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Statistical Monitoring of Process Capability Index Having One Sided Specification Under Repetitive Sampling Using an Exact Distribution

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ABSTRACT This paper proposes a control chart using repetitive sampling for monitoring process capability index having one-sided specification limit using an exact distribution. The performance of the proposed chart is evaluated in terms of average run lengths using the exact probability distribution of the process capability index. The result shows that the proposed chart is more efficient by indicating the quicker out-of-control signals than the one using single sampling.

INDEX TERMS Control chart, process capability indices, repetitive sampling, average run length.

I. INTRODUCTION

Shewhart [18] is the pioneer of control charts used in the statistical monitoring of manufacturing processes and improvement of service organizations. He designed the control charts to detect the assignable causes of variation in process mean or variance to avoid the unstable process. Unstable processes may result in poor quality products and services in the market.

Process capability index (PCI) plays a crucial role in evaluating the online process performance to produce better products according to the upper and/or the lower specification limits. Chen *et al.* [7] highlighted that there are many characteristics of industrial products such as tensile strength and compression strength that are better when they are larger. Similarly, defects in one square meter painting, the degree of radiation are desired to be better when they are smaller. Kane [8] designed the indices C_{pu} and C_{pl} to measure the capability of such processes having one-sided specification limits. Pearn and Chen [11] approved $\tilde{C}_{pu}$ and $\tilde{C}_{pl}$ as PCI's estimators for one-sided specification limits following the non-central t distribution. Use of the exact distribution gives authentic outcomes. Non-central t distribution has wide applications in statistical inference and robust modeling of data. A traditional Shewhart control chart monitors only the process mean and variance but a control chart based on PCI provides a more comprehensive way to monitor the process performance. The latter not only monitors the stability of process's quality but also monitors the quality of the process, see (Chen *et al.* [5], Boyles [4], and Spring [12]).

A new one-sided control chart based on C_{pu} or C_{pl} with non-central t distribution is designed in this study and incorporates repetitive sampling to monitor the decrease in process capability more rapidly. Repetitive sampling is more efficient than single sampling and double sampling in decision making as it investigates the samples repetitively when the decision may not be obvious.

Control charts with PCIs can be seen in studies of Spiring [14], Boyles [4], Spring [12], Sarkar and Pal [13], Montgomery [9], Subramani [15] and [16], Chen *et al.* [5] and Subramani and Balamurali [17]. Ahmad *et al.* [3] designed X-bar control charts based on process capability index C_p using repetitive sampling and Ahmad *et al.* [2] designed a repetitive sampling control chart based on process capability index C_{pk}. Aslam *et al.* [1] designed a t-chart for process capability index C_{pk}.

Chen *et al.* [5] developed the control limits based on C_{pU} and C_{pl} by fixing type I error $\alpha = 0.01$. In literature, there is no work on developing natural tolerance limits by using the first two moments of PCI's C_{pU} and C_{pl} for one-sided specification limits. Proposed one-sided control chart based on C_{pu} and C_{pl} using repetitive sampling RS detects the decrease in C_{pU} and C_{pl} more quickly than with single sampling.

II. PROCESS CAPABILITY INDICES FOR ONE-SIDED SPECIFICATION

Processes capability indices C_{pl} and C_{pu} proposed by Kane [8] are given by

$$C_{pl} = \frac{\mu - LSL}{3\sigma}, \quad C_{pu} = \frac{USL - \mu}{3\sigma}$$

where USL and LSL are the upper and lower specification limits, respectively, μ is the process mean, and σ is the process standard deviation. For normally distributed processes with one-sided specification limits, C_{pl} or C_{pu} provides an effective measure for the process capability. In practice, sample data needs to be collected to estimate the true process capability. Suppose a random sample is taken from a stable process to estimate the indices. Then, the following natural estimators are considered

$$\hat{C}_{pu} = \frac{USL - \bar{x}}{3s} \tag{1a}$$

$$\hat{C}_{pl} = \frac{\bar{x} - LSL}{3s}, \tag{1b}$$

where $\bar{x}$ is the sample mean, and s is the sample standard deviation. Given that the observations are from a normal distribution, Chou and Owen [6] showed that the estimator $\hat{C}_{pl}$ and $\hat{C}_{pu}$ are distributed as $t_{n-1,\delta 1}/3\sqrt{n}$ and $t_{n-1,\delta 2}/3\sqrt{n}$, respectively, where $t_{n-1,\delta 1}$ is a non-central t distribution with $n-1$ degrees of freedom and non-central parameter $\delta 1 = 3\sqrt{n}C_{pl}$, and $t_{n-1,\delta 2}$ is denoted similarly.

