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Performance Analysis of a Threshold-Based Parallel Multiple Beam Selection Scheme for WDM FSO Systems

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ABSTRACT In this paper, we statistically analyze the performance of a threshold-based parallel multiple beam selection scheme for a free-space optical-based system with wavelength division multiplexing in cases where a pointing error has occurred under independent identically distributed Gamma–Gamma fading conditions. To simplify the mathematical analysis, we additionally consider Gamma turbulence conditions, which are a good approximation of Gamma–Gamma distribution. Specifically, we statistically analyze the characteristics in operation under conventional detection schemes (i.e., heterodyne detection and intensity modulation/direct detection techniques) for both adaptive modulation (AM) case in addition to non-AM case (i.e., coherent/non-coherent binary modulation). Then, based on the statistically derived results, we evaluate the outage probability of a selected beam, the average spectral efficiency (ASE), the average number of selected beams, and the average bit error rate. Selected results show that we can obtain higher spectral efficiency and simultaneously reduce the potential for increasing the complexity of implementation caused by applying the selection-based beam selection scheme without considerable performance loss. Especially for the AM case, the ASE can be increased further compared with the non-AM cases. Our derived results based on the Gamma distribution as an approximation of the Gamma–Gamma distribution can be used as approximated performance measure bounds, especially, they may lead to lower bounds on the approximated considered performance measures.

INDEX TERMS Free-space optical communication, wavelength division multiplexing, heterodyne detection, intensity modulation/direct detection, strong turbulence, pointing error.

I. INTRODUCTION

Free-space optical (FSO) communications is a promising technology to provide high-speed, improved-capacity, cost-effective, secure and easy-to-deploy wireless networks. Whereas radio frequency (RF)-based wireless communication links are vulnerable to degradation from multi-path propagation, FSO links are vulnerable to degradation from atmospheric effects and pointing errors [2]–[8]. Specifically, the atmospheric effects may lead to a significant degradation in the performance of the FSO communication systems together with misalignment (e.g., pointing error) effects. When an FSO beam propagates through the atmosphere, it experiences deterioration and deformation of its wave-front caused by small-scale, randomly localized changes in the atmospheric index of refraction [9]. Refraction index inhomogeneities from turbulent air lead to beam spreading, beam wander, and intensity fluctuations (i.e., scintillation). These distortions compromise link reliability [3]. The intensity of these distortions in FSO systems can be dependent upon the atmospheric attenuation due to fog, low clouds, rain, snow, dust, and various combinations of each [2]. The attenuation of an optical beam traveling through the atmosphere due to both absorption and scattering is determined by four atmospheric attenuation coefficients: the absorption coefficient, the scattering coefficient, and the molecular and aerosol processes. Here, when selecting wavelength for FSO communication systems, each coefficient need to be usually considered [9]. If atmosphere conditions are severe, the deterioration of link quality in some wavelength may become more severe and there may sometimes be no data transmission to the receiver due to the nature of FSO unlike traditional fiber-optic communications.

Since wavelength division multiplexing (WDM) has been shown to increase the capacity and bandwidth of conventional fiber-optical communication systems [10], WDM has been considered as one of the most promising techniques. In fiber-optic communications, WDM multiplexes a number of optical carrier signals onto a single optical fiber by using different wavelengths (i.e., colors) of laser light. By allowing multiple WDM channels to coexist on a single optical fiber, the simultaneous transmission of multiple high-speed signals becomes possible, which leads to expanded capacity of the network without installing more fiber. Based on [11] and [12], in FSO systems, a WDM access network is a realistic proposition and feasible because both conventional fiber-optic communication systems and FSO systems use similar system components. Recently, several studies have been conducted to apply WDM to FSO communication systems [12]–[17]. Note that by applying WDM to FSO systems, we can achieve the higher sum rate by allowing the simultaneous transmission of multiple high-speed signals while the ASE is the same with or without WDM. However, in FSO based systems, the received signals at the receiver generated based on WDM may not be valid or may not have an acceptable signal-to-noise ratio (SNR) due to atmospheric attenuation, whereas in conventional fiber-optical communication systems, all the received signals may be valid. Therefore, in FSO systems, selection of valid beams not all is required.

Here, we analyze the performance of a threshold-based parallel multiple-beam selection scheme (TPMBS) for FSO systems using WDM. Note that the advantage of the threshold-based scheme is that the processing power can be saved at the receiver end since only the optical chains corresponding to "acceptable/good" wavelengths are activated at any given time similar to the RF based parallel "scheduling" schemes [18], [19]. We statistically analyze the operation under conventional detection schemes (i.e., heterodyne detection (HD) and intensity modulation/direct detection (IM/DD) techniques). Then, based on these statistically derived results, as performance measures, we derive closed-form results of the outage probability (CDF), the average spectral efficiency (ASE), the average number of selected beams (ANSB), and the average bit error rate (BER) in cases where a pointing error has occurred under independent identically distributed (i.i.d.) Gamma-Gamma turbulence conditions, which are widely accepted conditions in the current literature. Further, to simplify the mathematical analysis, we also consider Gamma turbulence conditions, which are a good approximation of the Gamma-Gamma distribution. Note that the closed-form results based on the Gamma approximation model consist of a commonly used Gamma function. Therefore, with the composite channel model and the related closed-form results based on the tractable Gamma approximation model, we can significantly simplify the performance analysis of composite fading channels using measures and reduce the computational complexity.

Another factor that affects the reliability of FSO channels is building sway caused by thermal expansion, wind loads, and weak earthquakes. Building sway leads to a misalignment between the transmitter and the receiver, consequently causing pointing errors, which also seriously degrade system performance. For this reason, we also consider the effects of pointing errors in our performance analysis. More specifically, we consider the composite PDF and the composite CDF of the irradiance fluctuations suffering from both atmospheric turbulence and pointing errors.

II. SYSTEM AND CHANNEL MODELS

We consider a point-to-point FSO link using either intensity modulation/direct detection (IM/DD) or heterodyne detection (HD) [7], [20]–[23]. We assume a block-turbulence FSO channel in which the turbulence is assumed to be constant for one hybrid ARQ (HARQ) round but in which the turbulence independently changes in different rounds. Data transmission is affected by path loss, pointing errors, atmospheric turbulence, and additive white Gaussian noise (AWGN), which are the major factors leading to performance degradation of the proposed scheme in FSO communication systems. In this paper, we assume that an FSO link mainly experiences both atmospheric turbulence and pointing errors. Note that similar to [22] and [24], we consider that the path loss, h_l, is deterministic and it is assumed to be equal to 1 throughout this paper. However, the effect of the path loss can be considered by replacing A_0 with $A_0 \cdot h_l(d)$ in our derived results, where $h_l(d) = \exp(-\sigma d)$ and σ is the attenuation coefficient at distance d [24].

We also assume that a FSO link experiences a Gamma-Gamma turbulence with pointing error impairment, which is a commonly used channel model in FSO communication systems. Additionally, we also assume that induced FSO channel turbulence is modeled by the Gamma distribution as an approximation of the Gamma-Gamma distribution.[1] More specifically, we consider the Gamma distribution that accounts for pointing errors to characterize turbulence in FSO communication systems operating in a variety of atmospheric turbulence regimes [25], [26]. In [25] and [26], the Gamma probability density function (PDF) is found to be a good approximation of the Gamma-Gamma distribution by use of the moment-matching method.

With the TPMBS scheme under consideration, the scheduler selects all the valid optical signals for each time slot

[1] The Gamma-Gamma model has recently considered widely in FSO communication. However, further derivations using that model have shown to be analytically difficult or computationally involved due to the arising special functions. Therefore, approximation using the tractable Gamma distribution can significantly simplify the performance analysis of composite fading channels using measures such as probability of outage, average bit error rate, ergodic capacity, and so on.

with a link condition above a preselected threshold. More specifically, the receiver estimates the link quality of each beam and compares this quality to this preselected threshold. The receiver feeds this channel state information back to the transmitter through a reliable feedback path (i.e. RF feedback channel). Then, only the acceptable beams are selected for the subsequent transmission time.

For selected beams, we consider a rate-adaptive N multi-dimensional trellis coded M-quadrature amplitude modulation (M-QAM) in addition to non-adaptive modulation (AM) (i.e., coherentnon-coherent binary modulation).[2]

Existing adaptive modulation schemes for RF-based systems [27]–[30] are not suitable for high average SNR. In general, FSO systems operate over a wider range of SNR than do RF-based systems, especially in the high SNR regime [31]–[34]. Therefore, to evaluate the exact performance (the ASE and the average BER), the system needs to employ an appropriate type of adaptive modulation in the region of high average SNR. However, our derived closed-form results remain valid if the appropriate type of adaptive modulation scheme for high SNR is adopted, especially the proper values of coefficient (i.e., a_n and b_n in [28]) in fading region n.[3]

With the TPMBS scheme, the feedback rate through RF based feedback channel can be reduced significantly since the receiver needs to send only an information indicating the selected beams and their related AM region information to the transmitter instead of sending the full channel state information.

With this set-up, we analyze the statistical characteristics of a point-to-point FSO link in operation using both HD and IM/DD techniques over the proposed channel models. We derive expressions for the composite PDF and the composite CDF of the irradiance fluctuations suffering from both atmospheric turbulence and pointing errors. Moreover, we provide exact closed-form results of the ANSB, the ASE, the average BER for both adaptive modulation and coherent/non-coherent binary modulation.

III. A CDF-BASED PERFORMANCE ANALYSIS OF TPMBS WITH AM

A. STATISTICAL REPRESENTATIONS OF THE OUTPUT SNR OF THE SELECTED BEAM BASED ON TPMBS

Based on the mode of operation, the selected beam has a conditional PDF of a truncated (above the threshold, γ_T) random variable (RV). Therefore, the PDF and the CDF can be expressed as

$$f_{\gamma_{TB\_WDM}}(\gamma) = \begin{cases} \dfrac{f_\gamma(\gamma)}{1-F_\gamma(\gamma_T)} & \gamma \geq \gamma_T \\ 0 & \text{otherwise} \end{cases}, \quad (1)$$

and

$$F_{\gamma_{TB\_WDM}}(\gamma) = \frac{1}{1-F_\gamma(\gamma_T)}\int_{\gamma_T}^{\gamma} f_\gamma(x)\,dx = \frac{F_\gamma(\gamma)-F_\gamma(\gamma_T)}{1-F_\gamma(\gamma_T)}, \quad (2)$$

where $f_\gamma(\cdot)$ and $F_\gamma(\cdot)$ are the PDF and CDF of an instantaneous SNR at the receiver over the various channel models, respectively.

B. AVERAGE NUMBER OF SELECTED BEAMS (ANSB)

In this case, the number of selected beams (NSB) above the threshold value is random and takes values from 0 to L and each beam among the total L beams can either be selected or not selected. Therefore, the NSB follows a binomial distribution and the resulting ANSB is given by

$$ANSB = \sum_{l=0}^{L} l\binom{L}{l}\left[1-F_\gamma(\gamma_T)\right]^l\left[F_\gamma(\gamma_T)\right]^{L-l} = L\cdot\left[1-F_\gamma(\gamma_T)\right]. \quad (3)$$

C. AVERAGE SPECTRAL EFFICIENCY (ASE)

1) WITH AM

The ASE can be expressed as the sum of the all spectral efficiencies of the individual codes, weighted by the probability, F_n, that the SNR of a selected beam is assigned to the n-th region as

$$ASE = \sum_{n=1}^{N} R_n F_n, \quad (4)$$

where R_n is the spectral efficiency and γ_{T_n} is the SNR boundary,

$$F_n = \int_{\gamma_{T_n}}^{\gamma_{T_{n+1}}} f_{\gamma_{TB\_WDM}}(\gamma)\,d\gamma = \frac{1}{1-F_\gamma(\gamma_T)}\left[F_\gamma\left(\gamma_{T_{n+1}}\right)-F_\gamma\left(\gamma_{T_n}\right)\right]. \quad (5)$$

Then, by multiplying the ANSB and the above ASE, the ASE of selected beams for each time-slot can be written as

$$ASE_{TS} = L\cdot\left[1-F_\gamma(\gamma_T)\right] \times \sum_{n=1}^{N} R_n \frac{1}{1-F_\gamma(\gamma_T)}\left[F_\gamma\left(\gamma_{T_{n+1}}\right)-F_\gamma\left(\gamma_{T_n}\right)\right] = L\cdot\sum_{n=1}^{N} R_n\left[F_\gamma\left(\gamma_{T_{n+1}}\right)-F_\gamma\left(\gamma_{T_n}\right)\right]. \quad (6)$$

[2]The receiver may measure a respective SNR and/or a respective effective channel bandwidth associated with each wavelength. The receiver may transmit the feedback information (e.g., the respective SNR and effective channel bandwidth associated with each wavelength) back to the transmitter. Then, the transmitter can be adapted relative to the information on the channel conditions fed back from the receiver.

[3]In [28], the proper values of a_n and b_n have been determined by applying curve fitting techniques with the least squares method [35] based on the given M_n, γ_{T_n}, R_n, and BER_0.

2) WITH FIXED MODULATION (RATE = R)

In this case, the rate is fixed, R, which simplifies the ASE expression to

$$ASE_{FIX} = R\int_0^{\infty} f_{\gamma_{TB\_WDM}}(\gamma)\,d\gamma = R\int_{\gamma_T}^{\infty} \frac{f_\gamma(\gamma)}{1-F_\gamma(\gamma_T)}d\gamma = R. \tag{7}$$

Similar to the AM case, the ASE of selected beams for each time-slot can be written as

$$ASE_{FIX,TS} = R\cdot L\cdot\left[1-F_\gamma(\gamma_T)\right]. \tag{8}$$

D. AVERAGE BER ($\overline{BER}$)

The average BER for over all codes and SNRs of the selected beams can be expressed as the average number of bits in error divided by the average number of bits transmitted.

