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Availability of Parallel Triplicated Redundancy Models With Imperfect Switchovers and Interrupted Repairs

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ABSTRACT This paper presents the analysis of the three parallel triplicated redundancy models: model with one active and two standby components, model with one standby and two active components, and model with three active components. The time-to-failure and the time-to-repair of the components follow an exponential and a general distribution, respectively. The repairs of failed components are randomly interrupted. The time-to-interrupt is taken from an exponentially distributed random variable and the interrupt times are generally distributed. Using the supplementary variable method and integro-differential equations, we obtain the analytical expression of the availability for the redundancy models with imperfect switchovers and interrupted repairs. Numerical examples show the effect of failure rate of active components, repair interruption rate, and switchover failure probability on the steady-state availability. The triplicated redundancy model with one active component and two standby components has higher availability than the other triplicated models with more active components when the switchover failure probability is small.

INDEX TERMS Availability, parallel triplicated redundancy, imperfect switchover, interrupted repair.

I. INTRODUCTION

Availability is generally defined as the probability that a system is operational at a given point in time under a given set of environmental conditions. High availability refers to a system or component that is continuously operational for a desirably long length of time and is becoming a must in various fields such as computer, telecommunication, power plant, industrial, and manufacturing systems [1]–[9]. Thus, there have been many efforts to achieve high system availability.

Redundancy is typically used to improve availability. There are various redundancy models. Different redundancy models may offer different levels of availability to the service being provided [10]. When an active component fails, the workload is switched over from the failed component to a standby one [1]. The switchover process of a standby component can also be error-prone [11]. Ardakan and Hamadani [12] and Ardakan *et al.* [13] considered a redundancy model with multiple active and multiple standby components. However, they did not consider repair of failed components. In realistic environments both the component failures and the switchover errors are fixed by a repairer and the repairer is possible to become unavailable when it is repairing. Therefore, considering interrupted repairs in redundancy models is also practical and imperative.

The availability analysis of a system is based on analyzing the various states that the system undergoes during its life cycle. The analysis mainly focuses on capturing the failures that cause the system to switch to a faulty state and the repairs that shift the system back to a healthy state. Since the occurrence of failures is erratic by nature, stochastic models have been used to conduct the availability analysis. Markovian models have been extensively used for this purpose because of their expressiveness and their capability of capturing the complexity of real systems [14]–[17]. A major problem of using Markovian models is that a large number of states are required to represent the model accurately [14]. As an alternative, Kanso *et al.* [10] used Stochastic Reward Nets (SRNs) to model various redundant systems and evaluated the availability measures by using the analytic-numeric methods of the Stochastic Petri Net Package (SPNP) tool. Kim *et al.* [18] analyzed the networking service availability of $2N$ redundancy model with non-stop forwarding by using the SPNP. The analytic-numeric methods of SPNP provide the capabilities of solving the Markovian SRNs but fail for non-Markovian SRNs. There is no reason to assume that the repair time has an exponential distribution. Kuznetsov [19] evaluated the availability of repairable networks with general repair time distribution by fast simulation method. In this paper, we consider an analytic method to evaluate the availability of non-Markovian redundancy models with general repair time distribution.

For the concept of the standby switching failures, Lewis [11] first introduced it in the availability with standby system. Wang *et al.* [20] studied the availability of four different repairable systems with standby components and standby switching failures. Ke *et al.* [21] provided a Laplace transform method for developing the system probability and studied the availability of a Markovian repairable system with switching failures. Hsu *et al.* [22] investigated the profit analysis of a repairable system with switching failures. Sadjadi and Soltani [23] considered a series-parallel system with the choice of redundancy strategy, in which the switching from the standby components to the active components is imperfect. In the above-mentioned works [11], [20]–[23], it was assumed that time-to-failure and time-to-repair of the components are exponentially distributed. However, the assumption limits its use for solving real problems. In this paper, we assume that they are generally distributed.

Even though many studies have focused on uninterrupted repairs with exponentially distributed repair time, there has been very little research reported on redundancy models with interrupted repairs and generally distributed repair time. Lee [24] analyse the availability of a simple 1 + 1 redundancy model with one active and one standby component. Kuo and Ke [25] and Lee [26] studied the steady-state availability of a series system with switching failures, interrupted repairs, and generally distributed repair time. However, they did not distinguish between the repairs of the component failures and the switchover errors. Bosse *et al.* [27] estimated the availability of a redundancy model with imperfect switchovers and interrupted repairs by using a Petri net Monte Carlo simulation.

In this paper, we focus on the analytical expression of the avaiability for parallel triplicated redundancy models with imperfect switchovers, generally distributed repair times, and interrupted repairs. To obtain the analytical expression of the availability, we use supplementary variable method and integro-differential equations governing the steady-state behavior of the models. Through numerical examples we present the effect of failure rate of active components, repair interruption rate, and switchover failure probability on the steady-state availability.

II. MODELS

We discuss three different redundancy models: Model 1 with one active and two standby components, Model 2 with one standby and two active components, and Model 3 with three active components (Table 1). The models has many real applications: a network device, a server designed with multiple power supplies, a bank website deployed to a cloud platform, and a factory having multiple industrial robots.

TABLE 1. Different models.

Model	Components
Model 1	1 active component and 2 standby components
Model 2	2 active components and 1 standby component
Model 3	3 active components

The active components operate normally and the standby components are ready to assume the active role should the active components fail. The likelihood of a component failing is independent of the state of the other components. It is assumed that the time-to-failure of the active and the standby components follows exponential distributions with rate λ and μ, respectively. Whenever an active or a standby component fails, it is repaired immediately by a repairer. The repair time X is generally distributed with probability density function (PDF) $f(x)$ and cumulative distribution function (CDF) $F(x)$. In Model 1 and 2, when an active component fails, a standby, if any is available, automatically takes over system operations with neglibile switchover time. However, the automatic switchover from standby to active may fail due to hardware or software issues. The automatic switchover is assumed to fail with probability p. In this case, the repairer first switchs over non-automatically a standby component to active, then repairs the failed component. The non-automatic switchover time Y is generally distributed with PDF $g(y)$ and CDF $G(y)$. Moreover, the repairer may function wrongly or fail sometimes with an exponential failure rate δ in its busy period, i.e., when the repairer is repairing a failed component or switching over non-automatically a standby component. When the repairer is not available, its repair or non-automatic switchover process is interrupted. Once the repairer becomes available again, it resumes the interrupted process. The interrupted time Z is generally distributed with PDF $h(z)$ and CDF $H(z)$.

