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An Electric Vehicle Routing Optimization Model With Hybrid Plug-In and Wireless Charging Systems

CHENG LI[1], (Student Member, IEEE), TAO DING[1], (Member, IEEE), XIYUAN LIU[1], (Student Member, IEEE), AND CAN HUANG[2], (Member, IEEE)

[1]State Key Laboratory of Electrical Insulation and Power Equipment, Department of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, China
[2]Lawrence Livermore National Laboratory, Livermore, CA 94550, USA

Corresponding author: Tao Ding (tding15@mail.xjtu.edu.cn)

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ABSTRACT In order to address the inconvenience of having to stop to charge the battery of an electric vehicle, wireless on-road charging technology, also known as charge-while-driving, has garnered much attention in recent studies. However, wireless charging devices may have a higher charge cost than traditional plug-in charging devices. Therefore, a multi-objective route optimization model based on model predictive control is established in this paper to determine an optimal route for drivers to coordinate wireless and plug-in charging strategies. To reduce the complexity of the proposed model due to its bilinear terms, the Big-M approach is employed to exactly linearize the bilinear terms by introducing dummy variables and additional constraints, which leads to a mixed integer linear programming model that can be solved efficiently. Finally, two systems are tested, including a real-world road map in Xi'an city to demonstrate the effectiveness of the proposed model.

INDEX TERMS Electric vehicles, mixed integer linear programming, plug-in charging, route optimization, wireless charging.

I. INTRODUCTION

Electric vehicles (EVs) have been developing rapidly due to their economic, environmental, and social benefits [1]–[9]. Due to the large amounts of sustainable energy accommodated by electric power systems, electric power is considered environmentally benign [10]–[13] EVs significantly reduce greenhouse gas emissions compared with gasoline-driven vehicles. EVs are powered by electric motors, with the energy usually stored in batteries. The first EV appeared following the discovery of electromagnetism in the 1880s [14]. EVs were popular in the 1890s and early 1900s. However, with the invention of the internal combustion engine in the 20th century and the subsequent mass production of gasoline vehicles, the EV lost its position in the automobile market. During the last few decades, the environmental impact of petroleum-based vehicles, as well as increasing oil prices, have led to renewed interest in EVs. It is reported that the U.S. market share of plug-in EVs (PEVs) increased from 0.14% in 2011 to 0.81% during the first eight months of 2016. California, the largest PEV regional market in the U.S., has over 223,000 registered PEVs [15]. EVs are expected to be a viable alternative to diesel- and gasoline-burning vehicles.

Traditional EVs (e.g., PEVs) need to recharge their batteries through home charging or public charging (e.g., charging stations), a process which is impacted by the availability and efficiency of the charging facilities. The issues that make charging inconvenient, such as limited charging facilities and the long wait time at charging stations, greatly limit EV adoption [16].

On the other hand, inductive power transfer (IPT) and wireless power transfer (WPT) technologies have been well developed in the past decade. In 2007, researchers from

MIT designed one of the first pieces of WPT equipment which can transfer 60 watts with nearly 40% efficiency [17]. In 2010, Lee *et al* developed a 220 W WPT prototype with 95% efficiency [18], and Wu *et al.* [19] implemented an IPT lighting system with 96% efficiency. Later, in 2013, a 7 kW WPT prototype with 90% efficiency was developed [20]. The advancement of IPT and WPT technologies greatly promote the development of wireless charging EVs. For example, the researchers from Oak Ridge National Laboratory (ORNL) designed an 8 kW wireless charger prototype with a 200-mm gap and 95.66% efficiency [21], and developed a 6.6 kW dynamic wireless power transfer apparatus with 85% efficiency [22]. With the convenience of wireless charging technology, electric transit buses could reduce their battery size by twothirds [23]. For the electric cars with wireless charging technology on major roads, the wireless charging infrastructure would theoretically allow EVs to have a large travel range with a small battery [24]. The primary structure of a typical CWD system is shown in Fig. 1.

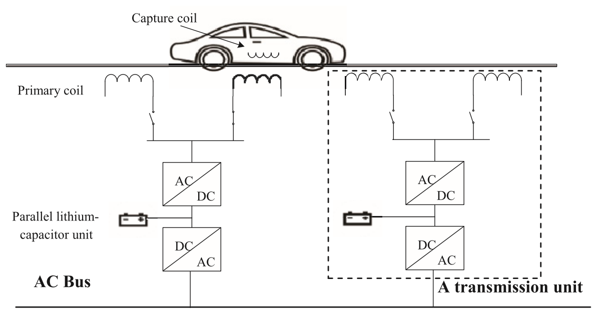

FIGURE 1. **A typical CWD system.**

In terms of EVs, the optimal routing scheme is one of the most popular research topics and many methods have been proposed. To obtain the optimal routing scheme for PEVs, Kobayashi *et al.* [25] divided the study into three cases: no need for recharging, recharging only once and recharging twice or more. Accordingly, the study developed three different models and used Dijkstra's algorithm to find the optimal route with the least cost. Siddiqi *et al.* [26] provided a new method: using particle swarm optimization (PSO) in the optimal routing problem of PEVs. Their main models are similar, in that they are both based on the shortest path (SP) problem. A. Artmeier *et al*. proposed energy-optimal routing methods based on the constrained shortest path (CSP) problem, solved with Dijkstra-like algorithms in [27] and [28]. Similar to these researchers, Lin *et al.* [29] proposed an optimal routing strategy for electric commercial vehicles to minimize time and energy cost. In their model, the electric vehicles were commercial vehicles so load effect on energy consumption was considered. To our knowledge, the main stream models of optimal routing of EVs are developed from a modified shortest path problem by adding several specific constraints and objectives, and most researchers currently focus on the optimal routing scheme of PEVs, while wireless charging EVs have been less frequently addressed.

However, with the development of wireless charging technology, it can be foreseen that hybrid plugin charging and wireless technology will coexist in the future. Moreover, both wireless charging and plug-in charging have advantages and disadvantages. Plug-in charging is a mature, higheffi-ciency technology, but it is inconvenient: EV Supply Equipments (EVSEs) or EV chargers are difficult to access and consume considerable time to charge. In contrast, wireless charging technology is convenient, but it is complicated and still in the early stages of development, which means efficiency is lower than traditional plug-in charging technology and the cost to charge is very high. Therefore, new problems will appear to choose the route while coordinating plug-in and wireless charging methods. These problems will appear in the near future when wireless charging technology is fully developed and exist for a long time until plug-in charging is completely replaced by wireless charging. However, few research studies have focused on the routing problem for coordinating plug-in and wireless charging.

Generally, the above two objectives often conflict due to the different characteristics of the hybrid plug-in charging and wireless charging schemes for EVs. Specifically, the wireless charging scheme can save time but increase cost, while the plug-in charging scheme is economical but time-consuming. Hence, it is advantageous to properly coordinate the two charging schemes.

In this research, we will develop a multi-objective optimization model to optimize electric vehicle routing with hybrid wireless and plugin charging systems. The main contributions are summarized as follows:

1) A mixed-integer, linear-programming-based route optimization model for EVs considering CWD systems is developed to coordinate the total time consumption and the charge cost of hybrid wireless and plug-in charging systems.
2) A Model Predictive Control (MPC) method is employed to address the uncertainties in the multi-stage problem, in which decisions are made sequentially with updated forecasting information.

