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Event-Triggered H∞ Fuzzy Filtering for Networked Control Systems With Quantization and Delays

ZHONG-DA LU[1,2], GUANG-TAO RAN[1], GUO-LIANG ZHANG[1], AND FENG-XIA XU[1]

[1]Department School of Computer and Control Engineering, Qiqihar University, Qiqihar 161006, China
[2]College of Computer Science and Technology, Harbin University of Science and Technology, Harbin 150080, China

Corresponding author: Zhong-da Lu (luzhongda@163.com)

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ABSTRACT This paper investigates the event-triggered H_∞ filtering for networked control systems (NCSs) with quantization and network-induced delays. With consideration of the limited capacity of the communication channels in NCSs, a quantizer is proposed to quantize control signals before being transmitted into the next node. First, an event-triggered scheme is addressed between the quantizer and the fuzzy filter, which aims to mitigate transmission rate and improve the usage of network resource. Based on the event-triggered scheme, the fuzzy filtering error system is established with quantization and delays. Second, a novel Lyapunov-Krasovskii functional is constructed, and the Writinger inequality is used to deal with the integral item of the derivative of the Lyapunov-Krasovskii functional, which can obtain more useful items. Then, a new stability criterion is addressed to ensure that the filtering error system is asymptotically stable with a prescribe H_∞ performance level. By introducing matrix decoupling technique, the fuzzy filter is designed without the coupling matrices. Finally, numerical simulations are given to show the effectiveness of the proposed method.

INDEX TERMS Event-triggered scheme, quantization, fuzzy filter, networked control systems, Writinger inequality.

I. INTRODUCTION

Networked control systems (NCSs) are such a class of systems connecting the sensor devices, control facilities and actuating units through a common communication network. There are many advantages in NCSs such as flexibility, low installation and high reliability which makes it has been widely applied in many fields. For example, the artificial intelligence, robots, smart grids, and Driver-less cars.

However, data packet transmission via communication network, the network time delays, packet dropouts and disorder may degrade the performance of NCSs and even destabilize the systems, which would be great challenges that the NCSs have to face. So, up to now, many efforts have been taken to modeling, analysis and design of NCSs in the undesired environment. The phenomenon of network-induced delays was studied to analysis and design control systems via an integral-inequality approach [1]. Aiming at the problem of data loss, the NCSs were modeled as the uncertain Markov jump linear system and designed fault detection filter in [2]. In view of the limited communication capacity of the network and the usage of the digital devices, a general logarithmic quantizer was proposed in [3].

In addition, the fuzzy control theory has been received considerable development because traditional linear systems theory can not be directly used for nonlinear systems. In 1985, Takagi and Sugeno firstly proposed T-S fuzzy system and then it became the most popular method to handle nonlinear systems in terms of its high capacity on model nonlinear systems [4]. Based on T-S fuzzy system, the author in [5] proposed a new stabilization condition to reduce conservatism. Non-fragile control with guaranteed cost for nonlinear systems and the networked induced delay was studied in [6] and [7], respectively. An and Li considered uncertain T-S fuzzy systems with interval time-varying delay, a novel Laypunov-Krasovskii function was employed to derive a new stability condition [8]. He *et al.* [9] investigated the problem of filtering for discrete-time nonlinear time-varying systems.

Note that the aforementioned statements are based on time-triggered. In time-triggered control environment, the sample data will be transmitted into the controller via network communication whether the data are desired or not, which leads to the waste of network resource and low efficiency [10]. Therefore, in recent years, an event-triggered scheme has been proposed to determine the data packet to transmit or not in network [11]. Static and dynamic event-triggered strategies were developed to reduce the utilization communication resources under packet losses [12]. Robust output feedback controller was designed base on an event-triggered scheme to save network resources and maintain desired performance in [13]. In the practical control loops, some state variables are not measured in disturbance systems. Therefore, filter is important for estimating the unmeasured states and eliminating the effects of extern disturbances. Yan *et al.* [14] discussed filtering for the linear system with time-varying delay. Zhang and Peng investigated filtering for networked T-S fuzzy systems [15]. For the NCSs with network-induced delays, how to propose an appropriate event-triggered scheme and design a suitable filter is practically valuable and still unresolved in [16]–[18]. Tackling these issues is one of the motivation of the current paper.

It is well known that the analog data signals must be converted into digital signals when the data signals were transmitted from sensor to controller. But in the limited precision of sensor and the limited networked bandwidth, the integrity of packet and the performance of systems can not be guaranteed. Hence, data signals must be quantized before transmission. To the best of our knowledge, little research has focused on the quantization for event-triggered control system, taking event-triggered scheme, filtering and quantization for networked control systems into account have not been investigated yet. Therefore, how to co-design the quantizer, filter and event-triggered scheme for NCSs is still needed. This is another motivation of the current paper.

Motivated by discussions above, this paper aims to handle the waste of network resources, quantization and filtering problems for NCSs with network-induced delays. The rest of this paper is organized as follows. The problem formulation is statemented in Section 2, a quantizer and an event-triggered scheme are employed to save network resources and energy. Stability analysis and fuzzy filter design are obtained in Section 3, by constructing a Lyapunov-Krasovskii functional, a new stability criterion is proposed to prove a less conservative than exist ones. An applicable fuzzy filter is co-designed with the quantization and the event-triggered scheme, which guarantee stability and a desire performance of the filtering error system. Simulation results are presented in Section 4 to show the effectiveness of the proposed method.

Notations:

Throughout this paper, $\Re^n$ denote the n dimensional Euclidean space, $\Re^{n\times m}$ is the set of $n\times m$ real matrices. Superscript $(\bullet)^T$ stands for the matrix transposition, I represents the identity matrix, $diag\{\ldots\}$ denotes the block-diagonal matrix. The notation $P > 0 (P < 0)$ means that the matrix P is a real symmetric positive define matrix. In symmetric block matrices, '$*$' is used as ellipsis for terms induced by symmetric.

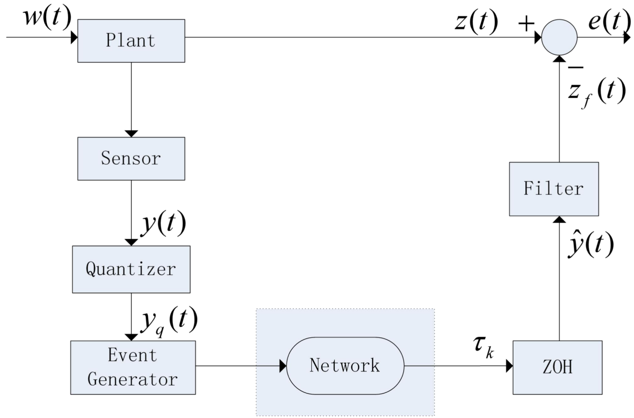

FIGURE 1. Framework of event-triggered filtering for NCSs with quantization and delays.

II. PROBLEM FORMULATION

In this section, we consider the NCSs with quantizer, event-generator and filter shown in Fig 1. The physical plant is given by:

$$\begin{cases} \dot{x}(t) = f_1(x(t)) + g_1(x(t))w(t) \\ z(t) = f_2(x(t)) + g_2(x(t))w(t) \\ y(t) = f_3(x(t)) \end{cases} \tag{1}$$

where $x(t)$ is the state vector, $y(t)$ is the measurement output, $z(t)$ is the signal to be estimated, $w(t)$ represent the input disturbance which belong to $L_2[0, \infty)$, and $f_i(x)$ $(i = 1, 2, 3)$ and $g_i(x)$ $(i = 1, 2)$ are continuous function of x.

The nonlinear plant is modeled by the following T-S fuzzy model,

Plant Rule: If $\theta_1(t)$ is M_{i1}, and $\theta_2(t)$ is M_{i2}, and, …, and $\theta_p(t)$ is M_{ip}.

