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Quasi-Synchronization of Coupled Nonlinear Memristive Neural Networks With Time Delays by Pinning Control

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ABSTRACT This paper formulates the models of systems of nonlinearly and diffusively coupled memristive neural networks (CMNNs) with time-varying delays and then investigates its dynamic behaviors. Particularly, a simple yet a generic sufficient condition for quasi-synchronization of drive-response CMNNs is derived based on the Lyapunov functional methods and matrix theories. The main result shows that quasi-synchronization of such CMNNs is guaranteed by suitably designing the memsitive mechanism, the coupling matrix, and the pinning control strategy. In addition, some applicable corollaries derived from the main result are drawn by considering other circumstances, such as the linearly coupling functions, the adjustable coupling strengths, the number of controlled nodes, and so on. Finally, some numerical simulations are presented to demonstrate the effectiveness of the results.

INDEX TERMS Quasi-synchronization, memristive neural network, nonlinear coupling, pinning control.

I. INTRODUCTION

Since memristor was first raised by Chua [1] in 1971, it has provoked considerable attention from scientists and engineers in various scientific fields [2]–[4]. As a new circuit element with memory and resistance properties, it has many implications in different areas, such as information storage, logical operation, neural networks studies, etc. Particularly, in terms of its applications in neural networks, memristor takes the place of conventional resistor as a connection between two neurons, so that the neural networks can enhance the ability of computation and information capacity. It is reported that the emergence of memristor provides an entirely new approach to simulate synaptic function, which could be used to fabricate brain-like neural network computers in the future [2], [5], [6].

In recent years, the studies on the collective behaviors of memristor-based neural networks have appealed to tremendous researchers, and many theoretical achievements on such analysis have been obtained, such as synchronization and consensus of the neural networks. For instance, Pershin and Di Ventra [6] have demonstrated experimentally the formation of associative memory in a simple neural network composed of three electronic neurons connected by two memristors. Wu *et al.* [7] probed into the exponential synchronization of memristor-based recurrent neural networks with time delays by a delay-dependent feedback controller. Afterwards, many further efforts have been devoted into the investigations of the dynamic analysis for memristive neural networks, amid which there are many results on different kinds of synchronization of two memsitive neural networks [3], [8]–[12], and some works have dealt with such issues between higher-dimensionally coupled systems consisting of these neural networks [13]–[16].

In neuroscience, synchronization of CMNNs has important relationship with pattern recognition, secure communication, the self-organization behaviors in the brain, and so on. Therefore, the research on these coupled systems, such as drive-response systems, is of significance. However, among those works that investigated the synchronization of the drive-response CMNNs, many of them have lost sight of the fact that some parameters between the two networks are not always identical due to the memristive mechanism and the differences between the initial conditions of different CMNNs. In this case, the drive-response CMNNs signify two heterogenous dynamic networks with mismatch parameters, yet some works just regarded them as homogeneous ones. Therefore, it is in great demand to give a comprehensive analysis of such systems, which provokes a motivation of this paper.

Generally speaking, neural networks can not realize synchronization spontaneously. To reaching synchronization, some control strategies are supposed to be applied in these neural networks. At present, many different kinds of control strategies have emerged successively in studying the synchronization and stability of networks, such as adaptive control [9], pinning control [17], [18], impulsive control [19], intermittent control [20], [21], distributed control [22], [23] and other newly emerged control approaches such as [24]. Since the configuration of the CMNNs is more massive and complicated, it is more difficult and expensive to apply controllers to all of the nodes,. One effective solution is to design a pinning control strategy, which means that only a fraction of nodes will be guided in the response networks. Therefore, in this paper a pinning control method will be applied, which is more economical and pragmatic.

Since the CMNNs can be considered as ones with mismatched parameters and the pinning control strategy is used in this paper, to the best of our knowledge, it may lead to possible destruction of synchronization. Fortunately, synchronization with an error level, referred to as quasi-synchronization can be realized. Up to now, there has been scarce literature dealing with this issue of CMNNs. However, many efforts have been taken into the inquiry of quasi-synchronization of heterogenous dynamic networks [25]–[28], which provide some assistance for us to analyze such issue of CMNNs.

Based on the aforementioned analysis, this paper aims to discuss the quasi-synchronization of the CMNNs with time-varying delays via pinning parts of the nodes. To tackle the difficulties in realizing the objective, some efforts have been made. First, the memsitive neural networks are expanded into the higher-dimensional ones, denoted as CMNNs, in which case the analysis of the characteristics of both the memristors and the neural networks becomes more difficult. Second, since the CMNNs are state-dependent, they are transferred into traditional neural networks with mismatch parameters. Third, with the consideration of both the nonlinear coupling and time-varying delays in our models at the meantime, this paper applies some differential inequality techniques and matrix theories and introduces a type of Lyapunov function and pinning algorithm to realize the quasi-synchronization. After that, some sufficiently useful criteria are derived.

The rest of this paper is organized as follows. In Section 2, the model of CMNNs is described, and then some preliminaries and pinning strategy are provided. In Section 3, the main results of quasi-synchronization are derived with some discussions. Moreover, some corollaries are then expanded from the main result. Section 4 presents some numerical simulations to demonstrate the results. Finally, the conclusion is drawn in section 5.

Notations: Some notations will be used in the following paper: $\mathbb{R}$ denotes the set of real numbers; $\mathbb{R}^n$ denotes the the $n-$dimensional real number space; $\not{\Bbbk}_n$ presents the $n \times 1$ real vectors consisting of 1s; $\|\cdot\|_2$ denotes the standard 2-norm of a vector or matrix; $|\cdot|$ means the absolute value of each element of a vector or matrix, i.e. $|A| = (|a_{ij}|)_{n\times n}$. For any vector $x \in \mathbb{R}^n$, $sign(x)$ means $diag\{sign(x_1), sign(x_2), \ldots, sign(x_n)\}$.

II. MODEL DESCRIPTION AND PRELIMINARIES

In this section, some preliminaries and concepts will be introduced, and then models of drive-response CMNNs will be formulated. Firstly, consider a class of isolate memristive neural networks with time delays, which can be described as (1) and has been analyzed in [3], [8]–[12].

$$\begin{aligned} \dot{x}_\iota(t) = -d_\iota(x_\iota(t))x_\iota(t) + \sum_{\kappa=1}^{n} a_{\iota\kappa}(x_\kappa(t))f_\kappa(x_\kappa(t)) \\ + \sum_{\kappa=1}^{n} b_{\iota\kappa}(x_\kappa(t))f_\kappa(x_\kappa(t-\tau_\iota(t))) + s_\iota, \\ \iota = 1, 2, \ldots, n, \quad (1) \end{aligned}$$

where $x_\iota(t) \in \mathbb{R}$ is the voltage of the capacitor C_i for the $\iota-th$ system. $\tau_\iota(t)$ is the time-varying delay. $f_\kappa(x_\kappa(t))$ and $f_\kappa(x_\kappa(t-\tau(t)))$ are the active functions of $x_\kappa(t)$ with and without time-varying delay respectively. s_ι is the external input or bias. $d_\iota(x_\iota(t)) > 0$ describes reset rate, and $a_{\iota\kappa}(x_\iota(t))$ and $b_{\iota\kappa}(x_\iota(t))$ describe the non-delayed and delayed memristive synaptic connection between neurons respectively, and they can be derived as following according to properties of memristor and the current-voltage characteristic.

$$\begin{aligned} d_\iota(x_\iota(t)) &= \frac{1}{C_\iota}\Big[\sum_{\kappa=1}^{N}(\frac{1}{R_{f\iota\kappa}} + \frac{1}{R_{g\iota\kappa}}) + W_\iota(x_\iota(t))\Big] \\ &= \begin{cases} \acute{d}_\iota, & |x_\iota(t)| \le T_\iota, \\ \grave{d}_\iota, & |x_\iota(t)| > T_\iota, \end{cases} \\ a_{\iota\kappa}(x_\iota(t)) &= \frac{W_{a\iota\kappa}(x_\kappa(t))}{C_\iota} sgn_{\iota\kappa} = \begin{cases} \acute{a}_{\iota\kappa}, & |x_\iota(t)| \le T_\iota, \\ \grave{a}_{\iota\kappa}, & |x_\iota(t)| > T_\iota, \end{cases} \\ b_{\iota\kappa}(x_\iota(t)) &= \frac{W_{b\iota\kappa}(x_\kappa(t))}{C_\iota} sgn_{\iota\kappa} = \begin{cases} \acute{b}_{\iota\kappa}, & |x_\iota(t)| \le T_\iota, \\ \grave{b}_{\iota\kappa}, & |x_\iota(t)| > T_\iota, \end{cases} \\ sign_{\iota\kappa} &= \begin{cases} 1, & \iota \ne \kappa, \\ -1, & \iota = \kappa, \end{cases} \quad s_\iota = \frac{I_\iota}{C_\iota}, \end{aligned}$$

where $R_{f\iota\kappa}$ and $R_{g\iota\kappa}$ are the resistors between $x_\iota(t)$ and feedback functions with and without time delay respectively. $W_\iota(x(t))$ denotes the memductance of the memristor parallel to the capacitor C_ι. $W_{a\iota\kappa}$ and $W_{b\iota\kappa}$ are the memductances of the memritors between the $x_\iota(t)$ and the feedback functions with and without time delays respectively. The positive constant T_ι is switching jump, and the constants $\acute{d}_\iota, \grave{d}_\iota, \acute{a}_{\iota\kappa}, \grave{a}_{\iota\kappa}, \acute{b}_{\iota\kappa}, \grave{b}_{\iota\kappa}, \iota, \kappa = 1, 2, \ldots, n$ satisfy $\acute{d}_\iota \ne \grave{d}_\iota$, $\acute{a}_{\iota\kappa} \ne \grave{a}_{\iota\kappa}$ and $\acute{b}_{\iota\kappa} \ne \grave{b}_{\iota\kappa}$.

