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# Distributed Control Method for Economic Dispatch in Islanded Microgrids With Renewable Energy Sources

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**ABSTRACT** The economic dispatch in islanded microgrids (MGs) is a thorny problem due to the low equivalent inertia and the uncertainty of environmental conditions. In this paper, a two-layer model is proposed for economic dispatch in islanded MGs, in which an MG is considered as the lower layer, while a communication network is considered as the upper layer. Furthermore, a systematic method is presented to derive fully distributed control laws from any given communication network. On the communication network, there are two subgraphs, $\tilde{G}$ and $\hat{G}$, one of which consists of all agents, while the other of which consists of only controllable agents. Correspondingly, two sets of control laws are derived from the two subgraphs, where the control laws from the subgraph $\tilde{G}$ ensure the supply-demand balance in the MG, while the control laws from the subgraph $\hat{G}$ realize the economic dispatch. Moreover, the economic operation in MGs can be satisfied, only if agents on the communication network regulate distributed generators iteratively according to the control laws. Finally, three cases are designed to evaluate the performance of the method, and the simulation results verify the effectiveness of the proposed method.

**INDEX TERMS** Distributed control, economic dispatch, incremental cost, microgrids.

## I. INTRODUCTION

In recent years, distributed generation has drawn much attention in the world, because it seems to be an effective way to reduce pollution. However, the outputs of distributed generators (DGs) using renewable resources such as wind and sunlight are uncertain due to the uncertainty of environmental conditions. Therefore, to decrease the negative impacts on the main grids, generally speaking, DGs and other equipment are integrated as microgrids (MGs) and then connected into the main grids. The control methods for MGs are shifting from centralized methods [1] to distributed ones [2], [3]. More recently, the economic operation or dispatch in MGs is considered as the extension and application of the control methods [4], [5]. Nevertheless, the economic dispatch (ED) in islanded MGs becomes more challenging due to no any support of the main grid and low equivalent inertia of MGs. Even worse, the uncertainty of environmental conditions and fluctuations of load demand are unavoidable [6]–[8].

Therefore, the ED is generally regarded as an optimization problem [9], [10]. Previous efforts to solve ED problem can be classified into two categories, namely centralized and distributed methods. The analytical methods such as the gradient search methods [11] and the heuristic methods such as the genetic algorithm [12] have been applied extensively to solve ED problems in a centralized way [13]. In this case, a central controller is necessary to collect information from the entire system for algorithm implementation [14]. However, in the future, systems will be much larger and have more variable topologies [15], so the possibility of single point failures of the central controller becomes higher because a huge amount of data needs to be collected and processed [16]–[18]. Hence, the response speed of the central controller will decrease a lot, which makes the applications of the real time control harder [19].

To improve the system performance, distributed methods have been proposed to satisfy the requirements for real-time applications and large scale systems. In distributed methods based on multi-agent systems, all the units only communicate with their local neighbors and no central controller exists [20]–[24]. The global optimal solution will be obtained iteratively by exchanging information among neighbors [25]. If the decision-making based on the distributed resource allocation is considered, it offers a way to deal with the single point failures [26]. Applying the equal incremental cost criterion, the operation costs of the system are minimized by a distributed method [27].

However, some of them are not fully distributed. For example, in [28], the leader node is needed to collect the global information, and some central measurement nodes are added to choose the leader node. Moreover, the intermittent resources are not taken into consideration [29]. In the distributed algorithm proposed in [30], the learning gain is used to accelerate the response speed of the algorithm, but it breaks the power balance during iterations of the algorithm. In [31], Mudumbai *et al.* use frequency deviations to eliminate power imbalance and the distributed algorithm can eventually converge to the optimal set points, but its convergence time is relatively long. The method in [32] converges almost in one second.

To the best of our knowledge, in some previous methods, due to the high complexity of algorithms, they cannot rapidly respond fluctuations of environmental conditions and/or load demand. Moreover, some of distributed methods break the supply-demand balance during iterations. Therefore, in this paper, a simple and fully distributed method for economic dispatch is proposed, which is a two layer model, where the lower layer is an MG, while the upper layer is a communication network. Next, two sets of control laws are derived from the communication network, where one set of laws ensures the supply-demand balance in the system during iterations, and the other reaches the minimal operation cost of the system in a distributed manner. Therefore, no any centralized or leader agents are needed, while only local information is used. Finally, a test bed is built in MATLAB/Simulink and different cases are designed to test the performance of our method. The simulation results show that the economic dispatch can be achieved, when constraints of DG capacities are considered, and the system can still remain stable even if the system fluctuates significantly. In this paper, the main contributions are summarized as follows.

(1) A simple and fully distributed method for the economic dispatch in MGs is proposed, which is a two layer model, where the rules of building a communication network are given.
(2) A systematic method is presented to obtain the distributed control laws, when a communication network is established.
(3) The control laws can respond the fluctuations rapidly in a distributed manner, so the economic dispatch in MGs can be obtained iteratively and the supply-demand balance is never broken during iterations.
(4) The theorems are proved, which ensure the convergence of our method.

The rest of this paper is organized as follows. Section II introduces the ED problem briefly. In Section III, the rules of forming a communication network over an MG are introduced first and then the control laws are derived. Further, the convergence of the control laws are analyzed. Section IV shows the structure of an MG and parameters of DGs for simulations. In Section V, the results are analyzed and compared. Section VI concludes the paper.

## II. FORMULATION OF ECONOMIC DISPATCH PROBLEM

This section introduces the ED problem for islanded MGs, which offers a way to minimize operation costs of the system. And the analytic solutions are given without and with constraints of DG capacities.

