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A Short-Range Range-Angle Dependent Beampattern Synthesis by Frequency Diverse Array

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ABSTRACT In this paper, we propose a new technique to generate a range-angle dependent transmit beampattern by introducing a nonlinear frequency modulation scheme based on arctangent function in the frequency diverse array (FDA) system. The propagation process of the transmitted signals is usually neglected in most of the existing range-angle dependent transmit beampattern synthesis schemes. More specifically, the time variable is considered to be a constant value or defined from 0 to T, where T is the pulse duration for pulsed FDA. As a result, such schemes cease to focus the signal power at a desired position for a period of time when this practical constraint is taken into account. The proposed methodology incorporates the propagation process of the transmitted signal and simultaneously focuses the signal power at a desired position and lasting for a period of time. We investigate the consequences of this constraint briefly and also explain the corrected model. Numerical simulations are implemented to validate the theoretical analysis of the proposed methodology.

INDEX TERMS Range-angle dependent transmit beampattern, frequency diverse array, signal power focusing, beampattern synthesis.

I. INTRODUCTION

The transmit beam of traditional phased-array radar is angle dependent, while the frequency diverse array (FDA) radar has the ability to generate a range-angle dependent transmit beampattern [1], [2]. On account of this feature, FDA technology provides potential technical support for wireless energy transmission, confidential communication or some other specific application scenarios. Therefore, some researchers strive to seek a transmit beampattern that can focus signal power at a desired position and lasting for a period of time [3]. However, there is a major problem that hinders the design of range-angle dependent transmit beampattern. The two features which are desired in FDAs beamforming are: focusing signal power at a desired position and the focused power will last for a period of time. But they are difficult to be achieved simultaneously.

The transmit-receive beampattern is always taken into consideration, when we use FDA radar to detect any target. For instance, in order to suppress the range-dependent clutter and interference, a pulsed-FDA system has been proposed in [4]. Furthermore, a high speed target is angle-Doppler-defocusing in the FDA-STAP radar, to tackle this issue, a robust adaptive beamforming scheme has been proposed in [5]. However, when it comes to the design of transmit beampattern, receiver's signal processing technology cannot be applied to the transmitter. Therefore, in order to focus transmit signal power at a dot-shaped position, a beam point focusing technology based on transmitter is desired. Under this circumstance, the propagation of electromagnetic waves cannot be ignored.

Frequency diverse array system with progressive frequency offset has been first introduced by Antonik *et al.* [6]. Some other range-angle dependent beampattern schemes based on linear frequency offset have been proposed in [7]–[9]. However, the signal power cannot be focused only at a desired position. An FDA system based on periodic triangular frequency modulated continuous waveform has been proposed in [10]. But it also suffers the same problem as [7]–[9]. Multiple subarrays have been applied in [11], in order to generate a range-angle-decoupled beampattern.

Logarithmic frequency offset and some other kinds of nonlinear frequency offset have been applied in [12]–[19], so that beampatterns with a single maximum at the desired position have been designed. However, in [12]–[19], the signal power will be only focused on the desired position at instant $t = 0$. Then, the position of focused power will move forward at the speed of light in the desired direction. They do not exploit the propagation process of the transmitted signal, hence, the time variable is considered to be a constant value. This constraint gives rise to certain limitations and consequently degrades the performance of FDA radar. Convex optimization algorithm has been used in [20] to optimize the complex weight coefficients of each sub-signal. However, in order to avoid the problem in [12]–[19], the weight coefficients of each signal needs to be optimized at each sampling point.

Different kinds of nonlinear frequency modulation (NLFM) have been used in [21]–[27], in order to focus the transmit energy at a desired position within the pulse duration. Unfortunately, the same practical constraint and performance deterioration of FDA radar prevails in [21]–[27]. Rather than assuming a fixed value, they define the time variable from 0 to T, where T is the pulse duration for pulsed FDA. This practical constraint is reported and analyzed briefly in the upcoming sections. Besides, a numerical simulation experiment is also implemented to visually show the consequences of this constraint. Nonlinear frequency offset applied in [12]–[27] makes the beampattern angle dependent. However, the NLFM applied in [21]–[27] try to make the beampattern range dependent but not successful. Therefore, the key to the successful design of a range-angle dependent beampattern synthesis is to design a novel modulation function and combine it with nonlinear frequency offset.

