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Triple-Mode and Triple-Band Cavity Bandpass Filter on Triplet Topology With Controllable Transmission Zeros

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ABSTRACT On the basis of triplet and its properties, a class of triple-mode and triple-band narrow-band bandpass filters is presented and designed by simultaneously exploiting the three resonant modes in a single rectangular cavity: TE_{101}, TE_{011}, and TM_{110} modes. The input/output ports of the proposed filter are formed by coupling a waveguide feed to a slot on the side wall of a rectangular cavity. These resonant modes are excited by making use of the position and shape of the slots at input/output of the rectangular cavity without any intra-cavity coupling structure. Besides three poles within the desired passband, a pair of transmission zeros (TZs) can be produced and controlled to appear at the inner or outer sides of this passband. Due to emergence of the three resonant modes and realization of a pair of TZs, a highly attenuated and sharpened filtering selectivity is achieved. In order to verify the proposed approach, two filter prototypes are in final fabricated and measured.

INDEX TERMS Triplet, triple-mode, triple-band, rectangular cavity, bandpass filter, slots.

I. INTRODUCTION

Since 1970s, multi-mode cavities have been widely used for designing various high-performance bandpass filters with multi-band functionality in many kinds of applications, such as satellite transponders, digital television broadcasting, base stations, and so on [1]–[5]. As one of the principal advantages, a multi-mode cavity can be easily used for size and mass reduction. In the recent years, multi-band system has been widely considered as one of the most useful techniques in this aspect as multi-channel signals could be implemented in most of involved devices. Hence, a large number of literatures have been focusing on development of various multi-band bandpass filters with multi-mode cavities in the past years [6]–[12].

Meanwhile, the high selectivity is highly demanded to create a highly sharpened response near cut-off frequencies so as to reject unwanted near-by noise signals. Triplet is firstly discussed in this paper as specific coupling topology as shown in Fig. 1, and its simplified topology doublet [13], [14] has exhibited the attractive zero-shifting feature for enhancement of filter's selectivity [15]. On the basis of doublet, some called extended and modified doublets have been realized not only in cavities, but also in substrate integrated waveguide (SIW) and microstrip line [16]–[19]. However, there are little literatures reported to more resonators, e.g., triplet [20]. Higher-order bandpass or bandstop filters with symmetric and asymmetric pseudo-elliptic responses can be designed

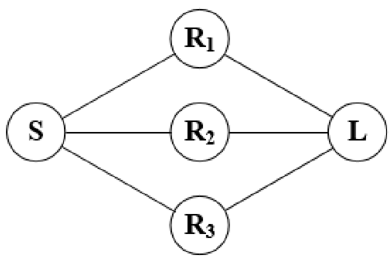


FIGURE 1. Basic coupling topology of a triplet for design of the proposed filter.

by cascading a few doublets with resonators [13] or non-resonators [10], [21]. As such, one can design a few separate sections as a module independently, thus providing much flexibility and convenience in filter design.

In this paper, a triplet topology is realized in a single multi- mode rectangular cavity where its first three resonant modes, namely, TE_{101}, TE_{011} and TM_{110} modes, are exploited. As well known, electric-field polarizations of these three modes are orthogonal to each other. By making effective use of a slot installed at the input/output sections of this cavity, these three resonant modes can be excited successfully. As such, a multi- band filter can be realized with virtue of these different modes. The proposed triplet is composed of different modes in a cavity, and it is then analyzed for filter design. The proposed filter shows good performance in both in-band and out-of-band, as well as simple geometry. Besides three poles within the passband, a pair of transmission zeros (TZs) is achieved. These TZs can be controlled by setting their positions at the inner or outer sides of the passband. Four cases of TZs are analyzed using coupling matrix: 1) Two TZs in lower-stopband; 2) Two TZs in upper-stopband; 3) One TZ in lower-stopband and the other in upper-stopband; 4) Two TZs inside the passband to separate three resonant modes into triple-band. In the remainder of this work, the working principle and design procedure of the presented multi-mode and multi-band cavity filter will be extensively described, and then two filters are fabricated and tested to validate the proposed technique in experiment. Both designs can be cascaded more cavities to form higher-order response with sharp cut-off and deep notch depth.

