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Decoupled Access in HetNets With Backhaul Constrained Small Base Stations

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ABSTRACT This paper investigates the uplink (UL) and downlink decoupled access in heterogeneous networks with backhaul constraints. A capacity-based user association policy is proposed for users with decoupled access, where small base stations (SBSs) connect to macro base stations via non-ideal backhaul. The UL association probabilities are derived under this policy, where the limitations of association probabilities are obtained when the density of SBSs grows to infinity. Then, the UL signal-to-interference-plus-noise ratio (SINR) coverage probabilities are derived and analyzed for different SINR thresholds. According to the analysis, the effects of the limited backhaul capacity on the average UL SINR coverage probabilities are presented, where a jump discontinuity, which can be considered as a certain SINR threshold, is shown to be brought by the backhaul capacity constraint. It is demonstrated that denser deployment of SBSs helps improve UL performance significantly for users with SINR lower than such SINR threshold while hardly benefits those with SINR higher than it. Finally, the theoretical results are validated via simulations.

INDEX TERMS Decoupled access, backhaul capacity, uplink association, SINR coverage probability, heterogeneous networks.

I. INTRODUCTION

Aiming to meet the tremendous traffic demands, a densification of base stations (BSs) in current networks is required. A growing number of low power and low cost small BSs (SBSs) are added to the existing macro BSs (MBSs) tier, converting the traditional single-tier cellular networks into heterogeneous networks (HetNets) [1]–[5]. In HetNets, the traditional MBSs provide full area coverage while the low-power SBSs, e.g., pico BSs and femto BSs, help offload MBSs as well as improve traffic capacity [6]–[8]. However, due to the considerable disparity among the downlink (DL) transmit powers of different types of BSs, the BS that offers the highest signal-to-interference-plus-noise ratio (SINR) in DL might be an MBS that is rather far from the user instead of a nearby SBS. In this way, keeping connecting to the far-away MBS in uplink (UL) may lead to severe performance losses. Hence, the traditional UL and DL coupled access, i.e., users associate with the same BSs in UL as those in DL, is no longer suitable in HetNets. Thus, the decoupled access, namely establishing UL and DL user associations independently, is highly demanded [9]–[11].

In HetNets, significant benefits have been proved to be provided by UL and DL decoupled access [12]–[26], where both BSs and users are distributed according to homogeneous Poisson Point Processes (PPPs) for tractability [27]–[29]. Remarkable gains in terms of UL throughput were shown to be achieved by decoupled access by using system level simulation tool Atool [12], while theoretical analysis in terms of the average UL throughput was given in [13]. Besides, impressive improvements in the spectral efficiency [14], [15] and the energy efficiency [14]–[16] brought by decoupled access were demonstrated via both theoretical analysis and simulations. Moreover, the analytical lower bounds on the ergodic UL capacity revealed the notable superiority of decoupled access over the traditional coupled access [17]. Furthermore, it was demonstrated that decoupled access helps enhance fairness for users associated with different types of BSs [18] and helps balance load in HetNets [19]. Besides UL performance, Singh *et al.* [20] also took DL performance into consideration, proving that decoupled access leads to remarkable improvement in the joint UL-DL rate coverage. The UL SINR and rate coverage probabilities were also proved to be

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enhanced by decoupled access when millimeter wave SBSs are deployed [21]. Additionally, BSs deployed with multiple antennas were considered when studying decoupled access in [22]–[24]. Gains in terms of both signal-to-interference ratio (SIR) coverage probability and rate coverage probability were shown to be offered by decoupled access in [22]. While higher UL signal-to-noise ratio (SNR) coverage probability was offered by decoupled access under a maximum SNR association policy, proposed in [23], than the traditional pathloss-based decoupled access. In addition, different from the single user scenario in the above works, decoupled access with BSs serving multiple users simultaneously was studied in [24], where considerable gains in the UL spectral efficiency were shown to be brought by decoupled access.

Nevertheless, all the above works assume that the backhaul between SBSs and MBSs is ideal, while the economic consideration due to the dense deployment of SBSs may limit the capacity of backhaul links [30], [31]. Thus, non-ideal backhaul which connects SBSs to MBSs with limited capacity should be concerned in decoupled access in HetNets. In [32], a user association algorithm involved with cell load, resource management and backhaul capacity constraints was presented. Similarly, limited backhaul capacity was also taken into consideration in [33] when studying user association problems. Moreover, Elshaer *et al.* [34] proposed an algorithm which extends the UL association policy of users with decoupled access to include cell load and backhaul capacity besides the link quality. However, algorithms in both [32] and [33] were based on DL without considering decoupled access while the study in [34], although focused on decoupled access, was not from a theoretical perspective.

Accordingly, we explore decoupled access in HetNets where SBSs connect to MBSs via non-ideal backhaul links and provide analytical results. In this paper, decoupled access in HetNets is investigated, where BSs and users are distributed as independent homogeneous PPPs. The maximum capacity user association policy is proposed. When making user association decisions, limited backhaul capacity is considered by the proposed policy as an upper bound on the capacity that an SBS can offer. Based on this association policy, user association probabilities are derived. Asymptotic results are then obtained in the ultra-dense network scenario where the density of SBSs grows towards infinity. After the analysis of association probabilities, the average UL SINR coverage probabilities are analyzed in two cases, namely 1) when the SINR threshold is lower than or equal to the SINR which corresponds to the backhaul capacity constraint and 2) when the SINR threshold is higher than that. Then, asymptotic results are derived in the ultra-dense network scenario for both cases. Besides, the effects of the non-ideal backhaul on the average SINR coverage probabilities are shown, where an interesting insight is that a discontinuity point of the average SINR coverage probability is caused by the limited backhaul capacity. Finally, simulation results are given to validate the analysis. The results show that due to the backhaul constraints, denser SBSs help improve the UL performance significantly for users with low capacity. While the density of SBSs does not affect the users with capacity which is higher than or equal to the limited backhaul capacity.

The rest of the paper is organized as follows. In Section II, the system model and the assumptions are described. In Section III, the maximum capacity association policy in HetNets with non-ideal backhaul is depicted and the user association probabilities are derived. Section IV analyzes the UL SINR coverage probabilities under the presented association policy. Then, in Section V, simulations and numerical results are conducted. Finally, Section VI concludes the paper.

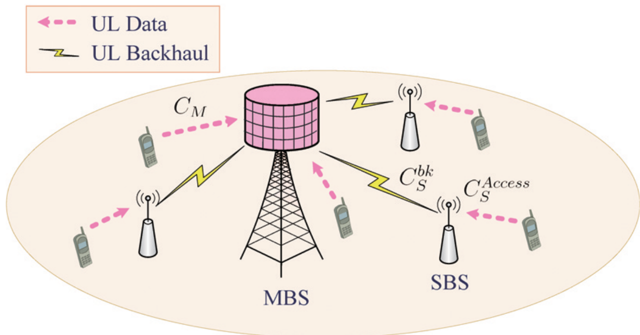

FIGURE 1. The illustration of UL user association in a two-tier HetNet where each SBS is connected to an MBS via wireless backhaul with limited capacity.

