IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received March 27, 2018, accepted April 23, 2018, date of publication April 30, 2018, date of current version June 5, 2018.

*Digital Object Identifier 10.1109/ACCESS.2018.2831205*

# Uplink Sum Rate Analysis of Massive Distributed MIMO Systems Over Composite Fading Channels

**LI WANG[1], JIANDONG LI[1], (Senior Member, IEEE), JIAN-KANG ZHANG[2], (Senior Member, IEEE), AND RUI CHEN[1]**

[1]State Key Laboratory of Integrated Service Networks, Xidian University, Xi'an 710071, China
[2]Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON L8S 4K1, Canada

Corresponding author: Jiandong Li (jdli@ieee.org)

This work was supported in part by the National Natural Science Foundation of China under Grant 91638202, Grant 61231008, Grant 61301168, and Grant 61701360, and in part by the Program for Changjiang Scholars and Innovative Research Team in University under Grant IRT0852.

**ABSTRACT** As the number of antennas goes large in the massive multiple-input multiple-output (MIMO) scenario, the large-scale fading plays an increasingly important role in the system performance. In this paper, we investigate the uplink sum rate of a massive distributed MIMO system over composite fading channels portrayed by the Rayleigh–Lognormal (RLN) model, where both zero forcing (ZF) and ZF decision-feedback receivers are considered. By exploring a novel and generalized method based on the integral and limit theories, the sum rate is expressed as a series with regard to the negative power of the transmit antennas, which is followed by a brief discussion on the asymptotic behavior when the number of transmit antennas goes infinite. Numeral experiments testify our theoretical analysis and prove the effectiveness even when the accuracy parameter is small. The influence of the number and location of the radio ports is also discussed.

**INDEX TERMS** Massive distributed MIMO, composite fading channel, sum rate, zero forcing

## I. INTRODUCTION

Massive Multiple-Input Multiple-Output (MIMO) is first introduced in the seminal work [1] as a promising technology for the next generation cellular networks, since it has the potential to improve space gains by increasing the number of antennas at the base station (BS). The typical scenario portrays the communication between a BS of massive antennas and some users of a single antenna [2]–[6]. In Massive MIMO systems, the influence of fast fading and thermal noise vanishes as the amount of antennas increases [7]–[9].

On account of a more realistic deployment, Massive Distributed MIMO (D-MIMO) systems emphasize on the distributed structure, where the users are geographically scattered over an area and may be equipped with several antennas, such as radio ports (RPs) [10]–[12]. In Massive D-MIMO systems, different RPs experience different path loss and large-scale fading, which results in different transmission features. Therefore, in order to evaluate the performance of precoders directly and clearly, it is necessary to derive the closed-form sum rate, where the composite fading (both small-scale and large-scale) is important and indispensible to be considered [9], [11].

In the related literatures, the Rayleigh-Lognormal (RLN) model is widely applied to characterize the composite fading channels [13]. Up to now, many efforts have been made to the large-scale (i.e., log-normal) part since it is hard-tackled mathematically.

- Simplification. Regardless of the accuracy, it is straightforward to take the large-scale fading factor as a constant or just estimate its final effects on the performance as in [14], [15].
- Substitution. Another way is to replace the log-normal part with an analytically friendlier model. For instance, the Gaussian distribution is used to imitate the log-normal distribution in [16]; the gamma distribution is adopted as the large-scale part of Generalized-K fading model in studies [17]–[19]. Besides, authors in [11] propose the upper and lower bounds instead.
- Discretization. With the help of Gauss-Hermite Quadrature Integration (GHQI), the log-normal section is transformed to a series in [20]–[22]. In this case, the more items are involved, the more accurate the result is.
- Special cases. Beside the aforementioned methods, authors in [23] focus on the average signal-to-noise

ratio (SNR), so that only the expectation of the large-scale factor is required. Similarly in [24], authors aim at the symbol error probability (SEP), where two alternative formulas for $Q$- and $Q^2$- functions are introduced.

In sum, simplification provides a quick solution to the large-scale fading, but it is generally a coarse estimation. Substitution avoids the direct analysis of the log-normal distribution, yet it is not the universal case. Compared with these methods, discretization works out the first way to tackle with the RLN model directly. However, a great many items (e.g. at least 10 in [25]) are demanded to obtain a satisfying approximation. For some special cases, there may exist smart means to study the composite fading channels, but unfortunately they are only the minority situation.

In this paper, we propose a general method to analyze the sum rate of the Massive D-MIMO systems over composite fading channels, where the integral and limit theories are utilized. The solution provides a direct and concise way to evaluate the performance of the transmission techniques, which is instructive for the future studies of precoder design. In addition, we extend the conclusions under the infinite antenna case. It enables us to learn the asymptotic behavior of the sum rate as the number of antennas goes infinite. Considering the efficiency of simple signal processing techniques in Massive D-MIMO systems, we apply zero-forcing (ZF) receivers in our discussion and introduce zero-forcing decision-feedback (ZF-DF) receivers to make a comparison.

*Notation:* The bold letters of lowercase and uppercase represent vectors and matrices respectively (e.g., $\mathbf{v}$, $\mathbf{M}$) where the corresponding elements are denoted by $v_i$ and $[\mathbf{M}]_{ij}$. Especially, the subscripts of the diagonal elements are shortened to a single letter for the diagonal matrices, i.e., $[\mathbf{\Lambda}]_{ii}$ is simplified to $[\mathbf{\Lambda}]_i$ for the diagonal matrix $\mathbf{\Lambda}$. Notations $\mathbf{A}^{\mathrm{H}}$ and $\mathbf{A}^{\dagger}$ stand for the hermitian transpose and pseudo-inverse of a complex matrix $\mathbf{A}$. $\mathbb{E}[\cdot]$ implies the expectation of a random variable or a random matrix. $I_M$ is an $M \times M$ identity matrix.

## II. PROBLEM STATEMENT

In this paper, we investigate the uplink sum rate of a Massive D-MIMO system over composite fading channels, where $M$ antennas are configured at each RP. The $P$ RPs are separately located and concurrently served by a BS with $N$ antennas for $N > PM >> 1$, as shown in Fig.1.

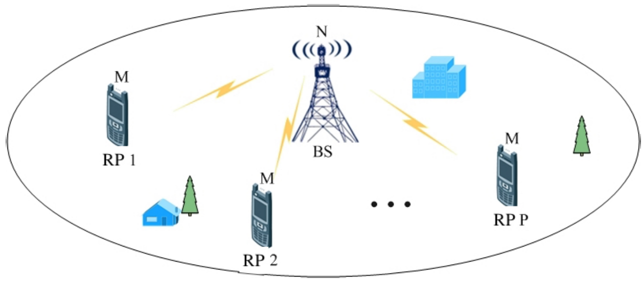


**FIGURE 1.** **A Massive D-MIMO System (*P* RPs each equipped with *M* antennas, a single BS equipped with *N* antennas, *N* > *PM* >> 1).**

Then the received signal is represented as

$$\mathbf{y} = \mathbf{H}\mathbf{\Sigma}^{\frac{1}{2}}\mathbf{\Xi}^{\frac{1}{2}}\mathbf{T}\mathbf{s} + \mathbf{n} \tag{1}$$

where the vectors $\mathbf{y} \in \mathbb{C}^{N\times 1}$, $\mathbf{s} \in \mathbb{C}^{PM\times 1}$ and $\mathbf{n} \in \mathbb{C}^{N\times 1}$ denote the received signal, the transmitted signal and the circularly-symmetric complex Gaussian noise respectively.