Next, regarding a memristive neural network as a node, a dynamic system can be constructed by consisting of N nonlinearly and diffusively coupled time-varying delayed neural networks. In that way, a configuration of the coupled memristive neural networks can be formulated as following.

$$\begin{aligned}\dot{x}_i(t) &= -D_i(x_i(t))x_i(t) + A_i(x_i(t))f(x_i(t)) \\ &\quad + B_i(x_i(t))f(x_i(t-\tau(t))) + s_i \\ &\quad + c_1\sum_{j=1}^{N} g_{ij}g(x_j(t)) + c_2\sum_{j=1}^{N}\bar{g}_{ij}g(x_j(t-\tau(t))), \quad (2)\end{aligned}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith memristive time-varying delayed neural networks. The external input or bias vector is $s_i = (s_{i1}, s_{i2}, \ldots, s_{iN})^T$. The active function vectors are $f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)), \ldots, f_n(x_{in}(t)))^T$ and $f(x_i(t-\tau(t))) = (f_1(x_{i1}(t-\tau(t))), f_2(x_{i2}(t-\tau(t))), \ldots, f_n(x_{in}(t-\tau(t))))^T$. The nonlinear coupling functions have the form that $g(x_i(t)) = (g_1(x_{i1}(t)), g_2(x_{i2}(t)), \ldots, g_n(x_{in}(t)))^T$, $g(x_i(t-\tau(t))) = (g_1(x_{i1}(t-\tau(t))), g_2(x_{i2}(t-\tau(t))), \ldots, g_n(x_{in}(t-\tau(t))))^T$. The reset rate matrices are $D_i(x_i(t)) = diag\{d_{i1}(x_{i1}(t)), \ldots, d_{in}(x_{in}(t))\} \in \mathbb{R}^{n\times n}$. $A_i(x_i(t)) = (a^i_{jk}(x_{ij}(t)))_{n\times n}$ and $B_i(x_i(t)) = (b^i_{jk}(x_{ij}(t)))_{n\times n}$ are the inner coupled matrices, and

$$d_{ik}(x_{ik}(t)) = \begin{cases} \acute{d}_{ik}, & |x_{ik}(t)| \le T_{ik}, \\ \grave{d}_{ik}, & |x_{ik}(t)| > T_{ik}, \end{cases} \quad (3)$$

$$a^i_{hk}(x_{ik}(t)) = \begin{cases} \acute{a}^i_{hk}, & |x_{ik}(t)| \le T_{ik}, \\ \grave{a}^i_{hk}, & |x_{ik}(t)| > T_{ik}, \end{cases} \quad (4)$$

$$b^i_{hk}(x_{ik}(t)) = \begin{cases} \acute{b}^i_{hk}, & |x_{ik}(t)| \le T_{ik}, \\ \grave{b}^i_{hk}, & |x_{ik}(t)| > T_{ik}. \end{cases} \quad (5)$$

Denote $\acute{D}_i = diag\{\acute{d}_{i1}, \ldots, \acute{d}_{in}\}$, $\grave{D}_i = diag\{\grave{d}_{i1}, \ldots, \grave{d}_{in}\}$, $\acute{A}_i = (\acute{a}^i_{hk})_{n\times n}$, $\grave{A}_i = (\grave{a}^i_{hk})_{n\times n}$, $\acute{B}_i = (\acute{b}^i_{hk})_{n\times n}$, $\grave{B}_i = (\grave{b}^i_{hk})_{n\times n}$. $T_{ij} > 0$ are the switching jumps, and $T_i = (T_{i1}, T_{i2}, \ldots, T_{in})^T$. The constants $\acute{d}_{ij}$, $\grave{d}_{ij}$, $\acute{a}^i_{ij}$, $\grave{a}^i_{ij}$, $\acute{b}^i_{ij}$, $\grave{b}^i_{ij}$, $i, j = 1, 2, \ldots, n$ satisfy $\acute{d}_{ij} \neq \grave{d}_{ij}$, $\acute{a}^i_{ij} \neq \grave{a}^i_{ij}$ and $\acute{b}^i_{ij} \neq \grave{b}^i_{ij}$. c_1 and c_2 are the external coupled strengths with and without time delayed. The external coupled matrices $G = (g_{ij})_{N\times N} \in \mathbb{R}^{N\times N}$ and $\bar{G} = (\bar{g}_{ij})_{N\times N} \in \mathbb{R}^{N\times N}$ describe the coupled configurations of the dynamic systems. What's more, assume that $G = (g_{ij})_{n\times n}$ and $\bar{G} = (\bar{g}_{ij})_{N\times N}$ are diffusive matrices, which means $g_{ij} = g_{ji}, g_{ij} \ge 0, \bar{g}_{ij} = g_{ji}, \bar{g}_{ij} \ge 0$ for $i \neq j$ and $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}, \bar{g}_{ii} = -\sum_{j=1, j\neq i}^{N}\bar{g}_{ij}$. Therefore, $\sum_{j=1}^{N} g_{ij} = 0, \sum_{j=1}^{N}\bar{g}_{ij} = 0$ for $i = 1, 2, \ldots, N$. Moreover, the $rank(G) = N - 1$, and the eigenvalues of G satisfy that $0 = \lambda_1(G) < \lambda_2(G) \le \lambda_3(G) \le \ldots \le \lambda_N(G)$. $\nVdash = (1, 1, \ldots, 1)^T \in \mathbb{R}^N$ is the right eigenvector of the eigenvalue 0 of G, i.e. $G \cdot \nVdash = 0$. The initial conditions of (2) are $x_i(t) = \phi_i(t) \in C([-\tau, 0], \mathbb{R}^n)$, $i = 1, 2, \ldots, N$.

Throughout this paper, there are some assumptions needed as follows.

$(H1)$ Assume that there exist positive constants l_i, $i = 1, \ldots, n$, such that for any $x, y \in \mathbb{R}$, $x \neq y$, it holds that

$$0 \le \frac{f_i(x) - f_i(y)}{x - y} \le l_i.$$

$(H2)$ Assume that there exist positive constants M_ι, such that for any $x \in \mathbb{R}$, $i = 1, \ldots, n$, it holds that

$$|f_i(x)| \le m_i.$$

$(H3)$ [29] Assume that $g(\cdot) : \mathbb{R} \to \mathbb{R}$ belongs to the acceptable nonlinear coupling function class, which is denoted as $g \in NCF(\alpha, \beta)$. That is, there exist two nonnegative scalars α and β, such that $g(\omega) - \alpha\omega$ satisfies the following Lipschitz conditions:

$$|g(\omega_1) - g(\omega_2) - \alpha(\omega_1 - \omega_2)| \leqslant \beta|\omega_1 - \omega_2|.$$

holds for any $\omega_1, \omega_2 \in \mathbb{R}$.

$(H4)$ Assume that $0 < \tau_i(t) \le \tau$, $\dot{\tau}_i(t) \le \theta < 1$, where θ and τ are constants.

Suppose the response memristive neural networks with control strategy can be described as

$$\begin{aligned}\dot{y}_i(t) &= -D_i(y_i(t))y_i(t) + A_i(y_i(t))f(y_i(t)) \\ &\quad + B_i(y_i(t))f(y_i(t-\tau(t))) + s_i + U_i(t) \\ &\quad + c_1\sum_{j=1}^{N} g_{ij}g(y_j(t)) + c_2\sum_{j=1}^{N}\bar{g}_{ij}g(y_j(t-\tau(t))), \quad (6)\end{aligned}$$

where $D_i(y_i(t)), A_i(y_i(t)), B_i(y_i(t))$ are defined similarly to (34) (4) and (5) respectively. $U_i(t)$ is the controller applied in (6). Out of consideration for economical and pragmatic implementation, only parts of the nodes are controlled. Without loss of generality, let the nodes $i_1, i_2, \ldots, i_l$ be selected as the pinned nodes, and the pinning strategy is designed as follows.

$$U_i(t) = \begin{cases} -c_1u_i(t)e_i(t), & i = 1, 2, \ldots, l, \\ 0, & i = l+1, \ldots, N, \end{cases} \quad (7)$$

where $u_i(t)$ is the time-invariant feedback gains, and denote $U(t) = \{u_1(t), \ldots, u_l(t), 0, \ldots, 0\}$.