II. PROPERTIES OF TRIPLET TOPOLOGY

Fig. 1 depicts the basic triplet, inclusive of three resonators which are all coupled to the source and load in parallel and not interacted with each other. Because of the three separate transmission paths between the source and load, the triplet topology can generate two TZs on either side of the desired passband at maximum. According to the coupling topology, a general coupling matrix of a triplet can be written as followed

$$M=\begin{bmatrix} 0 & M_{S1} & M_{S2} & M_{S3} & 0 \\ M_{S1} & M_{11} & 0 & 0 & M_{L1} \\ M_{S2} & 0 & M_{22} & 0 & M_{L2} \\ M_{S3} & 0 & 0 & M_{33} & M_{L3} \\ 0 & M_{L1} & M_{L2} & M_{L3} & 0 \end{bmatrix} \tag{1}$$

The scattering parameters or S-parameters of this triplet can then be expressed using the following two equations:

$$S_{21}=-\frac{k_1\Omega^2+k_2\Omega+k_3}{\det}*2\mathrm{i} \tag{2}$$

$$S_{11}=1-\frac{\Omega^3\,\mathrm{i}+d\Omega^2+e\Omega+f}{\det}*2\mathrm{i} \tag{3}$$

In (2) and (3), the symbol, Ω, denotes the normalized frequency of the low-pass prototype, and *det* is the determinant of the coupling matrix. The other parameters, k_1, k_2, k_3, d, e, and f are the constants related to the coupling coefficients in (1), the detailed expression is derived in Appendix. For a prescribed filtering specification, the coupling matrix can be first synthesized based on the method in [22] and then the filtering responses can be calculated from (2) and (3).

To determine the number and position of TZs, (2) is enforced $|S_{21}|=0$ so as to derive two solutions, such that

$$\Omega_{Z1}=-\frac{q+\sqrt{p}}{2k_1} \tag{4}$$

$$\Omega_{Z2}=-\frac{q-\sqrt{p}}{2k_1} \tag{5}$$

In (4) and (5), the parameters, k_1, q, and p are the constants related to the coupling coefficients in (1), the detailed

expression is derived in Appendix. By adjusting the values of coupling matrix, two TZs can thus be set at specific positions to meet a variety of filtering requirements.

III. REALIZATION OF CAVITY FILTER ON TRIPLET

In this part, we firstly put forward two types of triplets: triplet with out-of-band TZs and triplet with in-band TZs, and two cavity filter prototypes are presented by excitation of first three resonant modes in a multi-mode cavity. The cavity used herein is made up of silver plated aluminum, and filled with air. Each side of a rectangular cavity is set as about one half wavelength for its respective resonant mode. The port feeds used are the WR284 rectangular waveguide.

A. TRIPLET WITH OUT-OF-BAND TZS

From (2), two solutions are obtained for the two transmission zeros if $|S_{21}|=0$. In general, there are three cases for the triplet topology in Fig. 1 when out-of-band TZs are produced. For the first and second cases, two TZs can be located at one side of the passband under the assumption of $M_{L2}=-M_{S2}$, and $M_{L1}=M_{S1}$, and $M_{L3}=M_{S3}$. For the first case with $M_{11}<0$ and $M_{33}>0$, two TZs can be both excited at left side of the passband; whereas, for the second case with $M_{11}>0$ and $M_{33}<0$, two TZs are both at right side of the passband. As shown in Fig. 2, two curves show that two TZs can be shifted from the left to the right side.

For the third case, two TZs are separately placed at the left and right sides. According to the triplet topology in Fig. 1, the values of relevant coupling matrix are obtained as depicted in eq. (6). For a specification of center frequency at 2.99 GHz, bandwidth of 30 MHz, 15-dB return loss, and two TZs at 2.93/3.04 GHz, the matrix can be formed and synthesized. For simplification, we set M_{22} approximately equal to zero, and $M_{33}=-M_{11}$, $M_{L2}=M_{S2}$, and $M_{L1}=-M_{S1}=M_{L3}=-M_{S3}$, as depicted in (6) according to [22].

$$M=\begin{bmatrix} 0 & -0.53 & 0.78 & -0.53 & 0 \\ -0.53 & 1.44 & 0 & 0 & 0.53 \\ 0.78 & 0 & -0.04 & 0 & 0.78 \\ -0.53 & 0 & 0 & -1.44 & 0.53 \\ 0 & 0.53 & 0.78 & 0.53 & 0 \end{bmatrix} \tag{6}$$

Then, a prototype, named as $TE_{011}/TE_{101}/TM_{110}$ triplet, is proposed as shown in Fig. 3. Herein, the central portion is a

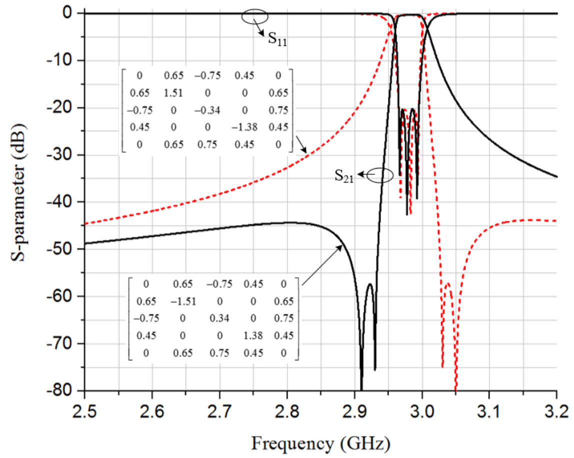


FIGURE 2. Calculated S-parameters of the proposed filter in the 1st and 2nd cases.