Then :

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i w(t) \\ z(t) = C_i x(t) + D_i w(t) \\ y(t) = E_i x(t) \end{cases} \tag{2}$$

where M_{ij} are the fuzzy sets, $i = 1, 2, \ldots, r, j = 1, 2, \ldots, p$ is the number of fuzzy rules,$\theta_1(t), \theta_2(t), \ldots, \theta_p(t)$ are the premise variables. A_i, B_i, C_i, D_i, E_i are known parameter matrices with appropriate dimensions.

By employing product inference, singleton fuzzifier and centre defuzzifier in system (2). The fuzzy system is inferred as :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\theta(t))(A_i x(t) + B_i w(t)) \\ z(t) = \sum_{i=1}^{r} \mu_i(\theta(t))(C_i x(t) + D_i w(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\theta(t))E_i x(t) \end{cases} \tag{3}$$

where $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_p(t)]^T$, the fuzzy basis function are given by

$$\mu_i(\theta(t)) = \frac{\partial_i(\theta(t))}{\sum_{i=1}^{r} \partial_i(\theta(t))}, \quad \partial_i(\theta(t)) = \prod_{j=1}^{p} M_{ij}(\theta_j(t))$$

$M_{ij}(\theta_j(t))$ represents the grade of membership for $\theta_j(t)$ in M_{ij},$0 \leq \partial_i(\theta(t)) \leq 1, (i = 1, 2, \ldots, r)$.

Obviously, we have

$$\mu_i(\theta(t)) \geq 0, (i = 1, 2, \ldots, r), \quad \sum_{i=1}^{r} \mu_i(\theta(t)) = 1 \quad (4)$$

To be able to further development, we adopt the following assumptions.

Assumption 1: The measurement output are sampled at kh by sampler with a constant period h.

Assumption 2: Network-induced delays from the sensor to the quantizer, the quantizer to the event generator, the event generator to the filter, and the waiting delay are lumped together as τ_k, where

$$0 < \tau_m \leq \tau_k \leq \tau_M \quad (5)$$

where τ_m and τ_M are the lower and upper bound of τ_k, respectively.

Assumption 3: The packets are transmitted with a single packet, and the packets losses do not occur in whole control process.

A. QUANTIZER

As depicted in Fig 1, we consider the limited capacity of the network and the usage of the digital devices, the sampled signals are quantized before transmit into the digital devices. We first construct a quantizer between the sampler and the event-generator. The quantizer is defined as

$$q(y_q) = [q_1(y_1)q_2(y_2)\ldots q_n(y_n)]^T \quad (6)$$

the quantizer is a logarithmic quantizer with quantization levels :

$$\begin{aligned}\mu &= \{\pm u_i : u_i = \rho_i u_0, i = \pm 1, \pm 2, \ldots\} \cup \{\pm u_0\} \\ &\cup \{0\}, 0 < \rho_i < 1, u_0 > 0 \end{aligned} \quad (7)$$

The corresponding quantizer of q are defined as follows :

$$q(v) = \begin{cases} u_i, & if \ \frac{1}{1+\sigma_i} u_i < v < \frac{1}{1-\sigma_i} u_i, \quad v > 0 \\ 0, & if \ v = 0 \\ -q(-v), & if \ v < 0 \end{cases} \quad (8)$$

where

$$\sigma_i = \frac{1-\rho_i}{1+\rho_i} \quad (9)$$

ρ_i denote the quantization density. Without loss of generality, we define

$$\Delta_q = diag\{\Delta_{q_1}, \Delta_{q_2}, \ldots, \Delta_{q_n}\} \quad (10)$$

where $\Delta_{q_i} \in [-\sigma_i, \sigma_i]$.

Inspired by [19] and [20], the quantized signal $q(y_q)$ is represented as:

$$y_q(t) = q(y_q) = (I + \Delta_q)y(t) \quad (11)$$

B. EVENT-TRIGGERED SCHEME

To reduce the data transmission rate and save the limited network resource. Inspired by [11], we address an event-triggered communication scheme, which can be expressed as:

$$t_{k+1}h = t_k h + \min_{j\geq 1}\{o(t_k h + jh)\} \quad (12)$$

for

$$o(t_k h + jh) = e_k(t_k h + jh)^T V_1 e_k(t_k h + jh) \geq \varepsilon y(t_k h)^T V_2 y(t_k h),$$

where $0 < \varepsilon < 1$ and $V_i > 0, i = 1, 2$ are triggered parameters. $e(t_k h+jh) = y_e(t_k h+jh)-y(t_k h)$ is the threshold error. $y(t_k h)$ is the last transmitted data, $y_e(t_k h + jh)$ is the current sampling data via quantized, which is expressed as

$$y_e(t_k h + jh) = y_q(t_k h + jh) = (I + \Delta_q)y(t_k h + jh) \quad (13)$$

Remark 1: Note that all the quantized data are transmitted periodically into the communication network for traditional time-triggered control system. But in this paper, the event-triggered scheme is addressed to determine the current quantized data should be transmitted or not be. Only the current quantized data and the last transmitted data satisfied the triggered threshold, the current quantized data will transmit to the filter. In comparison with traditional time-triggered control system, it is obvious that the data transmission rate can be reduced. Meanwhile, the network resource can be saved.

Remark 2: If $\varepsilon = 0$, the system is degraded as periodically transmitted system. It should be mentioned that $V_1 \neq V_2$>0 is different from the works in [11]. When $V_1 = V_2 \neq 0$, the event-triggered scheme is the same as in [11]. Therefore, the event-triggered scheme design method is more flexible in this paper. Note that the next triggered instant depend not only on the trigger parameter ε, V_1 and V_2, but also depend on the quantized output data.

C. FUZZY FILTER

In this section, consider the effect of quantizer, event-triggered scheme and networked induced delays, we supposed that a full-order fuzzy filter is designed to estimate the signal $z(t)$ as the following form:

Plant rule: If $\theta_1(t_k h)$ is W_{i1}, and $\theta_2(t_k h)$ is W_{i2}, and, ..., and $\theta_p(t_k h)$ is W_{ip}.

Then:

$$\begin{cases} \dot{x}_f(t) = A_{fj}x_f(t) + B_{fj}\hat{y}(t) \\ z_f(t) = C_{fj}x_f(t) + D_{fj}\hat{y}(t) \end{cases} \quad (14)$$

where $x_f(t) \in \Re^n$ is the filter state, $\hat{y}(t)$ is the real input of the filter, $z_f(t)$ is the estimate of $z(t)$. A_{fj}, B_{fj}, C_{fj}, D_{fj} are filter parameters to be determined.

Using the same method as in (3), we can infer the fuzzy filter as the following form:

$$\begin{cases} \dot{x}_f(t) = \sum_{j=1}^{r} u_j(\theta(t_k h))(A_{fj}x_f(t) + B_{fj}\hat{y}(t)) \\ z_f(t) = \sum_{j=1}^{r} u_j(\theta(t_k h))(C_{fj}x_f(t) + D_{fj}\hat{y}(t)) \end{cases} \tag{15}$$

Remark 3: Due to the effects of the event-triggered scheme, the premise variables $\theta(t)$ are available in equation (3), but not available in equation (15). The premise variables for the fuzzy filter are $\theta(t_k h)$, which is different from $\theta(t)$ in equation (3).

Note that $\hat{y}(t) \neq y(t)$, because the effects of the event-triggered communication scheme and the networked induced delay. Therefore, consider the effect of ZOH and the event-triggered communication scheme, the input of filter is modeled as $\hat{y}(t) = y(t_k h)$. The ZOH holds the last transmitted signal until the next signal update from the event-triggered mechanism.

