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A Hybrid Fuzzy Soft Sets Decision Making Method in Medical Diagnosis

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ABSTRACT The existing approaches for fuzzy soft sets decision-making are mainly based on different types of level soft sets. How to deal with such kinds of fuzzy soft sets decision-making problems via decreasing the uncertainty resulting from human's subjective cognition is still an open issue. To address this issue, a hybrid method for utilizing fuzzy soft sets in decision-making by integrating a fuzzy preference relations analysis based on the belief entropy with the Dempster–Shafer evidence theory is proposed. The proposed method is composed of four procedures. First, we measure the uncertainties of parameters by leveraging the belief entropy. Second, with the fuzzy preference relations analysis, the uncertainties of parameters are modulated by making use of the relative reliability preference of parameters. Third, an appropriate basic probability assignment in terms of each parameter is generated on the modulated uncertainty degrees of parameters basis. Finally, we adopt Dempster's combination rule to fuse the independent parameters into an integrated one; thus, the best one can be obtained based on the ranking candidate alternatives. In order to validate the feasibility and effectiveness of the proposed method, a numerical example and a medical diagnosis application are implemented. From the experimental results, it is demonstrated that the proposed method outperforms the related methods, because the uncertainty resulting from human's subjective cognition can be reduced; meanwhile, the decision-making level can also be improved with better performance.

INDEX TERMS Fuzzy soft set, decision making, Dempster–Shafer evidence theory, belief entropy, fuzzy preference relations, belief function, variance of entropy, medical diagnosis.

I. INTRODUCTION

It is inevitable for the uncertainty in the real world. How to model and cope with the uncertainty information is still an open issue. To overcome this problem, many mathematical tools are proposed and extended, like the rough sets theory [1], fuzzy sets theory [2]–[5], evidence theory [6]–[10], evidential reasoning [11], Z numbers [12], D numbers theory [13]–[15], and so on [16]–[18]. In addition, the approaches with hybrid intelligent algorithms are used for forecasting time series [19], fault diagnosis [20], [21], supplier selection [22], human reliability analysis [23], decision making [24], [25], and other optimization problems [26], [27].

Soft set theory which gives a parametric view for soft computing and uncertainty modelling was firstly presented by Molodtsov [28] in 1999. Because of having the loose and general set of characteristics, soft set theory is not limited by the inadequate parametric tools of those theories, like rough sets, probability theory and fuzzy sets. Hence, it is a general math tool to cope with objects and is widely applied in a variety of fields, like rule mining [29], forecasting [30], etc. Afterwards, through combining the theories of soft set and fuzzy set, the fuzzy soft set was firstly proposed by *Maji et al.* [31] in 2001. Due to the capability of dealing with imprecise and fuzzy parameters, fuzzy soft set theory was extended and extensively applied in the decision making problems. Roy and Maji [32] attempts to find a best object on fuzzy soft sets based on the assessment basis of score value. Then, Hou [33] takes advantage of the grey relational analysis to make decisions on fuzzy soft sets by taking into account both of the choice and score value assessment bases. Feng *et al.* [34] applies the level soft sets to make decisions in fuzzy soft sets. Furthermore, Jiang *et al.* [35] presents an adjustable method to make decisions on fuzzy soft sets by leveraging the intuitionistic fuzzy soft sets' level soft sets. Çağman and Enginoğlu [36] and Feng *et al.* present the uni-int decision making method that can select an optimal element set from the candidate alternatives.

From the above mentioned approaches, different results for the same decision problem may be obtained on the basis of suitable level soft sets and various assessment bases. Consequently, it is difficult for the decision makers to determine which alternative is the best one. Hence, the main issue is how to deal with such kinds of fuzzy soft sets-based decision making problems via decreasing the uncertainty resulting from the subjective cognition of human, so that it can improve the level of decision-making. Later on, by taking the above-mentioned issue into consideration, Li *et al.* [38] presents a method by using the grey relational analysis and Dempster–Shafer (D–S) evidence theory to make decisions on fuzzy soft sets. On the other hand, Wang *et al.* [39] proposes using fuzzy soft sets in decision making on the basis of the ambiguity measure and Dempster–Shafer evidence theory. Both of [38] and [39] reduce the uncertainty resulting from the subjective cognition of human and achieve a preferable decision-making level.

D–S evidence theory was proposed by Dempster [40] first, and it had been developed by Shafer *et al.* [41] later. As an efficient reasoning tool for the uncertainties, it has the advantage to represent the "uncertainty" directly by assigning the probability to the subsets of the set that includes multi-objects, rather than to an individual object. Moreover, D–S evidence theory is able to combine multiple evidences to produce an integrate evidence. On account of both of the flexibility and effectiveness in modelling the uncertainty and imprecision without relying on prior information, D–S evidence theory is extensively applied in a lot of areas [42]–[44].

The uncertainty measure can indicate the quality and clarity of the evidences. For better reflecting the uncertainty, we make use of a novel belief entropy [45] to measure the uncertainties of evidences. This belief entropy can not only measure the uncertainty of evidence that is expressed by a probability distribution, but also can measure the uncertainty of evidence that is expressed by a basic probability assignment. Hence, it is an effective method to measure the uncertainties of evidences which has been successfully utilised in decision-making problems [46].

Fuzzy preference relations which plays a base role in most decision-making processes was firstly presented by Tanino [47] in 1984. Because as the uncertainty increases in the course of information collection, the anarchy's degree which is involved in the systems is rising, so that the Dempster's rule of combination may not be able to use due to violating the essential condition. Employing the information that are ordered can improve the robustness of the D–S evidence theory-based systems. Thus, the fuzzy preference relations analysis are taken into account to further improve the decision-making level.

Therefore, a hybrid fuzzy soft sets decision making method by integrating the belief entropy, fuzzy preference relations analysis, with D–S evidence theory is proposed in this paper. The proposed method considers the uncertainty measure of the evidences, as well as the impact of evidences' relative reliability, so that it can obtain more appropriate basic probability assignments of alternatives. Finally, we illustrate three numerical examples and a medical diagnosis application to demonstrate that the proposed method is more efficient than the related works. Meanwhile, the uncertainties resulting from human's subjective cognition can be decreased and the level of decision-making can also be improved with more better performance.

The remainder of this paper is organized as follows. Section "Preliminaries" briefly introduces this paper's preliminaries. A hybrid fuzzy soft sets decision making method by integrating the belief entropy, fuzzy preference relations analysis, with D–S evidence theory is proposed in Section "The proposed method". Section "Experiment" illustrates a numerical example which show the effectiveness of the proposal. In Section "Application", the proposal is adopted to a practical application in medical diagnosis. Finally, Section "Conclusion" gives a conclusion.

II. PRELIMINARIES

A. FUZZY SOFT SETS

Definition 1 (Soft Set [28], [38]): Let U be a universe set, E be a set of parameters related to the objects in U, and $B \subseteq E$. The power set of U is represented by 2^U. A pair (F, B) is called a soft set over the universe set U, in which F is a mapping from B to 2^U defined as $F : B \rightarrow 2^U$.

Hence, the soft set over U is a parameterized family of subsets of U. For $e \in B$, $F(e)$ may be considered as a set of e-approximate elements of (F, B).

Example 1: Let $U = \{g_1, g_2, g_3, g_4, g_5\}$ and $B = \{e_1, e_2, e_3, e_4\}$. Let (F, B) be a soft set over U which are denoted as follows:

$$F(e_1) = \{g_1/1, g_2/1, g_3/0, g_4/0, g_5/0\},$$
$$F(e_2) = \{g_1/1, g_2/0, g_3/0, g_4/0, g_5/1\},$$
$$F(e_3) = \{g_1/0, g_2/0, g_3/1, g_4/1, g_5/0\},$$
$$F(e_4) = \{g_1/0, g_2/1, g_3/0, g_4/1, g_5/0\}.$$

Then, the soft set (F, B) is represented by Table 1.