II. SYSTEM MODEL

A two-tier HetNet consisting of an MBSs tier and an SBSs tier is considered. The locations of both MBSs and SBSs are modeled as homogeneous PPPs Φ_v with density λ_v, where $v = M$ for MBSs and $v = S$ for SBSs. The users are also assumed to be following the homogeneous PPP distribution Φ_u with density λ_u. The distributions of MBSs, SBSs and users are independent. Each SBS is connected to the core network (usually MBSs) via a wireless backhaul of limited capacity C_S^{bk}. To be specific, the backhaul capacity C_S^{bk} is assumed to be scheduled over time to satisfy users' data requests [33]. The illustration of UL user associations in such a two-tier HetNet is shown in Fig. 1 where each user is allowed to associate with its nearest MBS or SBS.[1] Let SINR_v and C_v be the SINR and capacity at vBSs, respectively. Then, by using the Shannon formula, the capacity that an MBS can offer to its serving user is given as

$$C_M = \ln\left(1 + \text{SINR}_M\right). \tag{1}$$

The MBS tier is assumed to be interference limited for simplicity, that is, the interference is much higher than the noise

[1] In this paper, we mainly focus on the UL user associations with backhaul constraints, while the DL user associations are not concerned for simplicity due to two reasons. For one thing, the effects of non-ideal backhaul on DL can be analyzed in a similar way. For another, decoupled access only affects UL while DL user associations of users with decoupled access are exactly the same with those for traditional coupled access, which are based on the DL received signal power [13], [20].

power [21]. Hence, it is assumed that $\mathrm{SINR}_M \approx \mathrm{SIR}_M$ with SIR_M being the SIR at MBSs. Then, the capacity of users associated to MBSs can be simplified as

$$C_M = \ln\left(1 + \mathrm{SIR}_M\right). \tag{2}$$

However, due to the backhaul constraints, the capacity that an SBS can offer to a user associated to it is

$$C_S = \min\left\{C_S^{bk}, C_S^{Access}\right\}, \tag{3}$$

where C_S^{Access} is the achievable capacity at SBSs, given by

$$C_S^{Access} = \ln\left(1 + \mathrm{SINR}_S\right). \tag{4}$$

Note that, the SBS tier can be assumed to be noise limited due to the following reasons.

1) No intra-cell interference exists since orthogonal time-frequency resources are used to serve intra-cell users. This is the typical case in [13], [14], and [20]–[23].
2) No inter-cell interference exists [17]. This can be explained in two aspects. One is that the inter-tier interference is removed since the two tiers operate on different frequency bands. The other is that frequency reuse is considered, which means the spectrum of the SBSs tier is partitioned into several subbands and each small cell is assigned with a subband. In this way, adjacent small cells are allowed to use different frequency subbands of the available spectrum while two small cells far apart operate on the same frequency subband. Along with the fact that the transmit power of users is low and for simplicity, the co-channel interference from a far-away small cell can be ignored.

Hence, it is assumed that $\mathrm{SINR}_S \approx \mathrm{SNR}_S$ with SNR_S being the SNR at SBSs. Then, the achievable capacity in (4) can be rewritten as

$$C_S^{Access} = \ln\left(1 + \mathrm{SNR}_S\right). \tag{5}$$

Let S_v be the received signal at vBSs, where

$$S_v = Qh_v r_{v,0}^{-\alpha} \tag{6}$$

in which Q is the transmit power of users, h_v is the Rayleigh fading with $h_v \sim \exp(1)$ and $r_{v,0}^{-\alpha}$ is the pathloss between a typical user and its tagged vBS with $r_{v,0}$ being the distance between them and α being the pathloss exponent. Considering the fact that both users and vBSs are PPP distributed, the probability density function (PDF) of $r_{v,0}$, as given in [27], is

$$f_{r_{v,0}}(r) = 2\pi\lambda_v r e^{-\lambda_v \pi r^2}. \tag{7}$$

Since there exists no inter-tier interference, the interfering users are those associated with other MBSs using the same time-frequency resource block as the typical user. Thus, the cumulative interference, denote by I, is given by

$$I = \sum_{j\in\Phi_{M_u}, j\neq 0} Qg_j r_{M,j}^{-\alpha}, \tag{8}$$

where Φ_{M_u} is the set of interfering users and the jth ($j \neq 0$) user is at a distance $r_{M,j}$ with Rayleigh fading value $g_j \sim \exp(1)$. Then, according to (6) and (8), the expressions for SIR_M and SNR_S in (2) and (5) can be obtained as

$$\mathrm{SIR}_M = \frac{Qh_M r_{M,0}^{-\alpha}}{\sum_{j\in\Phi_{M_u}, j\neq 0} Qg_j r_{M,j}^{-\alpha}}, \tag{9}$$

$$\mathrm{SNR}_S = \frac{Qh_S r_{S,0}^{-\alpha}}{\sigma^2}, \tag{10}$$

respectively, where σ^2 is the power of noise at SBSs.

Based on the system model, UL user association policy based on UL capacity will be studied for users with decoupled access where limited backhaul capacity is taken into consideration later on. Then, association probabilities will be investigated and the UL SINR coverage probabilities will be derived to evaluate the system performance. Some important notations in this paper are listed in TABLE 1.

TABLE 1. List of important notations.

Notation	Description
λ_v, λ_u	density of vBSs, density of users
Φ_u	PPP of users
SIR_M	SIR at MBSs
SNR_S	SNR at SBSs
C_v	capacity that a vBS can offer to its serving user
C_S^{Access}	achievable capacity of users associated with SBSs
C_S^{bk}, γ^{bk}	backhaul capacity constraint and the corresponding SNR_S
Q	transmit power of users
h_v	Rayleigh fading between vBS and its serving user
g_j	Rayleigh fading between MBS and the jth interfering user
$r_{v,0}$	distance between vBS and its serving user
$r_{M,j}$	distance between MBS and the jth interfering user
α	pathloss exponent
σ^2	power of noise
$A_{M,1}$	probability of associating to MBS when $\mathrm{SNR}_S < \gamma^{bk}$
$A_{M,2}$	probability of associating to MBS when $\mathrm{SNR}_S \geq \gamma^{bk}$
A_v	probability of associating to vBS
γ	SINR threshold
$\mathbb{P}_v(\gamma)$	UL SINR coverage probability at vBS
$\mathbb{P}(\gamma)$	average UL SINR coverage probability

III. UPLINK USER ASSOCIATIONS

In this section, the UL user associations based on UL capacity with limited backhaul is investigated. Then, the association probabilities are derived.

To improve the UL performance, it is assumed that each user with decoupled access associates with the BS that offers the highest UL capacity. Hence, a user associates with an MBS rather than an SBS in UL when

$$C_M > \min\left\{C_S^{bk}, C_S^{Access}\right\}. \tag{11}$$

In this way, the UL capacity the user can achieve is

$$\max\left\{C_M, \min\left\{C_S^{bk}, C_S^{Access}\right\}\right\}. \tag{12}$$

Then, based on the association policy, we investigate the UL user association probabilities. Let A_v denote the probability that users will associate with vBSs. The probability of associating to MBSs can be represented as

$$A_M \triangleq \Pr\left(C_M > \min\left\{C_S^{bk}, C_S^{Access}\right\}\right). \tag{13}$$

By using the law of total probability, the association probability A_M can be rewritten as

$$A_M = A_{M,1} + A_{M,2} \tag{14}$$

where

$$A_{M,1} \triangleq \Pr\left(C_M > C_S^{Access}, C_S^{Access} < C_S^{bk}\right), \tag{15}$$

$$A_{M,2} \triangleq \Pr\left(C_M > C_S^{bk}, C_S^{Access} \geq C_S^{bk}\right). \tag{16}$$

Similar with [21], instantaneous capacity is used for user association determination. Let $\gamma^{bk} = e^{C_S^{bk}} - 1$ be the SNR_S which corresponds to the limited backhaul capacity C_S^{bk}. Thus, the probabilities in (15) and (16) can be transformed into

$$A_{M,1} = \Pr\left(\mathrm{SIR}_M > \mathrm{SNR}_S, \mathrm{SNR}_S < \gamma^{bk}\right), \tag{17}$$

$$A_{M,2} = \Pr\left(\mathrm{SIR}_M > \gamma^{bk}, \mathrm{SNR}_S \geq \gamma^{bk}\right), \tag{18}$$

respectively. Accordingly, based on the definitions of $A_{M,1}$ and $A_{M,2}$, the probability of associating to MBSs can be derived in Theorem 1.

