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Optimization of Spare Parts Varieties Based on Stochastic DEA Model

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ABSTRACT Accurate inventory management starts with the scientific and rational classification of numerous varieties of spare parts. This paper presents a stochastic data envelopment analysis (DEA) model to address the problem of optimization of spares varieties under uncertainty. An index system is proposed in terms of product life-cycle process, which contains five design indexes, four operation indexes and five support indexes. Then, the quantification method of the index system is briefly discussed in preparation for mathematical calculation. A stochastic spares optimization model (SSOM) is proposed based on stochastic DEA with the constraints of 14 factors of the index system. The SSOM could be converted into equivalent deterministic models by probability theory, which overcomes the difficulty in solving non-linear programming. A numerical example is given to illustrate the proposed method in terms of ability to provide reasonable inventory management policies.

INDEX TERMS Uncertain environment, spare parts, optimization, stochastic, data envelopment analysis.

I. INTRODUCTION

Spares parts, which are stocked to replace failed parts, are the indemnification goods for plants to maintain normal functioning. Stocking strategy for spare parts has a pivotal influence on the productivity and efficiency of industrial plants. Conservative strategy may lead to overstocking and high cost of inventory, which will reduce the profit of industrial plants. On the other hand, optimistic strategy may result in shortage of necessary spare parts, long machine downtime and decrement in productivity. Therefore, optimization of spare parts varieties plays an important role in inventory management.

There are considerable existing literatures on optimizing stocking strategy of spare parts. It is important to have a reasonable index system before optimizing the controlling strategy. Some researchers tried to solve this problem by a single attribute. For example, Nahmias [1] proposed a one-dimensional approach that is only based on total cost. However, strategy with only one single attribute cannot solve the problem when the spare parts are competing. Many multi-attribute evaluation systems were then developed to overcome this difficulty. As was reported in Ng [2], Zhou and Fan [3], Hadi-Vencheh [4] and Lin *et al.* [5] a classification scheme including annual dollar usage, lead time and average unit cost was proposed. Multi-attribute decision making techniques were employed by Molenaers *et al.* [6], Almeida and Erel [7], and Sharaf and Helmy [8] to provide reasonable decision-making proposals. Deterministic attribute models were gradually developed into random attribute models in terms of parameter dispersion. Quantitative analyses for indeterministic variables were then carried out. Wang [9] established a stochastic model for joint spare parts inventory and planned maintenance optimization considering the random nature of plant failures and then applied stochastic dynamic programming to find the optimal solutions over a finite time horizon. Godoy *et al.* [10] presented a graphical technique which considered a stochastic lead time and a reliability threshold to enhance spare parts ordering decision-making. Gu *et al.* [11] assumed that the probability density distribution functions of lifetime and the number of failures follow normal distributions, then worked out the optimal order quantities by minimizing the total cost. Li *et al.* [12] proposed a stochastic programming model to seek a optimal spare parts ordering and pricing policy from a distributor's view. Zamar *et al.* [13] developed a quantile-based scenario analysis approach for stochastic supply chain optimization under uncertainty. However, the above research mainly focus on factors that influence the demand of spare parts in the normal operational stage or supporting stage and few articles investigate the factors in the whole product life-cycle process. In this paper, we will establish a comprehensive index system in terms of the whole product life-cycle process, including design factors, operation factors and support factors.

Once the evaluation indexing system is established, it will need to choose a kind of evaluation method that can evaluate the importance of these factors. Many methods have emerged in this field. The well-known ABC classification [3], [4], [14] is simple to use and easy to understand. However, ABC analysis is based on only single measurement such as annual dollar usage. Other criteria have gradually been recognized to be important in inventory management. Analytic hierarchy process (AHP) was adopted to determine the weights of factors for multi-attribute evaluation system. For example, Braglia *et al.* [15] employed AHP on an inventory policy matrix based on the reliability centered maintenance (RCM) to identify the best control strategy. Molenaers *et al.* [6] firstly presented the multi-criteria classification issue in a logic decision tree based on item criticality, then used AHP at different nodes of the diagram and converted relevant criteria into a single scalar to represent the criticality of the part. However, AHP requires subjective judgment when making pair-wise comparisons. Heuristic algorithms like genetic algorithms [7], [16] and artificial neural networks [17], [18] were also utilized to evaluate the importance of index system. However, they are complex and difficult in application.

Data Envelopment Analysis (DEA) is a Linear Programming based technique for the analysis of efficiency of organizations with multiple inputs and outputs and is proposed by Charnes *et al.* in 1978 [19]. In DEA, the organization under study is called a DMU (decision-making unit). Suppose there are n DMUs in a DEA model: DMU_0 is target DMU, DMU_i is the ith DMU ($i = 1, 2, \cdots, n$), $\boldsymbol{x_i} = (x_{i1}, x_{i2}, \ldots, x_{ip})$ is the inputs vectors of DMU_i, $\boldsymbol{x_0} = (x_{01}, x_{02}, \ldots, x_{0p})$ is the inputs vector of DMU_0, $\boldsymbol{y_i} = (y_{i1}, y_{i2}, \ldots, y_{ip})$ is the outputs vectors of DMU_i, $\boldsymbol{y_0} = (y_{01}, y_{02}, \ldots, y_{0p})$ is the outputs vector of DMU_0, $\boldsymbol{u} \in \boldsymbol{R}^{p\times 1}$ is the vector of input weights, $\boldsymbol{v} \in \boldsymbol{R}^{p\times 1}$ is the vector of output weights. Then a typical basic DEA model called CCR is represented as follows:

$$\begin{cases} \max\ \theta = \dfrac{\boldsymbol{v}^t \boldsymbol{y_0}}{\boldsymbol{u}^t \boldsymbol{x_0}} \\ \textit{subject to}: \\ \boldsymbol{v}^t \boldsymbol{y_i} \le \boldsymbol{u}^t \boldsymbol{x_i}, \quad i = 1, 2, \ldots, n, \\ \boldsymbol{u} \ge 0 \\ \boldsymbol{v} \ge 0. \end{cases} \tag{1}$$