Generally speaking, the initial conditions of the response memristive neural networks are not the same as those of the drive neural networks, and in this paper they are denoted by $y_i(t) = \psi(t) \in C([-\tau, 0], \mathbb{R}^n)$, $i = 1, 2, \ldots, N$.

From the above description and analysis, it can be seen that CMNNs are different from traditional neural networks, since the dynamic functions of the CMNNs are state-dependent while those of traditional ones are time-dependent. To realize the quasi-synchronization of drive and response memristive neural networks with time-varying delays in (2) and (6), this paper transfers the CMNNs to a traditional neural networks with uncertain and mismatch parameters. First of all, some definitions and lemmas are given, which will be used throughout the paper.

Definition 1: The drive memeistive dynamic networks (2) and response memeistive dynamic networks(6) are said to exponentially realize quasi-synchronization with an error bound $\epsilon > 0$ if there exists a compact set $\mathbb{E}$ and positive constants M and β, such that

$$\sum_{i=1}^{N} ||y_i(t) - x_i(t)|| \le M\exp^{-\beta t} + \epsilon, \quad t > 0,$$

and

$$\mathbb{E} = \Big\{ \sum_{i=1}^{N}(y_i(t) - x_i(t)) \in \mathbb{R}^n \Big| \sum_{i=1}^{N} \|y_i(t) - x_i(t)\| \le \epsilon \Big\}, \text{ as } t \to \infty.$$

Definition 2 (Filippov Regularization [30]): The Flippov set-valued map of $f(x)$ at $x \in \mathbb{R}^n$ is defined as follows:

$$F(x) = \bigcap_{\delta>0} \bigcap_{\mu(\Omega)} \overline{co}[f(B(x,\delta) \setminus \Omega)],$$

where $B(x, \delta) = y : \|y - x\| \le \delta$, and $\mu(\Omega)$ is Lebesgue measure of set Ω, $\overline{co}[E]$ is the closure of the convex hull of the set E.

Lemma 1 [31]: If $W(t) \ge 0, t \in (-\infty, +\infty)$,

$$D^+W(t) \le \gamma(t) + \alpha(t)W(t) + \beta(t) \sup_{t-\tau(t)\le s\le t} W(s), \quad t > t_0,$$

where $D^+W(t) = \overline{lim}_{h\to 0^+}(W(t+h) - W(t))/h$, and $\gamma(t) \ge 0, \alpha(t) \le 0, \beta(t) \ge 0$ are continuous functions and $\tau(t) > 0$. If there exists δ such that

$$\alpha(t) + \beta(t) \le -\delta < 0, t \ge t_0.$$

Then we have

$$W(t) \le \frac{\gamma^*}{\delta} + \sup_{-\infty\le s\le t_0} W(s)e^{-\mu^*}(t - t_0),$$

where $\gamma^* = \sup_{t_0\le t<\infty} \gamma(t)$ and $\mu^* = \inf_{t\ge t_0}\{\mu(t) : \mu(t) + \alpha(t) + \beta(t)e^{\mu(t)\tau(t)} = 0\}$.

Based on the theory of differential inclusion and definition 2, it can be derived that

$$\begin{aligned}\dot{x}_i(t) \in & -\overline{co}[\acute{D}_i, \grave{D}_i]x_i(t) + \overline{co}[\acute{A}_i, \grave{A}_i]f(x_i(t)) \\ & + \overline{co}[\acute{B}_i, \grave{B}_i]f(x_i(t - \tau(t))) + s_i \\ & + c_1 \sum_{j=1}^{N} g_{ij}g(x_j(t)) + c_2 \sum_{j=1}^{N} \bar{g}_{ij}g(x_j(t - \tau(t))). \end{aligned} \quad (8)$$

Definite function matrix $\Lambda_i^\iota = diag\{\lambda_{i1}^\iota(t), \ldots, \lambda_{in}^\iota(t)\}$, where $\iota = 1, 2, \ldots, 6.$ and $\lambda_{ij}^\iota(t) \in \overline{co}[0, 1]$, then $x_i(t)$ can be written as

$$\begin{aligned}\dot{x}_i(t) = & -\big(\acute{D}_i\Lambda^{i1}(t) + \grave{D}_i(1 - \Lambda^{i1}(t))\big)x_i(t) \\ & + \big(\acute{A}_i\Lambda^{i2}(t) + \grave{A}_i(1 - \Lambda^{i2}(t))\big)f(x_i(t)) \\ & + \big(\acute{B}_i\Lambda^{i3}(t) + \grave{B}_i(1 - \Lambda^{i3}(t))\big)f(x_i(t - \tau(t))) + s_i \\ & + c_1 \sum_{j=1}^{N} g_{ij}g(x_j(t)) + c_2 \sum_{j=1}^{N} \bar{g}_{ij}g(x_j(t - \tau(t))). \end{aligned} \quad (9)$$

With the same analysis, the response memristive dynamic networks can be derived as

$$\begin{aligned}\dot{y}_i(t) = & -\big(\acute{D}_i\Lambda^{i4}(t) + \grave{D}_i(1 - \Lambda^{i4}(t))\big)y_i(t) + \big(\acute{A}_i\Lambda^{i5}(t) \\ & + \grave{A}_i(1 - \Lambda^{i5}(t))\big)f(y_i(t)) + \big(\acute{B}_i\Lambda^{i6}(t) \\ & + \grave{B}_i(1 - \Lambda^{i6}(t))\big)f(y_i(t - \tau(t))) + s_i \\ & + c_1 \sum_{j=1}^{N} g_{ij}g(y_j(t)) + c_2 \sum_{j=1}^{N} \bar{g}_{ij}g(y_j(t - \tau(t))). \end{aligned} \quad (10)$$

To realize the quasi-synchronization between the response memristive neural networks (2) and drive memristive neural networks (6), a control strategy is applied in the response neural networks.

Denote $e_i(t) = y_i(t) - x_i(t)$ as the synchronization error under the drive system (1) and the response system (3), then $e_i(t) = (e_{i1}(t), e_{i2}(t), \ldots, e_{in}(t))^T$ and the error system can be represented as

$$\begin{aligned}\dot{e}_i(t) = & -\big(\acute{D}_i\Lambda^{i4}(t) + \grave{D}_i(1 - \Lambda^{i4}(t))\big)y_i(t) \\ & - \big(\acute{D}_i\Lambda^{i1}(t) + \grave{D}_i(1 - \Lambda^{i1}(t))\big)x_i(t) \\ & + \hat{A}_i(t)\tilde{f}(e_i(t)) + \hat{B}_i(t)\tilde{f}(e_i(t - \tau(t))) \\ & + U_i(t) + \Delta\hat{A}_i(t)f(x(t)) + \Delta\hat{B}_i(t)f(x_i(t - \tau(t))) \\ & + c_1 \sum_{j=1}^{N} g_{ij}\tilde{g}(e_j(t)) + c_2 \sum_{j=1}^{N} \bar{g}_{ij}\tilde{g}(e_j(t - \tau(t))), \end{aligned} \quad (11)$$

where $\tilde{f}(e_j(t)) = f(x_j(t)) - f(y_j(t))$, $\tilde{g}(e_j(t)) = g(x_j(t)) - g(y_j(t))$, $\hat{A}_i(t) = \acute{A}_i\Lambda^{i5}(t) + \grave{A}_i(1 - \Lambda^{i5}(t))$, $\Delta\hat{A}(t) = (\Lambda^{i5}(t) - \Lambda^{i2}(t))(\acute{A}_i - \grave{A}_i)$, $\hat{B}_i(t) = \acute{B}_i\Lambda^{i6}(t) + \grave{B}_i(1 - \Lambda^{i6}(t))$, $\Delta\hat{B}(t) = (\Lambda^{i6}(t) - \Lambda^{i3}(t))(\acute{B}_i - \grave{B}_i)$. Denotes $\hat{D}_i = diag\big\{\min\{\acute{d}_{i1}, \grave{d}_{i1}\}, \ldots, \min\{\acute{d}_{in}, \grave{d}_{in}\}\big\}$, $\bar{D}_i(t) = (|\acute{d}_{ij} - \grave{d}_{ij}|)_{n\times n}$, $T_i = (T_{i1}, T_{i2}, \ldots, T_{in})^T$. Obviously, there exist positive constants $\mu(A), \mu(B), \rho(A)$, and $\rho(B)$, such that $\|\hat{A}_i(t)\|_2^2 \le \mu(A)$, $\|\hat{B}_i(t)\|_2^2 \le \mu(B)$, $\|\Delta\hat{A}_i(t)\|_2 \le \rho(A)$, $\|\Delta\hat{B}_i(t)\|_2 \le \rho(B)$.

