IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received March 30, 2018, accepted May 10, 2018, date of publication May 21, 2018, date of current version July 12, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2839186

Reconstruction of Signals From Level-Crossing Samples Using Implicit Information

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This work was supported by the Polish National Center of Science under Grant DEC-2012/05/E/ST7/01143.

ABSTRACT This paper focuses on the recovery of bandlimited signals from level-crossing samples by exploiting not only the knowledge of when given levels are crossed by the input, but also the implicit information that the signal stays between neighboring levels in the time intervals between the level crossings. We propose to use the technique of projection onto convex sets (POCS) for perfect signal reconstruction from either the level crossings or the associated implicit information. Two POCS algorithms are proposed: iterative POCS and one-step POCS. While the one-step POCS is based on matrix inversion, the iterative projections can be implemented using a chain of standard circuit operations: a low-pass filter and a clipping circuit, respectively. The perfect signal recovery of the infinite projection iteration can be viewed as the completion of the nonperfect input reconstruction achieved in continuous-time digital signal processing from level crossings. The comparative analysis of simulation results for both iterative and one-step POCS algorithms show the importance of a good selection of the initial guess for the POCS reconstruction.

INDEX TERMS Level-crossing sampling, signal reconstruction, projection onto convex sets, event-based signal processing.

I. INTRODUCTION

Level-crossing sampling (LCS) is a central concept of event-based signal processing (EBSP). LCS has been adopted in various areas of EBSP such as Continuous-Time Digital Signal Processing (CT-DSP) [1], as well as the design of level-crossing analog-to-digital converters [2]–[4], and level-crossing digital filters [5]. This sampling scheme provides the time instants and amplitude values of the input crossing points with given reference levels in the amplitude domain. LCS was introduced to the signal processing literature in [6] although a similar concept was earlier laid for the foundation of asynchronous delta modulation [7]. The rate of level crossing is higher when the signal varies rapidly, and lower when it changes slowly. The mean level-crossing rate is a function of the power spectral density of the sampled signal and directly depends on the signal bandwidth. Furthermore, the level-crossing rate gives a rich information about signal concentration in the frequency domain and may be used for bandwidth estimation [8]. The concept of level-crossing sampling has been also applied in statistical signal processing for hypothesis testing [9]. On the other hand, LCS with hysteresis known also as Lebesgue sampling, or send-on-delta scheme [10], [11], provides the basis for event-based control and communication aimed at efficient utilization of computation and network system resources [12], [13].

Event-based signal processing (EBSP) is inherently associated with the time encoding of event instants (e.g., level crossings) [12]. From the perspective of continuous downscaling of VLSI technology, EBSP provides an efficient alternative to the conventional signal processing techniques based on uniform sampling. Modern nanoscale CMOS technology requires low supply voltage, which makes the fine quantization of the amplitude increasingly difficult. On the other hand, in a deep-submicron CMOS process, the resolution of digital event timestamping is superior to the voltage resolution of an analog signal [14]. The techniques of encoding signals in time instead of in amplitude is expected to be further improved by advances in chip fabrication technology. Finally, the event-based signal processing systems are characterized by activity-dependent power consumption, and energy savings at the circuit level [12].

A. METHODS OF SIGNAL RECOVERY FROM LCS

One of the main research issues with LCS is the recovery of an original signal from its level crossings. Usually, the widespread bandlimited model of the signal is assumed, with no frequency components above the Nyquist frequency. In CT-DSP, real time signal recovery is performed by lowpass filtering an initial continuous-time piecewise-constant [1], or a piecewise-linear approximation of the input signal created from the level crossings [15]. Because LCS is a nonlinear operation, it will not be sufficient to use a linear operation such as a lowpass filter to achieve perfect reconstruction [16]. This method however owes its success to its simple implementation, its potential for real-time operation and its acceptable accuracy for many applications.

In fact, LCS can be considered as a special way to acquire nonuniform samples. It is known (see e.g. [17]) that perfect recovery of bandlimited signal from nonuniform samples is possible when the mean sampling rate exceeds the Nyquist rate. References [18]–[21] provide methods that can potentially achieve this reconstruction. However, this recovery is perfect only at the limit of infinite computation complexity. To allow finite computation, one only aims at a certain level of precision. The presence of long time intervals between samples is one of the main difficulties of this task [22], [23].

In this paper, we address the problem of the recovery of bandlimited signals from level-crossing samples by exploiting not only the knowledge of the samples thus acquired, but also the expected amplitude behaviour of the input between the samples with respect to the thresholding levels.

B. IMPLICIT INFORMATION IN EVENT-BASED SAMPLING

The input signal representation by level-crossing samples gives the time instants when the relevant levels are crossed but also implies that the signal stays between neighboring levels during the time between the level crossings. The extra knowledge of the signal behavior between the sampling instants is inherently connected to event-based sampling and called the *implicit information* to distinguish it from the plain sample data considered as the *explicit information* [12]. The notion of implicit information results directly from the definition of event-based sampling: since the signal is sampled at every event occurrence, it is evident that no event occurs between consecutive sampling instants [13]. While the samples provide the information of discrete-time equality constraints satisfied by the input signal, the implicit information consists of inequalities in continuous time. The role of the implicit information grows when the temporal distance between the samples increases and is of particular impact when this distance is locally larger than the Nyquist period. The techniques currently used for signal recovery from level crossings are classical methods of signal reconstruction from nonuniform samples that only deal with discrete-time equality constraints and are unable to incorporate information from continuous-time inequalities [18], [21]. The development of effective event-based signal processing techniques calls for approaches capable of integrating both continuous-time and discrete-time information from the event-based sampling.

C. PAPER CONTRIBUTION

We propose to use the technique of Projection Onto Convex Sets (POCS) for bandlimited signal reconstruction from either the level crossings, or the implicit information resulting from LCS. The POCS method was previously used for signal recovery from nonuniform samples [24], and for image reconstruction [17] from level crossings [25]. This paper is an extended version of [26], which has presented preliminary results on POCS-based recovery from level crossings including implicit information. Two POCS algorithms are proposed: iterative POCS and one-step POCS. In the first algorithm, the implicit information is incorporated both in the amplitude-domain projection and in the choice of initial estimate. In the second algorithm, the implicit information is only used in the initial estimate to be projected.

The iterative POCS algorithm alternates projections onto two sets. One set is defined by the *a priori* frequency knowledge of bandlimitation of the input, and the other set results from the level-crossing information and consists of amplitude constraints. Alternating projections between constraints in the frequency domain and constraints in the amplitude domain due to encoding information was conceptually introduced in [25] and [27]. As will be shown, the projections on these two convex sets can be implemented using standard circuit operations: a lowpass filter and a clipping circuit, respectively. Furthermore, there is a strong relation between the iterative POCS algorithm and the CT-DSP technique. The perfect signal recovery of the infinite projection iteration can be viewed as the completion of the nonperfect input reconstruction achieved in CT-DSP from level crossings. The preliminary piecewise constant [1] (or linear [15]) approximation of the input performed in CT-DSP from the level crossings corresponds to the initial estimate of the POCS algorithm, and the second operation of lowpass filtering in CT-DSP plays the role of the first projection in the POCS approach, with its abstract interpretation as orthogonal projection onto the space of bandlimited signals.