TABLE 1. Tabular representation of the soft set (F, B).

	g_1	g_2	g_3	g_4	g_5
e_1	1	1	0	0	0
e_2	1	0	0	0	1
e_3	0	0	1	1	0
e_4	0	1	0	1	0

Definition 2 (Fuzzy Soft Set [31], [38]): Let U be a universe set and E be a set of parameters related to the objects in U, where $B \subseteq E$. I^U is denoted as a set of all fuzzy subsets of the universe set U. A pair (F, B) is called a fuzzy soft set

TABLE 2. Tabular representation of the fuzzy soft set (F, B).

	h_1	h_2	h_3
e_1	0.6	0.2	0.2
e_2	0.7	0.2	0.1
e_3	1/3	1/3	1/3

over U, in which F is a mapping from B to I^U defined as $F : B \to I^U$.

We can notice that every soft set can be considered as a fuzzy soft set [35]. Let $e \in B$, $x \in U$, and $F(e)$ be a fuzzy subset of the universe set U, which is called the parameter e's fuzzy value set. When $F(e)$ is a crisp subset of the universe set U, (F, B) will degenerate into a soft set. Then, let $F(e)(x)$ be a membership value, where the object x holds the parameter e, a fuzzy set $F(e)$ can be expressed as $F(e) = \{x/F(e)(x)|x \in U\}$.

Example 2: Let $U = \{h_1, h_2, h_3\}$ and $B = \{e_1, e_2, e_3\}$. Let (F, B) be a fuzzy soft set over U which are denoted as follows:

$$F(e_1) = \{h_1/0.6, h_2/0.2, h_3/0.2\},$$
$$F(e_2) = \{h_1/0.7, h_2/0.2, h_3/0.1\},$$
$$F(e_3) = \{h_1/\frac{1}{3}, h_2/\frac{1}{3}, h_3/\frac{1}{3}\}.$$

Then, the fuzzy soft set (F, B) is represented by Table 2.

Definition 3 (Fuzzy Intersection Operation [38]): Let (F, B) and (G, C) be two fuzzy soft sets, the fuzzy soft set "(F, B) AND (G, C)" which is denoted as $(F, B) \wedge (G, C)$ can be defined by $(F, B) \wedge (G, C) = (H, B \times C)$, in which for $\alpha \in B$ and $\beta \in C$, $H(\alpha, \beta) = F(\alpha)\widetilde{\cap}G(\beta)$ and $\widetilde{\cap}$ represents the fuzzy intersection operation between two fuzzy sets.

Definition 4 (Performance Measure [39]): The performance measure of a M method, denoted as γ_M is supposed to satisfy the optimal criteria for resolving a fuzzy soft set decision making problem. It is defined as the sum of the inverse of the summation of the non-negative differences between the membership values of the optimal object for the choice parameters and the choice value of the optimal object. Its mathematical form is defined by

$$\gamma_M = \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n}|F(e_i)(O_p)-F(e_j)(O_p)|} + \sum_{i=1}^{n} F(e_i)(O_p), \tag{1}$$

where n denotes the number of choice parameters and $F(e_i)(O_p)$ represents the membership value of the optimal object O_p for the choice parameter e_i.

Given two methods M_1 and M_2 that satisfy the optimal criteria, their corresponding performance measures are γ_{M_1} and γ_{M_2}, respectively. If $\gamma_{M_1} > \gamma_{M_2}$, then M_1 is better than M_2. If $\gamma_{M_1} < \gamma_{M_2}$, then M_2 is better than M_1. If $\gamma_{M_1} = \gamma_{M_2}$, then M_1 is equal to M_2.

B. DEMPSTER–SHAFER EVIDENCE THEORY

Because of the flexibility and effectiveness in modelling both of the uncertainty and imprecision without prior information, D–S evidence theory [40], [41] is more applicable to deal with uncertain information than the Bayesian probability theory. Under such a situation where probabilities are clear, D–S evidence theory could convert into Bayesian theory, hence D–S evidence theory is considered as the generalization of the Bayesian probability theory.

Definition 5 (Frame of Discernment): Let U be a set of collectively exhaustive and mutually exclusive events, indicted by

$$U = \{x_1, x_2, \ldots, x_i, \ldots, x_t\}. \tag{2}$$

The set U represents a frame of discernment. The power set of U is denoted by 2^U, where

$$2^U = \{\emptyset, \{x_1\}, \{x_2\}, \ldots, \{x_t\}, \{x_1, x_2\}, \ldots, \{x_1, x_2, \ldots, x_i\}, \ldots, U\}, \tag{3}$$

and $\emptyset$ is an empty set. If $A \in 2^U$, A is called a proposition.

Definition 6 (Mass Function): For a frame of discernment U, a mass function is a mapping m from 2^U to $[0, 1]$, formally defined as

$$m : 2^U \to [0, 1], \tag{4}$$

which satisfies the following condition:

$$m(\emptyset) = 0 \; and \sum_{A \in 2^U} m(A) = 1. \tag{5}$$

In the D–S evidence theory, the mass function can be also called as a basic probability assignment (BPA). If $m(A)$ is greater than 0, A will be called as a focal element, and the union of all of the focal elements is called as the core of the mass function.

Definition 7 (Belief Function): For a proposition $A \subseteq U$, the belief function $Bel : 2^U \to [0, 1]$ is defined as

$$Bel(A) = \sum_{B \subseteq A} m(B). \tag{6}$$

The plausibility function $Pl : 2^U \to [0, 1]$ is defined as

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B), \tag{7}$$

where $\bar{A} = U - A$.

Definition 8 (Dempster's Rule of Combination): Let m_1 and m_2 be two independent BPAs in the frame of discernment U, the Dempster's rule of combination is defined as below:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{B \cap C = A} m_1(B)m_2(C), & A \neq \emptyset, \\ 0, & A = \emptyset, \end{cases} \tag{8}$$

TABLE 3. An example of the Dempster's combination rule.

BPAs	$m(h_1)$	$m(h_2)$	$m(h_3)$
$e_1 : m_1$	0.60	0.20	0.20
$e_2 : m_2$	0.70	0.20	0.10
Fusing results	0.88	0.08	0.04

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C), \tag{9}$$

where $B \in 2^U$, $C \in 2^U$, and $K \in [0, 1]$ is the coefficient of conflict between two BPAs.

Notice that, the Dempster's combination rule is only practicable for the two BPAs with the condition $K < 1$.

Take the Example 2 as an instance, two BPAs m_1 and m_2 in terms of e_1 and e_2 in the frame of discernment $U = \{h_1, h_2, h_3\}$ can be obtained as follows

$$e_1 : m_1(h_1) = 0.60, \quad m_1(h_2) = 0.20, \; m_1(h_3) = 0.20;$$
$$e_2 : m_2(h_1) = 0.70, \quad m_2(h_2) = 0.20, \; m_2(h_3) = 0.10.$$

Then, we can produce a new BPA by using the Dempster's combination rule, where the fusing results are displayed in Table 3.

C. DENG ENTROPY

Recently, Kang and Deng [45] proposes a novel belief entropy which is named as the Deng entropy. Comparing with the Shannon entropy [48], Deng entropy is more efficient to measure the uncertain information, because Deng entropy can not only measure the uncertainty expressed by a probability distribution, but also can measure the uncertainty expressed by a basic probability assignment. Hence, the Deng entropy is considered as the generalization of Shannon entropy.

Definition 9 (Deng Entropy [45]): Let A_i be a hypothesis of the belief function m, $|A_i|$ is the cardinality of set A_i. Deng entropy E_d of set A_i is defined as follows:

$$E_d = -\sum_i m(A_i) \, log \, \frac{m(A_i)}{2^{|A_i|} - 1}. \tag{10}$$

When the belief value is only allocated to the single elements, Deng entropy degenerates to Shannon entropy, i.e.,

$$\begin{aligned} E_d &= -\sum_i m(A_i) \, log \, \frac{m(A_i)}{2^{|A_i|} - 1} \\ &= -\sum_i m(A_i) \, log \, m(A_i). \end{aligned} \tag{11}$$

The following examples show the effectiveness of the Deng entropy.

