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Stabilization of Discrete-Time Systems via a Partially Disabled Controller Experiencing Forced Dwell Times

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ABSTRACT This paper focuses on the stabilization problem of discrete-time systems via exploiting a partially disabled controller, where a stochastic variable with two values is used to describe a controller useful or not. Particularly, the adopted stochastic variable is not the traditional Bernoulli variable, but a variable has forced dwell times on the basis of traditional Bernoulli variable. Due to such a stochastic variable included, two kinds of dwell times, named as fixed and random dwell times, respectively are included in the closed-loop system. It will be shown that the stochastic stability of the original closed-loop system could be guaranteed by the established auxiliary system with state jumps. More importantly, sufficient conditions for the existence of the desired controller are presented with linear matrix inequalities forms. Finally, a numerical example is used to demonstrate the effectiveness of the proposed methods.

INDEX TERMS Discrete-time systems, stabilization, partially disabled controller, forced dwell times, stochastic stability, linear matrix inequalities (LMIs).

I. INTRODUCTION

As we know, stabilization problem is to design a particular controller for a system to guarantee its stability or maintain a desired performance, see, e.g., [1]–[4]. Since the stabilization problem is important in theory and application, it has been widely studied in recent years. Up to now, many kinds of control strategies have been proposed to realize stabilization, such as adaptive control [5]–[9], robust control [10]–[14], state feedback control [15]–[19], dissipative control [20], output feedback control [21]–[23], sliding mode control [24]–[29], synchronization control [30]–[34] and so on. Among these methods, state feedback control is a common one because of its concise form and easier installation.

On the other hand, it is obvious that any system is inevitable to experience faults in practice [35]–[39]. From some existing references such as [40] and [41], it is known that the case of a controller having a fault could be described by its information accessible or not. When a controller is useful or without any fault, it denotes that its information is transmitted successfully. To the contrary, it means that the related data is missed. Based on these facts, the phenomenon of a data available or not could be described by the Bernoulli variable.

Particularly, one value is used to demonstrate a data accessible, while the other one denotes data packet lost. During the past decades, many problems have been fully considered, for example, stability [42], stabilization [43]–[46], filtering [47]–[49], synchronization [50], state estimation [51], and so on. By investigating these references, it is seen that the used Bernoulli variable is very usual, where the dwell time of each value is simply random. In other words, no fixed dwell times such as ones in deterministic switched systems [52] happen to such two values. However, in some practical applications such as networked control systems (NCSs), it is reasonable that the dwell time of a data transmitted successfully or not is a constant one. After this duration occurs, the follow-up switching between the two values is driven by a traditional Bernoulli variable. Based on the above descriptions, it is seen that a more general Bernoulli variable considered here will have not only a random dwell time but also a fixed one. For this general case, it is said that the traditional methods cannot be applied directly, and new techniques should be proposed to deal with it. Very recently, the stochastic stability of continuous-time Markovian jump systems was firstly considered in [53], where both fixed and random dwell times

were included. Unfortunately, it was only concerned about continuous-time case and different from discrete-time case. More importantly, only stability problem was studied in this reference. When system synthesis problems are considered, some terms related to the fixed dwell time will be nonlinear and bring big difficulties to be done. Based on the above facts, it is necessary and meaningful to study the stabilization problem by a controller experiencing forced dwell times. How to describe the above phenomenon suitably will be the first problem to be considered. Second, how to obtain the stabilization conditions within a concise form is also necessary and more important. Up to now, to our best knowledge, very few results are available to study similar problems. All the facts motivate the current research.

In this paper, the stabilization problem of discrete-time systems by a partially disabled controller having forced dwell times is studied. The main contributions of this paper are summarized as follows: 1) A kind of partially disabled controller with forced dwell times is proposed, in which a stochastic variable instead of the traditional Bernoulli variable is introduced to describe the controller useful or not; 2) It will be shown that the stochastic stability of the original closed-loop system could be guaranteed by an auxiliary system with state jumps; 3) Sufficient LMI conditions for the desired controller are established and could be solved directly; 4) The effects of available probability and dwell time of a controller are fully considered, which are proved to be very important to system analysis and synthesis.

Notation: \mathbb{R}^n denotes the n -dimensional Euclidean space, $\mathbb{R}^{q \times n}$ is the set of all $q \times n$ real matrices. Interval $[a, b)$ denotes a integer set where any integer k satisfies $a \leq k < b$. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm. Ω is the sample space, \mathcal{F} is the σ -algebras of subsets of the sample space and \mathbb{P} is the probability measure on \mathcal{F} . $\mathcal{E}\{\cdot\}$ stands for the expectation. $\beta_{\min}(\cdot)$ is the minimum singular values of square matrices. In symmetric block matrices, we use “ $*$ ” as an ellipsis for the terms induced by symmetry, $\text{diag}\{\cdot\cdot\cdot\}$ for a block-diagonal matrix, and $(M)^* \triangleq M + M^T$.