Remark 1: Since the initial conditions of the CMNNs (1) and (2) are always different, according to the memristive mechanism, the measurable functions $\Lambda^{i\iota}(t)$ for node i cannot be guaranteed to be identical, which means when $a_{hk}^i(x_{ik}) = \acute{a}_{hk}^i$, the value of $a_{hk}^i(y_{ik})$ may be $\acute{a}_{hk}^i$ or $\grave{a}_{hk}^i$. In this case, the CMNNs can be considered as a traditional neural networks with mismatch parameters, which many other works always make a mistake on. Therefore, with the pinning strategy applied in this paper, the two CMNNs can eventually realize quasi-synchronization rather than complete synchronization.

III. MAIN RESULT

In this part, the exponentially quasi-synchronization of the drive system and the response system will be studied. By using Lyapunov-Krasovkii functions and matrix analysis techniques, some sufficient conditions will be derived and the main results are presented as follows.

In what follows, let $L = diag\{l_1, l_2, \ldots, l_n\}$, $M_i = m_iI_n$, $U(t) = diag\{u_1(t), \ldots, u_l(t), 0, \ldots, 0\}$, $\bar{D}_i = (|\acute{d}_{ij} - \grave{d}_{ij}|)_{n\times n}$, $\hat{D}_i = diag\big\{\min\{\acute{d}_{i1}, \grave{d}_{i1}\}, \ldots, \min\{\acute{d}_{in}, \grave{d}_{in}\}\big\}$.

Theorem 1: Suppose that $H(1) - H(4)$ hold. Using the control strategy designed as (7), the trajectory of the error system (11) converges exponentially to the set

$$E = \Big\{e_i \in \mathbb{R}^n, i = 1, \ldots, N \Big| \sum_{i=1}^{N} \|e_i\| \le \frac{1}{\delta p_{\min}} \sum_{i=1}^{N} \omega_i^2\Big\},$$

where $\omega_i = \|\bar{D}_iT_i + \big(\rho(A) + \rho(B)\big)M_i\|_2$, that is, the drive memristive dynamic network (2) is said to be exponentially quasi-synchronized with the response dynamic network (6) if there exist a positive constant δ and a nonsingular matrix $P = diag\{p_1, \ldots, p_n\}$, such that

$$c_1\alpha G - c_1 U(t) + c_1 G^T G + 2c_2 G^T G + \big(\bar{\lambda}_i + c_2\alpha^2 + (c_1+c_2)\beta^2\big)I_n \leq -\delta I_n < \mathbf{0}, \quad (12)$$

where $\bar{\lambda}_i = \lambda_{\max}\big\{-\hat{D}_i P^{-1} + 3P + (\mu(A) + \mu(B))L^2 P^{-1}\big\}$, for $i = 1, 2, \ldots, N$.

Proof: Construct the Lyapunov function

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t) P e_i(t).$$

Differentiating $V(t)$ along the solution of the error system (11), it yields that

$$\begin{aligned}
\dot{V}(t) \leq & \sum_{i=1}^{N} e_i^T(t)P\Big(-\big(\acute{D}_i\Lambda^{i4}(t) + \grave{D}_i(1-\Lambda^{i4}(t))\big)y_i(t) \\
& + \big(\acute{D}_i\Lambda^{i1}(t) + \grave{D}_i(1-\Lambda^{i1}(t))\big)x_i(t) + \hat{A}_i(t)\tilde{f}(e_i(t)) \\
& + \hat{B}_i(t)\tilde{f}(e_i(t-\tau(t))) + U_i(t) \\
& + \Delta\hat{A}_i(t)f(x_i(t)) + \Delta\hat{B}_i(t)f(x_i(t-\tau(t)))\Big) \\
& + c_1\sum_{i=1}^{N}\sum_{j=1}^{N} g_{ij}e_i^T(t)P\tilde{g}(e_j(t)) \\
& + c_2\sum_{i=1}^{N}\sum_{j=1}^{N} \bar{g}_{ij}e_i^T(t)P\tilde{g}(e_j(t-\tau(t))) \\
\leq & \sum_{i=1}^{N} e_i^T(t)P\Big(-\hat{D}_i e_i(t) + sgn(e_i^T(t))\bar{D}_i T_i \\
& + \hat{A}_i(t)\tilde{f}(e_i(t)) + \hat{B}_i(t)\tilde{f}(e_i(t-\tau(t))) + U_i(t) \\
& + \Delta\hat{A}_i(t)f(x_i(t)) + \Delta\hat{B}_i(t)f(x_i(t-\tau(t)))\Big) \\
& + c_1\sum_{i=1}^{N}\sum_{j=1}^{N} g_{ij}e_i^T(t)P\tilde{g}(e_j(t)) \\
& + c_2\sum_{i=1}^{N}\sum_{j=1}^{N} \bar{g}_{ij}e_i^T(t)P\tilde{g}(e_j(t-\tau(t))),
\end{aligned} \quad (13)$$

in which one derives from Lemma 1 and Assumption $H(2)$ that

$$\begin{aligned}
& \sum_{i=1}^{N} e_i^T(t)P\big(\Delta\hat{A}_i(t)f_i(x_i(t)) + \Delta\hat{B}_i(t)f_i(x_i(t-\tau(t))) \\
& \quad + sgn(e_i^T(t))\bar{D}_i T_i\big) \\
& \leq \sum_{i=1}^{N} |e_i^T(t)|P\Big[\bar{D}_i T_i + \big(|\Delta\hat{A}_i(t) + \Delta\hat{B}_i(t)|\big)M_i\Big] \\
& \leq \sum_{i=1}^{N} e_i^T(t)P^T P e_i(t) + \sum_{i=1}^{N}\omega_i^2.
\end{aligned} \quad (14)$$

By using Assumption $H(3)$, it can be obtained that

$$\begin{aligned}
& c_1\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)P\Gamma g_{ij}\tilde{g}(e_j(t)) - \sum_{i=1}^{N} e_i^T P U_i(t) \\
& = c_1\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)P g_{ij}\tilde{g}(e_j(t)) - c_1\sum_{i=1}^{l} e_i^T P u_i(t)e_i(t) \\
& \leq c_1\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)P g_{ij}(\alpha e_j(t) + \tilde{g}(e_j(t)) - \alpha e_j(t)) \\
& \quad - c_1\sum_{i=1}^{l} e_i^T P u_i(t)e_i(t) \\
& = c_1\sum_{k=1}^{n} p_k\tilde{e}_k(t)(\alpha G - U(t))\tilde{e}_k(t) \\
& \quad + c_1\sum_{k=1}^{n} p_k\tilde{e}_k(t)G(\tilde{g}(\tilde{e}_k(t)) - \alpha\tilde{e}_k(t)) \\
& \leq c_1\sum_{k=1}^{n} p_k\tilde{e}_k^T(t)(\alpha G - U(t) + G^T G + \beta^2 I_n)\tilde{e}_k(t),
\end{aligned} \quad (15)$$

and

$$\begin{aligned}
& \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)P\bar{g}_{ij}\tilde{g}(e_j(t-\tau(t))) \\
& = \alpha\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)P\bar{g}_{ij}e_j(t-\tau(t) \\
& \quad + \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)P\bar{g}_{ij}\tilde{g}(e_j(t-\tau(t))) - \alpha e_j(t-\tau(t))) \\
& = \alpha\sum_{k=1}^{n} p_k\tilde{e}_k^T(t)\bar{G}\tilde{e}_k(t-\tau(t)) \\
& \quad + \sum_{k=1}^{n} p_k\tilde{e}_k^T(t)\bar{G}\tilde{g}(\tilde{e}_k(t-\tau(t))) - \alpha\tilde{e}_k(t-\tau(t))) \\
& \leq \sum_{k=1}^{n} p_k\tilde{e}_k^T(t)(\alpha^2 I_n + \bar{G}^T\bar{G})\tilde{e}_k(t) \\
& \quad + \sum_{k=1}^{n} p_k\tilde{e}_k^T(t-\tau(t))(\bar{G}^T\bar{G} + \beta^2 I_n)\tilde{e}_k(t-\tau(t)).
\end{aligned} \quad (16)$$

It follows from $H(1)$ that

$$\begin{aligned}
& \sum_{i=1}^{N} e_i^T(t)P\hat{A}_i(t)\tilde{f}(e_i(t)) + \sum_{i=1}^{N} e_i^T(t)P\hat{B}_i(t)\tilde{f}(e_i(t-\tau(t))) \\
& \leq \sum_{i=1}^{N} e_i^T(t)P^T P e_i(t) + u(A)\sum_{i=1}^{N} f^T(e_i(t))f(e_i(t)) \\
& \quad + \sum_{i=1}^{N} e_i^T(t)P^T P e_i(t)
\end{aligned}$$

$$+\,u(B)\sum_{i=1}^{N}f^T(e_i(t-\tau(t)))f(e_i(t-\tau(t)))$$
$$\le \sum_{i=1}^{N}e_i^T(t)(2P^TP+\mu(A)L^2)e_i(t)$$
$$+\sum_{i=1}^{N}e_i^T(t-\tau(t)))\mu(B)L^2e_i(t-\tau(t)). \quad (17)$$