In view of these issues, this paper proposes a new NLFM scheme based on arctangent function to achieve a range-angle dependent beampattern in frequency diverse array radar. The new technique considers the practical constraint of propagation process mentioned above and the inefficiencies in the existing methodologies have been avoided.

The rest of this paper is organized as follows. Section 2 reports and analyzes the neglected constraint in [21]–[27] in detail. Section 3 presents the proposed NLFM scheme based on arctangent function. Section 4 reports and discusses a set of simulation examples. Finally, conclusions are drawn in Section 5.

II. ANALYSIS OF FDA BEAMPATTERN SYNTHESIS

Let us consider a uniformly-spaced linear array system with $2N + 1$ transmit elements, as shown in Fig. 1. The inter-element spacing is d and the center element is the reference element. The observation point P is in the far field. r_n is the range between the nth element and the observation point. θ is the azimuth angle between the observation point and the reference element. P^d is a desired focusing position, r_n^d is the distance between P^d and the nth element, and θ^d is the azimuth angle between P^d and the reference element.

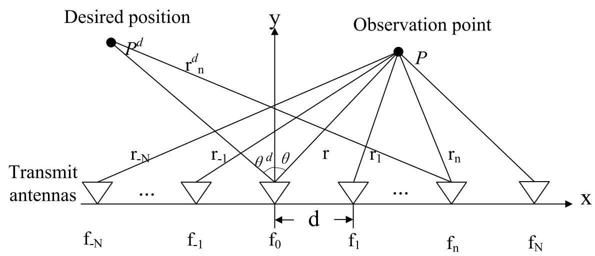

FIGURE 1. A uniform linear array with $2N + 1$ elements.

The pulse signal transmitted by the nth element is

$$s_n(t) = w_n^* e^{-j2\pi f_n(t)\cdot t}, \quad (-N \le n \le N, t \in [0, T]) \tag{1}$$

where T is the pulse duration, $f_n(t)$ and w_n represent the frequency and the complex weight of the nth signal, respectively. Although $f_n(t)$ and w_n are different in [21]–[27], $f_n(t)$ and w_n can be expressed in a general term as

$$\begin{cases} f_n(t) = f_0 + \dfrac{\Delta f_n}{t - r_n^d/c} \\ w_n = e^{j2\pi\left(f_0 r_n^d/c - \Delta f_n\right)} \end{cases} \tag{2}$$

where f_0 is the center carrier frequency, Δf_n is the frequency offset of the nth signal and c is the speed of light in free space. The differences among [21]–[27] are different methods used to generated the frequency offset Δf_n. For instance, the artificial intelligence optimization algorithm has been applied in [21] while logarithmic function has been used in [26]. Then, the resulting array factor observed at an arbitrary point (r, θ) can be derived as

$$AF(t; r, \theta) = \sum_{n=-N}^{N} s_n(t - r_n/c) \tag{3}$$

According to [21]–[27], the far-field approximation equation $r_n \approx r - nd\sin\theta$ has been taken into account. However, the array factor has been derived as (4) in [21]–[27].

$$\begin{aligned} AF(t; r, \theta) &= \sum_{n=-N}^{N} s_n(t - (r - nd\sin\theta)/c) \\ &= \sum_{n=-N}^{N} w_n^* e^{-j2\pi f_n(t)\cdot[t-(r-nd\sin\theta)/c]} \end{aligned} \tag{4}$$

It is observed from (4), that $f_n(t)$ has been directly substituted in (3). Even under the premise of narrowband system, it is not in conformity with practical situation. Therefore, all the time variable t should subtract the corresponding time delay. The correct array factor should be derived as

$$AF(t; r, \theta) = \sum_{n=-N}^{N} w_n^* e^{-j2\pi f_n(t-(r-nd\sin\theta)/c)\cdot[t-(r-nd\sin\theta)/c]} \tag{5}$$

It is important to remark that, once a signal is transmitted, the signal frequency will not change actively during the propagation, in the absence of Doppler Effect and other effects.