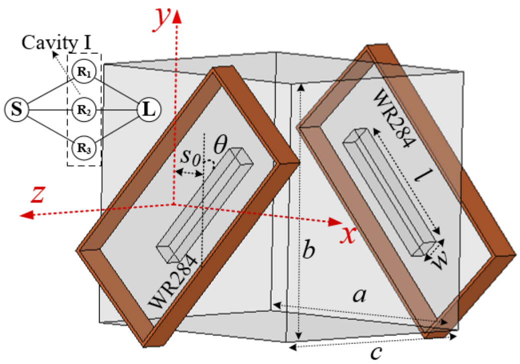


FIGURE 3. Geometrical structure of the designed triple-mode bandpass filter.

standard rectangular cavity, whose width, height, and length are marked as a, b, and c, respectively. A pair of symmetric shifted WR-284 waveguide ports is placed at the left- and right-hand sides of the cavity, while a pair of rectangular slots is coupled from two rectangular ports on the side wall of the cavity. The slots at the left and right sides of the cavity are both shifted along the positive X-axis with the offset to be marked as$s$. The slot at the left-hand side is rotated clockwise with an angle θ along the Z-axis, whereas the slot at the right-hand side is rotated counterclockwise with θ along the Z-axis. The two slots are geometrically symmetric with respect to the XOY plane. In our design, the rotated angle in the XOY plane of the slots is employed to excite the resonant modes with electric field in the X- and Y-axis. By offsetting the slot position from the origin, the mode with electric field in the Z-axis could be further excited. Fig. 4 depicts electric field distributions in the cavity of these three modes. Their resonant frequencies can be simply derived as:

$$w_{0,1,1}^2 = \frac{v^2}{\varepsilon_r u_r}\left[\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2\right] \tag{7a}$$

$$w_{1,0,1}^2 = \frac{v^2}{\varepsilon_r u_r}\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{c}\right)^2\right] \tag{7b}$$

$$w_{1,1,0}^2 = \frac{v^2}{\varepsilon_r u_r}\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right] \tag{7c}$$

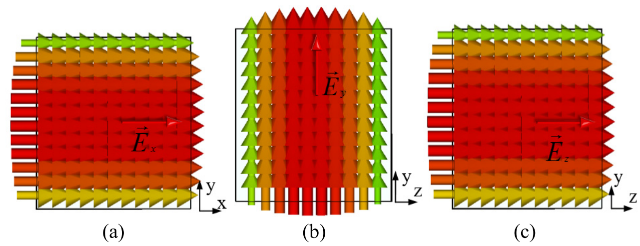


FIGURE 4. Electric field distributions of three modes. (a) TE_{011}, (b) TE_{101}, (c) TM_{110}.

where $w_{\mathrm{m,n,p}}$ is the resonant radian frequency of the specific mode (m, n, p are 0 and 1). v is the velocity of light in air. The three equations are a linear system equation that relates the three resonant frequencies as a function of the three cavity dimensions. a, b, and c represent the length, width, and height of the rectangular cavity as denoted in Fig. 3.

According to the triplet in Fig. 1, the $TE_{011}/TE_{011}/TM_{110}$ triple-mode bandpass filter on a rectangular cavity is formed up, where there is no intra-cavity coupling and the electric-field polarizations of the three modes in cavity are orthogonal to each other. Plus, there exist three in-band poles and two out-of-band TZs. TE_{011} mode and TE_{101} mode are excited by coupling electromagnetic (EM) wave from the rectangular waveguide to the slot on the side wall of the cavity. In the slot, electric field components are divided into two components: along X-axis $\vec{E}_x$, and Y-axis $\vec{E}_y$, and they have the same polarizations as the electric fields of TE_{011} mode and TE_{101} mode, resulting to effective excitation of TE_{011} mode and TE_{101} mode. As for the excitation of TM_{110} mode, the offset arrangement of this slot is used as a perturbation for electric field along the Z-axis to excite the resonant mode with electric component along the Z-axis. Since the TM_{110} mode has this electric field direction in the Z-axis, it can be excited to resonate in this cavity.