The one-step POCS algorithm consists in a single projection onto a set defined by the explicit information. The main complexity of this method lies in the inversion of a matrix. The comparative analysis of the iterative and the one-step algorithm shows the importance of a good selection of the initial guess for the final reconstruction.

The paper is organized as follows. Section I gives the formal definitions of explicit and implicit information in LCS. In Section II, the framework of POCS for LCS is presented. The dependence of the rate of convergence and performance of the POCS algorithm with the initial guess is discussed in Section III. Simulation results for both iterative and one-step POCS schemes with performance analysis are reported in Section IV. Related work is discussed in Section V.

II. EXPLICIT AND IMPLICIT INFORMATION IN LCS

Level-crossing sampling is a type of event-triggered sampling, where each event is the crossing of a signal $x(t)$ with one of the levels $\Theta = \{\theta_1, \theta_2, \ldots, \theta_L\}$, where $\theta_1 < \theta_2 < \ldots < \theta_L$. The convention $\theta_0 = -\infty$ and $\theta_{L+1} = \infty$ will be used. The event time instants $\{t_n,\ n \in \mathbb{Z}\}$ are defined as

$$t_n = \min\left\{t > t_{n-1},\ x(t) \in \Theta,\ x(t) \text{ is not local extremum of } x(\cdot)\right\} \qquad (1)$$

Excluding the local extrema from the above definition implies that $x(t)$ must strictly cross one of the levels at each $t = t_n$. The output of the level-crossing sampler is the sequence of pairs $(t_n, x(t_n))$. This data constitutes what we call the *explicit information*. However, this data does not contain the *implicit information* that the signal $x(t)$ does not cross any level between consecutive sampling instants t_n, t_{n+1}. This can be written in form

$$\theta^-(t) \le x(t) < \theta^+(t) \qquad (2)$$

where $\theta^-(t)$ and $\theta^+(t)$ are the bounds shown in Fig.1. At each instant t they can be defined as

$$\begin{aligned}\theta^+(t) &= \min\{\theta_\ell > x(t),\ \theta_\ell \in \Theta\}\\ \theta^-(t) &= \max\{\theta_\ell \le x(t),\ \theta_\ell \in \Theta\}.\end{aligned} \qquad (3)$$

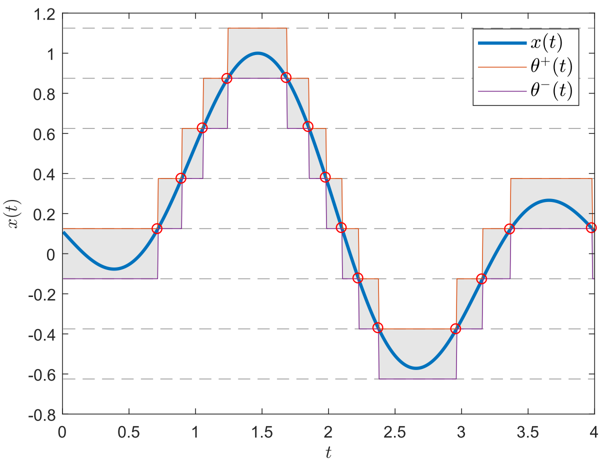

FIGURE 1. Level-crossing sampling.

The above functions can be also deduced solely from the sampling points $(t_n, x(t_n))$, provided that at least two levels are crossed. Since the sample amplitudes are known to belong to the set Θ, then there exists a sequence ℓ_n of level indices, such that

$$x(t_n) = \theta_{\ell_n}. \qquad (4)$$

The sign of the signal slope at t_n can be expressed as

$$s_n = \begin{cases}\ell_n - \ell_{n-1}, & \ell_n \ne \ell_{n-1}\\ -s_{n-1}, & \ell_n = \ell_{n-1},\end{cases} \qquad (5)$$

assuming that $\ell_m \ne \ell_{m-1}$ for some $m \le n$. Then in the interval $t \in [t_n, t_{n+1})$ we have

$$\theta^+(t) = \begin{cases}\theta_{\ell_n+1}, & s_n = 1\\ \theta_{\ell_n}, & s_n = -1\end{cases}$$

$$\theta^-(t) = \begin{cases}\theta_{\ell_n}, & s_n = 1\\ \theta_{\ell_n-1}, & s_n = -1.\end{cases}$$

III. RECONSTRUCTION USING PROJECTIONS ONTO CONVEX SETS

A. SETS CORRESPONDING TO LEVEL-CROSSING SAMPLES

When level-crossing samples $\{(t_n, x(t_n), n \in \mathbb{Z}\}$ are obtained from a bandlimited signal $x(t)$, the precise information that is available about $x(t)$ mathematically takes the form of set memberships of $x(t)$. The sets are as follows:

1) From the explicit information of the samples $\{(t_n, x(t_n)), n \in \mathbb{Z}\}$, $x(t)$ is known to belong to the set

$$\mathcal{E} = \{u(t) \in \mathcal{C}(\mathbb{R}) :\ u(t_n) = x(t_n) \text{ for all } n \in \mathbb{Z}\}. \qquad (6)$$

2) From the implicit information of (2) $x(t)$ is known to belong to the set

$$\mathcal{I} = \left\{u(t) \in \mathcal{C}(\mathbb{R}) :\ \theta^-(t) \le u(t) < \theta^+(t) \text{ for all } t \in \mathbb{R}\right\}. \qquad (7)$$

3) From its maximum frequency Ω, $x(t)$ is known to belong to the set

$$\mathcal{B} = \left\{u(t) \in \mathcal{L}^2(\mathbb{R}) :\ \forall|\omega| > \Omega,\ \int_{-\infty}^{+\infty} u(t)e^{-j\omega t}dt = 0\right\}. \qquad (8)$$

$\mathcal{C}(\mathbb{R})$ is used to denote set of continuous function.