Generally speaking, if there are $n$ DGs, an ED problem can be expressed as follows,

$$\text{Min } F = \sum_{i=1}^{n} F_i(p_i) \tag{1}$$

$$s.t.\ \sum_{i=1}^{n} p_i = P_{\text{load}} \tag{2}$$

$$p_i^{\min} \le p_i \le p_i^{\max} \tag{3}$$

where $P_{\text{load}}$ is the total load demand satisfying $\sum_{i=1}^{n} p_i^{\min} < P_{\text{load}} < \sum_{i=1}^{n} p_i^{\max}$. $p_i$ is the outputs of a DG$_i$. $p_i^{\min}$ and $p_i^{\max}$ are the minimal and maximal outputs of the DG$_i$, respectively. $F_i(p_i)$ is the cost function of the DG$_i$.

For conventional diesel generators, the cost function $F_i(p_i)$ is defined as a convex quadratic function of active power outputs [33],

$$F_i(p_i) = a_i p_i^2 + b_i p_i + c_i, \tag{4}$$

where $a_i \ge 0$, $b_i > 0$ and $c_i > 0$, and they are all the cost parameters of the DG$_i$.

For the sake of notational simplicity throughout the paper, if assuming $\alpha_i = \frac{b_i}{2a_i}$, $\beta_i = \frac{1}{2a_i}$, and $\gamma_i = c_i - \frac{b_i^2}{4a_i}$, the cost function can be rewritten as follows,

$$F_i(p_i) = \frac{(p_i + \alpha_i)^2}{2\beta_i} + \gamma_i. \tag{5}$$

Further, the incremental cost of a generator $i$ is defined as the first derivative of the cost function $F_i$ [30],

$$\lambda = \frac{\partial F_i(p_i)}{\partial p_i} = \frac{p_i + \alpha_i}{\beta_i}. \tag{6}$$

### A. MODEL WITHOUT CONSTRAINTS OF DG CAPACITIES

According to the equal incremental cost criterion, the optimal incremental costs without constraints of DG capacities will be obtained, if the incremental costs of all DGs are equal [34].

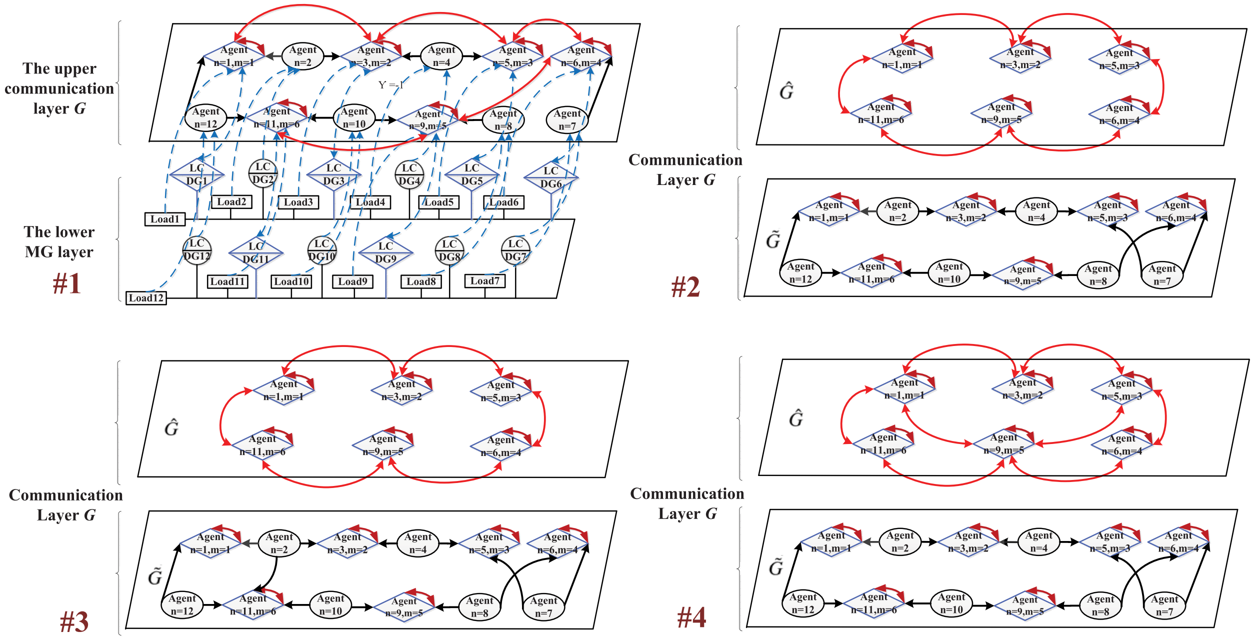

**FIGURE 1.** The two-layer model for the economic dispatch in MGs, where on the communication network *G* uncontrollable agents and partially controllable agents are represented by ovals, while controllable agents are represented by diamonds. (#1) is the two-layer model that consists of an MG and a communication network. (#2) is the communication network 1. (#3) is the communication network 2. (#4) is the communication network 3.