Suppose each RP is independent with each other, the corresponding segment of signal $\mathbf{s}$ is uncorrelated with each other and uncorrelated with the noise $\mathbf{n}$. The original SNR for each RP is assumed to be $\rho$.

Note that a composite fading model is applied in (1) to capture the full view of the channel state. $\mathbf{H} \in \mathbb{C}^{N\times PM}$ is an uncorrelated complex Gaussian matrix whose elements are independent identically distributed (i.i.d) with $\mathcal{CN}(0, 1)$. The covariance matrix $\mathbf{\Sigma} \in \mathbb{C}^{PM\times PM}$ is block diagonal, which implies that the links are correlated among the same RP but independent between different RPs. Hence $\mathbf{H}$ together with $\mathbf{\Sigma}$ characterizes the small-scale fading. $\mathbf{\Xi} \in \mathbb{C}^{PM\times PM}$ is also block diagonal, comprising $P$ diagonal matrices $\mathbf{\Xi}_p = diag\{\frac{\xi_p}{D_p^{\nu}}\} \in \mathbb{C}^{M\times M}$, $(p = 1, 2, ..., P)$. $\mathbf{T} \in \mathbb{C}^{PM\times PM}$ is a precoder.

For the small-scale fading part, we know $\left(\mathbf{H}\mathbf{\Sigma}^{\frac{1}{2}}\right)^{\mathrm{H}}\left(\mathbf{H}\mathbf{\Sigma}^{\frac{1}{2}}\right)$ is of Wishart distribution, in other words, $\left(\mathbf{H}\mathbf{\Sigma}^{\frac{1}{2}}\right)^{\mathrm{H}}\left(\mathbf{H}\mathbf{\Sigma}^{\frac{1}{2}}\right) \sim \mathcal{W}_{PM}(N, \mathbf{\Sigma})$. For the large-scale fading part, $D_p$ is the distance between the $p^{th}$ RP and the BS, $\nu$ is the path loss factor, and $\xi_p$ is modeled by the log-normal distribution with the probability density function (PDF)

$$p(\xi_p) = \frac{1}{\sqrt{2\pi}\sigma_{\xi_p}\cdot \xi_p} e^{-\frac{(\ln \xi_p - \mu_{\xi_p})^2}{2\sigma_{\xi_p}^2}}. \tag{2}$$

In order to focus on the sum rate analysis, we apply a precoder $\mathbf{T} = \frac{1}{\sqrt{M}}\cdot \mathbf{V}$ for simplicity, where $M$ is used to form the normalized SNR and $\mathbf{V}$ is from the eigenvalue decomposition of the covariance matrix $\mathbf{\Sigma} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{\mathrm{H}}$.

Therefore the transmission model (1) can be reorganized as

$$\mathbf{y} = \frac{1}{\sqrt{M}}\mathbf{H}\mathbf{V}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{\Xi}^{\frac{1}{2}}\mathbf{s} + \mathbf{n} \tag{3}$$

In the following part, we aim at the ZF receivers first, and then focus on the ZF-DF case. For simplicity of expression, we name the latter receivers 'DF' for short.

For the ZF receivers, we have

$$\mathbf{z} = \mathbf{s} + (\frac{1}{\sqrt{M}}\mathbf{H}\mathbf{V}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{\Xi}^{\frac{1}{2}})^{\dagger}\mathbf{n} \tag{4}$$

such that the sum rate with the predefined precoder is

$$R_{ZF} = \sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\mathbb{E}\{\log_2(1 + \frac{\rho[\mathbf{\Lambda}]_m[\mathbf{\Xi}]_p}{M\cdot[(\widetilde{\mathbf{H}}^{\mathbf{H}}\widetilde{\mathbf{H}})^{-1}]_{mm}})\} \tag{5}$$

where $\widetilde{\mathbf{H}} = \mathbf{H}\mathbf{V}$, whose covariance matrix is a $PM \times PM$ identity matrix. By defining $\widetilde{h}_m = \frac{1}{[(\widetilde{\mathbf{H}}^{\mathbf{H}}\widetilde{\mathbf{H}})^{-1}]_{mm}}$, we get the

PDF of $\widetilde{h}_m$ is

$$p(\widetilde{h}_m) = \begin{cases} \dfrac{1}{\Gamma(N-PM+1)} \widetilde{h}_m^{N-PM} \cdot e^{-\widetilde{h}_m}, & x \geq 0; \\ 0, & otherwise. \end{cases} \tag{6}$$

According to the PDFs of fading coefficients (2) and (6), we formulate the sum rate (5) in a double integral form as below.

$$R_{ZF} = \sum_{p=1}^{P} \sum_{m=(p-1)M+1}^{pM} \frac{\mathcal{T}_1}{(N-PM)!\sqrt{2\pi}\sigma_{\xi p}} \tag{7}$$

where

$$\mathcal{T}_1 = \int_0^\infty \int_0^\infty \left( \log_2(1 + \frac{\rho \cdot \lambda_m \cdot \xi_p}{M \cdot D_p^v} \widetilde{h}_m) \right) \cdot \widetilde{h}_m^{N-PM} \cdot e^{-\widetilde{h}_m} \xi_p^{-1} e^{-\frac{(\ln \xi_p - \mu_{\xi p})^2}{2\sigma_{\xi p}^2}} d\widetilde{h}_m d\xi_p$$

and $\lambda_m$ is the $m^{th}$ eigenvalue of channel covariance matrix $\mathbf{\Sigma}$, thus the $m^{th}$ element of $\mathbf{\Lambda}$.

## III. SUM RATE ANALYSIS

Prior to further study, we list two lemmas here for the convenience to handle the inner and outer integral in (5) respectively.

*Lemma 1:* Define an integral

$$\delta(x) = \int_0^\infty \frac{x^a}{(b^{-1}+x)^{a+c+1}} \cdot e^{-x} dx \tag{8}$$

where $b \in \mathbb{R}^+$ and $a, c \in \mathbb{Z}^+$ are constants. We have the following inequality

$$\delta(x) \leq \int_0^\infty \frac{1}{(b^{-1}+x)^{c+2}} dx = \frac{b^{c+1}}{c+1}, \tag{9}$$

since $0 \leq \frac{x}{e^x} \leq 1$ and $0 \leq \frac{x}{b^{-1}+x} \leq 1$ hold for $x \in [0, \infty)$.

*Lemma 2:* Define an integral

$$\mathcal{J}_b = \int_0^\infty \xi_p^b e^{-\frac{(\ln \xi_p - \mu_{\xi p})^2}{2\sigma_{\xi p}^2}} d\xi_p \tag{10}$$

which is related to the power index $b$, we get

$$\mathcal{J}_b = \sqrt{\frac{\pi}{a_2}} \cdot \exp\left[\frac{(a_3+b+1)^2}{4a_2} - a_4\right] \tag{11}$$

where $a_2 = \frac{1}{2\sigma_{\xi p}^2}$, $a_3 = \frac{\mu_{\xi p}}{\sigma_{\xi p}^2}$ and $a_4 = \frac{\mu_{\xi p}^2}{2\sigma_{\xi p}^2}$.