Theorem 1: The probability that a user will associate with an MBS is given as

$$A_M = \int_0^{\gamma^{bk}} \frac{1}{1+\rho(t)} \times \int_0^{+\infty} \frac{\sigma^2\xi^{\alpha/2}}{Q} e^{-\frac{t}{Q}\sigma^2\xi^{\alpha/2}} \pi\lambda_S e^{-\lambda_S\pi\xi} d\xi dt + \frac{1}{1+\rho(\gamma^{bk})} \int_0^{+\infty} e^{-\frac{\gamma^{bk}}{Q}\sigma^2\xi^{\alpha/2}} \pi\lambda_S e^{-\lambda_S\pi\xi} d\xi \tag{19}$$

where $\rho(t)$ is

$$\rho(t) = t^{2/\alpha} \int_{t^{-2/\alpha}}^{+\infty} \frac{1}{1+\eta^{\alpha/2}} d\eta. \tag{20}$$

Proof: Based on (17) and (18), the probabilities can be developed by

$$A_{M,1} = \int_0^{\gamma^{bk}} \overline{F}_{\mathrm{SIR}_M}(t) f_{\mathrm{SNR}_S}(t) \mathrm{d}t, \tag{21}$$

$$A_{M,2} = \overline{F}_{\mathrm{SIR}_M}(\gamma^{bk}) \overline{F}_{\mathrm{SNR}_S}(\gamma^{bk}), \tag{22}$$

where $\overline{F}_{\mathrm{SIR}_M}(t)$ and $\overline{F}_{\mathrm{SNR}_S}(t)$ are the complementary cumulative distribution function (CCDF) of SIR_M and SNR_S, respectively, while $f_{\mathrm{SNR}_S}(t)$ denotes the PDF of SNR_S. Thus, aiming to calculate the probabilities, it is important to derive $\overline{F}_{\mathrm{SIR}_M}(t)$, $\overline{F}_{\mathrm{SNR}_S}(t)$ and $f_{\mathrm{SNR}_S}(t)$ at first. According to [27], the CCDF of SIR_M is

$$\overline{F}_{\mathrm{SIR}_M}(t) = \Pr\left(\mathrm{SIR}_M > t\right) = \frac{1}{1+\rho(t)}, \quad t > 0 \tag{23}$$

where $\rho(t)$ is as given in (20). Then, we only need to derive the CCDF and the PDF of SNR_S. Similar with the definition of the CCDF of SIR_M, the CCDF of SNR_S can be obtained as

$$\overline{F}_{\mathrm{SNR}_S}(t) = \Pr\left(\frac{Qh_S r_{S,0}^{-\alpha}}{\sigma^2} > t\right) = \Pr\left(h_S > \frac{r_{S,0}^{\alpha} t\sigma^2}{Q}\right). \tag{24}$$

Since the Rayleigh fading h_S obeys $h_S \sim \exp(1)$, we have

$$\overline{F}_{\mathrm{SNR}_S}(t) = \mathbb{E}_{r_{S,0}}\left[e^{-\frac{r_{S,0}^{\alpha} t\sigma^2}{Q}}\right] \tag{25}$$

where $\mathbb{E}_x[\cdot]$ is the expectation operator with respect to x. Accordingly, by using the PDF of $r_{v,0}$ given in (7) and employing a change of variable $\xi = r_{S,0}^2$, the CCDF of SNR_S is given as

$$\overline{F}_{\mathrm{SNR}_S}(t) = \int_0^{+\infty} e^{-\frac{t}{Q}\sigma^2\xi^{\alpha/2}} \pi\lambda_S e^{-\lambda_S\pi\xi} \mathrm{d}\xi. \tag{26}$$

Let the function $g(\xi, t)$ be

$$g(\xi, t) \triangleq e^{-\frac{t}{Q}\sigma^2\xi^{\alpha/2}} \pi\lambda_S e^{-\lambda_S\pi\xi}. \tag{27}$$

Since the CCDF $\overline{F}_{\mathrm{SNR}_S}(t)$ is no more than 1, the integral $\int_0^{+\infty} g(\xi, t)\mathrm{d}\xi$ is convergent. Thus, the partial derivative of $g(\xi, t)$ with respect to t can be given as

$$\frac{\partial g(\xi, t)}{\partial t} = -\frac{\sigma^2\xi^{\alpha/2}}{Q} e^{-\frac{t}{Q}\sigma^2\xi^{\alpha/2}} \pi\lambda_S e^{-\lambda_S\pi\xi}. \tag{28}$$

According to the Weierstrass criterion [35], the fact that

$$\left|\frac{\partial g(\xi, t)}{\partial t}\right| \leq \frac{\sigma^2\xi^{\alpha/2}}{Q} \pi\lambda_S e^{-\lambda_S\pi\xi} \tag{29}$$

and the integral

$$\int_0^{+\infty} \frac{\sigma^2\xi^{\alpha/2}}{Q} \pi\lambda_S e^{-\lambda_S\pi\xi}\, \mathrm{d}\xi = \frac{\Gamma\left(\frac{\alpha}{2}+1\right)\sigma^2}{(\lambda_S\pi)^{\frac{\alpha}{2}} Q} \tag{30}$$

is convergent leads to the uniform convergence of the integral $\int_0^{+\infty} \frac{\partial g(\xi,t)}{\partial t} \mathrm{d}\xi$. Consequently, when deriving the PDF of SNR_S, the integral and partial differential operators can be interchanged, which results in

$$f_{\mathrm{SNR}_S}(t) = -\frac{\mathrm{d}\overline{F}_{\mathrm{SNR}_S}(t)}{\mathrm{d}t} = \int_0^{+\infty} \frac{\sigma^2\xi^{\alpha/2}}{Q} e^{-\frac{t}{Q}\sigma^2\xi^{\alpha/2}} \pi\lambda_S e^{-\lambda_S\pi\xi} \mathrm{d}\xi. \tag{31}$$

By substituting (23), (26) and (31) into (21) and (22), the expressions for $A_{M,1}$ and $A_{M,2}$ can be obtained as

$$A_{M,1} = \int_0^{\gamma^{bk}} \frac{1}{1+\rho(t)} \times \int_0^{+\infty} \frac{\sigma^2\xi^{\alpha/2}}{Q} e^{-\frac{t}{Q}\sigma^2\xi^{\alpha/2}} \pi\lambda_S e^{-\lambda_S\pi\xi} \mathrm{d}\xi \mathrm{d}t, \tag{32}$$

$$A_{M,2} = \frac{1}{1+\rho(\gamma^{bk})} \int_0^{+\infty} e^{-\frac{\gamma^{bk}}{Q}\sigma^2\xi^{\alpha/2}} \pi\lambda_S e^{-\lambda_S\pi\xi}\, \mathrm{d}\xi, \tag{33}$$

respectively. Finally, plugging (32) and (33) into (14) gives the desired association probability A_M in Theorem 1. ∎

