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Fully Distributed Fault-Tolerant Consensus Protocols for Lipschitz Nonlinear Multi-Agent Systems

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ABSTRACT In this paper, we investigate the fault-tolerant consensus problems for multi-agent systems with actuator faults. Based on the relative state information, fully distributed adaptive protocols are designed for these agents described by Lipschitz nonlinear dynamics under leaderless and leader-follower communication structures. As the special case, for linear multi-agent systems, fault-tolerant consensus problems are also solved with the proposed protocols. First, for the leaderless multi-agent systems, all agents reach an agreement on a common value with undirected connected communication graphs. Second, for the leader-follower multi-agent systems, in which the state of the leader is only available to a subset of the followers, the state of each follower converges to the state of the leader, asymptotically. A distinctive feature of this paper is that the adaptive protocols here are independent of the eigenvalues of the Laplacian matrix associated with the whole communication graph, which means the protocols can be implemented by each agent in a fully distributed fashion. Simulation examples are provided to illustrate the theoretical results.

INDEX TERMS Fault-tolerant, consensus, linear and Lipschitz nonlinearity, multi-agent systems, actuator faults.

I. INTRODUCTION

Strongly stimulated by the development of advanced control technologies and the explosion in computation and communication capacities, the consensus control problems for multi-agent systems have been investigated by many researchers [1]–[13]. However, these works about the consensus problems do not take faults occurred in the systems into consideration. In reality, advances in control technologies, computation and communication have also increased the complexity of engineering systems. Compared to a single agent, multi-agent systems involve an increasing number of actuators, sensors, and other system components. These physical components may become faulty which can cause system performance deterioration or even lead to accidents. In order to overcome these weaknesses, fault-tolerant control is proposed to maintain desirable performance and stability properties in spite of faults [14].

In the past few years, a large number of results have been reported on fault-tolerant consensus problem for multi-agent systems [15]–[19]. In [20], distributed protocols were proposed for fault-tolerant consensus tracking problems of linear and Lipschitz nonlinear multi-agent systems. The protocols proposed in [20] required the knowledge of some eigenvalue information of the graph Laplacian matrix. Whereas the eigenvalue information of the Laplacian matrix which depends on the entire communication graph, is global information and unknown for all agents. In [21], observer-based adaptive protocols were proposed for fault-tolerant tracking problems of Lipschitz nonlinear nonstrict-feedback systems. There is only one leader and one follower considered in [21]. Fault-tolerant consensus problem of nonlinear multi-agent system was investigated in [22]. The consensus error exponentially converges to a bounded set. Note that in [22], Chen *et al.* assumed that for agent i, its all actuators have the same actuator efficiency factor. However, for an agent in the multi-agent systems, its different actuators may have different actuator efficiency factors in practice. Thus for each agent, the actuator efficiency of the control input is an efficiency diagonal matrix. The method applied in [22] could not solve this situation. In this paper, we assumed that for each agent i, its actuator efficiency factors may be different. In [22], adaptive control techniques are used to tackle the unknown nonlinear term and actuator faults. Actually, in recent years, adaptive control techniques have been widely used to solve the unknown terms in the system [23]–[28]. Motivated by [22], [29], fully distributed adaptive protocols are designed to achieve consensus in this paper. Fault-tolerant consensus tracking problem and containment control problem of multi-agent systems with actuator faults were considered in [30] and [31], respectively. In [30]–[32], the protocols need the information of the graph Laplace matrix, thus are not fully distributed protocols.

In this paper, we investigate the fault-tolerant consensus problems for Lipschitz nonlinear multi-agent systems with leaderless and leader-follower communication structures. Contributions of this paper can be summarized as follows.

First, this paper considers actuator faults containing stuck, outage, bias and loss of effectiveness faults. Moreover, we assume that for each agent, its different actuators may have different actuator efficiency factors. Thus in this paper, for each agent, the actuator efficiency of the control input is an efficiency diagonal matrix. Second, compared with [20], [30], [32], the adaptive protocols proposed in this paper can be implemented by each agent in a fully distributed fashion without using any global information. Third, we extend the protocols proposed in [33]. If the multi-agent system is failure-free, the adaptive protocols proposed in this paper will reduce to the protocols in [33].

The rest of this paper is organized as follows. Section II provides some preliminaries and problem formulation. Section III considers the fault-tolerant consensus problem for Lipschitz nonlinear multi-agent systems. The fault-tolerant consensus tracking problem with leader-follower communication structure is studied in Section IV. The effectiveness of the proposed protocols is illustrated by examples in Section V. Finally, some concluding remarks are given in Section VI.