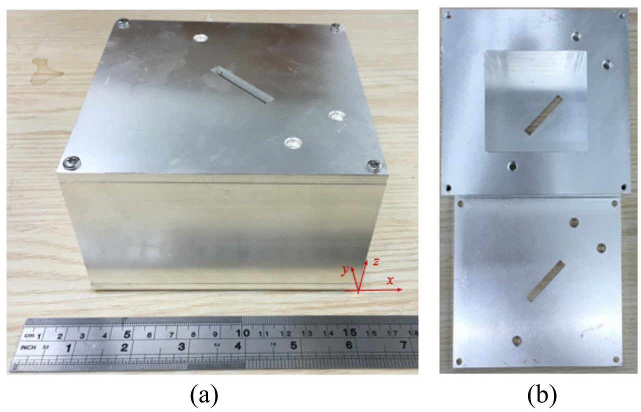


FIGURE 5. Photograph of the fabricated triple-mode cavity bandpass filter. (a) External view, (b) internal view.

The triple-mode cavity filter is then designed by using the commercial CST software and its fabricated circuit is depicted in Fig. 5, where a subset picture shows the internal view of the fabricated cavity. The dimensions of the rectangular cavity are determined as: $a = 71.2$ mm, $b = 71$ mm and $c = 67.5$ mm, and the specific parameters: offset $s_0 = 6$ mm, rotation $\theta = 43°$.

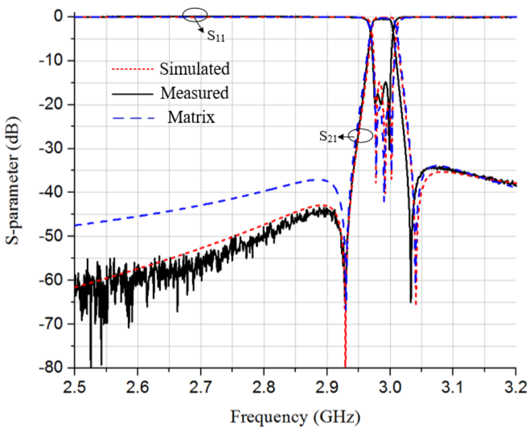


FIGURE 6. Comparison among the M-matrix calculated, measured, and simulated frequency responses of the proposed triple-mode cavity filter in Fig. 5.

Fig. 6 depicts the simulated, measured and coupling-matrix results for comparison. In measurement, the achieved passband has the center frequency of about 2.99 GHz, about 30.0 MHz absolute bandwidth, 0.5 dB insertion loss and more than 15.0 dB return loss. Moreover, two TZs are produced at about 2.93 and 3.04 GHz, respectively, and they are well correspond with the TZ's positions determined by the full-wave and matrix-based simulations. With the help of the two TZs, the out-of-band attenuation at both sides of the desired passband gets sharp skirt and achieves the level higher than 30.0 dB.

B. TRIPLET WITH IN-BAND TZS

Now, let's try to demonstrate how two TZs can be properly shifted inside the passband and make use of them to form a triple-band filter. To simplify our analysis, it is assumed that $M_{L1} = M_{S1}$, $M_{L2} = M_{S2}$, and $M_{L3} = M_{S3}$. Fig. 7(a) illustrates that two TZs emerge between three poles and their frequency locations can be determined by using (4) and (5). This phenomenon inspires us to separately allocate these three resonant modes into three individual passbands while creating each TZ between any of the two adjacent passbands.

In Fig. 7(b), the rotation θ_0 of the feeding slot is utilized to implement the frequency offset of the right TZ while the left TZ keeps unchanged, which the same as the tendency of M_{S1}. In Fig. 7(c), the length l_0 of the feeding slot is utilized to implement the frequency offset of both TZs, which the same as the tendency of M_{S3}. Owing to these TZs, each resonant mode contributes to one passband, thus three modes produce the three passbands with single poles in each passband. By cascading these two triple-mode cavities, a triple-band 2nd- order bandpass filter can be realized as will be demonstrated later on.