Substituting inequalities (14)(15)(16)(17) into (13), one can derive that

$$\begin{aligned}\dot V(t) \le& \sum_{i=1}^{N}e_i^T(t)\big(-\hat D_i+3P^TP+\mu(A)L^2\big)e_i(t)\\
&+\sum_{i=1}^{N}e_i^T(t-\tau(t)))\mu(B)L^2e_i(t-\tau(t))+\sum_{i=1}^{N}\omega_i^2\\
&+\sum_{k=1}^{n}p_k\tilde e_k^T(t)\big[c_1\alpha G-c_1U(t)+c_1G^TG+c_2\bar G^T\bar G\\
&+(c_1\beta^2+c_2\alpha^2)I_n\big]\tilde e_k(t)\\
&+\sum_{k=1}^{n}p_k\tilde e_k^T(t-\tau(t))(c_2\bar G^T\bar G+c_2\beta^2I_n)\tilde e_k(t-\tau(t))\\
\le& \sum_{k=1}^{n}p_k\tilde e_k^T(t)\big[c_1\alpha\tilde G-c_1U(t)+c_1G^TG+c_2\bar G^T\bar G\\
&+\bar\lambda I_n\big]\tilde e_k(t)+\sum_{k=1}^{n}p_k\tilde e_k^T(t-\tau(t))\big[c_2\bar G^T\bar G\\
&+\big(c_2\beta^2+\mu(B)\frac{l_k^2}{p_k}\big)I_n\big]\tilde e_k(t-\tau(t))+\sum_{i=1}^{N}\omega_i^2\\
\le& -a(t)V(t)+b(t)V(t-\tau(t))+\sum_{i=1}^{N}\omega_i^2. \quad (18)\end{aligned}$$

where $a(t)=\lambda_{\max}\{c_1\alpha G-c_1U(t)+c_1G^TG+c_2\bar G^T\bar G\}+\max\{\bar\lambda_i\}$, $b(t)=\lambda_{\max}\{c_2\bar G^T\bar G+\mu(B)L^2P^{-1}\}+c_2\beta^2$.

Combining (12) and Lemma 3, the inequality implies that

$$V(t)\le \sup_{-\infty\le s\le t_0}V(s)e^{-\mu(t-t_0)}+\frac{1}{\delta}\sum_{i=1}^{N}\omega_i^2, \quad (19)$$

where $\mu=\inf_{t\ge t_0}\{\mu(t):\mu(t)-a(t)+b(t)e^{\mu(t)\tau(t)}=0\}$.

Consequently, it can be obtained that

$$\sum_{i=1}^{N}\|e_i(t)\|^2\le\frac{1}{p_{\min}}\sup_{-\infty\le s\le t_0}e^{-\mu(t-t_0)}+\frac{1}{\delta p_{\min}}\sum_{i=1}^{N}\omega_i^2,\ t\ge t_0,$$

from which it can be concluded that the error system converges exponentially to the set $\mathbb{E}$, where

$$\mathbb{E}=\Big\{e_i\in\mathbb{R}^n, i=1,\dots,N\Big|\sum_{i=1}^{N}\|e_i\|\le\frac{1}{\delta p_{\min}}\sum_{i=1}^{N}\omega_i^2\Big\},$$

which implies that the drive CMNNs (1) and the response CMNNs (2) eventually achieve quasi-synchronization with an error level $\frac{1}{\delta p_{\min}}\sum_{i=1}^{N}\omega_i^2$. The proof is completed. □

To make the Theorem 1 more applicable, some corollaries are derived.

Remark 2: From Theorem 1 and Corollary 1, one can see that the properties of the nonlinear coupling functions $g_i(\cdot)$ have a great impact on the error level. The smaller the β is, the less nonlinear the $g_i(t)$ will be, and the easier and faster the error level will be achieved, and vise versa. Now let us take an extreme example, i.e., suppose $g(\cdot)\in NCF(\alpha,0)$, and then (2) and (6) become linearly coupled systems.

$$\begin{aligned}\dot x_i(t)=&-D_i(x_i(t))x_i(t)+A_i(x_i(t))f(x_i(t))\\
&+B_i(x_i(t))f(x_i(t-\tau(t)))+s_i\\
&+c_1\sum_{j=1}^{N}g_{ij}x_j(t)+c_2\sum_{j=1}^{N}\bar g_{ij}x_j(t-\tau(t)). \quad (20)\end{aligned}$$

$$\begin{aligned}\dot y_i(t)=&-D_i(y_i(t))y_i(t)+A_i(y_i(t))f(y_i(t))\\
&+B_i(y_i(t))f(y_i(t-\tau(t)))+s_i+u_i(t)\\
&+c_1\sum_{j=1}^{N}g_{ij}y_j(t)+c_2\sum_{j=1}^{N}\bar g_{ij}y_j(t-\tau(t)). \quad (21)\end{aligned}$$

$$\begin{aligned}\dot e_i(t)=&-\big(\acute D_i\Lambda^{i4}(t)+\grave D_i(1-\Lambda^{i4}(t))\big)y_i(t)\\
&-\big(\acute D_i\Lambda^{i1}(t)+\grave D_i(1-\Lambda^{i1}(t))\big)x_i(t)\\
&+\hat A_i(t)\tilde f(e_i(t))+\hat B_i(t)\tilde f(e_i(t-\tau(t)))\\
&+U_i(t)+\Delta\hat A_i(t)f(x(t))+\Delta\hat B_i(t)f(x_i(t-\tau(t)))\\
&+c_1\sum_{j=1}^{N}g_{ij}\Gamma e_j(t)+c_2\sum_{j=1}^{N}\bar g_{ij}\Gamma e_j(t-\tau(t)). \quad (22)\end{aligned}$$

Corollary 1: Suppose that $g(\cdot)\in NCF(\alpha,0)$, using the control strategy designed as (7), the trajectory of the error system (22) converges exponentially to the set

$$E=\Big\{e_i\in\mathbb{R}^n, i=1,\dots,N\Big|\sum_{i=1}^{N}\|e_i\|\le\frac{1}{\breve\delta p_{\min}}\sum_{i=1}^{N}\omega_i^2\Big\},$$

that is, the drive memristive dynamic network (20) is said to be exponentially quasi-synchronized with the response dynamic network (21) if there exist positive constants $\breve\delta$ and a nonsingular matrix $P=diag\{p_1,\dots,p_n\}$, such that

$$c_1(G-U(t))+c_2\bar G^T\bar G+\bar\lambda I_n\le-\breve\delta I_n<\mathbf{0},$$

where $\bar\lambda_i=\lambda_{\max}\{-\hat D_iP^{-1}+3P+(\mu(A)+\mu(B))L^2P^{-1}\}$, for $i=1,2,\dots,N$.

Remark 3: In the above models, the coupling strengths are constants. However, the networks structure will not always stay the same in practice. Therefore, an adaptive adjustment can be added to the coupling strengths in the drive and response memristive neural networks described in (23) and (24). In this case, a more concise condition will be obtained.

$$\begin{aligned}\dot{x}_i(t) = & -D_i(x_i(t))x_i(t) + A_i(x_i(t))f(x_i(t)) \\ & + B_i(x_i(t))f(x_i(t-\tau(t))) + s_i \\ & + c_1(t)\sum_{j=1}^{N} g_{ij}g(x_j(t)) + c_2(t)\sum_{j=1}^{N} \bar{g}_{ij}g(x_j(t-\tau(t))).\end{aligned} \quad (23)$$

$$\begin{aligned}\dot{y}_i(t) = & -D_i(y_i(t))y_i(t) + A_i(y_i(t))f(y_i(t)) \\ & + B_i(y_i(t))f(y_i(t-\tau(t))) + s_i + u_i(t) \\ & + c_1(t)\sum_{j=1}^{N} g_{ij}g(y_j(t)) + c_2(t)\sum_{j=1}^{N} \bar{g}_{ij}g(y_j(t-\tau(t))).\end{aligned} \quad (24)$$

$$\begin{aligned}\dot{e}_i(t) = & -\big(\acute{D}_i\Lambda^{i4}(t) + \grave{D}_i(1-\Lambda^{i4}(t))\big)y_i(t) \\ & -\big(\acute{D}_i\Lambda^{i1}(t) + \grave{D}_i(1-\Lambda^{i1}(t))\big)x_i(t) \\ & + \hat{A}_i(t)\tilde{f}(e_i(t)) + \hat{B}_i(t)\tilde{f}(e_i(t-\tau(t))) \\ & + U_i(t) + \Delta\hat{A}_i(t)f(x(t)) + \Delta\hat{B}_i(t)f(x_i(t-\tau(t))) \\ & + c_1(t)\sum_{j=1}^{N} g_{ij}\tilde{g}(e_j(t)) + c_2(t)\sum_{j=1}^{N} \bar{g}_{ij}\tilde{g}(e_j(t-\tau(t))).\end{aligned} \quad (25)$$

where $c_1(t) = \gamma c(t)$, $c_2(t) = \frac{1}{\gamma}c(t)$, and the derivative of $c(t)$ is designed as $\dot{c}(t) = \sum_{i=1}^{N} e_i(t)^T e_i(t)$.