Notations: $\mathbb{R}^{n\times m}$ denotes a set of $n \times m$ real matrices and I_n represents the identity matrix of dimension n. $\mathbf{1}_n = [1, 1, \dots, 1]^T \in \mathbb{R}^{n\times 1}$. $A \otimes B$ denotes the Kronecker product of matrices A and B. For a vector $x \in \mathbb{R}^n$, $\| x \|$ denotes its Euclidean norm. For real symmetric matrix G, $G > 0$ means that G is positive definite.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. GRAPH THEORY

For the leaderless multi-agent systems, assume that each agent is a node and the information exchange of N agents is denoted by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ represent, respectively, a nonempty finite set of nodes (i.e., agents) and a set of edges (i.e., communication links). An edge (v_i, v_j) means that agents v_i and v_j can obtain information from each other. The neighborhood of the ith agent is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}\}$. A path $\mathcal{P}$ in $\mathcal{G}$ is a sequence $\{v_{i_0}, v_{i_1}, \dots, v_{i_k}\}$ where $(v_{i_{j-1}}, v_{i_j}) \in \mathcal{E}$ for $j = 1, 2, \dots, k$ and the nodes are distinct. The weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ associated with $\mathcal{G}$ is defined by $a_{ii} = 0$, $a_{ij} = a_{ji} > 0$ if there is an edge between agent v_i and v_j, i.e., $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = a_{ji} = 0$ otherwise. The Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}]_{N\times N} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}\{d_1, d_2, \dots, d_N\}$ is called the degree matrix of $\mathcal{G}$ with $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$, $i = 1, 2, \dots, N$. An undirected graph $\mathcal{G}$ is connected if there exists a path between every pair of distinct nodes, otherwise is disconnected.

For the leader-follower multi-agent systems consist of N follower agents and a leader agent, assume that the leader is represented by node v_0 and the followers are represented by nodes $v_1, v_2, \dots, v_N$. The information exchange between these agents is represented by a directed graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \mathcal{V} \cup \{v_0\}$ and $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$. A node is a leader if it only sends out information and receives no information from any other node. The leader adjacency matrix is defined as a diagonal matrix $\mathcal{B} = \text{diag}\{a_{10}, a_{20}, \dots, a_{N0}\}$, where $a_{i0} > 0$ if the follower agent i has access to the leader's information and $a_{i0} = 0$ otherwise. The information exchange between different follower agents is represented by the undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of order N. $\mathcal{L} = [l_{ij}]_{N\times N}$ is the Laplacian matrix of $\mathcal{G}$.

Lemma 1 [5]: For a connected undirected graph $\mathcal{G}$, the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ has the following properties. $x^T\mathcal{L}x = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i - x_j)^2$ for any $x = [x_1, x_2, \dots, x_N]^T \in \mathbb{R}^N$, which implies that $\mathcal{L}$ is positive semi-definite. 0 is a simple eigenvalue of $\mathcal{L}$ and $\mathbf{1}_n$ is the associated eigenvector. Assume that the eigenvalues of $\mathcal{L}$ are denoted by $0, \lambda_2, \lambda_3, \dots, \lambda_N$ satisfying $0 \le \lambda_2 \le \lambda_3 \le \cdots \le \lambda_N$. Then the second smallest eigenvalue $\lambda_2 > 0$.

Assumption 1: For leader-follower multi-agent systems, the undirected graph $\mathcal{G}$ for the followers is connected and there is at least one follower agent i, which has access to the leader's information, i.e., $\mathcal{B} \neq 0$.

Lemma 2 [5]: Denote $\mathcal{H} = \mathcal{L} + \mathcal{B}$. For leader-follower multi-agent systems, if Assumption 1 holds, then $\mathcal{H} > 0$.

B. NODES DYNAMICS

Consider Lipschitz nonlinear multi-agent systems which consist of N identical follower agents and one leader agent, the dynamic of the ith follower is described by:

$$\dot{x}_i = Ax_i + f(x_i, t) + Bu_i, \quad i = 1, 2, \dots, N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^m$ is the control input, and A, B are constant matrices with compatible dimensions. The nonlinear function $f(x, t)$ is assumed to be continuous in t and Lipschitz in x with a Lipschitz constant $l > 0$, i.e.,

$$\|f(x, t) - f(y, t)\| \le l\|x - y\|, \quad \forall x, y \in \mathbb{R}^n. \tag{2}$$

The dynamic of the leader can be expressed as:

$$\dot{x}_0 = Ax_0 + f(x_0, t) + Bu_0. \tag{3}$$

Assumption 2: The leader's control input u_0 is continuous and bounded, i.e., $\|u_0\|_\infty \le D$ where D is an unknown positive constant.