Here, we will find the solution of the model. Assuming $p_i^{\max} = \infty$ and $p_i^{\min} = -\infty$, and then using Lagrangian function, the ED problem can be formulated as

$$U = \sum_{i=1}^{n} F_i(p_i) + \lambda(\sum_{i=1}^{n} p_i - P_{\text{load}}). \tag{7}$$

According to the first order optimality conditions, we have

$$\lambda^* = \frac{\partial U}{\partial p_i} = \frac{p_i^* + \alpha_i}{\beta_i}, \tag{8}$$

where $p_i^*$ is the optimal output of a DG$_i$ and $\lambda^*$ is the incremental cost. Applying (2), it yields

$$\lambda^* = \frac{P_{\text{load}} + \sum_{i=1}^{n} \alpha_i}{\sum_{i=1}^{n} \beta_i}, \tag{9}$$

which gives the optimal incremental cost, if the total active power is equal to $P_{\text{load}}$. Thus the optimal outputs of the DG$_i$ can be obtained by

$$p_i^* = \beta_i \lambda^* - \alpha_i. \tag{10}$$

### B. MODEL WITH CONSTRAINTS OF DG CAPACITIES

In reality, the constraints of DG capacities (3) are always considered. If the optimal references for outputs of the DG$_i$ are greater than or less than the maximal or minimal capacity of the generator, the constraints of DG capacities will be violated, which means the optimal incremental costs are not equal no matter how outputs of DGs are regulated. In this model, we will deal with this situation.

Assume $\Omega_p$ is the set of DGs whose outputs violate the inequality constraints (3), where their set points of outputs should be set at the upper or lower bounds, namely, $p_i^* = p_i^{\min}$ or $p_i^{\max}$. For the other DGs, the optimal incremental cost is still

$$\lambda^* = \frac{p_i^* + \alpha_i}{\beta_i}, \quad i \notin \Omega_p, \tag{11}$$

but (9) should be changed to

$$\lambda^* = \frac{P_{\text{load}} - \sum\limits_{i \in \Omega_p} p_i + \sum\limits_{i \notin \Omega_p} \alpha_i}{\sum\limits_{i \notin \Omega_p} \beta_i}. \tag{12}$$

Therefore, the optimal outputs of all DGs can be expressed as

$$p_i^* = \begin{cases} \beta_i \lambda^* - \alpha_i, & i \notin \Omega_p, \\ p_i^{\min} \text{ or } p_i^{\max}, & i \in \Omega_p. \end{cases} \tag{13}$$

## III. DISTRIBUTED CONTROL METHOD FOR ECONOMIC DISPATCH IN MGS

In this section, a two-layer model is proposed to solve the ED problem and the rules of building a communication network are introduced. Later, two sets of control laws are obtained based on a given communication network. Further, the method is extended to solve the ED problem with constraints of DG capacities.

### A. CONTROL MODEL

In general, an islanded MG consists of DGs, loads and local controllers, while a communication network consists of agents, which forms a two-layer model, as shown in Fig. 1.

In this model, the lower layer is the MG, in which DGs, like microturbines, are considered as controllable DGs, while DGs, like photovoltaic (PV) systems or wind turbines (WTs), are considered as uncontrollable DGs. In an islanded MG, generally a DG working in voltage and frequency control (V/F control) mode is needed to provide the system losses immediately, where a battery energy storage system (BESS) is used as the V/F DG, also called a partially controllable DG. Moreover, in this paper, the controllable DGs work in the active and reactive power control (PQ control) mode, while the uncontrollable DGs work in the maximum power point tracking (MPPT) control mode.

The upper layer over the MG is a communication network that consists of three types of agents, i.e., controllable agents, uncontrollable agents and partially controllable agents. As the links between two layers are shown, each agent connects to a DG and a load. Moreover, an agent is called a controllable agent, when it connects to a controllable DG, while the agents connecting to an uncontrollable DG or a partially controllable DG is called an uncontrollable or partially controllable agent, respectively. On the communication network, uncontrollable agents (denoted by ovals) only have outgoing links that are indicated by black arrow lines, because they only send information, such as the values of active power and reactive power. In contrast, the controllable agents (denoted by diamonds) may have outgoing and ingoing links that are indicated by red arrow lines, because they can send and process information. Also, they can collect their own information indicated by the self loops. Noting that the dashed lines between two layers indicate the information flow. If the communication network is built, distributed control laws will be derived, which regulate the outputs of DGs and finally make the economic dispatch obtained.

### B. DISTRIBUTED CONTROL LAWS FOR ECONOMIC DISPATCH WITHOUT CONSTRAINTS OF DG CAPACITIES

The communication network $G$ with $n$ agents consists of two subgraphs, where both controllable and uncontrollable agents form the subgraph $\tilde{G}$, while only $m$ controllable agents are on the subgraph $\hat{G}$. It is worth noting that the controllable agents in both subgraphs are the same. Moreover, the subgraph $\tilde{G}$ is a bi-directed graph, while the subgraph $\hat{G}$ is a directed graph, as shown in Fig. 1. From the communication network $G$, two sets of control laws can be derived. The first set of control laws derived from the subgraph $\tilde{G}$ maintains the supply-demand balance in the MG, while the second set of control laws based on the subgraph $\hat{G}$ reassigns the outputs of controllable DGs to obtain the economic dispatch.

On the subgraph $\tilde{G}$, a weighted matrix $W = [w_{ij}]_{n\times n}$ is applied to describe the relationships among all agents. If there is a link from an agent $i$ to an agent $j$, the weight on this link should be

$$w_{ij} = \frac{1}{g_i}, \tag{14}$$

where $g_i$ is the outgoing degree of an agent $i$ on the subgraph $\tilde{G}$. Moreover, the weight $w_{ii}$ on the self-loop is one and the sum of each row in $W$ is one,

$$\sum_{i=1}^{n} w_{ij} = 1. \tag{15}$$

Further, a diagonal matrix $V = [v_{ii}]_{n\times n}$ is applied to indicate the type of agents, i.e., if an agent is controllable, then $v_{ii} = 1$. Otherwise, $v_{ii} = 0$.