In addition, the input signal of the fuzzy filter instant can be described as

$$t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{16}$$

Then, we divide (16) into sub-intervals as

$$[t_k h + \tau_{tk}, t_{k+1}h + \tau_{t_{k+1}}) = \bigcup_{l=0}^{m} \Omega_l \tag{17}$$

where

$$\Omega_i = [t_k h + lh + \tau_{t_{k+i}}, t_k h + lh + \tau_{t_{k+i+1}}), \quad l = 0, 1, \ldots, m.$$

Next, defining function $\tau(t)$ as

$$\tau(t) = \begin{cases} t - t_k h & t \in \Omega_0 \\ t - t_k h - h & t \in \Omega_1 \\ \vdots & \vdots \\ t - t_k h - (m-1)h & t \in \Omega_m \end{cases} \tag{18}$$

Note that $\tau(t)$ is a piecewise function, which satisfies

$$0 < \tau_m = \tau_1 \leq \tau(t) \leq h + \tau_M = \tau_3, \quad \dot{\tau}(t) = 1, \; t \in \Omega_i,$$

and $\tau(t) = t - i_k h$, where $i_k h = t_k h + lh$.

Hence, the final input of the fuzzy filter is rewrote as

$$\hat{y}(t) = y(t_k h) = (I + \Delta_q)y(t - \tau(t)) - e_k(t - \tau(t)) \tag{19}$$

D. FUZZY FILTERING ERROR SYSTEM

In this section, we construct a filtering error system according to the aforementioned. Denote

$$\tilde{x} = col\{x(t), x_f(t)\}, \quad e(t) = z(t) - z_f(t).$$

Combing (3), (15) and (19), the filtering error system is represented as

$$\begin{cases} \dot{\tilde{x}}(t) = \sum_{i=i}^{r}\sum_{j=1}^{r} u_i u_j\{\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij1}x(t-\tau(t)) \\ +\tilde{B}_{ij2}e_k(t-\tau(t)) + \tilde{B}_{wij}w(t)\} \\ e(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} u_i u_j\{\tilde{C}_{ij}\tilde{x}(t) + \tilde{D}_{ij1}x(t-\tau(t)) \\ +\tilde{D}_{ij2}e_k(t-\tau(t)) + D_i w(t)\} \end{cases} \tag{20}$$

where

$$\tilde{A}_{ij} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fi} \end{bmatrix}, \quad \tilde{B}_{ij1} = \begin{bmatrix} 0 \\ B_{fi}(I+\Delta_q)E_i \end{bmatrix},$$

$$\tilde{B}_{ij2} = \begin{bmatrix} 0 \\ -B_{fi} \end{bmatrix}, \quad \tilde{B}_{wij} = \begin{bmatrix} B_i \\ 0 \end{bmatrix},$$

$$\tilde{C}_{ij} = \begin{bmatrix} C_i & -C_{fi} \end{bmatrix}, \quad \tilde{D}_{ij1} = -D_{fi}(I+\Delta_q)E_i, \; \tilde{D}_{ij2} = D_{fi}$$

Before end of this section, we make the following definition and some lemmas, which will be made the theoretical development easier.

Definition 1: The fuzzy filtering error system is asymptotically stable with an H_∞ performance, if the following two conditions hold:

1) The filtering error system is asymptotically stable with $w(t) = 0$.

2) Under zero initial condition, the filtering error $e(t)$ satisfies

$$\|e(t)\|_2 \leq \gamma \, \|w(t)\|_2$$

for any nonzero $\omega(t) \in L_2[0, \infty)$and a prescribed $\gamma > 0$.

Lemma 1 (Schur complement [21]): For a given symmetric matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

where $a_{11} \in R^{r \times r}$, the following conditions are equivalent:

1) $A < 0$

2) $a_{11} < 0, \quad a_{22} - a_{12}^T a_{11}^{-1} a_{12} < 0$

3) $a_{22} < 0, \quad a_{11} - a_{21}^T a_{22}^{-1} a_{21} < 0$

*Lemma 2 (Seuret and Gouaisbaut [22]):*For a given symmetric and positive matrix $Z > 0$ of appropriate dimensions and different signal x in $[a, b] \to R^n$, the following inequality holds:

$$-\int_a^b \dot{x}^T(s)Z\dot{x}(s)ds \leq -\frac{1}{b-a}\begin{bmatrix} \dot{x}(b) \\ \dot{x}(a) \\ v \end{bmatrix}^T \vartheta(Z) \begin{bmatrix} \dot{x}(b) \\ \dot{x}(a) \\ v \end{bmatrix}$$

where

$$v = \frac{1}{b-a}\int_a^b x(s)ds, \quad \vartheta(Z) = \begin{bmatrix} a_1 Z & a_2 Z & a_3 Z \\ * & a_1 Z & a_3 Z \\ * & * & a_4 Z \end{bmatrix}$$

$$a_1 = \frac{\pi^2}{4} + 1, a_2 = \frac{\pi^2}{4} - 1, a_3 = -\frac{\pi^2}{2}, a_4 = \pi^2$$

III. STABILITY ANALYSIS AND FUZZY FILTER DESIGN

In this section, we will propose a Lyapunov-Krasovskii functional to analyze the filtering error system. A new filter design criterion with less conservative is provided by using Writinger inequality.

A. STABILITY ANALYSIS

Theorem 1 : For given positive parameters $\tau_1, \tau_3, \gamma > 0$, and $0 < \varepsilon < 1$, the filtering error system in equation (20) is asymptotically stable with H_∞ performance γ under the quantization (11) and the event-triggered scheme (12), if there exists matrices $Q_i > 0 (i = 1, 2)$, $Z_i > 0 (i = 1, 2, 3)$, $V_i > 0$ $(i = 1, 2)$, $a_i > 0 (i = 1, 2, 3, 4)$ $P > 0$, and matrices A_{fj}, B_{fj}, C_{fj}, D_{fj} with appropriate dimensions such that the following matrix inequities hold

$$\begin{bmatrix} \Xi_{ii}^{11} & \Xi_{ii}^{12} \\ * & \Xi_{22} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r \tag{21}$$

$$\begin{bmatrix} \Xi_{ij}^{11} + \Xi_{ji}^{11} & \Xi_{ij}^{12} + \Xi_{ji}^{12} \\ * & \Xi_{22} \end{bmatrix} < 0, \quad 1 \le i < j \le r \tag{22}$$

for

$$\Xi_{ij}^{11} = \begin{bmatrix} \Lambda_1 & \Lambda_2 \\ * & \Lambda_3 \end{bmatrix},$$

$$\Lambda_1 = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} \\ * & \varphi_{22} & \varphi_{23} & 0 & \varphi_{25} \\ * & * & \varphi_{33} & \varphi_{34} & 0 \\ * & * & * & \varphi_{44} & 0 \\ * & * & * & * & \varphi_{55} \end{bmatrix},$$

$$\Lambda_2 = \begin{bmatrix} 0 & 0 & \varphi_{18} & \varphi_{19} & \varphi_{110} \\ \varphi_{26} & 0 & 0 & 0 & 0 \\ \varphi_{36} & \varphi_{37} & 0 & 0 & 0 \\ 0 & \varphi_{47} & \varphi_{48} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Lambda_3 = \begin{bmatrix} \varphi_{66} & 0 & 0 & 0 & 0 \\ * & \varphi_{77} & 0 & 0 & 0 \\ * & * & \varphi_{88} & 0 & 0 \\ * & * & * & \varphi_{99} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