C. FAULT MODEL

For a multi-agent system which consists of N identical agents, where each agent has m actuators. For the jth actuator of agent i, $v_{ij}(t)$ represents the input, $u_{ij}^{F_h}(t)$ represents the output that has failed under the hth faulty mode. In this paper, similar to [20], the actuator fault under the hth fault model is defined as

$$u_{ij}^{F_h} = \rho_{ij}^h(t)v_{ij}(t) + \sigma_{ij}^h u_{ij}^{S_h}(t), \tag{4}$$

where $i = 1, 2, \ldots, N,\ j = 1, 2, \ldots, m,\ h = 1, 2, \ldots, H$, $\rho_{ij}^h(t)$ are the unknown time-varying actuator efficiency factors, σ_{ij}^h are unknown constants. Here, the index H is the total fault modes. For the hth fault mode, there exist known scalars $\underline{\rho}_{ij}^h$ and $\overline{\rho}_{ij}^h$ with $0 \leq \underline{\rho}_{ij}^h \leq \rho_{ij}^h(t) \leq \overline{\rho}_{ij}^h \leq 1$. The stuck fault $u_{ij}^{S_h}(t)$ is time varying and bounded. According to the practical case, ρ_{ij}^h and σ_{ij}^h are defined as

$$0 \leq \rho_{ij}^h(t) \leq 1, \quad \sigma_{ij}^h = 0 \text{ or } 1. \tag{5}$$

Remark 1: For the jth actuator of agent i under the hth fault mode, equation (5) implies the following cases (fault modes): (i) $\underline{\rho}_{ij}^h = \overline{\rho}_{ij}^h = 1$ and $\sigma_{ij}^h = 0$ indicates the failure-free case; (ii) $\underline{\rho}_{ij}^h = \overline{\rho}_{ij}^h = 1$ and $\sigma_{ij}^h = 1$ is the actuator bias fault; (iii) $\underline{\rho}_{ij}^h = \overline{\rho}_{ij}^h = 0$ and $\sigma_{ij}^h = 0$ means that the outage fault; (iv) $\underline{\rho}_{ij}^h = \overline{\rho}_{ij}^h = 0$ and $\sigma_{ij}^h = 1$ represents the stuck fault; (v) $0 < \underline{\rho}_{ij}^h \leq \rho_{ij}^h(t) \leq \overline{\rho}_{ij}^h < 1$ and $\sigma_{ij}^h = 0$ is the partial loss of effectiveness fault; (vi) $0 < \underline{\rho}_{ij}^h \leq \rho_{ij}^h(t) \leq \overline{\rho}_{ij}^h < 1$ and $\sigma_{ij}^h = 1$ contains the partial loss of effectiveness fault and bias fault.

For convenience, we use a uniform actuator fault model for all possible fault modes H in the following discussion:

$$u_i = u_i^F = \Lambda_i v_i + \sigma_i u_i^S, \quad i = 1, 2, \ldots, N, \tag{6}$$

where $\Lambda_i \in \{\Lambda_i^1, \Lambda_i^2, \ldots, \Lambda_i^H\}$, $\Lambda_i^h = \mathrm{diag}\{\rho_{i1}^h, \rho_{i2}^h, \ldots, \rho_{im}^h\}$, $\rho_{ij}^h \in [\underline{\rho}_{ij}^h, \overline{\rho}_{ij}^h]$, and $\sigma_i \in \{\sigma_i^1, \sigma_i^2, \ldots, \sigma_i^H\}$, $\sigma_i^h = \mathrm{diag}\{\sigma_{i1}^h, \sigma_{i2}^h, \ldots, \sigma_{im}^h\}$, $\sigma_{ij}^h = 0$ or 1, $H = 1, 2, \ldots, H$.

To facilitate our analysis, we need the following mild assumptions.