II. MATHEMATICAL FORMULATION

There are two main objectives for an EV routing optimization problem: minimize the total time and minimize the total cost. These two objectives can be achieved through an optimization model and solved with the classic shortest path approach, in which the traffic network can be modeled with graph theories and constrained with routing constraints. To achieve the first objective, a mathematical model can be developed, in which the optimal route is constrained by the fact that the optimal route must be one connecting the start node and the terminal node. Moreover, traffic information should also be considered as contributing to the time consumption due to traffic congestion.

To achieve the second objective, the battery charge of the EV for the whole trip should be bounded by the battery's physical constraints, such as the battery capacity limits.

In this section, the optimization model is formulated, including the constraints and objective which has not been previously reported in the literature.

A. TRAFFIC CONSTRAINTS

In graph theory, a set of nodes is modeled as $\mathcal{N}$ and a set of pairs of nodes or a set of branches is modeled as $\mathcal{A}$. The traffic network is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{A} = \{(i,j)|i,j \in \mathcal{N}\}$ [30].

Since the optimization problem seeks the optimal route $\mathcal{R}$ between two terminal nodes, $\mathcal{R}$ is composed of a set of optimal directed branches. Let the directed branch and their set be $\boldsymbol{x}_{ij}$ and $X = \{\boldsymbol{x}_{ij}|(i,j) \in \mathcal{A}\}$, respectively. The optimal route can be written as $\mathcal{R} = \{(i,j)|(i,j) \in \mathcal{A}\}$. The constraints of the variables involved with the traffic network are discussed below.

First, let $\boldsymbol{x}_{ij}$ be binary variables. If the optimal route $\mathcal{R}$ includes road (i,j), then $\boldsymbol{x}_{ij} = 1$, otherwise, $\boldsymbol{x}_{ij} = 0$. Hence, $\boldsymbol{x}_{ij}$ should be constrained by

$$\boldsymbol{x}_{ij} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{R} \\ 0 & \text{otherwise} \end{cases} \quad \forall (i,j) \in \mathcal{A} \tag{1}$$

Second, let $S = \{s_i|i \in \mathcal{N}\}$ be the set of departing vectors of a node, such that

$$s_i = \sum_{j|(i,j)\in\mathcal{A}} \boldsymbol{x}_{ij} \quad \forall i \in \mathcal{N} \tag{2}$$

The set $Y = \{y_i|i \in \mathcal{N}\}$ is referred to as a divergence vector in which each element y_i is the total flow departing from node i minus the total flow arriving at node i. Thus, y_i can be expressed as

$$y_i = \sum_{j|(i,j)\in\mathcal{A}} \boldsymbol{x}_{ij} - \sum_{j|(j,i)\in\mathcal{A}} \boldsymbol{x}_{ji} \quad \forall i \in \mathcal{N} \tag{3}$$

It is reported in [36] that for a route, we have $y_i = 1$ for the starting node, $y_i = -1$ for the ending node, and $y_i = 0$ for other nodes, yielding

$$\sum_{j|(i,j)\in\mathcal{A}} \boldsymbol{x}_{ij} - \sum_{j|(j,i)\in\mathcal{A}} \boldsymbol{x}_{ji} = \begin{cases} 1 & \text{(i is source)} \\ -1 & \text{(i is destination)} \\ 0 & \text{(others)} \end{cases} \forall i \in \mathcal{N} \tag{4}$$

B. ELECTRIC CONSTRAINTS

The constraints of the traffic-network variables are presented above, and the constraints of the electrical-network variables (e.g., battery variables) are discussed here.

When an EV travels from node i to node j, the state of charge (SOC) of EV batteries can be expressed as

$$c_j = c_i + c_{ij}^r + c_i^n \quad \forall (i,j) \in \mathcal{A} \tag{5}$$

where c_i and c_j are the SOCs of EV batteries at node i and node j, respectively. c_{ij}^r is the change in the battery states on the road (i,j) via wireless charging, and c_i^n is the change of battery states at node i via plug-in charging.

Note that EV batteries have capacity limits during the whole trip, and the SOC of EV batteries has the following constraints

$$c_i^{min} \leq c_i \leq c_i^{max} \quad \forall i \in \mathcal{N} \tag{6}$$

where c_i^{min} and c_i^{max} are the lower and upper bounds of EV battery capacities at node i.

When an EV chooses plug-in charging at node i, the change in battery states can be written as

$$c_i^n = s_i z_i^c z_i t_i^z P_z \eta_z \quad \forall i \in \mathcal{N} \tag{7}$$

$$z_i^c \in \{0, 1\} \quad \forall i \in \mathcal{N} \tag{8}$$

where z_i^c is a binary variable representing whether plug-in charging is used at node i If plug-in charging is used, $z_i^c = 1$, otherwise $z_i^c = 0 z_i$ is a binary constant indicating whether plug-in charging is available at node i If it is available $z_i = 1$, otherwise $z_i = 0$. Additionally, t_i^z is the plug-in charging time, P_z is the plug-in charging power, and η_z is the efficiency.

The change in battery states on each road can be written as

$$c_{ij}^r = \boldsymbol{x}_{ij}(c_{ij}^y - c_{ij}^d) \quad \forall (i,j) \in \mathcal{A} \tag{9}$$

where c_{ij}^y is the charge injected into the EV when considering inroad wireless charging (i,j) and c_{ij}^d is the battery charge consumed on road (i,j).

In addition, when an EV selects wireless charging while driving, the change in battery states can be expressed as

$$c_{ij}^y = y_{ij} t_{ij}^y P_y \eta_y \quad \forall (i,j) \in \mathcal{A} \tag{10}$$

where y_{ij} is a binary parameter representing the availability of wireless charging on road (i,j) If road (i,j) is available for wireless charging, $y_{ij} = 1$, otherwise, $y_{ij} = 0$. t_{ij}^y is the wireless charging time on road (i,j), P_y is the wireless charging power, and η_y is the efficiency of wireless charging, which is inversely related to the vehicle speed. In this model, we suppose $\eta_y = \eta_0 e^{-v_{ij}}$ where η_0 is the transfer efficiency at the static condition.

Here, the time t_{ij}^y should be limited by

$$0 \leq t_{ij}^y \leq t_{ij}^r \quad \forall (i,j) \in \mathcal{A} \tag{11}$$

where t_{ij}^r is the total time consumed on road (i,j).

Let the battery charge consumed on road (i,j) be c_{ij}^d. This intermediary variable can be expressed as

$$c_{ij}^d = l_{ij} K \quad \forall (i,j) \in \mathcal{A} \tag{12}$$

where l_{ij} is the length of road (i,j) and K is the battery charge consumed per unit length.

C. OBJECTIVE

The optimization model is expected to minimize both the total time (i.e., *Time*) and the total cost (i.e., *Cost*) of the EV over the entire trip. The objective can be represented as

$$\text{Minimize } (1-\rho)\alpha \mathit{Time} + \rho \mathit{Cost} \tag{13}$$

where α is the value of time and ρ is the weight to adjust the two sub-objectives *Time* and *Cost*.

