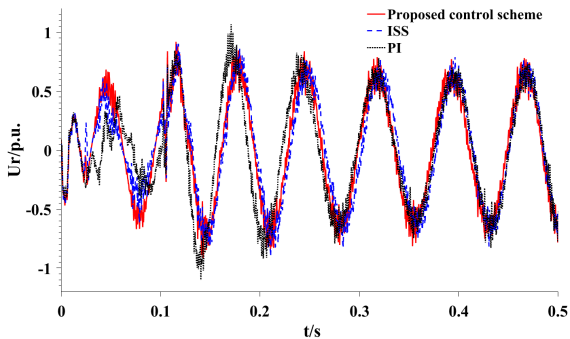


FIGURE 9. The instantaneous values of the RSC's output voltage U_r .

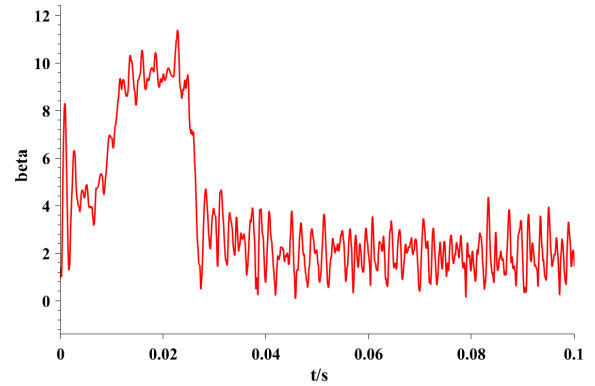


FIGURE 10. The values of β optimized by SDRE during the transient.

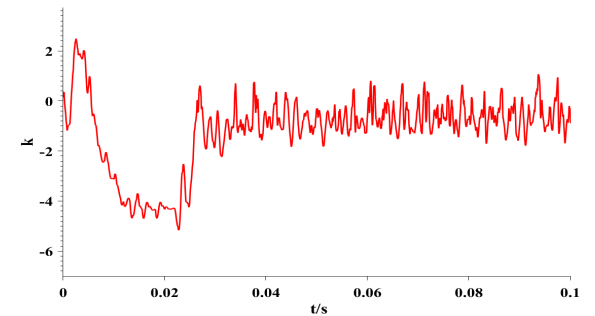


FIGURE 11. The values of k_e optimized by SDRE during the transient.

From Fig. 5, the proposed SDRE based auxiliary control provides more active power than those with conventional PI controller and original ISS controller during the transient period. It's beneficial for the operation of DFIG, since more active power can avoid large fluctuation of rotor speed as shown in Fig. 8. From Fig. 6, more reactive power is generated with the proposed SDRE based auxiliary control during the transient, which results in a less voltage dip after regaining the converter control as shown in Fig. 7. It can be concluded from Fig. 4-8 that better dynamic behavior and voltage support ability are achieved by applying the proposed SDRE based control.

To show the mechanism of the proposed SDRE based controller, the instantaneous values of the RSC's output voltage U_r is depicted in Fig. 9. The control parameters β and k are tuned online by the SDRE approach, of which the adjusting process are depicted in Fig. 10-11.

It can be seen from Fig. 9 that based on the SDRE control the output voltage of RSC is larger than those with conventional PI controllers and original ISS controller. The mechanism of proposed SDRE based controller is similar to the strong excitation control system for the synchronous generator. Once a voltage dip happens, the RSC's output voltage is controlled to increase, which results in the increase of rotor current and power to enhance the dynamic behavior of the DFIG based wind generation system. The parameters of the original ISS controller are tuned online to obtain a better control performance through solving the state-dependent

riccati equation as discussed in III-B, and the parameters tuning process is depicted in Fig. 10-11.

Case 2: The fault duration, condition, and location are the same as Case 1, while the corresponding grounding resistance is selected as 4Ω .