According to Theorem 1, it can be concluded that γ^{bk} as well as the density of SBSs have notable effects on the association probabilities. Let $f(\xi, t)$ be the integrand in (32), i.e.,

$$f(\xi, t) \triangleq \frac{1}{1+\rho(t)} \cdot \frac{\sigma^2 \xi^{\alpha/2}}{Q} e^{-\frac{t}{Q}\sigma^2 \xi^{\alpha/2}} \pi \lambda_S e^{-\lambda_S \pi \xi}, \tag{34}$$

we have $f(\xi, t) \geq 0$, $\forall(\xi, t) \in [0, +\infty; 0, \gamma^{bk}]$. Then, it can be concluded that $\int_0^{+\infty} f(\xi, t) \mathrm{d}\xi \geq 0$. Thereby, the integral $\int_0^{\gamma^{bk}} \int_0^{+\infty} f(\xi, t) \mathrm{d}\xi \mathrm{d}t$ increases with the growth of the upper limit of integration, i.e., γ^{bk}. Besides, since the integrand in (33) decreases while $\rho(\gamma^{bk})$ increases with the increasing γ^{bk}, the probability $A_{M,2}$ decreases as γ^{bk} grows. Next, we consider an ultra-dense network where λ_S grows towards infinity, the asymptotic results of the association probabilities can be derived in the following corollary.

Corollary 1: In the ultra-dense network scenario, as λ_S grows towards infinity, it can be concluded that

$$\lim_{\lambda_S \to +\infty} A_{M,1} = 0, \tag{35}$$

$$\lim_{\lambda_S \to +\infty} A_{M,2} = \frac{1}{1+\rho\left(\gamma^{bk}\right)}. \tag{36}$$

Hence, the asymptotic result of A_M is

$$\lim_{\lambda_S \to +\infty} A_M = \frac{1}{1+\rho\left(\gamma^{bk}\right)}. \tag{37}$$

Proof: Firstly, we calculate the asymptotic result of $A_{M,1}$. According to the integrand $f(\xi, t)$ in (34), the fact that $1 + \rho(t) \geq 1$ and $e^{-\frac{t}{Q}\sigma^2 \xi^{\alpha/2}} \leq 1$, $\forall t \in [0, +\infty)$ results in

$$f(\xi, t) \leq \frac{\sigma^2 \xi^{\alpha/2}}{Q} \pi \lambda_S e^{-\lambda_S \pi \xi}. \tag{38}$$

Hence, the upper bound on the integral in (32) is given as

$$\begin{aligned} A_{M,1} &\leq \int_0^{\gamma^{bk}} \int_0^{+\infty} \frac{\sigma^2 \xi^{\alpha/2}}{Q} \pi \lambda_S e^{-\lambda_S \pi \xi} \mathrm{d}\xi \mathrm{d}t \\ &= \frac{\gamma^{bk} \sigma^2 \Gamma\left(\frac{\alpha}{2}+1\right)}{Q\left(\lambda_S \pi\right)^{\frac{\alpha}{2}}} \end{aligned} \tag{39}$$

with the limitation being

$$\lim_{\lambda_S \to +\infty} \frac{\gamma^{bk} \sigma^2 \Gamma\left(\frac{\alpha}{2}+1\right)}{Q\left(\lambda_S \pi\right)^{\frac{\alpha}{2}}} = 0. \tag{40}$$

Besides, since the integrand in (32) satisfies $f(\xi, t) \geq 0$, $\forall(\xi, t) \in [0, +\infty; 0, \gamma^{bk}]$, it can be concluded that

$$A_{M,1} \geq 0. \tag{41}$$

By using the squeeze theorem, the desired result in (35) can be derived.

Then, the asymptotic result of $A_{M,2}$ is calculated. Since $\lambda_S \pi$ is a nonzero constant, the term $\lambda_S \pi \xi$ in (33) can be replaced by ζ. Thus, the expression of $A_{M,2}$ can be converted into

$$A_{M,2} = \frac{1}{1+\rho(\gamma^{bk})} \int_0^{+\infty} e^{-\frac{\gamma^{bk}}{Q}\sigma^2 (\lambda_S \pi)^{-\alpha/2} \zeta^{\alpha/2}} e^{-\zeta} \mathrm{d}\zeta. \tag{42}$$

Let $f(\zeta, \lambda_S)$ be the integrand, which is

$$f(\zeta, \lambda_S) = e^{-\frac{\gamma^{bk}}{Q}\sigma^2 (\lambda_S \pi)^{-\alpha/2} \zeta^{\alpha/2}} e^{-\zeta}. \tag{43}$$

Thereby, we have

$$|f(\zeta, \lambda_S)| \leq e^{-\zeta}. \tag{44}$$

Note that

$$\int_0^{+\infty} e^{-\zeta} \mathrm{d}\zeta = 1 \tag{45}$$

is convergent. According to the Weierstrass criterion, the integral $\int_0^{+\infty} f(\zeta, \lambda_S) \mathrm{d}\zeta$ is uniformly convergent. Therefore, the asymptotic value of $A_{M,2}$ when λ_S grows towards infinity can be obtained as

$$\begin{aligned} &\lim_{\lambda_S \to +\infty} A_{M,2} \\ &= \lim_{\lambda_S \to +\infty} \frac{1}{1+\rho(\gamma^{bk})} \int_0^{+\infty} e^{-\frac{\gamma^{bk}}{Q}\sigma^2 (\lambda_S \pi)^{-\frac{\alpha}{2}} \zeta^{\frac{\alpha}{2}}} e^{-\zeta} \mathrm{d}\zeta \\ &= \frac{1}{1+\rho(\gamma^{bk})} \int_0^{+\infty} \lim_{\lambda_S \to +\infty} e^{-\frac{\gamma^{bk}\sigma^2 \zeta^{\frac{\alpha}{2}}}{Q(\lambda_S \pi)^{\frac{\alpha}{2}}}} e^{-\zeta} \mathrm{d}\zeta. \end{aligned} \tag{46}$$

Consequently, the desired result in (36) is derived as

$$\begin{aligned} \lim_{\lambda_S \to +\infty} A_{M,2} &= \frac{1}{1+\rho(\gamma^{bk})} \int_0^{+\infty} e^{-\zeta} \mathrm{d}\zeta \\ &= \frac{1}{1+\rho(\gamma^{bk})}. \end{aligned} \tag{47}$$

Finally, the summation of the asymptotic results of $A_{M,1}$ and $A_{M,2}$ gives A_M in (37). ∎

As shown in Corollary 1, the limitation of $A_{M,1}$ is 0 as λ_S grows to infinity. The reason is that denser deployment of SBSs leads to higher SNR of users associated to SBSs, thereby decreasing the probability that $\mathrm{SNR}_S < \gamma^{bk}$. Hence, in a HetNet where SBSs are ultra-densely deployed, the probability that $\mathrm{SNR}_S < \gamma^{bk}$ approximates 0, which leads to the fact that $A_{M,1}$ approximates 0 based on (17). On the contrary, the probability that $\mathrm{SNR}_S \geq \gamma^{bk}$ increases as SBSs get denser and finally converges to 1, which means $A_{M,2}$ finally equals to the probability that $\mathrm{SIR}_M > \gamma^{bk}$ according to (18) and so does A_M. Besides, the variation of $A_{M,2}$ with the increase of λ_S can also be theoretically analyzed according to the proof of Corollary 1 by calculating $\frac{\partial A_{M,2}}{\partial \lambda_S}$. Since it is proved that the integral in $A_{M,2}$ is uniformly convergent, the integral and differential operators can be interchanged. As a result, the partial derivative of $A_{M,2}$ with respect to λ_S turns into the integral of the partial derivative of $f(\zeta, \lambda_S)$ in (43). Since the partial derivative of $f(\zeta, \lambda_S)$ with respect to λ_S is positive, the partial derivative of $A_{M,2}$ is positive, which means $A_{M,2}$ increases with the increase of λ_S.