II. PROBLEM FORMULATION

Consider a kind of discrete-time system described as

$$x(k + 1) = Ax(k) + Bu(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state, $u(k) \in \mathbb{R}^m$ is the control input. Matrices A and B are known matrices with appropriate dimensions. Here, the designed controller is referred to be a partially disabled controller and described by

$$u(k) = \alpha(k)Kx(k) \tag{2}$$

where K is the control gain to be determined, and $\alpha(k)$ is a random variable and indicates the controller useful or not. In particular, its detailed value is given as follows:

$$\alpha(k) = \begin{cases} 1, & \text{if controller is useful} \\ 0, & \text{if controller is disabled} \end{cases} \tag{3}$$

Define a finite set $\mathbb{S} \triangleq \{1, 2\}$, the above system is equivalent to

$$x(k + 1) = A_{r(k)}x(k) \tag{4}$$

where

$$\begin{aligned} A_1 &= A + BK_1, & K_1 &= K, & \text{if } r(k) = 1 \text{ or } \alpha(k) = 1 \\ A_2 &= A + BK_2, & K_2 &= 0, & \text{if } r(k) = 2 \text{ or } \alpha(k) = 0 \end{aligned}$$

Here, $r(k)$ takes values in a finite set \mathbb{S} . It represents the switching signal and decides the current system operation mode. However, it is worth mentioning that stochastic variable $\alpha(k)$ is not the traditional Bernoulli variable. Suppose time instant t_k being an integer. Then, it indicates that the system switches to another mode operation at $r(k) = i \in \mathbb{S}$, and integer $d_i > 0$ represents a fixed dwell time of the resulting closed-loop system (4) with mode i . The characteristic of the switching signal $r(k)$ is described as follows. For any time k satisfying $t_k \leq k < t_k + d_i$, the operation mode doesn't change almost surely, which is described as

$$\Pr\{r(k + 1) = j | r(k) = i\} = \begin{cases} 0, & \text{if } j \neq i \\ 1, & \text{if } j = i \end{cases} \tag{5}$$

For any time k satisfying $t_k + d_i \leq k < t_{k+1}$, the random characteristic of the switching signal $r(k)$ will be described by a transition probability matrix. As for the integer interval $t_k + d_i \leq k < t_{k+1}$, the property of stochastic variable $\alpha(k)$ is described as

$$\Pr\{\alpha(k) = 1\} = \alpha, \quad \Pr\{\alpha(k) = 0\} = 1 - \alpha \tag{6}$$

where parameter α is a scalar constant and satisfies $0 \leq \alpha \leq 1$. Based on descriptions (4) and (6), the corresponding transition probability of mode switching on $t_k + d_i \leq k < t_{k+1}$ is given by

$$\Pr\{r(k + 1) = j | r(k) = i\} = \pi_{ij} \tag{7}$$

where $\pi_{ij} \in [0, 1], \forall i, j \in \mathbb{S}$. Time instant t_{k+1} is supposed to represent the next switching. Then, one could define $\eta_i \triangleq t_{k+1} - (t_k + d_i)$ to represent the random dwell time of system (4) with mode i . The total dwell time of system (4) under mode i is defined as $\delta_i \triangleq t_{k+1} - t_k = d_i + \eta_i$. So, integer time interval $[t_k, t_{k+1})$ could be divided into two parts: $[t_k, t_{k+1}) = [t_k, t_k + d_i) \cup [t_k + d_i, t_{k+1})$. Moreover, it is known that $t_0 = 0, t_{k+1} > t_k$ and $\lim_{k \rightarrow \infty} t_k = \infty$. The transition probability matrix is established as

$$\Pi \triangleq \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \alpha & 1 - \alpha \\ \alpha & 1 - \alpha \end{bmatrix} \tag{8}$$

where

$$\begin{aligned} \pi_{11} &= \Pr\{r(k + 1) = 1 | r(k) = 1\} = \alpha \\ \pi_{12} &= \Pr\{r(k + 1) = 2 | r(k) = 1\} = 1 - \alpha \\ \pi_{21} &= \Pr\{r(k + 1) = 1 | r(k) = 2\} = \alpha \\ \pi_{22} &= \Pr\{r(k + 1) = 2 | r(k) = 2\} = 1 - \alpha \end{aligned}$$

Definition 1: The closed-loop system (4) is stochastically stable if

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} \|x(k)\|^2 \mid x_0, r_0\right\} < \infty$$

holds for all initial conditions $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathbb{S}$.

Remark 1: In expression (2), a stochastic variable with two values is used to denote a controller useful or not. Though similar variables were introduced in some references such as [40], [42], [44]–[47], and [49]–[51], they are quite different. In these references, the stochastic variable is a Bernoulli variable, while the stochastic variable introduced here is not a traditional Bernoulli variable. In other words, not only random dwell time but also fixed dwell time are involved in partially disabled controller (2). Compared with the some existing models, it is more general and suitable in some problem descriptions. However, such two dwell times included simultaneously will make the corresponding closed-loop system analysis complicated. Especially, when system synthesis problem is considered, how to make the obtained conditions with solvable forms becomes very difficult. The main reason is unknown variables or matrices to be solved exist in nonlinear terms.