1) AVERAGE BER WITH AM

The average BER with AM is

$$\overline{BER} = \frac{\sum_{n=1}^{N} R_n\overline{BER}_n}{\sum_{n=1}^{N} R_nF_n} = \frac{\sum_{n=1}^{N} R_n\overline{BER}_n}{\sum_{n=1}^{N} R_n\left[\frac{F_\gamma\left(\gamma_{T_{n+1}}\right)-F_\gamma\left(\gamma_{T_n}\right)}{1-F_\gamma(\gamma_T)}\right]}. \tag{9}$$

In (9), $\overline{BER}_n$ can be obtained as

$$\overline{BER}_n = \int_{\gamma_{T_n}}^{\gamma_{T_{n+1}}} BER_n f_{\gamma_{TB\_WDM}}(\gamma)\,d\gamma, \tag{10}$$

where $BER_n = a_n\exp\left(-\frac{b_n\cdot\gamma}{M_n}\right)$, a_n and b_n are code-dependent constants, and M_n is a constellation size in [28]. Here, by using an alternate definition of the exponential function based on a Taylor series for mathematical convenience (i.e., $\exp\left(-\frac{b_n}{M_n}\gamma\right) = \sum_{l=0}^{\infty}\frac{\left(-\frac{b_n}{M_n}\gamma\right)^l}{l!}$), (10) can be rewritten as

$$\overline{BER}_n = \overline{BER}_{n,I} - \overline{BER}_{n,II}, \tag{11}$$

where

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\int_{\gamma_{T_n}}^{\infty}\gamma^l f_\gamma(\gamma)\,d\gamma, \tag{12}$$

and

$$\overline{BER}_{n,II} = \sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\int_{\gamma_{T_{n+1}}}^{\infty}\gamma^l f_\gamma(\gamma)\,d\gamma. \tag{13}$$

The expressions in (12) and (13) involve an infinite summation of l. However, the summands in (12) and (13) decay exponentially (or slightly faster) with the increase of l, because Stirling's approximation specifies that $l!$ grows as $\exp(l\ln l)$ [36]. As a result, a truncated summation with a finite number of terms can reliably achieve the required accuracy.

2) FOR COHERENT AND NON-COHERENT BINARY MODULATION

With coherent and non-coherent binary modulation, BER_n can be written as

$$BER_n(\gamma) = \frac{\Gamma(p, q\gamma)}{2\Gamma(p)}, \tag{14}$$

where p and q represent the parameters defining the type of detection mechanism and modulation type, respectively, as

$$p = \begin{cases}\frac{1}{2} & \text{for coherent detection}\\ 1 & \text{for non-coherent or differentially,}\end{cases} \tag{15}$$

$$q = \begin{cases}\frac{1}{2} & \text{for FSK}\\ 1 & \text{for PSK.}\end{cases} \tag{16}$$

Then, the average BER can be obtained with (14) based on the given turbulence distribution as

$$\overline{BER} = \int_0^{\infty} BER(\gamma) f_{\gamma_{TB\_WDM}}(\gamma)\,d\gamma = BER(\gamma)F_{\gamma_{TB\_WDM}}(\gamma)\big|_{\gamma_T}^{\infty} - \int_{\gamma_T}^{\infty}F_{\gamma_{TB\_WDM}}(\gamma)\,dBER(\gamma). \tag{17}$$

In (17), with the help of [37, eq. (6.5.25)], $BER(\gamma)$ and its derivative term can be written as

$$BER(\gamma) = \frac{\Gamma(p, q\gamma)}{2\Gamma(p)}, \quad \frac{d}{d\gamma}BER(\gamma) = -\frac{q(q\cdot\gamma)^{p-1}}{2\Gamma(p)}\exp(-q\cdot\gamma). \tag{18}$$

Therefore, the average BER can be written as

$$\overline{BER} = \frac{q^p}{2\Gamma(p)}\int_{\gamma_T}^{\infty}F_{\gamma_{TB\_WDM}}(\gamma)\,\gamma^{p-1}\exp(-q\cdot\gamma)\,d\gamma, \tag{19}$$

or an alternative expression based on a Taylor series,

$$\overline{BER} = \sum_{n=0}^{\infty}\frac{(-1)^n q^{n+p}}{2n!\Gamma(p)}\int_{\gamma_T}^{\infty}F_{\gamma_{TB\_WDM}}(\gamma)\,\gamma^{n+p-1}d\gamma. \tag{20}$$

Then, substituting $F_{\gamma_{TB\_WDM}}(\gamma)$ in (20) for (2), (20) can be rewritten for mathematical convenience as

$$\overline{BER} = \overline{BER}_I - \overline{BER}_{II}, \tag{21}$$

where

$$\overline{BER}_I = \frac{q^p}{2\Gamma(p)}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)F_\gamma(\gamma)\,d\gamma, \tag{22}$$

or an alternative expression based on a Taylor series,

$$\overline{BER}_I = \sum_{n=0}^{\infty}\frac{(-1)^n q^{n+p}}{2n!\Gamma(p)}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\int_{\gamma_T}^{\infty}\gamma^{n+p-1}F_\gamma(\gamma)\,d\gamma, \tag{23}$$

and

$$\overline{BER}_{II} = \frac{q^p}{2\Gamma(p)} \cdot \frac{F_\gamma(\gamma_T)}{1-F_\gamma(\gamma_T)} \int_{\gamma_T}^{\infty} \gamma^{p-1} \exp(-q\gamma)\, d\gamma. \tag{24}$$

Here, with the help of [37, eq. (3.381.3)], (24) can be rewritten as

$$\overline{BER}_{II} = \frac{\Gamma(p, q, \gamma_T)}{2\Gamma(p)} \cdot \frac{F_\gamma(\gamma_T)}{1-F_\gamma(\gamma_T)}. \tag{25}$$

The infinite summation in (23) for the term of n can be achieved with a finite number of terms with the help of Stirling's approximation [36].

Note that with the CDF-based performance analysis framework, we now need to characterize the statistics of the SNR over strong turbulence channels with pointing errors using the HD and IM/DD techniques. In Table 1, the common parameters representing the characteristics of the turbulence and the pointing errors are tabulated.

TABLE 1. Common parameters for the characteristics of the turbulence and the pointing errors.

Notation	Description
$\xi = \frac{W_e}{2\sigma_s}$	the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter)at the receiver
A_0	the fraction of the collected power when both beam and detector centers are matched
α and β	turbulence/scintillation parameters related to the atmospheric turbulence conditions
η	the effective photoelectric conversion ratio
μ_{xx}	the electrical average SNR based on xx detection techniques
I	the optical receiver irradiance

IV. STATISTICAL ANALYSIS BASED ON HD OVER GAMMA-GAMMA TURBULENCE CHANNELS

A. CLOSED-FORM STATISTICAL CHARACTERISTICS

To derive the closed-form expression of the composite PDF of the output SNR of the selected beam based on TPMBS, $f_{\gamma_{TB\_WDM}}(\gamma)$, we need to derive both PDF and CDF of an instantaneous SNR at the receiver over Gamma-Gamma turbulence channels. In this case, from [8], [38], [39], the PDF of the optical receiver irradiance, I, can be written as

$$f_I(I) = \frac{\xi^2}{I \cdot \Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left[\alpha\beta \frac{I}{A_0} \middle| \begin{matrix} \xi^2+1 \\ \xi^2, \alpha, \beta \end{matrix}\right], \tag{26}$$

where ξ is the ratio between the equivalent beam radius at the receiver, W_e, and the pointing error displacement standard deviation, σ_s, [7], [8] at the receiver (i.e., when $\xi \to \infty$, the problem converges to the non-pointing error case), A_0 represents the fraction of the collected power when both beam and detector centers are matched, α and β are turbulence/scintillation parameters related to the atmospheric turbulence conditions, $\Gamma(\cdot)$ is the Gamma function, and $G(\cdot)$ is Meijer's G function.

For the HD case, the electrical average SNR can be developed as

$$\mu_{HD} = \begin{cases} \dfrac{\eta E_I\{I\}}{N_0} = \dfrac{\eta A_0 \xi^2}{(1+\xi^2) N_0} \\ \dfrac{\eta A_0}{N_0} \qquad \text{for } \xi^2 \gg 1, \end{cases} \tag{27}$$

where η is the effective photoelectric conversion ratio. Here, with the definition of the instantaneous electrical SNR, $\gamma = \frac{\eta I}{N_0}$,

$$I = \frac{\gamma N_0}{\eta} = \begin{cases} \dfrac{A_0 \gamma \xi^2}{(1+\xi^2)\mu_{HD}} \\ \dfrac{A_0\gamma}{\mu_{HD}} \qquad \text{for } \xi^2 \gg 1, \end{cases} \tag{28}$$

and

$$\frac{dI}{d\gamma} = \begin{cases} \dfrac{A_0 \xi^2}{(1+\xi^2)\mu_{HD}} \\ \dfrac{A_0}{\mu_{HD}} \qquad \text{for } \xi^2 \gg 1. \end{cases} \tag{29}$$

Therefore, the resulting SNR can be written as

$$\gamma = \begin{cases} \dfrac{(1+\xi^2)\mu_{HD} I}{A_0 \xi^2} \\ \dfrac{\mu_{HD} I}{A_0} \qquad \text{for } \xi^2 \gg 1. \end{cases} \tag{30}$$

Then, by applying a simple power transformation of RV I, the resulting PDF of the electrical SNR under HD can be given as

$$\begin{aligned} f_\gamma(\gamma) &= f_I(I(\gamma)) \left|\frac{dI}{d\gamma}\right| \\ &= \frac{\xi^2}{\gamma \cdot \Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left[\alpha\beta \frac{\gamma\xi^2}{(1+\xi^2)\mu_{HD}} \middle| \begin{matrix} \xi^2+1 \\ \xi^2, \alpha, \beta \end{matrix}\right]. \end{aligned} \tag{31}$$

With (31), the CDF of γ can be obtained as

$$\begin{aligned} F_\gamma(\gamma) &= \int_0^\gamma f_\gamma(x)\, dx = \frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} \\ &\quad \times \int_0^\gamma \frac{1}{x} G_{1,3}^{3,0}\left[\alpha\beta \frac{\xi^2}{(1+\xi^2)\mu_{HD}} x \middle| \begin{matrix} \xi^2+1 \\ \xi^2, \alpha, \beta \end{matrix}\right] dx. \end{aligned} \tag{32}$$

Then, utilizing [40, eq. (07.34.21.0084.01)] with (31) where $a_1 = 1+\xi^2$, $b_1 = \xi^2$, $b_2 = \alpha$, and $b_3 = \beta$ in [40, eq. (07.34.21.0084.01)]), the closed-form expression of (32) can be obtained as

$$\begin{aligned} F_\gamma(\gamma) &= \frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} G_{2,4}^{3,1}\left[\frac{\alpha\beta\xi^2}{(1+\xi^2)\mu_{HD}}\gamma \middle| \begin{matrix} 1, \frac{a_1}{1} \\ \frac{b_1}{1}, \frac{b_2}{1}, \frac{b_3}{1}, 0 \end{matrix}\right] \\ &= \frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} G_{2,4}^{3,1}\left[\frac{\alpha\beta\xi^2}{(1+\xi^2)\mu_{HD}}\gamma \middle| \begin{matrix} 1, \xi^2+1 \\ \xi^2, \alpha, \beta, 0 \end{matrix}\right]. \end{aligned} \tag{33}$$

Note that directly adapting the closed-form expression result given in (33) to (3), (6), and (8), the closed-form expressions of ANSB and ASE for TPMBS over Gamma-Gamma turbulence channels with HD can be directly obtained.

For $\xi^2 \gg 1$, we can assume that the pointing error is negligible. Therefore, we can also assume that $I \approx I_a$, which denotes the atmospheric turbulence and then $f_I(I)$ becomes

$$f_I(I) = \frac{2(\alpha\beta)^{\frac{(\alpha\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} I^{\frac{(\alpha\beta)}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right), \quad (34)$$

where $K_\nu(\cdot)$ is the modified Bessel function of the second kind. Then, by applying the random variable transformation with (28) and (29), we can obtain the PDF of γ as

$$f_\gamma(\gamma) = \frac{2(\alpha\beta)^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{A_0}{\mu_{HD}}\right)^{\frac{(\alpha+\beta)}{2}} (\gamma)^{\frac{(\alpha+\beta)}{2}-1} \times K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\frac{A_0}{\mu_{HD}}\gamma}\right). \quad (35)$$

Here, we can rewrite (35) in terms of Meijer's G-function as

$$f_\gamma(\gamma) = \frac{(A)^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} (\gamma)^{\frac{(\alpha+\beta)}{2}-1} \times G_{0,2}^{2,0}\left[A\gamma \,\middle|\, \begin{matrix} - \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2} \end{matrix}\right], \quad (36)$$

where $A = \frac{\alpha\beta A_0}{\mu_{HD}}$. With (36), the CDF expression is given as

$$\begin{aligned} F_\gamma(\gamma) &= \int_0^\gamma f_\gamma(x)\,dx \\ &= \frac{(A)^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\gamma (x)^{\frac{(\alpha+\beta)}{2}-1} G_{0,2}^{2,0}\left[Ax \,\middle|\, \begin{matrix} - \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2} \end{matrix}\right] dx. \end{aligned} \quad (37)$$

Here, utilizing [40, eq. (07.34.21.0084.01)], the closed-form expression of the integral part in (37) can be obtained as

$$\begin{aligned} &\int_0^\gamma (x)^{\frac{(\alpha+\beta)}{2}-1} G_{0,2}^{2,0}\left[Ax \,\middle|\, \begin{matrix} - \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2} \end{matrix}\right] dx \\ &= \frac{1}{\gamma^{-\frac{(\alpha+\beta)}{2}}} G_{1,3}^{2,1}\left[A\gamma \,\middle|\, \begin{matrix} 1-\frac{(\alpha+\beta)}{2} \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2}, -\frac{(\alpha+\beta)}{2} \end{matrix}\right]. \end{aligned} \quad (38)$$

Therefore, the final closed-form expression of $F_\gamma(\gamma)$ can be obtained as

$$F_\gamma(\gamma) = \frac{(A)^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \gamma^{\frac{(\alpha+\beta)}{2}} \times G_{1,3}^{2,1}\left[A\gamma \,\middle|\, \begin{matrix} 1-\frac{(\alpha+\beta)}{2} \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2}, -\frac{(\alpha+\beta)}{2} \end{matrix}\right]. \quad (39)$$

B. CLOSED-FORM AVERAGE BER FOR AM

To obtain the closed-form expression of (9), we need to derive the closed-form expressions of $F_\gamma(\cdot)$, (12), and (13). Here, the closed-form expression of $F_\gamma(\cdot)$ is given in (33). Therefore, we focus on the closed-form derivation of both (12) and (13).