*Proof:* With the variable substitution of $x = \ln \xi_p$, we arrive at

$$\mathcal{J}_b = \int_{-\infty}^\infty e^{-a_2 x^2 + (a_3+b+1)x - a_4} dx \tag{12}$$

Next we complete the square on the power of $e$ and let $y = \sqrt{a_2}x - \frac{a_3+b+1}{2\sqrt{a_2}}$, thus we attain

$$\mathcal{J}_b = \frac{\exp\left[\frac{(a_3+b+1)^2}{4a_2} - a_4\right]}{\sqrt{a_2}} \int_{-\infty}^\infty e^{-y^2} dy \tag{13}$$

Finally, transform the integral part to the polar coordinates and the lemma is proved. ■

In the following, we first analyze the inner integral, and then move on to the outer part for the sum rate of ZF receivers.

*Property 1:* Define

$$\mathcal{I} = \frac{1}{(N-PM)!} \int_0^\infty \left( \log_2(1 + a_{p,m} \cdot \xi_p \cdot \widetilde{h}_m) \right) \cdot \widetilde{h}_m^{N-PM} \cdot e^{-\widetilde{h}_m} d\widetilde{h}_m \tag{14}$$

where $a_{p,m} = \frac{\rho \cdot \lambda_m}{M \cdot D_p^v}$. Let $b_{p,m} = a_{p,m} \cdot \xi$ and $c_{p,m} = \frac{\rho \cdot \lambda_m}{D_p^v}$, we have

$$\begin{aligned} \mathcal{I} =& \frac{1}{\ln 2} \sum_{k=0}^{N-PM} \sum_{t=1}^{L} \frac{(-c_{p,m} \cdot \xi_p)^t}{k!} \\ &\cdot \left[ \frac{(L+k)!(t-1)!}{L!} - (k+t-1)! \right] M^{-t} \\ &+ \frac{1}{\ln 2} \sum_{k=0}^{N-PM} \frac{(L+k)!}{L!\,k!} \int_0^\infty \frac{e^{-\widetilde{h}_m}}{b_{p,m}^{-1} + \widetilde{h}_m} d\widetilde{h}_m \\ &+ \frac{1}{\ln 2} \sum_{k=0}^{N-PM} \frac{f_{m,k}}{k!} \cdot \mathbf{O}\left(M^{-(L+1)}\right) \end{aligned} \tag{15}$$

*Proof:* See Appendix A. ■

On the basis of Property 1, we present the theorem bellow demonstrating the system sum rate under the ZF case, where a critical step is to deal with the large-scale fading factor.

*Theorem 1:* For a Massive D-MIMO system consisting of $P$ RPs each equipped with $M$ antennas and a single ZF receiver configured with $N$ antennas ($N > PM >> 1$), the uplink sum rate over composite RLN fading channel is

$$\begin{aligned} R_{ZF} =& -a_5 \sum_{p=1}^{P} \sum_{m=(p-1)M+1}^{pM} \sum_{k=0}^{N-PM} \sum_{t=1}^{L} \\ &\times \frac{(k+t-1)!}{k!} \cdot (-c_{p,m})^t \cdot \exp\left[\frac{(a_3+t)^2}{4a_2}\right] \cdot M^{-t} \\ &+ \mathbf{O}\left[M^{-(L+1)}\right] \end{aligned} \tag{16}$$

where $a_2 = \frac{1}{2\sigma_{\xi p}^2}$, $a_3 = \frac{\mu_{\xi p}}{\sigma_{\xi p}^2}$, $a_4 = \frac{\mu_{\xi p}^2}{2\sigma_{\xi p}^2}$, $a_5 = \frac{e^{-a_4}}{\ln 2\sqrt{2\, a_2}\sigma_{\xi p}}$ and $c_{p,m} = \frac{\rho \cdot \lambda_m}{D_p^v}$.

*Proof:* Referring to the sum rate (7), we obtain

$$R_{ZF} = \frac{1}{\sqrt{2\pi}\sigma_{\xi p}} \sum_{p=1}^{P} \sum_{m=(p-1)M+1}^{pM} \int_0^\infty \mathcal{I} \cdot \xi_p^{-1} e^{-\frac{(\ln \xi_p - \mu_{\xi p})^2}{2\sigma_{\xi p}^2}} d\xi_p \tag{17}$$

Substituting the outcome of Property 1 into it, the sum rate is acquired as bellow by applying Lemma 2

$$\begin{aligned}R_{ZF} &= \frac{1}{\ln 2\sqrt{2\pi}\sigma_{\xi p}}\sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\sum_{k=0}^{N-PM}\sum_{t=1}^{L} \\ &\times\frac{(-c_{p,m})^t\mathcal{J}_{t-1}}{k!}\left[\frac{(L+k)!(t-1)!}{L!}-(k+t-1)!\right]M^{-t} \\ &+\frac{1}{\ln 2\sqrt{2\pi}\sigma_{\xi p}}\sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\sum_{k=0}^{N-PM}\frac{(L+k)!}{L!\,k!} \\ &\cdot\int_0^\infty\left(\int_0^\infty\frac{e^{-\widetilde{h}_m}}{b_{p,m}^{-1}+\widetilde{h}_m}d\widetilde{h}_m\right)\xi_p^{-1}e^{-\frac{(\ln\xi_p-\mu_{\xi p})^2}{2\sigma_{\xi p}^2}}d\xi_p \\ &+\frac{1}{\ln 2\sqrt{2\pi}\sigma_{\xi p}}\sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\sum_{k=0}^{N-PM} \\ &\frac{(2^k-1)(L+k)!c_{p,m}^{L+1}\mathcal{J}_L}{(L+1)k!}\mathbf{O}\left[M^{-(L+1)}\right] \qquad (18)\end{aligned}$$

Note that the key of processing the sum rate is to solve the $E_1$-function-involved double integral, which is a challenge when studying the composite fading channels. In this work, we expand $E_1$ according to [26, eq. 8.357] for $|b_{p,m}^{-1}| \to \infty$ when $M$ approaches infinite. Therefore, we achieve

$$\begin{aligned}&\int_0^\infty e^{\frac{1}{b_{p,m}}}\left(\int_{\frac{1}{b_{p,m}}}^\infty\frac{e^{-x}}{x}dx\right)\xi_p^{-1}e^{-\frac{(\ln\xi_p-\mu_{\xi p})^2}{2\sigma_{\xi p}^2}}d\xi_p \\ &=\sum_{t=1}^{L}(-1)^{t-1}\cdot c_{p,m}^t\cdot(t-1)!\cdot\mathcal{J}_{t-1}\cdot M^{-t}+\mathbf{O}\left[M^{-(L+1)}\right] \\ &\qquad (19)\end{aligned}$$

Finally by substituting (19) into (18), the theorem is proved. ■

Based on the above derivation, we would like to draw some conclusions on Theorem 1:

- The sum rate is estimated by the series sum of the former $L$ items, where $L$ stands for the accuracy. That is to say, with a given $M$, the larger $L$ is selected, the more accurate the estimation is.
- In fact, this estimation converges fast due to the large $M$, which is also confirmed by simulation. According to the enormous experiments in Section IV, $L = 3$ is enough to get a satisfying estimation.
- Note that, in order to get a certain accuracy $L$, it is required $K \geq L + k$, which implies at least $L + k$ times of integration by parts is carried out in (31).