where

$$\begin{aligned}
\varphi_{11} &= P\tilde{A}_{ij} + \tilde{A}_{ij}^T P + H^T(Q_1 + Q_2)H \\
&\quad - a_1 H^T(Z_1 + Z_3)H, \\
\varphi_{12} &= -a_2 H^T Z_1, \varphi_{13} = P\tilde{B}_{ij1}, \quad \varphi_{14} = -a_2 H^T Z_3, \\
\varphi_{15} &= -a_3 H^T Z_1, \quad \varphi_{18} = -a_3 H^T Z_3, \quad \varphi_{19} = -P\tilde{B}_{ij2}, \\
\varphi_{110} &= P\tilde{B}_{wij}, \varphi_{22} = -Q_1 - a_1 Z_1 - a_1 Z_2, \quad \varphi_{23} = -a_2 Z_2, \\
\varphi_{25} &= -a_3 Z_1, \varphi_{26} = -a_3 Z_2, \\
\varphi_{33} &= -2a_1 Z_2 - \varepsilon_1 E_i^T V_2 E_i, \quad \varphi_{34} = -a_2 Z_2, \\
\varphi_{36} &= -a_3 Z_2, \quad \varphi_{37} = -a_3 Z_2, \\
\varphi_{44} &= -Q_2 - a_1 Z_2 - a_1 Z_3, \quad \varphi_{47} = -a_3 Z_3, \\
\varphi_{48} &= -a_3 Z_3, \\
\varphi_{55} &= -a_4 Z_1, \quad \varphi_{66} = -a_4 Z_2, \ \varphi_{77} = -a_4 Z_2, \\
\varphi_{88} &= -a_4 Z_3, \quad \varphi_{99} = \varepsilon_1 V_2 - V_1,
\end{aligned}$$

$$\begin{aligned}
\phi_{ij}^1 &= \begin{bmatrix} \tilde{A}_{ij} & 0 & \tilde{B}_{ij1} & 0 & 0 & 0 & 0 & 0 & \tilde{B}_{ij2} & \tilde{B}_{wij} \end{bmatrix}, \\
\phi_{ij}^2 &= \begin{bmatrix} \tilde{C}_{ij} & 0 & \tilde{D}_{ij1} & 0 & 0 & 0 & 0 & 0 & D_{fj}(I + \Delta q) & D_i \end{bmatrix}, \\
\Xi_{ij}^{12} &= \begin{bmatrix} \tau_1(\phi_{ij}^1)^T Z_1 & \tau_2(\phi_{ij}^1)^T Z_2 & \tau_3(\phi_{ij}^1)^T Z_3 & (\phi_{ij}^2)^T \end{bmatrix}, \\
\Xi_{22} &= diag\left\{ -Z_1 \ -Z_2 \ -Z_3 \ -I \right\}, \\
\tau_2 &= \tau_3 - \tau_1, \quad H = \begin{bmatrix} I & 0 \end{bmatrix}.
\end{aligned}$$

Proof : We construct a Lyapunov-Krasovskii functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{23}$$

where

$$\begin{aligned}
V_1(t) &= \tilde{x}^T(t) P \tilde{x}(t), \\
V_2(t) &= \int_{t-\tau_1}^{t} x^T(s) Q_1 x(s) ds + \int_{t-\tau_3}^{t} x^T(s) Q_2 x(s) ds, \\
V_3(t) &= \tau_1 \int_{-\tau_1}^{0} \int_{t+s}^{t} \dot{x}^T(v) Z_1 \dot{x}(v) dv ds \\
&\quad + \tau_2 \int_{-\tau_3}^{-\tau_1} \int_{t+s}^{t} \dot{x}^T(v) Z_2 \dot{x}(v) dv ds \\
&\quad + \tau_3 \int_{-\tau_3}^{0} \int_{t+s}^{t} \dot{x}^T(v) Z_3 \dot{x}(v) dv ds.
\end{aligned}$$

Taking the time derivation of $V(t)$ for t, we have

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \tag{24}$$

for

$$\dot{V}_1(t) = 2\tilde{x}^T(t) P \dot{\tilde{x}}(t) \tag{25}$$

$$\begin{aligned}
\dot{V}_2(t) &= x^T(t) Q_1 x(t) - x^T(t-\tau_1) Q_1 x(t-\tau_1) \\
&\quad + x^T(t) Q_2 x(t) - x^T(t-\tau_3) Q_2 x(t-\tau_3)
\end{aligned} \tag{26}$$

$$\begin{aligned}
\dot{V}_3(t) &= \tau_1^2 \dot{x}^T(t) Z_1 \dot{x}(t) - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(v) Z_1 \dot{x}(v) dv \\
&\quad + \tau_2^2 \dot{x}^T(t) Z_2 \dot{x}(t) - \tau_2 \int_{t-\tau_3}^{t-\tau_1} \dot{x}^T(v) Z_2 \dot{x}(v) dv \\
&\quad + \tau_3^2 \dot{x}^T(t) Z_3 \dot{x}(t) - \tau_3 \int_{t-\tau_3}^{t} \dot{x}^T(v) Z_3 \dot{x}(v) dv
\end{aligned} \tag{27}$$

Note that the integral items in $\dot{V}_3(t)$, we apply Lemma 2 to deal with it and obtain

$$-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(v) Z_1 \dot{x}(v) dv \le -\eta_1^T(t) \vartheta_1(Z_1) \eta_1(t) \tag{28}$$

$$-\tau_3 \int_{t-\tau_3}^{t} \dot{x}^T(v) Z_3 \dot{x}(v) dv \le -\eta_4^T(t) \vartheta_4(Z_3) \eta_4(t) \tag{29}$$

$$\begin{aligned}
&-\tau_2 \int_{t-\tau_3}^{t-\tau_1} \dot{x}^T(v) Z_2 \dot{x}(v) dv \\
&\le -\tau_2 \left(\int_{t-\tau(t)}^{t-\tau_1} \dot{x}^T(v) Z_2 \dot{x}(v) dv + \int_{t-\tau_3}^{t-\tau(t)} \dot{x}^T(v) Z_2 \dot{x}(v) dv \right) \\
&\le -\eta_2^T(t) \vartheta_2(Z_2) \eta_2(t) - \eta_3^T(t) \vartheta_3(Z_2) \eta_3(t)
\end{aligned} \tag{30}$$

where

$$\eta_1^T = \begin{bmatrix} x^T(t) & x^T(t-\tau_1) & \upsilon_1^T \end{bmatrix},$$
$$\eta_2^T = \begin{bmatrix} x^T(t-\tau_1) & x^T(t-\tau(t)) & \upsilon_2^T \end{bmatrix},$$
$$\eta_3^T = \begin{bmatrix} x^T(t-\tau(t)) & x^T(t-\tau_3) & \upsilon_3^T \end{bmatrix},$$
$$\eta_4^T = \begin{bmatrix} x^T(t) & x^T(t-\tau_3) & \upsilon_4^T \end{bmatrix},$$
$$\vartheta_1(Z_1) = \begin{bmatrix} a_1Z_1 & a_2Z_1 & a_3Z_1 \\ * & a_1Z_1 & a_3Z_1 \\ * & * & a_4Z_1 \end{bmatrix},$$
$$\vartheta_2(Z_2) = \vartheta_3(Z_2) = \begin{bmatrix} a_1Z_2 & a_2Z_2 & a_3Z_2 \\ * & a_1Z_2 & a_3Z_2 \\ * & * & a_4Z_2 \end{bmatrix},$$
$$\vartheta_4(Z_3) = \begin{bmatrix} a_1Z_3 & a_2Z_3 & a_3Z_3 \\ * & a_1Z_3 & a_3Z_3 \\ * & * & a_4Z_3 \end{bmatrix},$$
$$\upsilon_1 = \frac{1}{\tau_1}\int_{t-\tau_1}^{t} x(s)ds, \quad \upsilon_2 = \frac{1}{\tau(t)-\tau_1}\int_{t-\tau(t)}^{t-\tau_1} x(s)ds$$
$$\upsilon_3 = \frac{1}{\tau_3-\tau(t)}\int_{t-\tau_3}^{t-\tau(t)} x(s)ds, \quad \upsilon_4 = \frac{1}{\tau_3}\int_{t-\tau_3}^{t} x(s)ds$$

By adding the right -side of (28), (29), (30) to (27), and combing with (24), (25), (26) and (27), we obtain

$$\dot V(t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) \le \xi^T(t)\Xi\xi(t) \tag{31}$$

where

$$\xi(t) = col\{\tilde x(t), x(t-\tau_1), x(t-\tau(t)), x(t-\tau_3), \upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, e_k(t-\tau(t)), w(t)\} \tag{32}$$
$$\Xi = \sum_{i=1}^{r} \mu_i^2(\theta(t))\bar\Xi_{ii} + \sum_{i=1}^{r-1}\sum_{j=i+1}^{r} \mu_i(\theta(t))\mu_j(\theta(t_kh))(\bar\Xi_{ij}+\bar\Xi_{ji}) \tag{33}$$
$$\bar\Xi_{ij} = \Xi_{ij}^{11} - (\Xi_{ij}^{12})^T\Xi_{22}^{-1}\Xi_{ij}^{12} \tag{34}$$

Ξ_{ij}^{11}, Ξ_{ij}^{12}, Ξ_{22} are defined above.