Total Time: The total time in (13) consists of two parts: the time consumed on the roads and the time consumed at the nodes for plug-in charging. Thus, the total time can be calculated as

$$Time = \sum_{ij\in\mathcal{A}} t_{ij}^{r} + \sum_{i\in\mathcal{N}} t_{i}^{z} \tag{14}$$

where t_{ij}^{r} is the time consumed on road (i, j) and t_i^z is the time consumed at node i for plug-in charging.

Considering the uncertainty of traffic congestion, a parameter called wait time (t_{ij}^{w}) is introduced to indicate the time spent due to random traffic issues. Then, the time t_{ij}^{r} becomes the sum of the predicted time t_{ij}^{0} and the expectation of the uncertain wait time t_{ij}^{w}. For example, for an EV travelling over distance l_{ij} it is predicted to move at an average speed v_{ij} and wait for the time t_{ij}^{w} due to a traffic jam. The time consumed on the road can be calculated as

$$t_{ij}^{r} = t_{ij}^{0} + t_{ij}^{w} \quad \forall (i, j) \in \mathcal{A} \tag{15}$$

The time consumption (t_{ij}^{0}) when traveling over a distance l_{ij} at a designed speed can be calculated by

$$t_{ij}^{0} = \frac{l_{ij}}{v_{ij}} \quad \forall (i, j) \in \mathcal{A} \tag{16}$$

The time consumed at the plug-in charging nodes can be calculated by (17)

$$t_{i}^{z} = (t_{i}^{z} + t_{i}^{zw}) z_{i}^{c} s_{i} \quad \forall i \in \mathcal{N} \tag{17}$$

where t_i^{zw} is the extra waiting time to access the charge facility due to the limited number of EVSEs, and t_i^{zw} is a dynamic parameter similar to t_{ij}^{w}.

Total Cost: The total cost $(Cost)$ also includes two parts: the cost of wireless charging $(Cost_y)$ and the cost for plug-in charging $(Cost_z)$, which can be expressed as

$$Cost = Cost_y + Cost_z \tag{18}$$

The wireless charging and plug-in charging costs can be written in the following ways

$$Cost_y = \sum_{ij\in\mathcal{A}} \boldsymbol{x}_{ij} y_{ij} t_{ij}^{y} P_y \boldsymbol{u}_y \tag{19}$$

$$Cost_z = \sum_{i\in\mathcal{N}} s_i z_i^c z_i t_i^z P_z \boldsymbol{u}_z \tag{20}$$

where $\boldsymbol{u}_y$ is electricity price of wireless charging and $\boldsymbol{u}_z$ is electricity price of plug-in charging.

III. MODEL PREDICTIVE CONTROL METHOD

In the optimal routing problem for an EV considering CWD systems, there may be some uncertain components. In the field of control science, Model Predictive Control (MPC) is widely used in process control to address the uncertainties [31]. In [32], a nonlinear model predictive control technique was used to control the emission of nitrogen oxides. In [33], a nonlinear MPC algorithm and its application to petroleum refining and the petrochemical industry were studied. In this paper, an MPC-based optimization model is formulated, including the constraints and objective, which has not been previously reported in the literature.

In an intelligent transportation system environment, all vehicles in the system have new technologies and infrastructure to obtain information flow from a traffic control center. Communication with the traffic control center provides the advantage of useful information and data accessed by vehicles.

Many methods have been proposed for traffic flow and electric price forecasting. The traffic control center can obtain shortterm forecast information for traffic flow and electric price by these forecasting methods. Based on the forecasted information from the traffic control center, the vehicle could determine its operation at each stage. The forecasting method is not covered in this paper, so the traffic and electric price forecasting information is considered as a given value.

The MPC is based on an iterative, finite-horizon optimization problem. Utilizing the output feedback results of the actual system, the MPC obtains the optimal control measures in the future for a period of stages through the repeated rolling optimization of the control targets. At each optimization stage, there is an optimal performance index for the future finite domain. To prevent control errors that result from model inaccuracies or other disturbances in the control process, only the control commands from the first stage are executed in the solution sequence. The rolling optimization strategy based on actual output feedback corrects the effect of the prediction error in real time, making the approach more robust.

The characteristics of a general system can be described by the following state-space equations: $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\omega}, t)$, where $\boldsymbol{x}$ is the state of the system, $\boldsymbol{u}$ is the control decision, $\boldsymbol{\omega}$ is the disturbance variable, and t is the time variable. To meet the requirements of online applications, the actual control process usually uses discrete model. The discrete state-space equations can be expressed as $\boldsymbol{x}_{k+1} = g(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{\omega}_k)$, where k is the decision time stage. The control decisions $\boldsymbol{u}_k$ are supposed to be made at the discrete times $k = 0, 1, 2, \cdots$. At decision stage k, the controller solves the following optimization problem to identify the control decisions:

$$\min_{\mathbf{U}_k} \quad J(\mathbf{X}_k, \mathbf{U}_k) \tag{21}$$

where

$$\mathbf{X}_k = \{\boldsymbol{x}_k(k+1), \boldsymbol{x}_k(k+2), \cdots, \boldsymbol{x}_k(k+N)\} \tag{22}$$

$$\mathbf{U}_k = \{\boldsymbol{u}_k(k), \boldsymbol{u}_k(k+1), \cdots, \boldsymbol{u}_k(k+N-1)\} \tag{23}$$

s.t.

$$\boldsymbol{x}_k(k+i+1) = g(\boldsymbol{x}_k(k+i), \boldsymbol{u}_k(k+i), \boldsymbol{\omega}_k(k+i)) \quad \forall i \in 0, 1, \cdots, N-1 \tag{24}$$

$$L(\mathbf{X}_k, \mathbf{U}_k) \le 0 \tag{25}$$

$$\boldsymbol{x}_k(k) = Z_k \tag{26}$$

where $J(\mathbf{X}_k, \mathbf{U}_k)$is the objective function of the optimization model, $g(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{\omega}_k)$ is the forecasting model, $L(\mathbf{X}_k, \mathbf{U}_k)$ is the set of constraints and Z_k is the actual measured value of the system state. Assuming that the optimal solution can be expressed as $\{\boldsymbol{u}_k^*(k), \boldsymbol{u}_k^*(k+1), \cdots, \boldsymbol{u}_k^*(k+N-1)\}$, then only $\boldsymbol{u}_k^*(k)$ will be actually executed after this optimization. This optimization is repeated every sampling period.

Based on this MPC approach, the EV routing problem at the k-th step can be expressed as:

1) Obtain the real-time and forecast traffic flow information and real-time and forecast electricity price information from the traffic control center.
2) Utilize the traffic information and electricity price information to solve the optimization problem, yielding the optimal route.
3) Implement the optimal route for the first stage until approaching a point in the road map (e.g. 30 s to reach a point).
4) Update the forecasting information over the next several stages from the traffic center, and go to step 2, continuing until the EV reaches the destination.

IV. MATHEMATICAL REFORMULATION

Unfortunately, there are many bilinear terms in equations (7), (9), (10), (17), (19) and (20), which present challenges for the computation of the mixed integer programming. Fortunately, these bilinear terms have a special structure in that each bilinear term is constructed by multiplying a binary variable and a continuous/binary variable. To simplify this model, we take Big-M approach to exactly linearize the bilinear terms in these constraints by introducing dummy variables and additional constraints.