For mathematical analysis, we define the following supplementary variables: the random process $X_-(t)$ denotes the amount of repair time already received by a failed component in repair at time t. We sometimes call $X_-(t)$ the elapsed repair time. The random processes $Y_-(t)$ and $Z_-(t)$ denote the elapsed non-automatic switchover time and the elapsed interrupted time, respectively, at time t. We also introduce

$$\alpha(x) \equiv \frac{f(x)}{1-F(x)}, \quad \beta(y) \equiv \frac{g(y)}{1-G(y)}, \quad \gamma(z) \equiv \frac{h(z)}{1-H(z)}.$$

The function $\alpha(x)$ is a PDF for the repair time X on condition that $X > x$: $\alpha(x)dx = \mathrm{P}\{x < X < x + dx \,|X > x\}$. Note that $\alpha(x)$ is called the hazard rate or the age-specific failure rate in renewal theory. The functions $\beta(y)$ and $\gamma(z)$ are the hazard rates of the random variables Y and Z, respectively: $\beta(y)dy = P\{y < Y < y + dy \,|Y > y\}$ and $\gamma(z)dz = \mathrm{P}\{z < Z < z + dz \,|Z > z\}$. Throughout this paper, $b^*(s)$ is the Laplace transform of a function $b(t)$.

III. STEADY STATE AVAILABILITY FOR MODEL 1

First, we consider Model 1. Let $N(t)$ and $M(t)$ be the state of three components and the state of the repairer, respectively, at time t:

$$N(t) = \begin{cases} 0 & \text{if there are 3 failed components at time } t, \\ 1 & \text{if there are 1 standby component} \\ & \qquad \text{and 2 failed components at time } t, \\ 2 & \text{if there are 2 standby components} \\ & \qquad \text{and 1 failed component at time } t, \\ 3 & \text{if there are 1 active component} \\ & \qquad \text{and 2 failed components at time } t, \\ 4 & \text{if there are 1 active, 1 standy,} \\ & \qquad \text{and 1 failed component at time } t, \\ 5 & \text{if there are 1 active component} \\ & \qquad \text{and 2 standby components at time } t, \end{cases}$$

$$M(t) = \begin{cases} 0 & \text{if the repairer is idle at time} t, \\ 1 & \text{if the repairer is busy at time} t, \\ 2 & \text{if the repairer is failed at time} t. \end{cases}$$

Note that when $N(t) = 0$ the system is unavailable and the repairer, if available, is repairing one of the three failed components; when $N(t) = 1$ the system is unavailable and the repairer, if available, is switching over non-automatically the standby component to active; when $N(t) = 2$ the system is unavailable and the repairer, if available, is switching over non-automatically one of the two standby components to active; when $N(t) = 3$ the system is available and the repairer, if available, is repairing one of the two failed components; when $N(t) = 4$ the system is available and the repairer, if available, is repairing the failed component; and when $N(t) = 5$ the system is available and the repairer is idle. Let us define

$$\begin{aligned} &P_n(x,z)dxdz \\ &\quad \equiv \lim_{t\to\infty} P\{N(t){=}n, M(t){=}2, x < X_-(t) < x{+}dx, \\ &\qquad z < Z_-(t) < z+dz\}, \quad n = 0, 3, 4, \\ &P_n(x,z)dxdz \\ &\quad \equiv \lim_{t\to\infty} P\{N(t){=}n, M(t){=}2, x < Y_-(t) < x+dx, \\ &\qquad z < Z_-(t) < z+dz\}, \quad n = 1, 2, \\ Q_n(x)dx &\equiv \lim_{t\to\infty} P\{N(t) = n, M(t) = 1, \\ &\qquad x < X_-(t) < x+dx\}, \quad n = 0, 3, 4, \\ Q_n(x)dx &\equiv \lim_{t\to\infty} P\{N(t) = n, M(t) = 1, \\ &\qquad x < Y_-(t) < x+dx\}, \quad n = 1, 2, \\ Q_5 &\equiv \lim_{t\to\infty} P\{N(t) = 5, M(t) = 0\}, \\ P_n &\equiv \int_0^\infty \int_0^\infty P_n(x,z)dxdz, \quad n = 0, 1, 2, 3, 4, \\ Q_n &\equiv \int_0^\infty Q_n(x)dx, \quad n = 0, 1, 2, 3, 4. \end{aligned}$$

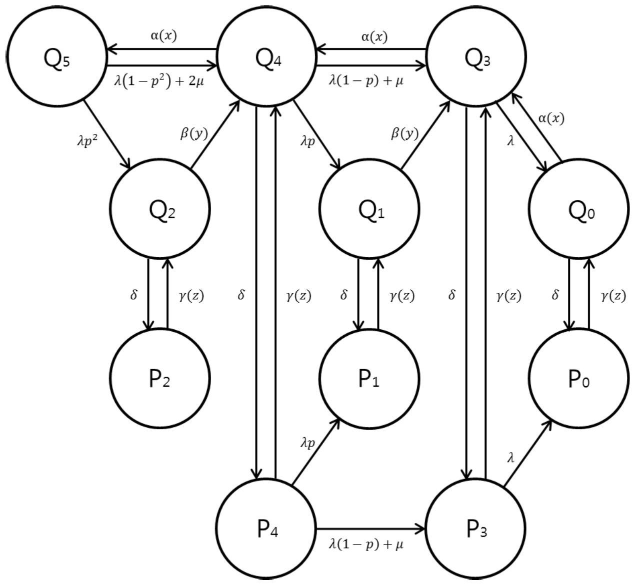

FIGURE 1. State transition diagram of Model 1.