The unbiased estimators $\tilde{C}_{\mathrm{pu}}$ and $\tilde{C}_{\mathrm{pl}}$ with a correction factor was recommended by Pearn and Chen [11]:

$$\tilde{C}_{\mathrm{pu}} = \mathrm{b}_{\mathrm{n}-1}\hat{C}_{\mathrm{pu}} \tag{2a}$$

$$\tilde{C}_{\mathrm{pl}} = \mathrm{b}_{\mathrm{n}-1}\hat{C}_{\mathrm{pl}} \tag{2b}$$

where the correction factor is given by

$$\mathrm{b}_{\mathrm{n}-1} = [2/(\mathrm{n}-1)]^{1/2}\,\Gamma[(\mathrm{n}-1)/2]/\Gamma[(\mathrm{n}-2)/2].$$

For the convenience of presentation either $\tilde{C}_{\mathrm{pu}}$ or $\ldots\tilde{C}_{\mathrm{pl}}$ is denoted as $\tilde{C}_s$ in the subsequent sections. Also, we express C_{pu} or C_{pl} as C_s. Then, the probability density function of $\tilde{C}_s$ can be expressed as

$$\mathrm{f}_{\tilde{C}_s}(\mathrm{x}) = \frac{3\sqrt{\mathrm{n}}/(\mathrm{n}-1)\,2^{-\mathrm{n}/2}}{\mathrm{b}_{\mathrm{n}-1}\sqrt{\pi}\,\Gamma[(\mathrm{n}-1)/2]}\int_0^\infty \mathrm{t}^{(\mathrm{n}-2)/2} \times\exp\left\{\frac{-1}{2}\left[\mathrm{t}+\left(\frac{3\mathrm{x}\sqrt{\mathrm{nt}}}{\mathrm{b}_{\mathrm{n}-1}\sqrt{\mathrm{n}-1}}-\delta\right)^2\right]\right\}\mathrm{dt} \tag{3}$$

where $\delta = 3\sqrt{\mathrm{n}}\mathrm{C_s}$. Since $\hat{C}_s$ is distributed as $t_{n-1,\,3\sqrt{n}C_s}/3\sqrt{n}$, Pearn and Chen [11] derived the variance of $\tilde{C}_s = b_{n-1}\hat{C}_s$ as

$$Var\left(\tilde{C}_s\right) = a_n C_s^2 + \frac{1}{9}a_n\,(4) \tag{4}$$

where

$$a_n = \frac{\Gamma[(n-1)/2]\,\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^2}$$

III. PROPOSED CONTROL CHART FOR C_s USING REPETITIVE SAMPLING

A. CONTROL LIMITS

We propose a control chart for monitoring process capability having the following charting procedure:

Step 1: Select a random sample of size n from the manufacturing process. Calculate the sample mean $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ and the sample variance $s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1)$. Also calculate $\tilde{C}_s = \mathrm{b}_{\mathrm{n}-1}\hat{C}_{pl}$ or $\tilde{C}_s = \mathrm{b}_{\mathrm{n}-1}\hat{C}_{pu}$ depending on the use of LSL or USL, where $\hat{C}_{pl} = \frac{\bar{x}-LSL}{3s}$, $\hat{C}_{pu} = \frac{USL-\bar{x}}{3s}$, $\mathrm{b}_{\mathrm{n}-1} = [2/(\mathrm{n}-1)]^{1/2}\,\Gamma[(\mathrm{n}-1)/2]/\Gamma[(\mathrm{n}-2)/2]$.

Step 2: The process will be declared as in control if $\tilde{C}_s > LCL_1$ and out-of-control if $\tilde{C}_s < LCL_2$. The process will be declared in-decision state if $LCL_2 < \tilde{C}_s < LCL_1$, and, go to Step 1 and repeat the process.