With respect to the Big-M reformulations, the expression (9) can be transformed into

$$\begin{cases} c_{ij}^r = \delta_{ij} y_{ij} P_y \eta_y - \boldsymbol{x}_{ij} l_{ij} K \\ t_{ij}^y - (1-\boldsymbol{x}_{ij})M \le \delta_{ij} \le t_{ij}^y + (1-\boldsymbol{x}_{ij})M \\ -\boldsymbol{x}_{ij} M \le \delta_{ij} \le \boldsymbol{x}_{ij} M \end{cases} \tag{27}$$

where δ_{ij} is a dummy variable to replace the bilinear term $\boldsymbol{x}_{ij} t_{ij}^y$.

In constraint (7), the bilinear term $s_i z_i^c$ can be replaced by θ_i with the additional constraints.

$$\theta_i \le s_i, \quad \theta_i \le z_i^c, \; c\theta_i \ge s_i + z_i^c - 1, \; \theta_i \in \{0, 1\} \tag{28}$$

Furthermore, the expression of c_i^n in (7) can be recast as

$$c_i^n = \theta_i t_i^z z_i P_z \eta_z \tag{29}$$

Obviously, there is still a bilinear term in (25) with the binary variable θ_i multiplied by the real variable t_i^z. This bilinear term can be further replaced by the dummy variable λ_i with the additional constraints

$$\begin{cases} t_i^z - (1-\theta_i)M \le \lambda_i \le t_i^z + (1-\theta_i)M \\ -\theta_i M \le \lambda_i \le \theta_i M \end{cases} \tag{30}$$

Finally, the expression c_i^ncan be expressed as

$$c_i^n = \lambda_i z_i P_z \eta_z \tag{31}$$

When the constraints are converted by Big-M reformulation, three dummy variables are introduced into this model:

δ_{ij}, a binary variable replacing $\boldsymbol{x}_{ij} t_{ij}^y$
θ_i, a real variable replacing $s_i z_i^c$
λ_i, a real variable replacing $\theta_i t_i^z$

Thus, the optimal model could be expressed as the following mixed integer linear programming:

$$\begin{aligned}
&\text{Minimize } (1-\beta)\alpha Time + \beta Cost \\
&\text{s.t. } \sum_{j|(i,j)\in\mathcal{A}} \boldsymbol{x}_{ij} - \sum_{j|(j,i)\in\mathcal{A}} \boldsymbol{x}_{ji} = \begin{cases} 1 & \text{(i is source)} \\ -1 & \text{(i is destination)} \\ 0 & \text{(others)} \end{cases} \quad \forall i \in \mathcal{N} \\
&\boldsymbol{x}_{ij} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{R} \\ 0 & \text{otherwise} \end{cases} \quad \forall (i,j) \in \mathcal{A} \\
&s_i, z_i^c \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}\, \forall i \in \mathcal{N} \\
&c_i^{min} \le c_i \le c_i^{max} \quad \forall i \in \mathcal{N} \\
&0 \le t_{ij}^y \le \frac{l_{ij}}{v_{ij}} + t_{ij}^w, \quad t_i^z \ge 0 \; \forall (i,j) \in \mathcal{A}\, \forall i \in \mathcal{N} \\
&s_i = \sum_{j|(i,j)\in\mathcal{A}} \boldsymbol{x}_{ij} \quad \forall i \in \mathcal{N} \\
&c_j = c_i + \delta_{ij} y_{ij} P_y \eta_y - \boldsymbol{x}_{ij} l_{ij} K + \lambda_i z_i P_z \eta_z \quad \forall (i,j) \in \mathcal{A} \\
&Time = \sum_{ij\in\mathcal{A}} \left(\frac{l_{ij}}{v_{ij}} + t_{ij}^w\right) + \sum_{i\in\mathcal{N}} (\lambda_i + t_i^{zw}\theta_i) \\
&Cost = \sum_{ij\in\mathcal{A}} \delta_{ij} y_{ij} P_y \eta_y \boldsymbol{u}_y + \sum_{i\in\mathcal{N}} \lambda_i z_i P_z \eta_z \boldsymbol{u}_z \\
&\begin{cases} t_{ij}^y - (1-\boldsymbol{x}_{ij})M \le \delta_{ij} \le t_{ij}^y + (1-\boldsymbol{x}_{ij})M \\ -\boldsymbol{x}_{ij} M \le \delta_{ij} \le \boldsymbol{x}_{ij} M \end{cases} \quad \forall (i,j) \in \mathcal{A} \\
&\begin{cases} t_i^z - (1-\theta_i)M \le \lambda_i \le t_i^z + (1-\theta_i)M \\ -\theta_i M \le \lambda_i \le \theta_i M \end{cases} \quad \forall i \in \mathcal{N} \\
&\theta_i \le z_i^c, \quad \theta_i \ge s_i + z_i^c - 1, \; \theta_i \in \{0,1\} \; \forall i \in \mathcal{N}
\end{aligned}$$

V. CASE STUDY

To better explain the proposed optimization model, two cases are presented in this section. The proposed model was programmed in MATLAB in which the mixed integer linear programming was solved via CPLEX 12.5. The computational tasks were performed on a 2.0 GHz personal computer with 4 GB RAM.

A. CASE STUDY 1

In Case 1, a simple traffic network is used with starting node 1, ending node 6, and four traffic nodes as shown in Fig. 2. Plug-in charging is available at nodes 2 and 5, and wireless charging is available on roads (1, 2), (1, 3), (2, 5), and (3, 4). The corresponding parameters are provided in TABLE 1. The detailed traffic information is available in [34].

Note that a per-unit (p.u.) system is used in Table 1 to quantify a variety of variables in the hybrid electric and traffic system. For example, the EV consumes 4 p.u. electricity in one kilometer, and the power of wireless charging is 1.6 p.u.

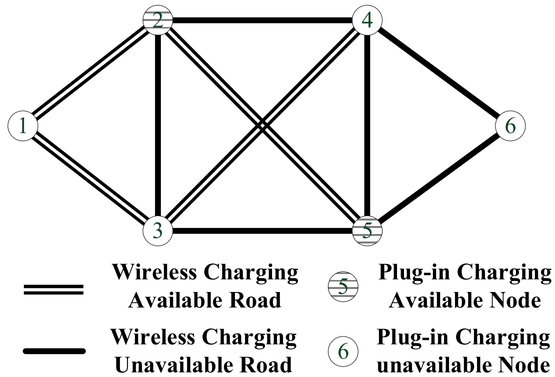

FIGURE 2. Traffic network of test system 1.

TABLE 1. Parameter values for case 1.

Description	Parameter	Value	Unit
Charge consumed each kilometer	K	4	p.u.
Power of wireless charging	P_y	1.6	p.u.
Price of wireless charging	u_y	1.2	RMB/p.u.
Efficiency of wireless charging	η_y	0.8	
Power of plug-in charging	P_z	2	p.u.
Price of plug-in charging	u_z	1	RMB/p.u.
Efficiency of plug-in charging	η_z	1	
Initial battery charge	C_0	4	p.u.
Maximum battery charge	C_{max}	20	p.u.
Minimum battery charge	C_{min}	0	p.u.
Value of time	α	1	RMB/p.u.