In this paper, instead of proving the stability of system (4) directly, an auxiliary system is constructed to guarantee its stability. It is supposed that the system switches to mode $r(k) = i \in \mathbb{S}$ at time instant t_k . Then, similar to [53], system state $x(k)$ in the time interval $[t_k, t_k + d_i]$ will evolve from $x(t_k)$ to $x(t_k + d_i) = (A_i)^{d_i} x(t_k)$. If the time interval $[t_k, t_k + d_i]$ is compressed to a time point t_k , in that way the system state will directly jump from $x(t_k)$ to $(A_i)^{d_i} x(t_k)$. In this case, the resulting closed-loop system (4) could be rewritten to a randomly switched system with state jumps

$$\begin{cases} \xi(\tilde{k} + 1) = A_{\theta(\tilde{k})} \xi(\tilde{k}) \\ \xi(t_{\tilde{k}}) = (A_{\theta_k})^{d_{\theta_k}} \xi(t_{\tilde{k}-1}) \end{cases} \quad (9)$$

where $\xi(\tilde{k})$ is the auxiliary system state, $t_{\tilde{k}}, \tilde{k} = 0, 1, 2, \dots$, represents time instants in which the system switches to the corresponding operation modes. $\theta(\tilde{k})$ also takes values in finite set \mathbb{S} and is followed by the following transition probability

$$\Pr\{\theta(\tilde{k} + 1) = j \mid \theta(\tilde{k}) = i\} = \rho_{ij} \quad (10)$$

where ρ_{ij} is consistent with π_{ij} in (7). Suppose that the auxiliary system (9) jumps to the mode $\theta_k \triangleq \theta(t_{\tilde{k}})$ at time instant $t_{\tilde{k}}$, it is clearly that the total dwell times of the auxiliary system (9) are defined as $\delta_{\theta_k} \triangleq t_{\tilde{k}+1} - t_{\tilde{k}} = \eta_{\theta_k}$. According to (10), η_{θ_k} is a geometric distribution random variable with parameter ρ_{ij} . Similarly, it is known that $t_0 = 0, t_{\tilde{k}+1} > t_{\tilde{k}}$ and $\lim_{\tilde{k} \rightarrow \infty} t_{\tilde{k}} = \infty$. By comparing system (4) with its auxiliary system (9), it is seen that the fixed dwell time in original system will bring a big difficulty to make system analysis. To the contrary, though its state of auxiliary system (9) jumps at some time instants, the stability analysis will be easier. The main reason is the effect of fixed dwell time in

each subsystem is transformed to a state with jumps. Based on this transformation, the next process is to how to get its stability condition of system (4) by guaranteeing its auxiliary system stable.

Lemma 1: The stochastic stability of the closed-loop system (4) could be guaranteed by the stochastic stability of the auxiliary system (9), where $A_i, \forall i \in \mathbb{S}$, is nonsingular.

Proof: Let \mathcal{F}_k be the σ -field by $\{x(t), r(t); t = 0, \dots, k\}$ for the closed-loop system (4) and define $\hat{\mathcal{F}}_{\tilde{k}}$ as the σ -field by $\{\xi(t), \theta(t); t = 0, \dots, \tilde{k}\}$ for the auxiliary system (9). Then, one could get some properties given as follows:

$$r_k = r(t_k) = \theta(t_{\tilde{k}}) = \theta_k \quad (11)$$

$$\xi(t_{\tilde{k}} + \tau) = x(t_k + d_{r_k} + \tau) \quad (12)$$

where integer parameter τ satisfies $0 \leq \tau \leq \eta_{\theta_k}$. Since matrix A_{r_k} is a nonsingular matrix, there is always a scalar $\lambda_{r_k} > 0$ such that $A_{r_k}^T A_{r_k} \geq \lambda_{r_k} I$. When the auxiliary system (9) is stochastically stable, it is concluded that η_{r_k} is a geometric distribution random variable with parameter π_{ij} such that

$$\begin{aligned} & \mathcal{E}\left\{\sum_{t_k+d_{r_k}}^{t_{k+1}} \|x(k)\|^2 \mid \mathcal{F}_k\right\} \\ &= \mathcal{E}\left\{\sum_{t_k+d_{r_k}}^{t_{k+1}} \|x(k)\|^2 \mid x(t_k), r(t_k) = r_k\right\} \\ &= \mathcal{E}\left\{\sum_{t_k+d_{r_k}}^{t_k+d_{r_k}+\eta_{r_k}} \|A_{r_k}^{(t-t_k)} x(t_k)\|^2\right\} \\ &= \mathcal{E}\left\{\sum_{d_{r_k}}^{d_{r_k}+\eta_{r_k}} \|(A_{r_k})^\tau x(t_k)\|^2\right\} \\ &= \mathcal{E}\left\{\sum_{d_{r_k}}^{d_{r_k}+\eta_{r_k}} x^T(t_k) (A_{r_k}^T)^\tau (A_{r_k})^\tau x(t_k)\right\} \\ &\geq \mathcal{E}\left\{\sum_{d_{r_k}}^{d_{r_k}+\eta_{r_k}} (\lambda_{r_k})^\tau \|x(t_k)\|^2\right\} \end{aligned} \quad (13)$$

for any $x(t_k) \neq 0$ and $r_k \in \mathbb{S}$. Define $\eta_{\min} \triangleq \min_{i \in \mathbb{S}} \{\eta_i\}$, $\lambda_{\min} \triangleq \min_{i \in \mathbb{S}} \{\lambda_i\}$ and $\alpha_{\min} \triangleq \min_{i \in \mathbb{S}} \{\eta_i (\lambda_i)^{\eta_i}\}$, it is obtained that

$$\begin{aligned} & \mathcal{E}\left\{\sum_{d_{r_k}}^{d_{r_k}+\eta_{r_k}} (\lambda_{r_k})^\tau \|x(t_k)\|^2\right\} \\ &= \mathcal{E}\left\{\sum_{d_{r_k}}^{d_{r_k}+\eta_{r_k}} (\lambda_{r_k})^\tau \|x(t_k)\|^2\right\} \\ &= \sum_{k=1}^{\infty} \sum_{d_{r_k}}^{d_{r_k}+\eta_{r_k}} (\lambda_{r_k})^\tau \pi_{ij} (1 - \pi_{ij})^{k-1} \|x(t_k)\|^2 \\ &\geq \eta_{\min} (\lambda_{\min})^{\eta_{\min}} \sum_{k=1}^{\infty} \pi_{ij} (1 - \pi_{ij})^{k-1} \|x(t_k)\|^2 \\ &= \alpha_{\min} \|x(t_k)\|^2 \end{aligned} \quad (14)$$