It should be noted here that two one-sided control limits are proposed here, whose forms are given as below:

$$LCL_1 = C_s - k_1\sqrt{a_n C_s^2 + \frac{1}{9}a_n} \tag{5a}$$

$$LCL_2 = C_s - k_2\sqrt{a_n C_s^2 + \frac{1}{9}a_n} \tag{5b}$$

Here, LCL_1 is lower control limit and LCL_2 upper control limit such that $LCL_1 \geq LCL_2$, a_n is constant defined in Eq. (4) and $C_s = \tilde{\tilde{C}}_{pl}$ is average of process capability index. When $LCL_1 = LCL_2$ or $k_1 = k_2$, it reduces to a control chart without using repetitive sampling. The control coefficients k_1 and k_2 should be determined by considering the target in-control average run length, which will be discussed later. It is assumed that C_s is known for evaluating the chart performance. In practice, however, it should be estimated from a preliminary sample data.

B. PERFORMANCE OF THE CONTROL CHART

Performance of the proposed control chart is evaluated through average run lengths (ARLs) as usual. In simple words, ARL is the expected number of samples taken before the shift in process is detected (Montgomery, 2013).

The probability of declaring as in-control P_{in} for the proposed control chart is calculated as follows:

$$P_{in}^0 = \frac{\beta}{1-\mathrm{P}_{\mathrm{rep}}^0} \tag{6}$$

where β is the probability that the process is declared as in-control based on the single sample

$$\beta = P\left\{\tilde{C}_s > LCL_1\right\} = \mathrm{P}\left\{3\sqrt{n}\tilde{C}_s\Big/b_{n-1} > \frac{3\sqrt{\mathrm{n}}\mathrm{LCL}_1}{b_{n-1}}\right\} = \mathrm{P}\left\{\mathrm{t}_{\mathrm{n}-1,\delta} > \frac{3\sqrt{\mathrm{n}}\mathrm{LCL}_1}{b_{n-1}}\right\}$$

where $\delta = 3\sqrt{n}C_s$

In Eq.(6), $\mathrm{P}_{\mathrm{rep}}^0$ is the repetition probability given by

$$\mathrm{P}_{\mathrm{rep}}^0 = \mathrm{P}\left\{t_{n-1,\delta} < \frac{3\sqrt{n}\mathrm{LCL}_1}{b_{n-1}}\right\} - \mathrm{P}\left\{t_{n-1,\delta} < \frac{3\sqrt{n}\mathrm{LCL}_2}{b_{n-1}}\right\}$$

TABLE 1. ARL_1 for proposed control chart with RS when $\acute{C}_s = mC_s, C_s = 2$.

m (Shift)	$n = 5$			$n = 10$			$n = 15$		
	k_1, k_2								
	0.7030 1.1702	0.7059 1.1586	0.6048 1.1404	1.1853 1.6859	1.1868 1.6626	1.2233 1.6140	1.1849 1.8900	1.1857 1.8608	1.1791 1.8016
	ARL								
1	370.37	300.06	200.58	370.01	300.11	200.66	370.27	300.12	200.25
0.9	83.99	71.28	49.65	61.45	52.56	39.41	44.89	38.62	28.89
0.8	22.04	19.55	14.19	13.07	11.77	9.88	7.65	6.99	5.87
0.7	6.92	6.41	4.92	3.79	3.59	3.33	2.18	2.11	1.97
0.6	2.81	2.70	2.25	1.70	1.67	1.65	1.22	1.21	1.19
0.5	1.58	1.56	1.42	1.17	1.17	1.17	1.04	1.03	1.03
0.4	1.19	1.19	1.14	1.04	1.04	1.04	1.004	1.004	1.004
0.3	1.06	1.06	1.04	1.006	1.006	1.006	1.00	1.00	1.00
0.2	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.003	1.00	1.002	1.00	1.00	1.00	1.00	1.00	1.00

Therefore,

$$P_{in}^{0} = \frac{P\left\{t_{n-1,\delta} > \frac{3\sqrt{n}LCL_1}{b_{n-1}}\right\}}{1 - P\left\{t_{n-1,\delta} < \frac{3\sqrt{n}LCL_1}{b_{n-1}}\right\} + P\left\{t_{n-1,\delta} < \frac{3\sqrt{n}LCL_2}{b_{n-1}}\right\}} \quad (7)$$