III. PROPOSED SYSTEM DESIGN

In this paper, we propose a different NLFM function which is designed based on arctangent function. The array structure is the same as mentioned in section 2. Since a short-range beampattern is taken into consideration, the far-field approximation will no longer apply. And r_n is exactly calculated as $r_n = \sqrt{(r)^2 + (nd)^2 - 2rnd\sin\theta}$. The frequency modulation function is designed as

$$f_n(t) = f_0 + \Delta f_n \cdot \frac{2}{\pi} \cdot \arctan\left[\alpha \cdot \frac{2}{T} \cdot (t - T/2)\right] \tag{6}$$

where f_0 is the carrier frequency, Δf_n is the frequency offset of the nth element, T is the pulse duration and α is an adjustable parameter. The nonlinear frequency offset is generated by sinusoidal function as

$$\Delta f_n = \frac{B}{2} \cdot \sin(|n|), \quad (-N \le n \le N) \tag{7}$$

where B is the system bandwidth. The transmitted signal of each element can be expressed as

$$s_n(t) = w_n^* e^{-j2\pi[f_0 \cdot t + \psi_n(t)]}, \quad (-N \le n \le N, t \in [0, T]) \tag{8}$$

Since $f_n(t)$ is the frequency function of $s_n(t)$, the phase function $\psi_n(t)$ can be derived as

$$\begin{aligned}\psi_n(t) &= \int \Delta f_n \cdot \frac{2}{\pi} \cdot \arctan\left[\alpha \cdot \frac{2}{T} \cdot (t - T/2)\right] dt \\ &= \Delta f_n \cdot \frac{2}{\pi} \cdot \left(t - \frac{T}{2}\right) \cdot \arctan\left[\frac{\alpha}{T} \cdot \left(t - \frac{T}{2}\right)\right] \\ &\quad - \Delta f_n \cdot \frac{T}{\alpha\pi} \cdot \ln\left[\frac{\alpha^2}{T^2} \cdot \left(t - \frac{T}{2}\right)^2 + 1\right]\end{aligned} \tag{9}$$

Then taking the desired focusing position (r^d, θ^d) into account, the transmitted signal is finally designed as

$$\begin{cases} S(t) = \sum_{n=-N}^{N} s_n(t) = \sum_{n=-N}^{N} w_n^* e^{-j2\pi[f_0 \cdot t + \psi_n(t + \Delta\tau_n)]} \\ w_n = e^{j2\pi f_0 \Delta\tau_n} \\ \psi_n(t) = \Delta f_n \cdot \frac{2}{\pi} \cdot \left(t - \frac{T}{2}\right) \cdot \arctan\left[\frac{\alpha}{T} \cdot \left(t - \frac{T}{2}\right)\right] \\ \qquad - \Delta f_n \cdot \frac{T}{\alpha\pi} \cdot \ln\left[\frac{\alpha^2}{T^2} \cdot \left(t - \frac{T}{2}\right)^2 + 1\right] \\ \Delta\tau_n = \left(r_n^d - r^d\right)/c \\ \Delta f_n = \frac{B}{2} \cdot \sin(|n|) \end{cases} \tag{10}$$

where w_n, Δf_n, $\psi_n(t)$ and $\Delta\tau_n$ represent the complex weight, frequency offset, phase function and the phase offset of the nth signal, respectively. Besides, $-N \le n \le N$ and $t \in [0, T]$.

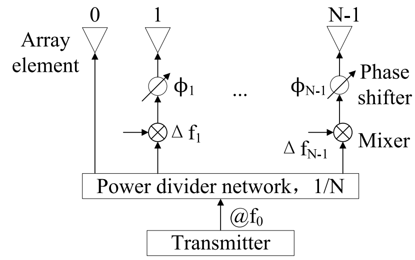

FIGURE 2. The array structure used in [23].