Corollary 2: Suppose that $H(1) - H(4)$ hold. Using the control strategy designed as (7), the trajectory of the error system (25) converges exponentially to the set

$$E = \Big\{e_i \in \mathbb{R}^n, i = 1, \ldots, N \Big| \sum_{i=1}^{N} \|e_i\| \le \frac{1}{\delta p_{\min}} \sum_{i=1}^{N} \omega_i^2 \Big\},$$

that is, the drive memristive dynamic network (23) is said to be exponentially quasi-synchronized with the response dynamic network (24) if there exist positive constants $\bar{\delta}$ and γ, such that

$$\begin{aligned}\gamma^2\big(\alpha\tilde{G} - c_1U(t) + \beta^2 I_n\big) + 2G^TG + (\alpha^2+\beta^2)I_n \\ \le -\bar{\delta}I_n < \mathbf{0}.\end{aligned} \quad (26)$$

Proof: Construct Lyapunov function

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)Pe_i(t).$$

With the same analysis as Theorem 1, it can be obtained that

$$\begin{aligned}\dot{V}(t) \le & \sum_{i=1}^{N} e_i^T(t)P\Big(-\hat{D}_ie_i(t) + sgn(e_i^T(t))\bar{D}_iT_i \\ & + \hat{A}_i(t)f(e_i(t)) + \Delta\hat{B}_i(t)f(e_i(t-\tau(t))) + u_i(t) \\ & + \Delta\hat{A}_i(t)f(x_i(t)) + \Delta\hat{B}_i(t)f(x_i(t-\tau(t)))\Big) \\ & + \gamma c(t)\sum_{i=1}^{N}\sum_{j=1}^{N} g_{ij}e_i^T(t)P\tilde{g}(e_j(t)) \\ & + \frac{1}{\gamma}c(t)\sum_{i=1}^{N}\sum_{j=1}^{N} \bar{g}_{ij}e_i^T(t)P\tilde{g}(e_j(t-\tau(t))) \\ \le & \sum_{i=1}^{N} e_i^T(t)\big(-\hat{D}_i + 3P^TP + \mu(A)L^2\big)e_i(t) \\ & + \sum_{i=1}^{N} e_i^T(t-\tau(t))\mu(B)L^2e_i(t-\tau(t)) + \sum_{i=1}^{N}\omega_i^2 \\ & + \frac{1}{\gamma}c(t)\sum_{k=1}^{n} p_k\tilde{e}_k^T(t)\big[\gamma^2\big(\alpha G - U(t) + G^TG + \beta^2 I_n\big) \\ & + \alpha^2 I_n + \bar{G}^T\bar{G}\big]\tilde{e}_k(t) \\ & + \frac{1}{\gamma}c(t)\sum_{k=1}^{n} p_k\tilde{e}_k^T(t-\tau(t))(\bar{G}^T\bar{G} + \beta^2 I_n)\tilde{e}_k(t-\tau(t)) \\ \le & \sum_{k=1}^{n} p_k\tilde{e}_k^T(t)\big[c(t)\gamma\big(\alpha\tilde{G} - U(t) + G^TG + \beta^2 I_n\big) \\ & + \frac{1}{\gamma}c(t)(\alpha^2 I_n + \bar{G}^T\bar{G}) \\ & + \lambda_{\max}\big(-\hat{D}_iP^{-1} + 3P + \mu(A)L^2P^{-1}\big)I_n\big]\tilde{e}_k(t) \\ & + \sum_{k=1}^{n} p_k\tilde{e}_k^T(t-\tau(t))\big[\frac{1}{\gamma}c(t)(\bar{G}^T\bar{G} + \beta^2 I_n) \\ & + \mu(B)\frac{l_k^2}{p_k}I_n\big]\tilde{e}_k(t-\tau(t)) + \sum_{i=1}^{N}\omega_i^2 \\ \le & -\bar{a}(t)V(t) + \bar{b}(t)V(t-\tau(t)) + \sum_{i=1}^{N}\omega_i^2,\end{aligned} \quad (27)$$

where $\bar{a}(t) = -\lambda_{\max}\Big[c(t)\gamma\big(\alpha\tilde{G} - U(t) + G^TG + \beta^2 I_n\big) + \frac{1}{\gamma}c(t)(\alpha^2 I_n + \bar{G}^T\bar{G}) + \lambda_{\max}\big(-\hat{D}_iP^{-1} + 3P + \mu(A)L^2P^{-1}\big)I_n\Big]$, $\bar{b}(t) = \lambda_{\max}\Big[\frac{1}{\gamma}c(t)(\bar{G}^T\bar{G} + \beta^2 I_n) + \mu(B)\frac{l_k^2}{p_k}I_n\Big]$, then $-\bar{a}(t) + \bar{b}(t) = \frac{1}{\gamma}c(t)\Big[\gamma^2\lambda_{\max}\big(\alpha G - U(t) + G^TG\big) + \gamma^2\beta^2 + 2\lambda_{\max}\big(\bar{G}^T\bar{G}\big) + \alpha^2 + \beta^2\Big] + \bar{\lambda}$. where $\bar{\lambda} = \lambda_{\max}\big\{+\hat{D}_iP^{-1} + 3P + (\mu(A)+\mu(B))L^2P^{-1}\big\}$, for $i = 1, 2, \ldots, N$.

Since $\dot{c}(t)$ is nonnegative, that is, $c(t)$ is monotone nondecreasing, then the value of $c(t)$ will either be bounded or monotone nondecreasing to infinite.

Case 1: There exists a $t^* > 0$, such that $c(t^*) > c^*$, where $c^* > 0$ is a scalar and satisfying

$$\begin{aligned}c^* > \gamma(\bar{\delta} - \bar{\lambda}_i)\Big[\gamma^2\lambda_{\max}\big(\alpha G - U(t) + G^TG + \frac{2}{\gamma^2}\bar{G}^T\bar{G}\big) \\ + \gamma^2\beta^2 + \lambda_{\max}\big(\bar{G}^T\bar{G}\big) + \alpha^2 + \beta^2\Big]^{-1}.\end{aligned}$$

Then according to (26), it can be easily obtained that

$$-\bar{a}(t) + \bar{b}(t) < 0.$$

where $\mu = \inf_{t \ge t_0}\{\mu(t) : \mu(t) - \bar{a}(t) + \bar{b}(t)e^{\mu(t)\tau(t)} = 0\}$.

With the similar analysis with Theorem 1, it can derive that the error system (25) will converge exponentially to the set

$$E = \left\{ e_i \in \mathbb{R}^n, i = 1, \ldots, N \left| \sum_{i=1}^{N} \|e_i\| \le \frac{1}{\delta p_{\min}} \sum_{i=1}^{N} \omega_i^2 \right.\right\}.$$

Case 2: For all $t > t^*$, $c(t) < c^*$. In this case, one has

$$\int_{t_0}^{+\infty} \sum_{i=1}^{N} e_i^T(t) e_i(t) dt < \infty.$$

Obviously, $\sum_{i=1}^{N} e_i^T(t) e_i(t) dt \to 0, t \to \infty$.

In conclusion, the error system (25) will converge exponentially to the set E. The proof is completed. □

Remark 4: It is clear that it is the memristive mechanism that attributes to the mismatch system matrices. Using pinning strategies and the above mathematical methods, the effects of mismatch system matrices can not be eliminated, that is, the memristive neural networks are only able to achieve quasi-synchronization due to mismatch system matrices. From the analysis process in the above main results, it is easy to find that the larger the couple strengths and pinning control strength are, the smaller the error level is. In addition, if more nodes in the neural networks are controlled, it will offer more advantages in condensing the error level. However, according to the mathematical analysis of the properties of the memristive neural network systems and Lyaponuv functions, it can be found that it is impossible the synchronize the drive-response systems with such kind of pinning strategies since the synaptic connection matrices $A(x_i(t))$ and $B(x_i(t))$ are not coupled. Nevertheless, if all the nodes can be controlled, then applying the control strategy shown in (28), the memristive neural systems (2) and (8) can realize exponential synchronization.

Corollary 3: Suppose that $H(1) - H(4)$ hold. With the control strategy design as following,

$$U_i(t) = -u_i(t)e_i(t) - sgn(e_i(t))\eta_i(t), \tag{28}$$

where $\dot{\eta}_i(t) = |e_i(t)|, i = 1, 2, \ldots, N$, the drive memristive dynamic network (2) is said to be globally exponentially synchronized with the response dynamic network (6) if there exist a positive constant β and the next conditions are satisfied:

$$c_1\alpha G - c_1 U(t) + c_1 G^T G + c_2 \bar{G}^T \bar{G} + (c_1\beta^2 + c_2\alpha^2) I_n + \frac{e^{\sigma}\tau}{p_{\min}} Q + \bar{\lambda} I_n \le \mathbf{0}, \tag{29}$$

where $\bar{\lambda} = \lambda_{\max}\{-\hat{D}_i P^{-1} + 3P + (\mu(A) + \mu(B)) L^2 P^{-1}\}$.