We construct the following integro-differential equations governing the steady-state behavior of the system (Fig. 1):

$$\frac{dP_0(x,z)}{dz} = -\gamma(z)P_0(x,z) + \lambda P_3(x,z), \tag{1}$$

$$\frac{dP_1(x,z)}{dz} = -\gamma(z)P_1(x,z), \tag{2}$$

$$\frac{dP_1(0,z)}{dz} = -\gamma(z)P_1(0,z) + \lambda p \int_0^\infty P_4(x,z)dx, \tag{3}$$

$$\frac{dP_2(x,z)}{dz} = -\gamma(z)P_2(x,z), \tag{4}$$

$$\begin{aligned} \frac{dP_3(x,z)}{dz} &= -\left[\lambda + \gamma(z)\right] P_3(x,z) \\ &\quad + \left[\lambda(1-p) + \mu\right] P_4(x,z), \end{aligned} \tag{5}$$

$$\frac{dP_4(x,z)}{dz} = -\left[\lambda + \mu + \gamma(z)\right] P_4(x,z), \tag{6}$$

$$\begin{aligned} \frac{dQ_0(x)}{dx} &= -\left[\delta + \alpha(x)\right] Q_0(x) + \lambda Q_3(x) \\ &\quad + \int_0^\infty \gamma(z)P_0(x,z)dz, \end{aligned} \tag{7}$$

$$\frac{dQ_1(x)}{dx} = -\left[\delta{+}\beta(x)\right] Q_1(x) + \int_0^\infty \gamma(z)P_1(x,z)dz, \tag{8}$$

$$\frac{dQ_2(x)}{dx} = -\left[\delta{+}\beta(x)\right] Q_2(x) + \int_0^\infty \gamma(z)P_2(x,z)dz, \tag{9}$$

$$\begin{aligned} \frac{dQ_3(x)}{dx} &= -\left[\lambda{+}\delta{+}\alpha(x)\right] Q_3(x) + \left[\lambda(1{-}p) + \mu\right] Q_4(x) \\ &\quad + \int_0^\infty \gamma(z)P_3(x,z)dz, \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{dQ_4(x)}{dx} &= -\left[\lambda + \mu + \delta + \alpha(x)\right] Q_4(x) \\ &\quad + \int_0^\infty \gamma(z)P_4(x,z)dz, \end{aligned} \tag{11}$$

$$0 = -\left(\lambda + 2\mu\right) Q_5 + \int_0^\infty \alpha(x)Q_4(x)dx \tag{12}$$

with boundary conditions

$$P_1(0,0)=0, \tag{13}$$
$$P_n(x,0)=\delta Q_n(x), \quad n=0,1,2,3,4, \tag{14}$$
$$Q_0(0)=0, \tag{15}$$
$$Q_1(0)=\lambda p\int_0^\infty Q_4(x)dx+\int_0^\infty \gamma(z)P_1(0,z)dz, \tag{16}$$
$$Q_2(0)=\lambda p^2 Q_5, \tag{17}$$
$$Q_3(0)=\int_0^\infty \beta(x)Q_1(x)dx+\int_0^\infty \alpha(x)Q_0(x)dx, \tag{18}$$
$$Q_4(0)=\left[\lambda(1-p^2)+2\mu\right]Q_5+\int_0^\infty \alpha(x)Q_3(x)dx+\int_0^\infty \beta(x)Q_2(x)dx. \tag{19}$$

Solving the above integro-differential equations (1)-(6) with boundary condition (13) and (14), we obtain

$$P_4(x,z)=\delta e^{-(\lambda+\mu)z}\bar{H}(z)Q_4(x), \tag{20}$$
$$P_3(x,z)=\delta e^{-\lambda z}\bar{H}(z)Q_3(x)+\frac{\lambda(1-p)+\mu}{\mu}\times\delta\left[e^{-\lambda z}-e^{-(\lambda+\mu)z}\right]\bar{H}(z)Q_4(x), \tag{21}$$
$$P_2(x,z)=\delta\bar{H}(z)Q_2(x), \tag{22}$$
$$P_1(x,z)=\delta\bar{H}(z)Q_1(x), \tag{23}$$
$$P_1(0,z)=\frac{\lambda p}{\lambda+\mu}\delta\left[1-e^{-(\lambda+\mu)z}\right]\bar{H}(z)\int_0^\infty Q_4(x)dx, \tag{24}$$
$$P_0(x,z)=\delta\bar{H}(z)Q_0(x)+\delta\left(1-e^{-\lambda z}\right)\bar{H}(z)Q_3(x)+\frac{\lambda(1-p)+\mu}{\mu}\lambda\delta\left[\frac{1-e^{-\lambda z}}{\lambda}-\frac{1-e^{-(\lambda+\mu)z}}{\lambda+\mu}\right]\times\bar{H}(z)Q_4(x), \tag{25}$$

where $\bar{H}(z)\equiv 1-H(z)$. Substituting (20)-(25) into (7)-(11), we get

$$Q_4(x)=e^{-C_{\lambda+\mu}x}\bar{F}(x)Q_4(0), \tag{26}$$
$$Q_3(x)=e^{-C_\lambda x}\bar{F}(x)Q_3(0)+\frac{\lambda(1-p)+\mu}{\mu}\times\left(e^{-C_\lambda x}-e^{-C_{\lambda+\mu}x}\right)\bar{F}(x)Q_4(0), \tag{27}$$
$$Q_2(x)=\bar{G}(x)Q_2(0), \tag{28}$$
$$Q_1(x)=\bar{G}(x)Q_1(0), \tag{29}$$
$$Q_0(x)=\left(1-e^{-C_\lambda x}\right)\bar{F}(x)\left[Q_3(0)+\frac{\lambda(1-p)+\mu}{\mu}Q_4(0)\right]-\frac{\lambda(1-p)+\mu}{\mu}\frac{\lambda}{\lambda+\mu}\left(1-e^{-C_{\lambda+\mu}x}\right)\bar{F}(x)Q_4(0), \tag{30}$$

where $\bar{F}(z)\equiv 1-F(z)$, $\bar{G}(z)\equiv 1-G(z)$, $C_\lambda\equiv\lambda+\delta-\delta h^*(\lambda)$, and $C_{\lambda+\mu}\equiv\lambda+\mu+\delta-\delta h^*(\lambda+\mu)$. From (12) and (26), we obtain

$$Q_5=\frac{f^*\left(C_{\lambda+\mu}\right)}{\lambda+2\mu}Q_4(0), \tag{31}$$

Substituting (24) and (26)-(31) into (16)-(18), we get

$$Q_3(0)=\frac{Q_4(0)}{f^*\left(C_\lambda\right)}\left[\lambda p\left\{1+\delta h^*(\lambda+\mu)\right\}\bar{F}^*\left(C_{\lambda+\mu}\right)+\frac{\lambda(1-p)+\mu}{\mu}\left\{1-f^*\left(C_\lambda\right)-\frac{\lambda}{\lambda+\mu}\left(1-f^*\left(C_{\lambda+\mu}\right)\right)\right\}\right], \tag{32}$$
$$Q_2(0)=\lambda p^2\frac{f^*\left(C_{\lambda+\mu}\right)}{\lambda+2\mu}Q_4(0), \tag{33}$$
$$Q_1(0)=\lambda p\left[1+\delta h^*(\lambda+\mu)\right]\bar{F}^*\left(C_{\lambda+\mu}\right)Q_4(0). \tag{34}$$