First, let's consider the case that the beliefs are only allocated to single elements. Take the Example 2 as an instance, a BPA m_3 in terms of e_3 in the frame of discernment $U = \{h_1, h_2, h_3\}$ can be obtained as $m_3(h_1) = m_3(h_2) = m_3(h_3) = 1/3$; thus its corresponding Shannon entropy, denoted as H and Deng entropy, denoted as E_d, can be calculated as shown in Table 4.

TABLE 4. The Shannon entropy and Deng entropy of e_3 in Example 2.

BPA	$m(h_1)$	$m(h_2)$	$m(h_3)$	$\lvert A_i \rvert$	H	E_d
$e_3 : m_3$	1/3	1/3	1/3	1	1.5850	1.5850

TABLE 5. The Shannon entropy and Deng entropy of m in Example 3.

BPA	$m(h_1, h_2, h_3)$	$\lvert A_i \rvert$	H	E_d
m	1	3	–	2.8074

Furthermore, let's consider the other case that the belief is assigned to multiple elements below.

Example 3: Supposing there exists the mass function $m(h_1, h_2, h_3) = 1$ in the frame of discernment $U = \{h_1, h_2, h_3\}$, its corresponding Deng entropy, denoted as E_d, can be obtained as shown in Table 5.

From the above two examples, we can notice that when the belief is only allocated to the single elements, the Deng entropy and Shannon entropy are the same. However, when the belief is assigned to the multiple elements as shown in Example 3, the Deng entropy can efficiently measure the uncertainty, but the Shannon entropy is incapable of that.

D. FUZZY PREFERENCE RELATIONS

Definition 10 (Fuzzy Preference Relations [47], [49]): Let P be a fuzzy preference relation and $E = \{e_1, e_2, \ldots, e_n\}$ be a set of alternatives, the fuzzy preference relation is defined as below:

$$P = (p_{jk})_{n \times n} = \begin{bmatrix} 0.5 & \cdots & p_{1j} & \cdots & p_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{j1} & \cdots & 0.5 & \cdots & p_{jn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n1} & \cdots & p_{nj} & \cdots & 0.5 \end{bmatrix}, \tag{12}$$

where $p_{jk} \in [0, 1]$ denotes the preference value for alternative e_j over e_k, $p_{jk} + p_{kj} = 1$, $p_{jj} = 0.5$, $1 \leq j \leq n$ and $1 \leq k \leq n$. $p_{jk} = 0.5$ denotes indifference between e_j and e_k; $p_{jk} = 1$ denotes that e_j is absolutely preferred to e_k; $p_{jk} > 0.5$ denoted that e_j is preferred to e_k, where $1 \leq j \leq n$ and $1 \leq k \leq n$.

Definition 11 (Additive Consistency for the Fuzzy Preference Relation [47]): Let $P = (p_{jk})_{n \times n}$ be a fuzzy preference relation, the concept of the additive consistency for P is defined as:

$$p_{jl} = p_{jk} + p_{kl} - 0.5, \tag{13}$$

where $p_{jk} + p_{kj} = 1$, $p_{jj} = 0.5$, $1 \leq j \leq n$ and $1 \leq k \leq n$.

Definition 12 (The Consistency Matrix [49]): Given a complete fuzzy preference relation $P^* = (p_{jk})_{n \times n}$, where p_{jk} denotes the preference values for alternative e_j over alternative e_k, $p_{jk} + p_{kj} = 1$, $p_{jj} = 0.5$, $1 \leq j \leq n$ and $1 \leq k \leq n$. The consistency matrix $\overline{P}$ can be constructed based on the complete fuzzy preference relation P^*, shown as follows:

$$\overline{P} = (\overline{p}_{jl})_{n\times n} = \left(\frac{1}{n}\sum_{k=1}^{n}(p_{jk} + p_{kl}) - 0.5\right)_{n\times n}. \quad (14)$$

The consistency matrix $\overline{P} = (\overline{p}_{jl})_{n\times n}$ has the following properties:

(1) $\overline{p}_{jl} + \overline{p}_{lj} = 1$;
(2) $\overline{p}_{jj} = 0.5$;
(3) $\overline{p}_{jl} = \overline{p}_{jk} + \overline{p}_{kl} - 0.5$;
(4) $\overline{p}_{jl} \leq \overline{p}_{js}$ for all $j \in \{1, 2, \ldots, n\}$, where $l \in \{1, 2, \ldots, n\}$ and $s \in \{1, 2, \ldots, n\}$.

Given a consistency matrix $\overline{P} = (\overline{p}_{jl})_{n\times n}$, the ranking value $RV(e_j)$ of alternative e_j is defined as follows:

$$RV(e_j) = \frac{2}{n^2}\sum_{k=1}^{n}\overline{p}_{jk}, \quad (15)$$

where $1 \leq j \leq n$ and $\sum_{j=1}^{n} RV(e_j) = 1$.

III. THE PROPOSED METHOD

In this section, a hybrid method for utilizing fuzzy soft sets in decision making by integrating fuzzy preference relations analysis based on the belief entropy with Dempster–Shafer (D–S) evidence theory is proposed. We first measure the uncertainties of parameters by leveraging the belief entropy. Next, with the fuzzy preference relations analysis, the relative reliability preference among the parameters are indicated. After that, the uncertainties of parameters are modulated by making use of the relative reliability preference of parameters. Afterwards, an appropriate basic probability assignment (BPA) in terms of each parameter is generated on the modulated uncertainty degrees of parameters basis. Eventually, we adopt the Dempster's combination rule to fuse the independent parameters into an integrate one; thus, the best one can be obtained based on the ranking candidate alternatives. The flowchart of the proposed method is shown in Fig. 1.

A. MEASURE THE UNCERTAINTY OF THE PARAMETER

Although, ambiguity measure is widely applied in uncertainty measure, because of lacking of the information in the Pignistic probability conversion process, the belief entropy, Deng entropy can better measure the uncertainty of evidence compared with the ambiguity measure. The below examples depict the Deng entropy's effectiveness.

First, let's consider the case that the beliefs are only allocated to single elements. Also, take the Example 2 as an instance, for e_3, $m_3(h_1) = m_3(h_2) = m_3(h_3) = 1/3$; thus its corresponding ambiguity measure, denoted as AM and Deng entropy, denoted as E_d, can be obtained as shown in Table 6.

Moreover, let's consider the other case that the beliefs are assigned to not only single elements, but also multiple elements below.

Example 4: Supposing there exists the mass function $m(h_1) = 0.05$, $m(h_2) = 0.05$, $m(h_3) = 0.05$, $m(h_1, h_2, h_3) = 0.85$ in a frame of discernment $U = \{h_1, h_2, h_3\}$, its corresponding ambiguity measure, denoted as AM and Deng entropy, denoted as E_d, can be obtained as shown in Table 7.