In the above model, the objective is to obtain the ratio of the weighted output to the weighted input weights with the constraints that the ratio of virtual outputİ vs. virtual inputİ should not exceed 1 for every DMU. By virtue of the constraints, the optimal objective value θ^* is at most 1. The optimal solution θ^* yields an efficiency score for a particular DMU and the process is repeated for each DMU_i, $i = 1, 2, \ldots, n$. DEA method can be regarded as a production process with multiple inputs and outputs. As is known, all the manufactures hope to produce maximum outputs with the least inputs. This principle is also reflected in DEA model. The DMU will be more effective than other DMUs if it has a larger optimal value. Therefore, DMUs are regarded to be inefficient if $\theta^* < 1$, while DMUs are efficient if $\theta^* = 1$. Compared with the aforesaid methodology, DEA have the advantages in avoiding subjective factors, having simple algorithms and reducing errors. Moreover, it can contain controllable input (output) and non-controllable input (output). DEA can efficiently deal with fact that the numerical dimension is not unified. Several DEA models, i.e. BCC model and Additive model, have been developed to suit different application scenarios [20]–[23]. In view of the situation that inputs and outputs of the DMU cannot be accurately determined, many literatures have proposed opportunity constrained programming models [24]–[26], stochastic DEA models [27]–[30] and fuzzy DEA models [31]. In this paper, some factors in index system, i.e. corrective maintenance time, logistic delay time and mission time, are assumed to be random variables, considering that their values will change with real-life situations. Therefore, a stochastic DEA model is employed to address the problem of optimization of spare parts varieties.

The reminder of this paper is organized as follows. In section 2, a comprehensive index system is established in terms of the whole product life-cycle process. Section 3 briefly introduces the quantification method of the index systems with respect to qualitative factors and quantitative factors. Section 4 establishes the stochastic spares optimization model (SSOM) based on stochastic DEA method. Some algorithms are addressed to obtain the equivalent deterministic model in Section 5. A numerical example is given to illustrate the SSOM in Section 6. Section 7 summarizes the main work and contributions of this paper.

II. ESTABLISHMENT OF INDEX SYSTEM

In this section, an index system is established in terms of design, operation and support as shown in Fig. 1. Design factors are composed of mean time between failures (MTBF), consequences of failure (CoF), the number of stand-alone installation, replace-ability and standard part, which are determined in the design phase. Operation factors are the attributes that influence the inventory of spare parts in the operation phase, including operation environment, turn around time (TAT), mission time and the number of equipment. Support factors consist of corrective maintenance time, logistic delay time, the number of suppliers, purchase lead time and cost, which are related to logistics and maintenance.

A. DESIGN INDEXES

The subsection below describes the properties of five design indexes.

MTBF refers to the average amount of time that a device or product functions before failing, which is an important index in repairable system [32]. MTBF is an important parameter for measuring the reliability and availability [33], [34]. Spare parts with short MTBFs are

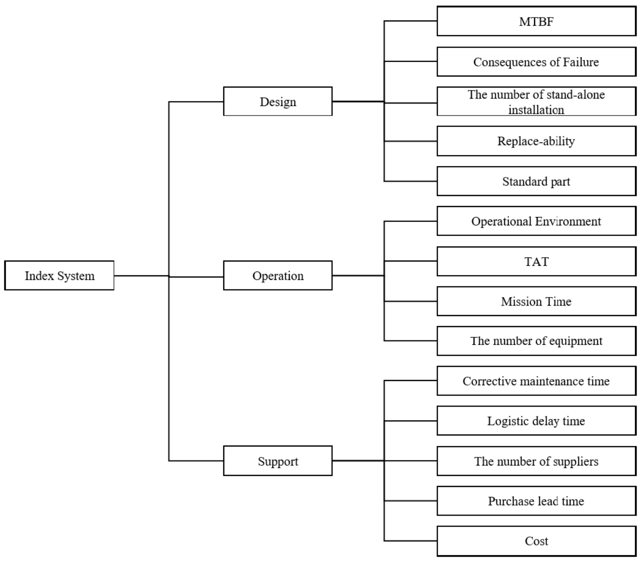

FIGURE 1. **Index system.**

always at a low level of reliability and availability and need frequent maintenance. Therefore, items with short MTBFs should be kept at high inventory levels.

CoF refers to losses or damages that are caused by a failure. Generally, consequences of failure could be classified into five types in terms of severity, i.e. catastrophic consequences, critical consequences, severe consequences, marginal consequences and negligible consequences. Items that are likely to cause serious consequences may high inventory levels because their shortage of spare parts will have a critical impact on the overall system.

The number of stand-alone installation is the number of a kind of component or unit installed on an equipment, which is clarified in the design phase. The number of stand-alone installation may, to some extent, reflect the demand for spare parts. Components with a large number of stand-alone installation are more likely to fail, so the demand for spare parts is higher.

Replace-ability is the capability of an item to be replaced by site workers. Components without replace-ability cannot be replaced in the current site and need to be sent to senior site. Thus, it is reasonable to reduce the inventory level of items with poor site replace-ability.