With the derived result in (31), (12) can be rewritten as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} \times \int_{\gamma_{T_n}}^{\infty} \gamma^{l-1} G_{1,3}^{3,0}\left[A'\gamma \,\middle|\, \begin{matrix} \xi^2+1 \\ \xi^2, \alpha, \beta \end{matrix}\right] d\gamma, \quad (40)$$

where the closed-form expression of $F_\gamma(\gamma_T)$ is given in (33) by replacing γ with γ_T and $A' = \frac{\xi^2}{(1+\xi^2)} \cdot \frac{\alpha\beta}{\mu_{HD}}$.

Therefore, with Theorem 1 in Appendix I, the closed-form expression of $\overline{BER}_{n,I}$ can be finally obtained as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} (\gamma_{T_n})^l \times G_{2,4}^{4,1}\left[A'\gamma_{T_n} \,\middle|\, \begin{matrix} \xi^2+1, -l+1 \\ -l, \xi^2, \alpha, \beta \end{matrix}\right], \quad (41)$$

where the closed-form expression of $F_\gamma(\gamma_T)$ is given in (33) with γ_T.

For $\xi^2 \gg 1$, similarly with (36), $\overline{BER}_{n,I}$ can be written as

$$\begin{aligned} &\overline{BER}_{n,I} \\ &= \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{A^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \\ &\quad \times \int_{\gamma_{T_n}}^{\infty} (\gamma)^{\frac{(\alpha+\beta)}{2}+l-1} G_{0,2}^{2,0}\left[A\gamma \,\middle|\, \begin{matrix} - \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2} \end{matrix}\right] d\gamma. \end{aligned} \quad (42)$$

Here, let $\frac{\gamma}{\gamma_{T_n}} = x$. Then, $\gamma = \gamma_{T_n}x$, $d\gamma = \gamma_{T_n}dx$, and the valid integral region of x becomes $1 < x < \infty$. Therefore, we can re-write the integral part in (42) as

$$\begin{aligned} &\int_{\gamma_{T_n}}^{\infty} (\gamma)^{\frac{(\alpha+\beta)}{2}-1} G_{0,2}^{2,0}\left[A\gamma \,\middle|\, \begin{matrix} - \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2} \end{matrix}\right] d\gamma \\ &= \left(\gamma_{T_n}\right)^{\frac{(\alpha+\beta)}{2}+l} \int_1^\infty (x)^{\frac{(\alpha+\beta)}{2}+l-1} \\ &\quad \times G_{0,2}^{2,0}\left[A\gamma_{T_n}x \,\middle|\, \begin{matrix} - \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2} \end{matrix}\right] dx. \end{aligned} \quad (43)$$

Then, with the help of the Erdelyi-Kober operator, the closed-form expression of (43) can be obtained as

$$\left(\gamma_{T_n}\right)^{\frac{(\alpha+\beta)}{2}+l} \times G_{1,3}^{3,1}\left[A\gamma_{T_n} \,\middle|\, \begin{matrix} -\frac{(\alpha+\beta)}{2}-l+1 \\ -\frac{(\alpha+\beta)}{2}-l, \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2} \end{matrix}\right]. \quad (44)$$

Therefore, by substituting (44) into (42), the final closed-from result can be obtained as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{A^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \times (\gamma_{T_n})^{\frac{(\alpha+\beta)}{2}+l} \times G_{1,3}^{3,1}\left[A\gamma_{T_n}\left| \begin{matrix} -\frac{(\alpha+\beta)}{2}-l+1 \\ -\frac{(\alpha+\beta)}{2}-l, \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2} \end{matrix}\right.\right]. \quad (45)$$

For $\overline{BER}_{n,II}$, similar to $\overline{BER}_{n,I}$, the closed-form expression can be obtained directly by replacing γ_{T_n} in (41) and (45) with $\gamma_{T_{n+1}}$.

C. CLOSED-FORM AVERAGE BER FOR NON-AM

In this case, we need to derive the closed-form results of (23) and (24). The closed-form expression of (24) is given in (25). Therefore, we focus on the closed-form derivation of (23). In this case, similar to Sec.IV-B, (23) can be rewritten as

$$\overline{BER}_I = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n+p}}{2n!\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} \times \int_{\gamma_T}^{\infty} \gamma^{n+p-1} G_{2,4}^{3,1}\left[A\gamma\left| \begin{matrix} 1, \xi^2+1 \\ \xi^2, \alpha, \beta, 0 \end{matrix}\right.\right] d\gamma, \quad (46)$$

where $A = \frac{\xi^2}{(1+\xi^2)} \cdot \frac{\alpha\beta}{\mu_{HD}}$. Here, similar to (41) with the help of Theorem 1, we can also obtain the closed-form expression of (46) as

$$\overline{BER}_I = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n+p}}{2n!\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} \times (\gamma_T)^{n+p} G_{3,5}^{4,1}\left[A\gamma_T\left| \begin{matrix} 1, \xi^2+1, -n-p+1 \\ -n-p, \xi^2, \alpha, \beta, 0 \end{matrix}\right.\right]. \quad (47)$$

The final closed-form expressions of (47) and the closed-form expression of $\overline{BER}_{II}$ given in (25) can be obtained by adopting the closed-form expression of $F_\gamma(\cdot)$, which is given in (33).

For $\xi^2 \gg 1$, similarly with (39), $\overline{BER}_I$ can be written as

$$\overline{BER}_I = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n+p}}{2n!\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{A^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \times \int_{\gamma_T}^{\infty} (\gamma)^{\frac{(\alpha+\beta)}{2}+p-1} \times G_{1,3}^{2,1}\left[A\gamma\left| \begin{matrix} 1-\frac{(\alpha+\beta)}{2} \\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2}, -\frac{(\alpha+\beta)}{2} \end{matrix}\right.\right] d\gamma. \quad (48)$$

Then, with the help of Theorem 1, we can obtain the closed-form result of (48) as

$$\overline{BER}_I = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n+p}}{2n!\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{A^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \times (\gamma_T)^{\frac{(\alpha+\beta)}{2}+p} G_{2,4}^{3,1} \times \left[A\gamma_T\left| \begin{matrix} 1-\frac{(\alpha+\beta)}{2}, -\frac{(\alpha+\beta)}{2}-p+1 \\ -\frac{(\alpha+\beta)}{2}-p, \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2}, -\frac{(\alpha+\beta)}{2} \end{matrix}\right.\right], \quad (49)$$

where $F_\gamma(\cdot)$ like for $\overline{BER}_{II}$ is given in (39).

V. STATISTICAL ANALYSIS BASED ON IM/DD OVER GAMMA-GAMMA TURBULENCE CHANNELS

A. CLOSED-FORM STATISTICAL CHARACTERISTICS

Similarly, for the IM/DD technique, the electrical average SNR can be written as

$$\mu_{IM/DD} = \begin{cases} \frac{\eta^2 E_I^2\{I\}}{N_0} = \frac{\eta^2 A_0^2 \xi^4}{(1+\xi^2)^2 N_0} \\ \frac{\eta^2 A_0^2}{N_0} & \text{for } \xi^2 \gg 1. \end{cases} \quad (50)$$

Note that for the IM/DD technique, $\mu_{IM/DD}$ is different from the average SNR, $\overline{\gamma} = \frac{\eta^2 E_I\{I^2\}}{N_0}$.

Here, with the definition of the instantaneous electrical SNR, $\gamma = \frac{\eta^2 I^2}{N_0}$, the optical receiver irradiance, I, and its derivative term can be written, respectively, as

$$I = \frac{\sqrt{\gamma N_0}}{\eta} = \begin{cases} \frac{A_0\xi^2}{(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}} \\ A_0\sqrt{\frac{\gamma}{\mu_{IM/DD}}} & \text{for } \xi^2 \gg 1 \end{cases}, \quad (51)$$

and

$$\frac{dI}{d\gamma} = \begin{cases} \frac{A_0\xi^2}{(1+\xi^2)} \cdot \frac{1}{2\mu_{IM/DD}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}} \\ A_0 \cdot \frac{1}{2\mu_{IM/DD}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}} & \text{for } \xi^2 \gg 1. \end{cases} \quad (52)$$

Therefore, the resulting SNR can be written as

$$\gamma = \begin{cases} \left(\frac{1+\xi^2}{\xi^2}\right)^2 \cdot \frac{\mu_{IM/DD}}{A_0^2} \cdot I^2 \\ \frac{\mu_{IM/DD}}{A_0^2} \cdot I^2 & \text{for } \xi^2 \gg 1. \end{cases} \quad (53)$$

Then, similar to the HD case, by applying a simple power transformation of RV I with (26), (51), and (52), the resulting

PDF of the electrical SNR under the IM/DD technique can be given as

$$f_\gamma(\gamma) = f_I(I(\gamma))\left|\frac{dI}{d\gamma}\right| = \frac{\xi^2}{2\gamma\cdot\Gamma(\alpha)\Gamma(\beta)} \times G_{1,3}^{3,0}\left[\alpha\beta\frac{\xi^2}{(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\middle|\begin{matrix}\xi^2+1\\ \xi^2,\alpha,\beta\end{matrix}\right]. \tag{54}$$

Thus, the CDF expression of (54) can be written as

$$F_\gamma(\gamma) = \frac{\xi^2}{2\gamma\cdot\Gamma(\alpha)\Gamma(\beta)}\int_0^\gamma \frac{1}{x} \times G_{1,3}^{3,0}\left[\alpha\beta\frac{\xi^2}{(1+\xi^2)\sqrt{\mu_{IM/DD}}}\sqrt{x}\middle|\begin{matrix}\xi^2+1\\ \xi^2,\alpha,\beta\end{matrix}\right]dx. \tag{55}$$

By utilizing [40, eq. (07.34.21.0084.01)], the closed-form expression of (55) can be obtained (56), as shown at the bottom of this page.

Similar to Sec. IV, by directly adapting the closed-form expression of $F_\gamma(\cdot)$ in (54) to (3), (6), and (8), we can obtain the closed-form expressions of ANSB and ASE over Gamma-Gamma turbulence channels with IM/DD.

For $\xi^2 \gg 1$, similar to Sec. IV with (51) and (52), the PDF expression can be obtained as

$$f_\gamma(\gamma) = \begin{cases} \dfrac{(\alpha\beta A_0)^{\frac{(\alpha+\beta)}{2}}}{\mu_{IM/DD}\Gamma(\alpha)\Gamma(\beta)}\left(\sqrt{\dfrac{\gamma}{\mu_{IM/DD}}}\right)^{\frac{(\alpha+\beta)}{2}-2} \\ \times K_{\alpha-\beta}\left(2\sqrt{\alpha\beta A_0\sqrt{\dfrac{\gamma}{\mu_{IM/DD}}}}\right) \\ \text{or equivalently} \\ \dfrac{(A)^{\frac{(\alpha+\beta)}{2}}}{2\mu_{IM/DD}\Gamma(\alpha)\Gamma(\beta)}\left(\sqrt{\dfrac{\gamma}{\mu_{IM/DD}}}\right)^{\frac{(\alpha+\beta)}{2}-2} \\ \times G_{0,2}^{2,0}\left[A\sqrt{\dfrac{\gamma}{\mu_{IM/DD}}}\middle|\begin{matrix}-\\ \frac{(\alpha-\beta)}{2}, -\frac{(\alpha-\beta)}{2}\end{matrix}\right], \end{cases} \tag{57}$$

where $A = \alpha\beta A_0$. Then, utilizing [40, eq. (07.34.21.0084.01)], the closed-from result of $F_\gamma(\gamma)$ can be obtained (58), as shown at the bottom of this page.

B. CLOSED-FORM AVERAGE BER FOR AM

Similar to Sec. IV, the closed-form expressions of $F_\gamma(\cdot)$ is given in (56). We focus on the derivation of the closed-form expressions for both (12) and (13). With the derived results in (54), (12) can be rewritten as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\cdot\frac{\xi^2}{2\Gamma(\alpha)\Gamma(\beta)} \times \int_{\gamma_{T_n}}^{\infty}\gamma^{l-1}G_{1,3}^{3,0}\left[A'\sqrt{\gamma}\middle|\begin{matrix}\xi^2+1\\ \xi^2,\alpha,\beta\end{matrix}\right]d\gamma, \tag{59}$$

where $A' = \frac{\xi^2}{(1+\xi^2)}\cdot\frac{\alpha\beta}{\sqrt{\mu_{IM/DD}}}$.