From (20)-(34), $P_n(x,z)$, $n=0,1,2,3,4$, $Q_n(x)$, $n=0,1,2,3,4$, and Q_5 can be clearly expressed by $Q_4(0)$. Now we need to find the expression of $Q_4(0)$. After doing some manipulations, we obtain

$$Q_0=\left[E(X)-\bar{F}^*\left(C_\lambda\right)\right]\left[Q_3(0)+\frac{\lambda(1-p)+\mu}{\mu}Q_4(0)\right]-\frac{\lambda(1-p)+\mu}{\mu}\frac{\lambda}{\lambda+\mu}\left[E(X)-\bar{F}^*\left(C_{\lambda+\mu}\right)\right]Q_4(0),$$
$$Q_1=E(Y)\lambda p\left[1+\delta h^*(\lambda+\mu)\right]\bar{F}^*\left(C_{\lambda+\mu}\right)Q_4(0).$$
$$Q_2=E(Y)\lambda p^2\frac{f^*\left(C_{\lambda+\mu}\right)}{\lambda+2\mu}Q_4(0),$$
$$Q_3=\bar{F}^*\left(C_\lambda\right)Q_3(0)+\frac{\lambda(1-p)+\mu}{\mu}\left[\bar{F}^*\left(C_\lambda\right)-\bar{F}^*\left(C_{\lambda+\mu}\right)\right]Q_4(0),$$
$$Q_4=\bar{F}^*\left(C_{\lambda+\mu}\right)Q_4(0),$$
$$Q_5=\frac{f^*\left(C_{\lambda+\mu}\right)}{\lambda+2\mu}Q_4(0),$$
$$P_0=\delta E(Z)Q_0+\delta\left[E(Z)-\bar{H}^*(\lambda)\right]\left[Q_3+\frac{\lambda(1-p)+\mu}{\mu}Q_4\right]-\frac{\lambda(1-p)+\mu}{\mu}\frac{\lambda\delta}{\lambda+\mu}\left[E(Z)-\bar{H}^*(\lambda+\mu)\right]Q_4,$$
$$P_1=\delta E(Z)Q_1+\frac{\lambda p}{\lambda+\mu}\delta\left[E(Z)-\bar{H}^*(\lambda+\mu)\right]Q_4,$$
$$P_2=\delta E(Z)Q_2,$$
$$P_3=\delta\bar{H}^*(\lambda)Q_3+\frac{\lambda(1-p)+\mu}{\mu}\delta\left[\bar{H}^*(\lambda)-\bar{H}^*(\lambda+\mu)\right]Q_4,$$
$$P_4=\delta\bar{H}^*(\lambda+\mu)Q_4.$$

By the normalization condition $\sum_{n=0}^{4}(P_n+Q_n)+Q_5=1$ with (32), we obtain

$$\frac{1}{Q_4(0)}=\frac{f^*\left(C_{\lambda+\mu}\right)}{\lambda+2\mu}+[1+\delta\mathrm{E}(Z)]\left[\frac{\lambda(1-p)+\mu}{\lambda+\mu}E(X)+\frac{\lambda p}{\lambda+\mu}\left\{1+\delta\left(1-h^*(\lambda+\mu)\right)(E(X)+E(Y))\right\}\times\bar{F}^*\left(C_{\lambda+\mu}\right)+\frac{\lambda(1-p)+\mu}{\mu}\left\{1-f^*\left(C_\lambda\right)-\frac{\lambda}{\lambda+\mu}\left(1-f^*\left(C_{\lambda+\mu}\right)\right)\right\}E(X)+\frac{\lambda p^2}{\lambda+2\mu}f^*\left(C_{\lambda+\mu}\right)E(Y)\right],$$

from which $Q_4(0)$ is obtained. Thus, we obtain Q_n, $n=0,1,2,3,4,5$, and P_n, $n=0,1,2,3,4$. Then, the steady-state availability Av_1 of Model 1 can be obtained as

$$Av_1 = Q_3 + Q_4 + Q_5 + P_3 + P_4. \tag{35}$$

IV. STEADY STATE AVAILABILITY FOR MODEL 2

Next, we consider Model 2. Let $N(t)$ and $M(t)$ be the state of three components and the state of the repairer, respectively, at time t:

$$N(t) = \begin{cases} 0 & \text{if there are 3 failed components at time } t, \\ 1 & \text{if there are 1 standby component} \\ & \quad \text{and 2 failed components at time } t, \\ 2 & \text{if there are 1 active component} \\ & \quad \text{and 2 failed components at time } t, \\ 3 & \text{if there are 1 active, 1 standby,} \\ & \quad \text{and 1 failed component at time } t, \\ 4 & \text{if there are 2 active components} \\ & \quad \text{and 1 failed component at time } t, \\ 5 & \text{if there are 2 active components} \\ & \quad \text{and 1 standby component at time } t, \end{cases}$$

$$M(t) = \begin{cases} 0 & \text{if the repairer is idle at time } t, \\ 1 & \text{if the repairer is busy at time } t, \\ 2 & \text{if the repairer is failed at time } t. \end{cases}$$