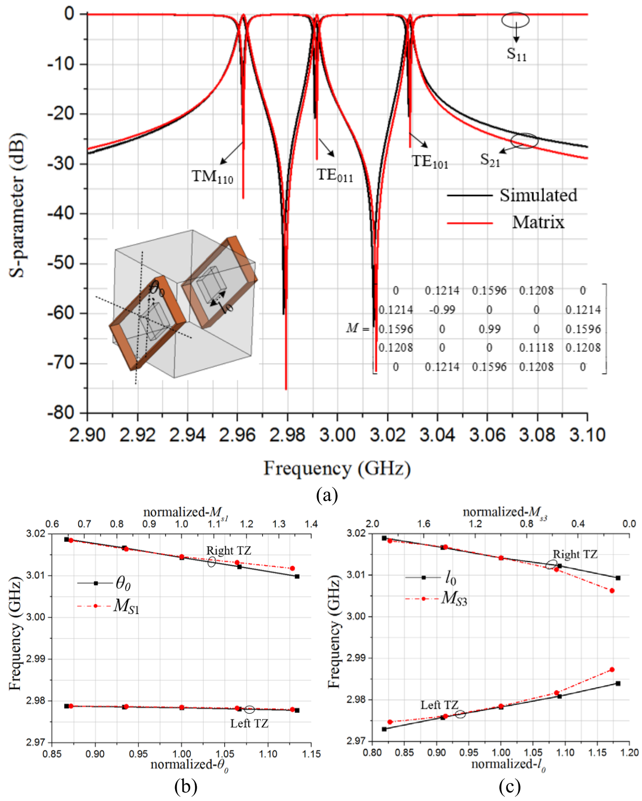


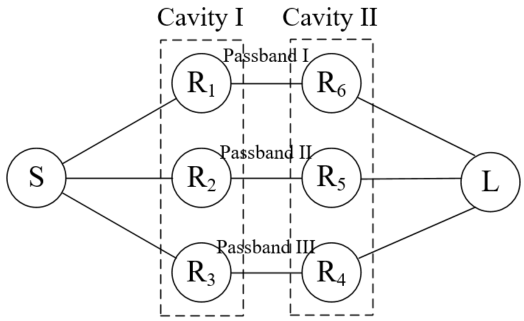
FIGURE 7. (a) Calculated S-parameter of the triplet with in-band TZs. (b) Variation in frequency of two TZs with respect to M_{S1} and θ_0. (c) Variation in frequency of two TZs with respect to M_{S3} and l_0.

Fig. 8(a) depicts the topology of a triple-band 2nd-order filter. For simplicity, it can be assumed that the first resonator of each band is coupled with the relevant second resonator without interaction with the resonator of other passbands. It means that no cross-coupling exists in the proposed topology. Fig. 8(b) depicts the geometrical structure of the proposed in-band-TZs triplet with dual-cavity. The two apertures in the left- and right-side walls with regard to input/output couplings share the same rotated angle marked as θ_1 and their offset distance are marked as s_1. The intra-cavity coupling slot has the offset marked as s_2 in opposite direction of the input/output slot with reference to the Y-axis, where the rotated angle is marked as θ_2. Herein, the synthesis method is employed to design this kind of triple-band filter. Consider the specification of three-passband fractional bandwidths of 0.44%, 0.17%, 0.15%, the element value of lowpass prototype is $g_1 = g_2 = 1.4142$. Actually, the bandwidths of each band can be determined by adjusting the values of their respective external quality factors. Thus, the external quality factors and coupling coefficients can be determined as

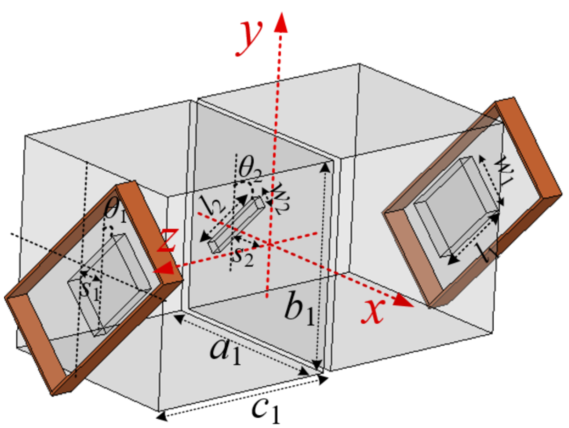
$$Q_e^I = \frac{g_1}{FBW^I} = 321.4 \tag{8a}$$

$$K_{12}^I = \frac{FBW^I}{\sqrt{g_1 g_2}} = 0.0031 \tag{8b}$$

$$Q_e^{II} = \frac{g_1}{FBW^{II}} = 831.9 \tag{8c}$$



(a)



(b)

FIGURE 8. **Schematic of the 2rd-order triplet with in-band-TZs. (a) Topology, (b) geometrical structure.**

$$K_{12}^{II} = \frac{FBW^{II}}{\sqrt{g_1 g_2}} = 0.0012 \tag{8d}$$

$$Q_e^{III} = \frac{g_1}{FBW^{III}} = 942.8 \tag{8e}$$

$$K_{12}^{III} = \frac{FBW^{III}}{\sqrt{g_1 g_2}} = 0.0011 \tag{8f}$$

where the superscripts represent the channel numbers, g_1 and g_2 are low-pass prototype element values of the second-order respond, FBW^I, FBW^{II}, and FBW^{III} stand for the fractional bandwidths of the first, second and third frequency channels, respectively.

The relationships among the coupling coefficients and external quality factors versus the physical dimensions of coupled resonators can be numerically extracted according to [23] and [24] by using:

$$K_{ij} = \pm\frac{f_{p2}^2 - f_{p1}^2}{f_{p2}^2 + f_{p1}^2} \tag{9a}$$

$$Q_e = \frac{f_0}{\Delta f_{3-dB}} \tag{9b}$$

where f_{p1} and f_{p2} are resonant frequencies of the resonant frequencies of the even and odd modes, respectively, and f_0 and Δf_{3-dB} are the bandwidth between $+/-90$ degrees phase shift of the resonant frequency.