A standard part refers to a part or material that conforms to an established industry published specification. The lack of non-standard parts is more difficult to handle than standard parts because standard parts are interchangeable and easier to obtain than non-standard parts. Therefore, it is necessary to maintain the inventory level of non-standard parts.

B. OPERATION INDEXES

This subsection gives a brief introduction of operation indexes, including mission time, operation environment, turnaround time (TAT) and the number of equipment.

Mission refers to the task, together with the purpose, that clearly indicates the action to be taken and the reason therefore [35]. Mission time is the length of time to complete the mission. Mission time has a direct impact on the demand for spare parts because more spare parts may need to be replaced within a longer mission time.

Operational environment is defined as a composite of the conditions, circumstances, and influences that affect the employment of capabilities and bear on the decisions of the commander [35]. The operating environment has a crucial influence on the life of components and the demand for components. For example, the operating environment of aircraft engines is more stringent than displays installed in cockpits in the same plane. The screws on the engine are more likely to fail than the screws on the display. Therefore, the spare parts for the screws on the engine are higher than those on the display.

Turnaround time (TAT) is defined as the length of time between arriving at a point and being ready to depart from that point [35]. TAT depends on the properties of equipment as well as maintainability and supportability in practice, and will change with the real scenarios. TAT has a obvious influence on the requirement of spare parts as there is not enough time to prepare spare parts in case of limited TAT.

The sum of equipments is the total number of the equipments participating in the mission. The sum of equipments is of interest because the requirement of inventory of spare parts is in a proportional relationship with it.

C. SUPPORT INDEXES

The subsection below describes the properties of five support indexes.

Corrective maintenance time is the time that begins with the observance of a malfunction of an item and ends when the item is restored to a satisfactory operating condition [36]. Corrective maintenance time could represent one item's maintainability. Components with long corrective maintenance time are poor in maintainability and thus should maintain high inventory levels.

Logistic delay time is the component of downtime during which no maintenance is being accomplished on the item because of technician alert and response time, supply delay, or administrative reasons [36]. As the shortage of these spare parts may cause huge time and money costs, spare parts with long logistics delay time should maintain a high inventory level.

Purchase lead time is defined as the time between the initiation and completion of a purchase process and could be determined by market research and historical experience. Parts with long purchase lead time are more likely to be in short supply. Therefore, spare parts with a long procurement cycle should maintain a stable inventory level.

The number of suppliers is the sum of suppliers who can provide required spare parts. According to MIL-STD-965B, components are selected from program parts selection list (PPSL) and their suppliers are then determined. The number of suppliers has a direct effect on the supply stability

TABLE 1. Quantification principles.

	1	3	5	7	9
OE	Particularly harsh	Severe	Moderate	Mild	Gentle
CoF	Catastrophic	Critical	Severe	Marginal	Negligible
RA	Cannot be replaced	Can be replaced			
SP	Standard part	Non-standard part			

because components with the large number of suppliers are at a low risk in shortage.

Cost include cost of purchase and cost of storage. The cost of spare parts is determined by their natural properties and is therefore determined during the design phase. It is intuitive to store the spare parts whose cost is at a low level with respect to costs-saving.

III. QUANTIFICATION METHOD OF INDEX SYSTEM

Quantification is an important step before we take the indexes into mathematical models. Based on the properties of indexes demonstrated in Section II, these indexes could be classified into two types, qualitative indexes and quantitative indexes.

Qualitative indexes include operational environment (OE), replace-ability (RA), standard part (SP), consequences of failure (CoF). To employ these factors in the mathematical model, Table 1 shows the qualification principles of these indexes.

Quantitative indexes could be further divided into two subtypes, deterministic factors and random factors. Deterministic factors could be represent by crisp values and include the sum of equipments, the number of stand-alone installation, purchase lead time, the number of suppliers and cost. Specifically, purchase lead time, the number of suppliers and cost could obtained by market research or historical experiences. The sum of equipments and the number of stand-alone installation are designed according to the requirements of mission. Random factors are the variables that will change with actual scenarios, including TAT, MTBF, mission time, corrective maintenance time and logistic delay time. The approach to determine these quantitative indexes is not demonstrated here as it is not the focus of this paper.

IV. MODELING SSOM BASED ON STOCHASTIC DEA

In this section, we develop a stochastic programming model based on Additive model proposed by Charnes *et al.* [21] in 1985 which considers the total slacks of inputs and outputs simultaneously in arriving at a point on the efficient frontier. In the proposed stochastic spares optimization model (SSOM), every candidate inventory item is regarded as a decision-making unit (DMU). The constraints derive from the index system. The objective is to find the optimum which simultaneously maximizes outputs and minimizes inputs in the sense of vector optimizations. The candidate DMU is efficient when the objective is zero, based on which we can rank all candidate DMUs. The section is organized as follows. Firstly, we give a brief introduction on the relative symbols and notations. Then we classify 14 factors into input variables and output variables. The SSOM is subsequently established with input vectors and output vectors. Finally, a ranking criterion is given and illustrated, based on which we can give priorities to all candidate DMUs.

