IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received April 6, 2018, accepted May 11, 2018, date of publication May 18, 2018, date of current version June 19, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2838121

Fixed-Time Leader-Follower Formation Control of Autonomous Underwater Vehicles With Event-Triggered Intermittent Communications

ZHENYU GAO[1] AND GE GUO[1,2], (Senior Member, IEEE)
[1]Department of Automation, Dalian Maritime University, Dalian 116026, China
[2]State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang 110004, China

Corresponding author: Ge Guo (geguo@yeah.net)

This work was supported by the Natural Science Foundation of China under Grants 61273107 and 61573077.

ABSTRACT This paper investigates a fixed-time leader-following formation control method for a set of autonomous underwater vehicles (AUVs) with event-triggered acoustic communications. First, an event-triggering communication strategy is developed to govern the communications between leader AUV and follower AUVs. Then, using a fixed-time control theory and a Lyapunov functional method, a compensator-based command filtered formation control algorithm is proposed, with which the follower AUVs can track the leader AUV in a given fixed time. Namely, the control method can guarantee all the signals in the formation control system to be globally fixed-time stabilized. With the presented fixed-time control scheme, the predesignated AUVs formation can be achieved within a fixed settling time under arbitrary initial system states, which is otherwise impossible under any existing methods including finite-time control. Furthermore, the compensator-based command filtered control technique makes the designed formation control law simple and easy to implement in practice. Simulation results demonstrate the effectiveness of the method.

INDEX TERMS Autonomous underwater vehicles (AUVs), event-triggered, leader-follower, formation control, fixed-time.

I. INTRODUCTION

In the past decades, autonomous underwater vehicles (AUVs) have been widely used to perform various underwater tasks, in both civilian and military applications such as deep sea inspections, long-duration surveys, oceanographic mapping, neutralizing undersea mines and offshore oil installations [1]–[5]. For some specific applications, e.g., ocean sampling, mapping and ocean floor survey, it is beneficial to carry out the missions through multiple AUVs cooperation to improve efficiency, increase service area, and provide redundancy in case of failure [6]–[8]. The fundamental idea of formation is to use relatively inexpensive, simple, and small AUVs instead of expensive specialized AUVs to cooperatively perform difficult or complex underwater tasks. In general, there are three main types of formation control strategies, namely, behavior-based strategy [9], virtual structure strategy [10], [11], and leader-follower formation strategy [12]–[14]. Each strategy has its own strengths and weaknesses. Among these strategies, the leader-follower formation strategy has been widely considered by many research, because of its advantages such as simplicity and scalability.

In leader-follower formation control of AUVs, there are some challenging issues worth mentioning. The first issue is the convergence speed of the formation tracking control system. An ideal formation control scheme should achieve satisfactory performance very quickly, i.e., the convergence time should be short enough. However, the AUVs formation control schemes in most existing literature can only guarantee asymptotic convergence of the formation tracking errors, namely, the convergence speed is at best exponential, which implies that the tracking errors will converge to zeros with infinite settling time [6], [12]–[14]. In practice, finite-time formation control is much more desirable and the problem has received great research attention in the control community [15], [17], [18]. In [15], the problem of finite-time formation control of a group of nonholonomic mobile robots was studied. In [17], a finite-time leader-follower formation control problem was investigated for a group of quadrotor aircrafts, and a similar finite-time fault tolerant leader-follower formation control scheme was presented for a group of autonomous surface vessels in [18]. Notice that, in these researches, the settling time of finite-time formation depends on the initial states of all the vehicles. Therefore, a predefined convergence time cannot be guaranteed since the initial conditions of the system is usually unavailable in advance. To deal with this limitation, some new studies appeared recently focusing on group coordination control with guaranteed settling time regardless of the initial conditions of the systems [19]–[22]. These research works are based on the notion of fixed-time stability [19], and are currently focused only on the consensus problem of general multi-agent systems with various dynamics. In this paper, we aim to seek for novel AUVs formation control algorithms using the interesting notion of fixed-time control.

Another challenging issue relates to the constraint of information exchange between the AUVs. For AUVs applications, acoustic signaling is the only viable option for long-range underwater communication, where electromagnetic and optical waves propagate quite poorly. Underwater acoustic communication has low propagation speed, limited bandwidth, and high energy consumption. Therefore, the data transmission between underwater vehicles should be kept to a minimum amount [23]. However, most of the results on AUVs formation control so far are based on the assumption that information exchanges between AUVs are conducted continuously [6], [11]–[14]. This assumption implies that a large amount of redundant data should be transmitted all the time, which is clearly impractical, or even infeasible, due to unbearable communication burden, energy consumption, and large communication delay. For this reason, a practical formation control method should demand less frequent communications between different systems. To this end, some new control strategies have been proposed [12], [24], [25]. For example, in [12], an impulsive system approach to formation control of AUVs was given, which reduces the total amount of data transmission by using a discrete impulsive signal sequences to control the communications between different nodes. The schemes in [24] and [25] reduce communication by omitting speed information transmission, and introducing a speed projection algorithm [24] and a velocity observer [25] for control respectively. However, these two schemes are still faced with large amount of data transmission, since continuous-time communication of other information is needed. In order to essentially get rid of heavy communication burden and make a control strategy practical for real application, event-triggered transmission control could be a promising solution. The basic idea of an event-triggered communication scheme is to sample and update the state and control input only when some variables exceed given thresholds. If these thresholds are set properly, an event-triggered transmission scheme can greatly reduce communication while maintaining satisfactory control performance. The readers are referred to [26]–[31] for event-triggered control studies of multiagent systems and vehicular platooning systems. Can we have a realistic AUVs formation control algorithm that runs on event-triggered acoustic communication?

Motivated by the above observations, this paper proposes a fixed-time AUVs formation control method with event-triggered acoustic communication scheduling. In comparison with the existing ones, the new method in this paper has the following features. Firstly, an event-triggered communication strategy is developed which can significantly reduce energy consumption and communication burden. Secondly, the fixed-time formation control algorithm can not only ensure the settling time regardless of the initial conditions of the system, but also can obtain higher accuracy performance and faster convergence speed of the system. To the best of the authors' knowledge, it is the first time in the literature event-triggered communication and fixed-time control are simultaneously employed in the formation control of autonomous underwater vehicles.

The organization of this paper is as follows. Some useful preliminaries and problem formulation are presented in the next section. Section III presents the main results. Section IV simulates the proposed control approach, and finally, we conclude this paper and propose some further work in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

In this section, firstly, some lemmas are given which will be used later. Secondly, the problem formulation is presented.

A. SOME LEMMAS

Lemma 1 [18]: The command filter is described as follows:

$$\begin{aligned} \dot{z}_1 &= \omega_n z_2, \\ \dot{z}_2 &= -2\zeta\omega_n z_2 - \omega_n(z_1 - \alpha_1), \end{aligned} \tag{1}$$

where $\omega_n > 0$ and $\zeta \in (0, 1]$.

Remark 1: If the input signal α_1 fulfills $|\dot{\alpha}_1| \leq \rho_1$ and $|\ddot{\alpha}_1| \leq \rho_2$ for all $t \in [0, +\infty)$, where $\rho_1 > 0,\ \rho_2 > 0$ and $z_1(0) = \alpha_1(0),\ z_2(0) = 0$, for any $\mu > 0$, there exist $\omega_n > 0$ and $0 < \zeta \leq 1$, so we have $|z_1 - \alpha_1| \leq \mu$, and $|\dot{z}_1|,\ |\ddot{z}_1|$ and $|\dddot{z}_1|$ are bounded. While, each command filter is designed to computer z_1 and $\dot{z}_1$ without differentiation.