Note that when $N(t) = 0$ the system is unavailable and the repairer, if available, is repairing one of the three failed components; when $N(t) = 1$ the system is unavailable and the repairer, if available, is switching over non-automatically the standby component to active; when $N(t) = 2$ the system is available and the repairer, if available, is repairing one of the two failed components; when $N(t) = 3$ the system is available and the repairer, if available, is switching over non-automatically the standby component to active; when $N(t) = 4$ the system is available and the repairer, if available, is repairing the failed component; and when $N(t) = 5$ the system is available and the repairer is idle. Let us define

$$\begin{aligned} &P_n(x,z)dxdz \\ &\quad \equiv \lim_{t\to\infty} P\{N(t)=n, M(t)=2, x<X_-(t)<x+dx, \\ &\qquad z<Z_-(t)<z+dz\}, \quad n=0,2,4, \\ &P_n(x,z)dxdz \\ &\quad \equiv \lim_{t\to\infty} P\{N(t)=n, M(t)=2, x<Y_-(t)<x+dx, \\ &\qquad z<Z_-(t)<z+dz\}, \quad n=1,3, \\ &Q_n(x)dx \equiv \lim_{t\to\infty} P\{N(t)=n, M(t)=1, \\ &\qquad x<X_-(t)<x+dx\}, \quad n=0,2,4, \\ &Q_n(x)dx \equiv \lim_{t\to\infty} P\{N(t)=n, M(t)=1, \\ &\qquad x<Y_-(t)<x+dx\}, \quad n=1,3, \\ &Q_5 \equiv \lim_{t\to\infty} P\{N(t)=5, M(t)=0\}, \\ &P_n \equiv \int_0^\infty \int_0^\infty P_n(x,z)dxdz, \quad n=0,1,2,3,4, \\ &Q_n \equiv \int_0^\infty Q_n(x)dx, \quad n=0,1,2,3,4. \end{aligned}$$

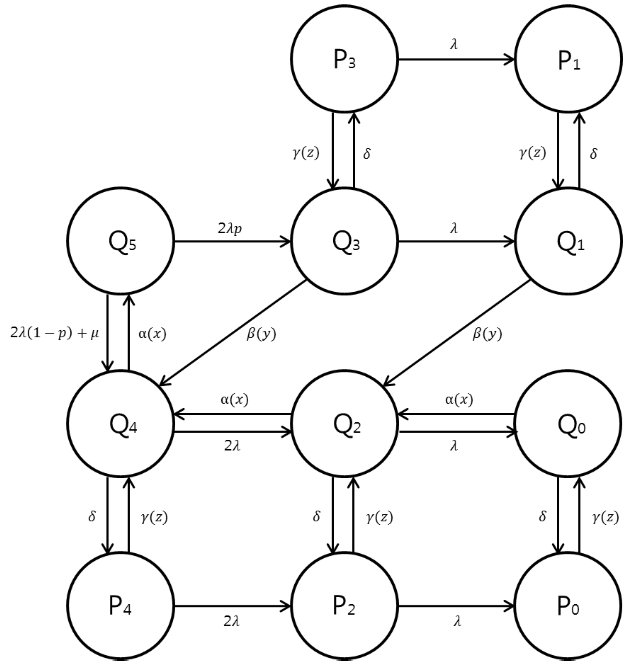

FIGURE 2. State transition diagram of Model 2.

We construct the following integro-differential equations governing the steady-state behavior of the system (Fig. 2):

$$\frac{dP_0(x,z)}{dz} = -\gamma(z)P_0(x,z) + \lambda P_2(x,z), \tag{36}$$

$$\frac{dP_1(x,z)}{dz} = -\gamma(z)P_1(x,z) + \lambda P_3(x,z), \tag{37}$$

$$\frac{dP_2(x,z)}{dz} = -\left[\lambda + \gamma(z)\right] P_2(x,z) + 2\lambda P_4(x,z), \tag{38}$$

$$\frac{dP_3(x,z)}{dz} = -\left[\lambda + \gamma(z)\right] P_3(x,z), \tag{39}$$

$$\frac{dP_4(x,z)}{dz} = -\left[2\lambda + \gamma(z)\right] P_4(x,z), \tag{40}$$

$$\frac{dQ_0(x)}{dx} = -\left[\delta + \alpha(x)\right] Q_0(x) + \lambda Q_2(x) + \int_0^\infty \gamma(z)P_0(x,z)dz, \tag{41}$$

$$\frac{dQ_1(x)}{dx} = -\left[\delta + \beta(x)\right] Q_1(x) + \lambda Q_3(x) + \int_0^\infty \gamma(z)P_1(x,z)dz, \tag{42}$$

$$\frac{dQ_2(x)}{dx} = -\left[\lambda + \delta + \alpha(x)\right] Q_2(x) + 2\lambda Q_4(x) + \int_0^\infty \gamma(z)P_2(x,z)dz, \tag{43}$$

$$\frac{dQ_3(x)}{dx} = -\left[\lambda + \delta + \beta(x)\right] Q_3(x) + \int_0^\infty \gamma(z)P_3(x,z)dz, \tag{44}$$

$$\frac{dQ_4(x)}{dx} = -\left[2\lambda + \delta + \alpha(x)\right] Q_4(x) + \int_0^\infty \gamma(z)P_4(x,z)dz, \tag{45}$$

$$0 = -(2\lambda + \mu) Q_5 + \int_0^\infty \alpha(x)Q_4(x)dx \tag{46}$$

with boundary conditions

$$P_n(x, 0) = \delta Q_n(x), \quad n = 0, 1, 2, 3, 4, \tag{47}$$
$$Q_0(0) = 0, \tag{48}$$
$$Q_1(0) = 0, \tag{49}$$
$$Q_2(0) = \int_0^\infty \beta(x)Q_1(x)dx + \int_0^\infty \alpha(x)Q_0(x)dx, \tag{50}$$
$$Q_3(0) = 2\lambda p Q_5, \tag{51}$$
$$Q_4(0) = [2\lambda(1-p) + \mu] Q_5 + \int_0^\infty \beta(x)Q_3(x)dx + \int_0^\infty \alpha(x)Q_2(x)dx. \tag{52}$$

Solving the above integro-differential equations (36)-(40) with boundary conditions (47), we obtain

$$P_4(x, z) = \delta e^{-2\lambda z}\bar{H}(z)Q_4(x), \tag{53}$$
$$P_3(x, z) = \delta e^{-\lambda z}\bar{H}(z)Q_3(x), \tag{54}$$
$$P_2(x, z) = \delta e^{-\lambda z}\bar{H}(z)Q_2(x) + 2\delta \left(e^{-\lambda z} - e^{-2\lambda z}\right) \bar{H}(z)Q_4(x), \tag{55}$$
$$P_1(x, z) = \delta\bar{H}(z)Q_1(x) + \delta \left(1 - e^{-\lambda z}\right) \bar{H}(z)Q_3(x), \tag{56}$$
$$P_0(x, z) = \delta\bar{H}(z) \left[Q_0(x) - \left(e^{-\lambda z} - e^{-2\lambda z}\right) Q_4(x) + \left(1 - e^{-\lambda z}\right) \{Q_2(x) + Q_4(x)\}\right]. \tag{57}$$