Originated from the traditional ZF receivers, DF receivers introduce the feedback scheme to enhance the received SNR. Hereafter, we analyze the sum rate of DF receivers, and then make a comparison between them.

*Corollary 1:* For a Massive D-MIMO system consisting of $P$ RPs each equipped with $M$ antennas and a single ZF-DF rceiver configured with $N$ antennas ($N > PM >> 1$), the uplink sum rate over composite RLN fading channel is

$$\begin{aligned}R_{DF} &= -a_5\sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\sum_{k=0}^{N-m}\sum_{t=1}^{L} \\ &\frac{(k+t-1)!}{k!}\cdot(-c_{p,m})^t\cdot\exp\left[\frac{(a_3+t)^2}{4a_2}\right]\cdot M^{-t} \\ &+\mathbf{O}\left[M^{-(L+1)}\right] \qquad (20)\end{aligned}$$

where $a_2 = \frac{1}{2\sigma_{\xi p}^2}$, $a_3 = \frac{\mu_{\xi p}}{\sigma_{\xi p}^2}$, $a_4 = \frac{\mu_{\xi p}^2}{2\sigma_{\xi p}^2}$, $a_5 = \frac{e^{-a_4}}{\ln 2\sqrt{2\ a_2}\sigma_{\xi p}}$ and $c_{p,m} = \frac{\rho\cdot\lambda_m}{D_p^v}$.

*Proof:* Applying QR decomposition to $\mathbf{HV}$ and premultiplying $\mathbf{Q}^{\mathrm{H}}$ to (3), we get

$$\mathbf{z} = \mathbf{Q}^{\mathrm{H}}\mathbf{y} = \frac{1}{\sqrt{M}}\mathbf{R}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{\Xi}^{\frac{1}{2}}\mathbf{s}+\mathbf{Q}^{\mathrm{H}}\mathbf{n} \qquad (21)$$

so that the sum rate is expressed as

$$R_{DF} = \sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\mathbb{E}\{\log_2(1+\frac{\rho[\mathbf{R}]_{mm}^2[\mathbf{\Lambda}]_m[\mathbf{\Xi}]_p}{M}\}. \qquad (22)$$

By considering the PDF of $\frac{r_{mm}^2}{d_m}$, we rearrange it as

$$R_{DF} = \sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\frac{\mathcal{T}_2}{(N-m)!\sqrt{2\pi}\sigma_{\xi p}} \qquad (23)$$

where

$$\begin{aligned}\mathcal{T}_2 = \int_0^\infty\int_0^\infty&\left(\log_2(1+\frac{\rho\cdot\eta_m\cdot\lambda_m\cdot\xi_p}{M\cdot D_p^v})\right)\cdot \\ &\eta_m^{N-m}e^{-\eta_m}\xi_p^{-1}e^{-\frac{(\ln\xi_p-\mu_{\xi p})^2}{2\sigma_{\xi p}^2}}d\eta_m d\xi_p.\end{aligned}$$

With Property 1, the sum rate is modified to

$$\begin{aligned}R_{DF} &= \frac{1}{\ln 2\sqrt{2\pi}\sigma_{\xi p}}\sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\sum_{k=0}^{N-m}\sum_{t=1}^{L} \\ &\frac{(-c_{p,m})^t\mathcal{J}_{t-1}}{k!}\left[\frac{(L+k)!(t-1)!}{L!}-(k+t-1)!\right]M^{-t} \\ &+\frac{1}{\ln 2\sqrt{2\pi}\sigma_{\xi p}}\sum_{p=1}^{P}\sum_{\substack{m=\\(p-1)M+1}}^{pM}\sum_{k=0}^{N-m}\frac{(L+k)!}{L!\,k!} \\ &\cdot\int_0^\infty e^{\frac{1}{b_{p,m}}}\left(\int_{\frac{1}{b_{p,m}}}^\infty\frac{e^{-x}}{x}dx\right)\xi_p^{-1}e^{-\frac{(\ln\xi_p-\mu_{\xi p})^2}{2\sigma_{\xi p}^2}}d\xi_p \\ &+\mathbf{O}\left[M^{-(L+1)}\right] \qquad (24)\end{aligned}$$

Finally, Corollary 1 is proved by substituting (19) into (24) and merging the first and second items with regard to the power of $M$. ∎

Recalling the sum rate (16) and (20), $\lambda_m$ is the only random variable. In the upcoming, we try to investigate the asymptotic features of the sum rates for both receivers with regard to infinite antennas.

*Corollary 2:* Given $\bar{\lambda}_{p,t} = \lim_{M\to\infty} \frac{1}{M}\sum_{m=1}^{M} \lambda_{p,m}^{t}$ (for $l = 1, 2, \cdots, L$), we have

$$R_{ZF-S} = -a_5 \sum_{p=1}^{P} \sum_{\substack{m=\\(p-1)M+1}}^{pM} \sum_{k=0}^{N-PM} \sum_{t=1}^{L} \frac{(k+t-1)!}{k!} \cdot \left(-\frac{\rho}{D_p^v}\right)^t \cdot \exp\left[\frac{(a_3+t)^2}{4a_2}\right] \cdot \bar{\lambda}_{p,t+1} \cdot M^{-t} + \mathbf{O}\left[M^{-(L+1)}\right] \tag{25}$$

$$R_{DF-S} = -a_5 \sum_{p=1}^{P} \sum_{\substack{m=\\(p-1)M+1}}^{pM} \sum_{k=0}^{N-m} \sum_{t=1}^{L} \frac{(k+t-1)!}{k!} \cdot \left(-\frac{\rho}{D_p^v}\right)^t \cdot \exp\left[\frac{(a_3+t)^2}{4a_2}\right] \cdot \bar{\lambda}_{p,t+1} \cdot M^{-t} + \mathbf{O}\left[M^{-(L+1)}\right] \tag{26}$$

where $a_2 = \frac{1}{2\sigma_{\xi p}^2}$, $a_3 = \frac{\mu_{\xi p}}{\sigma_{\xi p}^2}$, $a_4 = \frac{\mu_{\xi p}^2}{2\sigma_{\xi p}^2}$ and $a_5 = \frac{e^{-a_4}}{\ln 2\sqrt{2\, a_2}\sigma_{\xi p}}$.

*Proof:* Through [27, Th. 4.2], the corollary is directly obtained with the definition of $\bar{\lambda}_{p,t}$. ∎

In this part, the theorem and corollaries contribute to the Massive D-MIMO system from three aspects. First, the derivation of Theorem 1 and Corollary 1 provides an analysis framework for the sum rate over composite RLN channels. Second, the series sum of the former $L$ items in (16) and (20) gives an estimation of the system sum rate and can be applied in future study to evaluate the performance of precoder design. Third, the obtained sum rates have a good structure for us to learn the asymptotic behavior in Corollary 2. It explains the convergency of the sum rates against anntennas from a mathematical point of view while the simulation part testifies it in Fig.3.