Finally, the in-control ARL is obtained by

$$ARL_o = \frac{1}{1-P_{in}^{0}} \quad (8)$$

Now, let us consider a shifted process. Suppose that the process capability index C_s changes to

$\acute{C}_s = mC_s, (m \leq 1)$, when there is a shift in the process. Here $m \leq 1$ to see the downward shift in the process. Similar to the in-control ARL, the ARL for shifted process is obtained by

$$ARL_1 = \frac{1}{1-P_{in}^{1}} \quad (9)$$

where P_{in}^{1} is the probability that the process is declared as in-control for a shifted process, which is obtained by

$$P_{in}^{1} = \frac{P\left\{\tilde{C}_s > LCL_1 | mC_s\right\}}{1 - P\left\{\tilde{C}_s < LCL_1 | mC_s\right\} + P\left\{\tilde{C}_s < LCL_2 | mC_s\right\}}$$

$$= \frac{P\left\{t_{n-1,\delta'} > \frac{3\sqrt{n}LCL_1}{b_{n-1}}\right\}}{1 - P\left\{t_{n-1,\delta'} < \frac{3\sqrt{n}LCL_1}{b_{n-1}}\right\} + P\left\{t_{n-1,\delta'} < \frac{3\sqrt{n}LCL_2}{b_{n-1}}\right\}} \quad (10)$$

where $\delta' = 3m\sqrt{n}C_s$.

Above control chart coefficients are estimated using a program in R language for $C_s = 2$ with different sample sizes $n = 5, 10, 15$ and specified in-control average run length of $ARL_o = 370, 300, 200$. These control chart coefficients are shown in Table 1.

Table 1 shows that out-of-control ARL_1 decreases very fast as there is a decrease in the process capability index C_s. Proposed control chart detects the smaller shifts more quickly when the sample size is larger. For example, when $n = 5$, $ARL_o = 200.58$ proposed control chart shows the signal out of control after 49 samples on average but for $n = 15$ it shows the signal out-of-control after 28 samples on average.

In Table 2, the out-of-control ARL's of proposed control charts based on C_s using repetitive sampling (RS) and single sampling (SS) are being compared. Use of repetitive sampling saves the resources and detects the shifts quickly. For example, for $n = 5, ARL_o = 300.06$ proposed control chart with RS signals out of control after 71.28 samples on average but with SS it shows signals out of control after 82.58 samples on average.

Comparison of Table 1 and Table 2 shows that even for small sample size the use of repetitive sampling gives more chances to detect the shifts more quickly. The following algorithm is used to find the control chart coefficients.

1) specify the values of ARL_o, say r_0
2) find the suitable values of k_1 and k_2 such that $ARL_o \geq r_0$.
3) Find ARL_1 using k_1 and k_2 for various shifts.

TABLE 2. **ARL_1 for control chart of C_s with SS when $\acute{C_s} = mC_s$, $C_s = 2$.**

m (Shift)	$n = 5$			$n = 10$			$n = 15$		
	k								
	1.1605	1.1499	1.1260	1.6797	1.6563	1.6079	1.8792	1.8497	1.7898
	ARL								
1	370.13	300.85	206.16	370.39	300.51	200.04	370.63	300.00	200.65
0.9	97.33	82.58	61.21	67.86	57.91	42.56	52.55	45.05	33.57
0.8	30.47	26.86	21.34	16.76	14.95	11.98	11.25	10.13	8.30
0.7	11.37	10.36	8.75	5.59	5.18	4.47	3.64	3.42	3.03
0.6	5.07	4.75	4.22	2.52	2.40	2.20	1.75	1.69	1.59
0.5	2.69	2.58	2.40	1.50	1.47	1.40	1.19	1.18	1.15
0.4	1.70	1.66	1.59	1.14	1.13	1.11	1.03	1.03	1.02
0.3	1.26	1.25	1.22	1.03	1.02	1.02	1.003	1.002	1.002
0.2	1.08	1.07	1.07	1.003	1.002	1.002	1.00	1.00	1.00
0.1	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00

1) INDUSTRIAL EXAMPLE

An industrial example is taken to explain the practical use of the proposed control chart. Chen *et al.* [5] published 25 samples of wire strength in the pull test with each size of eleven (n=11) shown in Table 3. The lower specification limit of the wire strength is 5g.