Moreover, we consider a special case where noise is ignored. By letting $\sigma^2 = 0$ in (32) and (33), the simplified expressions for $A_{M,1}$ and $A_{M,2}$ can be obtained as $A_{M,1} = 0$ and $A_{M,2} = 1/(1+\rho(\gamma^{bk}))$, respectively. Therefore, the sum of $A_{M,1}$ and $A_{M,2}$ is given as $A_M = 1/(1+\rho(\gamma^{bk}))$. The reason is that $\text{SNR}_S \to +\infty$ when $\sigma^2 = 0$, thereby the probability that $\text{SNR}_S < \gamma^{bk}$ is 0 while $\text{SNR}_S \geq \gamma^{bk}$ is 1. Thus, according to (17) and (18), the desired results can be derived. Additionally, note that the association probabilities when $\sigma^2 = 0$ are equal to the asymptotic results when $\lambda_S \to +\infty$ (given in (35) and (36), respectively). This is due to the fact that both scenarios lead to high SNR at SBSs, thereby having similar effects on the UL association probabilities.

Furthermore, when $\alpha = 4$, the association probabilities can be simplified using the error function, i.e. $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$. The results are given in Corollary 2.

Corollary 2: When $\alpha = 4$, the association probability is derived according to (14) where $A_{M,2}$ can be simplified as

$$A_{M,2} = \frac{\lambda_S \pi^{\frac{3}{2}}\sqrt{Q}e^{\frac{\lambda_S^2\pi^2 Q}{4\gamma^{bk}\sigma^2}}\, erfc\left(\frac{\lambda_S\pi\sqrt{Q}}{2\sqrt{\gamma^{bk}}\sigma}\right)}{2\sqrt{\gamma^{bk}}\sigma\left[1+\rho(\gamma^{bk})\right]} \tag{48}$$

where $erfc(x)$ is the complementary error function with

$$erfc(x) = 1 - erf(x) \tag{49}$$

*and the **closed-form** $\rho(\gamma^{bk})$ is*

$$\rho(\gamma^{bk}) = \left(\gamma^{bk}\right)^{\frac{1}{2}}\left(\frac{\pi}{2} - \arctan\left(\gamma^{bk}\right)^{-\frac{1}{2}}\right). \tag{50}$$

Next, with the derived association probabilities, the UL performance in terms of the SINR coverage probability will be analyzed in the following section.

IV. UPLINK SINR COVERAGE PROBABILITY

In this section, the UL SINR coverage probabilities of users associated with MBSs and SBSs are analyzed and the average UL SINR coverage probability is derived afterwards.

The average UL SINR coverage probability, i.e., $\mathbb{P}$, is defined as the expectation of the probability that the UL SINR exceeds its threshold γ, which is,

$$\mathbb{P}(\gamma) \triangleq \mathbb{E}\left[\Pr\left(\text{SINR} > \gamma\right)\right]. \tag{51}$$

Note that, each user associates to vBS with the probability A_v. Hence, the average UL SINR coverage probability can be rewritten as

$$\mathbb{P}(\gamma) = \mathbb{P}_M(\gamma)A_M + \mathbb{P}_S(\gamma)A_S \tag{52}$$

where $\mathbb{P}_v(\gamma)$, $v \in \{M, S\}$, is the SINR coverage probability of a user, on the condition that the user is associated with vBS. Therefore, we primarily focus on the derivations of $\mathbb{P}_M(\gamma)$ and $\mathbb{P}_S(\gamma)$. Since it is declared in Section II that $\text{SINR}_M \approx \text{SIR}_M$ and $\text{SINR}_S \approx \text{SNR}_S$, the definitions of $\mathbb{P}_M(\gamma)$ and $\mathbb{P}_S(\gamma)$ are then given as

$$\mathbb{P}_M(\gamma) \triangleq \mathbb{E}\left[\Pr\left(\text{SIR}_M > \gamma\right)\right], \tag{53}$$

$$\mathbb{P}_S(\gamma) \triangleq \mathbb{E}\left[\Pr\left(\text{SNR}_S > \gamma\right)\right], \tag{54}$$

respectively. Considering the fact that the backhaul capacity of each SBS is limited to C_S^{bk} whose corresponding SNR_S is γ^{bk}, the derivations of $\mathbb{P}_M(\gamma)$ and $\mathbb{P}_S(\gamma)$ can be classified into two cases, namely $\gamma \leq \gamma^{bk}$ and $\gamma > \gamma^{bk}$. Then, the expressions for $\mathbb{P}_M(\gamma)$ and $\mathbb{P}_S(\gamma)$ are given as

- Case 1, $\gamma \leq \gamma^{bk}$
  When $\gamma \leq \gamma^{bk}$, the SIR coverage probability of a user, given that the user is associated to an MBS, is developed as

$$\begin{aligned}\mathbb{P}_M(\gamma) &= \frac{1}{A_M}\Big[\Pr\left(\text{SIR}_M > \gamma, \text{SIR}_M > \text{SNR}_S, \text{SNR}_S < \gamma^{bk}\right)\\ &\quad + \Pr\Big(\text{SIR}_M > \gamma, \text{SIR}_M > \gamma^{bk}, \text{SNR}_S \geq \gamma^{bk}\Big)\Big].\end{aligned} \tag{55}$$

  While the SNR coverage probability of a user, on the condition that it is associated with an SBS, is given by

$$\begin{aligned}\mathbb{P}_S(\gamma) &= \frac{1}{A_S}\Big[\Pr\left(\text{SNR}_S > \gamma, \text{SNR}_S > \text{SIR}_M, \text{SNR}_S < \gamma^{bk}\right)\\ &\quad + \Pr\left(\gamma^{bk} > \gamma, \text{SNR}_S > \gamma^{bk}, \gamma^{bk} \geq \text{SIR}_M\right)\Big].\end{aligned} \tag{56}$$

- Case 2, $\gamma > \gamma^{bk}$
  When $\gamma > \gamma^{bk}$, the SIR coverage probability of a user, given that it is associated with an MBS, can be developed as

$$\mathbb{P}_M(\gamma) = \frac{\Pr\left(\text{SIR}_M > \gamma\right)}{A_M}. \tag{57}$$

  However, the SNR coverage probability of a user associated with SBSs is

$$\mathbb{P}_S(\gamma) = 0 \tag{58}$$

  since the highest SNR that users associated with SBSs can achieve is γ^{bk} due to the limited backhaul capacity.

Then, the average SINR coverage probabilities in both cases are obtained in Theorem 2.

Theorem 2: The average SINR coverage probabilities in both cases are

- *Case 1, $\gamma \leq \gamma^{bk}$,*

$$\mathbb{P}(\gamma) = \frac{1+\rho(\gamma)\int_0^{+\infty} e^{-\frac{\gamma}{Q}\sigma^2\xi^{\frac{\alpha}{2}}}\pi\lambda_S e^{-\lambda_S\pi\xi}\,d\xi}{1+\rho(\gamma)}, \tag{59}$$

- *Case 2, $\gamma > \gamma^{bk}$,*

$$\mathbb{P}(\gamma) = \frac{1}{1+\rho(\gamma)}, \tag{60}$$

respectively.