Substituting (53)-(57) into (41)-(45), we get

$$Q_4(x) = e^{-C_{2\lambda}x}\bar{F}(x)Q_4(0), \tag{58}$$
$$Q_3(x) = e^{-C_\lambda x}\bar{G}(x)Q_3(0), \tag{59}$$
$$Q_2(x) = e^{-C_\lambda x}\bar{F}(x)Q_2(0) + 2\left(e^{-C_\lambda x} - e^{-C_{2\lambda}x}\right) \bar{F}(x)Q_4(0), \tag{60}$$
$$Q_1(x) = \left(1 - e^{-C_\lambda x}\right) \bar{G}(x)Q_3(0), \tag{61}$$
$$Q_0(x) = \left(1 - e^{-C_\lambda x}\right) \bar{F}(x) [Q_2(0) + Q_4(0)] - \left(e^{-C_\lambda x} - e^{-C_{2\lambda}x}\right) \bar{F}(x)Q_4(0), \tag{62}$$

where $C_\lambda \equiv \lambda + \delta - \delta h^*(\lambda)$ and $C_{2\lambda} \equiv 2\lambda + \delta - \delta h^*(2\lambda)$. From (46) and (58), we obtain

$$Q_5 = \frac{f^*(C_{2\lambda})}{2\lambda + \mu} Q_4(0). \tag{63}$$

Substituting (61)-(63) into (50) and (51), we get

$$Q_3(0) = 2\lambda p\frac{f^*(C_{2\lambda})}{2\lambda + \mu}Q_4(0), \tag{64}$$
$$Q_2(0) = \left[\frac{2\lambda p}{2\lambda + \mu}f^*(C_{2\lambda}) \frac{1 - g^*(C_\lambda)}{f^*(C_\lambda)} + \frac{2\{1 - f^*(C_\lambda)\} - \{1 - f^*(C_{2\lambda})\}}{f^*(C_\lambda)}\right] Q_4(0). \tag{65}$$

From (53)-(65), $P_n(x, z)$, $n = 0, 1, 2, 3, 4$, $Q_n(x)$, $n = 0, 1, 2, 3, 4$, and Q_5 can be clearly expressed by $Q_4(0)$. Now we need to find the expression of $Q_4(0)$. After doing some manipulations, we obtain

$$\begin{aligned}
Q_0 &= \left[E(X) - \bar{F}^*(C_\lambda)\right] [Q_2(0) + Q_4(0)] - \left[\bar{F}^*(C_\lambda) - \bar{F}^*(C_{2\lambda})\right] Q_4(0),\\
Q_1 &= \left[E(Y) - \bar{G}^*(C_\lambda)\right] Q_3(0),\\
Q_2 &= \bar{F}^*(C_\lambda) Q_2(0) + 2\left[\bar{F}^*(C_\lambda) - \bar{F}^*(C_{2\lambda})\right] Q_4(0),\\
Q_3 &= \bar{G}^*(C_\lambda) Q_3(0),\\
Q_4 &= \bar{F}^*(C_{2\lambda}) Q_4(0),\\
Q_5 &= \frac{f^*(C_{2\lambda})}{2\lambda + \mu}Q_4(0),\\
P_0 &= \delta E(Z)Q_0 + \delta\left[E(Z) - \bar{H}^*(\lambda)\right] (Q_2 + Q_4) - \delta\left[\bar{H}^*(\lambda) - \bar{H}^*(2\lambda)\right] Q_4.\\
P_1 &= \delta E(Z)Q_1 + \delta\left[E(Z) - \bar{H}^*(\lambda)\right] Q_3,\\
P_2 &= \delta\bar{H}^*(\lambda)Q_2 + 2\delta\left[\bar{H}^*(\lambda) - \bar{H}^*(2\lambda)\right] Q_4,\\
P_3 &= \delta\bar{H}^*(\lambda)Q_3,\\
P_4 &= \delta\bar{H}^*(2\lambda)Q_4.
\end{aligned}$$

By the normalization condition $\sum_{n=0}^{4} (P_n + Q_n) + Q_5 = 1$, with (64) and (65), we obtain

$$\frac{1}{Q_4(0)} = \frac{f^*(C_{2\lambda})}{2\lambda + \mu} + [1 + \delta E(Z)]\left[\frac{2\lambda p}{2\lambda + \mu}f^*(C_{2\lambda})E(Y) + \left\{\frac{2\lambda p}{2\lambda + \mu}f^*(C_{2\lambda}) \frac{1 - g^*(C_\lambda)}{f^*(C_\lambda)} + \frac{1 - f^*(C_\lambda) + f^*(C_{2\lambda})}{f^*(C_\lambda)}\right\} E(X)\right],$$

from which $Q_4(0)$ is obtained. Thus, we obtain Q_n, $n = 0, 1, 2, 3, 4, 5$, and P_n, $n = 0, 1, 2, 3, 4$. Then, the steady-state availability Av_2 of Model 2 can be obtained as

$$Av_2 = Q_2 + Q_3 + Q_4 + Q_5 + P_2 + P_3 + P_4. \tag{66}$$

V. STEADY STATE AVAILABILITY FOR MODEL 3

Now we consider Model 3. Let $N(t)$ and $M(t)$ be the number of active components and the state of the repairer, respectively, at time t:

$$M(t) = \begin{cases} 0 & \text{if the repairer is idle at time } t, \\ 1 & \text{if the repairer is busy at time } t, \\ 2 & \text{if the repairer is failed at time } t. \end{cases}$$

Let us define

$$\begin{aligned}
P_n(x, z)dxdz &\equiv \lim_{t\to\infty} P\{N(t) = n, M(t)=2, x < X_-(t) < x+dx, z < Z_-(t) < z + dz\}, \quad n = 0, 1, 2,\\
Q_n(x)dx &\equiv \lim_{t\to\infty} P\{N(t) = n, M(t) = 1, x < X_-(t) < x + dx\}, \quad n = 0, 1, 2,\\
Q_3 &\equiv \lim_{t\to\infty} P\{N(t) = 3, M(t) = 0\},\\
P_n &\equiv \int_0^\infty \int_0^\infty P_n(x, z)dxdz, \quad n = 0, 1, 2,
\end{aligned}$$