Additionally, the EV battery is assumed to be 4 p.u. power at the starting point, which needs to be charged during the trip.

In order to investigate the impact of the weighting on the optimal solution, different values of the weighting are selected (i.e., ρ=0.2, 0.5, and 0.8). We use the condition ρ=0.2 to simulate people who prefer to reach their destination as quickly as possible with little concern for cost. The condition ρ=0.8 represents people who are not in a hurry but try to save money consumed in this trip. The value of ρ=0.5 will produce a balanced choice between the wireless and plug-in charging methods.

For ρ=0.5, the optimal routes at each stage are shown in Fig. 3, where the final route is traveling along nodes 1 → 2 → 4 → 6, and the optimal charging strategy is presented in TABLE 2, with selecting wireless charging from nodes 1 to 2 (i.e., road 1-2) and using plug-in charging at node 2. As a result, the EV takes 15.74 p.u. of time and 12.96 p.u. of cost during the entire trip. It can be seen that the optimal route at stage 1 is different from that at stages 2 and 3. This is because the MPC method is used in this optimal strategy. When the EV receives the optimal route at time stage 1, it will travel along the route until it reaches node 2. At this time, the result of the optimal model shows that route 2 → 4 → 6 is better, so the EV will execute the control construction of this stage and travel from node 2 to 4. Then, the EV will optimize the route again and execute the optimal result to reach the destination.

TABLE 2. The optimal charging method of case 1.

ρ	Method	Location	Charging Time(min)	Time (min)	Cost (RMB)
0.2	Wireless	(1,2)	1.5	15.44	14.56
		(2,5)	2.5		
	Plug-in	2	3.44		
0.5	Wireless	(1,2)	1.5	15.74	12.96
	Plug-in	2	5.04		
0.8	Plug-in	2	6	16.7	12

For ρ=0.2, the optimal route is shown in Fig. 4, with the path being 1 → 2 → 5 → 6, which contains two pairs of nodes with wireless charging. In contrast, for ρ=0.8, the optimal route is shown in Fig. 5 with 1 → 2 → 5 → 6, where all the nodes have plug-in charging. Furthermore, the optimal charging strategies in TABLE 2 indicate that the proposed optimization model can work out the optimal route and optimal charging strategy to trade off the minimum time consumed with the minimum cost.

B. CASE STUDY 2

In Case 2, the proposed optimization model is tested on a real traffic system. The real road map of Xi'an, China, presented in Fig. 6 is used. The EV is designed to travel from Xi'an Jiaotong University (Node 1) to the northwest corner of the Xi'an City Wall (Node 32). At the beginning node, the SoC is assumed as 25% which is unable to provide enough energy for the whole trip. The wireless and plug-in charging powers are 6 kW and 12 kW, respectively. The other parameters are presented in TABLE 3. For convenience, the parameters are used in unit with respect to the percent of battery capacity and the value of time is 0.2 RMB/min considering the economic development in Xi'an City.

TABLE 3. Parameter values for case 2.

Description	Parameter	Value	Unit
Charge consumed each kilometer	K	0.8	kWh/km
Power of wireless charging	P_y	8	kW
Price of wireless charging	u_y	1.3	RMB/kWh
Base Efficiency of wireless charging	η_y	80	%
Power of plug-in charging	P_z	30	kW
Price of plug-in charging	u_z	0.5	RMB/kWh
Efficiency of plug-in charging	η_z	90	%
Initial battery charge	C_0	30	%
Maximum battery charge	C_{max}	100	%
Minimum battery charge	C_{min}	5	%
Battery capacity	B	20	kWh
Value of time	α	1.5	RMB/min

In addition, certain practical issues are considered. First, each road has a speed limit, such as 60 km/h, 50 km/h, or 30 km/h, and thus the road network with speed limit information is presented in Fig. 7. Second, each EV

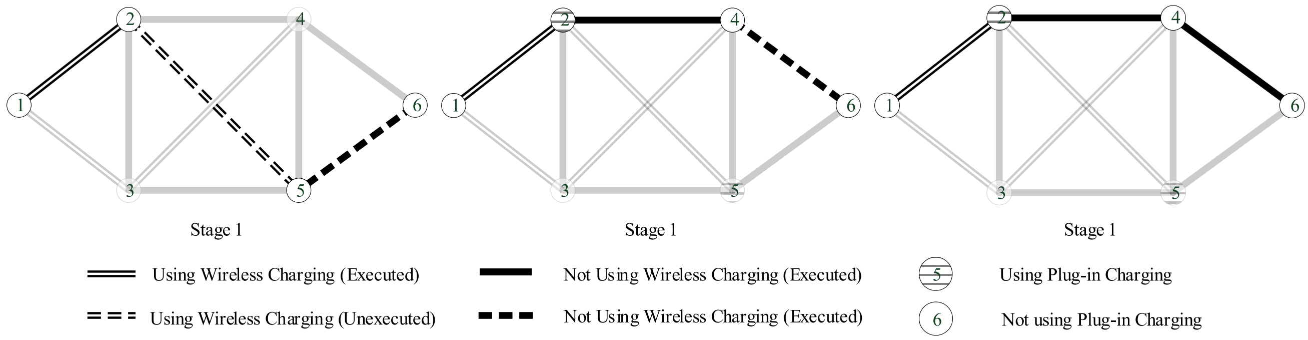

FIGURE 3. Optimal route while $\rho = 0.5$.

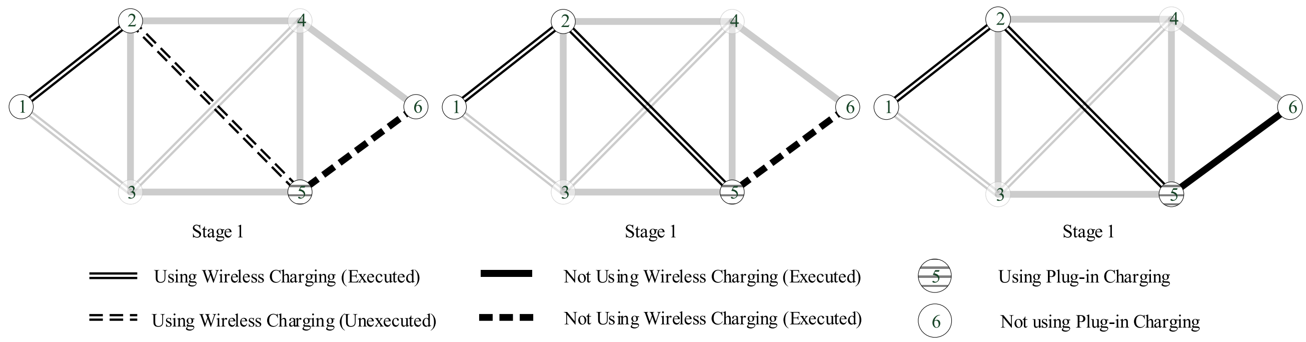

FIGURE 4. Optimal route while $\rho = 0.2$.