Define $d_{\max} \triangleq \max_{i \in \mathbb{S}} \{d_i\}$, $\eta_{\max} \triangleq \max_{i \in \mathbb{S}} \{\eta_i\}$, $\lambda_{\max} \triangleq \max_{i \in \mathbb{S}} \{\lambda_i\}$ and $\alpha_{\max} \triangleq \max_{i \in \mathbb{S}} \{\eta_i(\lambda_i)^{\eta_i}\}$, it is got

$$\begin{aligned} & \mathcal{E} \left\{ \sum_{t_k}^{t_k+d_{r_k}} \|x(k)\|^2 | \mathcal{F}_k \right\} \\ &= \mathcal{E} \left\{ \sum_{t_k}^{t_k+d_{r_k}} \|x(k)\|^2 | x(t_k), r(t_k) = r_k \right\} \\ &= \mathcal{E} \left\{ \sum_{t_k}^{t_k+d_{r_k}} \|A_{r_k}^{(t-t_k)} x(t_k)\|^2 \right\} \\ &= \mathcal{E} \left\{ \sum_0^{d_{r_k}} \|(A_{r_k})^T x(t_k)\|^2 \right\} \\ &\leq \mathcal{E} \left\{ \sum_0^{d_{r_k}} \alpha_{\max} \|x(t_k)\|^2 \right\} \\ &\leq \alpha_{\max} d_{\max} \|x(t_k)\|^2 \\ &\leq \frac{\alpha_{\max} d_{\max}}{\alpha_{\min}} \mathcal{E} \left\{ \sum_{t_k+d_{r_k}}^{t_{k+1}} \|x(k)\|^2 | \mathcal{F}_k \right\} \end{aligned} \quad (15)$$

where the last “ \leq ” in (15) establishes by means of (13) and (14). According to conditions (13)-(15), we have

$$\begin{aligned} & \mathcal{E} \left\{ \sum_{t_k}^{t_{k+1}} \|x(k)\|^2 | \mathcal{F}_k \right\} \\ &= \mathcal{E} \left\{ \sum_{t_k}^{t_k+d_{r_k}} \|x(k)\|^2 | \mathcal{F}_k \right\} + \mathcal{E} \left\{ \sum_{t_k+d_{r_k}}^{t_{k+1}} \|x(k)\|^2 | \mathcal{F}_k \right\} \\ &\leq \left[\frac{\alpha_{\max} d_{\max}}{\alpha_{\min}} + 1 \right] \mathcal{E} \left\{ \sum_{t_k+d_{r_k}}^{t_{k+1}} \|x(k)\|^2 | \mathcal{F}_k \right\} \\ &= \left[\frac{\alpha_{\max} d_{\max}}{\alpha_{\min}} + 1 \right] \mathcal{E} \left\{ \sum_{\tilde{k}}^{t_{\tilde{k}+1}} \|\xi(\tilde{k})\|^2 | \hat{\mathcal{F}}_{\tilde{k}} \right\} \end{aligned} \quad (16)$$

For the conditional expectation on both sides of (16) respectively, it is obtained that

$$\begin{aligned} & \mathcal{E} \left\{ \sum_{t_k}^{t_{k+1}} \|x(k)\|^2 | x_0, r_0 \right\} \\ &\leq \left[\frac{\alpha_{\max} d_{\max}}{\alpha_{\min}} + 1 \right] \mathcal{E} \left\{ \sum_{\tilde{k}}^{t_{\tilde{k}+1}} \|\xi(\tilde{k})\|^2 | \xi_0, \theta_0 \right\} \end{aligned} \quad (17)$$

Finally, the stochastic stability of the auxiliary system (9) could be deduced by

$$\begin{aligned} & \mathcal{E} \left\{ \sum_{k=0}^{\infty} \|x(k)\|^2 | x_0, r_0 \right\} \\ &= \mathcal{E} \left\{ \sum_{k=0}^{\infty} \sum_{t_k}^{t_{k+1}} \|x(k)\|^2 | x_0, r_0 \right\} \\ &= \sum_{k=0}^{\infty} \mathcal{E} \left\{ \sum_{t_k}^{t_{k+1}} \|x(k)\|^2 | x_0, r_0 \right\} \end{aligned}$$

$$\begin{aligned} & \leq \left[\frac{\alpha_{\max} d_{\max}}{\alpha_{\min}} + 1 \right] \sum_{k=0}^{\infty} \mathcal{E} \left\{ \sum_{t_k}^{t_{k+1}} \|x(k)\|^2 | x_0, r_0 \right\} \\ &= \left[\frac{\alpha_{\max} d_{\max}}{\alpha_{\min}} + 1 \right] \mathcal{E} \left\{ \sum_{\tilde{k}=0}^{\infty} \|\xi(\tilde{k})\|^2 | \xi_0, \theta_0 \right\} < \infty \end{aligned} \quad (18)$$

Then, the resulting closed-loop system (4) is stochastically stable. This completes the proof.