For the given data in Table 3, the proposed control limits for C_{pl} with $n = 11, ARL_o = 370.00, k_1 = 0.9012, k_2 = 1.7708$ are as follows

$$\text{Centerline} = C_s = \tilde{\tilde{C}}_{pl} = \frac{\sum_{i=1}^{25} \tilde{C}_{pl}}{25} = \frac{32.7920}{25} \approx 1.3$$

$$a_n C_s^2 + \frac{1}{9} a_n = \frac{\Gamma\left[(11-1)/2\right]\Gamma\left[(11-3)/2\right]}{\left(\Gamma\left[(11-2)/2\right]\right)^2} * 1.3^2 + \frac{1}{9}\frac{\Gamma\left[(11-1)/2\right]\Gamma\left[(11-3)/2\right]}{\left(\Gamma\left[(11-2)/2\right]\right)^2} = 0.1195$$

$$\text{LCL}_1 = 1.3 - (0.9012)\sqrt{0.1195} = 0.9885$$

$$\text{LCL}_2 = 1.3 - (1.7708)\sqrt{0.1195} = 0.6879$$

The above control limits are plotted in Figure 1. When the control statistics are drawn on Figure 1, there is no sign of an out-of-control process.

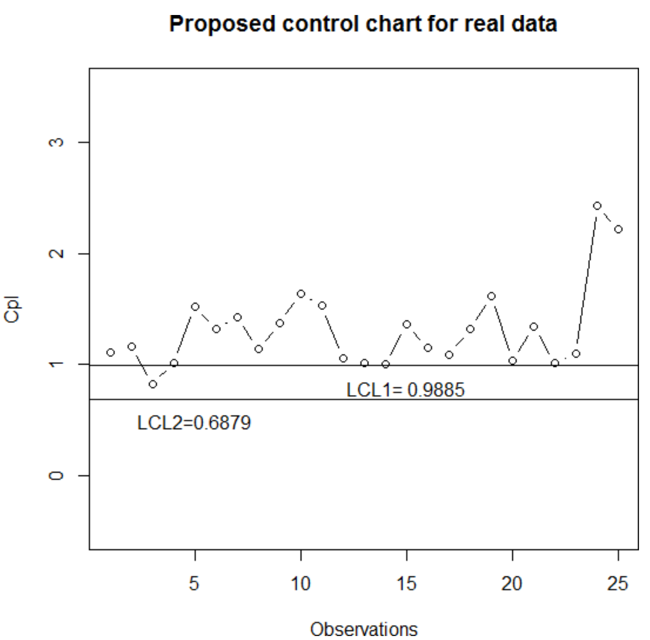

FIGURE 1. **The control chart for the proposed chart when $ARL_o = 370, n = 11$.**

IV. SIMULATION STUDY

In this section, the proposed method is explained through simulated data. The first 20 random samples of size 5 (=n) are generated from a non-central t distribution with degree of freedom $df = 4$ and the non-central parameter $\delta = 3\sqrt{n}C_s$, where $C_s = 2$. Then, the next 20 random samples of size 5 (= n) are generated from a non-central t distribution with degree of freedom f and non-central parameter $\delta = 3\sqrt{nm}C_s$, where m = 0.9.

The control coefficient from the in-control process with specified $ARL_o = 200.58, n = 5, C_s = 2$ is obtained

TABLE 3. Wire strength (*ingrams*) in the pull test.