Lemma 2 [19]: Consider a scalar syatem

$$\dot{x} = -\alpha x^{\frac{m}{n}} - \beta x^{\frac{p}{q}},\quad x(0) = x_0, \tag{2}$$

where $\alpha > 0,\ \beta > 0$, and $m,\ n,\ p,\ q$ are positive odd integers satisfying $m > n$ and $p < q$. Then the equilibrium of (2) is fixed-time stable and the setting T is bounded by

$$T < T_{\max} \triangleq \frac{1}{\alpha}\frac{n}{m-n} + \frac{1}{\beta}\frac{q}{q-p}. \tag{3}$$

Moreover, if $\varepsilon = [q(m-n)/n(q-p) \leq 1]$, then a less conservative upper-bound estimation for the settling time, instead of (3), can be obtained as

$$T < T_{\max} \triangleq \frac{q}{q-p}\left(\frac{1}{\sqrt{\alpha\beta}}\arctan\sqrt{\frac{\alpha}{\beta}} + \frac{1}{\alpha\varepsilon}\right). \tag{4}$$

Remark 2: The settling time function is upper bounded by a priori value that dose not rely on the system initial state x_0 but only on the design parameters, that is, m, n, p, q, α and β. This implies that the convergence time can be guaranteed in a prescribed manner.

Remark 3: Compared with the convergence rate of asymptotic results, the finite-time formation results have better dynamic property, for instance, higher accuracy and faster convergence rate. However, the settling time of the finite-time formation depends on the initial states of all the AUVs. Hence, the settling time can be sufficiently large if the initial states are very large. In this paper, the fixed-time consensus can ensure the settling time regardless of the initial states of the AUVs.

Lemma 3 [19]: If there have a continuous radially unbounded function $V : R^n \longrightarrow R_+ \cup \{0\}$ such that
(1) $V(x) = 0 \Leftrightarrow x = 0$;
(2) the solution $x(t)$ satisfied the inequality $V(x) \leq -(\alpha V^p(x) + \beta V^q(x))^k$ for some $\alpha, \beta, p, q, k > 0 : pk < 1$, and $qk > 1$;
then, the globally fixed-time stable can be achieved and the settling time T satisfies that $T(x_0) \leq \frac{1}{\alpha^k(1-pk)} + \frac{1}{\beta^k(qk-1)}$, $\forall x_0 \in R^n$. If $k = 1$, the globally fixed-time stable with settling time T bounded by

$$T \leq T_{\max} := \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)} \tag{5}$$

where $\alpha > 0$, $\beta > 0$, $0 < p < 1$, and $q > 1$.

Lemma 4 [13]: (Young's inequality) For $\forall x, y \in R$, the following inequality holds:

$$x^T y \leq \frac{a^m}{m}\|x\|^m + \frac{1}{na^n}\|y\|^n, \tag{6}$$

where $a > 0, m > 1, n > 1$ and $(m-1)(n-1) = 1$.

Lemma 5 [32]: Let $\epsilon_1, \epsilon_2, \cdots \epsilon_M \geqslant 0$. Then

$$\begin{aligned}
&\sum_{i=1}^{M} \epsilon_i^p \geq (\sum_{i=1}^{M} \epsilon_i)^p, 0 < p \leq 1,\\
&\sum_{i=1}^{M} \epsilon_i^p \geq M^{1-p}(\sum_{i=1}^{M} \epsilon_i)^p, 1 < p \leq \infty.
\end{aligned} \tag{7}$$

B. PROBLEM FORMULATION

1) AUTONOMOUS UNDERWATER VEHICLE (AUV) DYNAMICS

Suppose that there exists a team of two networked AUVs which are indexed by the subscript $i = l, f$. Among them, one is called the leader AUV and the other is called the follower AUV. The follower AUV is equipped with a sensor suite to receive the state information from the leader AUV and determine its own position in the global coordinate frame $\{E\}$. It is assumed that all the AUVs have fixed attitudes, such that the translational dynamics of the ith AUV can be described as [12]:

$$\begin{cases} \dot{p}_i = J(\Theta_i)\upsilon_i \\ M_i\dot{\upsilon}_i = -D(\upsilon_i)\upsilon_i - g_i(\Theta_i) + \tau_i \end{cases} \tag{8}$$

The vector $p_i = [x_i, y_i, z_i]^T$ and $\Theta_i = [\phi_i, \theta_i, \psi_i]^T$ denotes the generalized position and attitude (described by Euler angles, i.e., roll ϕ_i, pitch θ_i, and yaw ψ_i angles) in the global coordinate frame $\{E\}$, respectively. $J_i(\Theta_i)$ represents the kinematic transformation matrix from the body-fixed reference frame $\{B\}$ to the global coordinate frame $\{E\}$. The velocity vector $\upsilon_i = [u_i, v_i, \omega_i]^T$ denotes the generalized linear velocities. The vector $\tau_i = [\tau_{iu}, \tau_{iv}, \tau_{i\omega}]^T$ is the control forces. For simplicity, denote $s_a = \sin a$ and $c_a = \cos a$. M_i, $D_i(\upsilon_i)$ and $g_i(\Theta_i)$ represent the inertia matrix, the damping matrix and the restoring force vector, respectively, which are written as

$$M_i = \mathrm{diag}\{m_{i1}, m_{i2}, m_{i3}\}, \tag{9}$$

$$\begin{aligned} D_i(\upsilon_i) = \mathrm{diag}\{&d_{Li1} + d_{Qi1}|u_i|, d_{Li2} + d_{Qi2}|v_i|,\\ &d_{Li3} + d_{Qi3}|\omega_i|\}, \end{aligned} \tag{10}$$

$$\begin{aligned} g_i(\Theta_i) = [&(W_i - B_i)s_{\theta_i}, -(W_i - B_i)c_{\theta_i}s_{\phi_i},\\ &-(W_i - B_i)c_{\theta_i}c_{\phi_i}]^T. \end{aligned} \tag{11}$$

where $m_{ij}, d_{Lij}, d_{Qij} > 0, j = 1, 2, 3$. $J_i(\Theta_i)$ is given as

$$\begin{aligned}
J_i(\Theta_i) &= \begin{bmatrix} J_i(\Theta_i)_1 & J_i(\Theta_i)_2 \\ -s_{\theta_i} & J_i(\Theta_i)_3 \end{bmatrix},\\
J_i(\Theta_i)_1 &= \begin{bmatrix} c_{\psi_i}c_{\theta_i} & -s_{\psi_i}c_{\phi_i} + c_{\psi_i}s_{\theta_i}s_{\phi_i} \\ s_{\psi_i}c_{\theta_i} & c_{\psi_i}c_{\phi_i} + s_{\phi_i}s_{\theta_i}s_{\psi_i} \end{bmatrix},\\
J_i(\Theta_i)_2 &= \begin{bmatrix} s_{\psi_i}s_{\phi_i} + c_{\psi_i}c_{\phi_i}s_{\theta_i} \\ -c_{\psi_i}s_{\phi_i} + s_{\theta_i}s_{\psi_i}c_{\phi_i} \end{bmatrix},\\
J_i(\Theta_i)_3 &= \begin{bmatrix} c_{\theta_i}s_{\phi_i} & c_{\theta_i}c_{\phi_i} \end{bmatrix}.
\end{aligned} \tag{12}$$

2) TRIGGERING EVENT DESIGN

In the conventional leader-follower formation control of autonomous underwater vehicles, the leader AUV needs to transmit its state signals (i.e. position information and velocity information) to the follower AUV continuously. On one hand, this communication mode may often lead to sending many unnecessary state signals to the follower AUV, which in turn will increase fuel consumption and communication burden. This is not desirable especially when the fuel resources are limited. On the other hand, in the difficult underwater communication environment, continuously communication in long time is always too expensive or unavailable. Inspired by [28] and [30], we will design an event-triggered mechanism for the formation system. According to this mechanism, the follower AUV will decide which mode of communication (i,e. continuously communication or periodic communication) will be adopt by the leader AUV when to transmit its state signals to the follower AUV. In this way, continuously communication in long time between the leader AUV and the follower AUV is no longer needed, which can largely decrease the consumption of fuel resources and more suitable for underwater communication environment.