The bandlimited, irregularly sampled signal is characterized only by the membership $x(t) \in \mathcal{B} \cap \mathcal{E}$. In general, this characterization does not define $x(t)$ uniquely. A bandlimited signal that interpolates the given level crossings may still violate the bounds $\theta^-(t) \le x(t) < \theta^+(t)$. On the other hand, any bandlimited signal satisfying the bounds $\theta^-(t) \le x(t) < \theta^+(t)$ must go through the given level crossings by continuity. Therefore, the set $\mathcal{I}$ is a subset of $\mathcal{E}$,

$$\mathcal{I} \subset \mathcal{E}, \qquad (9)$$

and using the constraint $\hat{x}(t) \in \mathcal{I}$ can improve the precision of the signal recovery. Next it is sufficient to use the constraint $\hat{x}(t) \in \mathcal{B} \cap \mathcal{I}$ instead of $\hat{x}(t) \in \mathcal{B} \cap \mathcal{E} \cap \mathcal{I}$, since the latter is redundant. If $\mathcal{B} \cap \mathcal{E}$ is limited to a single signal (for example, in the case of uniform sampling above the Nyquist rate) the constraint $\hat{x}(t) \in \mathcal{I}$ does not bring any additional information. This shows that level crossing provides information on the signal that can be useful in the case of sampling under the Nyquist rate, or in signal recovery from a finite number of samples.

B. PROJECTIONS ONTO CONVEX SETS

The membership of a signal to a set has a great impact when the set is convex.

Let us assume that a signal x (the notation $x := x(t)$ will be used for brevity) belongs to the Hilbert space $\mathcal{L}^2(\mathbb{R})$ of finite-energy signals. The energy of the signal x is $\|x\|^2 = \langle x, x\rangle$, where $\langle u, v\rangle = \int_{-\infty}^{+\infty} u(t)\overline{v(t)}dt$ is the inner product of $\mathcal{L}^2(\mathbb{R})$ and $\overline{v(t)}$ is the conjugate of $v(t)$. Now consider a closed and convex set $\mathcal{C}$ containing signals with some predefined properties. The convexity of $\mathcal{C}$ means that the weighted averages

$$w = (1-\alpha)u + \alpha v, \quad \forall \alpha \in (0, 1) \tag{10}$$

of two arbitrary signals $u, v \in \mathcal{C}$ is also in $\mathcal{C}$ (Fig. 2).

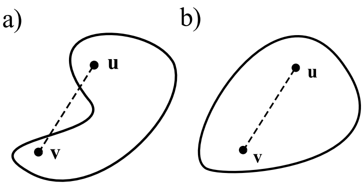

FIGURE 2. a) Non-convex set, b) convex set.

The projection of a signal g onto the set $\mathcal{C}$ is the element $\hat{x}$ of the set $\mathcal{C}$ which is closest to the signal g

$$\hat{x} = P_{\mathcal{C}}g = \arg\min_{y\in\mathcal{C}} \|g - y\|. \tag{11}$$

For each $g \notin \mathcal{C}$, the projection $P_{\mathcal{C}}g$ is closer to any vector $x \in \mathcal{C}$ than g,

$$\|P_{\mathcal{C}}g - x\| < \|g - x\|. \tag{12}$$

The projection (11) can be generalized using a relaxation parameter λ,

$$T_{\mathcal{C}} := I + \lambda(P_{\mathcal{C}} - I) \tag{13}$$

where I is the identity operator. For $0 < \lambda < 2$, the transformation $T_{\mathcal{C}}$ also ensures the decrease of the error [28],

$$\|T_{\mathcal{C}}g - x\| < \|g - x\|. \tag{14}$$

If the set $\mathcal{C}$ contains more than one element, the reconstruction is not unique, and a selection of a different g may result in a different $\hat{x}$. In particular, a selection of g close to the actual signal x can be a determinant factor for the precision of recovery.

The direct projection onto the set $\mathcal{C}$ may be difficult or computationally intractable. However if the set $\mathcal{C}$ is the intersection of the convex sets $\mathcal{C} = \mathcal{C}_1 \cap \mathcal{C}_2 \cap \cdots \cap \mathcal{C}_K = \bigcap_{k=1}^{K} \mathcal{C}_k$, then a signal $\hat{x} \in \mathcal{C}$ can be obtained by alternating projections $P_{\mathcal{C}_1}, \ldots, P_{\mathcal{C}_K}$:

$$g_{m+1} = P_{\mathcal{C}_1} \cdots P_{\mathcal{C}_K} g_m. \tag{15}$$

Provided that the sets $\mathcal{C}_k$, $k = 1, \ldots, K$ are convex and their intersection is non-empty, this iteration converges to $\hat{x} \in \bigcap_{k=1}^{K} \mathcal{C}_k$ for any arbitrary g_0. The method of projections onto convex sets (POCS) was proposed by Bregman [29] and was used in a variety of applications, including the recovery of signals from samples [20], [24], [27] or from part of the signal, as in the Papoulis-Gerchberg algorithm [30]. The convergence of POCS for two sets $\mathcal{C}_1$ and $\mathcal{C}_2$ in $\mathbb{R}^2$ is illustrated in Fig. 3. In the case of the transformation $T_{\mathcal{C}}$, in the iteration

$$g_{m+1} = T_{\mathcal{C}_1} T_{\mathcal{C}_2} \cdots T_{\mathcal{C}_K} g_m \tag{16}$$

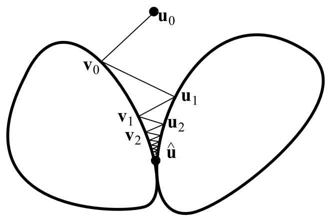

FIGURE 3. Iterative projections onto convex sets.

with $\lambda_k > 1$ (which is the relaxation coefficient used for $T_{\mathcal{C}_k}$) the estimate error decay can be faster than with (15) [31]. This aspect is interesting in practice since the actual number of iterations is often limited. More information on POCS can be found in [28], [31], and [32].

C. POCS USING SETS CORRESPONDING TO LEVEL-CROSSING SAMPLES

1) CONVEXITY

To use POCS for the signal recovery using the sets from Section III-A, the convexity of these sets must be proven. Fortunately, it is easy to show it for all sets $\mathcal{B}$, $\mathcal{E}$, and $\mathcal{I}$:

- The convexity of $\mathcal{E}$ and $\mathcal{I}$ results from the following fact. If $w = (1-\alpha)u + \alpha v$ for some $\alpha \in [0, 1]$, then at any given t, $w(t)$ belongs to the interval $[u(t), v(t)]$ or $[v(t), u(t)]$. When $u, v \in \mathcal{E}$, this interval at $t = t_n$ for any n is reduced to $\{x(t_n)\}$, so $w \in \mathcal{E}$. When $u, v \in \mathcal{I}$, this interval is at any instant t included in $[\theta^-(t), \theta^+(t))]$, so $w \in \mathcal{I}$.
- The set $\mathcal{B}$ is convex as the bandlimited signals form a linear space [33].

This allows to perform the projection onto each of above sets separately, as well as the projection onto their intersection, which is also convex.