In the system, it is deemed that the system is balanced, if the following equation [3] is satisfied,

$$\begin{cases}\sum[V\cdot P(t+1)] = \sum L^p(t) - \sum[(I-V)\cdot P(t)]\,,\\ \sum[V\cdot Q(t+1)] = \sum L^q(t) - \sum[(I-V)\cdot Q(t)]\,,\end{cases} \tag{16}$$

where $P(t+1) = [p_i(t+1)]_{n\times 1}$, $Q(t+1) = [q_i(t+1)]_{n\times 1}$, $L^p(t) = [l_i^p(t)]_{n\times 1}$ and $L^q(t) = [l_i^q(t)]_{n\times 1}$ are the active and reactive power column vector, and the active and reactive load demand column vector, respectively, while $I$ is an $n\times n$ identity matrix. Moreover, a parameter $\gamma$ is added between the V/F DG and its agent. If $\gamma = -1$, the outputs of the V/F DG is regarded as loads, so that they can be shared by controllable DGs. In this way, the V/F DG can inject or absorb power into or from the system immediately, and then its outputs return to zero gradually [22].

First, we will derive the first set of control laws from the subgraph $\tilde{G}$. According to the subgraph $\tilde{G}$, the set of control laws can be derived as follows,

$$\begin{cases}V\cdot P(t+1) = V\cdot P(t) + W^T\cdot[L^p(t) - P(t)]\,,\\ V\cdot Q(t+1) = V\cdot Q(t) + W^T\cdot[L^q(t) - Q(t)]\,,\end{cases} \tag{17}$$

where $(\cdot)^T$ denotes the transpose of a matrix. The control laws (17) can maintain the supply-demand balance, but the minimum operation costs of all controllable DGs cannot be ensured.

Therefore, the second set of control laws from the subgraph $\hat{G}$ is needed to realize the economic dispatch. Here, we define an $m\times m$ matrix $H = [h_{ij}]_{m\times m}$ to indicate the relationships among all controllable agents on $\hat{G}$, and the entries of $H$ denote the weights on links between agents. Assume there is a link from a controllable agent $i$ to a controllable agent $j$ on $\hat{G}$, so the weight on this link is

$$h_{ij} = \frac{1}{d_i} - \frac{1}{d_i}\cdot\frac{\beta_i/d_i}{\beta_i/d_i + \beta_j/d_j} \quad (j\in N_i, j\neq i), \tag{18}$$

where $\beta_i$ and $\beta_j$ are the cost parameters of the $\mathrm{DG}_i$ and $\mathrm{DG}_j$, which are defined in Section II, and $d_i$ and $d_j$ are the ingoing/outgoing degrees of the agents $i$ and $j$ on the subgraph $\hat{G}$. $N_i$ is the set of agents connecting to the agent $i$. On the other hand, the weight on the self loop of a controllable agent $i$ is

$$h_{ii} = 1 - \frac{1}{d_i}\cdot\sum_{j\in N_i}\frac{\beta_j/d_j}{\beta_i/d_i + \beta_j/d_j}. \tag{19}$$

Summarily, the second set of control laws for the ED in the MG is written as

$$\tilde{P}'(t+1) = H^T\cdot(\tilde{P}(t+1) + \boldsymbol{\alpha}) - \boldsymbol{\alpha}, \tag{20}$$

where $\tilde{P}(t+1) = [\tilde{p}_i(t+1)]_{m\times 1}$ is an $m \times 1$ vector that is formed by selecting the set points of all controllable DGs from $P(t+1) = [p_i(t+1)]_{n\times 1}$ in (17). The vector, $\tilde{P}'(t+1) = [\tilde{p}'_i(t+1)]_{m\times 1}$, is obtained, after active power is reassigned among controllable DGs economically. And $\boldsymbol{\alpha} = [\alpha_i]_{m\times 1}$ is the cost parameter vector. If assuming $\tilde{p}'^{\alpha}_i(t+1) = \tilde{p}'_i(t+1)+\alpha_i$ and $\tilde{p}^{\alpha}_i(t+1) = \tilde{p}_i(t+1)+\alpha_i$, then (20) is changed to

$$\tilde{P}'^{\alpha}(t+1) = H^T \cdot \tilde{P}^{\alpha}(t+1). \tag{21}$$

Therefore, one can find two sets of distributed control laws in terms of (17) and (21) to solve the ED problem in MGs.

### *C. CONVERGENCE ANALYSIS*

Applying the two sets of control laws, the economic dispatch will be obtained iteratively, which means the incremental costs of all controllable DGs will converge to a certain value. In this section, theorems are proven, which guarantee the convergence of the control laws. Fisrt, Theorem 1 is given as follows.

*Theorem 1:* Assume a communication network $G$ consists of two subgraphs, $\tilde{G}$ and $\hat{G}$, over an MG. On the subgraph $\tilde{G}$ there are $n$ agents, where the $k$-th agent is the partially controllable agent and the others are controllable agents or uncontrollable agents. If the set points of controllable DGs are calculated in terms of (17), then the power balance in MG will be ensured, i.e., (16) always holds.