The dynamics of the wind farm's terminal voltage, reactive power, rotor current are depicted in Fig. 12-14 respectively.

In this case, the fault is relatively small compared with the fault in case 1. From Fig. 14, the overcurrent does not occur due to this light fault. Thus, the active crowbar circuit is not activated, the rotor current is controlled by the proposed rotor current control mechanism during the transient. It can be seen from Fig. 12-13 that the proposed SDRE control method can provide more reactive power support for the DFIG, and a less voltage sag is obtained as a consequence.

Based on the above discussion, a performance index named integral of squared-error (ISE) [34] is presented here to further compare the control performance. The definition of ISE is presented as follows.

$$ISE = \int_{t_0}^{t_f} \left(\frac{x - x_e}{x_e} \right)^2 dt \quad (21)$$

where x_e denotes the equilibrium point, t_0 and t_f denote the initial time and the terminal time. In this paper, $t_0 = 0s$, and $t_f = 1s$.

The ISE of terminal voltages are illustrated in Table 1 with different controllers. It can be seen from Table 1 that better voltage support is achieved by using the proposed SDRE

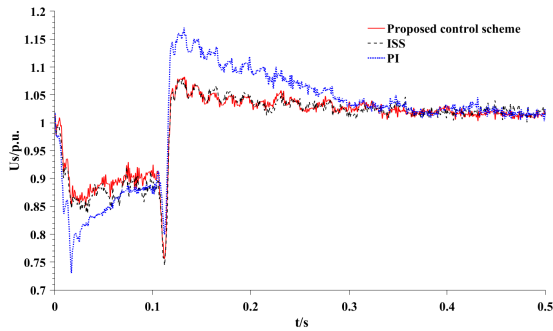


FIGURE 12. The terminal voltage of wind farm.

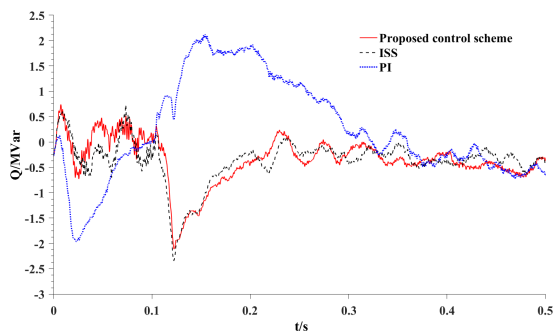


FIGURE 13. The output reactive power of wind farm.

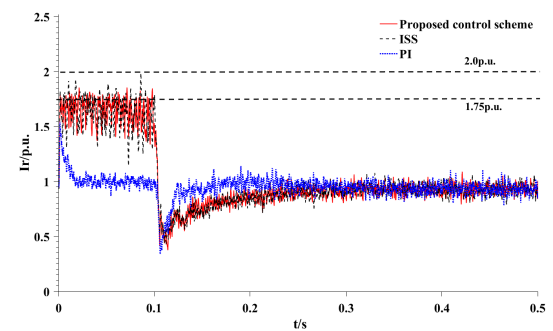


FIGURE 14. The rotor current of DFIG.

TABLE 1. ISE of terminal voltages in two cases.

Variable	PI	ISS	SDRE+ISS
U_s (case 1)	0.06629268	0.04618607	0.03856443
U_s (case 2)	0.00460399	0.00253181	0.00212785

based auxiliary control approach with a relatively smaller ISE value.

B. TESTS ON A 4-MACHINE-2-AREA SYSTEM

A 4-machine-2-area test system is used in this section, whose structure is depicted in Fig. 15. The parameters of each device in the four-machine system can be found in SimPowerSystems of MATLAB R2016a. A three phase to ground fault is applied at tie line 1 at $t = 0$ s. The corresponding grounding resistance is 0.01Ω and the fault lasts for 0.15 s.