According to the topology shown in Fig. 8(a) and the specification described above, the people can employ the coupling matrix for synthesis design of this triple-band filter. The proposed element values of the coupling matrix are determined according to [22], and the synthesized coupling matrix are provided in (10), as shown at the bottom of this page. Fig. 9 depicts the calculated S-parameter frequency responses of this triple-band coupling matrix.

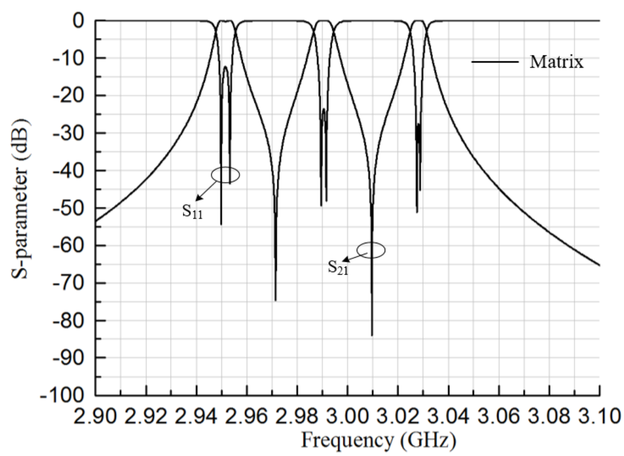


FIGURE 9. **Calculated S-parameter frequency responses from the triple-band coupling matrix (10).**

In order to verify the proposed design approach, a dual-cavity triple-band filter is fabricated and tested. Table 1 presents the detailed dimensions of the complete triple-band filter. The photograph of the fabricated filter is illustrated in the Fig. 10, which are prepared to illustrate the apertures and the relevant setup in measurement. The overall volume of this fabricated dual-cavity filter is about $1/2\lambda_0 \times 1/2\lambda_0 \times \lambda_0$, where λ_0 stands for the wavelength at the center frequency

$$M = \begin{bmatrix} 0 & 0.2334 & 0.2392 & 0.2569 & 0 & 0 & 0 & 0 \\ 0.2334 & -0.0204 & 0 & 0 & 0 & 0 & 1.0397 & 0 \\ 0.2392 & 0 & -0.0337 & 0 & 0 & -0.9109 & 0 & 0 \\ 0.2569 & 0 & 0 & -0.0498 & -0.0713 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0713 & -0.0498 & 0 & 0 & 0.2569 \\ 0 & 0 & -0.9109 & 0 & 0 & -0.0337 & 0 & 0.2392 \\ 0 & 1.0397 & 0 & 0 & 0 & 0 & -0.0204 & 0.2334 \\ 0 & 0 & 0 & 0 & 0.2569 & 0.2392 & 0.2334 & 0 \end{bmatrix} \tag{10}$$

TABLE 1. **Detailed dimensions of triple-band filter.**

a_1	70.8	s_2	10.5
b_1	72.5	l_1	29
c_1	68.2	w_1	20
θ_1	47	l_2	30
θ_2	45	w_2	0.8
s_1	12		

Unit: all in mm

of the second or middle passband. The Agilent N5230A Vector Network Analyzer (VNA) is used to test this two-port cavity bandpass filter. The simulated and measured results are plotted in Fig. 11 for comparison, and they are found in good agreement with each other. The subset in Fig. 11 displays the measured in-band insertion losses in all the three passbands of the fabricated cavity filter.

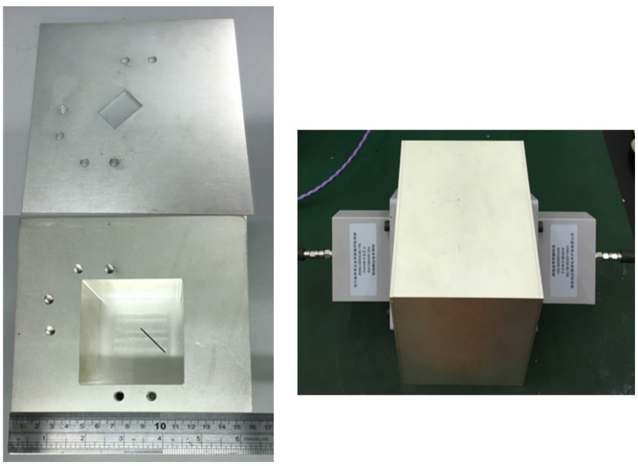


FIGURE 10. **Photographs of the fabricated dual-cavity triple-band cavity filter. (a) Internal view, (b) external view.**

Compared to the simulated return losses higher than 15.0 dB in all three passbands, the measured ones are found to be about 11.8 dB, 9.5 dB, and 13.2 dB in the three respective passbands. The maximum in-band insertion losses achieve 0.6 dB, 1.75 dB, and 1.2 dB in these three passbands against the 0.4 dB, 0.6 dB, and 0.7 dB in simulation. These unexpected discrepancies are primarily caused by unavoidable tolerance in fabrication, and it can be compensated by installing certain tuning elements at the cost of increased complexity in filter geometry. Nevertheless, reasonably good agreement between the simulated and measured results, as shown in Fig. 11, has evidently verified our proposed design approach.