Assume that there are n DMUs and then relative symbols and notations are introduced as follows:

DMU_k : the kth DMU, $k = 1, 2, \cdots, n$;
DMU_0 : target DMU;
$\widetilde{T}_k$: the random TAT of DMU_k;
$\widetilde{F}_k$: the random MTBF of DMU_k;
$\widetilde{W}_k$: the random mission time of DMU_k;
$\widetilde{M}_k$: the random corrective maintenance time of DMU_k;
$\widetilde{D}_k$: the random logistic delay time of DMU_k;
S_k : the number of suppliers of DMU_k;
E_k : the operational environment of DMU_k
G_k : the consequences of failure of DMU_k;
A_k : the replace-ability of DMU_k;
P_k : standard part or not of DMU_k;
N_k : the number of stand-alone installation of DMU_k;
Z_k : the number of equipments of DMU_k;
L_k : the purchase lead time of DMU_k;
C_k : the cost of DMU_k;
λ_k: the weght of kth DMU $i = 1, 2, \cdots, n$;
s_i^-: the slack of each ith input;
s_j^+: the slack of each jth output;
Pr is the probability measure;
α is belief degree which is a predetermined number between 0 and 1.

DEA method can be regarded as a production process with multiple inputs and outputs. As is known, all the manufactures hope to produce maximum outputs with the least inputs. Therefore, this principle is also reflected in DEA model. The DMU will be more effective than other DMUs if it has a smaller input as well as a larger output. According to the strategy tendency with smaller inputs and larger outputs, we divide all the parameters into input indexes and output indexes. It should be classified as input index if more attentions need to be paid to the smaller parameter, conversely it should be regarded as output index. For example, MTBF is regarded as an input index because items with shorter MTBF are more important with the aim of selecting pivotal spare parts varieties. By contrast, purchase lead time is regarded as an output variable as it is more likely to be short of the spare parts whose purchase lead time is long. The inputs and outputs are:

$$\begin{aligned} X_k &= \{\widetilde{T}_k, \widetilde{F}_k, S_k, E_k, G_k, A_k, C_k\}, \quad k = 1, 2, 3, \ldots, n; \\ Y_k &= \{\widetilde{W}_k, N_k, Z_k, \widetilde{M}_k, L_k, \widetilde{D}_k, P_k\}, \quad k = 1, 2, 3, \ldots, n. \end{aligned}$$

Then the SSOM is:

$$\begin{cases} \max\ \theta = \sum_{i=1}^{7} s_i^- + \sum_{j=1}^{7} s_j^+ \\ \textit{subject to}: \\ \Pr\left\{\sum_{k=1}^{n} \widetilde{T}_k\lambda_k \le \widetilde{T}_0 - s_1^-\right\} \ge \alpha, \\ \Pr\left\{\sum_{k=1}^{n} \widetilde{F}_k\lambda_k \le \widetilde{F}_0 - s_2^-\right\} \ge \alpha, \\ \Pr\left\{\sum_{k=1}^{n} \widetilde{W}_k\lambda_k \ge \widetilde{W}_0 + s_1^+\right\} \ge \alpha, \\ \Pr\left\{\sum_{k=1}^{n} \widetilde{M}_k\lambda_k \ge \widetilde{M}_0 + s_4^+\right\} \ge \alpha, \\ \Pr\left\{\sum_{k=1}^{n} \widetilde{D}_k\lambda_k \ge \widetilde{D}_0 + s_6^+\right\} \ge \alpha, \\ \sum_{k=1}^{n} S_k\lambda_k \le S_0 - s_3^-, \\ \sum_{k=1}^{n} E_k\lambda_k \le E_0 - s_4^-, \\ \sum_{k=1}^{n} G_k\lambda_k \le G_0 - s_5^-, \\ \sum_{k=1}^{n} A_k\lambda_k \le A_0 - s_6^-, \\ \sum_{k=1}^{n} C_k\lambda_k \le C_0 - s_7^-, \\ \sum_{k=1}^{n} N_k\lambda_k \ge N_0 + s_2^+, \\ \sum_{k=1}^{n} Z_k\lambda_k \ge Z_0 + s_3^+, \\ \sum_{k=1}^{n} L_k\lambda_k \ge L_0 + s_5^+, \end{cases} \tag{2a}$$

$$\begin{cases} \sum_{k=1}^{n} P_k\lambda_k \ge P_0 + s_7^+, \\ \sum_{k=1}^{n} \lambda_k = 1, \\ \lambda_k \ge 0, \quad k = 1, 2, \cdots, n \\ s_i^- \ge 0, \quad i = 1, 2, \cdots, 7 \\ s_j^+ \ge 0, \quad j = 1, 2, \cdots, 7 \end{cases} \tag{2b}$$

RANKING CRITERION

The closer θ is to zero, the more efficient the DMU_0 is ranked.

The θ is the sum of all the input slacks and output slacks for one DMU. To let θ close to 0, either the inputs are small, or the outputs are larger, or both of them. Thus the closer to 0 the θ is, the more potential the DMU has got to be reserved.

We can give priorities to DMUs by the ranking criterion, based on which inventory policies could be determined.

V. EQUIVALENT DETERMINISTIC MODEL

This section simplifies the constraints of random variables and develops equivalent deterministic models to overcome the difficulty in solving nonlinear programming.

Defination 1: (Liu [37]) Suppose that ξ is a random variable defined on probability space $(\Omega, \tilde{A}, \Pr)$. For any $\alpha \in (0, 1]$, the α-optimistic values of ξ are defined as

$$\xi_{\sup}(\alpha) = \sup\{r \,|\, \Pr\{\xi \ge r\} \ge \alpha\}.$$

Defination 2: (Liu [37]) Suppose that ξ is a random variable defined on probability space $(\Omega, \tilde{A}, \Pr)$. For any $\alpha \in (0, 1]$, the α-pessimistic values of ξ are defined as

$$\xi_{\inf}(\alpha) = \inf\{r \,|\, \Pr\{\xi \le r\} \ge \alpha\}.$$

Defination 3: A real-valued function f defined on a convex set $X \in R^n$ is said to be quasiconcave if

$$f(\lambda x + (1-\lambda) y) \ge \min\{f(x), f(y)\}$$

for any $x, y \in X$ and $0 < \lambda < 1$.