With (59), let $\sqrt{\frac{\gamma}{\gamma_{T_n}}} = x$. Then, $\gamma = \gamma_{T_n}x^2$ and $d\gamma = 2\gamma_{T_n}x dx$. Therefore, the integral part in (59) can be rewritten as

$$\int_{\gamma_{T_n}}^{\infty}\gamma^{l-1}G_{1,3}^{3,0}\left[A'\gamma\middle|\begin{matrix}\xi^2+1\\ \xi^2,\alpha,\beta\end{matrix}\right]d\gamma = 2\left(\gamma_{T_n}\right)^l\int_1^\infty x^{2l-1}G_{1,3}^{3,0}\left[A'\sqrt{\gamma_{T_n}}x\middle|\begin{matrix}\xi^2+1\\ \xi^2,\alpha,\beta\end{matrix}\right]dx. \tag{60}$$

Then, with the help of Theorem 1, the closed-form expression of $\overline{BER}_{n,I}$ can be obtained as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\cdot\frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} \times\left(\gamma_{T_n}\right)^l G_{2,4}^{4,1}\left[A'\sqrt{\gamma_{T_n}}\middle|\begin{matrix}\xi^2+1,-2l+1\\ -2l,\xi^2,\alpha,\beta\end{matrix}\right]. \tag{61}$$

For $\xi^2 \gg 1$, with the derived result in terms of Meijer's G-function in (57), (12) can be rewritten as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\cdot\frac{(A)^{\frac{(\alpha+\beta)}{2}}}{2\mu_{IM/DD}\Gamma(\alpha)\Gamma(\beta)}$$

$$F_\gamma(\gamma) = \frac{2^{\alpha+\beta-2}\xi^2}{2\pi\cdot\Gamma(\alpha)\Gamma(\beta)}\times G_{3,7}^{6,1}\left[\frac{(\alpha\beta)^2}{2^4\mu_{IM/DD}}\left\{\frac{\xi^2}{(1+\xi^2)}\right\}^2\gamma\middle|\begin{matrix}1,\frac{1+\xi^2}{2},\frac{2+\xi^2}{2}\\ \frac{\xi^2}{2},\frac{1+\xi^2}{2},\frac{\alpha}{2},\frac{1+\alpha}{2},\frac{\beta}{2},\frac{1+\beta}{2},0\end{matrix}\right]. \tag{56}$$

$$F_\gamma(\gamma) = \frac{1}{4\pi\Gamma(\alpha)\Gamma(\beta)}\left(\frac{A^2}{\mu_{IM/DD}}\right)^{\frac{(\alpha+\beta)}{4}}(\gamma)^{\frac{(\alpha+\beta)}{4}}\times G_{1,5}^{4,1}\left[A\gamma\middle|\begin{matrix}1-\frac{(\alpha+\beta)}{4}\\ \frac{(\alpha-\beta)}{4},\frac{(\alpha-\beta)}{4}+\frac{1}{2},-\frac{(\alpha-\beta)}{4},-\frac{(\alpha-\beta)}{4}+\frac{1}{2},-\frac{(\alpha+\beta)}{4}\end{matrix}\right]. \tag{58}$$

$$\cdot\left(\frac{1}{\mu_{IM/DD}}\right)^{\frac{(\alpha+\beta)}{4}-1}\times\int_{\gamma_{T_n}}^{\infty}(\gamma)^{\frac{(\alpha+\beta)}{4}-1}$$
$$\times G_{0,2}^{2,0}\left[\frac{A}{\sqrt{\mu_{IM/DD}}}\sqrt{\gamma}\,\middle|\,\begin{matrix}-\\ \frac{(\alpha-\beta)}{2},-\frac{(\alpha-\beta)}{2}\end{matrix}\right]d\gamma. \tag{62}$$

Then, by applying a similar approach to that used for the HD case, we can obtain the closed-form expression (63) as shown at the bottom of this page.

For $\overline{BER}_{n,II}$, similar to Sec. IV, the closed-form results can be obtained simply by replacing γ_{T_n} in the closed-from expression of $\overline{BER}_{n,I}$ in (61) and (63) with $\gamma_{T_{n+1}}$.

C. CLOSED-FORM AVERAGE BER FOR NON-AM

In this case, we need to derive the closed-form expression of (23) and (24) over Gamma-Gamma turbulence channels with IM/DD. For the closed-form derivation of (23), similar to Sec. IV, (23) can be rewritten (64) as shown at the bottom of this page, where $A=\frac{(\alpha\beta)^2}{2^4\mu_{IM/DD}}\cdot\left(\frac{\xi^2}{1+\xi^2}\right)^2$.

Then, by applying Theorem 1, the closed-form expression of (64) as shown at the bottom of this page.

For $\xi^2 \gg 1$, similarly with (58), $\overline{BER}_I$ can be written as shown in (66) at the top of the next page. Then, by applying Theorem 1, the closed-form result of (66) can be obtained as shown in (67) at the top of next page.

For $\overline{BER}_{II}$, similar to the HD case, the closed-form expression of (24), $\overline{BER}_{II}$, can be simply obtained by substituting the closed-form expression of $F_\gamma(\cdot)$ with γ_T in (56) and (58) into (24).

VI. STATISTICAL ANALYSIS BASED ON HD OVER GAMMA DISTRIBUTION AS AN APPROXIMATION OF THE GAMMA-GAMMA DISTRIBUTION

A. CLOSED-FORM STATISTICAL CHARACTERISTICS

The Gamma distribution is a good approximation of the Gamma-Gamma distribution through the use of the moment-matching method [25], [26]. Therefore, the composite PDF of the optical receiver irradiance, I, over the Gamma distribution as an approximation of the Gamma-Gamma distribution can be written as given in [26]:

$$f_I(I)=\frac{\xi^2\theta^{-\xi^2}}{\Gamma(k)A_0^{\xi^2}}I^{\xi^2-1}\Gamma\left(k-\xi^2,\frac{I}{\theta A_0}\right), \tag{68}$$

where ξ is the ratio between the equivalent beam radius and the pointing error displacement standard deviation (jitter) at the receiver. A_0 is the pointing loss. $k=\frac{\alpha\beta}{1+\alpha+\beta}$, $\theta=\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\alpha\beta}$, and α and β are the atmospheric turbulence conditions. Then, similar to the Gamma-Gamma results, we can obtain the PDF of SNR, γ, for both the HD and IM/DD techniques.

Then, similar to the Gamma-Gamma case, by applying a power transformation of RV I, the resulting PDF of the electrical SNR under HD can be given as

$$f_\gamma(\gamma)=\frac{\xi^2}{\Gamma(k)}\left\{\frac{\xi^2}{\theta\left(1+\xi^2\right)\mu_{HD}}\right\}^{\xi^2}\gamma^{\xi^2-1}$$
$$\times\Gamma\left(k-\xi^2,\frac{\xi^2}{\theta\left(1+\xi^2\right)\mu_{HD}}\gamma\right). \tag{69}$$

Thus, with (69), the CDF of γ can be written as

$$F_\gamma(\gamma)=\frac{\xi^2}{\Gamma(k)}\left\{\frac{\xi^2}{\theta\left(1+\xi^2\right)\mu_{HD}}\right\}^{\xi^2}$$
$$\times\int_0^{\gamma}x^{\xi^2-1}\Gamma\left(k-\xi^2,\frac{\xi^2}{\theta\left(1+\xi^2\right)\mu_{HD}}x\right)dx. \tag{70}$$

$$\overline{BER}_{n,I}=\sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\cdot\frac{1}{4\pi\Gamma(\alpha)\Gamma(\beta)}\cdot\left(\frac{A^2\gamma_{T_n}}{\mu_{IM/DD}}\right)^{\frac{(\alpha+\beta)}{4}}$$
$$\times G_{1,5}^{5,0}\left[\frac{A^2\gamma_{T_n}}{16\mu_{IM/DD}}\,\middle|\,\begin{matrix}1-\frac{(\alpha+\beta)}{4}\\ -\frac{(\alpha+\beta)}{4},\frac{(\alpha-\beta)}{4},\frac{(\alpha-\beta)}{4}+\frac{1}{2},-\frac{(\alpha-\beta)}{4},-\frac{(\alpha-\beta)}{4}+\frac{1}{2}\end{matrix}\right]. \tag{63}$$

$$\overline{BER}_I=\sum_{n=0}^{\infty}\frac{(-1)^n q^{n+p}2^{\alpha+\beta-2}\xi^2}{4\pi n!\Gamma(p)\Gamma(\alpha)\Gamma(\beta)}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\times\int_{\gamma_T}^{\infty}\gamma^{n+p-1}G_{3,7}^{6,1}\left[A\gamma\,\middle|\,\begin{matrix}1,\frac{\xi^2+1}{2},\frac{\xi^2+2}{2}\\ \frac{\xi^2}{2},\frac{\xi^2+1}{2},\frac{\alpha}{2},\frac{\alpha+1}{2},\frac{\beta}{2},\frac{\beta+1}{2},0\end{matrix}\right]d\gamma, \tag{64}$$

$$\overline{BER}_I=\sum_{n=0}^{\infty}\frac{(-1)^n q^{n+p}2^{\alpha+\beta-2}\xi^2}{4\pi n!\Gamma(p)\Gamma(\alpha)\Gamma(\beta)}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\times(\gamma_T)^{n+p}G_{4,8}^{7,1}\left[A\gamma_T\,\middle|\,\begin{matrix}1,\frac{\xi^2+1}{2},\frac{\xi^2+2}{2},-n-p+1\\ -n-p,\frac{\xi^2}{2},\frac{\xi^2+1}{2},\frac{\alpha}{2},\frac{\alpha+1}{2},\frac{\beta}{2},\frac{\beta+1}{2},0\end{matrix}\right]. \tag{65}$$

$$\overline{BER}_I = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n+p}}{2n!\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{1}{4\pi\Gamma(\alpha)\Gamma(\beta)} \cdot \left(\frac{A^2}{\mu_{IM/DD}}\right)^{\frac{(\alpha+\beta)}{4}} \times \int_{\gamma_T}^{\infty} (\gamma)^{\frac{(\alpha+\beta)}{4}+p-1} G_{1,5}^{4,1}\left[A\gamma \left| \begin{matrix} 1-\frac{(\alpha+\beta)}{4} \\ \frac{(\alpha-\beta)}{4}, \frac{(\alpha-\beta)}{4}+\frac{1}{2}, -\frac{(\alpha-\beta)}{4}, -\frac{(\alpha-\beta)}{4}+\frac{1}{2}, -\frac{(\alpha+\beta)}{4} \end{matrix}\right.\right] d\gamma. \quad (66)$$

$$\overline{BER}_I = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n+p}}{2n!\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{1}{4\pi\Gamma(\alpha)\Gamma(\beta)} \cdot \left(\frac{A^2}{\mu_{IM/DD}}\right)^{\frac{(\alpha+\beta)}{4}} \times (\gamma_T)^{\frac{(\alpha+\beta)}{4}+p} G_{3,6}^{5,1}\left[A\gamma_T \left| \begin{matrix} 1-\frac{(\alpha+\beta)}{4}, -\frac{(\alpha+\beta)}{4}-p+1 \\ -\frac{(\alpha+\beta)}{4}-p, \frac{(\alpha-\beta)}{4}, \frac{(\alpha-\beta)}{4}+\frac{1}{2}, -\frac{(\alpha-\beta)}{4}, -\frac{(\alpha-\beta)}{4}+\frac{1}{2}, -\frac{(\alpha+\beta)}{4} \end{matrix}\right.\right]. \quad (67)$$

Then as shown in Appendix III, the closed-form expression of (70) can be obtained as

$$F_\gamma(\gamma) = \frac{1}{\Gamma(k)}\left[\left\{\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma\right\}^{\xi^2}\Gamma\left(k-\xi^2, \frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma\right) + \Gamma(k) - \Gamma\left(k, \frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma\right)\right]. \quad (71)$$

For $\xi^2 \gg 1$ case, we can assume that $f_I(I)$ becomes

$$f_I(I) = \frac{\theta^{-k}}{\Gamma(K)} I^{k-1} \exp\left(-\frac{I}{\theta}\right). \quad (72)$$

Thus, with (72), by applying the RV transformation similar to the Gamma-Gamma case, $f_\gamma(\gamma)$ can be written as

$$f_\gamma(\gamma) = \frac{A^k}{\Gamma(K)} \gamma^{k-1} \exp(-A\cdot\gamma), \quad (73)$$

where $A = \frac{A_0}{\mu_{HD}}$. Then, with (73), the CDF expression of γ can be written as

$$F_\gamma(\gamma) = \frac{A^k}{\Gamma(K)} \int_0^\gamma x^{k-1} \exp(-A\cdot x)\, dx. \quad (74)$$

Here, by applying [37, eq. (3.381.1)] to (74), we can obtain the closed-form expression of (74) as

$$F_\gamma(\gamma) = \frac{\gamma(K, A\cdot\gamma)}{\Gamma(K)} = P(K, A\cdot\gamma), \quad (75)$$

where $P(a,z)$ is a regularized Gamma function. By adapting (71) and (75) to (3), (6), and (8), the closed-form expressions of ANSB and ASE over Gamma-Gamma turbulence channels with IM/DD can also be directly obtained.

B. CLOSED-FORM AVERAGE BER FOR AM

Similar to Sec. IV, we need to derive the closed-form expressions for both (12) and (13) over the Gamma distribution as an approximation of the Gamma-Gamma PDF with the HD technique. Here, in both (12) and (13), the closed-form expressions of $F_\gamma(\cdot)$ are given in (71) and (75). Therefore, we now focus on the derivation of the closed-form expressions of the integral terms in both (12) and (13). With the derived results in (69), (12) can be rewritten as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{\xi^2}{\Gamma(k)} \cdot (A')^{\xi^2} \times \int_{\gamma_{T_n}}^{\infty} \gamma^{l+\xi^2-1} \Gamma\left(k-\xi^2, A'\gamma\right) d\gamma, \quad (76)$$

where $A' = \frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}$.