2) ITERATIVE PROJECTION ONTO $\mathcal{B} \cap \mathcal{I}$

As shown in Section III-A, it is sufficient to use set $\mathcal{B} \cap \mathcal{I}$ as a constraint without using $\mathcal{E}$ explicitly. Since one-step projection onto set $\mathcal{B} \cap \mathcal{I}$ is intractable, it must be performed iteratively by the alternating the projections $P_{\mathcal{B}}$ and $P_{\mathcal{I}}$.

The projection operator $P_{\mathcal{B}}$ simply removes out-of-band components making the projected signal bandlimited, so it amounts to the ideal filtering

$$(P_{\mathcal{B}}g)(t) = (t) * \frac{\Omega}{\pi}\mathrm{sinc}\,(\Omega t) \tag{17}$$

$$= \int_{-\infty}^{+\infty} g(\tau)\frac{\Omega}{\pi}\mathrm{sinc}\,(\Omega(t-\tau))\,d\tau. \tag{18}$$

The sinc function is defined here as $\mathrm{sinc}\,(u) = \frac{\sin(u)}{u}$.

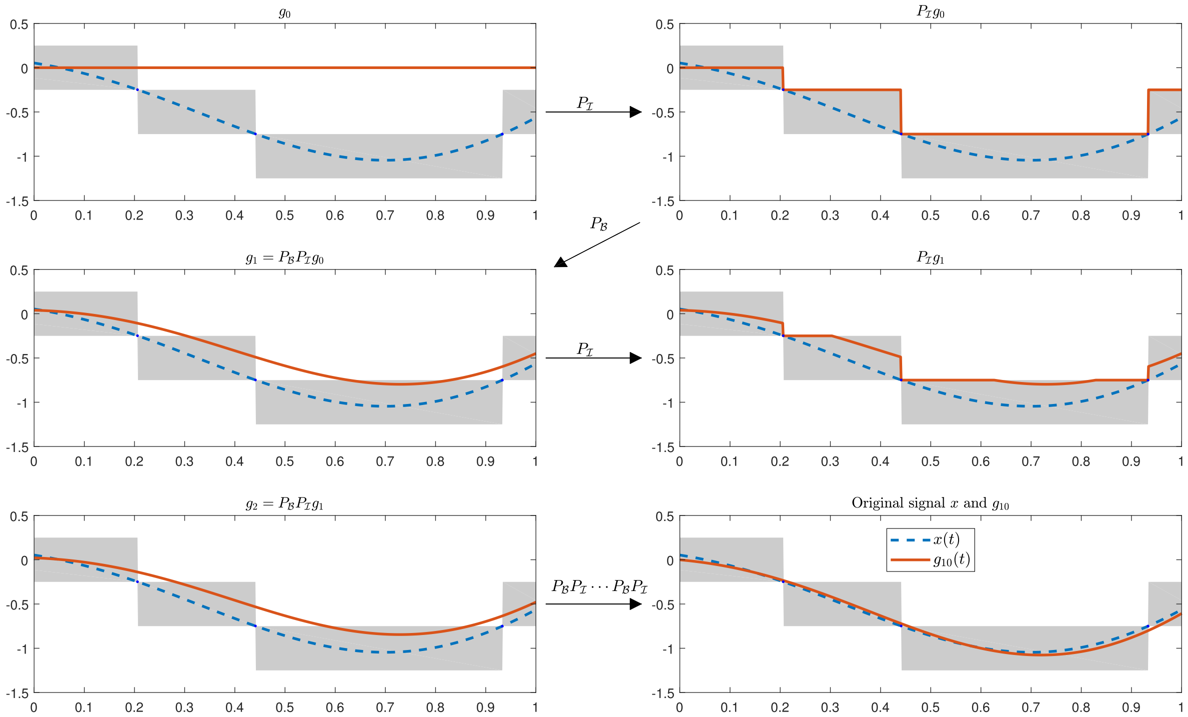

FIGURE 4. Initial iterations and the comparison of original signal $x(t)$ with $\hat{x}(t) = g_{10}(t)$.

The projection operator $P_{\mathcal{I}}$ clips the values of signal $x(t)$ which exceed the bounds $\theta^-(t), \theta^+(t)$

$$(P_{\mathcal{I}} g)(t) = \begin{cases} \theta^+(t), & g(t) > \theta^+(t) \\ g(t), & \theta^-(t) \le g(t) < \theta^+(t) \\ \theta^-(t), & g(t) < \theta^-(t). \end{cases} \tag{19}$$

Finally, the desired reconstruction using the implicit information on the bandlimited signal $x(t)$ is given by the iteration

$$g_{m+1} = P_{\mathcal{B}} P_{\mathcal{I}} g_m \tag{20}$$

or in the generalized version

$$g_{m+1} = T_{\mathcal{B}} T_{\mathcal{I}} g_m. \tag{21}$$

An example of an iterative reconstruction is shown in Fig. 4. To make it more illustrative, an initial signal $x_0(t) = 0$ was chosen.

3) ONE-STEP PROJECTION ONTO $\mathcal{B} \cap \mathcal{E}$

While the algorithm (20) (or (21)) is conceptually simple, its convergence is slow, and its natural implementation is by use of analog circuits. A digital implementation would require the use of high oversampling to accurately represent the non-bandlimited signals produced by the projection $P_{\mathcal{I}}$, which would further slow down the iteration. Meanwhile, in a purely digital implementation, an alternative to iterative projections is to limit the algorithm to a one-step projection onto $\mathcal{B} \cap \mathcal{E}$ and to use the implicit information only for the choice of the estimate $g(t)$ to be projected.

According to (11), the direct projection of a signal $g(t)$ onto the convex set $\mathcal{B} \cap \mathcal{E}$ can be formulated as

$$\hat{x}(t) = \arg\min_{u(t)\in\mathcal{B}} \|u(t) - g(t)\|^2 \quad \text{subject to } u(t_n) = x(t_n), \quad n = 1, \ldots, N \tag{22}$$