III. MAIN RESULTS

Theorem 1: Consider the closed-loop system (4) with given controller (2), it is stochastically stable if there exists matrix $P > 0$ satisfying

$$\begin{aligned} & \rho_{ii}(A + BK_i)^T P(A + BK_i) + \rho_{ij}[(A + BK_j)^T]^{d_j} \\ & \quad \times P(A + BK_j)^{d_j} - P < 0 \end{aligned} \quad (19)$$

for all $i \in \mathbb{S}$.

Proof: Based on Lemma 1, it is known that the stability of the closed-loop system (4) could be guaranteed by the auxiliary system (9). Choose the Lyapunov function for the auxiliary system as

$$V(\xi(\tilde{k}), \theta(\tilde{k})) = \xi^T(\tilde{k}) P \xi(\tilde{k}) \quad (20)$$

For each $\theta(\tilde{k}) = i \in \mathbb{S}$, we have

$$\begin{aligned} & \Delta V(\xi(\tilde{k}), \theta(\tilde{k})) \\ &= \mathcal{E} \{ V(\xi(\tilde{k} + 1), \theta(\tilde{k} + 1)) | \xi(\tilde{k}), \theta(\tilde{k}) \} - V(\xi(\tilde{k}), \theta(\tilde{k})) \\ &= \xi^T(\tilde{k}) [\rho_{ii} A_i^T P A_i + \rho_{ij} (A_j^T)^{d_j} P (A_j)^{d_j} - P] \xi(\tilde{k}) \\ &= \xi^T(\tilde{k}) \{ \rho_{ii} (A + BK_i)^T P (A + BK_i) \\ & \quad + \rho_{ij} [(A + BK_j)^T]^{d_j} P (A + BK_j)^{d_j} - P \} \xi(\tilde{k}) \\ &= \xi^T(\tilde{k}) \Psi_i \xi(\tilde{k}) < 0 \end{aligned} \quad (21)$$

where

$$\begin{aligned} \Psi_i &= \rho_{ii} (A + BK_i)^T P (A + BK_i) \\ & \quad + \rho_{ij} [(A + BK_j)^T]^{d_j} P (A + BK_j)^{d_j} - P \end{aligned}$$

Based on this, one could obtain that

$$\begin{aligned} & \Delta V(\xi(\tilde{k}), \theta(\tilde{k})) = \xi^T(\tilde{k}) \Psi_i \xi(\tilde{k}) \\ & \quad = -\xi^T(\tilde{k}) (-\Psi_i) \xi(\tilde{k}) \\ & \quad \leq -\gamma \|\xi(\tilde{k})\|^2 \end{aligned} \quad (22)$$

where

$$\gamma \triangleq \min_{i \in \mathbb{S}} \{ \beta_{\min}(-\Psi_i) \} > 0$$

Then, it is concluded that

$$\begin{aligned} & \mathcal{E} \{ V(\xi(\tilde{k} + 1), \theta(\tilde{k} + 1)) | \xi(\tilde{k}), \theta(\tilde{k}) \} \\ & \quad \leq V(\xi(\tilde{k}), \theta(\tilde{k})) - \gamma \xi^T(\tilde{k}) \xi(\tilde{k}) \end{aligned} \quad (23)$$

Taking the expectation on both sides with (23) and continuing the iterative procedure of (23), one gets

$$\begin{aligned} & \mathcal{E} \{ V(\xi(T + 1), \theta(T + 1)) | \xi_0, \theta_0 \} \\ & \quad \leq V(\xi_0, \theta_0) - \gamma \sum_{\tilde{k}=0}^T \mathcal{E} \{ \xi^T(\tilde{k}) \xi(\tilde{k}) | \xi_0, \theta_0 \} \end{aligned} \quad (24)$$

implying

$$\sum_{\tilde{k}=0}^T \mathcal{E}\{\xi^T(\tilde{k})\xi(\tilde{k})|\xi_0, \theta_0\} \leq \frac{1}{\gamma} V(\xi_0, \theta_0) < \infty \quad (25)$$

Then, it is obtained that the auxiliary system (9) is stochastically stable, which implies the closed-loop system (4) stochastically stable. This completes the proof.

Remark 2: When the partially disabled controller (2) is given beforehand, the stability of closed-loop system (4) could be tested by condition (19) conveniently. It is seen that both dwell times and probability α are important in system analysis. However, when controller (2) must be designed, the corresponding problem will be not easy. That is because nonlinear terms such as $(A + BK_j)^{d_j}$ in addition to unknown matrix P make the above studied problem very complicated. In other words, the existence conditions for controller (2) established with form (19) cannot be solved directly and easily.

Next, some existence conditions for controller (2) will be given with concise forms, which are presented in terms of LMIs and could be solved directly.