In the preceding discussion, the first and second order statistics of channel are assumed to be available at the RPs. This is often achieved through the channel state estimation with the help of training or pilot sequence, where the transmitters send the previously known signals to the receiver first and then feed them back. Although the estimation error is unavoidable, it is being improved by technologies like signal design and blind estimation. Therefore, it is our future investigation on how to design the precoders robust to the estimation error.

## IV. SIMULATION RESULTS

In this section, we examine the sum rates of both ZF and DF receivers over the number of transmit antennas, the number of receive antennas and SNR respectively, where several comparisons are made with regard to different parameters such as the number and location of RPs.

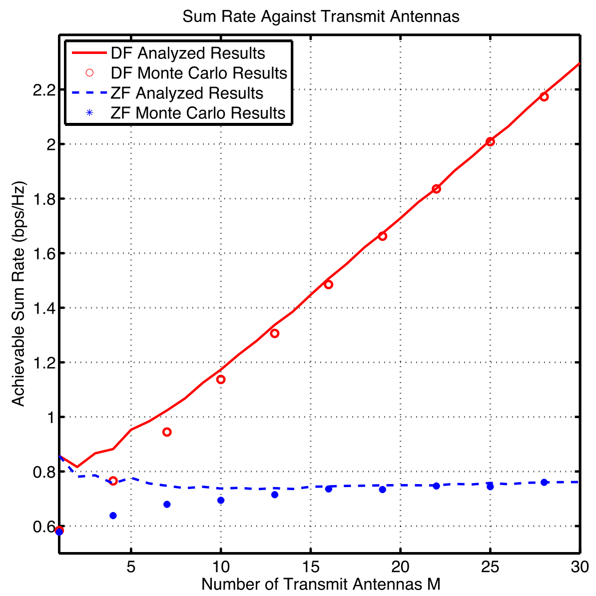


**FIGURE 2. Sum Rate of ZF and DF Receivers against Transmit Antennas ($P = 1, N - PM = 1, SNR = 30dB, D = 10km, L = 3$).**

Considering the sum rate expression is closely related to the transmit antennas, we first concentrate on the sum rate against the number of transmit antennas in Fig.2.

The curves testify Theorem 1 and Corollary 1 since both the analytical results approach well to the Mont Carlo outcome. It also suggests that the analytical results converge to the experimental outcome much better as the number of transmit antennas increases. It starts to gain a good approximation from $M = 18$ for DF receivers and $M = 16$ for ZF. Note that in order to keep a good convergence, the number of the transmit antennas $M$ should not be too small especially when $PM = 1$. It is not only a requirement for mathematical analysis of this paper but also testified by simulation, where on the one hand, the front ends of the curves are not smooth, on the other hand, the significant gaps are present between the analytical and the Mont Carlo results.

In Fig.3, we examine the sum rate against the number of transmit antennas with different SNR, where Fig.3(a) is for DF receivers and Fig.3(b) is for ZF.

It shows that our analytical result matches the Monte Carlo well even when $M$ is not a huge number. Besides, each group of curves has a slight growth. On the one hand, the sum rate grows owing to the space gains brought by the antennas. On the other hand, the growth is limit because the asymptotic features gradually take effect as the number of antennas increases. Additionally, the sum rate of DF receivers benefits greater compared to that of ZF with the help of feedback scheme. Therefore, we only present the rates of DF receivers in Fig.4 and Fig.5 to explicitly discuss the effect of location and antennas distribution.

From previous work, it is known that the location of RPs plays a prominent role in distributed systems. Hence we

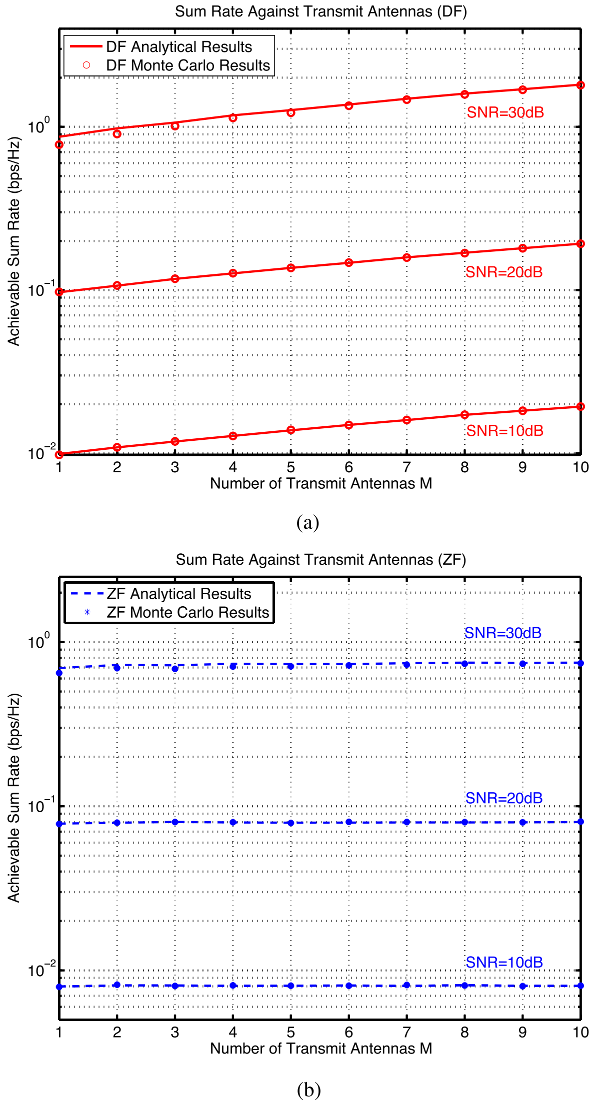


**FIGURE 3. Sum Rate of ZF and DF Receivers against Transmit Antenna ($P = 2, N - PM = 1, SNR = 10/20/30dB, D = [10\ 10]km, L = 3$). (a) DF Receiver. (b) ZF Receiver.**