Then, with (76), by applying Theorem 2 in Appendix II and then some algebraic simplifications, we can obtain a closed-form expression of (76) as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{\xi^2}{\Gamma(k)} \cdot \frac{(A')^{-l}}{l+\xi^2} \times \left[\Gamma\left(l+k, A'\gamma_{T_n}\right) - \left(A'\gamma_{T_n}\right)^{l+\xi^2} \Gamma\left(k-\xi^2, A'\gamma_{T_n}\right)\right]. \quad (77)$$

For $\overline{BER}_{n,II}$, similar to previous cases, by replacing γ_{T_n} in (79) with $\gamma_{T_{n+1}}$, the closed-form expression can be obtained directly.

For $\xi^2 \gg 1$, with (73), $\overline{BER}_{n,I}$ can be written as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{A^k}{\Gamma(K)} \times \int_{\gamma_{T_n}}^{\infty} \gamma^{k-1} \exp(-A\cdot\gamma)\, d\gamma. \quad (78)$$

Then, with the help of [37, eq. (3.381.3)], the closed-form expression of (76) can be obtained as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{\Gamma\left(K, A\cdot\gamma_{T_n}\right)}{\Gamma(K)}, \quad (79)$$

where $\frac{\Gamma(K, A\cdot\gamma_{T_n})}{\Gamma(K)} = 1 - P(a,z)$.

C. CLOSED-FORM AVERAGE BER FOR NON-AM

In this case, we need to derive the closed-form expression of (22) and (24) over the Gamma distribution as an approximation of the Gamma-Gamma PDF with the HD technique. For the closed-form derivation of (22), by substituting (71) into (22), we can rewrite (22) as

$$\begin{aligned}\overline{BER_I} = &\frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{1}{\Gamma(k)} \\ &\times\left[A^{\xi^2}\int_{\gamma_T}^{\infty} \gamma^{\xi^2+p-1}\exp(-q\gamma)\,\Gamma\left(k-v\xi^2, A\gamma\right)d\gamma\right. \\ &+\Gamma(k)\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)\,d\gamma \\ &\left.-\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)\,\Gamma(k, A\gamma)\,d\gamma\right], \quad (80)\end{aligned}$$

where $A = \frac{\xi^2}{(1+\xi^2)\theta\mu_{HD}}$.

Here, for the second integral term in (80), with the help of [37, eq. (3.381.3)], the closed-form expression can be obtained as

$$\Gamma(k)\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)\,d\gamma = \Gamma(k)\,q^{-p}\,\Gamma(p, q\gamma_T). \quad (81)$$

To the first and third integral terms, for mathematical convenience, we apply $\exp(-q\gamma) = \sum_{n=0}^{\infty}\frac{(-q\gamma)^n}{n!}$. Then, the first and third integral terms can respectively be rewritten as

$$\begin{aligned}&A^{\xi^2}\int_{\gamma_T}^{\infty}\gamma^{\xi^2+p-1}\exp(-q\gamma)\,\Gamma\left(k-\xi^2, A\gamma\right)d\gamma \\ &\quad= \sum_{n=0}^{\infty}\frac{(-q)^n A^{\xi^2}}{n!}\int_{\gamma_T}^{\infty}\gamma^{n+\xi^2+p-1}\Gamma\left(k-\xi^2, A\gamma\right)d\gamma, \end{aligned} \quad (82)$$

and

$$\begin{aligned}&\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)\,\Gamma(k, A\gamma)\,d\gamma \\ &\quad= \sum_{n=0}^{\infty}\frac{(-q)^n}{n!}\int_{\gamma_T}^{\infty}\gamma^{n+p-1}\Gamma(k, A\gamma)\,d\gamma. \quad (83)\end{aligned}$$

Therefore, by applying Theorem 2, we can obtain the closed-form expressions of (82) and (83), respectively as

$$\begin{aligned}&\sum_{n=0}^{\infty}\frac{(-q)^n}{n!A^{n+p}} \\ &\cdot\frac{\left\{\Gamma(n+p+k, A\gamma_T) - (A\gamma_T)^{\xi^2+n+p}\Gamma\left(k-\xi^2, A\gamma_T\right)\right\}}{\xi^2+n+p},\end{aligned} \quad (84)$$

and

$$\sum_{n=0}^{\infty}\frac{(-q)^n}{n!A^{n+p}} \cdot \frac{\left\{\Gamma(n+p+k, A\gamma_T) - (A\gamma_T)^{n+p}\Gamma(k, A\gamma_T)\right\}}{n+p}. \quad (85)$$

Similarly, for $\xi^2 \gg 1$, $\overline{BER_I}$ can be written with (75) as

$$\begin{aligned}\overline{BER_I} = &\frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{1}{\Gamma(K)} \\ &\times\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\cdot\gamma)\,\gamma(K, A\gamma)\,d\gamma. \quad (86)\end{aligned}$$

Here, for mathematical convenience, by using the alternate representation of $\gamma(\cdot,\cdot)$ and an alternate definition of the exponential function based on a Taylor series, we can rewrite the integral part in (86) as

$$\begin{aligned}&\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\cdot\gamma)\,\gamma(K, A\gamma)\,d\gamma \\ &\quad= \Gamma(K)\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\cdot\gamma)\,d\gamma \\ &\qquad-\sum_{n=0}^{\infty}\frac{(-q)^n}{n!}\int_{\gamma_T}^{\infty}\gamma^{n+p-1}\Gamma(K, A\gamma)\,d\gamma. \quad (87)\end{aligned}$$

Then, with the help of [37, eq. (3.381.3)] and Theorem 2, the closed-form result can be finally obtained as

$$\begin{aligned}\overline{BER_I} = &\frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \cdot \frac{1}{\Gamma(K)} \\ &\times\left[\Gamma(K)\,q^{-p}\,\Gamma(p, q\gamma_T) - \sum_{n=0}^{\infty}\frac{(-q)^n}{n!}\left(\frac{1}{A}\right)^{n+p}\right. \\ &\left.\frac{\left(\Gamma(n+p+K, A\gamma_T) - (A\gamma_T)^{n+p}\Gamma(K, A\gamma_T)\right)}{n+p}\right].\end{aligned} \quad (88)$$

For the closed-form expression of $\overline{BER_{II}}$, we can directly obtain a closed-form result over the Gamma distribution with the HD technique by substituting the closed-form results given in (71) and (75) into (25).

VII. STATISTICAL ANALYSIS BASED ON IM/DD OVER GAMMA DISTRIBUTION AS AN APPROXIMATION OF THE GAMMA-GAMMA DISTRIBUTION

A. CLOSED-FORM STATISTICAL CHARACTERISTICS

In this case, by adopting (53) and (54) with (68), the resulting PDF of the electrical SNR under the IM/DD technique can be given as

$$\begin{aligned}f_\gamma(\gamma) = &\frac{\xi^2}{\Gamma(k)}\left\{\frac{\xi^2}{\theta\left(1+\xi^2\right)}\right\}^{\xi^2}\frac{1}{2\mu_{IM/DD}}\left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2-2} \\ &\times\Gamma\left(k-\xi^2, \frac{\xi^2}{\theta\left(1+\xi^2\right)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) \quad (89)\end{aligned}$$

Then, with the closed-form results in (89), the CDF of γ can be written as

$$F_\gamma(\gamma) = \frac{\xi^2}{\Gamma(k)}\left\{\frac{\xi^2}{\theta(1+\xi^2)}\right\}^{\xi^2}\frac{1}{2\mu_{IM/DD}}\times\int_0^\gamma\left(\sqrt{\frac{x}{\mu_{IM/DD}}}\right)^{\xi^2-2} \times\Gamma\left(k-\xi^2,\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{x}{\mu_{IM/DD}}}\right)dx. \quad (90)$$

Then, as shown in Appendix IV, we can obtain the closed-form expression of (90) as

$$F_\gamma(\gamma) = \frac{1}{\Gamma(k)}\left[\left(\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2} \Gamma\left(k-\xi^2,\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) +\Gamma(k)-\Gamma\left(k,\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)\right]. \quad (91)$$

For $\xi^2 \gg 1$, similarly, by applying the random variable transformation with (72), the PDF expression can be written as

$$f_\gamma(\gamma) = \frac{A^k}{2\cdot\Gamma(K)}\left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{k-2}\exp\left(-A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right), \quad (92)$$

where $A = \frac{A_0}{\theta}$. Then, with (92), the CDF of γ can be expressed as

$$F_\gamma(\gamma) = \int_0^\gamma f_\gamma(x)\,dx = \frac{A^k}{2\cdot\Gamma(K)}\int_0^\gamma\left(\sqrt{\frac{x}{\mu_{IM/DD}}}\right)^{k-2}\exp\left(-A\sqrt{\frac{x}{\mu_{IM/DD}}}\right)dx. \quad (93)$$

Here, let $\sqrt{\frac{x}{\mu_{IM/DD}}} = y$. Then, $x = \mu_{IM/DD}y^2$, $dx = 2\mu_{IM/DD}ydy$, and the valid integral region of y becomes $0 < y < \sqrt{\frac{\gamma}{\mu_{IM/DD}}}$. Therefore, by applying [37, eq. (3.381.1)] and then some algebraic simplifications, we can obtain the closed-form result of (93) as

$$F_\gamma(\gamma) = \frac{\mu_{IM/DD}}{\Gamma(K)}\gamma\left(K, A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right), \quad (94)$$

where $\gamma(\cdot)$ is the incomplete Gamma function. Then, similarly, with (91) and (94), the closed-form expressions of ANSB and ASE over Gamma-Gamma turbulence channels with IM/DD can also be directly obtained.

B. CLOSED-FORM AVERAGE BER FOR AM

Similarly, we focus on the derivation of the closed-form expressions of both (12) and (13) over Gamma-Gamma turbulence channels with IM/DD. First, with the derived results in (89), (12) can be rewritten as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\cdot\frac{\xi^2}{\Gamma(k)}\cdot\frac{(A')^{\xi^2}}{2\mu_{IM/DD}} \times\int_{\gamma_{T_n}}^{\infty}\gamma^l\left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2-2}\Gamma\left(k-\xi^2,A'\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)d\gamma, \quad (95)$$

where $A' = \frac{\xi^2}{\theta(1+\xi^2)}$.

Here, let $A'\sqrt{\frac{\gamma}{\mu_{IM/DD}}} = y$. Then, $\sqrt{\frac{\gamma}{\mu_{IM/DD}}} = \frac{y}{A'}$, $\gamma = \frac{\mu_{IM/DD}}{A'^2}y^2$, $d\gamma = \frac{\mu_{IM/DD}}{A'^2}2ydy$, and the integral region of y becomes $A'\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}} < y < \infty$. Therefore, the integral term in (95) can be rewritten as

$$\int_{\gamma_{T_n}}^{\infty}\gamma^l\left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2-2}\Gamma\left(k-\xi^2,A'\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)d\gamma = \frac{2\left(\mu_{IM/DD}\right)^{l+1}}{(A')^{2l+\xi^2}}\int_{A'\sqrt{\frac{\gamma}{\mu_{IM/DD}}}}^{\infty}y^{2l+\xi^2-1}\Gamma\left(k-\xi^2,y\right)dy. \quad (96)$$

In (96), with the help of Theorem 2, the closed-form expression of (96) can be obtained as

$$\frac{2\left(\mu_{IM/DD}\right)^{l+1}}{(A')^{2l+\xi^2}}\cdot\frac{1}{2l+\xi^2}\left[\Gamma\left(2l+k,A'\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}\right) -\left(A'\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}\right)^{2l+\xi^2}\Gamma\left(k-\xi^2,A'\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}\right)\right]. \quad (97)$$

As a result, the closed-form result of (95) can be obtained as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\cdot\frac{\xi^2}{\Gamma(k)}\cdot\frac{\left(\mu_{IM/DD}\right)^l}{(A')^{2l}\left(2l+\xi^2\right)} \times\left[\Gamma\left(2l+k,A'\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}\right) -\left(A'\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}\right)^{2l+\xi^2}\Gamma\left(k-\xi^2,A'\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}\right)\right]. \quad (98)$$

For $\xi^2 \gg 1$, with (92), $\overline{BER}_{n,I}$ can be written as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty}\frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!}\cdot\frac{1}{1-F_\gamma(\gamma_T)}\cdot\frac{A^K}{2\cdot\Gamma(K)}$$

$$\times \int_{\gamma_{T_n}}^{\infty} \left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{K-2} \exp\left(-A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma. \tag{99}$$

Here, let $\sqrt{\frac{\gamma}{\mu_{IM/DD}}} = y$. Then, $x = \mu_{IM/DD}y^2$, $dx = 2\mu_{IM/DD}ydy$, and the valid integral region becomes $\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}} < y < \infty$. Therefore, the integral part in (99) can be rewritten as

$$\int_{\gamma_{T_n}}^{\infty} \left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{K-2} \exp\left(-A \cdot \sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma$$
$$= \mu_{IM/DD} \int_{\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}}^{\infty} y^{K-1} \exp\left(-A \cdot y\right) dy. \tag{100}$$

Then, with the help of [37, eq. (3.381.3)], the closed-form expression of (100) can be obtained as

$$\mu_{IM/DD} \int_{\sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}}^{\infty} y^{K-1} \exp\left(-A \cdot y\right) dy$$
$$= \mu_{IM/DD}A^{-K}\Gamma\left(K, A \cdot \sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}\right). \tag{101}$$

After substituting (101) into (99) and then some algebraic manipulations similar to (79), we can finally obtain the closed-form result of (99) as

$$\overline{BER}_{n,I} = \sum_{l=0}^{\infty} \frac{a_n\left(-\frac{b_n}{M_n}\right)^l}{l!} \cdot \frac{\mu_{IM/DD}}{2} \cdot \frac{1-P\left(K, A \cdot \sqrt{\frac{\gamma_{T_n}}{\mu_{IM/DD}}}\right)}{1-F_{\gamma}(\gamma_T)}. \tag{102}$$

In (98) and (102), the closed-form expressions of $F_{\gamma}(\cdot)$ are given in (91) and (94), respectively. Similarly, for $\overline{BER}_{n,II}$, by replacing γ_{T_n} in (98) and (102) with $\gamma_{T_{n+1}}$, the closed-form expressions can be obtained directly.