Proof: Construct Lyapunov function

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t) P e_i(t) + \frac{1}{2}\sum_{i=1}^{N} (\eta_i - \eta_i(t))^T (\eta_i - \eta_i(t)),$$

where $\eta_i = \|(\rho(A) + \rho(B)) M_i + \bar{D}_i T_i\|_2$.

Differentiating $V_1(t)$ and $V_2(t)$ along the solution of the error system (III), it yieds that

$$\begin{aligned}
\dot{V}_1(t) \le& \sum_{i=1}^{N} e_i^T(t) P e_i(t) + \sum_{i=1}^{N} e_i^T(t) P \dot{e}_i(t) \\
&- \sum_{i=1}^{N} (\eta_i - \eta_i(t))^T \dot{\eta}_i(t) \\
\le& \sum_{i=1}^{N} e_i^T(t) P \Big[\hat{D} e_i(t) + sgn(e_i^T(t)) \bar{D}_i T_i \\
&+ \hat{A}_i(t) \tilde{f}(e_i(t)) + \hat{B}_i(t) \tilde{f}(e_i(t - \tau(t))) \\
&+ \Delta\hat{A}_i(t) f(x_i(t)) + \Delta\hat{B}_i(t) f(x_i(t - \tau(t))) \\
&+ u_i(t) e_i(t) - sgn(e_i(t)) \eta_i(t) \Big] \\
&+ c_1 \sum_{i=1}^{N}\sum_{j=1}^{N} g_{ij} e_i^T(t) P \tilde{g}(e_j(t)) \\
&+ c_2 \sum_{i=1}^{N}\sum_{j=1}^{N} \bar{g}_{ij} e_i^T(t) P \tilde{g}(e_j(t - \tau(t))) \\
&- \sum_{i=1}^{N} (\eta_i - \eta(t))^T |e_i(t)| \\
\le& \sum_{i=1}^{N} e_i^T(t) (\sigma I_n - \hat{D}_i P + 2P^T P + \mu(A) L^2) e_i(t) \\
&+ \sum_{i=1}^{N} e_i^T(t - \tau(t)) \mu(B) L^2 e_i(t - \tau(t)) \\
&+ \sum_{i=1}^{N} |e_i(t)|^T \Big[P\big(|\Delta\hat{A}_i(t) + \Delta\hat{B}_i(t)|\big) M_i \\
&+ P\bar{D}_i T_i - \eta_i \Big] \\
&+ \sum_{k=1}^{n} p_k \tilde{e}_k^T(t) [c_1\alpha G - c_1 U(t) + c_1 G^T G + c_2 \bar{G}^T \bar{G} \\
&+ (c_1\beta^2 + c_2\alpha^2) I_n] \tilde{e}_k(t) \\
&+ \sum_{k=1}^{n} p_k \tilde{e}_k^T(t - \tau(t)) (c_2 \bar{G}^T \bar{G} + c_2 \beta^2 I_n) \tilde{e}_k(t - \tau(t)).
\end{aligned} \tag{30}$$

Since $\eta_i = \|(\rho(A) + \rho(B)) M_i + \bar{D}_i T_i\|_2$, one has

$$\begin{aligned}
\dot{V}(t) \le& \sum_{k=1}^{n} p_k \tilde{e}_k^T(t) [c_1 \alpha \tilde{G} - c_1 U(t) + c_1 G^T G \\
&+ c_2 \bar{G}^T \bar{G} + \bar{\lambda} I_n] \tilde{e}_k(t) + \sum_{k=1}^{n} p_k \tilde{e}_k^T(t - \tau(t)) \\
&\times [c_2 \bar{G}^T \bar{G} + \big(c_2\beta^2 + \mu(B) \frac{l_k^2}{p_k}\big) I_n] \tilde{e}_k(t - \tau(t)) \\
\le& -a(t) V(t) + b(t) V(t - \tau(t)).
\end{aligned} \tag{31}$$

Using Lemma 3 the condition (29), it can be obtained

$$V(t) \leq \sup_{-\infty \leq s \leq t_0} V(s) e^{-\mu(t-t_0)}, \tag{32}$$

where $\mu = \inf_{t \geq t_0}\{\mu(t) : \mu(t) - a(t) + b(t)e^{\mu(t)\tau(t)} = 0\}$.

Therefore, $\dot{V}(t) \leq 0$, which means that $V(t) \leq V(0)$ for all $t \geq 0$. What's more, it can be easily obtained that $e^{\beta t}\sum_{i=1}^{N} \| e_i(t) \|^2 \leq V(t)$, and

$$\begin{aligned} V(0) &= \sum_{i=1}^{N} e_i^T(0) e_i(0) \\ &\quad + \sum_{i=1}^{N} (\eta_i - \eta_i(0))^T (\eta_i - \eta_i(0)) \\ &\quad + \sum_{i=1}^{N} \int_{-\tau(0)}^{0} e^{\sigma(s+\tau(0))} q_i e_i^T(s) e_i(s) ds \\ &\leq \sum_{i=1}^{N} \|e_i(0)\|^2 \\ &\quad + q_{\max} \sum_{i=1}^{N} \int_{0-\tau(0)}^{0} e^{\sigma(s+\tau(0))} e_i^T(s) e_i(s) ds \\ &\leq \sum_{i=1}^{N} \left[1 + \sigma^{-1} q_{\max}(e^{\sigma\tau(0)} - 1)\right] \sup_{\tau(0) \leq \vartheta \leq 0} \|e_i(\vartheta)\|^2. \end{aligned} \tag{33}$$

Therefore,

$$\begin{aligned} e^{\sigma t} \sum_{i=1}^{N} \| e_i(t) \|^2 &\leq V(t) \\ &\leq \sum_{i=1}^{N} \left[1 + \sigma^{-1} q_{\max}(e^{\sigma\tau(0)} - 1)\right] \sup_{\tau(0) \leq \vartheta \leq 0} \|e_i(\vartheta)\|^2 \end{aligned}$$

and

$$\sum_{i=1}^{N} \|e_i(t)\|^2 \leq \sum_{i=1}^{N} \omega \sup_{\tau(0) \leq \vartheta \leq 0} \|e_i(\vartheta)\| e^{-\sigma t}, t \geq 0$$

where $\omega = 1 + \sigma^{-1} q_{\max}(e^{\sigma\tau(0)} - 1)$. The proof is completed. □

IV. NUMERICAL EXAMPLES

Example 1: In this section, an numerical example to demonstrate the main result of theorem 1 will be given. Consider CMNNS with 5 nodes and the follows parameters: $c_1 = c_2 = 1, s_i = 0, \tau(t) = e^t/(1 + e^t)$

$$D_i(t) = \begin{bmatrix} d_{i1}(t) & 0 \\ 0 & d_{i2}(t) \end{bmatrix}, \quad A_i(t) = \begin{bmatrix} a_{11}^i(t) & a_{12}^i(t) \\ a_{21}^i(t) & a_{22}^i(t) \end{bmatrix}$$

$$B_i(t) = \begin{bmatrix} b_{11}^i(t) & b_{12}^i(t) \\ b_{21}^i(t) & b_{22}^i(t) \end{bmatrix}$$

For node i, we assume that

$$d_{i1}(t)) = \begin{cases} 0.1, & |x_{i1}(t)| \leq 1.5, \\ 0.2, & |x_{i1}(t)| > 1.5, \end{cases}$$

$$d_{i2}(t)) = \begin{cases} 0.1, & |x_{i2}(t)| \leq 1.5, \\ 0.2, & |x_{i2}(t)| > 1.5, \end{cases}$$

$$a_{11}^i(t) = \begin{cases} 2, & |x_{i1}(t)| \leq 1.5, \\ -1.5, & |x_{i1}(t)| > 1.5, \end{cases}$$

$$a_{12}^i(t) = \begin{cases} -1, & |x_{i1}(t)| \leq 1.5, \\ -2, & |x_{i1}(t)| > 1.5, \end{cases}$$

$$a_{21}^i(t) = \begin{cases} 3.1, & |x_{i2}(t)| \leq 1.5, \\ -1.4, & |x_{i2}(t)| > 1.5, \end{cases}$$

$$a_{22}^i(t) = \begin{cases} -2.3, & |x_{i2}(t)| \leq 1.5, \\ 3.2, & |x_{i2}(t)| > 1.5, \end{cases}$$

$$b_{11}^i(t) = \begin{cases} -2.2, & |x_{i1}(t)| \leq 1.5, \\ -1.5, & |x_{i1}(t)| > 1.5, \end{cases}$$

$$b_{12}^i(t) = \begin{cases} -1.4, & |x_{i1}(t)| \leq 1.5, \\ -2.3, & |x_{i1}(t)| > 1.5, \end{cases}$$

$$b_{21}^i(t) = \begin{cases} -2, & |x_{i2}(t)| \leq 1.5, \\ -4, & |x_{i2}(t)| > 1.5, \end{cases}$$

$$b_{22}^i(t) = \begin{cases} -3.3, & |x_{i2}(t)| \leq 1.5, \\ 2.7, & |x_{i2}(t)| > 1.5, \end{cases}$$

$$G = \bar{G} = \begin{bmatrix} -3.62 & 1.56 & 0.35 & 0.98 & 0.73 \\ 1.56 & -4.28 & 1.12 & 0.92 & 0.68 \\ 0.35 & 1.12 & -2.57 & 0.70 & 0.40 \\ 0.98 & 0.92 & 0.70 & -4.16 & 1.56 \\ 0.73 & 0.68 & 0.40 & 1.56 & -3.37 \end{bmatrix}$$