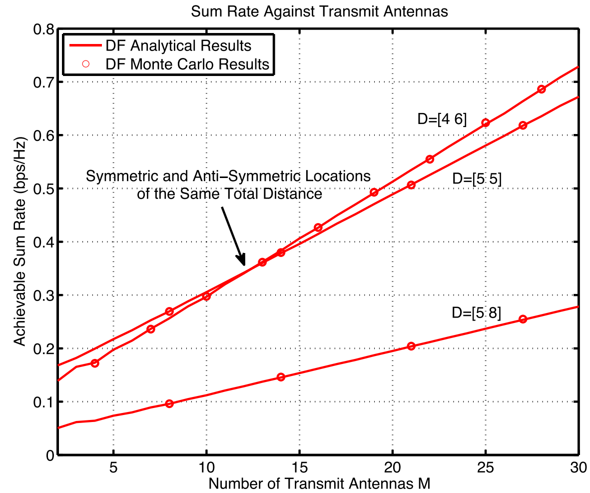


**FIGURE 4. DF Sum Rate against Transmit Antenna ($P = 2, N - PM = 1, SNR = 10dB, D = [4\ 6]/[5\ 5]/[5\ 8]km, L = 3$).**

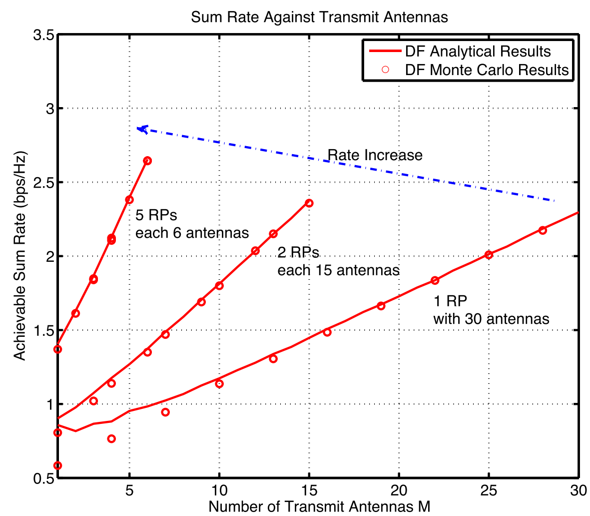


**FIGURE 5. DF Sum Rate against Transmit Antenna ($P = 1/2/5, M = 30/15/6, N = 31, SNR = 30dB, D = 10km, L = 3$).**

evaluate the sum rate with different locations in Fig.4, where both the same and different distances from each RP to the BS (symmetric location and anti-symmetric location) are considered.

We can conclude from Fig.4 that the shorter distance of $D = [5\ 5]$ results in higher sum rate than that of $D = [5\ 8]$ because a higher SNR is achieved. Note that when the sum distance of all the RPs is the same, for instance $D = [4\ 6]$ and $D = [5\ 5]$, the anti-symmetric location has a better performance than the symmetric case from $M = 12$ since a more abundant scattering environment is acquired from the anti-symmetric location when the number of transmit antenna rises.

Subsequently in Fig.5, the gain of multiple RPs is studied, where we remain a same number of total transmit antennas as $PM = 30$.

It shows that the case of 5 RPs each with 6 antennas has a greater rate than that of 2 RPs each 15 and 1 RP with 30. It is also because a more abundant scattering environment results in more independent subchannels.

In the following part, we inspect the sum rate against the number of receive antennas $N$ and the original SNR $\rho$ respectively.

Fig.6 characterizes the influence of the receive antennas, in which the analytical and simulation results of both receivers increase almost linearly. The configuration of 2 RPs each with 59 antennas is applied. However, it is worth notice that the transmit antennas should not be too few, so as to keep a good convergence according to the equations (16) and (20).

In Fig.7, the sum rate against the original SNR $\rho$ is estimated. As showed in this figure, the sum rates increase tremendously with the improving SNR, where more transmit antennas and DF receivers also result in higher rates.

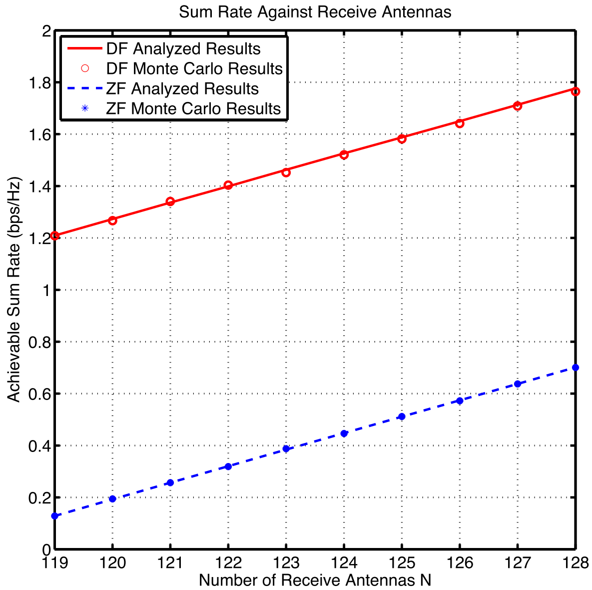


**FIGURE 6.** Sum Rate of both Receivers against Receive Antenna ($P = 2, M = 59, SNR = 10dB, D = [5\ 5]km, L = 3$).

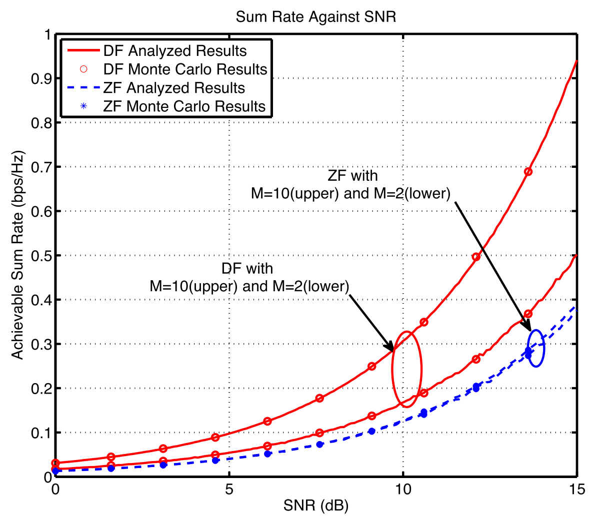


**FIGURE 7.** Sum Rate of both Receivers against SNR ($P = 2, M = 2/10, N = 5/21, D = [5\ 5]km, L = 3$).

## V. CONCLUSIONS

In this paper, we investigate the sum rate of Massive D-MIMO systems over composite RLN channels, where two receivers ZF and ZF-DF are considered successively. An analysis framework is proposed first to obtain the sum rates, where the accuracy requirement of $L = 3$ is enough to gain a satisfying estimation. Thus it suggests a direct and clear way for performance evaluation of Massive D-MIMO systems. Then the solution is extended to a steady form under the infinite antenna situation, where the asymptotic behavior of the sum rates is revealed. Our theoretical analysis is confirmed by the simulation experiments from multiple aspects. In order to concentrate on the sum rate, a simple precoder is assumed in this work. It remains further study of more relaxed conditions for precoder design.