To better describe the nature of the event-triggered mechanism given in this paper, firstly, a communication mode selection signal σ is introduced which sended by the follower AUV, and received by the leader AUV.

$$\sigma = \begin{cases} 1, & \text{if } f_i(t) \geq 0 \\ 0, & \text{else} \end{cases} \tag{13}$$

where $\sigma = 1$ indicates that the continuously communication mode is adopted, otherwise, periodic communication is adopted. $f_i(t)$ is the event triggering function which will be designed later. Secondly, the state tracking error $\tilde{e}(tk)$ is introduced.

$$\tilde{e}(tk) = p_l(t_{kh}) - p_f(t) - d_{lf}, \tag{14}$$

where $\{t_{kh}\}$, $k = 1, 2, \cdots N$ are the time instants at which the state information of the leader AUV will be sended to the follower AUV in periodic communication mode, h is the sampling period, and $d_{lf} \in R^3$ is a constant vector in the global coordinate frame $\{E\}$.

For convenience, denote $\{t^0, t^1, \cdots, t^k\}$ as a sequence of triggering instants for follower AUV, and the triggering instant sequence t^k for the follower AUV is defined iteratively by

$$t^{k+1} = inf\{t > t^k : f_i(t) \geq 0\}, \tag{15}$$

Here, the event triggering function is defined as follows:

$$f_i(t) = |\tilde{e}(tk) - d|, \tag{16}$$

where $d = [d_x, d_y, d_z]^T \in R^3$ is a positive definite matrix denoting the threshold of the triggering event.

Once the event is triggered, the continuously communication mode is selected, and to exclud the Zeno behavior, the continuously communication mode will be maintain h.

Remark 4: Here, the demand that the continuously communication mode will be maintain h, which not only can exclude the Zeno behavior, but also can reduce the number of selection signal.

Remark 5: Compared to the continuously state information p_l and υ_l used in the whole formation process [9], [12]–[14], only continuously state information is needed when the event is triggered. Therefore, the communication burden can be greatly reduced while guaranteeing the desired control performance.

Remark 6: Note that the continuous models are only used for the analysis and design purpose. Before implementation, a discretization of the model and algorithm is necessary. In the process of formation control, the feedback information is sampled periodically, and transmitted according to the event triggering condition. To obtain results and algorithms that are more suitable for practical applications, an extension of the method in this paper to the discrete time domain or sampled-data control framework will be given in a future paper.

3) CONTROL OBJECTIVE

In this paper, the control objective is to design a nonlinear control law τ_f for the follower AUV modeled by (8), under the event-triggered mechanism (14)-(16), a team of two networked AUVs can achieve formation in the global coordinate frame $\{E\}$, which means that the generalized position between the leader AUV and the follower AUV reaches to a defined distance in finite time, i.e.,

$$\lim_{t \longrightarrow T} \|p_l - p_f - d_{lf}\| = 0, \tag{17}$$

where T is the designed setting time, here, $T \leq h$.

Remark 7: According to the event-triggered mechanism given in this paper, once the event is triggered, the leader will transmit the state signals to the follower AUV in continuously communication mode, meanwhile, the follower AUV update its controller continuously until the formation is achieved. So, the follower AUV's controller will be designed under the continuously communication mode.

III. MAIN RESULTS

In this section, we will apply the fixed-time method, the command filtered techinique, and backstepping technique to design the control input $\tau_f = [\tau_{fu}, \tau_{fv}, \tau_{f\omega}]$ for the follower AUV to achieve the formation control objective.

In order to satisfy the demand for convergence time, the fixed-time theory is applied to the controller design process, and the whole process consists of the following two steps.

Step 1: Define the position tracking error surface vector $e_1 = [e_{1x}, e_{1y}, e_{1z}] \in R^3$ as

$$e_1 = p_l - p_f - d_{lf}, \tag{18}$$

where $d_{lf} \in R^3$ is the desired relative distance vector among the AUVs.

According to (8), the time derivative of (18) is

$$\dot{e}_1 = J(\Theta_l)\upsilon_l - J(\Theta_f)\upsilon_f, \tag{19}$$

where υ_f is viewed as the virtual control vector.

Here, we choose the nominal function v_f^c as

$$\upsilon_f^c = J^{-1}(\Theta_f)(k_1 e_1 + \alpha_1 e_1^{\frac{m_1}{n_1}} + \beta_1 e_1^{\frac{p_1}{q_1}} - \lambda_1 \xi_1 + J(\Theta_l)\upsilon_l), \tag{20}$$

where $k_1 \in R^{3\times3}$ and $\lambda_1 \in R^{3\times3}$ are positive definite design matrixs with $k_1 > 2\lambda_1$, and m_1, n_1, p_1, q_1 are positive odd integers satisfying $m_1 > n_1$ and $p_1 < q_1$. ξ_1 is the filter compensating signal to be designed later.

To generate the stabilizing function v_f^d and also its derivative $\dot{v}_f^d$, the nominal stabilizing function is then passed through a command filter (1), where the magnitude of the limit filters and bandwidth parameters are chosen based on the requirement of the system performance. Define the filtered error $\omega_1 = v_f^c - v_f^d$, and the compensating signal as in (20) is generated by the following system

$$\dot{\xi}_1 = \begin{cases} -\lambda_1\xi_1 - \alpha_1\xi_1^{\frac{m_1}{n_1}} - \beta_1\xi_1^{\frac{p_1}{q_1}} - f_1\xi_1 + \omega_1, & \text{if } |e_1| > \vartheta_1 \\ 0, & \text{if } |e_1| \leq \vartheta_1 \end{cases} \tag{21}$$

where $\vartheta_1 > 0$ is a small number, $\lambda_2 \in R^{3\times3}$ is a positive definite design matrix with $\lambda_2 > \lambda_1 + 1$, and $f_1 = \frac{|\omega_1 e_1| + \frac{1}{2}\lambda_2\omega_1^2}{\|\xi_1\|^2}$.

Remark 8: The differential term $\dot{\upsilon}_f^d$ can be directly acquired from the command filter (1), which can be used to replace the differential term $\dot{\upsilon}_f^c$ needed in the traditional backstepping design. As a result, the differential operation is replaced by the simple algebraic operation so that the designed formation control law is simple and easy to implement in practice. Meanwhile, the compensating signal is introduced to compensate the filtered error, which in turn can obtain a better control performance.