Then the overall signal arriving at position (r, θ) can be expressed as

$$\begin{aligned} S_{arrival}(t; r, \theta) &= \sum_{n=-N}^{N} s_n\left(t - \frac{r_n}{c}\right) \\ &= \sum_{n=-N}^{N} w_n^* e^{-j2\pi\left[f_0\left(t - \frac{r_n}{c}\right) + \psi_n\left(t - \frac{r_n}{c}\right) + \Delta\tau_n\right]} \end{aligned} \tag{11}$$

Consequently, the resulting array factor observed at position (r, θ) can be derived as

$$AF(t; r, \theta) = \sum_{n=-N}^{N} w_n^* \cdot e^{j2\pi f_0 r_n/c - j2\pi\psi_n(t - r_n/c + \Delta\tau_n)} \tag{12}$$

and the resulting power pattern at the desired position $\left(r^d, \theta^d\right)$ can be expressed as

$$\begin{aligned} P\left(t; r^d, \theta^d\right) &= \left|AF\left(t; r^d, \theta^d\right)\right|^2 \\ &= \left|\sum_{n=-N}^{N} e^{-j2\pi\psi_n\left(t - r_n^d/c + \Delta\tau_n\right)}\right|^2 \end{aligned} \tag{13}$$

IV. NUMERICAL SIMULATIONS

This section consists of two parts. In the first part, a set of simulation results are reported to graphically show the consequences caused by the neglected constraint reported in section 2. In the second part, a set of simulation results are reported and discussed to validate the theoretical model proposed in section 3.

A. SIMULATION RESULTS OF THE CORRECTED ARRAY FACTOR

For illustrative purposes, [23] is selected from [21]–[27]. The array structure designed in [23] is shown in Fig. 2 and the corresponding signal model named TMLFO is designed as

$$\begin{cases} S(t) = \sum_{n=0}^{N-1} e^{j\{2\pi f_n(t) \cdot t + \varphi_n\}} \\ f_n(t) = f_0 + \dfrac{\ln(n+1)^k - f_0 n d \sin\theta^d/c}{t - r^d/c} \\ \varphi_n = -2\pi \ln(n+1)^k \end{cases} \tag{14}$$

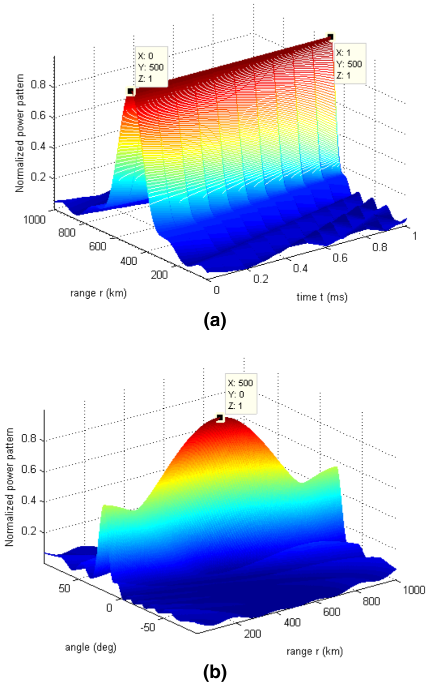

FIGURE 3. Simulation results of original array factor in [23]. (a) normalized power pattern at $\theta = 0°$ on time-range dimensions and (b) normalized power pattern at $t = 0$ on range-angle dimensions.

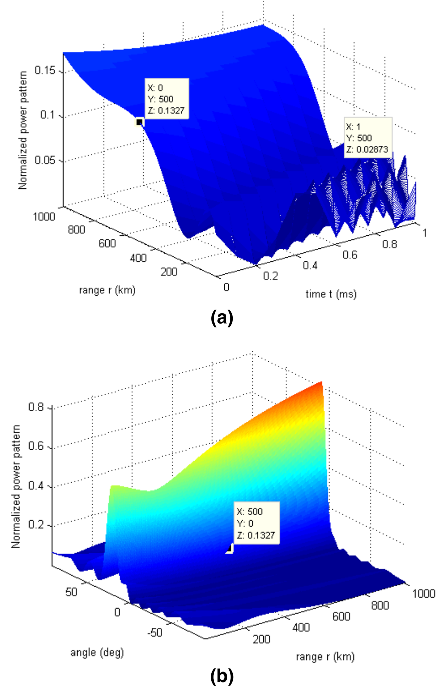

FIGURE 4. Simulation results of corrected array factor. (a) normalized power pattern at $\theta = 0°$ on time-range dimensions and (b) normalized power pattern at $t = 0$ on range-angle dimensions.