By the Lemma 1 (Schur complement) and from (21), (22), (33) and (34), we can conclude that the filtering error system (20) with $\omega(t) = 0$ is asymptotically stable. Therefore, the inequality (31) is rewrote as the following:

$$\dot V(t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) \le 0 \tag{35}$$

Integrating the right and left sides of (35) from 0 to ∞ on t, and considering under the zero initial condition, we derives

$$\int_0^\infty e^T(t)e(t)d(t) \le \gamma^2 \int_0^\infty w^T(t)w(t)d(t) \tag{36}$$

obviously, $\|e(t)\|_2 \le \gamma\,\|\omega(t)\|_2$. This completes the proof.

Remark 4: It is worth mentioning that in the previous studies (see [15]), some negative items are ignored when the process of stability analysis, which may lose much useful information and lead to conservative. In this paper, the negative terms are considered in inequality (28) to (30). Meanwhile, the integral terms $\upsilon_i(i = 1, 2, 3, 4)$ in (28) to (30) contain more useful information of the system. Therefore, by employing the Writinger inequality in Lyapunov-Krasovskii functional is more effective in conservatism reduction than traditionally inequality.

B. FUZZY FILTER DESIGN

Note that theorem 1 provides a sufficient stability condition, but the filter arguments with coupled matrix P which lead to calculating difficultly. Thus, by using matrix decoupling technique to tackle it, a suitable fuzzy filter is derived.

Theorem 2: For given positive parameters τ_1, τ_3, $\gamma > 0$, and $0 < \varepsilon < 1$, if there exists matrices $X > 0$, $W > 0$, $Q_i > 0(i = 1, 2)$, $Z_i > 0(i = 1, 2, 3)$, $V_i > 0(i = 1, 2)$, $a_i > 0(i = 1, 2, 3, 4)$, and matrices $\tilde A_{fj}$, $\tilde B_{fj}$, $\tilde C_{fj}$, $\tilde D_{fj}$ with appropriate dimensions in the following hold:

$$X - W > 0 \tag{37}$$
$$\begin{bmatrix} \tilde\Theta_{ii}^{11} & \tilde\Theta_{ii}^{12} \\ * & \tilde\Theta_{22} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r \tag{38}$$
$$\begin{bmatrix} \tilde\Theta_{ij}^{11} + \tilde\Theta_{ji}^{11} & \tilde\Theta_{ij}^{12} + \tilde\Theta_{ji}^{12} \\ * & \tilde\Theta_{22} \end{bmatrix} < 0, \quad 1 \le i < j \le r \tag{39}$$

for

$$\tilde\Theta_{ij}^{11} = \begin{bmatrix} \tilde\Lambda_1 & \tilde\Lambda_2 \\ * & \tilde\Lambda_3 \end{bmatrix},$$
$$\tilde\Lambda_1 = \begin{bmatrix} \tilde\varphi_{11} & \tilde\varphi_{12} & \tilde\varphi_{13} & \tilde\varphi_{14} & -a_2Z_3 & -a_3Z_1 \\ * & \tilde\varphi_{22} & 0 & \tilde\varphi_{24} & 0 & 0 \\ * & * & \varphi_{22} & \varphi_{23} & 0 & \varphi_{25} \\ * & * & * & \varphi_{33} & \varphi_{34} & 0 \\ * & * & * & * & \varphi_{44} & 0 \\ * & * & * & * & * & \varphi_{55} \end{bmatrix},$$
$$\tilde\Lambda_2 = \begin{bmatrix} 0 & 0 & -a_3Z_3 & -\tilde B_{fj} & XB_i \\ 0 & 0 & 0 & -\tilde B_{fj} & WB_i \\ \varphi_{26} & 0 & 0 & 0 & 0 \\ \varphi_{36} & \varphi_{37} & 0 & 0 & 0 \\ 0 & \varphi_{47} & \varphi_{48} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\tilde\Lambda_3 = \begin{bmatrix} \varphi_{66} & 0 & 0 & 0 & 0 \\ * & \varphi_{77} & 0 & 0 & 0 \\ * & * & \varphi_{88} & 0 & 0 \\ * & * & * & \varphi_{99} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}.$$

where

$$\tilde\varphi_{11} = XA_i + A_i^TX + Q_1 + Q_2 - a_1(Z_1 + Z_3),$$
$$\tilde\varphi_{12} = \tilde A_{fj} + A_i^TW, \quad \tilde\varphi_{22} = \tilde A_{fj} + \tilde A_{fj}^T, \ \tilde\varphi_{13} = -a_2Z_1,$$
$$\tilde\varphi_{14} = \tilde\varphi_{24} = \tilde B_{fj}(I + \Delta_q)E_i,$$
$$\tilde\phi_{ij}^1 = \begin{bmatrix} A_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_i \end{bmatrix},$$
$$\tilde\phi_{ij}^2 = \begin{bmatrix} C_i & -\tilde C_{fj} & 0 & -\tilde D_{fj}\tilde qE_i & \underbrace{0, \ldots, 0}_{5} & \tilde D_{fj}\tilde q & D_i \end{bmatrix},$$
$$\tilde\Theta_{ij}^{12} = \begin{bmatrix} \tau_1(\tilde\phi_{ij}^1)^TZ_1 & \tau_2(\tilde\phi_{ij}^1)^TZ_2 & \tau_3(\tilde\phi_{ij}^1)^TZ_3 & (\tilde\phi_{ij}^2)^T \end{bmatrix}$$
$$\tilde\Theta_{22} = diag\begin{Bmatrix} -Z_1 & -Z_2 & -Z_3 & -I \end{Bmatrix}, \tilde q = I + \Delta_q.$$

TABLE 1. Minimum performance level γ for different cases ($\tau_1 = 0.02$, τ_3 unknown).

τ_3	0.1	0.2	0.3	0.4	0.5
γ	0.2838	0.3095	0.3398	0.3641	0.3812

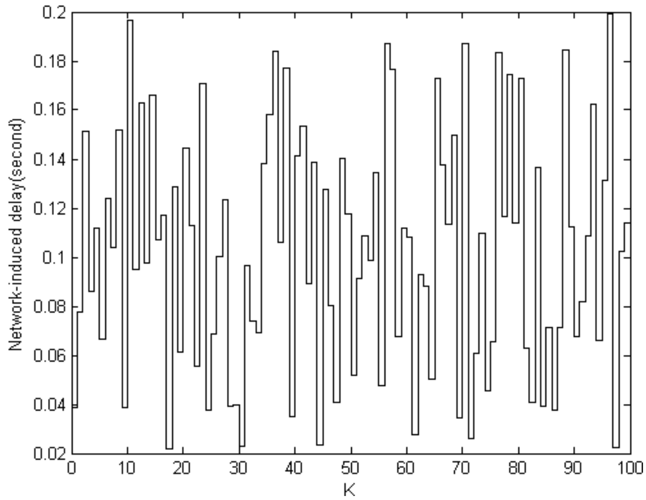

FIGURE 2. Network-induced delays.