The relative energy can be expanded as

$$\begin{aligned} \|u(t) - g(t)\|^2 &= \|u(t) - g_\Omega(t) + g_\Omega(t) - g(t)\|^2 \\ &= \|\underbrace{u(t) - g_\Omega(t)}_{\text{in-band}}\|^2 + \|\underbrace{g_\Omega(t) - g(t)}_{\text{out-of-band}}\|^2 \end{aligned} \tag{23}$$

where $g_\Omega(t) = P_{\mathcal{B}} g(t)$. The reason why the energy can be splitted into the in-band and the out-of-band components is that these two spaces of signals are orthogonal. This can be easily seen in the frequency domain via Parseval's equality. Since by definition $\hat{x}(t) \in \mathcal{B}$, then the argument of minimization is only the bandlimited component

$$v(t) = u(t) - g_\Omega(t). \tag{24}$$

This leads to the minimization

$$\hat{y}(t) = \arg\min_{v(t)\in\mathcal{B}} \|v(t)\|^2 \quad \text{subject to } v(t_n) = x(t_n) - g_\Omega(t_n), \quad n = 1, \ldots, N \tag{25}$$

and the solution of (22) is

$$\hat{x}(t) = \hat{y}(t) + g_\Omega(t). \tag{26}$$

This is standard least-squares recovery [34], [35] from the samples $\mathbf{y} = \mathbf{x} - \mathbf{g}_\Omega$, where $\mathbf{x}, \mathbf{g}_\Omega \in \mathbb{R}^N$ are the values of $x(t)$, $g_\Omega(t)$ at points $(t_1, \ldots, t_N)$. The solution is [24], [35]

$$\hat{y}(t) = \sum_{n=1}^{N} c_n \operatorname{sinc}\left(\Omega(t - t_n)\right), \tag{27}$$

where

$$\mathbf{c} = \mathbf{S}^{-1}(\mathbf{x} - \mathbf{g}_\Omega), \tag{28}$$

and $\mathbf{S}$ is the matrix of coefficients $S_{i,j} = \operatorname{sinc}\left(\Omega(t_i - t_j)\right)$. It is known that the inversion of the matrix $\mathbf{S}$ is usually ill-conditioned, especially for clustered samples [34]–[36]. To improve the stability of reconstruction, small positive values are added to the diagonal of $\mathbf{S}$ to obtain a regularized solution [36], [37]:

$$\mathbf{c} = (\mathbf{S} + \varepsilon \mathbf{I})^{-1}(\mathbf{x} - \mathbf{g}_\Omega). \tag{29}$$

For $g^\pm(t) = 0$ also $\mathbf{g}_\Omega = \mathbf{0}$ and (26) reduces to the standard least-squares solution [35].

IV. CHOICE OF THE INITIAL GUESS

A. MOTIVATION AND DESIGN REQUIREMENTS

In the signal reconstruction process, each sample has mostly an influence on the values of the reconstructed signal in its vicinity. In the case of iterative POCS, a large number of iterations is necessary to propagate the impact of samples to the regions of signal located in the large sampling gaps. In the case of one-step POCS, the remote impact of samples on the signal could be higher without the regularization (29). Unfortunately, with a high clustering of samples (which is common for the level-crossing sampling of bursty signal) regularization is required, because of the high impact of noise. Overall, these various reconstruction schemes have weak actions on the signal in large sampling gaps where, as a result, the initial guess $g(t)$ has a stronger influence on the final reconstruction. The choice of initial guess therefore plays an important role in the performance of reconstruction.

In plain irregular sampling, there is no prior information about the behavior of the signal between the samples. However in the case of level crossing, the additional implicit information can be used to choose an initial guess. There are a few criteria for the choice of $g(t)$:

1) It should be a good model of expected behavior of original signal in the large gaps between the level-crossing samples.
2) It must be consistent with the implicit information.
3) It should contain small energy out of the desired band $(-\Omega, \Omega)$. The more out-of-band energy, the more distortion is introduced by filtering and the more iterations are required for the iterative POCS to converge. In the same scenario for the one-step POCS, the energy of the signal $\hat{y}(t)$ is higher, and as this signal is not constrained by inequality bounds, it is more likely that the final $\hat{x}(t)$ is not consistent with implicit information.
4) The function $g(t)$ should be simple to implement. In the particular case of the one-step POCS, it should be designed such that $g_\Omega(t)$ yields a closed form formula in terms of the samples $(t_n, x(t_n))$ for computation feasibility.

To simplify the formulation of the initial guess function $g(t)$, we will assume that the sampled signal amplitude is upper and lower bounded (which is a common assumption in analog-to-digital conversion), and it is bounded by the outer levels

$$\theta_1 \leq x(t) \leq \theta_L. \tag{30}$$

B. PIECEWISE CONSTANT SIGNAL

The simplest guess of a signal staying between levels $\theta^+(t)$ and $\theta^-(t)$ is their average which is a piecewise constant function of t (see Fig. 5),

$$\theta^\pm(t) = \frac{\theta^+(t) + \theta^-(t)}{2}. \tag{31}$$

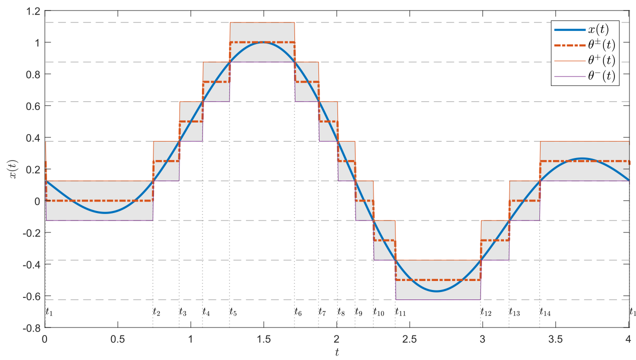

FIGURE 5. Piecewise constant guess signal $\theta^\pm(t)$.

The formula for $\theta^\pm_\Omega(t)$ can be found in Appendix A. The piecewise constant signal is used also in CT-DSP for signal recovery from level crossings by lowpass filtering [1].

The reconstruction using the piecewise constant initial guess is shown in Fig. 7 for the orginal signal presented in Fig. 6.

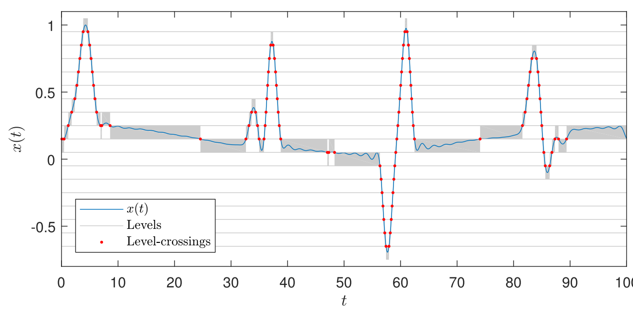

FIGURE 6. Example of bursty signal.

C. PIECEWISE LINEAR SIGNAL

A simple alternative for the piecewise initial guess is the piecewise linear function $\lambda(t)$ with the samples as knots.