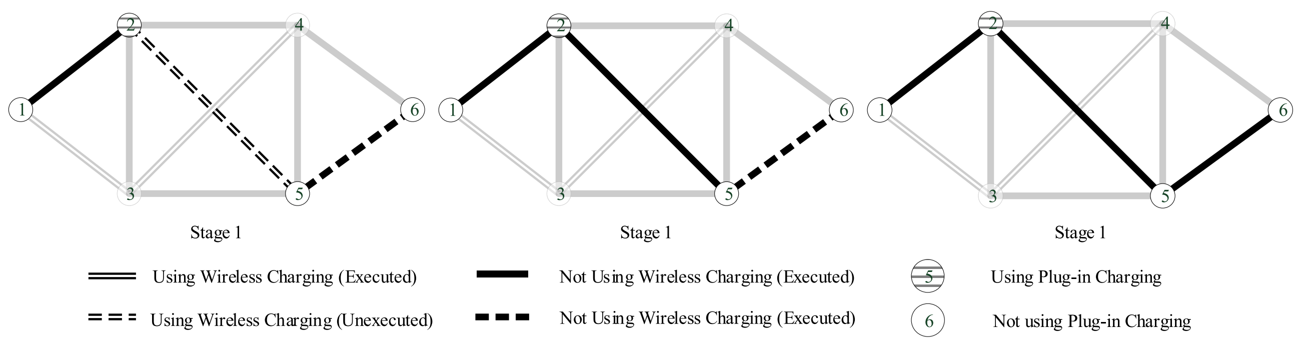

FIGURE 5. Optimal route while $\rho = 0.8$.

FIGURE 6. The real road map of Xi'an, China.

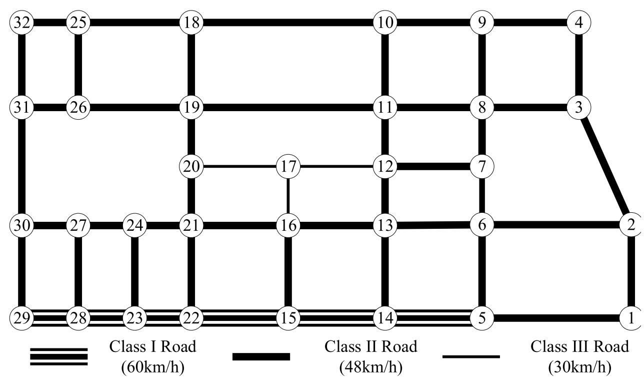

FIGURE 7. Case 2: road network and road class.

has a minimum SoC to avoid battery damage. We take 20% of battery capacity as the minimum SoC and reduce the battery capacity to 20 kWh. The detailed traffic information is available in [34].

In this case, we take three different conditions with ρ=0.2, 0.5, and 0.8 to observe the optimal solutions of the proposed model. The optimal routes are shown in Fig. 8, 9, and 10, and the optimal charging strategies are summarized in TABLE 4.

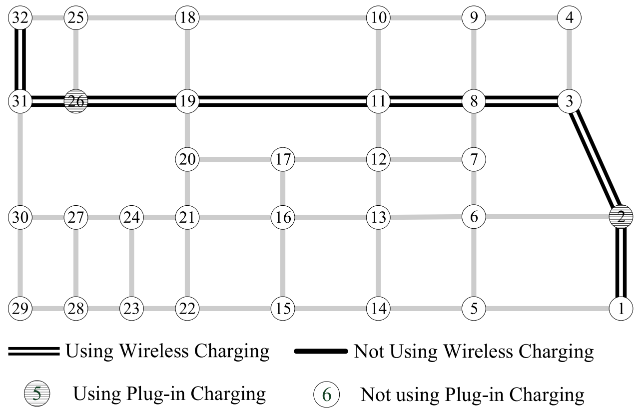

FIGURE 8. Optimal route while $\rho = 0.2$.

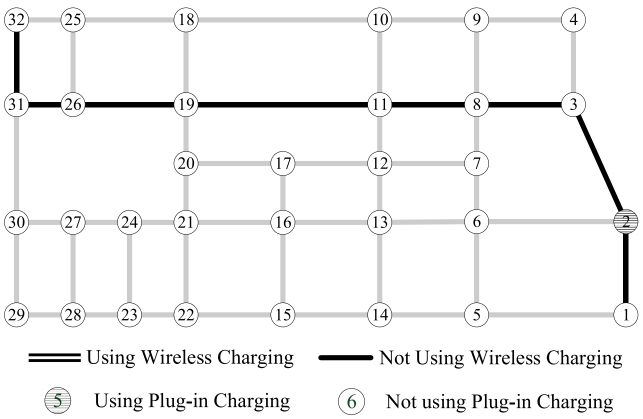

FIGURE 9. Optimal route while $\rho = 0.8$.

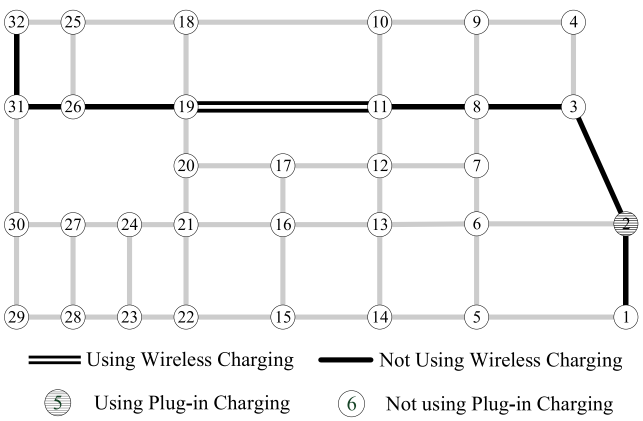

FIGURE 10. Optimal route while $\rho = 0.5$.

When ρ=0.2, the driver chooses a low weight on "cost" and expects to arrive at the destination in a timely manner. The optimal route as shown in Fig. 8 has a priority for wireless charging and takes 48.4 min and 17.35 RMB for a single trip. When ρ=0.8, the driver selects a high value on "cost" and expects the optimal route to have a low cost.

TABLE 4. The optimal charging method of case 2.

ρ	Method	Location	Charging Time(min)	Time (min)	Cost (RMB)
0.2	Wireless	(1,2)	1.4956	48.3596	17.3539
		(2,3)	3.7442		
		(3,8)	2.4863		
		(8,11)	5.4179		
		(11,19)	3.6663		
		(19,26)	3.5178		
		(26,31)	8.9444		
		(31,32)	5.8009		
	Plug-in	2	4.5507		
		26	0.9759		
0.5	Wireless	(11,19)	0.469	53.0192	6.2396
	Plug-in	2	12.09		
0.8	Plug-in	-	-	54.3196	6.0386
		2	13.1		

The optimal route as shown in Fig. 9 avoids the use of wireless charging and extends the time of plug-in charging. As a result, the total electricity cost is 6.04 RMB and the total time is 54.3 min.

Further, when ρ=0.5, the driver sets the same weight for cost and time and the corresponding optimal route is shown in Fig. 10. The optimal route includes one road for wireless charging and one node for plug-in charging. Compared with the other two cases, this case yields a balance between the time and the cost during the trip. The detailed optimal charging method for each stage is presented in TABLE 5 as an example demonstrating the flow of the MPC method. It can be found that at stage 3 the optimal route changes to 3→8→11→19→26→31→32 when the traffic condition is altered.