*Proof:* First, summing up both sides of the control laws (17) respectively, it yields

$$\begin{aligned}
&\sum V \cdot P(t+1)\\
&\quad = \sum V \cdot P(t) + \sum W^T \cdot \left[L^p(t) - P(t)\right]\\
&\quad = (v_{11} \cdot p_1(t) + \cdots + v_{kk} \cdot \gamma \cdot p_k(t) + \cdots + v_{nn} \cdot p_n(t))\\
&\qquad + (w_{11} + w_{12} + \cdots + w_{1n}) \cdot \left[l_1^p(t) - p_1(t)\right]\\
&\qquad + \cdots + (w_{k1} + w_{k2} + \cdots + w_{kn}) \cdot \left[l_k^p(t) - \gamma \cdot p_k(t)\right]\\
&\qquad + \cdots + (w_{n1} + w_{n2} + \cdots + w_{nn}) \cdot \left[l_n^p(t) - p_n(t)\right].
\end{aligned} \tag{22}$$

Applying (15) to (22), it yields

$$\begin{aligned}
&\sum V \cdot P(t+1)\\
&\quad = (v_{11} \cdot p_1(t) + \cdots + v_{kk} \cdot \gamma \cdot p_k(t) + \cdots + v_{nn} \cdot p_n(t))\\
&\qquad + \left[l_1^p(t) - p_1(t)\right] + \left[l_k^p(t) - \gamma \cdot p_k(t)\right] + \left[l_n^p(t) - p_n(t)\right]\\
&\quad = (v_{11} - 1) \cdot p_1(t) + \cdots + (v_{kk} - 1) \cdot \gamma \cdot p_k(t)\\
&\qquad + (v_{nn} - 1) \cdot p_n(t) + \sum L^p(t)\\
&\quad = \sum L^p(t) - \sum (I - V) \cdot P(t).
\end{aligned} \tag{23}$$

Therefore, if the control laws (17) are applied, (16) always holds to ensure the power balance in the system. ■

*Theorem 2:* Assume there is a communication network $G$ composed of two subgraphs, $\tilde{G}$ and $\hat{G}$ over an MG. On the subgraph $\hat{G}$ there are only $m$ controllable agents. If controllable agents on the subgragh $\hat{G}$ compute the set points of controllable DGs according to the control laws (20) or (21) in order to dispatch the power economically, the system will always maintain balanced during iterations.

*Proof:* First, we prove the sum of a row of the matrix $H$ is one. If summing up the $i$-th row of $H$, we have

$$\begin{aligned}
&h_{i1} + \cdots + h_{ii} + \cdots + h_{im}\\
&\quad = \frac{1}{d_i} - \frac{1}{d_i} \cdot \frac{\beta_i/d_i}{\beta_i/d_i + \beta_1/d_1} + \cdots + 1\\
&\qquad - \frac{1}{d_i} \cdot \left(\frac{\beta_1/d_1}{\beta_i/d_i + \beta_1/d_1} + \cdots + \frac{\beta_m/d_m}{\beta_i/d_i + \beta_m/d_m}\right) + \cdots\\
&\qquad + \frac{1}{d_i} - \frac{1}{d_i} \cdot \frac{\beta_i/d_i}{\beta_i/d_i + \beta_m/d_m}\\
&\quad = 1 + \frac{1}{d_i} + \cdots + \frac{1}{d_i} - \frac{1}{d_i}\\
&\qquad \cdot \left(\frac{\beta_i/d_i}{\beta_i/d_i + \beta_1/d_1} + \frac{\beta_1/d_1}{\beta_i/d_i + \beta_1/d_1}\right)\\
&\qquad - \cdots - \frac{1}{d_i} \cdot \left(\frac{\beta_i/d_i}{\beta_i/d_i + \beta_m/d_m} + \frac{\beta_m/d_m}{\beta_i/d_i + \beta_m/d_m}\right)\\
&\quad = 1 + \frac{1}{d_i} + \cdots + \frac{1}{d_i} - \frac{1}{d_i} - \cdots - \frac{1}{d_i}\\
&\quad = 1.
\end{aligned} \tag{24}$$

Next, if both sides of the control laws (20) are summed up, respectively, it has the form as follows,

$$\begin{aligned}
&\sum \tilde{P}'(t+1)\\
&\quad = \sum H^T \cdot (\tilde{P}(t+1) + \boldsymbol{\alpha}) - \boldsymbol{\alpha}\\
&\quad = h_{11} \cdot (\tilde{p}_1 + \alpha_1) + \cdots + h_{i1} \cdot (\tilde{p}_i + \alpha_i) + \cdots + h_{m1}\\
&\qquad \cdot (\tilde{p}_m + \alpha_m)\\
&\qquad + \cdots + h_{1i} \cdot (\tilde{p}_1 + \alpha_1) + \cdots + h_{ii} \cdot (\tilde{p}_i + \alpha_i) + \cdots\\
&\qquad + h_{mi} \cdot (\tilde{p}_m + \alpha_m) + \cdots + h_{1m} \cdot (\tilde{p}_1 + \alpha_1) + \cdots\\
&\qquad + h_{im} \cdot (\tilde{p}_i + \alpha_i) + \cdots + h_{mm} \cdot (\tilde{p}_m + \alpha_m)\\
&\qquad - \alpha_1 - \cdots - \alpha_i - \cdots - \alpha_m\\
&\quad = (h_{11} + \cdots + h_{1i} + \cdots + h_{1m}) \cdot (\tilde{p}_1 + \alpha_1) + \cdots\\
&\qquad + (h_{i1} + \cdots + h_{ii} + \cdots + h_{im}) \cdot (\tilde{p}_i + \alpha_i) + \cdots\\
&\qquad + (h_{m1} + \cdots + h_{mi} + \cdots + h_{mm}) \cdot (\tilde{p}_m + \alpha_m)\\
&\qquad - \alpha_1 - \cdots - \alpha_i - \cdots - \alpha_m.
\end{aligned} \tag{25}$$

Applying (24), the above equations can be transformed into

$$\begin{aligned}
&\sum \tilde{P}'(t+1)\\
&\quad = (\tilde{p}_1 + \alpha_1) + \cdots + (\tilde{p}_i + \alpha_i) + \cdots + (\tilde{p}_m + \alpha_m)\\
&\qquad - \alpha_1 - \cdots - \alpha_i \cdots - \alpha_m\\
&\quad = \sum_{i=1}^{m} \tilde{p}_i = \sum \tilde{P}(t+1).
\end{aligned} \tag{26}$$

So, the system is still balanced, after active power is reassigned among controllable DGs. ■

Summarily, it can be concluded that during iterations the control laws never break the supply-demand balance of the system. Finally, the economic dispatch can be obtained iteratively.