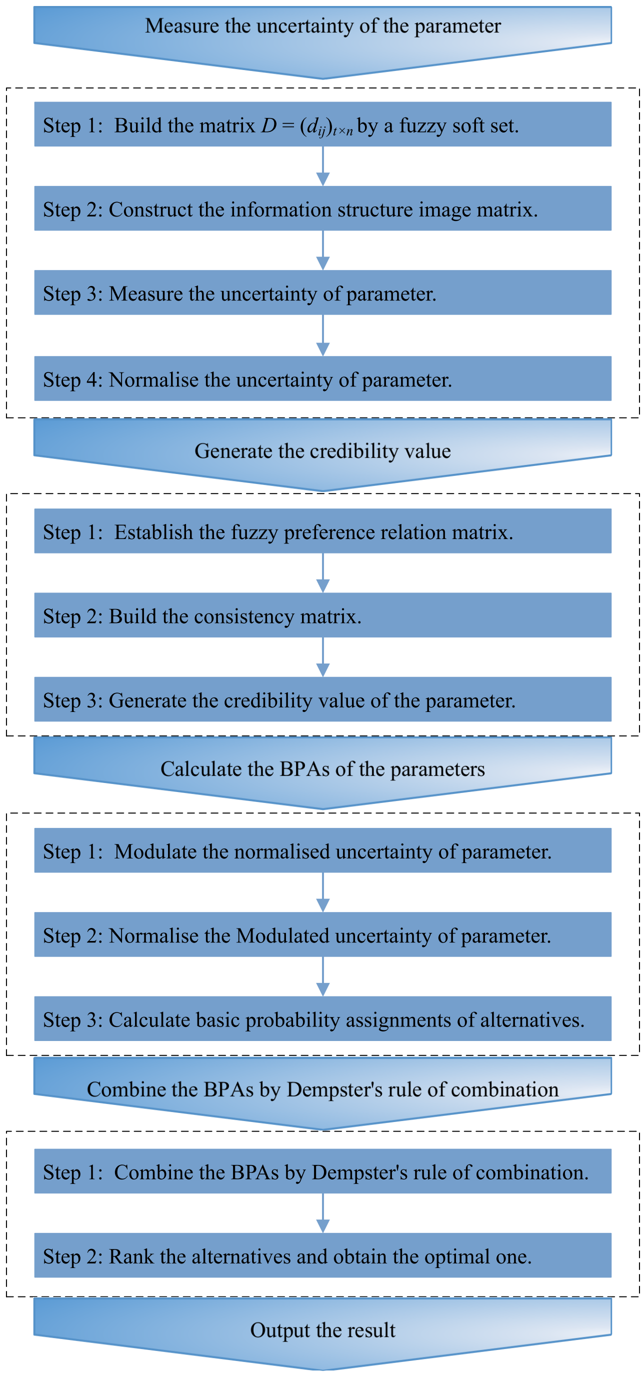

FIGURE 1. The flowchart of the proposed method.

TABLE 6. The ambiguity measure and Deng entropy of e_3 in Example 2.

BPA	$m(h_1)$	$m(h_2)$	$m(h_3)$	AM	E_d
$e_3 : m_3$	1/3	1/3	1/3	1.5850	1.5850

TABLE 7. The ambiguity measure and Deng entropy of *m* in Example 4.

BPAs	$m(h_1)$	$m(h_2)$	$m(h_3)$	$m(h_1, h_2, h_3)$	AM	E_d
m	0.05	0.05	0.05	0.85	1.5850	3.2338

From the above two examples, we can easy see the effectiveness of the Deng entropy which can better measure the uncertainties of evidences compared with the ambiguity measure. Specifically, m_3 is supposed to be more certainty than m in Example 4. However, the values of AM for m_3 and m are identical. Conversely, the m's Deng entropy $E_d = 3.2338$ is more bigger than m_3's Deng entropy $E_d = 1.5850$, where this result is consist with the intuition.

Let $\Theta = \{x_1, x_2, \ldots, x_i, \ldots, x_t\}$ be the frame of discernment and $B = \{e_1, e_2, \ldots, e_j, \ldots, e_n\}$ be the set of parameters, where x_i $(1 \leq i \leq t)$ represents the mutually exclusive alternatives and e_j $(1 \leq j \leq n)$ denotes the evaluation parameters. Then, $F : B \rightarrow I^{\Theta}$ is defined as $F(e_j)(x_i) = d_{ij}$. The uncertainties of parameters can be measured by the following steps:

Step 1: The matrix $D = (d_{ij})_{t\times n}$ is built by the aid of the fuzzy soft set (F, B) over Θ, in which d_{ij} denotes the membership value of x_i with e_j:

$$D = \begin{bmatrix} d_{11} & \cdots & d_{1j} & \cdots & d_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{i1} & \cdots & d_{ij} & \cdots & d_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{t1} & \cdots & d_{tj} & \cdots & d_{tn} \end{bmatrix}. \quad (16)$$

Step 2: The information structure image sequence with regard to the parameter e_j is generated by $d_j = \{\widetilde{d_{1j}}, \ldots, \widetilde{d_{ij}}, \ldots, \widetilde{d_{tj}}\}$, in which $\widetilde{d_{ij}} = \frac{d_{ij}}{\sum_{i=1}^{t} d_{ij}}$; thus, we construct the information structure image matrix $\widetilde{D}$ as follows:

$$\widetilde{D} = \begin{bmatrix} \widetilde{d_{11}} & \cdots & \widetilde{d_{1j}} & \cdots & \widetilde{d_{1n}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{d_{i1}} & \cdots & \widetilde{d_{ij}} & \cdots & \widetilde{d_{in}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{d_{t1}} & \cdots & \widetilde{d_{tj}} & \cdots & \widetilde{d_{tn}} \end{bmatrix}. \quad (17)$$

Step 3: The belief entropy of the parameter e_j is calculated by leveraging Eq. (10), which is denoted as $E_d(e_j)$:

$$E_d(e_j) = -\sum_{i=1}^{t} d_{ij} \, log \, \frac{d_{ij}}{2^{|x_i|} - 1}, \quad 1 \leq j \leq n. \quad (18)$$

Considering that the parameter's belief entropy may be zero in some certain case, we use the following formula to measure the uncertain of the parameter e_j for avoiding assigning zero weight to such a kind of parameter, which is denoted as $U(e_j)$:

$$U(e_j) = e^{E_d(e_j)} = e^{-\sum_{i=1}^{t} d_{ij} \, log \, \frac{d_{ij}}{2^{|x_i|}-1}}, \quad 1 \leq j \leq n. \quad (19)$$

Step 4: The uncertain of the parameter e_j is normalised as follows, which is represented as $\overline{U}(e_j)$:

$$\overline{U}(e_j) = \frac{U(e_j)}{\sum_{h=1}^{n} U(e_h)}, \quad 1 \leq j \leq n. \quad (20)$$

B. GENERATE THE CREDIBILITY VALUE BASED ON FUZZY PREFERENCE RELATIONS ANALYSIS

How to distinguish relatively credible evidences based on the obtained evidences plays an important role during the process of information fusion. Nevertheless, the uncertainty raises in the course of information collection, which results in the increasing of the anarchy degree involved in the systems. What is frustrating is that this behavior violates the essential condition of utilizing the Dempster's rule of combination. The robustness of the system based on the Dempster–Shafer evidence theory will become better when using ordered information. It is considered that the variance of entropy has capability to express the difference between evidences. In this context, the variance of entropy is taken into account to generate fuzzy preference relations. Then, through the fuzzy preference relations analysis, the relative reliability preference among the evidences are indicated, which can further be utilised to decrease the impact of anarchy's degree caused by the accessorial uncertainty in the course of information collection.