It is observed from Table 4 and Fig. 8, 9 and 10 that the optimal routes are the same, determined by the specific traffic condition. However, for the difference of the preference of cost or time, the charging strategies are different. The difference between those three cases is the time scheduled for wireless and plug-in charging. The proposed optimization model can optimize both the route and the charging schemes aligned with certain preferences.

Actually, Xi'an City has not installed wireless charging facilities for EVs. To investigate the potential economic and technical benefits of wireless charging, a comparison between scenarios with and without wireless charging is provided.

It can also be observed that the optimal route without wireless charging (ρ=0.8) present will take 13 minutes to utilize plug-in charging at node 2, while the presence of wireless charging devices will change the choice of optimal routes, reducing the plug-in charging time from 13 minutes to 6 minutes. Since wireless charging is performed during driving, this approach saves time in the trip. However, the total charging cost is a bit higher due to the high cost of wireless charging.

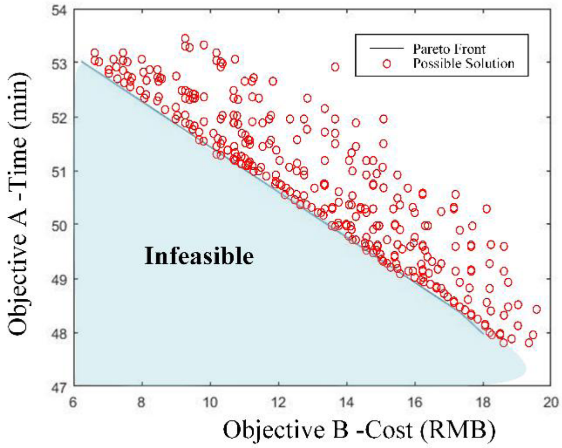

FIGURE 11. Pareto Frontier curve.

Fig. 11 depicts a Pareto frontier curve between time and costs, where the frontier takes on a linear relationship.

Finally, the total computational time is 29.6612 s for the scenario when ρ=0.5. It takes 2.4216 s to find the optimal route and charging strategy for the first stage. The convergence curve is shown in Fig. 12.

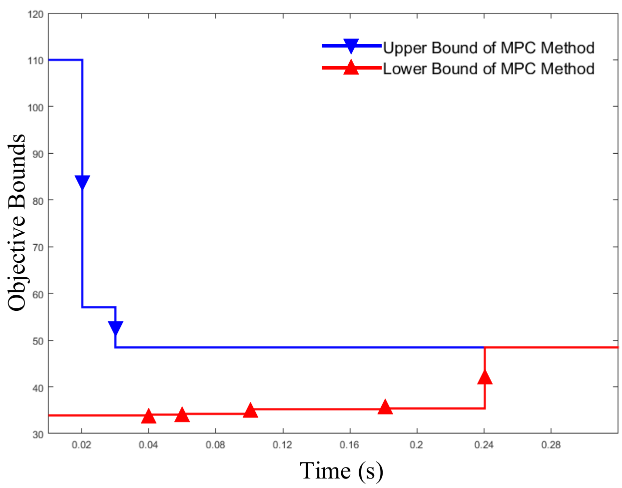

FIGURE 12. Convergence curve.

VI. CONCLUSION

This paper developed a multi-objective optimization model and a charging strategy considering hybrid wireless and plug-in charging to seek the optimal EV route to trade off the total time consumption and charging cost for a trip. Two cases including practical road traffic are examined to show that the proposed model can find the optimal route while trading off wireless and plug-in charging according to driver preference. Moreover, wireless charging is preferred by drivers who are hurrying to the destination, whereas the shortest path is more likely to be chosen by drivers who want to minimize charging cost.

APPENDIX

BIG-M REFORMULATION

Big-M reformulations are used to convert a logic constraint to a set of constraints describing the same feasible set, using dummy variables and additional constraints.

As an example, consider a bilinear term bc where b is binary, and c is a continuous variable. Let a be an introduced dummy variable to replace the bilinear term bc. Also introduce the additional constraints

$$\begin{cases} c-(1-b)M \leq a \leq c+(1-b)M \\ -bM \leq a \leq bM \end{cases} \tag{32}$$

where M is a sufficiently large number. Note that this Big-M reformulation is equivalent to the original formulation. Clearly, if b is 1, a is guaranteed to be c; and if b is 0, a is guaranteed to be 0.

When b and c are both binary variables, the bilinear term bc can be replaced by a with the additional constraints

$$a \leq b, \quad a \leq c, \ a \geq b+c-1, \ a \in \{0, 1\} \tag{33}$$

Here, only when b and c are both 1, a is equal to 1, otherwise, a is 0.

REFERENCES

[1] P. Guo and P. Liu, "Research on development of electric vehicles in China," in *Proc. Int. Conf. Future Inf. Technol. Manage. Eng. (FITME)*, Oct. 2010, pp. 94–96.
[2] S. Carley, R. M. Krause, B. W. Lane, and J. D. Graham, "Intent to purchase a plug-in electric vehicle: A survey of early impressions in large U.S. cites," *Transp. Res. D, Transp. Environ.*, vol. 18, pp. 39–45, Jan. 2013.
[3] G. Benysek and M. Jarnut, "Electric vehicle charging infrastructure in Poland," *Renew. Sustain. Energy Rev.*, vol. 16, no. 1, pp. 320–328, 2012.
[4] A. Poullikkas, "Sustainable options for electric vehicle technologies," *Renew. Sustain. Energy Rev.*, vol. 41, pp. 1277–1287, Jan. 2015.
[5] C. Chan and K. Chau, *Modern Electric Vehicle Technology*. London, U.K.: Oxford Univ. Press, 2001.
[6] C. C. Chan, "The state of the art of electric, hybrid, and fuel cell vehicles," *Proc. IEEE*, vol. 95, no. 4, pp. 704–718, Apr. 2007.
[7] A. C. Z. de Souza, D. Q. Oliveira, and P. F. Ribeiro, "Overview of plug-in electric vehicles technologies," in *Plug in Electric Vehicles in Smart Grids*. Singapore: Springer, 2015, pp. 1–24.
[8] D. Sperling, D. Gordon, and A. Schwarzenegger, *Two Billion Cars: Driving Toward Sustainability*. London, U.K.: Oxford Univ. Press, 2009.
[9] W. J. Mitchell, C. E. Borroni-Bird, and L. D. Burns, *Reinventing the Automobile: Personal Urban Mobility for the 21st Century*. Cambridge, MA, USA: MIT Press, 2010.
[10] T. Ding, R. Bo, F. Li, Y. Gu, Q. Guo, and H. Sun, "Exact Penalty function based constraint relaxation method for optimal power flow considering wind generation uncertainty," *IEEE Trans. Power Syst.*, vol. 30, no. 3, pp. 1546–1547, May 2015.
[11] T. Ding, R. Bo, H. Sun, F. Li, and Q. Guo, "A robust two-level coordinated static voltage security region for centrally integrated wind farms," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 460–470, Jan. 2016.
[12] T. Ding *et al.*, "Interval arithmetic based optimal curtailment for infeasible SCED considering wind power uncertainties," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, Jul. 2015, pp. 1–5.
[13] T. Ding, S. Liu, W. Yuan, Z. Bie, and B. Zeng, "A two-stage robust reactive power optimization considering uncertain wind power integration in active distribution networks," *IEEE Trans. Sustain. Energy*, vol. 7, no. 1, pp. 301–311, Jan. 2016.
[14] M. Guarnieri, "Looking back to electric cars," in *Proc. 3rd IEEE Hist. Electro-Technol. Conf. (HISTELCON)*, Sep. 2012, pp. 1–6.
[15] C. Jeff. (Sep. 23, 2016). *Americans Buy Their Half-Millionth Plug-in Car*. [Online]. Available: http://www.hybridcars.com/

[16] S. Amjad, S. Neelakrishnan, and R. Rudramoorthy, "Review of design considerations and technological challenges for successful development and deployment of plug-in hybrid electric vehicles," *Renew. Sustain. Energy Rev.*, vol. 14, no. 3, pp. 1104–1110, 2010.