Theorem 2: Consider system (1), there is a partially disabled controller (2) experiencing forced dwell times such that the closed-loop system (4) is stochastically stable, if for given positive scalars δ and μ , there exist matrices $X > 0$ and Y satisfying

$$\begin{bmatrix} -X & \Phi_1 & \Omega_1 \\ * & -X & 0 \\ * & * & -X \end{bmatrix} < 0 \quad (26)$$

$$\begin{bmatrix} -X & \Phi_2 & \Omega_2 \\ * & -X & 0 \\ * & * & -\mu I \end{bmatrix} < 0 \quad (27)$$

$$AX + BY \geq -\delta I \quad (28)$$

$$AX + BY \leq \delta I \quad (29)$$

$$X \geq \mu I \quad (30)$$

where

$$\begin{aligned} \Phi_1 &= \sqrt{\rho_{11}}(X^T A^T + Y^T B^T), & \Phi_2 &= \sqrt{\rho_{22}}X^T A^T \\ \Omega_1 &= \sqrt{\rho_{12}}X^T (A^T)^{d_2}, & \Omega_2 &= \sqrt{\rho_{21}}\left(\frac{\delta}{\mu}\right)^{d_1} X^T \end{aligned}$$

Thus, the gain of controller (2) is computed by

$$K = YX^{-1} \quad (31)$$

Proof: From the proof of Theorem 1, it is seen that the closed-loop system (4) is stochastically stable if the following conditions

$$\rho_{11}(A+BK)^T P(A+BK) + \rho_{12}(A^T)^{d_2} P(A)^{d_2} - P < 0 \quad (32)$$

and

$$\rho_{22}A^T P A + \rho_{21}[(A + BK)^T]^{d_1} P(A + BK)^{d_1} - P < 0 \quad (33)$$

hold respectively. By using the Schur complement lemma, it is known that (32) is equivalent to

$$\begin{bmatrix} -P & \hat{\Phi}_1 & \hat{\Omega}_1 \\ * & -X & 0 \\ * & * & -X \end{bmatrix} < 0 \quad (34)$$

where

$$\begin{aligned} \hat{\Phi}_1 &= \sqrt{\rho_{11}}(A + BK)^T \\ \hat{\Omega}_1 &= \sqrt{\rho_{12}}(A^T)^{d_2}, \quad X = P^{-1} \end{aligned}$$

By pre- and post-multiplying (34) with $\text{diag}\{X, I, I\}$, it is concluded that condition (26) with representation (31) is equivalent to condition (34). As for condition (33), it is known that it could be guaranteed by

$$\rho_{22}A^T P A + \rho_{21} \|(A + BK)^T\|^{d_1} \|P\| \|(A + BK)^{d_1}\| I - P < 0 \quad (35)$$

Then, it is obtained by

$$\begin{aligned} \rho_{22}A^T P A + \rho_{21} \|(A + BK)^T\|^{d_1} \|P\| \\ \times \|(A + BK)\|^{d_1} I - P < 0 \end{aligned} \quad (36)$$

By pre- and post-multiplying (36) with X , it is obtained

$$\begin{aligned} \rho_{22}X^T A^T P A X + \rho_{21}X^T \|(A + BK)^T\|^{d_1} \|P\| \\ \times \|(A + BK)\|^{d_1} X - X < 0 \end{aligned} \quad (37)$$

Based on conditions (28) and (29), one gets

$$\|AX + BY\| \leq \delta \quad (38)$$

From condition (30), it is concluded that

$$X^{-1} \leq \mu^{-1} I \quad (39)$$

which implies

$$\|X^{-1}\| \leq \mu^{-1} \quad (40)$$

Taking into account (31), (38) and (40), it is obtained

$$\begin{aligned} \|A + BK\| &= \|(A + BK)XX^{-1}\| \\ &\leq \|(AX + BY)\| \|X^{-1}\| \\ &\leq \frac{\delta}{\mu} \end{aligned} \quad (41)$$

Based on this, it is claimed that condition (37) is guaranteed by

$$\rho_{22}X^T A^T P A X + \rho_{21}X^T \left(\frac{\delta}{\mu}\right)^{d_1} \frac{1}{\mu} \left(\frac{\delta}{\mu}\right)^{d_1} X - X < 0 \quad (42)$$

Based on the Schur complement lemma, it is known that conditions (27) and (42) are equivalent. This completes the proof.

Remark 3: In order to get LMI conditions ultimately, some techniques are used to deal with the problems mentioned above, where some additional variables and inequalities are introduced. Moreover, because of the established conditions being LMIs, they could be extended to other cases easily. For example, when probability α is uncertain or unknown, similar

results could be obtained by applying methods given here and references [3], [54] simultaneously.

Remark 4: Though the existence conditions for controller (2) are presented in terms of LMIs, there are still some problems to be further studied. First, the Lyapunov function (20) constructed for system (9) has a constant matrix P , while the auxiliary system is a switching system. It is known that results based on a mode-dependent Lyapunov function will be less conservative than ones obtained by a common one. However, there will be an unavoidable contradiction between mode-dependent matrix P_i and common control gain K . Thus, how to select a suitably improved mode-dependent Lyapunov function is necessary to be considered; Second, some additional inequalities such as (35), (38), (40), are introduced to get LMI conditions. But, they also bring larger conservatism. It is necessary to find a better way to solve the above problems simultaneously; Third, it is better to compute scalars δ and μ directly instead of giving them beforehand. Similar to the former problem, how to make the conditions with LMI forms will be encountered for δ and μ . When some improved conditions are needed, all the above mentioned problems should be carefully considered.