Proof: The average SINR coverage probabilities in the two cases are derived separately.

- Case 1, $\gamma \leq \gamma^{bk}$

According to (55), we have

$$\begin{aligned}\mathbb{P}_M(\gamma) &= \frac{1}{A_M}\Big[\Pr\left(\mathrm{SIR}_M > \mathrm{SNR}_S, \mathrm{SNR}_S > \gamma, \mathrm{SNR}_S < \gamma^{bk}\right) \\ &\quad + \Pr\left(\mathrm{SIR}_M > \gamma, \mathrm{SNR}_S < \gamma\right) \\ &\quad + \Pr\left(\mathrm{SIR}_M > \gamma^{bk}, \mathrm{SNR}_S \geq \gamma^{bk}\right)\Big] \end{aligned} \tag{61}$$

which leads to

$$\begin{aligned}\mathbb{P}_M(\gamma) = \frac{1}{A_M}\Big[&\int_{\gamma}^{\gamma^{bk}} \overline{F}_{\mathrm{SIR}_M}(S) f_{\mathrm{SNR}_S}(S)\mathrm{d}S \\ &+ \overline{F}_{\mathrm{SIR}_M}(\gamma)\left(1-\overline{F}_{\mathrm{SNR}_S}(\gamma)\right) \\ &+ \overline{F}_{\mathrm{SIR}_M}(\gamma^{bk})\overline{F}_{\mathrm{SNR}_S}(\gamma^{bk})\Big]. \end{aligned} \tag{62}$$

Similarly, the SNR coverage probability of users associated with SBSs can be given by

$$\begin{aligned}\mathbb{P}_S(\gamma) = \frac{1}{A_S}\Big[&\int_{\gamma}^{\gamma^{bk}} \left(1-\overline{F}_{\mathrm{SIR}_M}(S)\right) f_{\mathrm{SNR}_S}(S)\,\mathrm{d}S \\ &+ \overline{F}_{\mathrm{SNR}_S}(\gamma^{bk})\left(1-\overline{F}_{\mathrm{SIR}_M}(\gamma^{bk})\right)\Big]. \end{aligned} \tag{63}$$

Substituting (62) and (63) into (52) yields the average SINR coverage probability as

$$\begin{aligned}\mathbb{P}(\gamma) &= \int_{\gamma}^{\gamma^{bk}} f_{\mathrm{SNR}_S}(S)\mathrm{d}S + \overline{F}_{\mathrm{SIR}_M}(\gamma) \\ &\quad - \overline{F}_{\mathrm{SIR}_M}(\gamma)\overline{F}_{\mathrm{SNR}_S}(\gamma) + \overline{F}_{\mathrm{SNR}_S}(\gamma^{bk}) \\ &= \overline{F}_{\mathrm{SIR}_M}(\gamma) + \overline{F}_{\mathrm{SNR}_S}(\gamma) F_{\mathrm{SIR}_M}(\gamma). \end{aligned} \tag{64}$$

Finally, based on the CCDF of SIR_M and SNR_S, given in (23) and (26), respectively, the desired average UL SINR coverage probability when $\gamma \leq \gamma^{bk}$ is as derived in (59).

- Case 2, $\gamma > \gamma^{bk}$
  The average SINR coverage probability in (60) is obtained in accordance with (57), (58) as well as the CCDF of SIR_M given in (23).

The proof is complete. ■

Based on Theorem 2, several remarks can be obtained as follows.

Remark 1: Since $\lambda_S\pi$ in (59) is a constant and $\lambda_S\pi \neq 0$, the term $\lambda_S\pi\xi$ can be replaced by ζ. Then, the average UL SINR coverage probability in Case 1 can be transformed into

$$\mathbb{P}(\gamma) = \frac{1+\rho(\gamma)\int_0^{+\infty} e^{-\frac{\gamma}{Q}\sigma^2\zeta^{\frac{\alpha}{2}}(\lambda_S\pi)^{-\frac{\alpha}{2}}} e^{-\zeta}\, d\zeta}{1+\rho(\gamma)}. \tag{65}$$

Note that, the integrand in (65) is nonnegative and increases as λ_S grows, whereas λ_S has no influence on $\rho(\gamma)$. Hence, there is an increase in $\mathbb{P}(\gamma)$ as λ_S increases.

Remark 2: Since λ_S is not involved in (60), the average UL SINR coverage probability in Case 2 remains constant with the variation of λ_S. This can be interpreted as follows. On one hand, when the SINR threshold γ satisfies $\gamma \leq \gamma^{bk}$, the distance between a user and its associated SBS decreases as SBSs get denser, which increases the UL SNR at SBSs. Thus, the SNR coverage probability of those associated with SBSs increases. Besides, since there exists no inter-tier interference, the increasingly deployment of SBSs has no effect on the SIR coverage probability at MBSs. As a result, the average UL SINR coverage probability improves as SBSs get denser. On the other hand, when $\gamma > \gamma^{bk}$, the SNR of each user associated to an SBS is always lower than γ due to the limited backhaul capacity. Moreover, the growing λ_S does not affect the SIR coverage probability at MBSs. Therefore, the average SINR coverage probability remains unchanged with more SBSs being deployed.

Additionally, the asymptotic results of the average UL SINR coverage probability as λ_S grows towards infinity is derived in the following corollary.

Corollary 3: In ultra-dense networks, the average SINR coverage probabilities when $\lambda_S \to +\infty$ in both cases are

- *Case 1, $\gamma \leq \gamma^{bk}$,*

$$\lim_{\lambda_S \to +\infty} \mathbb{P}(\gamma) = 1, \tag{66}$$

- *Case 2, $\gamma > \gamma^{bk}$,*

$$\lim_{\lambda_S \to +\infty} \mathbb{P}(\gamma) = \frac{1}{1+\rho(\gamma)}, \tag{67}$$

respectively.

Proof: Since the average SINR coverage probability in Case 2 is independent of λ_S, we only focus on the proof of the limitation in Case 1. According to (65) and similar with the proof of Corollary 1, in Case 1 where $\gamma \leq \gamma^{bk}$, the integral $\int_0^{+\infty} e^{-\frac{\gamma}{Q}\sigma^2\zeta^{\frac{\alpha}{2}}(\lambda_S\pi)^{-\frac{\alpha}{2}}} e^{-\zeta}\mathrm{d}\zeta$ is uniformly convergent. Hence, the limitation of the integral can be transformed into

$$\begin{aligned}\lim_{\lambda_S \to +\infty} \mathbb{P}(\gamma) &= \frac{1+\rho(\gamma)\int_0^{+\infty} \lim_{\lambda_S \to +\infty} e^{-\frac{\gamma}{Q}\sigma^2\zeta^{\frac{\alpha}{2}}(\lambda_S\pi)^{-\frac{\alpha}{2}}} e^{-\zeta}\, \mathrm{d}\zeta}{1+\rho(\gamma)} \\ &= \frac{1+\rho(\gamma)\int_0^{+\infty} e^{-\zeta}\mathrm{d}\zeta}{1+\rho(\gamma)} \\ &= 1. \end{aligned} \tag{68}$$

■

Based on Theorem 2, the simplified average SINR coverage probabilities can be obtained for a special case where noise at each SBS is neglected, i.e. $\sigma^2 = 0$. In this case, the SINR a user can achieve is the higher one between γ^{bk} and SIR_M. Then, we have $\mathbb{P}(\gamma) = 1$ when $\gamma \leq \gamma^{bk}$, while $\mathbb{P}(\gamma)$ is the same as that in (60) when $\gamma > \gamma^{bk}$. Besides, note that, the average SINR coverage probabilities when $\sigma^2 = 0$ are the same as those when $\lambda_S \to +\infty$. The reason is that the noise at SBSs is neglectable when it is much smaller than the desired signal power, which means high SNR. While the small pathloss, which is due to the ultra-dense deployment of SBSs, leads to high SNR as well.