The active functions and coupling function are $f_k(t) = \tanh(t)$ and $g_k(t) = \sin(t) + t/10, k = 1, 2$. The initial functions of the active and response memristive neural networks are

$$\begin{aligned} x1(t) &= 5 * [\sin(t/4); \cos(t/4)]; \\ x2(t) &= 5 * [\sin(t/4 + 1); \cos(t/4 + 1)]; \\ x3(t) &= 5 * [\sin(t/4.5 + 2); \cos(t/3 + 2)]; \\ x4(t) &= 5 * [2 * \sin(t/5 + 3); \cos(t/5 + 3)]; \\ x5(t) &= 5 * [\sin(t/3 + 4); \cos(t/3 + 4)]; \\ y1(t) &= 5 * [\cos(t/4 + 5); \cos(t/4 + 1)]; \\ y2(t) &= 5 * [2 * \cos(t/4 + 8); \sin(t/3 + 1)]; \\ y3(t) &= 5 * [\cos(t/4.5 + 2); \cos(t/5 + 2)]; \\ y4(t) &= 5 * [\cos(t/5); \cos(t/4 + 3)]; \\ y5(t) &= 5 * [\sin(t/3 + 5); \sin(t/3 + 4)]; \end{aligned}$$

from which one can calculate the norms: $\mu(A) = 3.9266, \mu(B) = 4.8296$, Suppose that the assumptions $(H1) - (H4)$ are satisfied with $l_1 = l_2 = 1, \alpha = 0.1, \beta = 1, \tau = 1$. Choose $u_1 = 15, u_i = 0, i = 2, \ldots, 5$, it is obtained that the the drive and response CMNNs can be exponentially synchronized by controlling a single first node. In the simulations, Figure 1 presents the error trajectories of the first states of the five nodes in the CMNNs with different initial values. Figure 2 presents the error trajectories of the second states of the five nodes in the CMNNs. It can be seen that

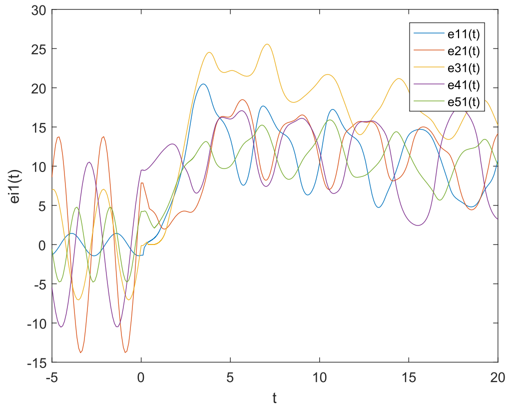

FIGURE 1. Time response of the error variables' modulus.

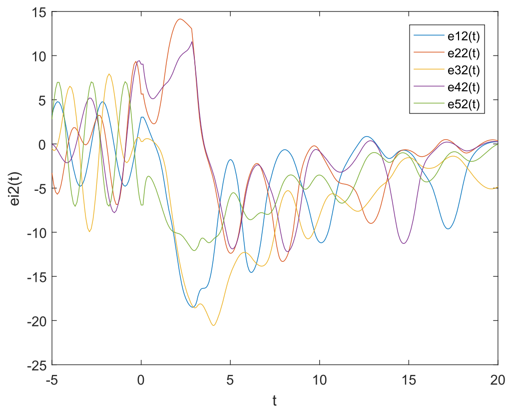

FIGURE 2. Time response of the error variables' modulus.

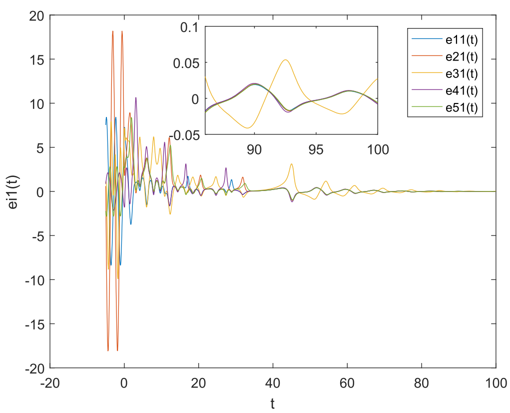

FIGURE 3. Time response of the error variables' modulus.

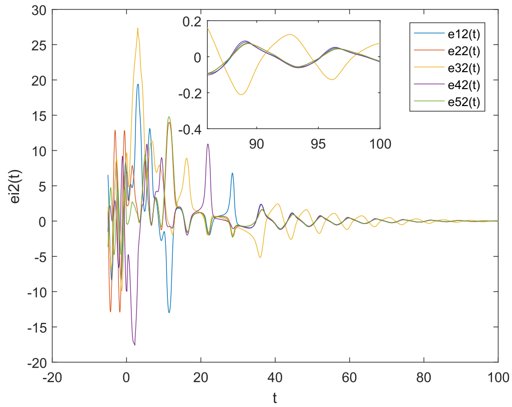

FIGURE 4. Time response of the error variables' modulus.

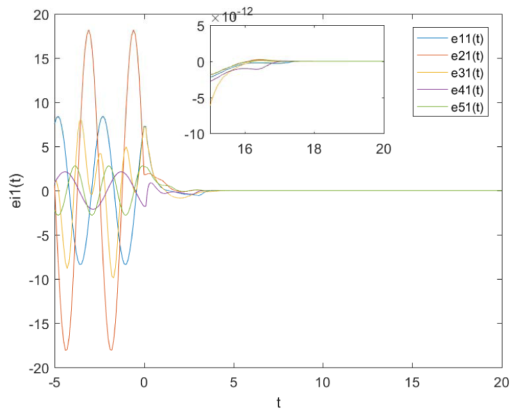

FIGURE 5. Time response of the error variables' modulus.

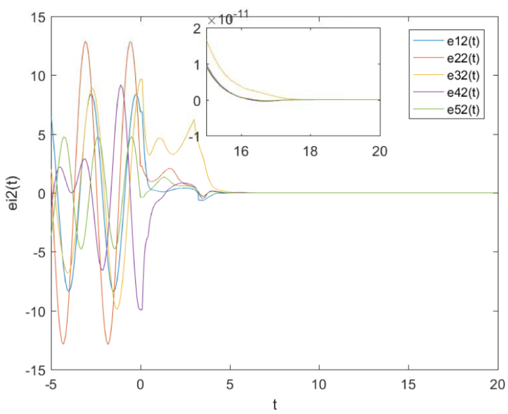

FIGURE 6. Time response of the error variables' modulus.

the trajectories is highly dependent on its initial values and it cannot realize synchronization spontaneously. After applying control to the first node in the response neural networks, the two memristive networks reach synchronization,which is shown in Figure 3 and Figure 4.

Example 2: In this example all the nodes will be controlled and the control strengths are also chosen as $u_i = 15, i = 1, \ldots, 5$. With the same parameters, initial conditions and memristive mechanism as Example 1 and choosing $p_1 = p_2 = 0.2$, eventually the error system converges to zero, which means the drive CMNNs and response CMNNs finally realize synchronization, and the figures are shown in Figure 5 and Figure 6.

V. CONCLUSION

In this paper, the model of a system of nonlinearly and diffusively coupled memristive neural networks (CMNNs) with time-varying delays is formulated, and its dynamics behaviors are then studied. First this class of CMNNs is transferred into traditional neural networks with mismatched parameters. Next by using the Lyapunov function and pinning control approach and referring to some lemmas and existing works, a sufficient condition for quasi-synchronization of drive-response CMNNs with time delays is derived. Furthermore, the main result is expanded by taking into account the another case of coupling function, coupling strength as well as the number of controlled nodes. However, thinking about the control approach in this paper, though it is more economic and practical than other approaches, it still has to control the objective nodes all the time. Hence, we will try to combine pinning control strategy with other feasible approach such as intermittent control or event trigger method, and more control strategies will be studies to compare their validity and practicability for this class of CMNNs. In addition, in the existing works on CMNNs, only the segmentation model of the meristor is considered, while there are many other models in neuroscience. Therefore, in future we will continue to adapt other models into the neural networks and analyze their properties.

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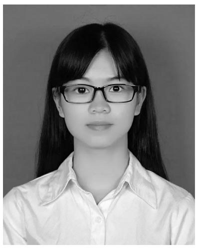

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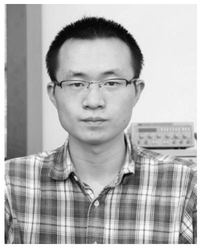

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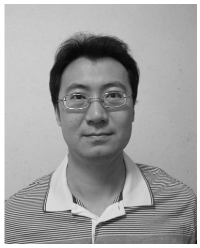

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