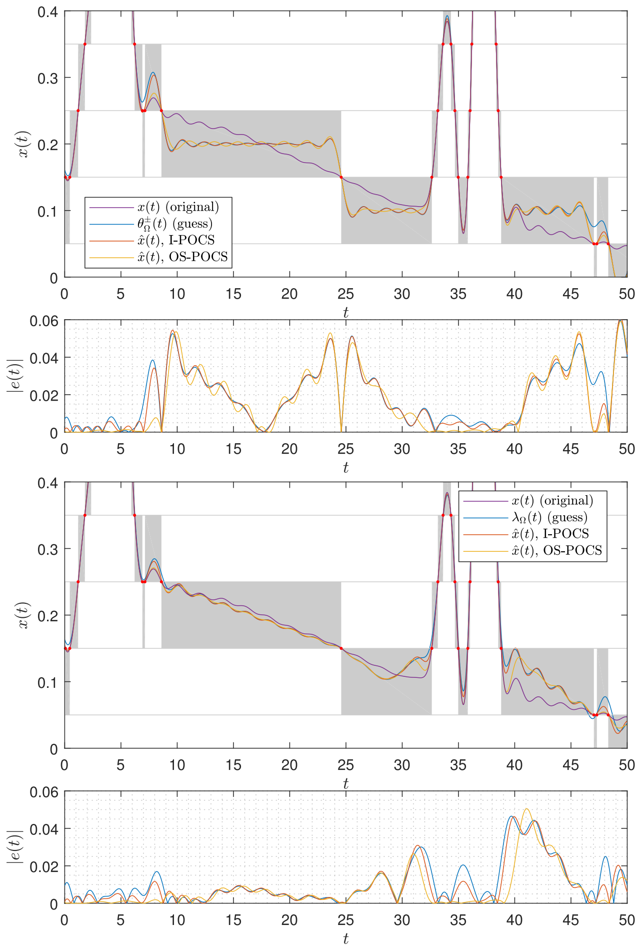

FIGURE 7. Example of bursty signal, initial guess and reconstructed signal obtained using iterative POCS ($M = 10$) and one-step POCS ($\varepsilon = 10^{-3}$). Left: piecewise constant guess; right: piecewice linear initial guess.

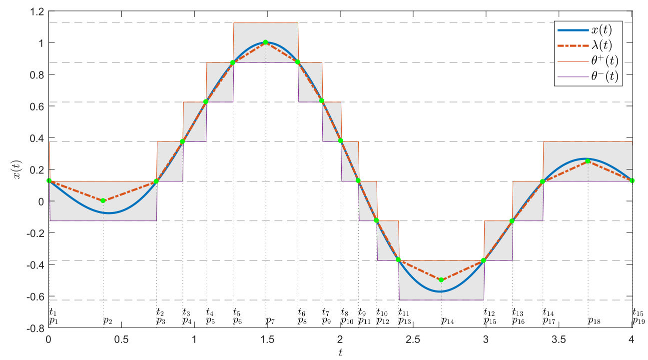

FIGURE 8. Piecewise linear guess signal $\lambda(t)$ and reenumareted time instants $\{p_i\}$.

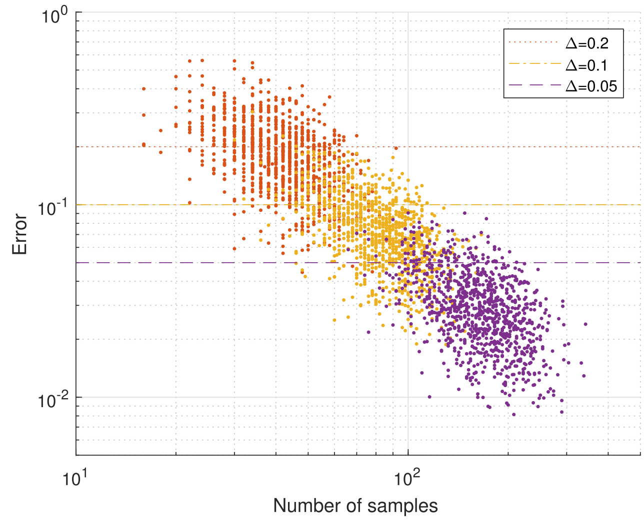

FIGURE 9. Reconstruction error vs. number of samples for one-step POCS with $\varepsilon \in 10^{-3}$ and piecewise linear initial guess. Lines correspond to the mean error.

Additional knots are placed in the center points of the intervals (t_n, t_{n+1}) for which $x(t_n) = x(t_{n+1})$. Calling (p_i, λ_i) the complete sequence of knots in increasing order of p_i, $\lambda(t)$ yields the formula

$$\lambda(t) = \lambda_i + (\lambda_{i+1} - \lambda_i)\,\frac{t - p_i}{p_{i+1} - p_i}, \quad \text{for } t \in (p_i, p_{i+1}). \tag{32}$$

The sequence $\{p_i\}$ constists of the original sampling instants $\{t_n\}$ and the points $t_{n,n+1} = \frac{t_n + t_{n+1}}{2}$ added at the center of some intervals (t_n, t_{n+1}), sorted in ascending order, as shown in Fig. 8. The sequence $\{\lambda_i\}$ is defined as

$$\lambda_i = \begin{cases} \theta_{\ell_n}, & \exists i, n : p_i = t_n \\ \theta^{\pm}(p_i), & \text{otherwise.} \end{cases} \tag{33}$$

The formula for $\lambda_\Omega(t)$ can be found in Appendix B. The example of reconstruction using such an initial guess is shown in Fig. 7. The piecewise linear signal has been proposed for signal recovery from level crossings by lowpass filtering in the framework of CT-DSP [15].

V. PERFORMANCE EVALUATION

A. TEST SIGNALS

In our experiments, the input signals belong to the space of bandlimited signals of Nyquist period $T = 1$. They are generated as the sum of two random components: impulsive bursts and slowly varying component,

$$x(t) = \underbrace{\sum_{k=1}^{K} b_k(t - t_k)}_{\text{impulsive bursts}} + \underbrace{\sum_{n=1}^{N} w_n \phi(t - n)}_{\text{slowly varying}} \tag{34}$$

where for each k, $b_k(t)$ is a bursty signal of the form

$$b_k(t) = \sum_{j=1}^{J} w_{k,j}\psi(t + t_{k,j}), \tag{35}$$

$\psi(t) = \text{sinc}^2\left(\frac{1}{2}t\right)$ and $\phi(t)$ is some bandlimited and slowly-varying kernel. The positions t_k of the bursts are drawn randomly according to the uniform distrubution $\mathcal{U}(0, N)$, where N defines the signal length. Within each burst $b_k(t)$, the relative positions $t_{k,j}$ of the kernels follow the