IV. NUMERICAL EXAMPLE

Example 1: Consider a discrete-time system of form (1), whose parameters are described as follows:

$$A = \begin{bmatrix} 0.5 & 0 \\ -0.1 & 1.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ -1 \end{bmatrix}$$

For this example, it is obvious that the open-loop system is unstable. Based on the proposed methods, one could design a partially disabled controller (2). Here, its probability of stochastic variable $\alpha(k)$ on random dwell times interval is $\alpha = 0.6$. Then, the transition probability matrix is given as

$$\Pi = \begin{bmatrix} 0.6 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}$$

Without loss of generality, the fixed dwell times of controller (2) are assumed to be $d_1 = 2$ and $d_2 = 3$ respectively. From Theorem 2 with $\delta = 0.25$ and $\mu = 0.5$, one has the parameters computed as

$$X = \begin{bmatrix} 0.6109 & -0.7243 \\ -0.7243 & 5.4146 \end{bmatrix}$$

$$Y = \begin{bmatrix} -0.6652 & 6.0267 \end{bmatrix}$$

Then, the gain of controller (2) is computed as

$$K = \begin{bmatrix} 0.2743 & 1.1497 \end{bmatrix}$$

It is concluded that both matrices $A_1 = A + B * K$ and $A_2 = A$ are nonsingular. In other words, the assumption about A_i in Lemma 1 is satisfied. Then, the stability of the resulting closed-loop system could be guaranteed by its auxiliary system, while the stability of the auxiliary system has been implied by Theorem 2. Under the initial condition $x_0 = [1 \ -1]^T$ and applying the desired controller, one has the state response of the resulting closed-loop system given

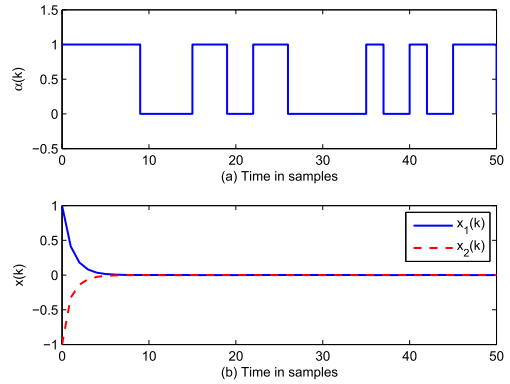


FIGURE 1. Simulations of the closed-loop system.

TABLE 1. The allowable maximum and minimum values of δ for different α .

α	0.446	0.6	0.7	0.8	1
δ_{\max}	0.462	0.480	0.487	0.492	0.499
δ_{\min}	0.250	0.235	0.233	0.231	0.228

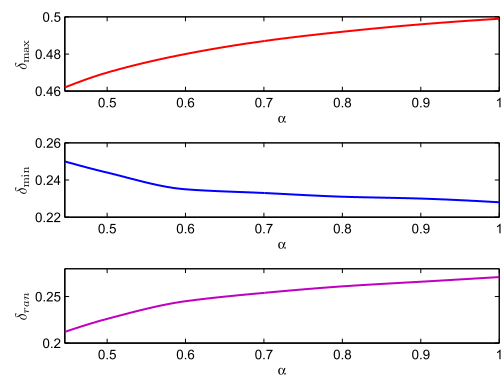


FIGURE 2. The curves of δ_{\max} , δ_{\min} and δ_{ran} along with α .

TABLE 2. The allowable maximum and minimum values of μ for different α .

α	0.446	0.6	0.7	0.8	1
μ_{\max}	0.5	0.532	0.537	0.541	0.549
μ_{\min}	0.271	0.261	0.257	0.254	0.251

in Fig. 1. (b), while Fig. 1. (a) is simulation of stochastic variable $\alpha(k)$ with forced dwell times $d_1 = 2$ and $d_2 = 3$. Since the closed-loop system is stable, it is said that the designed partially disabled controller with forced dwell times is useful. Moreover, it is obtained from Theorem 2 that the solvable range of probability α under the above selected parameters is $[0.446, 1]$.

In order to further demonstrate the effects of parameters, such as δ , μ , α , d_1 and d_2 , more work should be done. Firstly, the correlation between parameters α and δ are considered, while the other parameters are constant. By Theorem 2, the allowable maximum and minimum values of δ along with α could be obtained and given in Table 1, which are denoted as δ_{\max} and δ_{\min} respectively. Based on Table 1,

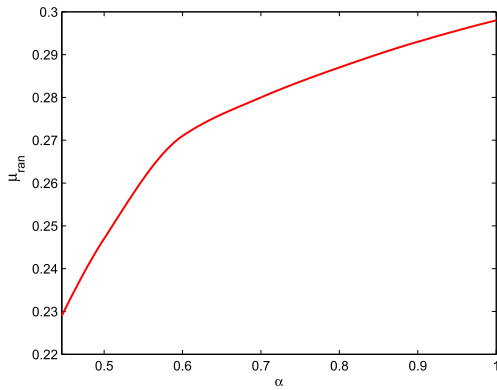


FIGURE 3. The curve of μ_{ran} along with α .

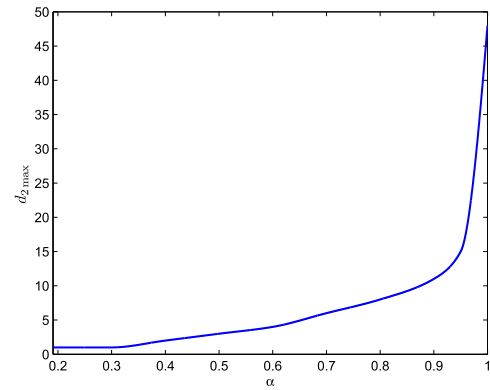


FIGURE 4. The curve of $d_{2\text{max}}$ along with α .