Theorem 1: Assume $\tilde{T}_1, \tilde{T}_2, \ldots, \tilde{T}_n$ are independent random variables defined on probability space $(\Omega, \tilde{A}, \Pr)$. If $\Pr\{\tilde{T}_k = x_k\}(k = 1, 2, \cdots, n)$ are quasiconcave, and any α is given in $(0.5, 1]$, $\lambda_k \in [0, 1]$, then for

$$Pr\left\{\sum_{k=1}^{n} \widetilde{T}_k\lambda_k \le \widetilde{T}_0 - s_1^-\right\} \ge \alpha, \tag{3}$$

we have

$$\sum_{k=1,k\neq 0}^{n} \{\lambda_k(\widetilde{T}_k)_{\inf}(\alpha)\} + \lambda_0[(\widetilde{T}_0)_{\sup}(\alpha)] \le (\widetilde{T}_0)_{\sup}(\alpha) - s_1^-. \tag{4}$$

Proof 1: Without loss of generality, let $n = 2$, $\lambda_1 = \lambda_0$ and $\tilde{T}_1 = \tilde{T}_0$, then we will consider the equation

$$Pr\left\{\widetilde{T}_0\lambda_0 + \widetilde{T}_2\lambda_2 \le \widetilde{T}_0 - s_1^-\right\} \ge \alpha. \tag{5}$$

Rewrite equation (5) as

$$Pr\left\{(1-\lambda_0)\,\widetilde{T}_0 + (-\lambda_2)\,\widetilde{T}_2 \le s_1^-\right\} \le 1 - \alpha. \tag{6}$$

Then we have

$$\begin{aligned} &\Pr\{(1-\lambda_0)\,\widetilde{T}_0 + (-\lambda_2)\,\widetilde{T}_2 \le s_1^-\} \\ &= 1 - \sup_{x_1+x_2 > s_1^-} \{\Pr\{(1-\lambda_0)\widetilde{T}_0 = x_1\} \wedge \Pr\{(-\lambda_2)\,\widetilde{T}_2 = x_2\}\} \\ &\le 1 - \alpha. \end{aligned} \tag{7}$$

TABLE 2. Quantification results of qualitative indexes .

Number	E	G	A	P
Item 1	7	7	3	3
Item 2	1	5	3	1
Item 3	9	7	3	1
Item 4	7	7	1	3
Item 5	5	9	1	3
Item 6	3	1	1	1
Item 7	7	5	1	3
Item 8	7	5	1	3
Item 9	7	7	3	3
Item 10	5	3	3	3

Hence,

$$\sup_{x_1+x_2>s_1^-}\{\Pr\{(1-\lambda_0)\widetilde{T}_0=x_1\}\wedge\Pr\{(-\lambda_2)\widetilde{T}_2=x_2\}\}\geq\alpha.$$

Suppose that $\left(x_1^*,x_2^*\right)=\arg\sup_{x_1+x_2\in R}\Pr\{\{(1-\lambda_0)\widetilde{T}_0=x_1\}\wedge\Pr\{(-\lambda_2)\widetilde{T}_2=x_2\}|\{x_1+x_2>s_1^-\}\}\geq\alpha\}$.

It follows that $\Pr\{(1-\lambda_0)\widetilde{T}_0=x_1^*\}\wedge\Pr\{(-\lambda_2)\widetilde{T}_2=x_2^*\}\geq\alpha$ *and* $x_1^*+x_2^*>s_1^-$.

Since $\Pr\{(1-\lambda_0)\widetilde{T}_0=x_1^*\}\wedge\Pr\{(-\lambda_2)\widetilde{T}_2=x_2^*\}\geq\alpha$ *implies that* $\Pr\{(1-\lambda_0)\widetilde{T}_0=x_1^*\}\geq\alpha$, $\Pr\{(-\lambda_2)\widetilde{T}_2=x_2^*\}\geq\alpha$.

From that the functions $\Pr\{(1-\lambda_0)\widetilde{T}_0=x_1\}$ *and* $\Pr\{(-\lambda_2)\widetilde{T}_2=x_2\}$ *are quasiconcave, we have*

$$x_1^*\leq((1-\lambda_0)\widetilde{T}_0)_{\sup}(\alpha),\ x_2^*\leq((-\lambda_2)\widetilde{T}_2)_{\sup}(\alpha).$$

Then we get

$$((1-\lambda_0)\widetilde{T}_0)_{\sup}(\alpha)+((-\lambda_2)\widetilde{T}_2)_{\sup}(\alpha)\geq s_1^-.$$

Otherwise,

$$((1-\lambda_0)\widetilde{T}_0)_{\sup}(\alpha)+((-\lambda_2)\widetilde{T}_2)_{\sup}(\alpha)<s_1^-,$$

$\Pr\{(1-\lambda_0)\widetilde{T}_0=((1-\lambda_0)\widetilde{T}_0)_{\sup}(\alpha)\}\leq\Pr\{(1-\lambda_0)\widetilde{T}_0=x_1^*\}$,

$\Pr\{(-\lambda_2)\widetilde{T}_2=((-\lambda_2)\widetilde{T}_2)_{\sup}(1-\alpha)\}\leq\Pr\{(-\lambda_2)\widetilde{T}_2=x_2^*\}$,

which are contradict with probability function $\Pr\{\xi\geq\xi_{\sup}(\alpha)\}\geq\alpha$.