Step 1: Based on the entropy that obtained from Section ''Measure the uncertainty of the parameter'', the fuzzy preference relation matrix for all the parameters e_j $(1 \leq j \leq n)$, denoted as $P = (p_{jk})_{n\times n}$ is established by the steps below:

Step 1-1: Based on the Definition 10, the diagonal elements p_{jj} is allocated to 0.5 as follows, because no preference relation exists for the parameter e_j itself.

$$P = (p_{jk})_{n\times n} = \begin{bmatrix} 0.5 & \cdots & p_{1j} & \cdots & p_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{j1} & \cdots & 0.5 & \cdots & p_{jn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n1} & \cdots & p_{nj} & \cdots & 0.5 \end{bmatrix}.$$

Step 1-2: When only two parameters exist which means $n = 2$, the off-diagonal elements p_{jk} and p_{kj} is allocated to 0.5, because there are no plenty parameters to judge how the parameters are preferred to each other. Hence, we build the fuzzy preference relation matrix as follows:

$$P = (p_{jk})_{n\times n} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}. \quad (21)$$

Step 1-3: If there are more than two parameters which means $n > 2$, the variance of entropy for the parameter e_j $(1 \leq j \leq n)$ is generated by:

$$Var(e_j) = Var(\{\overline{U}(e_1), \overline{U}(e_2), \ldots, \overline{U}(e_{j-1}), \overline{U}(e_{j+1}), \ldots, \overline{U}(e_n)\}). \quad (22)$$

If the parameter e_j highly conflicts with other parameters, the variance of entropy $Var(e_j)$ excepting oneself implies the degree of difference between this conflicting parameter and other parameters. The more conflict the parameter e_j has, the less value the variance of entropy $Var(e_j)$ has.

Step 1-4: The off-diagonal elements p_{jk} and p_{kj} is calculated as follows:

$$p_{jk} = \frac{Var(e_j)}{Var(e_j) + Var(e_k)}, \quad (23)$$

$$p_{kj} = \frac{Var(e_k)}{Var(e_j) + Var(e_k)}, \quad (24)$$

where $1 \leq j \leq n$ and $1 \leq k \leq n$.

It is obvious that if the parameter e_j is more conflicting comparing with the parameter e_k, the value of p_{jk} will be lower than p_{kj}, because the variance of entropy $Var(e_k)$ is more higher than that of $Var(e_j)$.

Step 2: On the basis of the obtained fuzzy preference relation matrix $P = (p_{jk})_{n \times n}$, we construct the consistency matrix $\overline{P}$ by using Eq. (14).

Step 3: Based on the consistency matrix $\overline{P}$ and Eq. (15), we define the credibility value of the parameter e_j by:

$$Crd(e_j) = \frac{2}{n^2} \sum_{k=1}^{n} \overline{p}_{jk}, \quad 1 \leq j \leq n; \ 1 \leq k \leq n. \quad (25)$$

where $\sum_{j=1}^{n} Crd(e_j) = 1$, such that the credibility values of parameters can be considered as the weights to indicate the relative reliability preference of parameters.

C. CALCULATE THE BPAS OF THE PARAMETERS

In this part, the final uncertainty of the parameters are determined. Based on the final uncertain degree of each parameter, we generate the basic probability assignments (BPAs) of the parameters which can be further utilised by the Dempster's rule of combination. The specific steps are as follows:

Step 1: On the credibility degree $Crd(e_j)$ basis, the normalised uncertain of the parameter e_j is modulated, represented as $MU(e_j)$:

$$MU(e_j) = Crd(e_j) \times \overline{U}(e_j), \quad 1 \leq j \leq n. \quad (26)$$

Step 2: The $MU(e_j)$ is normalised as below, denoted as $\overline{MU}(e_j)$ which is regarded as the final uncertainty measurement of the parameter e_j.

$$\overline{MU}(e_j) = \frac{MU(e_j)}{\sum_{h=1}^{n} MU(e_h)}, \quad 1 \leq j \leq n. \quad (27)$$

Step 3: The basic probability assignment of the alternative x_i and Θ with regard to the parameter e_j can be calculated by:

$$m_{e_j}(\emptyset) = 0, \quad (28)$$

$$m_{e_j}(x_i) = \widetilde{d}_{ij} \times (1 - \overline{MU}(e_j)), \quad (29)$$

$$m_{e_j}(\Theta) = 1 - \sum_{i=1}^{t} m_{e_j}(x_i), \quad (30)$$

where $1 \leq i \leq t$ and $1 \leq j \leq n$.

For $j = 1, 2, \ldots, n$, $\sum_{A \subseteq \Theta} m_{e_j}(A) = 1$ is obvious. Thus, m_{e_j} is the basic probability assignment on Θ.

To be specific, Eq. (26) represents that the uncertainty of parameter is adjusted by multiplying a factor $Crd(e_j)$, so that the effect of the parameter with less uncertainty will be enhanced via $(1-\overline{MU}(e_j))$ operation in Eq. (29), while the impact of the parameter with more uncertainty will be alleviated via $(1 - \overline{MU}(e_j))$ operation when constructing the BPAs of the parameters. Therefore, an appropriate basic probability assignment (BPA) in terms of each parameter is generated on the modulated uncertainty degrees of parameters basis.

D. COMBINE THE BPAS WITH THE DEMPSTER'S RULE OF COMBINATION

Step 1: The independent parameters will be fused into an integrate one by adopting the Dempster's combination rule based on Eq. (8); thus, the final BPA of the candidate alternative x_i $(1 \leq i \leq t)$ that is regarded as the belief measure of the alternative can be generated.

Step 2: The candidate alternatives can be ranked based on the final BPA of the alternative x_i and the best one will be obtained.

IV. EXPERIMENT

In this section, we illustrate a numerical example to show the effectiveness of the proposed method.

Example 5: Consider a decision making problem related to a fuzzy soft set (F, B) shown in [38, Table 8], where $\Theta = \{x_1, x_2, x_3\}$ is the frame of discernment and $B = \{e_1, e_2, e_3, e_4, e_5\}$ is the set of parameters which is considered as the set of evidences.

TABLE 8. The fuzzy soft set (F, B) in Example 5.

Alternatives	e_1	e_2	e_3	e_4	e_5
x_1	0.85	0.73	0.26	0.32	0.75
x_2	0.56	0.82	0.76	0.64	0.43
x_3	0.84	0.55	0.82	0.53	0.47

Step 1: Build the matrix $D = (d_{ij})_{t\times n}$ induced by the fuzzy soft set (F, B) over Θ as follows:

$$D = \begin{pmatrix} 0.85 & 0.73 & 0.26 & 0.32 & 0.75 \\ 0.56 & 0.82 & 0.76 & 0.64 & 0.43 \\ 0.84 & 0.55 & 0.82 & 0.53 & 0.47 \end{pmatrix}.$$

Step 2: Construct $\widetilde{D}$, i.e., the information structure image matrix as below:

$$\widetilde{D} = \begin{pmatrix} 0.3778 & 0.3476 & 0.1413 & 0.2148 & 0.4545 \\ 0.2489 & 0.3905 & 0.4130 & 0.4295 & 0.2606 \\ 0.3733 & 0.2619 & 0.4457 & 0.3557 & 0.2848 \end{pmatrix}.$$

Step 3: Measure the uncertainty of parameter e_j $(j = 1, 2, 3, 4, 5)$ as follows:

$$U(e_1) = 4.7617, \quad U(e_2) = 4.7870,$$
$$U(e_3) = 4.2435, \quad U(e_4) = 4.6214,$$
$$U(e_5) = 4.6585.$$

Step 4: Normalise the uncertainty of parameter e_j $(j = 1, 2, 3, 4, 5)$ as below:

$$\overline{U}(e_1) = 0.2064, \quad \overline{U}(e_2) = 0.2075,$$
$$\overline{U}(e_3) = 0.1839, \quad \overline{U}(e_4) = 0.2003,$$
$$\overline{U}(e_5) = 0.2019.$$

Step 5: Establish $P = (p_{jk})_{n\times n}$, i.e., the fuzzy preference relation matrix as follows:

$$P = \begin{pmatrix} 0.5000 & 0.5147 & 0.9043 & 0.4630 & 0.4664 \\ 0.4853 & 0.5000 & 0.8991 & 0.4485 & 0.4519 \\ 0.0957 & 0.1009 & 0.5000 & 0.0836 & 0.0847 \\ 0.5370 & 0.5515 & 0.9164 & 0.5000 & 0.5034 \\ 0.5336 & 0.5481 & 0.9153 & 0.4966 & 0.5000 \end{pmatrix}.$$