[17] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, "Wireless power transfer via strongly coupled magnetic resonances," *Science*, vol. 317, no. 5834, pp. 83–86, 2007.

[18] S.-H. Lee and R. D. Lorenz, "Development and validation of model for 95%-efficiency 220-W wireless power transfer over a 30-cm air gap," *IEEE Trans. Ind. Appl.*, vol. 47, no. 6, pp. 2495–2504, Nov./Dec. 2011.

[19] H. H. Wu, J. T. Boys, and G. A. Covic, "An AC processing pickup for IPT systems," *IEEE Trans. Power Electron.*, vol. 25, no. 5, pp. 1275–1284, May 2010.

[20] P. Ning, J. M. Miller, O. C. Onar, C. P. White, and L. D. Marlino, "A compact wireless charging system development," in *Proc. 28th Annu. IEEE Appl. Power Electron. Conf. Expo. (APEC)*, Mar. 2013, pp. 3045–3050.

[21] T.-D. Nguyen, S. Li, W. Li, and C. C. Mi, "Feasibility study on bipolar pads for efficient wireless power chargers," in *Proc. IEEE Appl. Power Electron. Conf. Expo. (APEC)*, Mar. 2014, pp. 1676–1682.

[22] J. M. Miller *et al.*, "Demonstrating dynamic wireless charging of an electric vehicle: The benefit of electrochemical capacitor smoothing," *IEEE Power Electron. Mag.*, vol. 1, no. 1, pp. 12–24, Mar. 2014.

[23] Z. Bi, L. Song, R. De Kleine, C. C. Mi, and G. A. Keoleian, "Plug-in vs. wireless charging: Life cycle energy and greenhouse gas emissions for an electric bus system," *Appl. Energy*, vol. 146, pp. 11–19, May 2015.

[24] J. M. Miller, P. T. Jones, J.-M. Li, and O. C. Onar, "ORNL experience and challenges facing dynamic wireless power charging of EV's," *IEEE Circuits Syst. Mag.*, vol. 15, no. 2, pp. 40–53, 2nd Quart., 2015.

[25] Y. Kobayashi, N. Kiyama, H. Aoshima, and M. Kashiyama, "A route search method for electric vehicles in consideration of range and locations of charging stations," in *Proc. IEEE Intell. Vehicles Symp. (IV)*, Jun. 2011, pp. 920–925.

[26] U. F. Siddiqi, Y. Shiraishi, and S. M. Sait, "Multi-constrained route optimization for electric vehicles (EVs) using particle swarm optimization (PSO)," in *Proc. 11th Int. Conf. Intell. Syst. Design Appl. (ISDA)*, Nov. 2011, pp. 391–396.

[27] A. Artmeier, J. Haselmayr, M. Leucker, and M. Sachenbacher, "The optimal routing problem in the context of battery-powered electric vehicles," in *Proc. CPAIOR Workshop Constraint Reasoning Optim. Comput. Sustain.*, 2010, pp. 1–13.

[28] M. Sachenbacher, M. Leucker, A. Artmeier, and J. Haselmayr, "Efficient energy-optimal routing for electric vehicles," in *Proc. AAAI*, 2011, pp. 1402–1407.

[29] J. Lin, W. Zhou, and O. Wolfson, "Electric vehicle routing problem," in *9th Int. Conf. City Logistics*, in Transportation Research Procedia, Tenerife, Spain, vol. 12, E. Taniguchi and R. G. Thompson, Eds. 2016, pp. 508–521, doi: 10.1016/j.trpro.2016.02.007.

[30] D. P. Bertsekas, *Network Optimization: Continuous and Discrete Methods*. Belmont, MA, USA: Athena Scientific, 1998.

[31] E. F. Camacho and C. B. Alba, *Model Predictive Control*. London, U.K.: Springer, 2013.

[32] F. Allgöwer and A. Zheng, *Nonlinear Model Predictive Control, Progress in System and Control Theory*, vol. 26. Basel, Switzerland: Birkhäuser Verlag, 2000.

[33] G. P. Liu and S. Daley, "Adaptive predictive control of combustor NO_x emissions," *Control Eng. Pract.*, vol. 9, no. 6, pp. 631–638, 2001.

[34] *Case Study Data*. Accessed: May 16, 2018. [Online]. Available: https://www.dropbox.com/s/3tetsxq5yixtpi6/Case.xlsx

CHENG LI (S'16) received the B.S. degree from the School of Electrical Engineering, Xi'an Jiaotong University, Xi'an, China, in 2016, where he is currently pursuing the M.S. degree. His major research interests include power system optimization and renewable energy integration.

TAO DING (S'13–M'15) received the B.S.E.E. and M.S.E.E. degrees from Southeast University, Nanjing, China, in 2009 and 2012, respectively, and the Ph.D. degree from Tsinghua University, Beijing, China, in 2015.

From 2013 to 2014, he was a Visiting Scholar with the Department of Electrical Engineering and Computer Science, The University of Tennessee, Knoxville, TN, USA. He is currently an Associate Professor with the State Key Laboratory of Electrical Insulation and Power Equipment, School of Electrical Engineering, Xi'an Jiaotong University, Xi'an, China. He has authored or co-authored over 60 technical papers and was published by Springer Theses recognizing outstanding Ph.D. research around the world and across the physical sciences *Power System Operation With Large Scale Stochastic Wind Power Integration*. His current research interests include electricity markets, power system economics and optimization methods, and power system planning and reliability evaluation.

Dr. Ding was a recipient of the Excellent Master and Doctoral Dissertation from Southeast University and Tsinghua University, respectively, and the Outstanding Graduate Award of Beijing City.

XIYUAN LIU (S'13) received the B.S. degree from the School of Electrical Engineering, Xi'an Jiaotong University, Xi'an, China, in 2016, where she is currently pursuing the Ph.D. degree. Her major research interests include the electric vehicle operation and planning.

CAN HUANG (GS'13–M'16) received the B.S.E.E. degree from Hohai University, Nanjing, China, in 2008, the M.S.E.E. degree from Southeast University, Nanjing, in 2011, and the Ph.D. degree from The University of Tennessee, Knoxville, TN, USA, in 2016. From 2011 to 2012, he was with the State Grid Electric Power Research Institute, NARI Group Corporation, Nanjing. He is currently a Post-Doctoral Research Staff Member with the Lawrence Livermore National Laboratory, Livermore, CA, USA. His current research interests include advanced control, computing, and communication for power and energy systems.