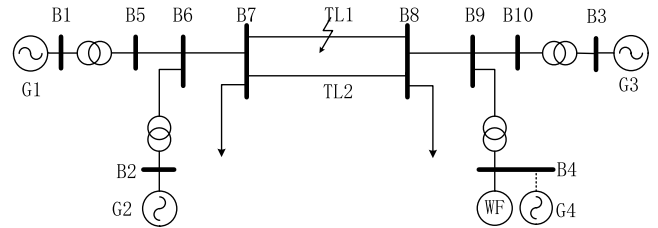


FIGURE 15. The 4-machine-2-area test system.

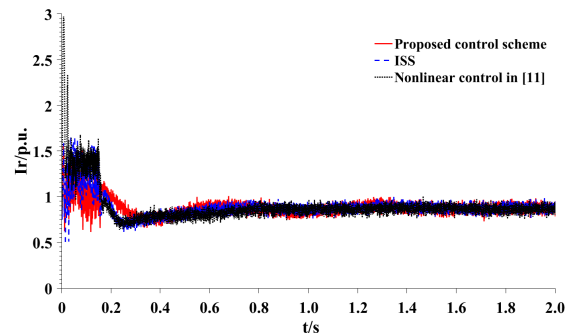


FIGURE 16. Rotor current of DFIG.

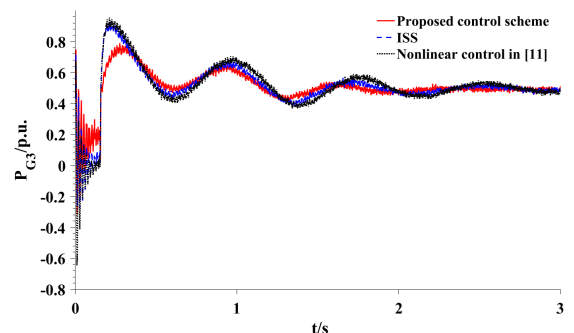


FIGURE 17. Active power output of G3.

TABLE 2. Comparison of ISE indexes.

Variable	Nonlinear control in [11]	ISS control	Proposed control
U_{B4}	0.01576466	0.01556443	0.00897467
U_{B8}	0.03847152	0.03647224	0.03141805
I_r	0.10984424	0.04480354	0.02161023
P_{G3}	0.38004031	0.29141596	0.16471570
P_{tl2}	0.40411133	0.22014353	0.14468179

The proposed SDRE based control is compared with the exact linearization based nonlinear control proposed in [11] and the original ISS control. The dynamics of rotor current, terminal voltage of B4 and B8, active power of generator G3, and active power of tie line 2 are depicted in Fig. 16-20 respectively.

It can be seen from Fig. 16-20 that better voltage dynamics can be achieved and the active power oscillations are better damped by the proposed SDRE based controller.

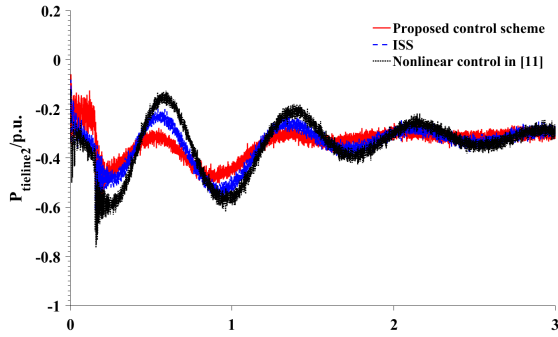


FIGURE 18. Active power on tie line 2.

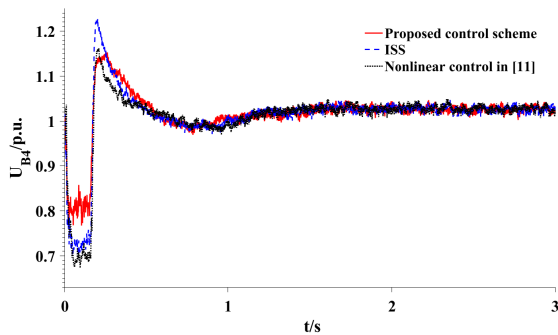


FIGURE 19. Voltage of bus 4.