In addition, if the above conditions are feasible, the parameter matrices of the fuzzy filter are given by

$$A_{fj} = W^{-1}\tilde{A}_{fj}, \quad B_{fj} = W^{-1}\tilde{B}_{fj}, \; C_{fj} = \tilde{C}_{fj}, \; D_{fj} = \tilde{D}_{fj}. \quad (40)$$

Proof: Define

$$P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}, \quad J_1 = \begin{bmatrix} I & 0 \\ 0 & P_3^{-1}P_2^T \end{bmatrix}, \; J_2 = \{I, I, I, I\}. \quad (41)$$

Where$P_1 > 0$ and $P_3 > 0$. For $P > 0$, we obtain that $P_3 + P_3^T > 0$ and P_3 is revertible.

Then, Pre- and post-multiply (21) and (22) by

$$J = diag\left\{J_1, \underbrace{I, \ldots I}_{9}, J_2\right\}$$

and their transposes. For the sake of convenience, we define

$$\begin{aligned} X &= P_1, \quad W = P_2P_3^{-1}P_2^T, \; \tilde{A}_{fj} = P_2A_{fj}P_3^{-1}P_2^T, \\ \tilde{B}_{fj} &= B_{fj}P_3^{-1}P_2^T, \quad \tilde{C}_{fj} = C_{fj}, \; \tilde{D}_{fj} = D_{fj}. \end{aligned} \quad (42)$$

Thus, (37) , (38) and (39) can be easily obtained from (21), (22), (40) and (41) respectively.

Moreover, it is worth mentioning that $P > 0$ is equaled to $P_1 - W > 0$ by using Lemma 1, so the form of fuzzy filter parameters are represented as follow:

$$\begin{bmatrix} \tilde{A}_{fj} & \tilde{B}_{fj} \\ \tilde{C}_{fj} & \tilde{D}_{fj} \end{bmatrix} = \begin{bmatrix} P_2 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{fj} & B_{fj} \\ C_{fj} & D_{fj} \end{bmatrix} \begin{bmatrix} P_3^{-1}P_2^T & 0 \\ 0 & I \end{bmatrix} \quad (43)$$

This completes the proof.

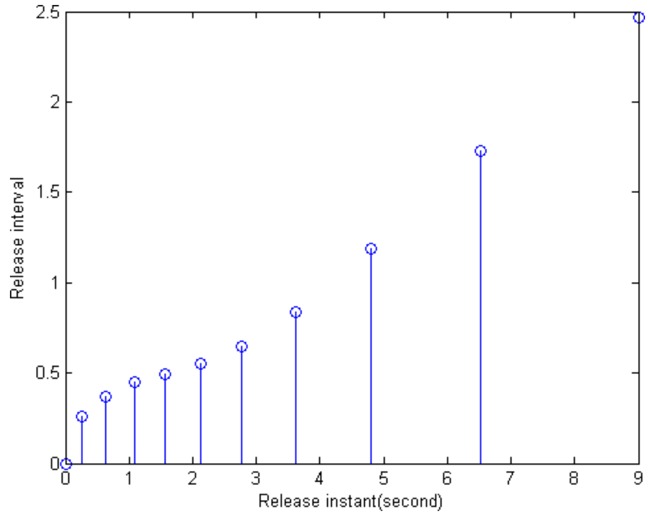

FIGURE 3. Release instant.

TABLE 2. Transmission rates and V_1, V_2 for differentε.

ε	V_1	V_2	transmission rates
0	86.5048	86.5499	100%
0.1	101.9005	149.0968	1.5%
0.2	112.9096	139.5361	1.1%
0.3	119.9453	121.0091	0.9%
0.4	124.6766	104.9026	0.9%
0.5	128.1039	92.0810	0.8%
0.6	130.7397	81.9106	0.8%
0.9	137.2800	56.6270	0.8%

IV. NUMERICAL SIMULATION

In this section, we provide two numerical examples to illustrate the effectiveness of the obtained results.

Example 1: Considering the fuzzy system in [21], the system parameters as follow:

$$\begin{aligned} A_1 &= \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -2.5 & 1 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.8 & 0.3 & 0 \end{bmatrix}, \quad D_1 = 0.2, \\ E_1 &= \begin{bmatrix} 0.5 & -0.1 & 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} -0.5 & 0.2 & 0.3 \end{bmatrix}, \quad D_2 = 0.5, \\ E_2 &= \begin{bmatrix} 0 & 1 & 0.6 \end{bmatrix}. \end{aligned}$$

Then, choosing the membership function and external disturbance as follows:

$$h_1 = \frac{1}{1 + e^{-x_1}}, \quad h_2 = 1 - h_1, \; w(t) = \frac{1}{1 + 3t^2}$$

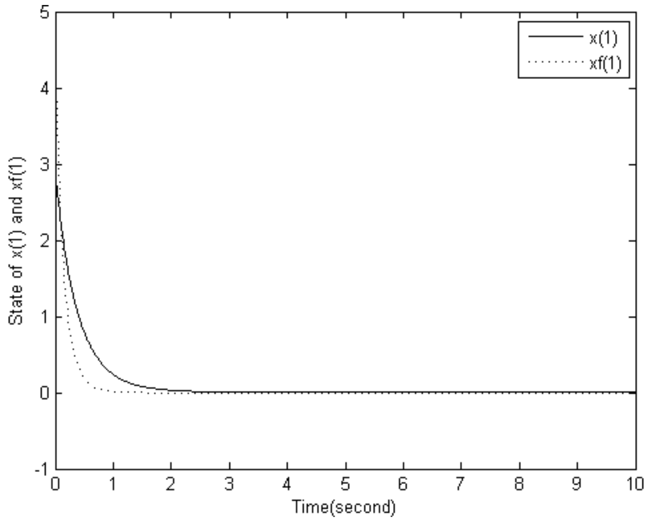

FIGURE 4. State trajectories of x_1.

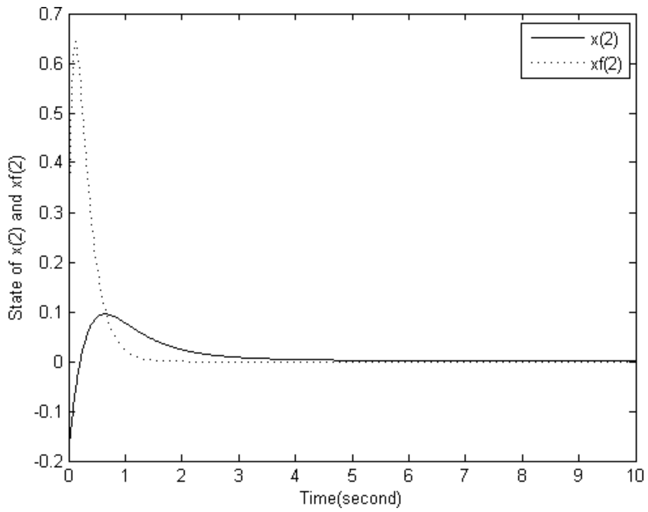

FIGURE 5. State trajectories of x_2.