Define the Lyapunov candidate function as follows:

$$V_1 = \frac{1}{2}e_1^T e_1 + \frac{1}{2}\xi_1^T \xi_1. \tag{22}$$

According to (7), (8), (19)-(21) and Young's inequality, the time derivative of V_1 is

$$\begin{aligned}
\dot{V}_1 &= e_1\dot{e_1} + \xi_1\dot{\xi}_1 \\
&= e_1(-k_1e_1 - \alpha_1 e_1^{\frac{m_1}{n_1}} - \beta_1 e_1^{\frac{p_1}{q_1}} + \lambda_1\xi_1) \\
&\quad + \xi_1(-\lambda_1\xi_1 - \alpha_1\xi_1^{\frac{m_1}{n_1}} - \beta_1\xi_1^{\frac{p_1}{q_1}} - f_1\xi_1 + \omega_1) \\
&= -k_1e_1^2 - \alpha_1 e_1^{\frac{m_1}{n_1}+1} - \beta_1 e_1^{\frac{p_1}{q_1}+1} + \lambda_1 e_1\xi_1 \\
&\quad -\lambda_1\xi_1^2 - \alpha_1\xi_1^{\frac{m_1}{n_1}+1} - \beta_1\xi_1^{\frac{p_1}{q_1}+1} - f_1\xi_1^2 + \xi_1\omega_1 \\
&\le -k_1e_1^2 - \alpha_1 e_1^{\frac{m_1}{n_1}+1} - \beta_1 e_1^{\frac{p_1}{q_1}+1} + \frac{\lambda_1}{2}(e_1^2 + \omega_1^2) \\
&\quad -\lambda_1\xi_1^2 - \alpha_1\xi_1^{\frac{m_1}{n_1}+1} - \beta_1\xi_1^{\frac{p_1}{q_1}+1} - |\omega_1 e_1| - \frac{1}{2}\lambda_2\omega_1^2 \\
&\quad + \frac{1}{2}(\xi_1^2 + \omega_1^2) \\
&\le -(k_1 - \frac{\lambda_1}{2})e_1^2 - \alpha_1 e_1^{\frac{m_1}{n_1}+1} - \beta_1 e_1^{\frac{p_1}{q_1}+1} - (\lambda_1 - \frac{1}{2})\xi_1^2 \\
&\quad -\alpha_1\xi_1^{\frac{m_1}{n_1}+1} - \beta_1\xi_1^{\frac{p_1}{q_1}+1} - \frac{\lambda_2 - \lambda_1 - 1}{2}\omega_1^2 \\
&\le -\alpha_1 2^{\frac{m_1+n_1}{2n_1}}\{(\frac{1}{2}e_1^2)^{\frac{m_1+n_1}{2n_1}} + (\frac{1}{2}\xi_1^2)^{\frac{m_1+n_1}{2n_1}}\} \\
&\quad -\beta_1 2^{\frac{p_1+q_1}{2q_1}}\{(\frac{1}{2}e_1^2)^{\frac{p_1+q_1}{2q_1}} + (\frac{1}{2}\xi_1^2)^{\frac{p_1+q_1}{2q_1}}\} \\
&\le -\alpha_1 2^{\frac{m_1+n_1}{2n_1}}(\frac{1}{2}e_1^2 + \frac{1}{2}\xi_1^2)^{\frac{m_1+n_1}{2n_1}} \\
&\quad -\beta_1 2^{\frac{p_1+q_1}{2q_1}}(\frac{1}{2}e_1^2 + \frac{1}{2}\xi_1^2)^{\frac{p_1+q_1}{2q_1}} \\
&= -\hat{\alpha}V_1^{\frac{m_1+n_1}{2n_1}} - \hat{\beta}V_1^{\frac{p_1+q_1}{2q_1}}.
\end{aligned} \tag{23}$$

where $\hat{\alpha} = \alpha_1 2^{\frac{m_1+n_1}{2n_1}}$ and $\hat{\beta} = \beta_1 2^{\frac{p_1+q_1}{2q_1}}$.

Step 2: Define the velocity error vector $e_2 \in R^3$ as

$$e_2 = \upsilon_f - \upsilon_f^d. \tag{24}$$

According to (8), the time derivative of (23) is

$$\dot{e}_2 = M_f^{-1}\{-D(\upsilon_f)\upsilon_f - g(\eta_f) + \tau_f\} - \dot{\upsilon}_f^d. \tag{25}$$

The actual nonlinear control law are designed as

$$\tau_f = M_f\{-k_2e_2 - \alpha_1 e_2^{\frac{m_1}{n_1}} - \beta_1 e_2^{\frac{p_1}{q_1}} + \chi\} + \dot{\upsilon}_f^d, \tag{26}$$

where $k_2 \in R^{3\times 3}$ is a positive definite design matrix and $\chi = D(\upsilon_f)\upsilon_f + g(\eta_f)$.

Define the Lyapunov candidate function as follows:

$$V_2 = \frac{1}{2}e_2^T e_2. \tag{27}$$

According to (8), (25), (26) and Young's inequality, the time derivative of V_2 is

$$\begin{aligned}
\dot{V}_2 &= e_2(-k_2e_2 - \alpha_1 e_2^{\frac{m_1}{n_1}} - \beta_1 e_2^{\frac{p_1}{q_1}}) \\
&= -k_2e_2^2 - \alpha_1 e_2^{\frac{m_1}{n_1}+1} - \beta_1 e_2^{\frac{p_1}{q_1}+1} \\
&\le -\alpha_1 e_2^{\frac{m_1}{n_1}+1} - \beta_1 e_2^{\frac{p_1}{q_1}+1} \\
&= -\alpha_1(e_2^2)^{\frac{m_1+n_1}{2n_1}} - \beta_1(e_2^2)^{\frac{p_1+q_1}{2q_1}}.
\end{aligned} \tag{28}$$

Select the Lyapunov function candidate for the whole system consisting of (8), (18), (21) and (24) as follows:

$$\begin{aligned}
V &= V_1 + V_2 \\
&= \frac{1}{2}e_1^T e_1 + \frac{1}{2}e_2^T e_2 + \frac{1}{2}\xi_1^T \xi_1.
\end{aligned} \tag{29}$$

According to (7), (19)-(21), (25), (26) and Young's inequality, the time derivative of V is

$$\begin{aligned}
\dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + \xi_1\dot{\xi}_1 \\
&= e_1(-k_1e_1 - \alpha_1 e_1^{\frac{m_1}{n_1}} - \beta_1 e_1^{\frac{p_1}{q_1}} + \lambda_1\xi_1) \\
&\quad + \xi_1(-\lambda_1\xi_1 - \alpha_1\xi_1^{\frac{m_1}{n_1}} - \beta_1\xi_1^{\frac{p_1}{q_1}} - f_1\xi_1 + \omega_1) \\
&\quad + e_2(-k_2e_2 - \alpha_1 e_2^{\frac{m_1}{n_1}} - \beta_1 e_2^{\frac{p_1}{q_1}}) \\
&\le -\alpha_1(e_1^2)^{\frac{m_1+n_1}{2n_1}} - \alpha_1(\xi_1^2)^{\frac{m_1+n_1}{2n_1}} - \alpha_1(e_2^2)^{\frac{m_1+n_1}{2n_1}} \\
&\quad -\beta_1(e_1^2)^{\frac{p_1+q_1}{2q_1}} - \beta_1(\xi_1^2)^{\frac{p_1+q_1}{2q_1}} - \beta_1(e_2^2)^{\frac{p_1+q_1}{2q_1}} \\
&= -\alpha_1\{(e_1^2)^{\frac{m_1+n_1}{2n_1}} + (\xi_1^2)^{\frac{m_1+n_1}{2n_1}} + (e_2^2)^{\frac{m_1+n_1}{2n_1}}\} \\
&\quad -\beta_1\{(e_1^2)^{\frac{p_1+q_1}{2q_1}} + (\xi_1^2)^{\frac{p_1+q_1}{2q_1}} + (e_2^2)^{\frac{p_1+q_1}{2q_1}}\} \\
&= -\alpha_1 2^{\frac{m_1+n_1}{2n_1}}\{(\frac{1}{2}e_1^2)^{\frac{m_1+n_1}{2n_1}} + (\frac{1}{2}\xi_1^2)^{\frac{m_1+n_1}{2n_1}} + (\frac{1}{2}e_2^2)^{\frac{m_1+n_1}{2n_1}}\} \\
&\quad -\beta_1 2^{\frac{p_1+q_1}{2q_1}}\{(\frac{1}{2}e_1^2)^{\frac{p_1+q_1}{2q_1}} + (\frac{1}{2}\xi_1^2)^{\frac{p_1+q_1}{2q_1}} + (\frac{1}{2}e_2^2)^{\frac{p_1+q_1}{2q_1}}\} \\
&\le -\alpha_1 2^{\frac{m_1+n_1}{2n_1}}(V)^{\frac{m_1+n_1}{2n_1}} - \beta_1 2^{\frac{p_1+q_1}{2q_1}} 3^{1-\frac{p_1+q_1}{2q_1}}(V)^{\frac{p_1+q_1}{2q_1}}.
\end{aligned} \tag{30}$$

Therefore, there is the following theorem.