Moreover, two transmission poles have been successfully produced in measurement to achieve the flat in-band frequency responses in all the three passbands. Meanwhile, two pairs of TZs have been expectedly excited between any of two adjacent passbands to achieve good band-to-band isolation, i.e., higher than 50.0 dB, as highly demanded. Three desired passbands have achieved about 11.0 MHz, 4.8 MHz, and 3.3 MHz absolute bandwidths at center frequencies

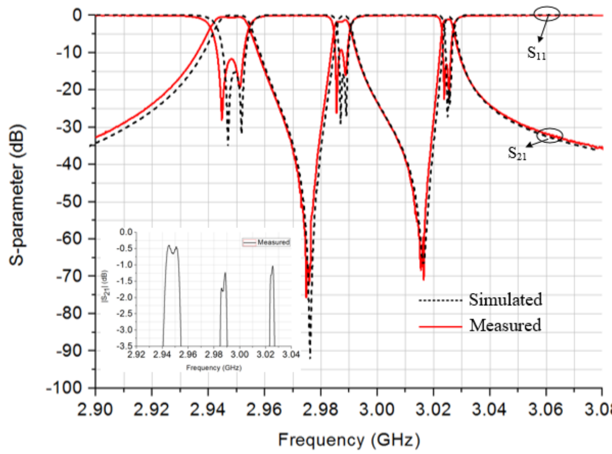


FIGURE 11. **Comparison between the simulated and measured frequency responses of the dual-cavity 2nd-order triple-band filter.**

of 2.95 GHz, 2.99 GHz, and 3.03 GHz in measurement. They are all reasonably matched with the predicted ones in simulation.

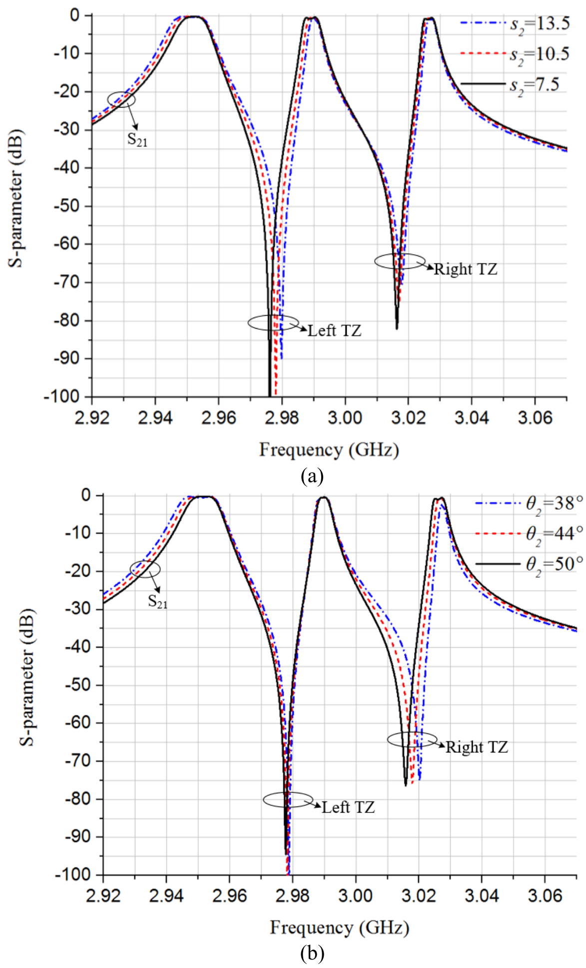


FIGURE 12. **(a) Variation in location of TZs with respect to the s_2 value. (b) Variation in location of TZs with respect to the θ_2 value.**

Fig. 12 indicates the variation in controllable locations of the two TZs. In Fig. 12(a), the slight shift of the left TZ is achieved by changing the value of the offset s_2. While in Fig. 12(b), the similar shift of the right TZ is achieved by changing the values of the rotation θ_2, and the location of left TZ keeps unchanged.