The correct power pattern turns out to be

$$P(t;r,\theta) = \left|\sum_{n=0}^{N-1} e^{j2\pi\left[f_0\frac{nd\sin\theta}{c} - \ln(n+1)^k + \frac{\ln(n+1)^k - f_0 nd\sin\theta^d/c}{t-\frac{r-nd\sin\theta}{c}-\frac{r^d}{c}}\left(t-\frac{r-nd\sin\theta}{c}\right)\right]}\right|^2 \tag{15}$$

The array system parameters are set to: $N = 10$, $f_0 = 5GHz$, $d = \lambda_0/2$, $T = 1ms$ and $k = 1$. The numerical simulation parameters are as follows: the whole range-angle region is $\Omega = \{(r, \theta)\,|0 \le r \le 1000km, -\pi/2 \le \theta \le \pi/2\}$, $r_{step} = 1km$ and $\theta_{step} = \pi/720$ are the accuracies of space grid, and the sampling frequency is $f_s = 2f_0$. Besides, the desired focusing position is $(500km, 0°)$.

Simulation results of original array factor in [23] and the corrected array factor are shown in Fig. 3 and Fig. 4 respectively. It can be seen from Fig. 4(b) that the signal power has not been focused at $(500km, 0°)$ when $t = 0$. And Fig. 4(a) shows that the signal power maximum has not been achieved at $(500km, 0°)$ during pulse duration. By comparing Fig. 3 and Fig. 4, we can find out that the simulation results of the correct array factor are quite different from those of the original array factor in [23]. Therefore, we can conclude that the signal model in [23] is impractical.

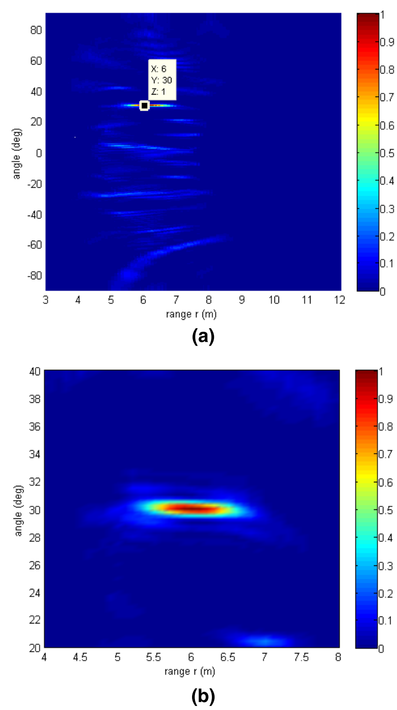

FIGURE 5. Projection of normalized power pattern on the range-angle dimensions at $t = 25ns$, (a) view of the whole simulation region, (b) the magnified view of the area around the desired position.

B. SIMULATION RESULTS OF THE PROPOSED SCHEME

In this part, the simulation results of the proposed scheme are reported and discussed in detail. The array system parameters are set to: $N = 16$, $f_0 = 6GHz$, $B = 400MHz$, $d = \lambda_0$, $T = 10ns$ and $\alpha = 30$. The numerical simulation parameters are set to: $\Omega = \{(r, \theta)\,|3m \le r \le 12m, -\pi/2 \le \theta \le \pi/2\}$,

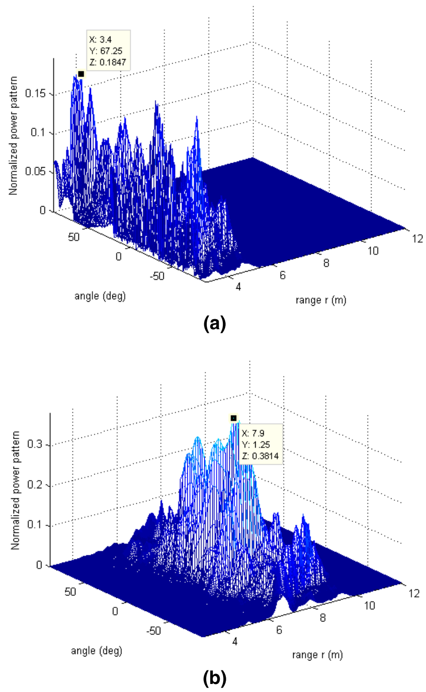

FIGURE 6. **The resulting normalized power pattern at (a) $t = 15ns$ and (b) $t = 30ns$.**