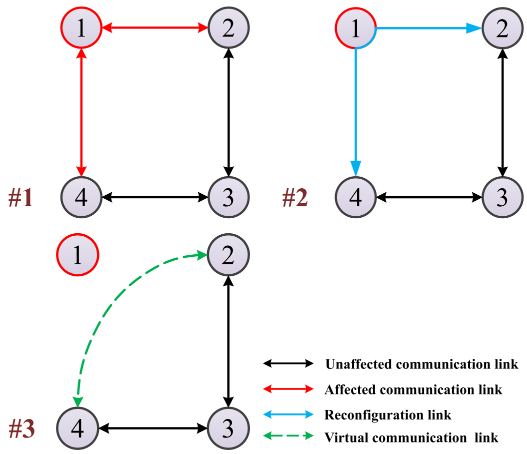

**FIGURE 2.** Illustration for virtual communication link.

### D. IMPROVED CONTROL LAWS FOR ECONOMIC DISPATCH WITH CONSTRAINTS OF DG CAPACITIES

In the previous section, the control laws (20) are derived without considering the constraints of DG capacities. However, in reality, the constraints of DG capacities always exist. Therefore, in this section, the control laws (20) are modified to adapt the constraints of DG capacities.

It is possible that the set points calculated by agents are greater than the maximal capacities or less than zero. Therefore, the outputs of DGs should be the maximum or zero due to the physical constraints of DGs. In this case, the agents do not update the set points of their DGs again, and at the same time they work like relays to forward the received information to their neighbors instead of dealing with it. For example, if the set point calculated by the agent 1 is greater than the maximal capacity of $\text{DG}_1$, then the output of $\text{DG}_1$ should be fixed at the maximum and its set points will not be updated. Meanwhile, it forwards the information received from the agent 2 to the agent 4 and forwards the information received from the agent 4 to the agent 2. In this way, it seems a virtual link is added between the neighbors of the agent 1, as shown in Fig. 2. Following this idea, the control laws (20) are changed into

$$\tilde{P}'(t+1)=\begin{cases}H^T\cdot(\tilde{P}(t+1)+\boldsymbol{\alpha})-\boldsymbol{\alpha}, & \text{if } i\notin\Omega_p,\\ p_i^{\min}\text{ or }p_i^{\max}, & \text{if } i\in\Omega_p.\end{cases} \tag{27}$$

## IV. MICROGRID SYSTEM ARCHITECTURE AND SETUP

To evaluate the performance of the control laws, a radial MG is built in Matlab/Simulink, where there are 12 DGs and 12 loads as shown in Fig. 3. Correspondingly, 12 agents are also built by MATLAB Functions that are blocks in the Simulink library of User-Defined Functions. In the islanded MG, $\{\text{DG}_i|i=1,3,5,6,9,11\}$ are controllable DGs that are modeled by ideal DC voltage sources $V_{\text{dc}}$, and $\{\text{DG}_i|i=2,7,8,10,12\}$ are uncontrollable DGs, in which $\text{DG}_7$ and $\text{DG}_{10}$ are PVs, while $\text{DG}_2$, $\text{DG}_8$ and $\text{DG}_{12}$ are WTs. Further, the partially controllable $\text{DG}_4$ is modeled by a BESS, which provides the frequency and the voltage references for the MG. All parameters are listed in Table 1.