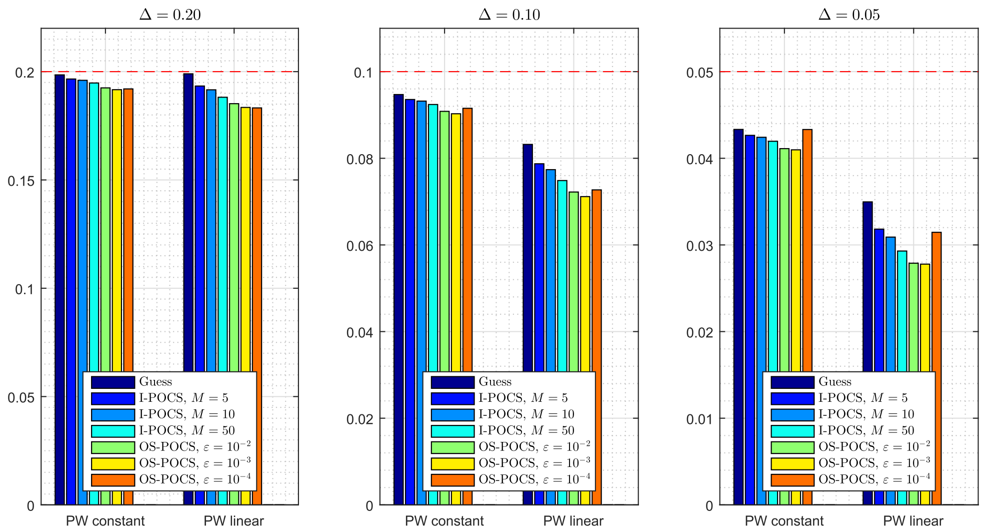

FIGURE 10. Comparison of nornmalized RMS reconstruction error between initial guess, iterative POCS (I-POCS) and one-step POCS (OS-POCS) depending on the type of initial guess. Strongest factor determining the performance is the choice of initial guess. OS-POCS with carefully chosen regularization constant outperforms the I-POCS, even with high number of iterations.

normal distribution $\mathcal{N}(0, \sigma_t^2)$. Also the weights $w_{k,j}$ are independent normal random variables of distribution $\mathcal{N}(0, \sigma_{burst}^2)$. The kernel $\psi(t)$ has been chosen with a faster decay than $\mathrm{sinc}(t)$ (narrow time localization of energy). The slowly varying component can be interpreted as the bandlimited signal of random Nyquist samples $w_n \sim \mathcal{N}(0, \sigma_{slow}^2)$ filtered by $\phi(t)$.

The signal (34) is finally normalized to keep unit amplitude, using

$$\overline{x}(t) = \frac{x(t)}{\max |x(t)|}. \tag{36}$$

An example of $\overline{x}(t)$ is shown in Fig. 6.

B. NUMERICAL EXPERIMENTS

To validate and compare of the proposed methods, the simulations are performed using a large number (10^3) of randomly generated signals of the form (34). The parameters of simulations are

- the distance between the levels $\Delta \in \{0.2, 0.1, 0.05\}$
- the type of initial guess (piecewise constant or piecewise linear)
- the type of method (iterative POCS or one-step POCS)
- the method parameter (iterative POCS - number of iterations $M \in \{5, 10, 50\}$, one-step POCS - regularization constant $\varepsilon \in \{10^{-2}, 10^{-3}, 10^{-4}\}$)

The parameters of bursty signal are $N = 100$, $K = 4$, $L = 5$, $\sigma_t^2 = 2$, $\sigma_{burst}^2 = 1$, $\sigma_{slow}^2 = 2$. The kernel $\phi(t)$ of the slowly varying component was obtained by twofold averaging

$$\phi(t) = \psi(t) * \Pi\left(\frac{t}{W}\right) * \Pi\left(\frac{t}{W}\right), \tag{37}$$

$$\Pi(t) = \begin{cases} 1, & |t| < \dfrac{1}{2} \\ 0, & \text{otherwise.} \end{cases} \tag{38}$$

with $W = 40$.

The error of a reconstruction estimate $\hat{x}(t)$ is measured as

$$e = \mathrm{NRMS}[\overline{x}(t), \hat{x}(t)] = \sqrt{\frac{\int_0^N \left(\overline{x}(t) - \hat{x}(t)\right)^2 dt}{\int_0^N \overline{x}(t)^2 dt}}. \tag{39}$$

The results indicate that the distance between the levels Δ has a strong impact on the final reconstruction error, whose value is of similar order, $e \lesssim \Delta$. For smaller Δ values, the relative error e/Δ is better because the number of samples is higher and the gaps between the samples are also smaller (see Fig. 9).

The comparison of the reconstruction error averaged over all generated random signals for various methods is shown in Fig. 10. The piecewise linear guess outperforms the piecewise constant guess especially with the smaller values of Δ (which is also reflected by shorter gaps between the samples). The input signal behavior between two neighboring levels is hard to predict when Δ is large ($\Delta = 0.2$). In this case, both types of guesses are not accurate. Despite this, the piecewise linear guess remains the better initial estimate for all values of Δ. With the iterative POCS, it enables a faster convergence, and with the one-step POCS it gives better reconstruction results.

In general the one-step POCS outperforms the iterative POCS only if a proper regularization constant ε is chosen. It was observed that a too small ε introduces oscillations that make the reconstructed signal $\hat{x}(t)$ violate the implicit information to a certain extent.

The performance of the iterative POCS in terms of the number of iterations is shown in Fig. 11. The results were also averaged over all reconstructions of random signals. The impact of the initial guess on the convergence is clearly visible. The iteration axis is logarithmic, and since the decay is close to linear then the error reduction in the first 10 iterations is almost the same as the reduction

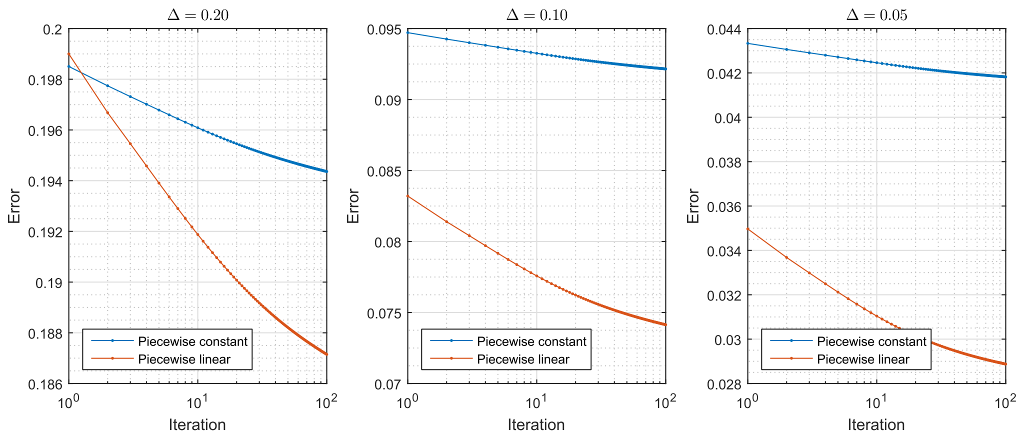

FIGURE 11. Convergence of the iterative POCS depending on the type of initial guess.

between 10 and 100 iteration. Therefore the increase of the number of iterations above a certain limit gives a small improvement of reconstruction performance.