Step 6: Build $\overline{P} = (\overline{p}_{jl})_{n\times n}$, i.e., the consistency matrix as below:

$$\overline{P} = \begin{pmatrix} 0.5000 & 0.5127 & 0.8967 & 0.4680 & 0.4710 \\ 0.4873 & 0.5000 & 0.8840 & 0.4553 & 0.4582 \\ 0.1033 & 0.1160 & 0.5000 & 0.0713 & 0.0743 \\ 0.5320 & 0.5447 & 0.9287 & 0.5000 & 0.5030 \\ 0.5290 & 0.5418 & 0.9257 & 0.4970 & 0.5000 \end{pmatrix}.$$

Step 7: Generate the credibility value of parameter e_j $(j = 1, 2, 3, 4, 5)$ as follows:

$$Crd(e_1) = 0.2279, \quad Crd(e_2) = 0.2228,$$
$$Crd(e_3) = 0.0692, \quad Crd(e_4) = 0.2407,$$
$$Crd(e_5) = 0.2395.$$

Step 8: Modulate the normalised uncertainty of parameter e_j $(j = 1, 2, 3, 4, 5)$ on the basis of the credibility value as below:

$$MU(e_1) = 0.0470, \quad MU(e_2) = 0.0462,$$
$$MU(e_3) = 0.0127, \quad MU(e_4) = 0.0482,$$
$$MU(e_5) = 0.0484.$$

TABLE 9. The BPAs of alternatives with regard to the parameters in Example 5.

BPA	e_1	e_2	e_3	e_4	e_5
$m(x_1)$	0.2901	0.2683	0.1324	0.1636	0.3460
$m(x_2)$	0.1911	0.3014	0.3871	0.3273	0.1984
$m(x_3)$	0.2866	0.2021	0.4176	0.2710	0.2168
$m(\Theta)$	0.2322	0.2282	0.0628	0.2380	0.2387

TABLE 10. The belief measure of the alternatives in terms of different methods in Example 5.

Method	$Bel(x_1)$	$Bel(x_2)$	$Bel(x_3)$
Li et al. [38]	0.2507	0.3410	0.3332
Wang et al. [39]	0.2420	0.3803	0.3726
Proposed method	0.1905	0.4029	0.4019

TABLE 11. The comparison of different methods in Example 5.

Method	Ranking	Optimum	$m(\Theta)$	γ
Li et al. [38]	$x_2 > x_3 > x_1$	x_2	0.0751	3.7202
Wang et al. [39]	$x_2 > x_3 > x_1$	x_2	0.0051	3.7202
Proposed method	$x_2 > x_3 > x_1$	x_2	0.0031	3.7202

Step 9: Normalise the modulated uncertainty of parameter e_j $(j = 1, 2, 3, 4, 5)$ as follows:

$$\overline{MU}(e_1) = 0.2322, \quad \overline{MU}(e_2) = 0.2282,$$
$$\overline{MU}(e_3) = 0.0628, \quad \overline{MU}(e_4) = 0.2380,$$
$$\overline{MU}(e_5) = 0.2387.$$

Step 10: Calculate the basic probability assignment of alternative x_i and Θ with regard to the parameter e_j as shown in Table 9.

Step 11: Combine the BPAs of Table 9 through the Dempster's rule of combination, and the fusing results, namely, the belief measures of alternatives are shown in Table 10 and Fig. 2.

Step 12: The final ranking of candidate alternatives on the basis of the final BPA of the alternative x_i is $x_2 > x_3 > x_1$. Hence, the optimal choice decision is x_2 which corresponds to the maximum.

Additionally, we compare the proposed method with the related methods [38] and [39] where the comparison results are displayed in Table 10, Table 11 and Fig. 2.

As shown in Table 11, it is obvious that the belief measures of the uncertainties that are obtained by Li *et al.* [38] and Wang *et al.* [39] methods are 0.0751 and 0.0051, respectively, whereas the uncertainty's belief measure falls to 0.0031 which are obtained by the proposed method. It indicates that the proposal by integrating fuzzy preference relations analysis based on the belief entropy with D–S evidence theory can reduce uncertainty resulting from human's subjective cognition, such that it can improve the decision-making level. Furthermore, according to the belief measure of the alternatives in Table 10 and Fig. 2, and the measure of performance γ in Table 11, the proposed method is

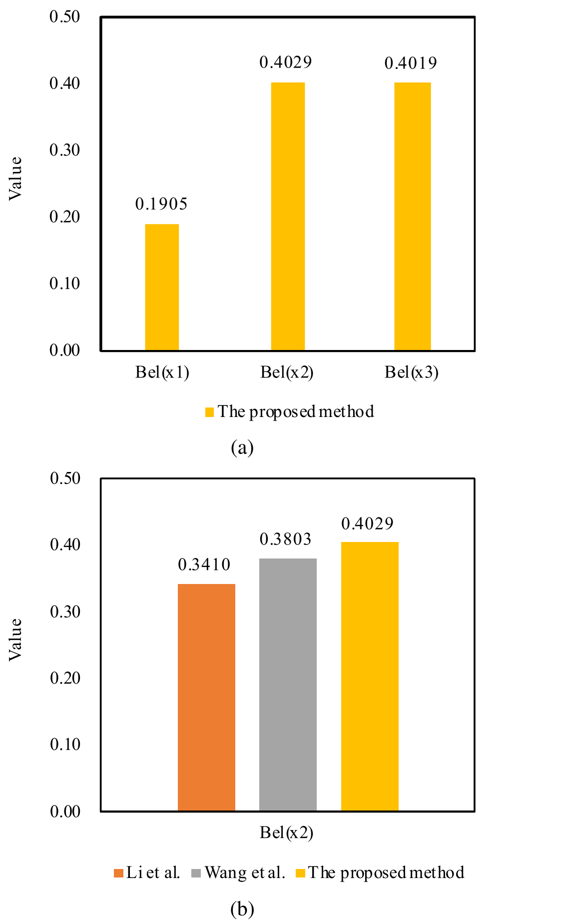

FIGURE 2. The comparison of the belief measure of different methods in Example 5.

more accurate and effective than the previous methods [38] and [39].

V. APPLICATION

As well as we know, it is critical to manage uncertainty in medical diagnosis. By considering the medical diagnosis problem from [38] and [39], we compare the proposed method with the related methods [38] and [39]. Finally, the experimental results illustrate that the proposed method is as efficient as the related methods.

Supposing that the universe set Θ that consists of four diseases is given by $\Theta = \{$*acute dental abscess*, *migraine*, *acute sinusitis*, *peritonsillar abscess*$\} = \{x_1, x_2, x_3, x_4\}$, and the set of parameters B is given by $B = \{$*fever*, *running nose*, *weakness*, *orofacial pain*, *nausea vomiting*, *swelling*, *trismus*, *history*, *physical examination*, *laboratory investigation*$\} = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, f_1, f_2, f_3\}$.

Let I_1 and I_2 be two subsets of E, given by $I_1 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and $I_2 = \{f_1, f_2, f_3\}$. Supposing that (F, I_1) is the fuzzy soft that expresses "symptoms of the diseases", and (G, I_2) is the fuzzy soft set that represents "decision making tools of the diseases". The fuzzy soft sets (F, I_1) and (G, I_2) are shown in Table 12 and Table 13, respectively.

TABLE 12. The fuzzy soft set (F, I_1).

Alternatives	e_1	e_2	e_3	e_4	e_5	e_6	e_7
x_1	0.60	0.00	0.60	0.90	0.00	0.70	0.80
x_2	0.20	0.00	0.10	0.90	0.80	0.00	0.00
x_3	0.30	0.70	0.30	0.80	0.30	0.40	0.00
x_4	0.40	0.00	0.20	0.70	0.10	0.60	0.50

TABLE 13. The fuzzy soft set (F, I_2).