Considering the induced delay maximum and minimum are $\tau_1 = 0.002$, $\tau_3 = 0.2$, respectively. Choosing $\varepsilon = 0.2$, $\rho = 0.8$, sampling period $h = 10ms$, by Theorem 2 we obtain the minimum performance level is $\gamma = 0.3095$ after 58 iterations, which is less than previous studied $\gamma = 0.6157$ in [21]. Then, we consider different τ_3 to find the minimum performance level, the results are listed in Table 1. The event-triggered arguments

$$V_1 = 112.9096, \quad V_2 = 139.5361,$$

and the fuzzy filter parameters are

$$A_{f1} = \begin{bmatrix} -7.4570 & 2.5462 & -1.0454 \\ 2.5872 & -7.3962 & 3.4256 \\ -1.0257 & 3.1925 & -9.6245 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -7.1526 & 2.4057 & -1.1008 \\ 2.3554 & -7.6137 & 2.9972 \\ -1.1641 & 3.2708 & -8.3402 \end{bmatrix},$$

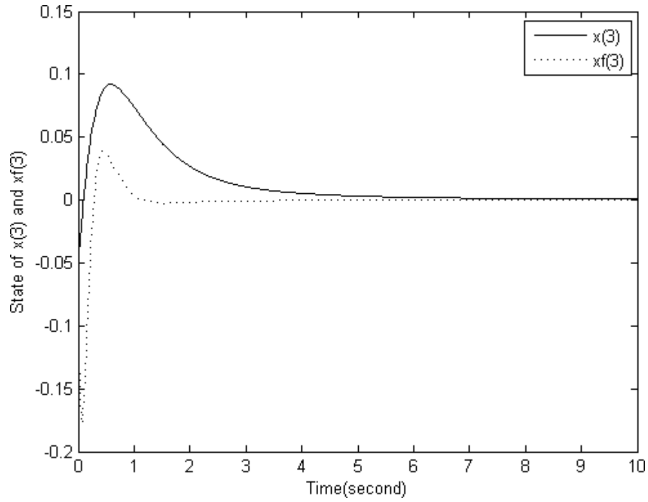

FIGURE 6. State trajectories of x_3.

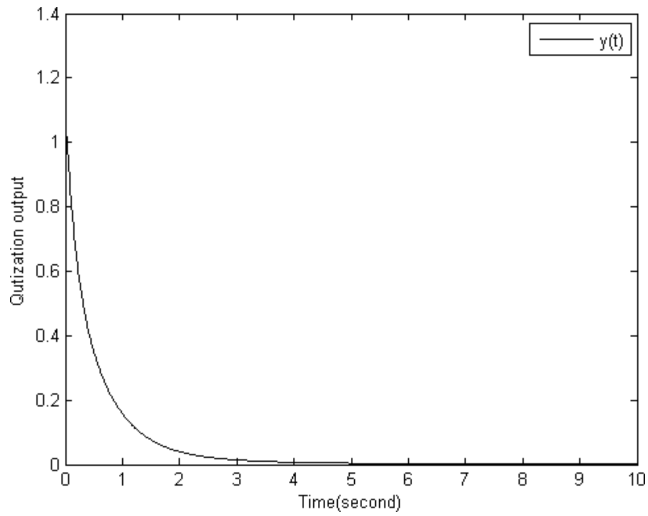

FIGURE 7. The trajectories of output signal after quantized.

$$B_{f1} = \begin{bmatrix} -0.1547 \\ 0.1701 \\ -0.2791 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -0.0280 \\ 0.0279 \\ -0.1878 \end{bmatrix}$$
$$C_{f1} = \begin{bmatrix} -3.3362 & -2.1929 & 0.4319 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} 0.1584 & -1.5944 & -0.9051 \end{bmatrix},$$
$$D_{f1} = 0.1659, \quad D_{f2} = 0.2125.$$

The distribution of network-induced delays (100 times) and the triggered time are described in Fig 2 and Fig 3, respectively. During the ten seconds, only 11 times are triggered in 1000 times. The transmission rates less than the existing ones in [21]. Meanwhile, we consider the different event-triggered scheme parameter ε to obtain corresponding trigger parameters matrix and transmission rate in Table 2. Based upon the Table 2, we can see that the event-triggered parameters V_1 nearly equal to V_2 when $\varepsilon = 0$, and the transmission rates is 100% because the event-triggered systems are degraded as time-triggered systems. On the contrary, the transmission rates are sharply decreased form 100% to 1.5 % in $\varepsilon = 0.1$. Observing ε change from 0.1 to 0.9, we can

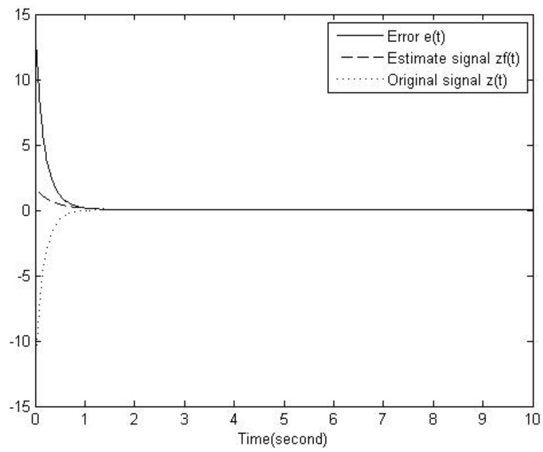

FIGURE 8. The trajectories of original signal, estimate signal and filter error.

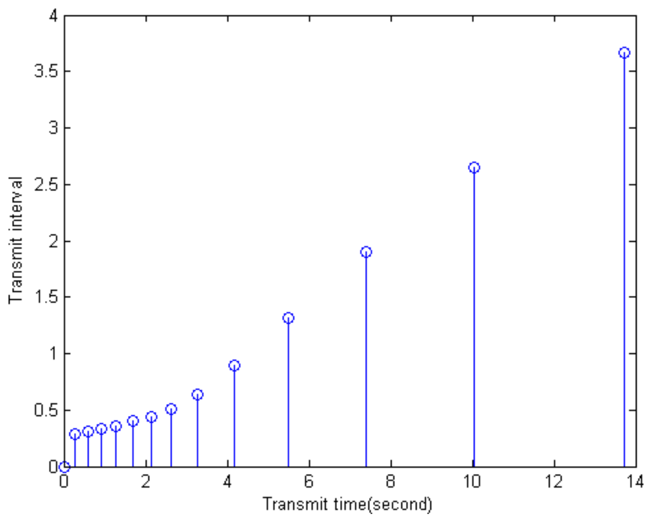

FIGURE 9. Release instant.

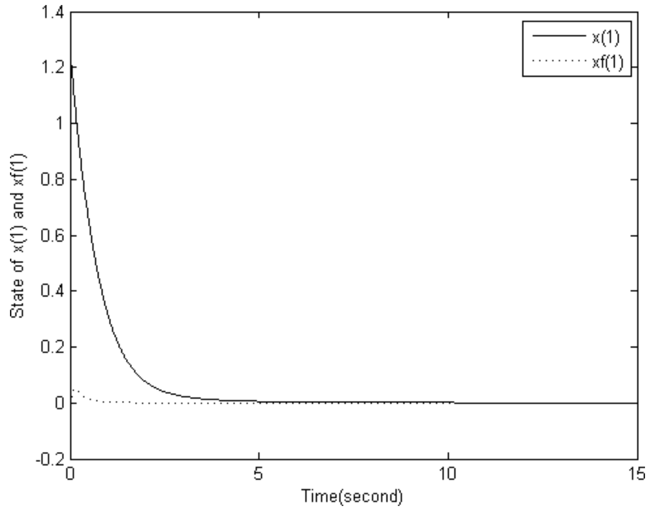

FIGURE 10. State trajectories of x_1.