Furthermore, it is worth noting that the average UL SINR coverage probability has a discontinuity point, which is elaborated in Corollary 4.

Corollary 4: There exists a point $\gamma = \gamma^{bk}$ at which the average SINR coverage probability $\mathbb{P}(\gamma)$ is discontinuous and the point $\gamma = \gamma^{bk}$ is a jump discontinuity.

Proof: According to (60), we have

$$\lim_{\gamma \to \gamma^{bk}} \mathbb{P}(\gamma) = \frac{1}{1+\rho(\gamma^{bk})}. \tag{69}$$

However, when $\gamma = \gamma^{bk}$, it is obtained from (59) that

$$\mathbb{P}(\gamma^{bk}) = \frac{1+\rho(\gamma^{bk})\int_0^{+\infty} e^{-\frac{\gamma^{bk}}{Q}\sigma^2\xi^{\frac{\alpha}{2}}}\pi\lambda_S e^{-\lambda_S\pi\xi}\, d\xi}{1+\rho(\gamma^{bk})} \tag{70}$$

where $\rho(\gamma^{bk})\int_0^{+\infty} e^{-\frac{\gamma^{bk}}{Q}\sigma^2\xi^{\frac{\alpha}{2}}}\pi\lambda_S e^{-\lambda_S\pi\xi} d\xi$ is a constant which is positive. Thus, it can be concluded that

$$\mathbb{P}(\gamma^{bk}) > \lim_{\gamma \to \gamma^{bk}} \mathbb{P}(\gamma) \tag{71}$$

which means $\gamma = \gamma^{bk}$ is a point of discontinuity. Besides, since the one-sided limit from the negative direction and that from the positive direction at γ^{bk} both exist and are finite while are not equal to each other, the point $\gamma = \gamma^{bk}$ is a jump discontinuity of the average UL SINR coverage probability. The proof is complete. ■

Based on Theorem 2, a special case where $\alpha = 4$ is considered. Consequently, the simplified average UL SINR coverage probability is obtained in the following corollary.

Corollary 5: When $\alpha = 4$, the average UL SINR coverage probability for Case 1 can be simplified using erfc(x), while the closed-form results for Case 2 can be derived. The results are as follows.

- *Case 1, $\gamma \le \gamma^{bk}$,*

$$\mathbb{P}(\gamma) = \frac{1+\frac{\sqrt{Q}\pi^{\frac{3}{2}}\lambda_S\left(\frac{\pi}{2}-\arctan\gamma^{-\frac{1}{2}}\right)}{2\sigma} e^{\frac{\pi^2\lambda_S^2 Q}{4\gamma\sigma^2}}\, erfc\left(\frac{\pi\lambda_S\sqrt{Q}}{2\sigma\sqrt{\gamma}}\right)}{1+\gamma^{1/2}\left(\frac{\pi}{2}-\arctan\gamma^{-1/2}\right)}, \tag{72}$$

- *Case 2, $\gamma > \gamma^{bk}$,*

$$\mathbb{P}(\gamma) = \frac{1}{1+\gamma^{1/2}\left(\frac{\pi}{2}-\arctan\gamma^{-1/2}\right)}. \tag{73}$$

Next, aiming to validate the analytical results, simulations will be given and the influence of the limited backhaul capacity on the average UL SINR coverage probability will be shown.

V. NUMERICAL RESULTS

In this section, the analytical results of association probabilities and the SINR coverage probabilities are validated via simulations. A HetNet consisting of an MBS tier and an SBS tier is considered. It is assumed that each user associates with its nearest BS for initialization. Then, user associations are established using the maximum UL capacity policy described in Section III. The main parameters and the default values for the simulations are listed in TABLE 2. Here, the results are based on 10000 Monte Carlo trials.

TABLE 2. Simulation parameters.

Parameter	Value
Q	0.1W
σ^2	10^{-12}W
λ_M	$1\times 10^{-6}/m^2$
α	4
C_S^{bk}	1.5 nats/s

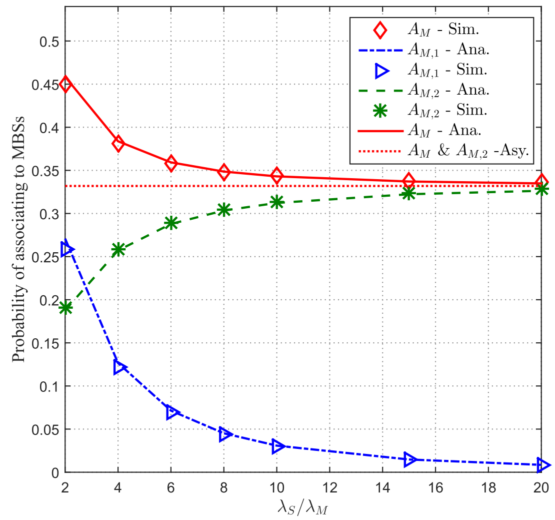

FIGURE 2. Probability of associating with MBSs versus the ratio of BSs' densities, i.e. λ_S/λ_M.

Firstly, Fig. 2 depicts the probability of associating with MBSs as a function of the ratio of SBSs' and MBSs' densities. It can be seen that with the increasingly deployment of SBSs, the probability $A_{M,2}$ increases while $A_{M,1}$ as well as A_M decreases. The reason is that as SBSs get denser, the distance between a user and its nearest SBS decreases, thereby boosting the achievable capacity at SBSs. Hence, the probability that the achievable capacity at SBSs is lower than that at MBSs and the limited backhaul capacity, i.e. $A_{M,1}$, decreases. However, since the achievable capacity at SBSs increases, the probability that it is higher than the backhaul capacity constraint increases. Besides, the probability that the achievable capacity at MBSs is higher than C_S^{bk} remains constant. Consequently, the probability $A_{M,2}$ increases. Moreover, in general, more SBSs being deployed implies higher capacity that each SBS can offer to its user. Thus, users are more likely to associate with SBSs. Furthermore, it is worth noting that the asymptotic results of A_M, $A_{M,1}$ and $A_{M,2}$ are expected to be $1/(1+\rho(\gamma^{bk}))$, 0 and $1/(1+\rho(\gamma^{bk}))$, respectively, according to the analysis in Section III. Indeed, the expected results can be observed in Fig. 2. The asymptotic bounds exist due to the fact that when λ_S further increases, the achievable capacity at SBSs is quite likely to be higher than the backhaul capacity. In this way, the further increase of λ_S barely affects the association probabilities and thus the association probabilities converge to their bounds.