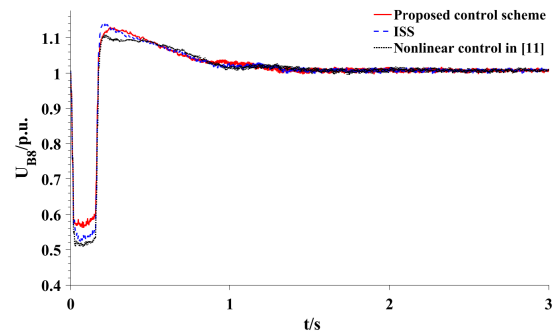


FIGURE 20. Voltage of bus 8.

The ISE indexes with respect to the variables in Fig. 16-20 are shown in Table 2 to further illustrate effectiveness of the proposed control scheme.

VI. SUMMARY OF CONTRIBUTIONS

This paper proposed a state-dependent Riccati equation (SDRE) based auxiliary control design approach to the performance and low voltage ride through (LVRT) ability enhancement of doubly fed induction generator (DFIG) based wind generation system. Based on the Input-to-State Stability (ISS) theory, the mathematical model of doubly fed wind generator with parameterized ISS controller is presented in Section II. The SDRE control method is presented in Section III of which the direct application is ensured by fully discussed conditions. The SDRE design process is proposed

in Section IV. The protection circuit as well as the rotor current control mechanism are presented to avoid overcurrent at the initial fault period and during the fault period. Time domain simulations on a test system are presented, and the results verify the effectiveness of the proposed method.

The benefit of the proposed SDRE based auxiliary control is its simplicity as well as the effectiveness in controller design, since there is no need to solve the Hamilton-Jacobi-Bellman (HJB) equation. By combining with ISS control approach, the conditions for applying SDRE are guaranteed. Thus, the controllability check for all states and times is not required. Also, the state-dependent coefficient (SDC) parameterizations give an extra freedom in selecting the ‘best’ SDC matrices that achieve a better controller performance.

APPENDIX I

Definitions of Comparison Functions [22]:

The definition of ISS is based on a series of comparison functions, and the definitions of \mathcal{K} , \mathcal{K}_∞ , and \mathcal{KL} functions are described as follows.

$$\mathcal{K} := \left\{ \begin{array}{l} \gamma : R_{\geq 0} \rightarrow R_{\geq 0} | \text{continuous,} \\ \text{strictly increasing, and } \gamma(0) = 0 \end{array} \right\}$$

$$\mathcal{K}_\infty := \{ \gamma \in \mathcal{K} | \gamma(s) \rightarrow \infty \text{ as } s \rightarrow \infty \}$$

$$\mathcal{KL} := \left\{ \begin{array}{l} \beta(s, t) : R_{\geq 0} \times R_{\geq 0} \rightarrow R_{\geq 0} | \text{continuous,} \\ \text{for any fixed } t \geq 0, \beta(\cdot, t) \in \mathcal{K} \\ \text{for any fixed } s \geq 0, \beta(s, \cdot) \rightarrow 0 \text{ as } t \rightarrow 0 \end{array} \right\}$$

APPENDIX II

Parameters of the Doubly Fed Wind Generator:

$$S_B = 1.67 \text{MVA}, V_B = 575 \text{V}, H = 0.685 \text{ s},$$

$$R_s = 0.023 \text{ p.u.}, L_s = 0.18 \text{ p.u.}, L_m = 2.9 \text{ p.u.}, R_r =$$

$$0.006 \text{ p.u.}, L_r = 0.16 \text{ p.u.}, L_{ss} = L_s + L_m, L_{rr} = L_r + L_m.$$

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