## APPENDIX
## PROOF OF PROPERTY 1

Regarding the integral $\mathcal{I}$ as a sequence $\mathcal{I}_{(N-PM)}$, we get an iterative expression through integration by parts.

$$\mathcal{I}_{(N-PM)} = \mathcal{I}_{(N-PM-1)} + \frac{1}{\ln 2}\int_0^\infty \frac{e^{-\widetilde{h}_m}}{b_{p,m}^{-1}+\widetilde{h}_m}\cdot\frac{\widetilde{h}_m^{N-PM}}{(N-PM)!}d\widetilde{h}_m \tag{27}$$

Applying this operation to (14) for $N - PM$ times, we come to

$$\mathcal{I} = \frac{1}{\ln 2}\int_0^\infty \frac{e^{-\widetilde{h}_m}}{b_{p,m}^{-1}+\widetilde{h}_m}\cdot \left(1+\widetilde{h}_m+\frac{\widetilde{h}_m^2}{2!}+\cdots+\frac{\widetilde{h}_m^{N-PM}}{(N-PM)!}\right)d\widetilde{h}_m. \tag{28}$$

Define a sequence

$$\mathcal{I}_k = \int_0^\infty \frac{e^{-\widetilde{h}_m}}{b_{p,m}^{-1}+\widetilde{h}_m}\widetilde{h}_m^k d\widetilde{h}_m, \quad (k = 0, 1, \ldots, N-PM), \tag{29}$$

such that

$$\mathcal{I} = \frac{1}{\ln 2}\sum_{k=0}^{N-PM}\frac{\mathcal{I}_k}{k!}. \tag{30}$$

Similarly, taking integration by parts with respect to $e^{-\widetilde{h}_m}$ and iterating for $K$ ($K > k$, illustrated later) times, we have

$$\mathcal{I}_k = -\sum_{t=0}^{K-1}\left(\frac{\widetilde{h}_m^k}{b_{p,m}^{-1}+\widetilde{h}_m}\right)^{(t)} e^{-\widetilde{h}_m}\Bigg|_0^\infty + \int_0^\infty\left(\frac{\widetilde{h}_m^k}{b_{p,m}^{-1}+\widetilde{h}_m}\right)^{(K)} e^{-\widetilde{h}_m}d\widetilde{h}_m. \tag{31}$$

Denote the former summation as $\mathcal{I}_{k1}$ and the latter integral as $\mathcal{I}_{k2}$.

Notice that $\mathcal{I}_{k1} = 0$ only holds for $t < k$ because from $t = k$ a constant appears in the numerator part. As a result, with Leibniz Formula we arrive at

$$\begin{aligned}\mathcal{I}_{k1} &= -\sum_{t=k}^{K-1}\sum_{s=0}^{t}\binom{t}{s}(\widetilde{h}_m^k)^{(s)}\left(\frac{1}{b_{p,m}^{-1}+\widetilde{h}_m}\right)^{(t-s)} e^{-\widetilde{h}_m}\Bigg|_0^\infty \\ &= -\sum_{t=k}^{K-1}\binom{t}{k}k!\frac{(-1)^{t-k}(t-k)!}{(b_{p,m}^{-1}+\widetilde{h}_m)^{t-k+1}}e^{-\widetilde{h}_m}\Bigg|_0^\infty\end{aligned} \tag{32}$$

It is worth notice that the second equality is acquired for $s = k$ since

$$\begin{cases}(\widetilde{h}_m^k)^{(s)} = 0, \ for\ s > k; \\ \dfrac{\widetilde{h}_m^{k-s}}{(b_{p,m}^{-1}+\widetilde{h}_m)^{t-s+1}}e^{-\widetilde{h}_m}\Bigg|_0^\infty = 0, \ for\ s < k.\end{cases} \tag{33}$$

Accordingly, we calculate and re-arrange (32) in an ascending order corresponding to the power of $b_{p,m}$, achieving

$$\mathcal{I}_{k1} = \sum_{t=1}^{K-k} (-1)^{t-1} \cdot b_{p,m}^{t} \cdot (k+t-1)!. \quad (34)$$

In order to deal with $\mathcal{I}_{k2}$, we first tackle the derivative part inside, and then focus on the integral with regard to $\widetilde{h}_m$. Through Leibniz Formula, this high-oder derivative is expanded as in (35) considering $(\widetilde{h}_m^k)^l = 0$ for $l > k$.

$$\left(\frac{\widetilde{h}_m^k}{b_{p,m}^{-1} + \widetilde{h}_m}\right)^{(K)} = \sum_{l=0}^{k} \binom{K}{l} \frac{k! \cdot \widetilde{h}_m^{k-l}}{(k-l)!} \cdot \frac{(-1)^{K-l}(K-l)!}{(b_{p,m}^{-1} + \widetilde{h}_m)^{K-l+1}} \quad (35)$$

Considering $K > k$, let us suppose a parameter $L \in \mathbb{Z}^+$ satisfying $K = L + k$. It is always true because we can appropriately select $K$, the times to do integration by parts in the previous operation (31).

Hence in the upcoming, we substitute $K$ by $L+k$, and point out the importance of $L$ which refers to the effectiveness of our final result.

Substituting (35) into $\mathcal{I}_{k2}$, we get

$$\mathcal{I}_{k2} = \mathcal{I}_{k21} + \mathcal{I}_{k22}. \quad (36)$$

where

$$\mathcal{I}_{k21} = \sum_{l=0}^{k-1} \frac{(-1)^{L+k-l} k!(L+k)!}{l!(k-l)!} \cdot \int_0^{\infty} \frac{\widetilde{h}_m^{k-l}}{(b_{p,m}^{-1} + \widetilde{h}_m)^{L+k-l+1}} \cdot e^{-\widetilde{h}_m} d\widetilde{h}_m \quad (37)$$

and

$$\mathcal{I}_{k22} = (-1)^L (L+k)! \int_0^{\infty} \frac{1}{(b_{p,m}^{-1} + \widetilde{h}_m)^{L+1}} \cdot e^{-\widetilde{h}_m} d\widetilde{h}_m \quad (38)$$

As a matter of fact, (36) is just a separation of all the items, in which $\mathcal{I}_{k21}$ involves the elements of $l \neq k$ and $\mathcal{I}_{k22}$ is just for $l = k$.

According to Lemma 1, we arrive at

$$|\mathcal{I}_{k21}| \leq |f_{m,k}| \cdot M^{-(L+1)} \quad (39)$$

where

$$f_{m,k} = \frac{(2^k - 1)(L+k)!(c_{p,m} \cdot \xi_p)^{L+1}}{L+1}, \quad (40)$$

which implies $\mathcal{I}_{k21} = \mathbf{O}\big(M^{-(L+1)}\big)$.

For $\mathcal{I}_{k22}$, we come to

$$\mathcal{I}_{k22} = \sum_{t=1}^{L} \frac{(-1)^t (L+k)!(t-1)! \cdot b_{p,m}^t}{L!} + \frac{(L+k)!}{L!} \int_0^{\infty} \frac{e^{-\widetilde{h}_m}}{b_{p,m}^{-1} + \widetilde{h}_m} d\widetilde{h}_m \quad (41)$$

through integration by parts for $L$ times, sorting it by the ascending power of $b_{p,m}$. Note that an upper incomplete gamma function is contained in the second part, which will be solved later in Theorem 1.

By combining (34) and (41) with the consideration of $L = K - k$, we get $\mathcal{I}_k$ through (30), and finally $\mathcal{I}$ in Property 1.

## REFERENCES

[1] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[2] "5G concept," IMT-2020 (5G) Promotion Group, Beijing, China, White Paper 2015-02, Feb. 2015

[3] "5G network technology architecture," IMT-2020 (5G) Promotion Group, Beijing, China, White Paper 2015-05, May 2015.

[4] A. Alexiou, "Wireless world 2020: Radio interface challenges and technology enablers," *IEEE Veh. Technol. Mag.*, vol. 9, no. 1, pp. 46–53, Mar. 2014.

[5] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[6] E. Björnson, L. Sanguinetti, J. Hoydis, and M. Debbah, "Optimal design of energy-efficient multi-user MIMO systems: Is massive MIMO the answer?" *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3059–3075, Jun. 2015.