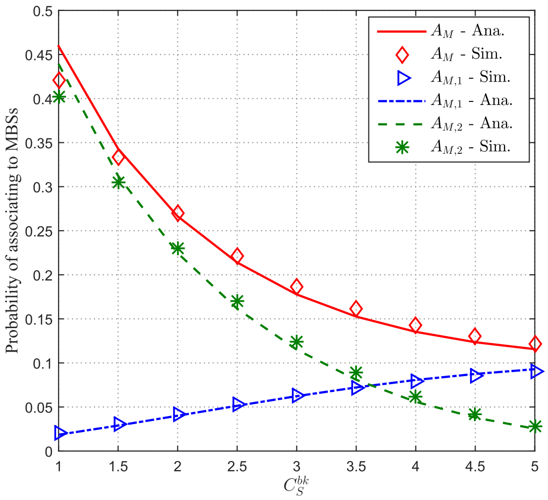

FIGURE 3. **Probability of associating with MBSs versus the backhaul capacity constraint, i.e. C_S^{bk} where $\lambda_S = 10\lambda_M$.**

Secondly, the probability of associating to MBSs with the variation of the limited backhaul capacity, i.e., C_S^{bk}, is illustrated in Fig. 3. As shown, the probability $A_{M,1}$ increases with the growth of C_S^{bk} while $A_{M,2}$ decreases. This phenomenon is caused by the increasing probability that $C_S^{Access} < C_S^{bk}$ as C_S^{bk} gets higher, which is consistent with the previous analysis in Section III. Besides, the sum of $A_{M,1}$ and $A_{M,2}$, i.e., A_M, namely the probability that users will associate with MBSs, decreases with the increase of C_S^{bk}. The reason is that as the backhaul capacity constraint grows, users associated to SBSs are allowed to achieve higher capacity, thus the probability of associating to SBSs boosts.

Next, we focus on the validation of the analytical average UL SINR coverage probabilities. The variation of the average UL SINR coverage probability is illustrated as a function of the SINR threshold γ for different C_S^{bk} ($C_S^{bk} = [1, 1.5, 2]$ nats/s) in Fig. 4. As Fig. 4 suggested, the average UL SINR coverage probability decreases with the increase of SINR threshold. Specifically, a steep fall exists between 1.7 and 1.8 when $C_S^{bk} = 1$ nats/s (3.4 and 3.5 when $C_S^{bk} = 1.5$ nats/s, 6.3 and 6.4 when $C_S^{bk} = 2$ nats/s). This coincides with the analysis in Section IV that the average SINR coverage probability is discontinuous and the point γ^{bk} where $\gamma^{bk} \approx 1.7183$ ($\gamma^{bk} \approx 3.4817$ and $\gamma^{bk} \approx 6.3891$), which corresponds to the backhaul capacity constraint $C_S^{bk} = 1$ nats/s ($C_S^{bk} = 1.5$ nats/s and $C_S^{bk} = 2$ nats/s), is a jump discontinuity. The discontinuity point exists due to the fact that when $\gamma \leq \gamma^{bk}$, both the SIR of users associated with MBSs and the SNR of users associated with SBSs might be higher than γ, while only the SIR of users associated with MBSs might exceed γ when $\gamma > \gamma^{bk}$. Moreover, it can be observed that as we expected, the growth of γ^{bk} has no

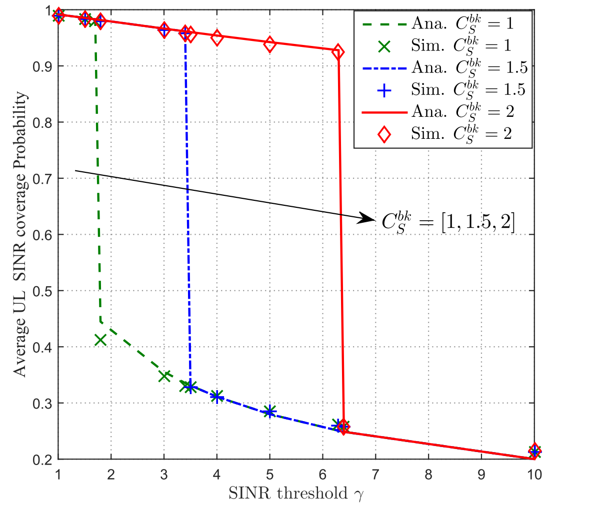

FIGURE 4. **The average UL SINR coverage probability versus the SINR threshold γ with $\lambda_S = 10\lambda_M$ for different C_S^{bk} ($C_S^{bk} = [1, 1.5, 2]$ nats/s).**

influence on the average UL SINR coverage probabilities in case 1 and case 2 but let the value of the discontinuity point increases.

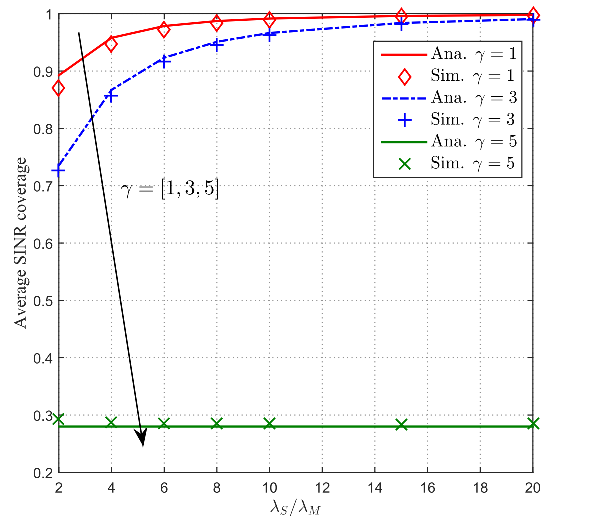

FIGURE 5. **The average UL SINR coverage probability versus the density of SBSs for different SINR thresholds ($\gamma = [1, 3, 5]$).**

Finally, Fig. 5 presents the variation of the average UL SINR coverage probability with the ratio of BSs' densities, where different SINR thresholds, i.e. $\gamma = [1, 3, 5]$, is considered. As shown, when $\gamma \leq \gamma^{bk}$, i.e. $\gamma = 1$ and $\gamma = 3$, the average UL SINR coverage probability boosts as SBSs get denser. This is due to the higher SNR at SBSs caused by the increasing λ_S. Besides, as λ_S further increases, the average SINR coverage probability approximates 1, which is pointed out in Corollary 3. However, when $\gamma > \gamma^{bk}$, namely $\gamma = 5$, the average UL SINR coverage probability remains constant with the variation of λ_S. This follows the discussions in Section IV where it is elaborated that the increasing λ_S does not affect the SIR coverage probability of users associated with MBSs and that the SNR at SBSs cannot exceed γ when $\gamma > \gamma^{bk}$.

As a final remark, with the increasing deployment of SBSs, users are more likely to associate with SBSs. Moreover, when the SINR threshold is lower than the SNR at SBSs that corresponds to the backhaul capacity, the decreasing distance between each user and its associated SBS leads to the enhanced average UL SINR coverage probability. Therefore, deploying more SBSs helps improve the UL performance significantly for users with low capacity, more specifically, those whose capacities at SBSs are lower than the backhaul capacity constraint.

VI. CONCLUSION

In this paper, the problem of decoupled access in a two-tier HetNet with non-ideal backhaul between MBSs and SBSs has been investigated. The capacity-based user association policy with limited backhaul capacity considered is proposed. The association probabilities and UL SINR coverage probabilities are studied where theoretical analysis is derived and numerical results are shown. Moreover, the main results reveal that denser SBSs are significantly beneficial to users with low capacity while are not very helpful in further enhancing UL performance for those with high capacity. Finally, the analytical and simulation results provide a guideline for the design of HetNets with backhaul constrained SBSs.

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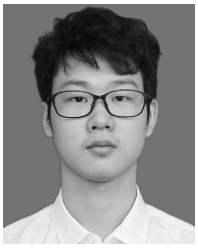

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