C. CLOSED-FORM AVERAGE BER FOR NON-AM

Similar to the Gamma-Gamma case, we need to derive the closed-form expression of both (22) and (24) over the Gamma distribution as an approximation of the Gamma-Gamma PDF with the IM/DD technique. For the closed-form derivation of (22), by substituting (91) into (22), we can rewrite (22) as

$$\begin{aligned}
&\overline{BER_I} \\
&= \frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1-F_{\gamma}(\gamma_T)} \cdot \frac{1}{\Gamma(k)} \\
&\quad \times \Bigg[A^{\xi^2} \int_{\gamma_T}^{\infty} \gamma^{p-1} \left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2} \\
&\quad\quad \times \exp(-q\gamma)\Gamma\left(k-\xi^2, A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma \\
&\quad\quad +\Gamma(k) \int_{\gamma_T}^{\infty} \gamma^{p-1} \exp(-q\gamma)\, d\gamma \\
&\quad\quad - \int_{\gamma_T}^{\infty} \gamma^{p-1} \exp(-q\gamma)\, \Gamma\left(k, A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma \Bigg],
\end{aligned} \tag{103}$$

where $A = \frac{\xi^2}{(1+\xi^2)\theta}$. Then, as shown in Appendix V, the closed-form expression of $\overline{BER_I}$ can be finally obtained as

$$\begin{aligned}
&\overline{BER_I} \\
&= \frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1-F_{\gamma}(\gamma_T)} \cdot \frac{1}{\Gamma(k)} \\
&\quad \times \Bigg[\sum_{n=0}^{\infty} \frac{2(-q)^n \left(\mu_{IM/DD}\right)^{n+p}}{n!A^{2(n+p)}\left(\xi^2+2(n+p)\right)} \\
&\quad \cdot \Bigg\{\Gamma\left(2(n+p)+k, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right) \\
&\quad - \left(A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right)^{\xi^2+2(n+p)} \Gamma\left(k-\xi^2, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right)\Bigg\} \\
&\quad + \Gamma(k)\, q^{-p}\Gamma(p, q\gamma_T) \\
&\quad - \sum_{n=0}^{\infty} \frac{(-q)^n \left(\mu_{IM/DD}\right)^{n+p}}{n!A^{2(n+p)}(n+p)} \cdot \Bigg\{\Gamma\left(2(n+p)+k, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right) \\
&\quad - \left(A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right)^{2(n+p)} \Gamma\left(k, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right)\Bigg\}\Bigg].
\end{aligned} \tag{104}$$

For $\xi^2 \gg 1$, after applying a similar approach as that used in the HD case with (94), $\overline{BER_I}$ can be written as

$$\begin{aligned}
\overline{BER_I} &= \frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1-F_{\gamma}(\gamma_T)} \cdot \frac{\mu_{IM/DD}}{\Gamma(K)} \\
&\quad \times \Bigg[\Gamma(K) \int_{\gamma_T}^{\infty} \gamma^{p-1} \exp(-q \cdot \gamma)\, d\gamma \\
&\quad\quad - \sum_{n=0}^{\infty} \frac{(-q)^n}{n!} \int_{\gamma_T}^{\infty} \gamma^{n+p-1} \gamma\left(K, A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma\Bigg].
\end{aligned} \tag{105}$$

Then, with the help of [37, eq. (3.381.3)] and Theorem 2, the final closed-form result can be obtained as

$$\begin{aligned}
&\overline{BER_I} \\
&= \frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1-F_{\gamma}(\gamma_T)} \cdot \frac{\mu_{IM/DD}}{\Gamma(K)} \\
&\quad \times \Bigg[\Gamma(K)\, q^{-p}\Gamma(p, q\gamma_T) - \sum_{n=0}^{\infty} \frac{2(-q)^n \left(\mu_{IM/DD}\right)^{n+p}}{n!A^{2(n+p)}} \\
&\quad\quad \times \frac{1}{2(n+p)} \Bigg\{\Gamma\left(2(n+p)+K, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right) \\
&\quad\quad - \left(A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right)^{2(n+p)} \Gamma\left(K, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right)\Bigg\}\Bigg].
\end{aligned} \tag{106}$$

Similarly, the closed-form results of $\overline{BER_{II}}$ can be obtained simply with (91) and (94).

VIII. NUMERICAL RESULTS

In this section, we present some selected results. Our main concern which is improvement in ASE in terms of the average number of selected beams, the ASE of these selected beams, and the average BER of the selected beams. We verified these results via computer-based Monte-Carlo simulations, we evaluated the analytical expressions presented in the previous sections numerically and show them under both varying turbulent FSO channel conditions and pointing errors. In the following figures, the marker is given for the analytical results and the line is given for the simulation results while each item defined in the legend represents both simulation and analytical results for each case.

The FSO link is modeled as a Gamma-Gamma turbulence channel based on the FSO channels in [38]. Specifically, we vary the fading/scintillation parameters (i.e., α and β, respectively) and the pointing error parameters (i.e., ξ). We consider strong turbulence (i.e., $\alpha = 2.064$ and $\beta = 1.342$), weak turbulence (i.e., $\alpha = 2.902$ and $\beta = 2.51$), strong pointing error (i.e., $\xi = 1$), and weak pointing error (i.e., $\xi = 8$).

For selected beams, a rate-adaptive N multidimensional trellis coded M-QAM with AWGN channels is employed [30].[4] More specifically, the constellation size, M_n, for $N = 8$ is restricted ($M_n = 2^{n+1}$) with spectral efficiency R_n ($R_n = n + 0.5$) [bits/s/Hz] for $n = 1, 2, \ldots, 8$, which is transmitted with the target BER (i.e., $\overline{BER_0} = 10^{-3}$).

Figs. 1-a and b and Figs. 2-a and b show the average number of selected beams and the ASE of selected beams for FSO with TPMBS under HD and IM/DD techniques with varying turbulence and pointing errors, respectively. Expectedly, as $\overline{\gamma}$ increases, the number of beams with acceptable performance gradually increases and eventually converges to the number of all beams (i.e., $L = 5$), which directly leads to improved ASE. More specifically, for the AM case, the ASE will eventually converge to $L \cdot R_N$, where $L = 5$ and $R_8 = 8.5$ as $\overline{\gamma}$ increases, whereas for the non-AM (i.e., coherentnon-coherent binary modulation) cases, ASE eventually converges to $L \cdot R$, where $L = 5$ and $R = 1$ as $\overline{\gamma}$ increases. In addition, as the effect of the pointing error or the effects of the atmosphere decrease (i.e., as the value of ξ increases or as the effect of turbulence decreases, respectively), the system performance improves due to the number of selected beams. Further, ASE of both AM and non-AM cases increase, but the AM case can provide a relatively better ASE performance compared with the non-AM case due to the nature characteristics of the AM scheme. Further, similar to conventional FSO systems, the TPMBS scheme under the HD technique slightly outperforms that under the IM/DD technique.

Figs. 1-c and 2-c present the average BER for FSO with TPMBS under HD and IM/DD techniques, respectively. Although AM cases under both HD and IM/DD techniques provide better ASE performance in Fig. 1 b and Fig. 2 b, the average BER performance still satisfies the BER requirement even under strong pointing error and strong turbulence conditions (i.e., the average BER is still below our target BER (i.e., $\overline{BER_0} = 10^{-3}$)). Further, similar to conventional FSO systems, the TPMBS scheme under the HD technique slightly outperforms that under the IM/DD technique. For non-AM cases, similarly to conventional digital communications systems, the coherent detection PSK scheme performs the best while the coherent detection PSK scheme provides the worst performance although the AM case provides the required BER performance. As a result, we confirm that the proposed scheme, especially with AM, can provide improved ASE while meeting the required system quality. We note that there is some fluctuation in the average BER curves because of the discrete nature of AM.

Figs. 3 and 4 show ANSB, ASE, and average BER for FSO with TPMBS over the Gamma distribution as an approximation of the Gamma-Gamma distribution under HD and IM/DD techniques, respectively. As expected, ANSB, ASE, and average BER performance results under the approximate Gamma turbulence model assumptions roughly have similar behaviors as those obtained on the Gamma-Gamma turbulence channels in Figs. 1 and 2. More specifically, under the same turbulence and pointing error conditions, the approximate Gamma turbulence model provides almost the same performance, indicating the tightness of the approximation compared with that of the Gamma-Gamma turbulence model. Our derived results based on the Gamma distribution as an approximation of the Gamma-Gamma distribution can be used as approximated performance measure bounds, especially, because they may lead to lower bounds for the approximated performance measures.

In Fig. 5, the average BER performance comparisons based on the Gamma distribution and the Gamma-Gamma distribution under HD technique, especially over strong turbulence with strong pointing error. The result shows that under the worst conditions (i.e., strong turbulence with strong pointing error), the approximate Gamma turbulence model provides almost the same performance. With this sufficiently accurate approximation using the Gamma distribution, we can simplify the performance analysis as a function of tractable function (i.e., Gamma function) such as probability of outage, average BER performance and so on. Note that for AM case, there exists the performance gap between the Gamma distribution and the Gamma-Gamma distribution. In general, the performance gap between Gamma distri-

[4]Here, M-QAM, commonly used in wireless communications, requires the DC bias to convert negative amplitudes to positive ones because the optical signal has to be non-negative [41]. Especially, we assume that a rate-adaptive N multidimensional trellis coded M-QAM is employed [28]. In [28], the constellation size, M_n, is restricted 2^{n+1} $(n = 1, 2, \ldots, N)$ for N different codes based on the QAM signal. In this case, rate adaptation is performed by dividing the SNR range into $N + 1$ turbulence regions, which are defined by the SNR thresholds, denoted by $0 < \gamma_{T_1} < \gamma_{T_2} < \cdots < \gamma_{T_N} < \gamma_{T_{N+1}} = \infty$. When the estimated SNR of the scheduled beam is in the n-th region $\left(i.e., \gamma_{T_n} \leq \gamma < \gamma_{T_{n+1}}\right)$, the constellation size, M_n, with spectral efficiency, R_n, is transmitted. The lower SNR boundary, γ_{T_n}, of each turbulence region is set to the lowest SNR required to achieve the predefined target BER_0.

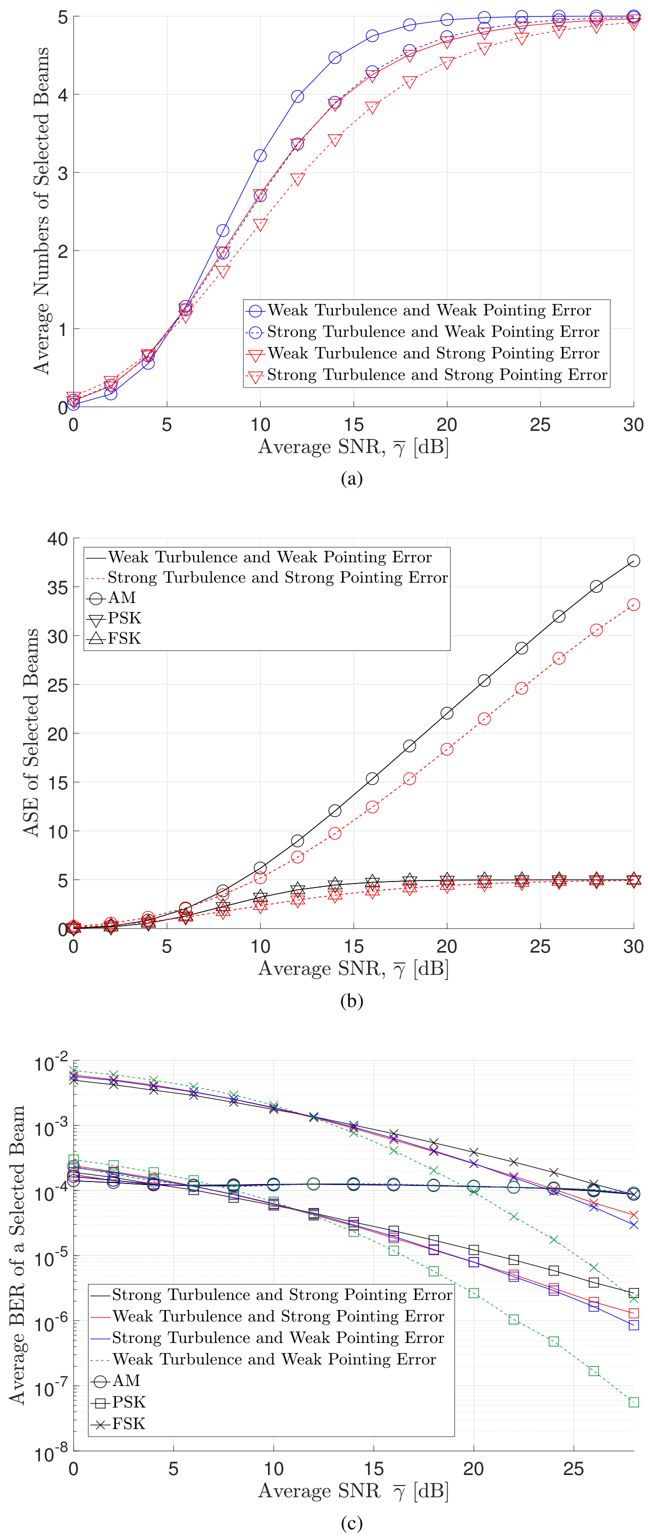

FIGURE 1. Performance of FSO under HD with $L = 5$, $\gamma_T = 7.1$ dB, and $\overline{BER}_0 = 10^{-3}$ with Gamma-Gamma turbulence conditions. (a) Average number of selected beams. (b) Average spectral efficiency of selected beams. (c) Average BER of a selected beam.