Theorem 1: With the kinematic and dynamic models of autonomous underwater vehicle which is given by (8), the event-triggered rule (14)-(16) and the control law (26) based on the intermediate control vector (20), the command filter (1), and the error compensating signal (21) has the following results:

1). Once the event is triggered, the formation position tracking error e_1 will converge to zero in finite time.

2). Closed-loop signals e_1, ξ_1, e_2 are all globally fixed-time stable.

3). Control signals $\tau_f = [\tau_{fu}, \tau_{fv}, \tau_{f\omega}]$ are both bounded.

Proof: The (30) can be rewritten as:

$$\begin{aligned}
\dot{V} &\le -\alpha_1 2^{\frac{m_1+n_1}{2n_1}}(V)^{\frac{m_1+n_1}{2n_1}} - \beta_1 2^{\frac{p_1+q_1}{2q_1}} 3^{\frac{q_1-p_1}{2q_1}}(V)^{\frac{p_1+q_1}{2q_1}} \\
&= -\tilde{\alpha}V^{\gamma_1} - \tilde{\beta}V^{\gamma_2},
\end{aligned} \tag{31}$$

where $\tilde{\alpha} = \alpha_1 2^{\frac{m_1+n_1}{2n_1}} > 0$, $\tilde{\beta} = \beta_1 2^{\frac{p_1+q_1}{2q_1}} 3^{\frac{q_1-p_1}{2q_1}} > 0$, $\gamma_1 = \frac{m_1+n_1}{2n_1} > 1$, and $0 < \gamma_2 = \frac{p_1+q_1}{2q_1} < 1$. According to *Lemma 3*, the fixed-time formation can be obtained, and the setting time T satisfies

$$T \le T_{\max} := \frac{1}{\tilde{\alpha}(\gamma_1 - 1)} + \frac{1}{\tilde{\beta}(1 - \gamma_2)}. \tag{32}$$

Remark 9: According to the theoretical proof, the convergence speed of formation can be chosen arbitrarily. In practical applications, this character can well satisfy the strict settling time requirement.

IV. SIMULATION

In this section, to illustrate the performance and effectiveness of the proposed control approach, numerical simulations and comparisons are performed for the formation control of two autonomous underwater vehicles by simulations. The simulations are carried out using MATLAB/Simulink environment.

A. PERFORMANCE OF PROPOSED FORMATION CONTROL LAW

In this subsection, the simulations are carried out using the proposed formation control law.

It is assumed that the dynamics for the AUVs are identical and all the AUVs have the same structure and the model parameters are $M_i = \mathrm{diag}\{150, 120, 120\}$, $D_i(\upsilon_i) = \mathrm{diag}\{100 + 80|u_i|, 80 + 60|v_i|, 80 + 60|\omega_i|\}$, and $\phi_i = -\pi/8, \theta_i = \pi/12, \psi_i = \pi/4, i = l, f$.

The formation of AUVs is placed in the global coordinate frame $\{E\}$, where the desired relative distance among the AUVs is given by $d_{lf} = [2, 6, 6]^T$.

The trajectory of the leader AUV is described as follows:

$$u_l = v_l = \omega_l = 2 \text{ m/s}, \tag{33}$$

with the initial position $p_l(0) = [0, 0, 0]$.

The initial position of the follower is $p_f(0) = [0, -2, -4]$, and the initial velocity is $v_f(0) = [0, 0, 0]$.

The control parameters are selected as $h = 4$, $k_1 = \mathrm{diag}(5, 5, 5)$, $k_2 = \mathrm{diag}(3, 3, 3)$, $\lambda_1 = \mathrm{diag}(2, 2, 2)$, $\lambda_2 = \mathrm{diag}(3.5, 3.5, 3.5)$, $\alpha_1 = 1$, $\beta_1 = 2$, $m_1 = 5$, $n_1 = 3$, $p_1 = 3$, and $q_1 = 7$.

According to the parameters given above, we can obtain the setting time T satisfies

$$T \le T_{\max} := 3 < h. \tag{34}$$

The simulation results are shown in Figs. 1-11.

Fig. 1 depicts the formation motion trajectories of two AUVs in the leader-follower strategy under the proposed control law, and it can be seen that the follower AUV is able to follow the leader AUV. Fig. 2 depicts the motion states of two AUVs in each motion direction. The formation position tracking errors e_{1x}, e_{1y}, and e_{1z} are plotted in Fig. 3, and converge to the origin after 2 seconds. In Figs. 4-5, the velocity input υ_f^c and the actual control input τ_f of the follower AUV are shown. Fig. 6 depicts the velocity tracking

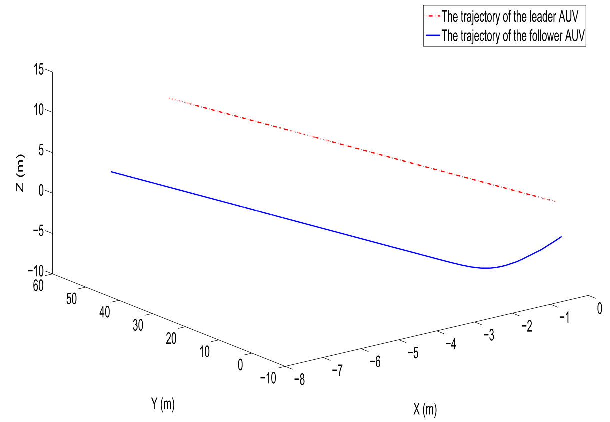

FIGURE 1. The trajectories of the leader AUV and the follower AUV.

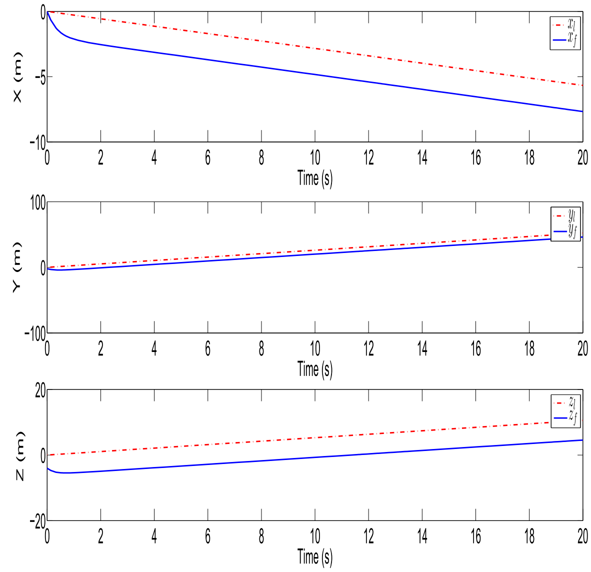

FIGURE 2. The position state p_l and p_f.