Conversely, if

$$((1-\lambda_0)\widetilde{T}_0)_{\sup}(\alpha)+((-\lambda_2)\widetilde{T}_2)_{\sup}(\alpha)\geq s_1^-,$$

we get

$$\Pr\{(1-\lambda_0)\widetilde{T}_0=a_1\}\geq\alpha,$$

$$\Pr\{(-\lambda_2)\widetilde{T}_2=a_2\}\geq\alpha.$$

since $a_1<((1-\lambda_0)\widetilde{T}_0)_{\sup}\alpha$, $a_2<((-\lambda_2)\widetilde{T}_2)_{\sup}(\alpha)$.

Consequently,

$\sup_{x_1+x_2\in R}\Pr\{\{(1-\lambda_0)\widetilde{T}_0=x_1\}\wedge\Pr\{(-\lambda_2)\widetilde{T}_2=x_2\}|\{x_1+x_2>s_1^-\}\}\geq\alpha\}$.

TABLE 3. Quantification results of deterministic indexes .

Number	Z	N	S	L(day)	C(million)
Item 1	6	12	8	16	0.7
Item 2	6	4	8	8	0.9
Item 3	8	2	8	8	0.5
Item 4	2	1	8	8	3.7
Item 5	2	1	8	8	9.57
Item 6	5	3	8	7	0.7
Item 7	7	19	8	7	0.4
Item 8	15	25	8	7	1.2
Item 9	4	8	8	10	0.2
Item 10	6	4	8	10	0.3

Then,

$$\begin{aligned}&\Pr\{(1-\lambda_0)\widetilde{T}_0+(-\lambda_2)\widetilde{T}_2\leq s_1^-\}\\&=1-\sup_{x_1+x_2>s_1^-}\{\Pr\{(1-\lambda_0)\widetilde{T}_0=x_1\}\wedge\Pr\{(-\lambda_2)\widetilde{T}_2=x_2\}\}\\&\leq1-\alpha.\end{aligned}$$

Finally, we can get

$$((1-\lambda_0)\widetilde{T}_0)_{\sup}(\alpha)+\Big(\sum_{k=1,k\neq0}^{n}\big((-\lambda_k)\widetilde{T}_k)_{\sup}(\alpha)\big)\geq s_1^-\quad(8)$$

If $\lambda_k=0$ *or* 1, *it is obvious that*

$$\begin{aligned}&((1-\lambda_0)\widetilde{T}_0)_{\sup}(\alpha)=(1-\lambda_0)(\widetilde{T}_0)_{\sup}(\alpha)\\&((-\lambda_k)\widetilde{T}_k)_{\sup}(\alpha)=(-\lambda_k)(\widetilde{T}_k)_{\inf}(\alpha)\end{aligned}$$

If $\lambda_k\in(0,1)$, *then* $1-\lambda_\theta>\theta,-\lambda_k<\theta$,

$$\begin{aligned}&((1-\lambda_0)\widetilde{T}_0)_{\sup}(\alpha)\\&=\sup\{r|\Pr\{(1-\lambda_0)\widetilde{T}_0\geq r\}\geq\alpha\}\\&=(1-\lambda_0)\sup\{r/(1-\lambda_0)\,|\Pr\{\widetilde{T}_0\geq r/(1-\lambda_0)\}\geq\alpha\}\\&=(1-\lambda_0)(\widetilde{T}_0)_{\sup}(\alpha),\\&((-\lambda_k)\widetilde{T}_k)_{\sup}(\alpha)\\&=\sup\{r|\Pr\{(-\lambda_k)\widetilde{T}_k\geq r\}\geq\alpha\}\\&=-\lambda_k\sup\{-r/\lambda_k|\Pr\{\widetilde{T}_k\leq-r/\lambda_k\}\geq\alpha\}\\&=(-\lambda_k)(\widetilde{T}_k)_{\inf}(\alpha).\end{aligned}$$

Then, equation (8) can be rewritten as:

$$\sum_{k=1,k\neq0}^{n}\{\lambda_k(\widetilde{T}_k)_{\inf}(\alpha)\}+\lambda_0[(\widetilde{T}_0)_{\sup}(\alpha)]\leq(\widetilde{T}_0)_{\sup}(\alpha)-s_1^-.\tag{9}$$

Similarly, we may simplify other random constraints in the same way, the model (2) can be rewrite as *model (10).*

$$\begin{cases} \max\ \theta = \sum_{i=1}^{7} s_i^- + \sum_{j=1}^{7} s_j^+ \\ subject\ to: \\ \sum_{k=1}^{n} \{\lambda_k(\widetilde{T}_k)_{\inf}(\alpha)\} + \lambda_0[(\widetilde{T}_0)_{\sup}(\alpha) - (\widetilde{T}_0)_{\inf}(\alpha)] \\ \le (\widetilde{T}_0)_{\sup}(\alpha) - s_1^-, \\ \sum_{k=1}^{n} \{\lambda_k(\widetilde{F}_K)_{\inf}(\alpha)\} + \lambda_0[(\widetilde{F}_0)_{\sup}(\alpha) - (\widetilde{F}_0)_{\inf}(\alpha)] \\ \le (\widetilde{F}_0)_{\sup}(\alpha) - s_2^-, \\ \sum_{k=1}^{n} \{\lambda_k(\widetilde{W}_k)_{\sup}(\alpha)\} + \lambda_0[(\widetilde{W}_0)_{\inf}(\alpha) - (\widetilde{W}_0)_{\sup}(\alpha)] \\ \ge (\widetilde{W}_0)_{\inf}(\alpha) + s_1^+, \\ \sum_{k=1}^{n} \{\lambda_k(\widetilde{M}_k)_{\sup}(\alpha)\} + \lambda_0[(\widetilde{M}_0)_{\inf}(\alpha) - (\widetilde{M}_0)_{\sup}(\alpha)] \\ \ge (\widetilde{M}_0)_{\inf}(\alpha) + s_4^+, \\ \sum_{k=1}^{n} \{\lambda_k(\widetilde{D}_k)_{\sup}(\alpha)\} + \lambda_0[(\widetilde{D}_0)_{\inf}(\alpha) - (\widetilde{D}_0)_{\sup}(\alpha)] \\ \ge (\widetilde{D}_0)_{\inf}(\alpha) + s_6^+, \\ \sum_{k=1}^{n} S_k\lambda_k \le S_0 - s_3^-, \\ \sum_{k=1}^{n} E_k\lambda_k \le E_0 - s_4^-, \\ \sum_{k=1}^{n} G_k\lambda_k \le G_0 - s_5^-, \\ \sum_{k=1}^{n} A_k\lambda_k \le A_0 - s_6^-, \\ \sum_{k=1}^{n} C_k\lambda_k \le C_0 - s_7^-, \\ \sum_{k=1}^{n} N_k\lambda_k \ge N_0 + s_2^+, \end{cases} \tag{10a}$$