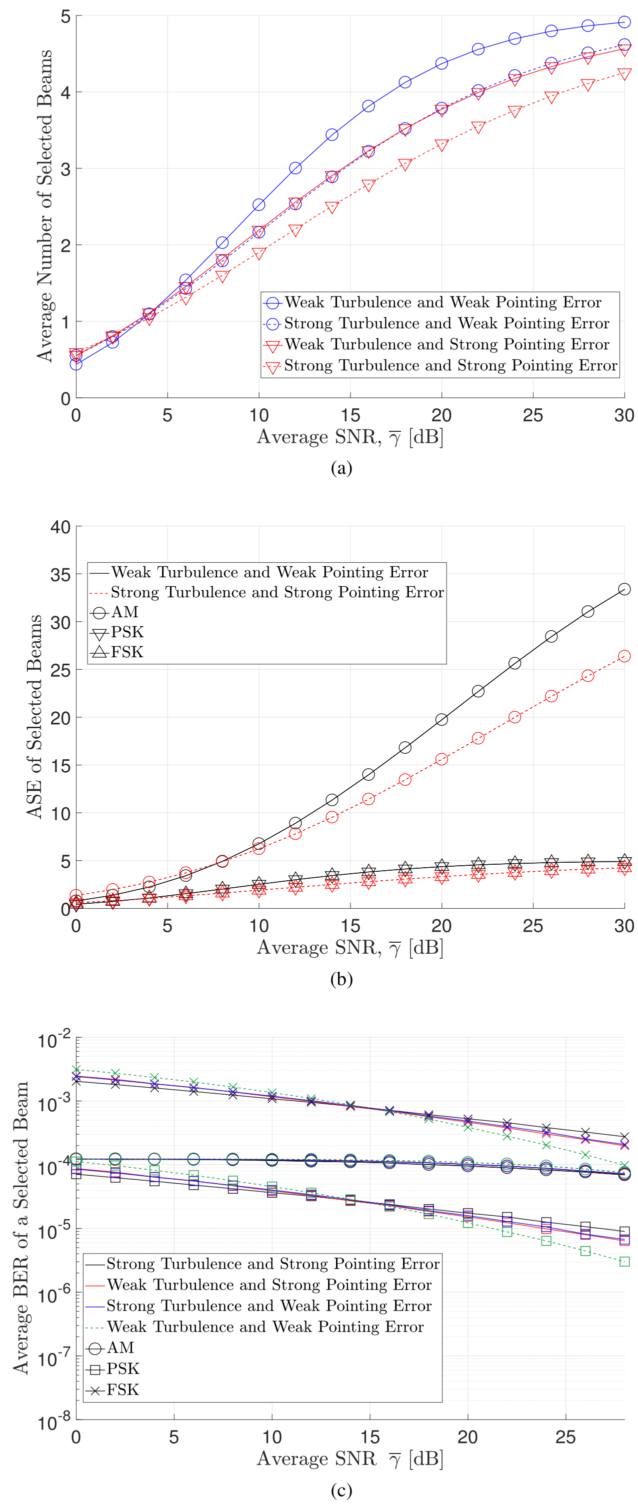

FIGURE 2. Performance of FSO under IM/DD with $L = 5$, $\gamma_T = 7.1$ dB, and $\overline{BER}_0 = 10^{-3}$ with Gamma-Gamma turbulence conditions. (a) Average number of selected beams. (b) Average spectral efficiency of selected beams. (c) Average BER of a selected beam with.

bution and the Gamma-Gamma distribution increases as the average SNR increases but for the AM case, this gap seems to be relatively more severe because the combined effect of the discrete nature of the adaptive modulation and the equivalent threshold is changing with the value of the average SNR. However, these results still satisfy the target BER, $\overline{BER}_0 = 10^{-3}$.

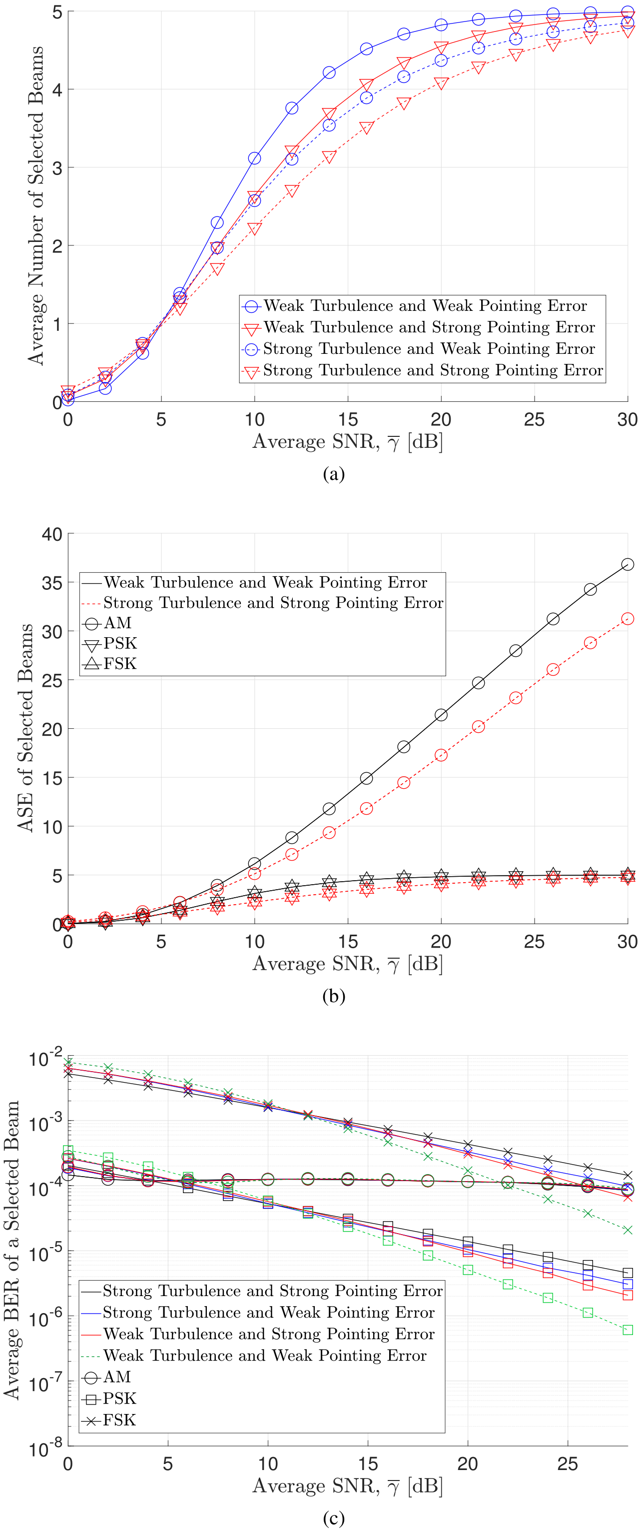

FIGURE 3. **Performance of FSO under HD with $L = 5$, $\gamma_T = 7.1$ dB, and $BER_0 = 10^{-3}$ based on the Gamma distribution as an approximation of the Gamma-Gamma distribution. (a) Average number of selected beams. (b) Average spectral efficiency of selected beams. (c) Average BER of a selected beam.**

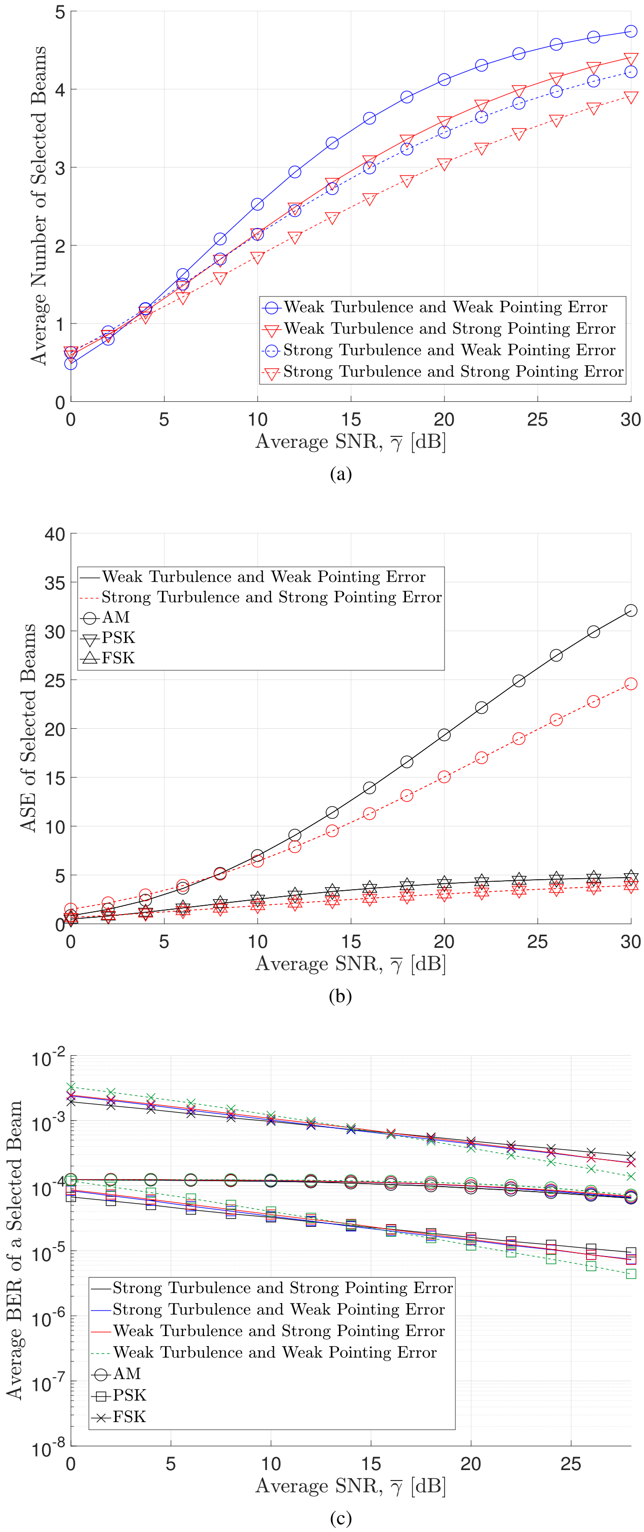

FIGURE 4. **Performance of FSO under HD with $L = 5$, $\gamma_T = 7.1$ dB, and $BER_0 = 10^{-3}$ based on the Gamma distribution as an approximation of the Gamma-Gamma distribution. (a) Average number of selected beams. (b) Average spectral efficiency of selected beams. Average BER of a selected beam.**

We employed a conventional AM [28] with $N = 8$ and investigated ASE and average BER to observe the trend of these performance measures according to changes in the environment (i.e., atmospheric turbulence and pointing errors). Here, the values of R_n and M_n for ASE and the values of a_n and b_n for average BER are improper, especially for

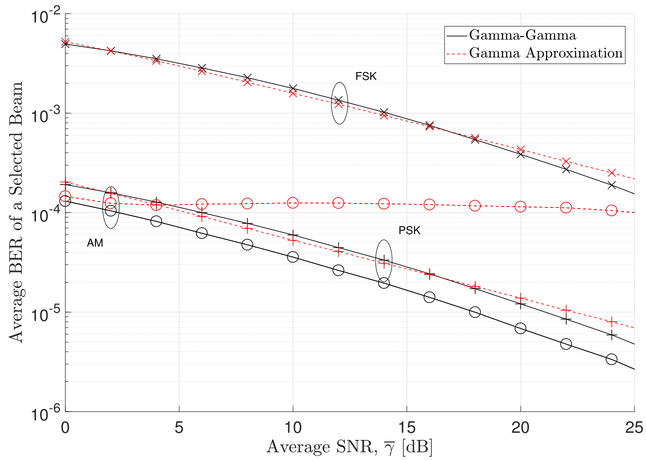

FIGURE 5. Average BER comparison between Gamma-Gamma and Gamma approximation over strong turbulence with strong pointing error under HD with $L = 5$, $\gamma_T = 7.1$ dB, and $\overline{BER_0} = 10^{-3}$.

FSO-based systems. However, the closed-form expressions of the performance measures derived in this paper are still valid if we apply the proper values of R_n, M_n, a_n and b_n. To take full advantage of our results, the research on AM schemes suitable for FSO needs to continue, especially with high average SNR.

IX. CONCLUSIONS AND DISCUSSIONS

In this paper, we statistically analyzed the performance of a threshold-based parallel multiple beam selection scheme for WDM FSO systems in cases where a pointing error occurred under Gamma-Gamma turbulence conditions. Further, to simplify the mathematical analysis, we also considered Gamma turbulence conditions, which are a good approximation of the Gamma-Gamma distribution. Specifically, we statistically analyzed the characteristics of conventional detection schemes (i.e., HD and IM/DD techniques) for both AM and non-AM cases (i.e., coherent/non-coherent binary modulation). Based on the statistically derived results for both the Gamma-Gamma turbulence model and the approximate Gamma turbulence model, we evaluated the outage probability (CDF) of a selected beam, the average spectral efficiency, the average number of selected beams, and the average bit error rate. We confirm that higher spectral efficiency and reduced complexity of implementation are possible. Especially, for the AM case compared to the non-AM case, the relatively better ASE performance can be obtained with TPMBS based WDM FSO system due to the nature characteristics of AM scheme while the minimum average BER requirement is satisfied. Further, the performance results under both Gamma-Gamma turbulence and approximate Gamma turbulence have a similar behavior, especially, provide almost the same performance results. Note that our derived results based on the Gamma distribution as an approximation of the Gamma-Gamma distribution can still be used as approximated performance measure bounds, especially, they may lead to lower bounds on the approximated considered performance measures.