Alternatives	f_1	f_2	f_3
x_1	0.60	0.80	0.40
x_2	0.80	0.30	0.60
x_3	0.80	0.40	0.70
x_4	0.60	0.80	0.30

Supposing that a patient who is suffering from a disease has three symptoms $P = \{$*fever*, *runny noise*, *orofacial pain*$\}$. A doctor requires to make the most suitable diagnosis regarding to the the patient's symptoms, physical examination, history, and laboratory investigation. To figure out this problem, "$(F, P) \wedge (G, I_2)$" is constructed as shown in Table 14. There are four diseases, i.e., x_1, x_2, x_3, x_4, and nine pairs of parameters, i.e., $s_1 = (e_1, f_1)$, $s_2 = (e_1, f_2)$, $s_3 = (e_1, f_3)$, $s_4 = (e_2, f_1)$, $s_5 = (e_2, f_2)$, $s_6 = (e_2, f_3)$, $s_7 = (e_4, f_1)$, $s_8 = (e_4, f_2)$, $s_9 = (e_4, f_3)$, where $s = (e, f)$ denotes the pair of one symptom and one decision making tool.

Aforementioned, the frame of discernment is $\Theta = \{x_1, x_2, x_3, x_4\}$ that is constructed by four diseases, and the set of parameters which is considered as the set of evidence is $Q = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9\}$ that is constructed by the pairs of the symptoms and the decision-making tools. Afterwards, we propose a method to identify the most suitable symptoms of each disease. The concrete steps are given as below.

Step 1: Build the matrix $D = (d_{ij})_{t \times n}$ induced by the fuzzy soft set $(F, P) \wedge (G, I_2)$ over Θ as shown in Eq. (31), at the bottom of the next page.

Step 2: Construct $\widetilde{D}$, i.e., the information structure image matrix as shown in Eq. (32), at the bottom of the next page.

Step 3: Measure the uncertainty of parameter s_j $(j = 1, 2, \ldots, 9)$ as follows:

$$U(s_1) = 6.6141, \quad U(s_2) = 6.6141,$$
$$U(s_3) = 7.0936, \quad U(s_4) = 1.0000,$$
$$U(s_5) = 1.0000, \quad U(s_6) = 1.0000,$$
$$U(s_7) = 7.2805, \quad U(s_8) = 6.6559,$$
$$U(s_9) = 6.8627.$$

Step 4: Normalise the uncertainty of parameter s_j $(j = 1, 2, \ldots, 9)$ as below:

$$\overline{U}(s_1) = 0.1499, \quad \overline{U}(s_2) = 0.1499,$$
$$\overline{U}(s_3) = 0.1608, \quad \overline{U}(s_4) = 0.0227,$$
$$\overline{U}(s_5) = 0.0227, \quad \overline{U}(s_6) = 0.0227,$$

TABLE 14. The fuzzy soft set $(F, P) \wedge (G, I_2)$.

Alternatives	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
x_1	0.60	0.60	0.40	0.00	0.00	0.00	0.60	0.80	0.40
x_2	0.20	0.20	0.20	0.00	0.00	0.00	0.80	0.30	0.60
x_3	0.30	0.30	0.30	0.70	0.40	0.70	0.80	0.40	0.70
x_4	0.40	0.40	0.30	0.00	0.00	0.00	0.60	0.70	0.30

TABLE 15. The BPAs of alternatives in terms of the parameters.

BPA	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
$m(x_1)$	0.3391	0.3391	0.2798	0	0	0	0.1792	0.3080	0.1686
$m(x_2)$	0.1130	0.1130	0.1399	0	0	0	0.2389	0.1155	0.2530
$m(x_3)$	0.1696	0.1696	0.2098	0.9796	0.9796	0.9796	0.2389	0.1540	0.2951
$m(x_4)$	0.2261	0.2261	0.2098	0	0	0	0.1792	0.2695	0.1265
$m(\Theta)$	0.1522	0.1522	0.1607	0.0204	0.0204	0.0204	0.1638	0.1530	0.1568

$$\overline{U}(s_7) = 0.1650, \quad \overline{U}(s_8) = 0.1509,$$
$$\overline{U}(s_9) = 0.1555.$$

Step 5: Establish $P = (p_{jk})_{n\times n}$, i.e., the fuzzy preference relation matrix which is shown in Eq. (33), at the bottom of the next page.

Step 6: Build $\overline{P} = (\overline{p}_{jk})_{n\times n}$, i.e., the consistency matrix which is shown in Eq. (34), at the bottom of the next page.

Step 7: Generate the credibility value of parameter s_j ($j = 1, 2, \ldots, 9$) as follows:

$$Crd(s_1) = 0.1161, \quad Crd(s_2) = 0.1161,$$
$$Crd(s_3) = 0.1143, \quad Crd(s_4) = 0.1030,$$
$$Crd(s_5) = 0.1030, \quad Crd(s_6) = 0.1030,$$
$$Crd(s_7) = 0.1134, \quad Crd(s_8) = 0.1159,$$
$$Crd(s_9) = 0.1152.$$

Step 8: Modulate the normalised uncertainty of parameter s_j ($j = 1, 2, \ldots, 9$) on the basis of the credibility value as below:

$$MU(s_1) = 0.0174, \quad MU(s_2) = 0.0174,$$
$$MU(s_3) = 0.0184, \quad MU(s_4) = 0.0023,$$
$$MU(s_5) = 0.0023, \quad MU(s_6) = 0.0023,$$
$$MU(s_7) = 0.0187, \quad MU(s_8) = 0.0175,$$
$$MU(s_9) = 0.0179.$$

Step 9: Normalise the modulated uncertainty of parameter s_j ($j = 1, 2, \ldots, 9$) as follows:

$$\overline{MU}(s_1) = 0.1522, \quad \overline{MU}(s_2) = 0.1522,$$
$$\overline{MU}(s_3) = 0.1607, \quad \overline{MU}(s_4) = 0.0204,$$
$$\overline{MU}(s_5) = 0.0204, \quad \overline{MU}(s_6) = 0.0204,$$
$$\overline{MU}(s_7) = 0.1638, \quad \overline{MU}(s_8) = 0.1530,$$
$$\overline{MU}(s_9) = 0.1568.$$

Step 10: Calculate the basic probability assignment of alternative x_i and Θ with regard to the parameter s_j ($j = 1, 2, \ldots, 9$) as shown in Table 15.

Step 11: Combine the BPAs of Table 15 through the Dempster's rule of combination, and the fusing results, namely, the belief measures of alternatives are shown in Table 16 and Fig. 3.

Step 12: The final ranking of candidate alternatives on the basis of the final BPA of the alternative x_i is $x_3 > x_1 > x_4 > x_2$. Hence, the optimal choice decision is x_3 which corresponds to the maximum.

In addition, we compare the proposed method with the related methods [38] and [39] where the comparison results are displayed in Table 16, Table 17 and Fig. 3.