$$\begin{cases} \sum_{k=1}^{n} Z_k\lambda_k \ge Z_0 + s_3^+, \\ \sum_{k=1}^{n} L_k\lambda_k \ge L_0 + s_5^+, \\ \sum_{k=1}^{n} P_k\lambda_k \ge P_0 + s_7^+, \\ \sum_{k=1}^{n} \lambda_k = 1, \\ \lambda_k \ge 0, \quad k = 1, 2, \cdots, n \\ s_i^- \ge 0, \quad i = 1, 2, \cdots, 7 \\ s_j^+ \ge 0, \quad j = 1, 2, \cdots, 7 \end{cases} \tag{10b}$$

Especially, when random variables obey normal distributions and they are independent, the equivalent model can be rewrite as model (11),

$$\begin{cases} \max\ \theta = \sum_{i=1}^{7} s_i^- + \sum_{j=1}^{7} s_j^+ \\ subject\ to: \\ \sum_{k=1,k\neq 0}^{n} \overline{T}_k + \Phi^{-1}(\alpha)\sigma_{Tk}^I\lambda_k + \lambda_0[\overline{T}_0 - \sigma_{T0}^I\Phi^{-1}(\alpha)] \\ \le [\overline{T}_0 - \sigma_{T0}^I\Phi^{-1}(\alpha)] - s_1^-, \\ \sum_{k=1,k\neq 0}^{n} \overline{F}_k + \Phi^{-1}(\alpha)\sigma_{Fk}^I\lambda_k + \lambda_0[\overline{F}_0 - \sigma_{F0}^I\Phi^{-1}(\alpha)] \\ \le [\overline{F}_0 - \sigma_{F0}^I\Phi^{-1}(\alpha)] - s_2^-, \\ \sum_{k=1,k\neq 0}^{n} (\overline{W}_k - \Phi^{-1}(\alpha)\sigma_{Wk}^O)\lambda_k + \lambda_0[\overline{W}_0 + \sigma_{W0}^O\Phi^{-1}(\alpha)] \\ \ge [\overline{W}_0 + \sigma_{W0}^O\Phi^{-1}(\alpha)] + s_1^+, \\ \sum_{k=1,k\neq 0}^{n} (\overline{M}_k - \Phi^{-1}(\alpha)\sigma_{Mk}^O)\lambda_k + \lambda_0[\overline{M}_0 + \sigma_{M0}^O\Phi^{-1}(\alpha)] \\ \ge [\overline{M}_0 + \sigma_{M0}^O\Phi^{-1}(\alpha)] + s_4^+, \\ \sum_{k=1,k\neq 0}^{n} (\overline{D}_k - \Phi^{-1}(\alpha)\sigma_{Dk}^O)\lambda_k + \lambda_0[\overline{D}_0 + \sigma_{D0}^O\Phi^{-1}(\alpha)] \\ \ge [\overline{D}_0 + \sigma_{D0}^O\Phi^{-1}(\alpha)] + s_6^+, \\ \sum_{k=1}^{n} S_k\lambda_k \le S_0 - s_3^-, \\ \sum_{k=1}^{n} E_k\lambda_k \le E_0 - s_4^-, \\ \sum_{k=1}^{n} G_k\lambda_k \le G_0 - s_5^-, \\ \sum_{k=1}^{n} A_k\lambda_k \le A_0 - s_6^-, \\ \sum_{k=1}^{n} C_k\lambda_k \le C_0 - s_7^-, \\ \sum_{k=1}^{n} N_k\lambda_k \ge N_0 + s_2^+, \\ \sum_{k=1}^{n} Z_k\lambda_k \ge Z_0 + s_3^+, \\ \sum_{k=1}^{n} L_k\lambda_k \ge L_0 + s_5^+, \\ \sum_{k=1}^{n} P_k\lambda_k \ge P_0 + s_7^+, \\ \sum_{k=1}^{n} \lambda_k = 1, \end{cases} \tag{11a}$$

$$\begin{cases} \lambda_k \ge 0, \quad k = 1, 2, \cdots, n \\ s_i^- \ge 0, \quad i = 1, 2, \cdots, 7 \\ s_j^+ \ge 0, \quad j = 1, 2, \cdots, 7 \end{cases} \tag{11b}$$

TABLE 4. Distributions of random variables.