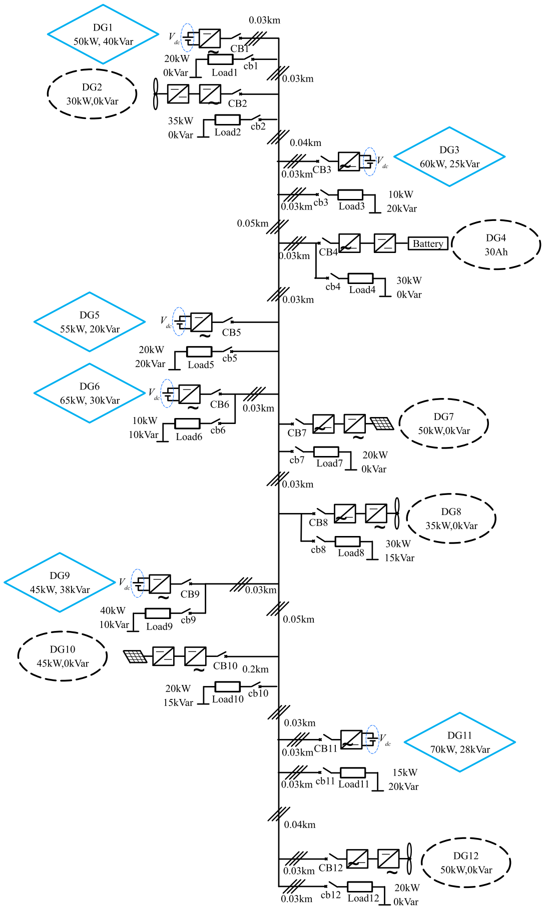

**FIGURE 3.** Topology of a radial MG.

**TABLE 1.** Setup and parameters of DGs.

| Sources | Capacities | Control | $a$ | $b$ | $c$ |
|---|---|---|---|---|---|
| $\text{DG}_1$ | 50 kW, 40 kVar | PQ | 0.059 | 6.71 | 80 |
| $\text{DG}_2$ | 30 kW, 0 kVar | MPPT | - | - | - |
| $\text{DG}_3$ | 60 Ah, 25 kVar | PQ | 0.047 | 7.08 | 56 |
| $\text{DG}_4$ | 30 Ah | V/F | - | - | - |
| $\text{DG}_5$ | 55 kW, 20 kVar | PQ | 0.066 | 6.29 | 43 |
| $\text{DG}_6$ | 65 kW, 30 kVar | PQ | 0.031 | 7.53 | 35 |
| $\text{DG}_7$ | 50 kW, 0 kVar | MPPT | - | - | - |
| $\text{DG}_8$ | 35 kW, 0 kVar | MPPT | - | - | - |
| $\text{DG}_9$ | 45 kW, 38 kVar | PQ | 0.069 | 4.57 | 48 |
| $\text{DG}_{10}$ | 45 kW, 0 kVar | MPPT | - | - | - |
| $\text{DG}_{11}$ | 70 kW, 28 kVar | PQ | 0.038 | 5.86 | 91 |
| $\text{DG}_{12}$ | 50 kW, 0 kVar | MPPT | - | - | - |

Throughout all simulations, the outputs of PVs and WTs fluctuate with the environmental conditions, as shown in Fig. 4. Moreover, the load demand is scheduled as follows,

- $t=2$ s: all active power and reactive power loads rise by 20%,
- $t=4$ s: $\text{load}_4$ and $\text{load}_8$ are cut from the MG,
- $t=6$ s: all active power and reactive power loads reduce by 20%,
- $t=8$ s: $\text{load}_4$ and $\text{load}_8$ are reconnected to the MG,

which is applied to all the cases.

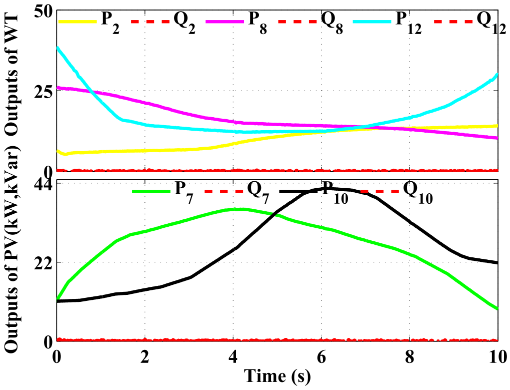

**FIGURE 4.** Active power and reactive power outputs of all uncontrollable DGs.

Moreover, the line voltage is set at 380 V, the frequency in the system is set at 50 Hz, and the line impedance is set at $0.169 + j0.07$ Ω/km. In simulations, the line losses are compensated by the outputs of the V/F DG first, and then the outputs of the V/F DG are shared by controllable DGs by means of the derived control laws. Additionally, to save the simulation time, we only use 10 seconds to simulate the significant fluctuations of load demand and those of the outputs of uncontrollable DGs (e.g. WTs) in a day. Therefore, agents on the subgraph $\tilde{G}$ exchange information at every 45 ms, while at every 1 ms on the subgraph $\hat{G}$. Certainly, the time for information exchange in reality is much longer than that in the simulations. Initially, the system works in a steady state.

## V. RESULTS

In this section, three cases are designed to test the performance of the proposed method, when both the environment conditions and load demand change at the same time. In Case 1 and Case 2, the effectiveness of the method is investigated without and with the constraints of DG capacities, respectively. In Case 3, the impacts of different topologies on our method are studied.

### A. CASE 1: DISTRIBUTED ECONOMIC DISPATCH WITHOUT CONSTRAINTS OF DG CAPACITIES

In this case, the economic dispatch is investigated without the constraints of DG capacities (3), when both load demand and environmental conditions fluctuate. Simulations are carried out on the network 1 according to the settings in Section IV and results are shown in Fig. 5. From Fig. 5(#1) and (#2), with the loads increasing by 20% at $t = 2$ s, the outputs of all controllable DGs increase. Meanwhile, the incremental costs of all the controllable DGs also increase and finally reach almost the same value, which means the near optimal incremental costs are obtained.

From Fig. 5(#3), $DG_4$, the BESS, absorbs or injects power from or into the system to balance the system immediately when the system fluctuates. After that, the outputs of $DG_4$ decreases to zero gradually when they are shared by controllable DGs according to the derived control laws. In Fig. 5(#4) and (#5), the fluctuations of the line voltage and the system frequency are presented. It is clear that the line voltage and frequency fluctuations can settle down quickly regardless of how large the fluctuations are. Here, it is worth noting that all the outputs of controllable DGs are within the generator ranges.

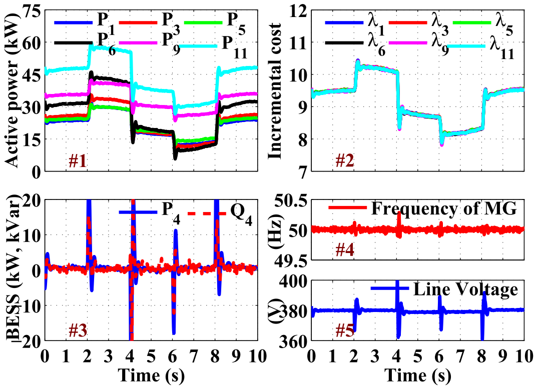

**FIGURE 5.** Simulation results of economic dispatch without constraints of DG capacities, when environmental conditions and load demand change over time. (#1) The active power outputs of controllable DGs. (#2) The incremental costs of controllable DGs. (#3) The power outputs of $DG_4$. (#4) The frequency in the MG. (#5) The line voltage of MG.