VI. RELATED WORK

There is very little research work on the use of inequalities in the framework of signal recovery. Some of it employs some implicit information in an event-based approach without mentioning this concept. In [24], the knowledge of the maximum and minimum signal values is utilized to improve signal reconstruction. The use of inequalities for image reconstruction from level crossings is shown in [25]. This reference *de facto* presents an early use of implicit information associated with LCS for the reconstruction of two dimensional signals. The notion of implicit information is exploited also in [38] to improve signal reconstructions from level crossings by conventional algorithms.

The other point to be emphasized is that the use of implicit information in signal reconstruction involves nonlinear operations, as is the case of the projections corresponding to amplitude constraints. By contrast, the pure sampling approach to signal reconstruction is inherently linear [18]. The recent work [39] reports a new approach to signal reconstruction from level crossings based on random projections in the split-projection least squares (SPLS) algorithm which belongs to compressed sensing framework. The level crossings are encoded using continuous-time ternary encoding (CT-TE) that modulates amplitude variation to ternary timing information. The SPLS algorithm uses only the knowledge of level crossings and has been implemented in FPGA.

VII. CONCLUSIONS

When continuous-time signals are discretized in time by event-based sampling and stored for example in computer memory, the pure samples no longer represent how the signal behaves during the time between them. But the information on the expected signal behavior between the events can still be deduced if one knows how the samples have been captured. This knowledge is however not included in the samples, and is given implicitly.

Most approaches to signal reconstruction in event-based sampling adopt conventional methods of signal recovery from plain (nonuniform) samples. Such methods do not include the implicit information on the expected behaviour of the input between the samples. CT-DSP is a technique that consciously uses this information, but the signal reconstruction it proposes by lowpass filtering a simple approximation of the input is nonperfect. In this paper, we have presented the perfect reconstruction of bandlimited signals from level-crossings including the implicit information using POCS. We have also shown that the signal reconstruction in CT-DSP can be interpreted as a truncated version of the iterative POCS.

The iterative alternating projections have the advantage to be implementable with a chain of standard circuit operations: a lowpass filter and a clipping circuit, respectively. A problem of high practical interest will be the design of hardware-based POCS signal-recovery systems for perspectives of fast signal reconstruction.

APPENDIX A
PIECEWISE CONSTANT INITIAL GUESS

We derive the explicit expression of $g_\Omega(t)$ in terms of $(t_n, x(t_n))$ for the various initial guesses $g(t)$ presented in Section IV.

Let $h_\Omega(t) := \frac{\Omega}{\pi}\text{sinc}(\Omega t) = \frac{\sin(\Omega t)}{\pi t}$ and let $\mathbf{1}(t)$ be the unit step function. The piecewise constant guess (31) can be reformulated as

$$\theta^{\pm}(t) = \sum_n \theta_{\ell_n} \left(\mathbf{1}(t - t_n) - \mathbf{1}(t - t_{n+1})\right) \tag{40}$$

and its filtered version is

$$\theta^{\pm}_{\Omega}(t) = \sum_n \theta_{\ell_n} \left(\mathbf{1}_\Omega(t - t_n) - \mathbf{1}_\Omega(t - t_{n+1})\right) \tag{41}$$

where

$$\mathbf{1}_\Omega(t) = (h_\Omega * \mathbf{1})(t) = \int_{-\infty}^{t} h_\Omega(\tau)d\tau = \frac{1}{2} + \frac{1}{\pi}\text{Si}(\Omega t) \tag{42}$$

and $\text{Si}(t)$ denotes sine integral function.

APPENDIX B
PIECEWISE LINEAR INITIAL GUESS

The piecewise linear initial guess (32) can be represented by

$$\lambda(t) = \sum_i \lambda_i \left(\Lambda_{i-1}(t) - \Lambda_i(t)\right) \tag{43}$$

which after filtering gives

$$\lambda_\Omega(t) = \sum_i \lambda_i \left(\Lambda_{\Omega,i-1}(t) - \Lambda_{\Omega,i}(t)\right), \tag{44}$$

where

$$\Lambda_i(t) = \frac{1}{p_{i+1} - p_i} \mathbf{1}(t) * \left(\mathbf{1}(t - p_i) - \mathbf{1}(t - p_{i+1})\right) \tag{45}$$

and

$$\begin{aligned}\Lambda_{\Omega,i}(t) &= (h_\Omega * \Lambda_i)(t)\\ &= \frac{1}{p_{i+1} - p_i} \mathbf{1}_\Omega(t) * \left(\mathbf{1}(t - p_i) - \mathbf{1}(t - p_{i+1})\right)\end{aligned} \tag{46}$$

$$\Lambda_{\Omega,i}(t) = \frac{1}{p_{i+1} - p_i} \int_{t-p_{i+1}}^{t-p_i} \mathbf{1}_\Omega(\tau) d\tau. \tag{47}$$

Let us use notation

$$\Lambda_{\Omega,i}(t) = \mathbf{1}_\Omega \Big|_{t-p_{i+1}}^{t-p_i} \quad \text{where } f \Big|_a^b := \text{mean of } f(t) \text{ on } [a, b] \tag{48}$$

Since

$$\lim_{t \to -\infty} \mathbf{1}_\Omega(t) = 0 \quad \text{and} \quad \lim_{t \to +\infty} \mathbf{1}_\Omega(t) = 1 \tag{49}$$

it is easy to see that also

$$\lim_{t \to -\infty} \Lambda_{\Omega,i}(t) = 0 \quad \text{and} \quad \lim_{t \to +\infty} \Lambda_{\Omega,i}(t) = 1 \tag{50}$$

and hence

$$\lim_{t \to \pm\infty} \left(\Lambda_{\Omega,i-1}(t) - \Lambda_{\Omega,i}(t)\right) = 0. \tag{51}$$

Explicitly

$$\mathbf{1}_\Omega \Big|_a^b = \frac{1}{b-a} \int_a^b \mathbf{1}_\Omega(\tau) d\tau \tag{52}$$

$$= \frac{1}{b-a} \left([\tau \mathbf{1}_\Omega(\tau)]_a^b - \int_a^b \tau h_\Omega(\tau) d\tau \right). \tag{53}$$

Note that $\tau h_\Omega(\tau) = \frac{1}{\tau} \sin(\Omega t)$. Then

$$\mathbf{1}_\Omega \Big|_a^b = \frac{1}{b-a} [\varphi(\tau)]_a^b \text{ where } \varphi(t) := t \mathbf{1}_\Omega(t) + \frac{\cos(\Omega t)}{\pi \Omega}, \tag{54}$$

and finally

$$\Lambda_{\Omega,i}(t) = \frac{\varphi(t - p_i) - \varphi(t - p_{i+1})}{p_{i+1} - p_i}. \tag{55}$$

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