[7] J. Hoydis, S. ten Brink, and M. Debbah, "Massive MIMO in the UL/DL of cellular networks: How many antennas do we need?" *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 160–171, Feb. 2013.

[8] E. Bjornson, L. Sanguinetti, J. Hoydis, and M. Debbah, "Designing multi-user MIMO for energy efficiency: When is massive MIMO the answer?" in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Apr. 2014, pp. 242–247.

[9] S. Lakshminarayana, M. Assaad, and M. Debbah, "Coordinated multicell beamforming for massive MIMO: A random matrix approach," *IEEE Trans. Inf. Theory*, vol. 61, no. 6, pp. 3387–3412, Jun. 2015.

[10] D. Wang, J. Wang, X. You, Y. Wang, M. Chen, and X. Hou, "Spectral efficiency of distributed MIMO systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 10, pp. 2112–2127, Oct. 2013.

[11] M. Matthaiou, C. Zhong, M. R. McKay, and T. Ratnarajah, "Sum rate analysis of ZF receivers in distributed MIMO systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 180–191, Feb. 2013.

[12] H. Yin, D. Gesbert, and L. Cottatellucci, "Dealing with interference in distributed large-scale MIMO systems: A statistical approach," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 942–953, Oct. 2014.

[13] T. S. Rappaport, *Wireless Communications: Principles and Practice*. London, U.K.: Dorling Kindersley, 2009.

[14] Y.-S. Jeon, Y.-J. Kim, M. Min, and G.-H. Im, "Distributed block diagonalization with selective zero forcing for multicell MU-MIMO systems," *IEEE Signal Process. Lett.*, vol. 21, no. 5, pp. 605–609, May 2014.

[15] H. Q. Ngo, M. Matthaiou, T. Q. Duong, and E. G. Larsson, "Uplink performance analysis of multicell MU-SIMO systems with ZF receivers," *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4471–4483, Nov. 2013.

[16] D. Wang, X. You, J. Wang, Y. Wang, and X. Hou, "Spectral efficiency of distributed MIMO cellular systems in a composite fading channel," in *Proc. IEEE Int. Conf. Commun.*, May 2008, pp. 1259–1264.

[17] M. Matthaiou, N. D. Chatzidiamantis, G. K. Karagiannidis, and J. A. Nossek, "On the capacity of generalized-*K* fading MIMO channels," *IEEE Trans. Signal Process.*, vol. 58, no. 11, pp. 5939–5944, Nov. 2010.

[18] M. Matthaiou, N. D. Chatzidiamantis, and G. K. Karagiannidis, "A new lower bound on the ergodic capacity of distributed MIMO systems," *IEEE Signal Process. Lett.*, vol. 18, no. 4, pp. 227–230, Apr. 2011.

[19] M. Matthaiou, N. D. Chatzidiamantis, G. K. Karagiannidis, and J. A. Nossek, "ZF detectors over correlated K fading MIMO channels," *IEEE Trans. Commun.*, vol. 59, no. 6, pp. 1591–1603, Jun. 2011.

[20] M. S. Alouini and A. J. Goldsmith, "A unified approach for calculating error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. Commun.*, vol. 47, no. 9, pp. 1324–1334, Sep. 1999.

[21] C. Yoon, H. Lee, and J. Kang, "Performance evaluation of space-time block codes from coordinate interleaved orthogonal designs in shadowed fading channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 3, pp. 1289–1295, Mar. 2011.

[22] M. Bacha and S. A. Hassan, "Performance analysis of linear cooperative multi-hop networks subject to composite shadowing-fading," *IEEE Trans. Wireless Commun.*, vol. 12, no. 11, pp. 5850–5858, Nov. 2013.

[23] X. C. Gao, J. K. Zhang, Z. Y. Wang, and J. Jin, "Optimal precoder design maximizing the worst-case average received SNR for massive distributed MIMO systems," *IEEE Commun. Lett.*, vol. 19, no. 4, pp. 589–592, Apr. 2015.

[24] X.-C. Gao, J.-K. Zhang, J. Jin, and Z.-Y. Wang, "Optimal diagonal power loading minimizing asymptotic symbol error probability for distributed MIMO systems with ZF receivers," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 7818–7823, Sep. 2016.

[25] M. Park, C.-B. Chae, and R. W. Heath, "Ergodic capacity of spatial multiplexing MIMO systems with ZF receivers for log-normal shadowing and Rayleigh fading channels," in *Proc. IEEE 18th Int. Symp. Pers., Indoor Mobile Radio Commun.*, Dec. 2007, pp. 1–5.

[26] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Amsterdam, The Netherlands: Elsevier, 2007.

[27] R. M. Gray, "Toeplitz and circulant matrices: A review," *Found. Trends Commun. Inf. Theory*, vol. 2, no. 3, pp. 155–239, Nov. 2006.

**LI WANG** received the B.E. degree in communications engineering from Shandong University, Jinan, China, in 2009, and M.S. degree in communications engineering from Xidian University, Xi'an, China, in 2012, where she is currently pursuing the Ph.D. degree. From 2013 to 2014, she was a Visiting Student funded by the China Scholarship Council at the Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON, Canada. Her research interests include MIMO-based systems, 5G cellular networks, and interference management.

**JIANDONG LI** (SM'05) received the B.E., M.S., and Ph.D. degrees from Xidian University, Xi'an, China, in 1982, 1985, and 1991, respectively, all in communications engineering. He has been a Faculty Member with the School of Telecommunications Engineering, Xidian University, since 1985, where he is currently a Professor and the Vice Director of the academic committee of State Key Laboratory of Integrated Service Networks. He was a Visiting Professor with the Department of Electrical and Computer Engineering, Cornell University, Ithaca, NY, USA, from 2002 to 2003. His research interests include wireless communication theory, cognitive radio, and signal processing. He served as the General Vice Chair for ChinaCom 2009 and the TPC Chair of the IEEE ICCC 2013. He was a recipient of the Distinguished Young Researcher from NSFC and Changjiang Scholar from the Ministry of Education, China.

**JIAN-KANG ZHANG** (M'04–SM'09) received the B.S. degree in information science (mathematics) from Shaanxi Normal University, Xi'an, China, the M.S. degree in information and computational science (mathematics) from Northwest University, Xi'an, and the Ph.D. degree in electrical engineering from Xidian University, Xi'an. He has held research positions at Harvard University and McMaster University. He is currently an Associate Professor with the Department of Electrical and Computer Engineering, McMaster University. He has co-authored of the paper, which received the IEEE Signal Processing Society Best Young Author Award in 2008. He is currently serving as an Associate Editor for the IEEE Transactions on Signal Processing and the *Journal of Electrical and Computer Engineering*. He has served as an Associate Editor for the IEEE Signal Processing Letters.

**RUI CHEN** received the B.S., M.S., and Ph.D degrees from Xidian University, Xian, China, in 2005, 2007, and 2011, respectively, all in communications and information systems. He is currently an Associate Professor with the School of Telecommunications Engineering, Xidian University. His research interests include MIMO-OFDM techniques and MAC layer protocol design.