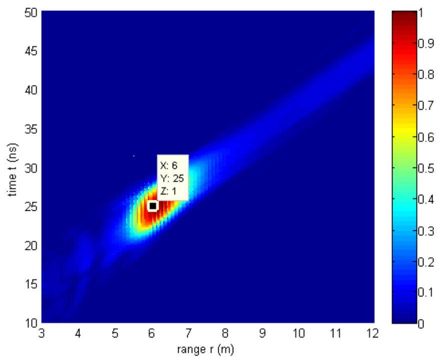

FIGURE 7. **Projection of normalized power pattern on the range-time dimensions at $\theta = 30°$.**

$r_{step} = 0.1m$ and $\theta_{step} = \pi/720$ are the accuracies of space grid, and the sampling frequency is $f_s = 4f_0$. Besides, the simulation time belongs to $[0, 50ns]$, where $t = 0$ means the instant that the center element starts transmitting signals. And the desired focusing position is $(6m, \pi/6)$.

The resulting normalized power pattern at instant $t = 25ns$ is shown in Fig. 5(a), and Fig. 5(b) shows the magnified view of the area around the desired position. Besides, the normalized power pattern at instants $t = 15ns$ and $t = 30ns$ are shown in Fig. 6(a) and Fig. 6(b) respectively. It is observed from Fig. 5 and Fig. 6 that the signal power can be focused at the desired position at $t = 25ns$ and cannot be focused at any other position at $t = 15ns$ and $t = 30ns$.

The projection of normalized power pattern on the range-time dimensions at $\theta = 30°$ is shown in Fig. 7. It can be seen from Fig. 7 that signal power can be focused at desired

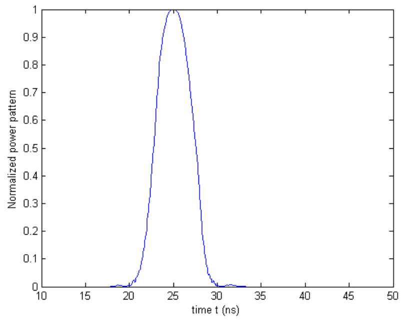

FIGURE 8. **Normalized power pattern time curve of the desired position.**

position when pulse signals reach the desired position. When the pulse signals reach other points in the desired direction, signal power is not focused. Normalized power pattern time curve of the desired position is shown in Fig. 8. Considering the half-power width is about half the pulse width, we can conclude that the time for signal power to be focused is about $T/2$.

V. CONCLUSION

In this paper, a new range-angle dependent beampattern synthesis method for frequency diverse array radar has been proposed. The new method incorporates a nonlinear frequency modulation scheme based on arc-tangent function. It has been proven that schemes proposed in [12]–[19] and [21]–[27] neglect a practical constraint condition in their models that is the propagation process of the transmitted signal is ignored. This neglected constraint condition has been reported and discussed in detail. The methods presented in [12]–[19] assume the time variable t to be a fixed value while those in [21]–[27] define it in the range from 0 to T, where T is pulse duration. Simulation results demonstrate that these methods cannot focus signal power at a desired position for a period of time when the propagation process is considered. While taking this practical constraint into consideration, the proposed method can focus signal power at a desired position and last for a period of time simultaneously. The numerical evaluation of the proposed method confirms the superiority of the proposed method over existing techniques. Besides, the focusing performance in the proposed scheme is related to system bandwidth and the number of elements. Distance focusing precision is related to the bandwidth of the system. The wider the bandwidth of the system, higher the focus accuracy of the signal power in the distance dimension. And more the number of array elements, better will be the focusing performance of signal power in angle dimension.

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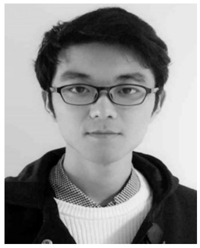

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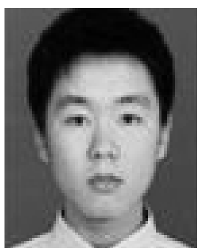

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