Number	T(h)	F(h)	W(h)	M(h)	D(h)
Item 1	N(11, 0.78)	N(49103, 1054)	N(60724, 1067.6)	N(14.5, 2.77)	N(7.3, 0.83)
Item 2	N(7.9, 0.46)	N(134081 ,1659.5)	N(159089, 1089.7)	N(22.0, 3.02)	N(9.9, 2.02)
Item 3	N(0.5, 0.25)	N(78495,1347)	N(239800 , 3409.9)	N(17.6, 2.97)	N(3.7, 0.87)
Item 4	N(20, 0.38)	N(50466,1289)	N(678006 , 4590.6)	N(3.5 , 1.33)	N(11.4, 1.73)
Item 5	N(4.5, 1.32)	N(154077,2531.9)	N(149800 , 2063.2)	N(11.0, 0.80)	N(7.8, 0.96)
Item 6	N(2.7, 0.35)	N(70290, 1356.8)	N(132079, 1340.7)	N(2.5, 0.46)	N(12.1, 0.98)
Item 7	N(5.9, 2.32)	N(130839, 1567.5)	N(4765000, 3985.7)	N(10.5, 1.65)	N(15.6, 2.86)
Item 8	N(18.6, 5.22)	N(93675, 1029.6)	N(158900, 2183.4)	N(16.7, 2.77)	N(13.6, 2.18)
Item 9	N(1.9, 0.28)	N(165009, 1342.8)	N(367284, 3026.4)	N(17.9, 2.65)	N(8.7, 1.35)
Item 10	N(1.8, 0.27)	N(134200,1507.6)	N(529099, 4515.7)	N(4.5, 1)	N(5.3, 0.66)

TABLE 5. Optimal results.

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}
θ	$6.23 * 10^4$	$1 * 10^5$	$4.95 * 10^5$	0	0	0	0	0	0	0

where $\Phi^{-1}(\alpha)$ is the inverse function of the standard normal distribution, σ^I_{TK} is the standard deviation of T_k $(k = 1, 2, ..., n)$, $\sigma^I_{T_0}$ is the standard deviation of T_0, $\overline{T}_k$ is the average value of T_k $(k = 1, 2, ..., n)$, $\overline{T}_0$ is the average value of T_0. $\sigma^O_{W_k}$ is the standard deviation of W_k $(k = 1, 2, ..., n)$, $\sigma^O_{W_0}$ is the standard deviation of W_0, $\overline{W}_k$ is the average value of W_k $(k = 1, 2, ..., n)$, $\overline{W}_0$ is the average value of W_0.

VI. A NUMERICAL EXAMPLE

In this section, we apply and evaluate the performance of the proposed method to address the problem of optimization of spare parts varieties. Firstly, we introduce the background on this simple system and the information of input and output factors. Then we calculate the optimal value of each DMU under SSOM. Finally, we provide some decision proposals on inventory management.

We focus on a depot which support eight airplanes in a mission. Ten items are selected randomly from the airplane parts list as an example to illustrate the proposed approach. The quantification results of qualitative indexes are obtained from expert elicitation as shown table 2. The quantification results of deterministic indexes are determined by mission requirements and historical data, as shown in table 3. The distributions of random variables are supposed to be normal distributions to reduce the computational complexity and the relative data are shown in table 4.

We regard each type of item as a DMU to calculate the optimal value in SSOM and then select the appropriate spare parts based on these optimal results. The input values and output values are taken from the quantification results as shown in table 2 to 4. Specifically, the belief degree α is 0.80, which means that the target DMU could meet the restrictions with the probability of 0.80. The optimal results solved by the SSOM is shown in table 5.

According to the ranking criteria, the basic ranking order is DMU_4, DMU_5, DMU_6, DMU_7, DMU_8, DMU_9, DMU_{10}, DMU_1, DMU_2, DMU_3, in which DMU_4 to DMU_{10} are equally important. Based on the ranking order, the basic inventory policy is to store item 4 to item 10. Moreover, we could distinguish the equally important items by a single index. For example, cost is an important factor in the inventory management. Priority is usually given to low-cost items. Then the ranking order in terms of cost could be given as follows: DMU_9, DMU_{10}, DMU_7, DMU_6, DMU_8, DMU_4, DMU_5. The inventory policy could be adjusted based on the new ranking order when the budget is limited.

VII. CONCLUSIONS

We presented a stochastic DEA model for optimization under uncertainty, developed equivalent deterministic models to overcome difficulty in solving non-linear programming and then applied our approach to address the problem of optimizing the inventory policy of spare parts varieties. We proposed a comprehensive index system from the perspective of the product life-cycle process, which consisted of design factors, operation factors and support factors. Then we established a stochastic DEA model called SSOM considering the uncertain nature in parameters to select spare parts varieties with the constraints of multi-criteria. Some algorithms were employed to obtain the equivalent deterministic model of SSOM. In particular, the equivalent deterministic model of normal distributions was discussed. Finally, we applied our approach on a depot which serves eight airplanes for illustration and provided reasonable inventory management proposals based on the optimal results. We also provided a single criterion for equally important DMUs in terms of cost.

Decision makers can assign different values of belief degree α to different factors in terms of actual demand, which keeps the same for all random factors in the current model. Although the model in this paper deals with the optimization problem in spare parts varieties, it is a general model and can be applied in the other fields of multi-criteria optimization where random factors need to be considered. In actual implementation, decision makers can simplify the model as needed.

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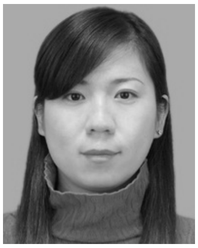

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