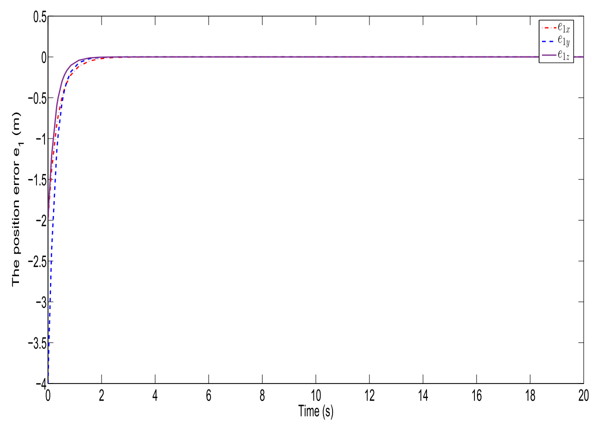

FIGURE 3. The position tracking error vector e_1.

error e_2 which converge to the origin after 2 seconds, too. According to Fig. 6, we can obtain that the actual input τ_f can drive the follower AUV sailing at the designed velocity υ_f^c.

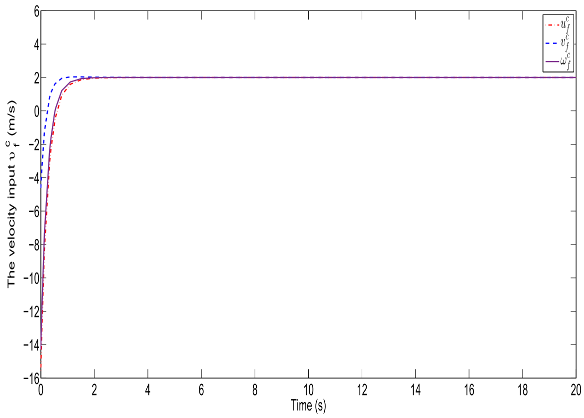

FIGURE 4. The velocity vector υ_f^c of the follower AUV.

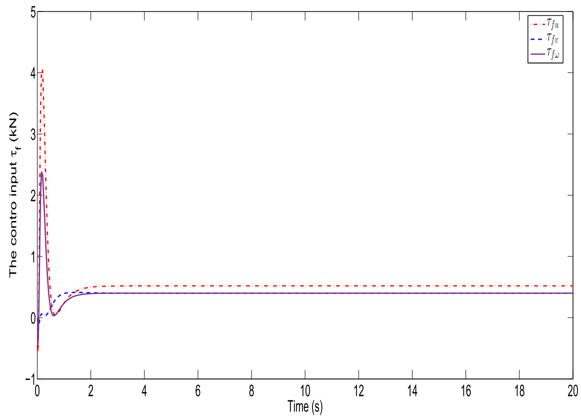

FIGURE 5. The control input τ_f of the follower AUV.

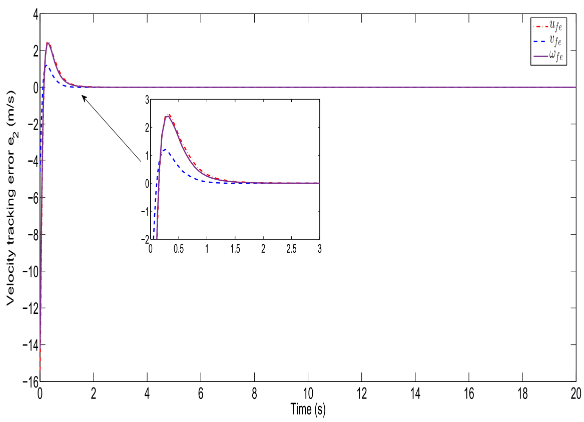

FIGURE 6. The velocity tracking error vector e_2.

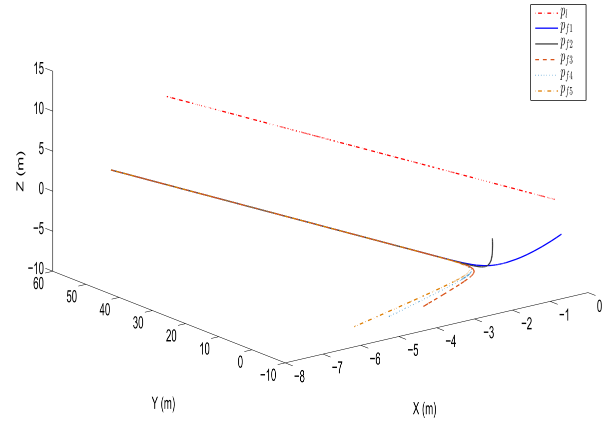

FIGURE 7. The trajectories of the leader AUV and the follower AUV with different initinal states.

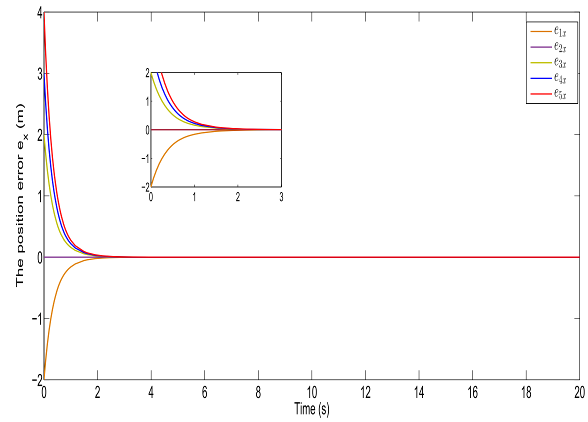

FIGURE 8. The position tracking error e_x.

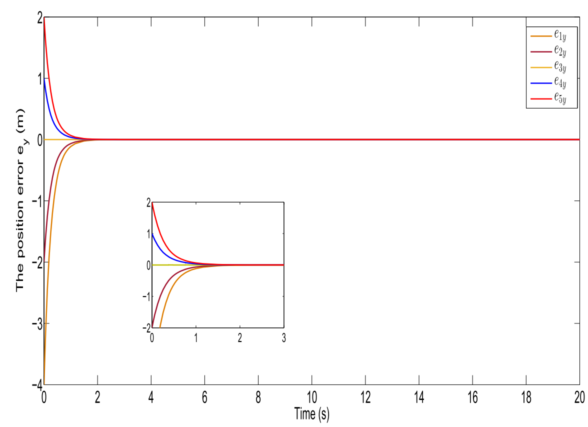

FIGURE 9. The position tracking error e_y.

The simulation results show that the proposed state feedback controller (26) can ensure that the desired formation can be achieved within the time required.

In order to demonstrate that the settling time is independent on the initial states of the system, a simulation is also conducted with different initial states of the follwer AUV $p_{fx}(0) = [0, -2, -4, -5, -6]$, $p_{fy}(0) = [-2, -4, -6, -7, -8]$, $p_{fz}(0) = [-4, -2, -8, -7, -5]$ for the event-triggered formation control algorithms with the same dynamic models, and control gains.

The results can be proved according to Figs. 7-10.