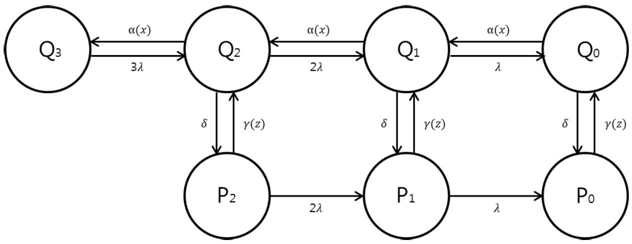

FIGURE 3. State transition diagram of Model 3.

$$Q_n \equiv \int_0^\infty Q_n(x)dx, \quad n=0,1,2.$$

We construct the following integro-differential equations governing the steady-state behavior of the system (Fig. 3):

$$\frac{dP_0(x,z)}{dz} = -\gamma(z)P_0(x,z) + \lambda P_1(x,z), \tag{67}$$

$$\frac{dP_1(x,z)}{dz} = -\left[\lambda+\gamma(z)\right]P_1(x,z) + 2\lambda P_2(x,z), \tag{68}$$

$$\frac{dP_2(x,z)}{dz} = -\left[2\lambda+\gamma(z)\right]P_2(x,z), \tag{69}$$

$$\frac{dQ_0(x)}{dx} = -\left[\delta+\alpha(x)\right]Q_0(x) + \lambda Q_1(x) + \int_0^\infty \gamma(z)P_0(x,z)dz, \tag{70}$$

$$\frac{dQ_1(x)}{dx} = -\left[\lambda+\delta+\alpha(x)\right]Q_1(x) + 2\lambda Q_2(x) + \int_0^\infty \gamma(z)P_1(x,z)dz, \tag{71}$$

$$\frac{dQ_2(x)}{dx} = -\left[2\lambda+\delta+\alpha(x)\right]Q_2(x) + \int_0^\infty \gamma(z)P_2(x,z)dz, \tag{72}$$

$$0 = -3\lambda Q_3 + \int_0^\infty \alpha(x)Q_2(x)dx, \tag{73}$$

with boundry conditions

$$P_n(x,0) = \delta Q_n(x), \quad n=0,1,2, \tag{74}$$

$$Q_0(0) = 0, \tag{75}$$

$$Q_1(0) = \int_0^\infty \alpha(x)Q_0(x)dx, \tag{76}$$

$$Q_2(0) = 3\lambda Q_3 + \int_0^\infty \alpha(x)Q_1(x)dx. \tag{77}$$

Solving the above integro-differential equations (67)-(69) with the bounday conditions (74), we obtain

$$P_2(x,z) = \delta e^{-2\lambda z}\bar{H}(z)Q_2(x), \tag{78}$$

$$P_1(x,z) = \delta e^{-\lambda z}\bar{H}(z) \times \left[2\left(1-e^{-\lambda z}\right)Q_2(z) + Q_1(x)\right], \tag{79}$$

$$P_0(x,z) = \delta\bar{H}(z)\left[\left(1-e^{-\lambda z}\right)^2 Q_2(x) + \left(1-e^{-\lambda z}\right)Q_1(x) + Q_0(x)\right], \tag{80}$$

where $\bar{H}(z) \equiv 1-H(z)$. Substituting (78)-(80) into (70)-(72), we get

$$Q_2(x) = e^{-C_{2\lambda}x}\bar{F}(x)Q_2(0), \tag{81}$$

$$Q_1(x) = e^{-C_\lambda x}\bar{F}(x)Q_1(0) + 2\left(e^{-C_\lambda x} - e^{-C_{2\lambda}x}\right)\bar{F}(x)Q_2(0), \tag{82}$$

$$Q_0(x) = \left(1-e^{-C_\lambda x}\right)\bar{F}(x)\left[Q_1(0)+Q_2(0)\right] - \left(e^{-C_\lambda x} - e^{-C_{2\lambda}x}\right)\bar{F}(x)Q_2(0), \tag{83}$$

where $C_\lambda \equiv \lambda+\delta-\delta h^*(\lambda)$, $C_{2\lambda} \equiv 2\lambda+\delta-\delta h^*(2\lambda)$, and $\bar{F}(y) \equiv 1-F(y)$. From (73) and (81), we obtain

$$Q_3 = \frac{f^*\left(C_{2\lambda}\right)}{3\lambda}Q_2(0). \tag{84}$$

Substituting (83) into (76), we get

$$Q_1(0) = \frac{1-2f^*(C_\lambda)+f^*(C_{2\lambda})}{f^*(C_\lambda)}Q_2(0). \tag{85}$$

From (78)-(85), $P_n(x,z)$, $n=0,1,2$, $Q_n(x)$, $n=0,1,2$, and Q_3 can be clearly expressed by $Q_2(0)$. After doing some manipulations, we obtain

$$\begin{aligned}
Q_0 &= \left[E(X)-\bar{F}^*\left(C_\lambda\right)\right]\left[Q_1(0)+Q_2(0)\right] - \left[\bar{F}^*\left(C_\lambda\right)-\bar{F}^*\left(C_{2\lambda}\right)\right]Q_2(0),\\
Q_1 &= \bar{F}^*\left(C_\lambda\right)Q_1(0) + 2\left[\bar{F}^*\left(C_\lambda\right)-\bar{F}^*\left(C_{2\lambda}\right)\right]Q_2(0),\\
Q_2 &= \bar{F}^*\left(C_{2\lambda}\right)Q_2(0),\\
Q_3 &= \frac{f^*\left(C_{2\lambda}\right)}{3\lambda}Q_2(0),\\
P_0 &= \delta E(Z)Q_0 + \delta\left[E(Z)-\bar{H}^*(\lambda)\right](Q_1+Q_2) - \delta\left[\bar{H}^*(\lambda)-\bar{H}^*(2\lambda)\right]Q_2,\\
P_1 &= \delta\bar{H}^*(\lambda)Q_1 + 2\delta\left[\bar{H}^*(\lambda)-\bar{H}^*(2\lambda)\right]Q_2,\\
P_2 &= \delta\bar{H}^*(2\lambda)Q_2.
\end{aligned}$$