No.	Observations							$\bar{X}$	S	$\tilde{C}_s$
	1	2	3	4	……	10	11			
1	8.930	6.073	7.714	7.845	……	9.259	8.080	8.071	0.926	1.106
2	7.893	7.837	7.227	7.119	……	8.466	8.358	8.167	0.905	1.166
3	7.657	8.102	5.655	8.216	……	6.179	8.253	7.627	1.066	0.822
4	8.542	8.086	8.888	7.717	……	7.497	9.241	7.774	0.910	1.017
5	7.923	7.856	7.846	8.300	……	6.620	8.455	8.085	0.677	1.519
6	8.671	7.402	8.961	9.858	……	9.345	9.211	8.412	0.862	1.320
7	7.432	6.921	8.137	8.423	……	9.149	9.157	8.192	0.746	1.427
8	7.241	8.898	7.184	8.116	……	8.755	6.523	8.016	0.881	1.141
9	9.201	7.065	7.411	8.139	……	7.747	7.791	7.791	0.679	1.371
10	7.974	7.348	7.724	7.296	……	7.884	8.952	8.157	0.643	1.637
11	6.781	7.094	8.616	8.457	……	7.409	7.675	7.725	0.593	1.532
12	7.027	8.782	7.793	8.305	……	8.714	9.087	8.390	1.070	1.057
13	7.623	7.728	7.434	8.133	……	9.157	7.029	7.610	0.859	1.013
14	8.540	8.314	9.271	6.156	……	7.508	9.826	8.178	1.059	1.001
15	6.744	8.312	8.084	7.167	……	7.504	9.165	7.875	0.704	1.361
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
24	8.157	8.693	8.216	7.916	……	8.483	8.748	8.207	0.441	2.426
25	7.815	8.223	8.150	7.850	……	8.329	8.431	7.870	0.431	2.219

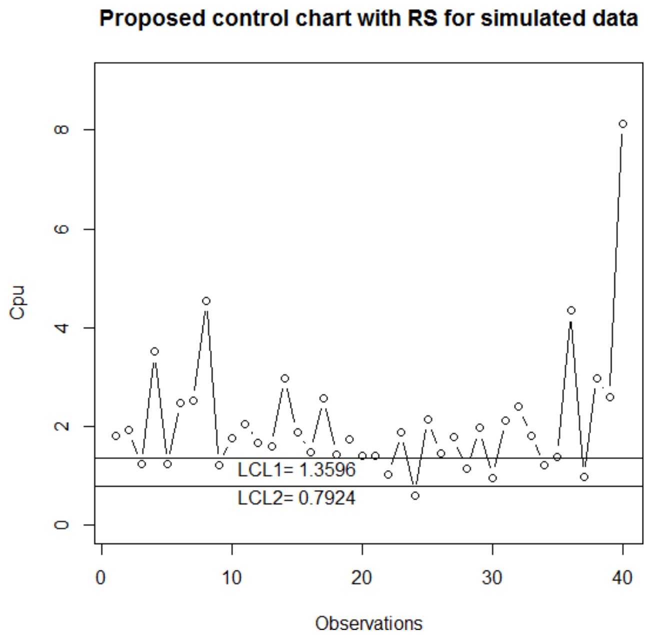

FIGURE 2. **The control chart for the simulated data.**

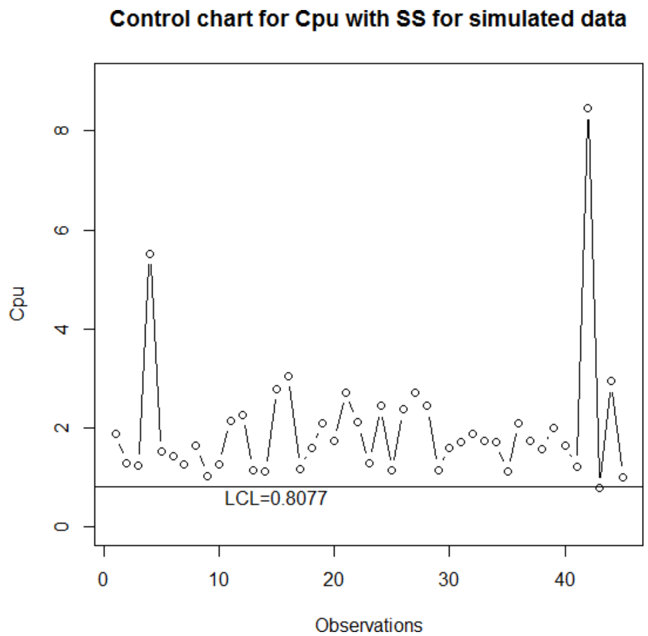

FIGURE 3. **The existing control chart for simulated data.**

from Table 2 by $k_1 = 0.6048, k_2 = 1.1404$. Therefore, the proposed control limits with RS are obtained as follows:

$$LCL_1 = 2 - 0.6048\sqrt{1.1213} = 1.3596,$$
$$LCL_2 = 2 - 1.1404\sqrt{1.1213} = 0.7924$$

When 40 simulated estimates $\tilde{C}_{s_i}$ are plotted with the proposed control limit in Figure 2, it shows the shift at 24^{th} observation (4^{th} observation after shift). While, the existing control chart in Figure 3 shows no shift in the process.