To prove the given event-triggered communication strategy can reduce the amount of data significantly that needed to be transmitted in the process of formation control than continuously communication, here, we define the amount of data

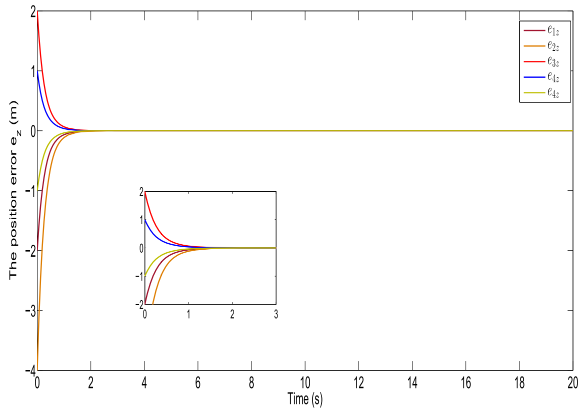

FIGURE 10. The position tracking error e_z.

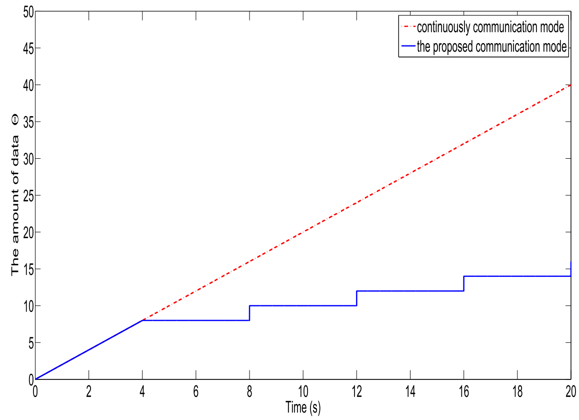

FIGURE 11. The amount of data Θ with $\varepsilon = 2$.

transferred per unit time as ε among AUVs, the amount of data in the whole formation process as Θ, and the simulation result is shown in Fig. 11.

As described in the paper, the continuously communication mode only be adopt when the event is triggered (i.e. in 0-4s) and the periodic communication is adopt in the remaining time (i.e. the event is not triggered). The simulation result show that the proposed event-triggered communication scheme can reduce amount of data significantly. As is well known, energy consumption is proportional to the amount of data, so the simulation result can indicate that energy consumption can be reduced significantly, too.

B. COMPARISON WITH PID CONTROL LAW

In this subsection, comparisons with a PID controller is made, and the PID control law is given as follows:

$$\tau_{fpid} = K_p e_1(t) + K_i \int_0^t e_1(\tau_t) d\tau_t + K_d \dot{e}_1(t), \qquad (35)$$

where $K_p = \mathrm{diag}(10, 10, 10)$, $K_i = \mathrm{diag}(5, 5, 5)$, $K_d = \mathrm{diag}(5, 5, 5)$.

The simulation results are shown in Figs. 12-14.

Figs. 12-13 depict the comparison of formation performance among the proposed fixed-time controller and the PID controller. It is observed that the proposed fixed-time controller can achieve formation with the better performance.

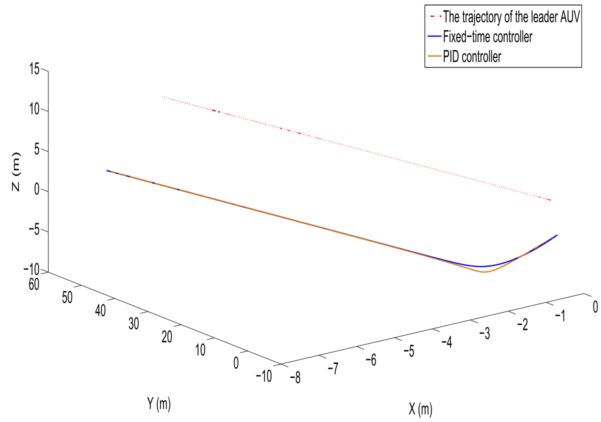

FIGURE 12. Comparison of formation performance.

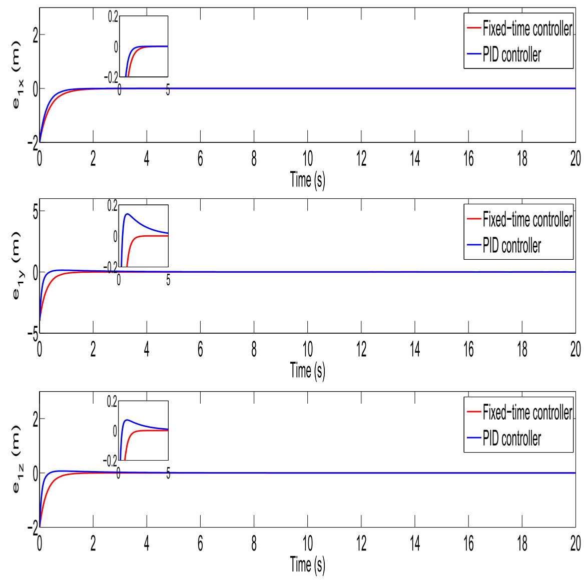

FIGURE 13. Comparison of formation performance in the direction of X, Y, Z, respectively.

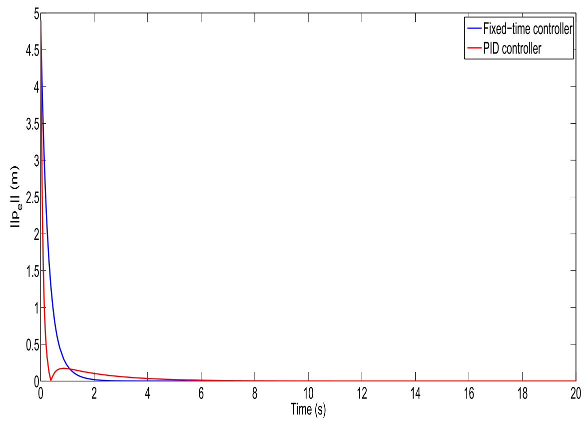

FIGURE 14. Comparison of formation error norm $\|p_e\|$.

To show the efficiency of the proposed controller better, define the error norm as

$$\|p_e\| = \sqrt{e_{1x}^2 + e_{1y}^2 + e_{1z}^2}. \tag{36}$$

Fig. 14 depicts the comparison results of formation error norm $\|p_e\|$ among the two controllers. Notice that the PID controller has a good steady-state performance, but less satisfactory transient performance, i.e., the formation error suddenly has a peak at about 0.5s. By contrast, the proposed fixed-time controller have better transient performance and the formation of two AUVs can be achieved more smoothly.

V. CONCLUSION

In this paper, we consider a fixed-time leader-following formation control method for a set of autonomous underwater vehicles (AUVs) with event-triggered acoustic communications. Compared with continuously communication mode, the event-triggered mechanism relieves the occupation of network abundantly and has better system property. With the proposed event-triggered control approach, the energy consumption and communication burden can be reduced significantly. In order to satisfy the demand for convergence time, the fixed-time based formation control algorithm is proposed which can ensure the convergence time meets our requirement. Furthermore, the settling time regardless of the initial states of the system. Finally, simulations and comparisons of our proposed control law and PID control law have demonstrated the performance of the proposed formation control scheme.

It is worth mentioning that this paper does not take into account of the problem of model uncertainties and external disturbance. In order to apply the proposed approach in practice more widely, it will be interesting to study the performance of the controller under model uncertainties and external disturbance, which will be concerned in the future work.

ACKNOWLEDGMENT

The authors would like to thank the associate editor and reviewers for all their very valuable suggestions and comments which have helped to improve the presentation and quality of the paper.

REFERENCES

[1] K. Y. Pettersen and H. Nijmeijer, "Underactuated ship tracking control: Theory and experiments," *Int. J. Control*, vol. 74, no. 14, pp. 1435–1446, Sep. 2001.