By the normalization condistion $\sum_{n=0}^{2}\left(P_n+Q_n\right)+Q_3=1$ with (85), we obtain

$$Q_2(0) = \frac{3\lambda f^*(C_\lambda)}{\left[\begin{array}{c} f^*(C_\lambda)f^*(C_{2\lambda}) + 3\lambda E(X)\left\{1+\delta E(Z)\right\} \\ \times\left\{1-f^*(C_\lambda)+f^*(C_{2\lambda})\right\}\end{array}\right]}.$$

Thus, we obtain $P_n, n=0,1,2$, and $Q_n, n=0,1,2,3$. Then, the steady-state availability Av_3 of Model 3 can be obtained as

$$Av_3 = Q_1+Q_2+Q_3+P_1+P_2. \tag{86}$$

VI. NUMERICAL EXAMPLES

We compare the steady-state availability for different parameter values. It is assumed that the repair time X of failed components and the interrupted time Z follow a Weibull distribution with shape parameter $k=2$ and scale parameter $a=\frac{2}{\sqrt{\pi}}$ and the non-automatic switchover time Y follows a Weibull distribution with shape parameter $k=2$ and scale parameter $a=\frac{1}{\sqrt{\pi}}$. The pdf of the Weibull random variable is

$$f(x) \equiv \frac{k}{a}\left(\frac{x}{a}\right)^{k-1}e^{-\left(\frac{x}{a}\right)^k}, \quad x \geq 0, \tag{87}$$

TABLE 2. Values of parameters.

Case	Parameters	Variable
Case 1	$\lambda \in [0, 1]$, $\mu = 0.2\lambda$, $\delta = 0.5$, $p = 0.1$	λ, μ
Case 2	$\lambda = 0.5$, $\mu = 0.2\lambda$, $\delta \in [0, 1]$, $p = 0.1$	δ
Case 3	$\lambda = 0.5$, $\mu = 0.2\lambda$, $\delta = 0.5$, $p \in [0, 1]$	p

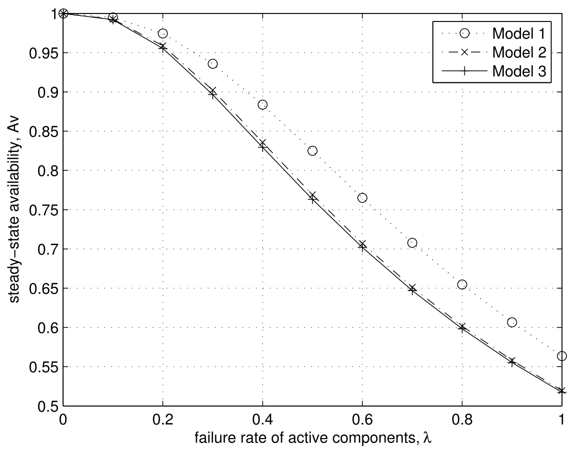

FIGURE 4. Availability *Av* versus failure rate λ of active components.

and the mean of the random variable is $\frac{\sqrt{\pi}}{2a}$. The random variables X, Y, and Z are independent of each other.

We carry out comparative experiments for three different parallel triplicated redundancy models shown in Table 1. As shown in Table 2, we provide three cases. Note that all parameters are given in dimensionless units for illustration purposes and can be modified to reflect other situations.

Figure 4 shows the effect of the failure rates of active components on the steady-state availability and Figure 5 shows the effect of the repair interruption rates on the availability. Model 1 with one active component and two standby components has higher availability than the others with more active components and less stanby components. It is because the failure rate of standby components is less than that of active components and the switchover failure probability is small. As expected, the availability decreases as the failure rate or the repair interruption rate increases.

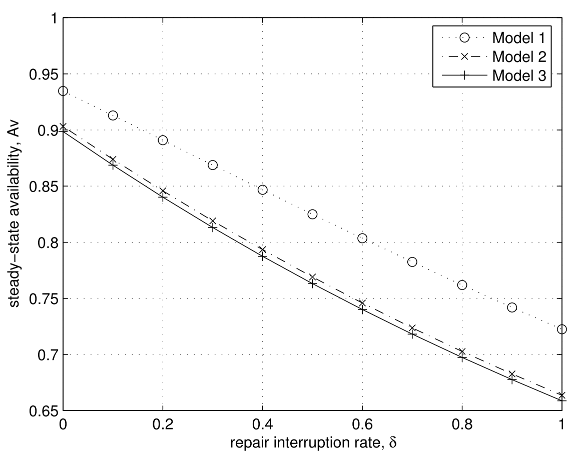

FIGURE 5. Availability *Av* versus repair interruption rate δ.

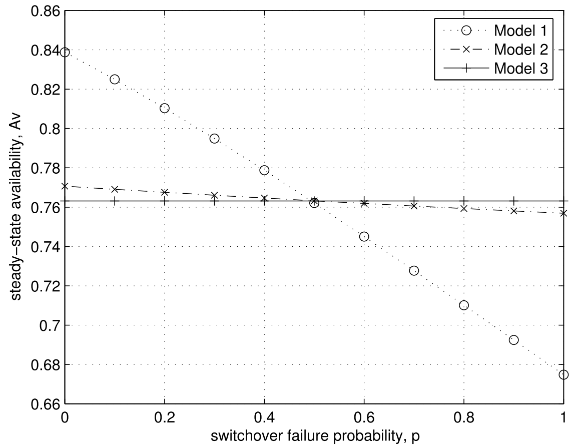

FIGURE 6. Availability *Av* versus swichingover failure probability *p*.

Figure 6 shows the effect of the switchover failure probability on the steady-state availability. The availabilities of Model 1 and 2 decrease as the switchover failure probability increases. The availability of Model 3 is constant because it has no standby components and the switchovers from active to standby do not occur. When the switchover failure probability is small, Model 1 with less active and more standby components has higher availability. When the switchover failure probability is large, Model 3 with the more active has higher availability.

VII. CONCLUSIONS

By using supplementary variables and integro-differential equations, we have obtained the analytical expression of the steady-state availability for parallel triplicated redundancy models with imperfect switchovers, generally distributed repair times, and interrupted repairs. Numerical examples show the effect of failure rate of active components, repair interruption rate, and switchover failure probability on the steady-state availability. The triplicated redundancy model with one active and two standby components has higher availability than the other triplicated models with more active components when the switchover failure probability is small. It is because the failure rate of standby components is less than that of active components and the switchover failure probability is small. When the switchover failure probability is large, the model with the more active components has higher availability than the models with less active components.

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