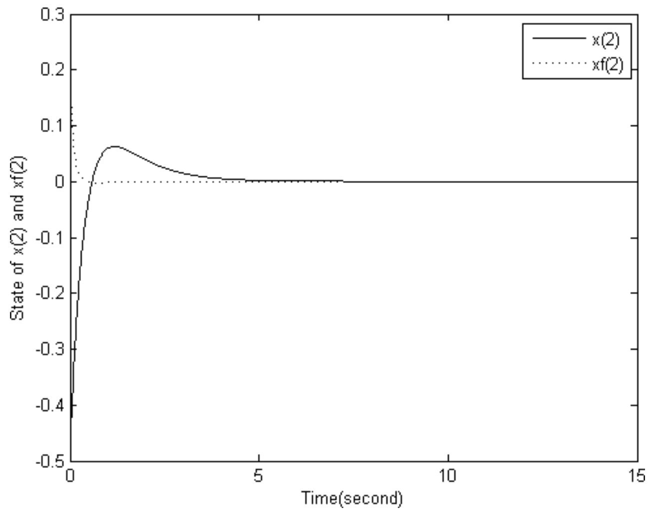

FIGURE 11. State trajectories of x_2.

find that the event-triggered parameters $V_i(i = 1, 2)$ are automatic adjustment and the transmission rates are nearly maintain to 0.8%. It is obvious that the network resource are greatly saved by employing the event-triggered scheme and the quantizer.

Moreover, the response of the original state is shown in Fig 4, and the filter state are depicted in Fig 5 and Fig 6. It is clear that the fuzzy filter system is asymptotically stable. The output signal of quantizer is shown in Fig 7. The trajectory of estimate signal error, original signal and estimate signal are shown in Fig 8. Obviously, the curve of estimate error tends to zero, which demonstrates the effectiveness of the proposed method.

Example 2: Considering the following fuzzy system [23]:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} \mu_i(\theta(t))(A_i x(t) + B_i w(t)) \\ y(t) = \sum\limits_{i=1}^{r} \mu_i(\theta(t)) E_i x(t) \\ z(t) = \sum\limits_{i=1}^{r} \mu_i(\theta(t))(C_i x(t) + D_i w(t)) \end{cases} \tag{44}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ D_1 &= 0.3, E_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}, \\ D_2 &= 0.6, E_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}. \end{aligned}$$

To compare with ones existing in [23]. We choose

$$\tau_1 = 0.002, \quad \tau_3 = 0.2, \ \varepsilon = 0.2, \ \rho = 0.85.$$

The membership function and disturbance are the same as in Example 1. Using Theorem 2, we obtain the event-triggered parameters $V_1 = 23.6026$, $V_2 = 27.0494$, and the parameters

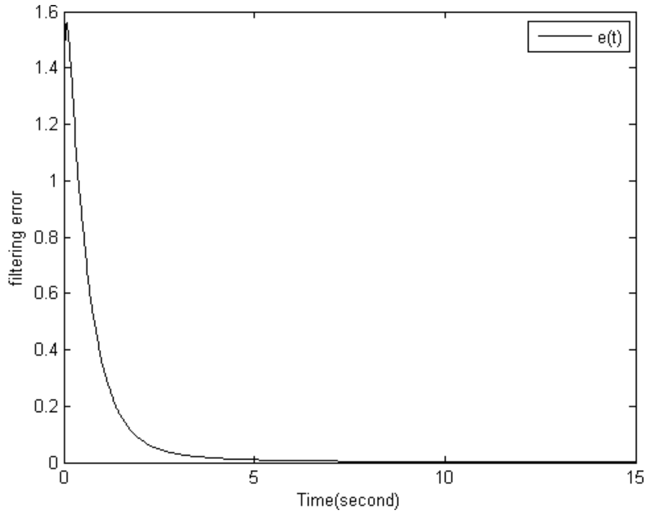

FIGURE 12. The trajectories of estimate error.

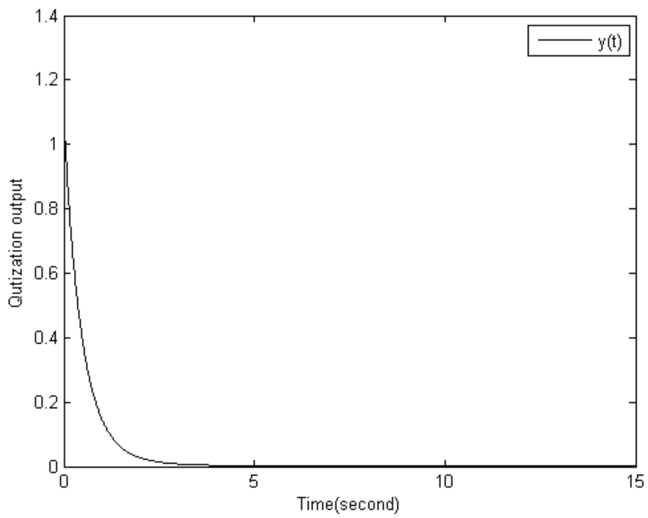

FIGURE 13. The trajectories of quantized output.

of the fuzzy filter are given by

$$A_{f1} = \begin{bmatrix} -17.4366 & 8.5268 \\ 7.7146 & -11.2453 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 0.4852 \\ -0.3343 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} -2.3715 & 0.2475 \end{bmatrix}, D_{f1} = -0.0727,$$
$$A_{f2} = \begin{bmatrix} -16.9835 & 8.5268 \\ 7.7146 & -9.1581 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.2106 \\ -0.1013 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} -1.8903 & 1.1024 \end{bmatrix}, D_{f2} = -0.0290.$$

In addition, we obtain a less minimum performance level $\gamma = 0.3856$ than in [23]. It should be mentioned that if the event-triggered scheme is not considered, the transmission rate should be 100%(every quantized data should be transmitted). But From Fig 9, one can see that the addressed event-triggered scheme is effective in reducing transmission rate (only about 1% data should be transmitted) and saving the network resource. It is more effective than some previously known results [22]–[23]. From Fig 10 and Fig 11, it is clear that the state of filter is stable only five seconds. The trajectories of estimate error are depicted in Fig 12, one can see that the fuzzy filter is effective in estimating unmeasured states. From Fig 13, one can see that the proposed quantizer is effective. The finding from Fig 9 to Fig 13 shows that the proposed novel fuzzy filtering is effective, which is especially important for wireless communication and industrial control.

V. CONCLUSION

In this paper, the event-triggered H_∞ fuzzy filtering stability analysis and design problem have been investigated for NSCs with quantization and delays. The Writinger inequality has been applied to deal with the derivation of the Lyapunov-Krasovskii functional and a less conservative stability conditions are derived. The event-triggered scheme and the quantizer has been proposed to improve the network resources utilization and save network bandwidth. Numerical simulations are given to demonstrate the effectiveness of the proposed approach.

Future research includes event-triggered fuzzy fault filter design for NCSs considering packet dropout and network-induced delays. Moreover, event-triggered type-2 fuzzy filter design for NCSs with network-induced delays also can be further considered for the future investigation.

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ZHONG-DA LU was born in Harbin, Heilongjang, China, in 1970. He received the B.E. degree in automation from Qiqihar University, Qiqihar, China, in 1993 and the M.S. degree in communication and information system from Harbin Engineering University, Harbin, in 2007. He is currently a Professor with Qiqihar University. His research interests include robot control nonlinear system, pattern recognition, and mechatronics.

GUANG-TAO RAN received the B.E. degree in automation from Qiqihar University, Qiqihar, China, in 2016, where he is currently pursuing the M.S. degree. His research interests include fuzzy control, networked control systems, and robust control.

GUO-LIANG ZHANG received the B.E. degree in automation from Qiqihar University, Qiqihar, China, in 2014. His research interests include networked control systems, nonlinear control, and robust control.

FENG-XIA XU received the B.E. degree in automation from Qiqihar University, Qiqihar, China, in 1993, and the M.S. and Ph.D. degrees in control science and control engineering from Harbin Institute of Technology, China, in 2002 and 2006, respectively. She is currently a Professor and the Dean with the School of Computer and Control Engineering, Qiqihar University. Her research interests include networked control systems, descriptor systems, and nonlinear control.