From Table 17, we can see that the belief measures of the uncertainties that are obtained by Li *et al.* [38] and Wang *et al.* [39] methods are 0.0069 and 0.0001, respectively, whereas the uncertainty's belief measure falls to

$$D = \begin{pmatrix} 0.60 & 0.60 & 0.40 & 0.00 & 0.00 & 0.00 & 0.60 & 0.80 & 0.40 \\ 0.20 & 0.20 & 0.20 & 0.00 & 0.00 & 0.00 & 0.80 & 0.30 & 0.60 \\ 0.30 & 0.30 & 0.30 & 0.70 & 0.40 & 0.70 & 0.80 & 0.40 & 0.70 \\ 0.40 & 0.40 & 0.30 & 0.00 & 0.00 & 0.00 & 0.60 & 0.70 & 0.30 \end{pmatrix}. \tag{31}$$

$$\widetilde{D} = \begin{pmatrix} 0.4000 & 0.4000 & 0.3333 & 0 & 0 & 0 & 0.2143 & 0.3636 & 0.2000 \\ 0.1333 & 0.1333 & 0.1667 & 0 & 0 & 0 & 0.2857 & 0.1364 & 0.3000 \\ 0.2000 & 0.2000 & 0.2500 & 1 & 1 & 1 & 0.2857 & 0.1818 & 0.3500 \\ 0.2667 & 0.2667 & 0.2500 & 0 & 0 & 0 & 0.2143 & 0.3182 & 0.1500 \end{pmatrix}. \tag{32}$$

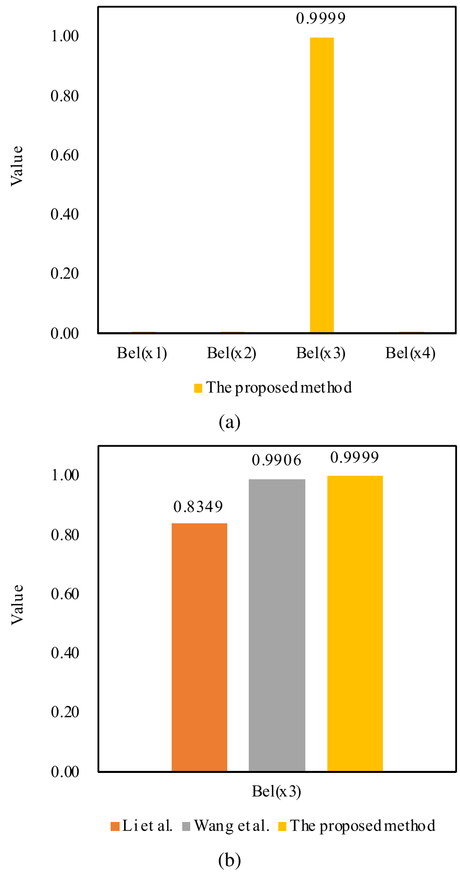

FIGURE 3. The comparison of the belief measure of different methods in the application.

0.0000001 which are obtained by the proposed method. It indicates that the proposal by integrating fuzzy preference relations analysis based on the belief entropy with D–S evidence theory can reduce uncertainty resulting from human's subjective cognition, such that it can improve the decision-making level. Furthermore, according to the belief measure of the alternatives in Table 16 and Fig. 3, and the measure of performance γ in Table 17, the proposal is more accurate and efficient compared with the previous methods [38] and [39].

TABLE 16. The belief measures of alternatives in terms of different methods.

Method	$Bel(x_1)$	$Bel(x_2)$	$Bel(x_3)$	$Bel(x_4)$
Li et al. [38]	0.0827	0.0284	0.8349	0.0471
Wang et al. [39]	0.0071	0.0004	0.9906	0.0018
Proposed method	0.0000216	0.0000037	0.9999661	0.0000086

TABLE 17. The comparison of different methods.

Method	Ranking	Optimum	$m(\Theta)$	γ
Li et al. [38]	$x_3 > x_1 > x_4 > x_2$	x_3	0.0069	4.716
Wang et al. [39]	$x_3 > x_1 > x_4 > x_2$	x_3	0.0001	4.716
Proposed method	$x_3 > x_1 > x_4 > x_2$	x_3	0.0000001	4.716

VI. CONCLUSION

In this paper, a hybrid method for utilizing fuzzy soft sets in decision making by integrating fuzzy preference relations analysis based on the belief entropy with Dempster–Shafer (D–S) evidence theory was proposed. The proposed method regarded not only the uncertainty measure of parameters, but also the impact of the relative reliability of parameters. The proposed hybrid method was composed of four parts. Firstly, the belief entropy was utilised to measure the uncertainties of parameters. Then, the uncertainties of parameters were

$$P = \begin{pmatrix} 0.5000 & 0.5000 & 0.5082 & 0.5589 & 0.5589 & 0.5589 & 0.5120 & 0.5006 & 0.5039 \\ 0.5000 & 0.5000 & 0.5082 & 0.5589 & 0.5589 & 0.5589 & 0.5120 & 0.5006 & 0.5039 \\ 0.4918 & 0.4918 & 0.5000 & 0.5509 & 0.5509 & 0.5509 & 0.5038 & 0.4925 & 0.4958 \\ 0.4411 & 0.4411 & 0.4491 & 0.5000 & 0.5000 & 0.5000 & 0.4529 & 0.4417 & 0.4450 \\ 0.4411 & 0.4411 & 0.4491 & 0.5000 & 0.5000 & 0.5000 & 0.4529 & 0.4417 & 0.4450 \\ 0.4411 & 0.4411 & 0.4491 & 0.5000 & 0.5000 & 0.5000 & 0.4529 & 0.4417 & 0.4450 \\ 0.4880 & 0.4880 & 0.4962 & 0.5471 & 0.5471 & 0.5471 & 0.5000 & 0.4887 & 0.4920 \\ 0.4994 & 0.4994 & 0.5075 & 0.5583 & 0.5583 & 0.5583 & 0.5113 & 0.5000 & 0.5033 \\ 0.4961 & 0.4961 & 0.5042 & 0.5550 & 0.5550 & 0.5550 & 0.5080 & 0.4967 & 0.5000 \end{pmatrix}. \quad (33)$$

$$\overline{P} = \begin{pmatrix} 0.5000 & 0.5000 & 0.5081 & 0.5589 & 0.5589 & 0.5589 & 0.5119 & 0.5006 & 0.5039 \\ 0.5000 & 0.5000 & 0.5081 & 0.5589 & 0.5589 & 0.5589 & 0.5119 & 0.5006 & 0.5039 \\ 0.4919 & 0.4919 & 0.5000 & 0.5508 & 0.5508 & 0.5508 & 0.5038 & 0.4925 & 0.4958 \\ 0.4411 & 0.4411 & 0.4492 & 0.5000 & 0.5000 & 0.5000 & 0.4530 & 0.4417 & 0.4450 \\ 0.4411 & 0.4411 & 0.4492 & 0.5000 & 0.5000 & 0.5000 & 0.4530 & 0.4417 & 0.4450 \\ 0.4411 & 0.4411 & 0.4492 & 0.5000 & 0.5000 & 0.5000 & 0.4530 & 0.4417 & 0.4450 \\ 0.4881 & 0.4881 & 0.4962 & 0.5470 & 0.5470 & 0.5470 & 0.5000 & 0.4887 & 0.4920 \\ 0.4994 & 0.4994 & 0.5075 & 0.5583 & 0.5583 & 0.5583 & 0.5113 & 0.5000 & 0.5033 \\ 0.4961 & 0.4961 & 0.5042 & 0.5550 & 0.5550 & 0.5550 & 0.5080 & 0.4967 & 0.5000 \end{pmatrix}. \quad (34)$$

modulated via the relative reliability preference of parameters by making use of the fuzzy preference relations analysis. After that, an appropriate BPA in terms of each parameter was generated on the basis of the modulated uncertainty degrees of parameters. On this basis, we adopted the Dempster's combination rule to fuse the independent evidences, i.e., parameters into an integrate evidence; thus, the best one could be obtained based on the ranking candidate alternatives. Afterwards, the proposed method was compared with the related works through a numerical example. The results showed that the uncertainty's belief measure fell from 0.0051 to 0.0031 in Example 5; meanwhile, the belief value of the best candidate increased from 0.3803 to 0.4029, so that the decision-making level was improved. Additionally, the proposed method was implemented in a medical diagnosis application, where the uncertainty's belief measure reduced from 0.0001 to 0.0000001; at the same time, the belief value of the best candidate increased from 0.9906 to 0.9999661 which also improved the decision-making level. Consequently, it can be concluded that the proposed method was more efficient than the related works, because the uncertainty resulting from human's subjective cognition was reduced and the decision-making level was improved with better performance by using the proposed method.

VII. CONFLICT OF INTEREST

The authors state that there are no conflicts of interest.

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