TABLE 3. The allowable maximum value of $d_{2\text{max}}$ for different α .

α	0.2	0.4	0.7	0.9	0.95
$d_{2\text{max}}$	1	2	6	11	15

TABLE 4. The allowable range of ζ for different pair (d_1, d_2) .

(d_1, d_2)	$d_1 = 1$	$d_1 = 2$	$d_1 = 3$	$d_1 = 4$
$d_2 = 1$	[-0.768, 0.569]	[-0.768, 0.570]	[-0.768, 0.570]	[-0.768, 0.570]
$d_2 = 2$	[-0.472, 0.379]	[-0.472, 0.379]	[-0.472, 0.379]	[-0.472, 0.379]
$d_2 = 3$	[-0.292, 0.275]	[-0.292, 0.275]	[-0.292, 0.275]	[-0.292, 0.275]
$d_2 = 4$	[-0.126, 0.176]	[-0.126, 0.176]	[-0.126, 0.176]	[-0.126, 0.176]

the curves of parameters δ_{max} , δ_{min} and δ_{ran} along with α are simulated in Fig. 2, where $\delta_{\text{ran}} \triangleq \delta_{\text{max}} - \delta_{\text{min}}$. Similarly, the correlation between parameters α and μ could be obtained in Table 2, where μ_{max} and μ_{min} are the allowable maximum and minimum values of μ along with α . The simulation of correlation between parameters α and $\mu_{\text{ran}} \triangleq \mu_{\text{max}} - \mu_{\text{min}}$ is given in Fig. 3. Based on these simulations, it is seen that higher probability α could lead to less conservative results in terms of larger ranges of δ_{ran} and μ_{ran} . In other words, the partially disable property of controller (2) plays a negative effect in system stabilization. This phenomenon is consistent with facts. Next, we will demonstrate the correlation between dwell times and probability α . Without loss of generality, the allowable maximum value of d_2 along with α is defined as $d_{2\text{max}}$ and given in Table 3, where the other parameters are same to the ones mentioned above. The simulation of correlation presented in Table 3 is shown in Fig. 4. From this simulation, it is concluded that probability α has a positive effect on dwell times d_2 . In other words, larger dwell time d_2 could be guaranteed by a higher probability. It means the resulting closed-loop system could allow the unstable subsystem suffering a larger dwell time. In order to further demonstrate the effect of dwell times, more simulations will be done in the following. Without loss of generality, system matrix A is assumed to be

$$A = \begin{bmatrix} 0.5 & \zeta \\ -0.1 & 1.1 \end{bmatrix}$$

where ζ is a scalar. Table 4 presents the allowable range of ζ for different pair (d_1, d_2) . Based on this table, the simulation of correlation between $\zeta_{\text{ran}} \triangleq \zeta_{\text{max}} - \zeta_{\text{min}}$ and (d_1, d_2) is shown in Fig. 5, where ζ_{max} and ζ_{min} are the allowable maximum and minimum values of ζ for different pair (d_1, d_2) . From this simulation, it is seen that dwell time d_1 plays a positive effect on system stabilization, while dwell time d_2 is negative to system stabilization. It is because d_1 is related

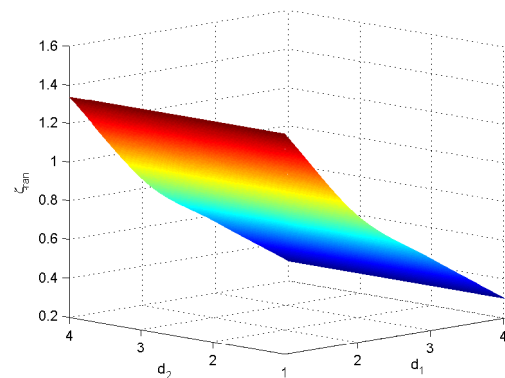


FIGURE 5. The simulation of correlation between pair (d_1, d_2) and ζ_{ran} .

to the designed controller useful, while d_2 is mentioned to the controller disabled. This phenomenon is also in according to the facts, which comes from the properties of dwell times d_1 and d_2 .

V. CONCLUSION

In this paper, the stabilization problem of discrete-time systems has been realized via applying a partially disabled controller, where the partial action of the desired controller is illustrated by a stochastic variable. Though the used variable has two values, it is different from the traditional Bernoulli variable and has forced dwell times. Because of containing the fixed and random dwell times, an auxiliary system with state jumps has been constructed to study the stochastic stability of the original closed-loop system. Moreover, the existence conditions for the partially disabled controller have been given in terms of LMIs, which could be solved easily. Then, a numerical example has been used to demonstrate the utility of the proposed methods. Finally, because the

considered system is normal and without time delay, it is more challenging to study a similar stabilization problem for singular systems with time delay and could be our future work.

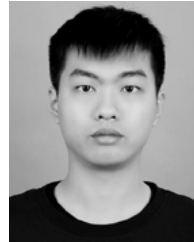
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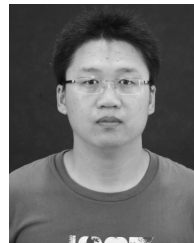
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