[2] F. A. Papoulias, "Cross track error and proportional turning rate guidance of marine vehicles," *J. Ship Res.*, vol. 38, no. 2, pp. 123–132, Jun. 1994.

[3] F. A. Papoulias and Z. O. Oral, "Hopf bifurcations and nonlinear studies of gain margins in path control of marine vehicles," *Appl. Ocean Res.*, vol. 17, no. 1, pp. 21–32, Jan. 1995.

[4] E. Lefeber, K. Y. Pettersen, and H. Nijmeijer, "Tracking control of an underactuated ship," *IEEE Trans. Control Syst. Technol.*, vol. 11, no. 1, pp. 52–61, Jan. 2003.

[5] Z.-P. Jiang, "Global tracking control of underactuated ships by Lyapunov's direct method," *Automatica*, vol. 38, no. 2, pp. 301–309, 2002.

[6] R. Skjetne, S. Moi, and T. I. Fossen, "Nonlinear formation control of marine craft," in *Proc. 41st IEEE Conf. Decis. Control*, Las Vegas, NV, USA, Dec. 2002, pp. 1699–1704.

[7] D. J. Stilwell and B. E. Bishop, "Platoons of underwater vehicles," *IEEE Control Syst.*, vol. 20, no. 6, pp. 45–52, Dec. 2000.

[8] E. Fiorelli, N. E. Leonard, P. Bhatta, D. A. Paley, R. Bachmayer, and D. M. Fratantoni, "Multi-AUV control and adaptive sampling in Monterey Bay," *IEEE J. Ocean. Eng.*, vol. 31, no. 4, pp. 935–948, Oct. 2006.

[9] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot teams," *IEEE Trans. Robot. Autom.*, vol. 14, no. 6, pp. 926–939, Dec. 1998.

[10] K. D. Do, "Practical control of underactuated ships," *Ocean Eng.*, vol. 37, no. 13, pp. 1111–1119, Sep. 2010.

[11] M. A. Lewis and K.-H. Tan, "High precision formation control of mobile robots using virtual structures," *Auto. Robots*, vol. 4, no. 4, pp. 387–403, 1997.

[12] Z. Hu, C. Ma, L. Zhang, A. Halme, T. Hayat, and B. Ahmad, "Formation control of impulsive networked autonomous underwater vehicles under fixed and switching topologies," *Neurocomputing*, vol. 147, pp. 291–298, Jan. 2015.

[13] R. Cui, S. S. Ge, B. V. E. How, and Y. S. Choo, "Leader–follower formation control of underactuated autonomous underwater vehicles," *Ocean Eng.*, vol. 37, nos. 17–18, pp. 1491–1502, Dec. 2010.

[14] D. B. Edwards, T. A. Bean, D. L. Odell, and M. J. Anderson, "A leader-follower algorithm for multiple AUV formations," in *Proc. IEEE Conf. Auto. Underwater Vehicles*, Sebasco, ME, USA, Jun. 2004, pp. 40–46.

[15] M. Ou, H. Du, and S. Li, "Finite-time formation control of multiple nonholonomic mobile robots," *Int. J. Robust Nonlinear Control*, vol. 24, no. 1, pp. 140–165, Jan. 2014.

[16] Y. Cao, W. Ren, and Z. Meng, "Decentralized finite-time sliding mode estimators and their applications in decentralized finite-time formation tracking," *Syst. Control Lett.*, vol. 59, no. 9, pp. 522–529, Sep. 2010.

[17] H. Du, W. Zhu, G. Wen, and D. Wi, "Finite-time formation control for a group of quadrotor aircraft," *Aerosp. Sci. Technol.*, vol. 69, pp. 609–616, Oct. 2017.

[18] X. Jin, "Fault tolerant finite-time leader-follower formation control for autonomous surface vessels with LOS range and angle constraints," *Automatica*, vol. 68, pp. 228–236, Jun. 2016.

[19] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2106–2110, Aug. 2012.

[20] Y. Zuo and L. Tie, "A new class of finite-time nonlinear consensus protocols for multi-agent systems," *Int. J. Control*, vol. 87, no. 2, pp. 363–370, Feb. 2014.

[21] Z. Zuo, "Nonsingular fixed-time consensus tracking for second-order multi-agent networks," *Automatica*, vol. 54, pp. 305–309, Apr. 2015.

[22] M. Defoort, A. Polyakov, G. Demesure, M. Djemai, and K. Veluvolu, "Leader-follower fixed-time consensus for multi-agent systems with unknown non-linear inherent dynamics," *IET Control Theory*, vol. 9, no. 14, pp. 2165–2170, Sep. 2015.

[23] R. Campos, N. Gracias, and P. Ridao, "Underwater multi-vehicle trajectory alignment and mapping using acoustic and optical constraints," *Sensors*, vol. 16, no. 3, p. 387, Mar. 2016.

[24] B. S. Park, "Adaptive formation control of underactuated autonomous underwater vehicles," *Ocean Eng.*, vol. 96, pp. 1–7, Mar. 2015.

[25] J. Ghommam and M. Saad, "Adaptive leader–follower formation control of underactuated surface vessels under asymmetric range and bearing constraints," *IEEE Trans. Veh Technol.*, vol. 67, no. 2, pp. 852–865, Feb. 2018.

[26] M. Zhao, C. Peng, W. He, and Y. Song, "Event-triggered communication for leader-following consensus of second-order multiagent systems," *IEEE Trans. Cybern.*, vol. 48, no. 6, pp. 1888–1897, Jun. 2018.

[27] L. Xing, C. Wen, F. Guo, Z. Liu, and H. Su, "Event-based consensus for linear multiagent systems without continuous communication," *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 2132–2142, Aug. 2017.

[28] H. Li, X. Liao, T. Huang, and W. Zhu, "Event-triggering sampling based leader-following consensus in second-order multi-agent systems," *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1998–2003, Jul. 2015.

[29] J. Liu, Y. Yu, Q. Wang, and C. Sun, "Fixed-time event-triggered consensus control for multi-agent systems with nonlinear uncertainties," *Neurocomputing*, vol. 260, pp. 497–504, Oct. 2017.

[30] X. Ge and Q.-L. Han, "Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism," *IEEE Trans. Ind. Electron.*, vol. 64, no. 10, pp. 8118–8127, Oct. 2017.

[31] W. Yue, L. Wang, and G. Guo, "Event-triggered platoon control of vehicles with time-varying delay and probabilistic faults," *Mech. Syst. Signal Process.*, vol. 87, pp. 96–117, Mar. 2017.

[32] Z. Zuo and L. Tie, "Distributed robust finite-time nonlinear consensus protocols for multi-agent systems," *Int. J. Syst Sci.*, vol. 47, no. 6, pp. 1366–1375, Apr. 2016.

ZHENYU GAO was born in Liaocheng, Shandong, China. He received the B.S. degree in automation from the Shandong University of Technology in 2013. He is currently pursuing the Ph.D. degree with the Department of Automation, Dalian Maritime University. His current research interests include optimal control and autonomous surface/underwater vehicle control.

GE GUO was born in Lanzhou, Gansu, China. He received the B.S. degree in automatic instrument and equipment and the Ph.D. degree in control science and engineering from Northeastern University, Shenyang, in 1994 and 1998, respectively. His current research interests include networked control systems, multi-agent systems, sensor/actuator networks, vehicular cooperative control, and autonomous surface/underwater vehicle control. He has published over 160 Journal papers within these areas.

He is currently a Managing Editor of the *International Journal of Systems, Control and Communications*, an Associate Editor of the *Information Sciences* and the *IEEE Intelligent Transportation Systems Magazine*, and an Editorial Board Member of *ACTA Automatica Sinica*.