V. CONCLUSION

A control chart based on process capability indices C_{pu} or C_{pl} for a one-sided specification limit using repetitive sampling has designed in this study. Performance of the proposed control chart has been evaluated using the exact distribution through ARL_1. Use of repetitive sampling has been proved to be more efficient in detecting the shifts in C_{pu} or C_{pl} than using single sampling (SS). The proposed control chart can be extended for other types of process capability indices.

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REFERENCES

[1] M. Aslam, M. Mohsin, and C.-H. Jun, "A new t-chart using process capability index," *Commun. Statist. Simul. Comput.*, vol. 46, no. 7, pp. 5141–5150, 2016.
[2] L. Ahmad, M. Aslam, and C.-H. Jun, "The design of a new repetitive sampling control chart based on process capability index," *Trans. Inst. Meas. Control*, vol. 38, no. 8, pp. 971–980, 2015.
[3] L. Ahmad, M. Aslam, and C.-H. Jun, "Designing of X-bar control charts based on process capability index using repetitive sampling," *Trans. Inst. Meas. Control*, vol. 36, no. 3, Pp. 367–374, 2013.
[4] R. A. Boyles, "The Taguchi capability index," *J. Quality Technol.*, vol. 23, no. 1, pp. 17–26, 1991.
[5] K. S. Chen, H. L. Huang, and C. T. Huang, "Control charts for one-sided capability indices," *Quality Quantity*, vol. 41, no. 3, pp. 413–427, 2007.
[6] Y.-M. Chou and D. B. Owen, "On the distributions of the estimated process capability indices," *Commun. Statist.-Theory Methods*, vol. 18, no. 12, pp. 4549–4560, 1989.
[7] K. S. Chen, R. K. Li, and S. J. Liao, "Capability evaluation of a product family for processes of the larger-the-better type," *Int. J. Adv. Manuf. Technol.*, vol. 20, no. 11, pp. 824–832, 2002.
[8] V. E. Kane, "Process capability indices," *J. Quality Technol.*, vol. 18, no. 1, pp. 41–52, 1986.
[9] D. C. Montgomery, *Introduction to Statistical Quality Control*, 6th ed. New York, NY, USA: Wiley, 2009.
[10] D. C. Montgomery, *Introduction to Statistical Quality Control*. New York, NY, USA: Wiley, 2013.
[11] W. L. Pearn and K. S. Chen, "One-sided capability indices C_{PU} and C_{PL}: Decision making with sample information," *Int. J. Quality Reliab. Manage.*, vol. 19, no. 3, pp. 221–245, 2002.
[12] F. A. Spiring, "Process capability: A total quality management tool," *Total Quality Manage.*, vol. 6, no. 1, pp. 21–34, 1995.
[13] A. Sarkar and S. Pal, "Process control and evaluation in the presence of systematic assignable cause," *Quality Eng.*, vol. 10, no. 2, pp. 383–388, 1997.
[14] F. A. Spiring, "Assessing process capability in the presence of systematic assignable cause," *J. Quality Technol.*, vol. 23, no. 2, pp. 125–134, 1991.
[15] J. Subramani, "Process control in the presence of linear trend," *Model Assisted Statist. Appl.*, vol. 5, no. 4, pp. 272–282, 2010.
[16] J. Subramani, "Application of systematic sampling in process control, statistics and applications," *J. Soc. Statist., Comput. Appl.*, vol. 2, no. 2, pp. 7–17, 2004.
[17] J. Subramani and S. Balamurali, "Control charts for variables with specified process capability indices," *Int. J. Probab. Statist.*, vol. 1, no. 4, pp. 101–110, 2012.
[18] W. A. Shewhart, *Economic Control of Quality of Manufactured Product*. New York, NY, USA: Van Nostrand, 1931.

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