APPENDIX I: THEOREM 1

Theorem 1:

$$\int_u^{\infty} x^{\upsilon} G_{k,l}^{m,n}\left[\mu x\middle|\begin{matrix}\mathcal{A}_k\\ \mathcal{B}_l\end{matrix}\right]dx = u^{\upsilon+1} G_{k+1,l+1}^{m+1,n}\left[\mu u\middle|\begin{matrix}\mathcal{A}_k,-\upsilon\\ -\upsilon-1,\mathcal{B}_l\end{matrix}\right]. \tag{107}$$

Proof: With the integral expression on the left side of (107), let $\frac{x}{u} = y$. Then, $x = uy$ and $dx = udy$. Thus, the integral expression on the left side of (107) can be rewritten as

$$\int_u^{\infty} x^{\upsilon} G_{k,l}^{m,n}\left[\mu x\middle|\begin{matrix}\mathcal{A}_k\\ \mathcal{B}_l\end{matrix}\right]dx = u^{\upsilon+1}\int_1^{\infty} y^{\upsilon} G_{k,l}^{m,n}\left[\mu uy\middle|\begin{matrix}\mathcal{A}_k\\ \mathcal{B}_l\end{matrix}\right]dy. \tag{108}$$

Then, with the help of the Erdelyi-Kober operator, the closed-form expression of this Euler-type integral of Meijer G function can be obtained as

$$\begin{aligned}&\int_1^{\infty} x^{-\alpha}(x-1)^{\alpha-\beta-1} G_{k,l}^{m,n}\left[ax\,\middle|\,\begin{matrix}\mathcal{A}_k\\ \mathcal{B}_l\end{matrix}\right]dx\\ &\quad = \Gamma(\alpha-\beta)\, G_{k+1,l+1}^{m+1,n}\left[a\middle|\begin{matrix}\mathcal{A}_k,\alpha\\ \beta,\mathcal{B}_l\end{matrix}\right].\end{aligned} \tag{109}$$

Therefore, by adopting (109) with $\alpha = -\upsilon$, $\beta = -\upsilon - 1$, and $a = \mu u$, we can obtain the closed-form expression of the integral expression on the left side of (107) as

$$\begin{aligned}&u^{\upsilon+1}\Gamma(-\upsilon-(-\upsilon-1))\, G_{k+1,l+1}^{m+1,n}\left[\mu x\middle|\begin{matrix}\mathcal{A}_k,-\upsilon\\ -\upsilon-1,\mathcal{B}_l\end{matrix}\right]\\ &\quad = u^{\upsilon+1} G_{k+1,l+1}^{m+1,n}\left[\mu u\middle|\begin{matrix}\mathcal{A}_k,-\upsilon\\ -\upsilon-1,\mathcal{B}_l\end{matrix}\right].\end{aligned} \tag{110}$$

■

In (40), the closed-form expression of the integral term can be obtained with the help of Theorem 1. More specifically, with (40), let $\frac{\gamma}{\gamma_{T_n}} = x$. Then, $\gamma = \gamma_{T_n} x$ and $d\gamma = \gamma_{T_n} dx$. Then, the integral term (40) can be rewritten as

$$\begin{aligned}&\int_{\gamma_{T_n}}^{\infty} \gamma^{l-1} G_{1,3}^{3,0}\left[A'\gamma\middle|\begin{matrix}\xi^2+1\\ \xi^2,\alpha,\beta\end{matrix}\right]d\gamma\\ &\quad = \left(\gamma_{T_n}\right)^l \int_1^{\infty} x^{l-1} G_{1,3}^{3,0}\left[A'\gamma_{T_n}x\middle|\begin{matrix}\xi^2+1\\ \xi^2,\alpha,\beta\end{matrix}\right]dx.\end{aligned} \tag{111}$$

After that, the closed-form expression of this Euler-type integral for the Meijer G function can be obtained with the help of the Erdelyi-Kober operator as

$$\left(\gamma_{T_n}\right)^l G_{2,4}^{4,1}\left[A'\gamma_{T_n}\middle|\begin{matrix}\xi^2+1,-l+1\\ -l,\xi^2,\alpha,\beta\end{matrix}\right]. \tag{112}$$

APPENDIX II: THEOREM 2

Theorem 2:

$$\int_u^{\infty} x^{\nu-1}\Gamma(a,\mu x)\,dx=\left(\frac{1}{\mu}\right)^{\nu}\frac{\Gamma(\nu+a,\mu u)-(\mu u)^{\nu}\Gamma(a,\mu u)}{\nu},\quad \text{for } \mu>0 \text{ and } u>0. \tag{113}$$

Proof: With the integral expression in (113), let $\mu x = y$. Then, $x=\frac{y}{\mu}$, $dx=\frac{1}{\mu}dy$, and the integral region of y becomes $\mu u < y < \infty$. Therefore, the integral expression in (113) can be rewritten as

$$\int_u^{\infty} x^{\nu-1}\Gamma(a,\mu x)\,dx=\left(\frac{1}{\mu}\right)^{\nu}\int_{\mu u}^{\infty} y^{\nu-1}\Gamma(a,y)\,dy. \tag{114}$$

Then, by applying [40, eq. (06.06.21.0002.01)], we can obtain a closed-form expression as

$$\left(\frac{1}{\mu}\right)^{\nu}\frac{\Gamma(\nu+a,\mu u)-(\mu u)^{\nu}\Gamma(a,\mu u)}{\nu}. \tag{115}$$

■

APPENDIX III: DERIVATION OF (71)

In (70), let $\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}x=y$. Then, $x=\frac{\theta(1+\xi^2)\mu_{HD}}{\xi^2}y$, $dx=\frac{\theta(1+\xi^2)\mu_{HD}}{\xi^2}dy$, and $\gamma\rightarrow\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma$. Therefore, the integral part of (70) can be rewritten as

$$\int_0^{\gamma} x^{\xi^2-1}\Gamma\left(k-\xi^2,\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}x\right)dx =\left(\frac{\theta(1+\xi^2)\mu_{HD}}{\xi^2}\right)^{\xi^2}\int_0^{\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma} y^{\xi^2-1}\Gamma\left(k-\xi^2,y\right)dy. \tag{116}$$

Utilizing [40, eq. (06.06.21.0002.01)], the closed-form expression of (116) can be obtained as

$$\begin{aligned}&\left(\frac{\theta(1+\xi^2)\mu_{HD}}{\xi^2}\right)^{\xi^2}\int_0^{\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma} y^{\xi^2-1}\Gamma\left(k-\xi^2,y\right)dy\\ &=\left(\frac{\theta(1+\xi^2)\mu_{HD}}{\xi^2}\right)^{\xi^2}\frac{1}{\xi^2}\\ &\quad\times\left[\left\{\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma\right\}^{\xi^2}\Gamma\left(k-\xi^2,\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma\right)\right.\\ &\quad\left.+\Gamma(k)-\Gamma\left(k,\frac{\xi^2}{\theta(1+\xi^2)\mu_{HD}}\gamma\right)\right].\end{aligned} \tag{117}$$

Then, after substituting (117) into (70) and some mathematical manipulations, the closed-form expression of (70) can be obtained as shown in (71).

APPENDIX IV: DERIVATION OF (91)

In (90), let $\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{x}{\mu_{IM/DD}}}=y$. Then, $x=\mu_{IM/DD}\left\{\frac{\theta(1+\xi^2)}{\xi^2}\right\}y^2$, $dx=2\mu_{IM/DD}\left\{\frac{\theta(1+\xi^2)}{\xi^2}\right\}ydy$, and $\gamma\rightarrow\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}$. Therefore, the integral term in (90) can be rewritten as

$$\begin{aligned}&\int_0^{\gamma}\left(\sqrt{\frac{x}{\mu_{IM/DD}}}\right)^{\xi^2-2}\Gamma\left(k-\xi^2,\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{x}{\mu_{IM/DD}}}\right)dx\\ &=2\mu_{IM/DD}\left\{\frac{\theta(1+\xi^2)}{\xi^2}\right\}^{\xi^2}\\ &\quad\times\int_0^{\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}} y^{\xi^2-1}\Gamma\left(k-\xi^2,y\right)dy.\end{aligned} \tag{118}$$

Then, with the help of [40, eq. (06.06.21.0002.01)], the closed-form expression of (118) can be obtained as

$$\begin{aligned}&\int_0^{\gamma}\left(\sqrt{\frac{x}{\mu_{IM/DD}}}\right)^{\xi^2-2}\Gamma\left(k-\xi^2,\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{x}{\mu_{IM/DD}}}\right)dx\\ &=2\mu_{IM/DD}\left\{\frac{\theta(1+\xi^2)}{\xi^2}\right\}^{\xi^2}\\ &\quad\times\left[\frac{1}{\xi^2}\left\{\left(\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2}\right.\right.\\ &\quad\times\Gamma\left(k-\xi^2,\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)\\ &\quad\left.\left.-\Gamma\left(k,\frac{\xi^2}{\theta(1+\xi^2)}\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)\right\}-\frac{1}{\xi^2}\{0-\Gamma(k)\}\right].\end{aligned} \tag{119}$$

Finally, by substituting (119) into (90), we can obtain the closed-form expression of (90) as shown in (91).

APPENDIX V: DERIVATION OF (104)

In (103), for the second integral term, by applying similar approach used in HD case, we can obtain the closed-form result as

$$\Gamma(k)\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)\,d\gamma=\Gamma(k)\,q^{-p}\Gamma(p,q\gamma_T). \tag{120}$$

For the first and third integral terms in (103), we can re-write similar to the HD case, respectively, as

$$\begin{aligned}&A^{\xi^2}\int_{\gamma_T}^{\infty}\gamma^{p-1}\left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2}\\ &\quad\times\exp(-q\gamma)\,\Gamma\left(k-\xi^2,A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)d\gamma\\ &=\sum_{n=0}^{\infty}\frac{(-q)^n A^{\xi^2}}{n!}\int_{\gamma_T}^{\infty}\gamma^{n+p-1}\left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2}\\ &\quad\times\Gamma\left(k-\xi^2,A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)d\gamma,\end{aligned} \tag{121}$$

and

$$\int_{\gamma_T}^{\infty} \gamma^{p-1} \exp(-q\gamma)\, \Gamma\left(k, A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma = \sum_{n=0}^{\infty} \frac{(-q)^n}{n!} \int_{\gamma_T}^{\infty} \gamma^{n+p-1} \Gamma\left(k, A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma. \quad (122)$$

In (121) and (122), let $y = A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}$. Then, $\sqrt{\frac{\gamma}{\mu_{IM/DD}}} = \frac{y}{A}$, $\gamma = \frac{\mu_{IM/DD}}{A^2} y^2$, $d\gamma = \frac{\mu_{IM/DD}}{A^2} 2y dy$, and the integral region of y becomes $A\sqrt{\frac{\gamma}{\mu_{IM/DD}}} < y < \infty$. Therefore, the first integral term can be rewritten as

$$\begin{aligned} &\sum_{n=0}^{\infty} \frac{(-q)^n A^{\xi^2}}{n!} \int_{\gamma_T}^{\infty} \gamma^{n+p-1} \left(\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right)^{\xi^2} \\ &\quad \times \Gamma\left(k - \xi^2, A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma \\ &= \sum_{n=0}^{\infty} \frac{2(-q)^n \left(\mu_{IM/DD}\right)^{n+p}}{n! A^{2(n+p)}} \int_{A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}}^{\infty} y^{\xi^2+2(n+p)-1} \\ &\quad \times \Gamma\left(k - \xi^2, y\right) dy. \end{aligned} \quad (123)$$

Then, by applying Theorem 2, the closed-form result of (123) can be obtained as

$$\begin{aligned} &\sum_{n=0}^{\infty} \frac{2(-q)^n \left(\mu_{IM/DD}\right)^{n+p}}{n! A^{2(n+p)}} \int_{A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}}^{\infty} y^{\xi^2+2(n+p)-1} \Gamma\left(k - \xi^2, y\right) dy \\ &= \sum_{n=0}^{\infty} \frac{2(-q)^n \left(\mu_{IM/DD}\right)^{n+p}}{n! A^{2(n+p)}} \cdot \frac{1}{\xi^2 + 2(n+p)} \\ &\quad \times \left\{ \Gamma\left(2(n+p) + k, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right) \right. \\ &\quad \left. - \left(A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right)^{\xi^2+2(n+p)} \Gamma\left(k - \xi^2, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right) \right\}. \end{aligned} \quad (124)$$

Similar to the first integral derivation, the closed-form result of third integral term can be obtained as

$$\begin{aligned} &\sum_{n=0}^{\infty} \frac{(-q)^n}{n!} \int_{\gamma_T}^{\infty} \gamma^{n+p-1} \Gamma\left(k, A\sqrt{\frac{\gamma}{\mu_{IM/DD}}}\right) d\gamma \\ &= \sum_{n=0}^{\infty} \frac{2(-q)^n \left(\mu_{IM/DD}\right)^{n+p}}{n! A^{2(n+p)}} \cdot \frac{1}{2(n+p)} \\ &\quad \times \left\{ \Gamma\left(2(n+p) + k, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right) \right. \\ &\quad \left. - \left(A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right)^{2(n+p)} \Gamma\left(k, A\sqrt{\frac{\gamma_T}{\mu_{IM/DD}}}\right) \right\}. \end{aligned} \quad (125)$$

As a result, the closed-form expression of $\overline{BER_I}$ can be finally obtained as shown in (104).

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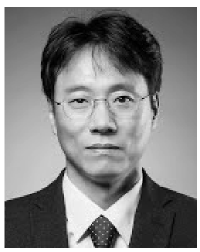

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