### B. CASE 2: DISTRIBUTED ECONOMIC DISPATCH WITH CONSTRAINTS OF DG CAPACITIES

In this case, the inequality constraints (3) are considered. To make the inequality constraint (3) violated, the parameters $a$ of $DG_9$ in Table 1 is decreased from 0.069 \$/MW$^2$h to 0.05 \$/MW$^2$h, and all other parameters follow those in Case 1. Simulation results are shown in Fig. 6.

From Fig. 6(#1), it can be found that the outputs of $DG_9$ is a straight line from $t = 0$ s to $t = 4$ s, which indicates that its set point is greater than its maximal capacity, but the output is fixed at the maximum and it is not updated. On the other

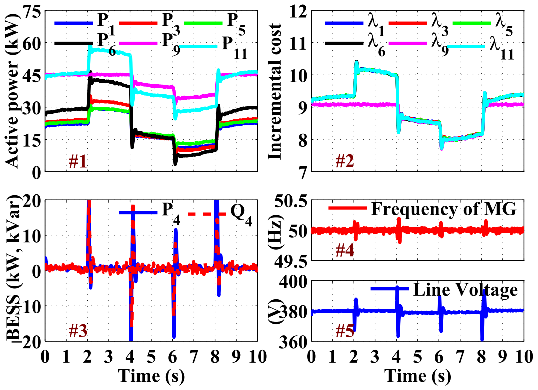

**FIGURE 6.** Simulation results of economic dispatch with constraints of DG capacities, when environmental conditions and load demand change over time. (#1) The active power outputs of controllable DGs. (#2) The incremental costs of controllable DGs. (#3) The power outputs of $DG_4$. (#4) The frequency in the MG. (#5) The line voltage of MG.

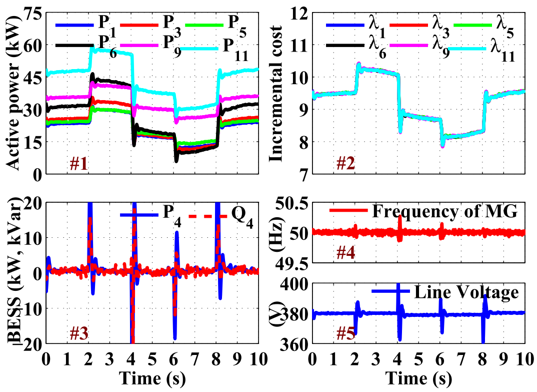

**FIGURE 7.** **Simulation results on the network 2, when environmental conditions and load demand change over time. (#1) The active power outputs of controllable DGs. (#2) The incremental costs of controllable DGs. (#3) The power outputs of DG$_4$. (#4) The frequency in the MG. (#5) The line voltage of MG.**

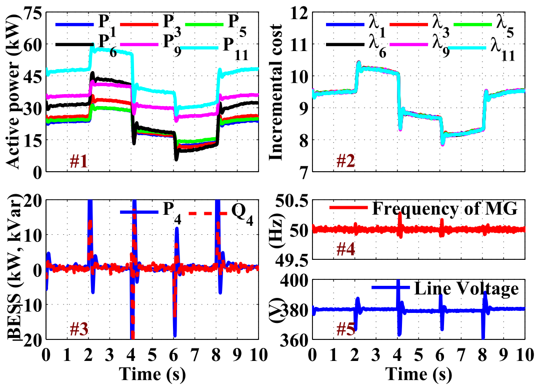

**FIGURE 8.** **Simulation results on the network 3, when environmental conditions and load demand change over time. (#1) The active power outputs of controllable DGs. (#2) The incremental costs of controllable DGs. (#3) The power outputs of DG$_4$. (#4) The frequency in the MG. (#5) The line voltage of MG.**

hand, the incremental cost of other controllable DGs reach the same value, for the set points of other controllable DGs are within their capacities. This means the control laws still work even if outputs of some DGs reach saturation. From the above two cases, it can be concluded that our method can deal with the ED problem with or without the constraints of DG capacities.

### *C. CASE 3: IMPACTS OF DIFFERENT TOPOLOGIES ON SYSTEM PERFORMANCE*

Two different topologies, the network 2 and the network 3, shown in Fig. 1(#3) and (#4), are designed to test the performance of the proposed method. Compared with the network 1, one link is added between the agent 2 and the agent 11 on the subgraph $\tilde{G}$ of the network 2. On the network 3, compared with the network 1, two links are added between the agent 1 and the agent 5, and between the agent 3 and the agent 5 on the subgraph $\hat{G}$. Other parameters follow those in Case 1.

The simulation results on two networks are shown in Fig. 7 and Fig. 8, respectively. It is shown that our control method is not sensitive to different network topologies, so it is easier for users to design different communication networks.

## VI. CONCLUSION

In this paper, a two-layer model has been proposed and two sets of control laws are derived to obtain the economic dispatch of the islanded MG in a distributed manner. In the model, there is a communication network composed of agents as the upper layer over an MG as the lower layer. The communication network is composed of two subgraphs, where the subgraph $\tilde{G}$ consists of all agents, while the subgraph $\hat{G}$ consists of only controllable agents. Further, two sets of control laws are derived from these two subgraphs in order to realize the economic dispatch. The control laws based on the subgraph $\tilde{G}$ guarantee supply-demand balance, while the control laws derived from the subgraph $\hat{G}$ obtain the economic dispatch in terms of the equal incremental cost criterion. It is worth noting that the control laws can work with or without the constraints of DG capacities. Meantime, the system can keep supply-demand balance during iterations. Finally, three cases are designed to test the control laws. Simulation results show that the system can work well, i.e, the frequency and the voltage stay close to the prescribed value, when environmental conditions and load demand change over time. And the economic dispatch also can be achieved, when constraints of DG capacities are considered.

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