IV. CONCLUSION

In this paper, three fundamental resonant modes, i.e., TE_{011}, TE_{101} and TM_{110}, in a single rectangular metal cavity are applied to present the two kinds of triplet for design and exploration of a triple-mode filter with two out-of-band TZs and a triple-band filter with two in-band TZs. By virtue of these Two TZs, the designed filters have achieved highly sharpened responses near the cut-off frequencies. By cascading the two single cavities via an aperture on the middle metallic wall, the dual-cavity filters with triple passbands, i.e., triple-band filter. In this context, two transmission poles are achieved in each of three passbands, while one TZ is produced in each of stopbands between any of two adjacent passbands. In final, two kinds of filters are fabricated and measured. Both the measured results of these two fabricated filters are found in good agreement with those derived from full-wave simulation and M-matrix, thereby demonstrating the validity of our design method. And the proposed design methodology can be further explored to higher-order response by cascading more cavities.

APPENDIX

Here, the expressions of the coefficients involved in the scattering parameters in (2) and (3) are deduced and provided. The triplet coupling matrix is assumed to be of the form as

$$M = \begin{bmatrix} 0 & x_1 & x_2 & x_3 & 0 \\ x_1 & y_1 & 0 & 0 & x_4 \\ x_2 & 0 & y_2 & 0 & x_5 \\ x_3 & 0 & 0 & y_3 & x_6 \\ 0 & x_4 & x_5 & x_6 & 0 \end{bmatrix}$$

Using this matrix, the coefficients in (2) and (3) are given by

$$\begin{aligned}
k_1 &= x_1x_4 + x_2x_5 + x_3x_6 \\
k_2 &= x_1x_4(y_2+y_3) + x_2x_5(y_1+y_3) + x_3x_6(y_1+y_2) \\
k_3 &= x_1x_4y_2y_3 + x_2x_5y_1y_3 + x_3x_6y_1y_2 \\
d &= x_4^2 + x_5^2 + x_6^2 + (y_1+y_2+y_3)\mathrm{i} \\
e &= x_4^2(y_2+y_3) + x_5^2(y_1+y_3) + x_6^2(y_1+y_2) \\
&\quad + (y_1y_2 + y_2y_3 + y_1y_3)\mathrm{i} \\
f &= x_4^2y_2y_3 + x_5^2y_1y_3 + x_6^2y_1y_2 + y_1y_2y_3\mathrm{i} \\
\det &= -\Omega^3 + g\Omega^2 + h\Omega + j \\
g &= -(y_1+y_2+y_3) + (x_1^2 + x_2^2 + x_3^2 \\
&\quad + x_4^2 + x_5^2 + x_6^2)\mathrm{i} \\
h &= x_1^2(x_5^2 + x_6^2) + x_2^2(x_4^2 + x_6^2) + x_3^2(x_4^2 + x_5^2) \\
&\quad - (y_1y_2 + y_1y_3 + y_2y_3) - 2(x_1x_2x_4x_5 + x_1x_3x_4x_6 \\
&\quad + x_2x_3x_5x_6) + ((x_1^2 + x_4^2)(y_2+y_3) + \\
&\quad + (x_2^2x_5^2)(y_1+y_3) + (x_3^2 + x_6^2)(y_1+y_2))\mathrm{i} \\
j &= x_1^2(x_5^2y_3 + x_6^2y_2) + x_2^2(x_4^2y_3 + x_6^2y_1) \\
&\quad + x_3^2(x_4^2y_2 + x_5^2y_1) - 2(x_1x_2x_4x_5y_3 + x_2x_3x_5x_6y_1 \\
&\quad + x_1x_3x_4x_6y_2) - y_1y_2y_3 + ((x_1^2 + x_4^2)y_2y_3 \\
&\quad + (x_2^2 + x_5^2)y_1y_3 + (x_3^2 + x_6^2)y_1y_2)\mathrm{i} \\
q &= x_1x_4(y_2+y_3) + x_2x_5(y_1+y_3) + x_3x_6(y_1+y_2) \\
p &= x_1^2x_4^2(y_2-y_3)^2 + x_2^2x_5^2(y_1-y_3)^2 + x_3^2x_6^2(y_1-y_2)^2 \\
&\quad + 2x_1x_2x_4x_5(y_1y_2 + y_3^2 - y_1y_3 - y_2y_3) \\
&\quad + 2x_1x_3x_4x_6(y_1y_3 + y_2^2 - y_1y_2 - y_2y_3) \\
&\quad + 2x_2x_3x_5x_6(y_2y_3 + y_1^2 - y_1y_2 - y_1y_3)
\end{aligned}$$

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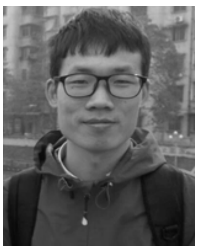

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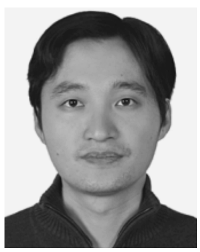

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