Assumption 3: For the stuck faults, there exists unknown scalar $\overline{u}_i^S > 0$, such that

$$\|u_i^S(t)\| \leq \overline{u}_i^S. \tag{7}$$

Assumption 4: For multi-agent system, $\mathrm{rank}[B] = \mathrm{rank}[B\Lambda_i]$, where $\Lambda_i \in \{\Lambda_i^1, \Lambda_i^2, \ldots, \Lambda_i^H\}$, $i = 1, 2, \ldots, N$.

Assumption 4 is necessary for compensating the actuator faults of stuck or outage. The following lemma presents a useful property of multi-agent systems with Assumption 4.

Lemma 3 [31]: Suppose that Assumption 4 holds, then there exist constants $\mu_1 > 0, \mu_2 > 0, \ldots, \mu_N > 0$, such that

$$B\Lambda_i B^T \geq \mu_i BB^T, \quad i = 1, 2, \ldots, N, \tag{8}$$

where $\mu_1, \mu_2, \ldots, \mu_N$ are unknown constants.

According to the Assumption 3, there exist positive constants δ_i, such that

$$\|\sigma_i u_i^S\| \leq \overline{u}_i^S = \mu_i \delta_i, \quad i = 1, 2, \ldots, N, \tag{9}$$

where μ_i is defined in Lemma 3.

Definition 1 (Fault-Tolerant Consensus Problem): For the leaderless multi-agent systems, design fully distributed fault-tolerant consensus protocols $v_i(t), i = 1, 2, \ldots, N$ based on the local relative information between ith agent and its neighboring agents such that for any initial conditions $x_i(0)$,

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall\, i, j = 1, 2, \ldots, N. \tag{10}$$

Definition 2 (Fault-Tolerant Consensus Tracking Problem): For the leader-follower multi-agent systems, design fully distributed fault-tolerant consensus protocols $v_i(t)$, $i = 1, 2, \ldots, N$ based on the local relative information between ith agent and its neighboring agents such that for any initial conditions $x_i(0)$, $i = 0, 1, \ldots, N$, the state of each follower converges to the state of the leader, that is,

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, \quad \forall\, i = 1, 2, \ldots, N. \tag{11}$$

III. LEADERLESS FAULT-TOLERANT CONSENSUS

Consider a group of N identical agents with Lipschitz nonlinear dynamics (1). With actuator faults (6), we rewrite the multi-agent systems (1) as

$$\dot{x}_i = Ax_i + f(x_i, t) + B[\Lambda_i v_i + \sigma_i u_i^S]. \tag{12}$$

Let ξ_i denote the local relative state information between ith agent and its neighboring agents that can be described by

$$\xi_i = \sum_{j=1}^{N} a_{ij}(x_i - x_j). \tag{13}$$

For agent i, $i = 1, 2, \ldots, N$, based on ξ_i, we propose the following distributed adaptive protocol:

$$\begin{aligned} &v_i = v_{i,1} + v_{i,2}, \quad v_{i,1} = c_i K\xi_i, \quad v_{i,2} = \frac{K\xi_i\hat{\delta}_i^2}{\|K\xi_i\|\hat{\delta}_i + \sigma(t)}, \\ &\dot{c}_i = \gamma_{i,1}\xi_i^T\Gamma\xi_i, \quad \dot{\hat{\delta}}_i = \gamma_{i,2}\|K\xi_i\| - \gamma_{i,2}\sigma(t)\hat{\delta}_i, \end{aligned} \tag{14}$$

where $c_i(t)$ denotes the time-varying coupling weight associated with the ith agent with $c_i(0) > 0$. $\hat{\delta}_i$ is the estimate of δ_i with $\hat{\delta}_i(0) > 0$. $\gamma_{i,1}$ and $\gamma_{i,2}$ are positive constants. $\sigma(t) \in \mathbb{R}^+$ is uniform continuous function and satisfies the following inequality:

$$\lim_{t\to\infty} \int_{t_0}^{t} \sigma(\tau)d\tau \leq \bar{\sigma} < +\infty, \tag{15}$$

where $\bar{\sigma}$ is an unknown positive constant. $K = -B^T P^{-1}$, $\Gamma = P^{-1}BB^T P^{-1}$, where matrix $P > 0$ and scalar $\tau > 0$ satisfy the following linear matrix inequality (LMI):

$$\begin{bmatrix} AP + PA^T - \tau BB^T + lI & \sqrt{l}P \\ \sqrt{l}P & -I \end{bmatrix} < 0. \tag{16}$$

Theorem 1: Suppose that Assumptions 3 and 4 hold. If the undirected graph $\mathcal{G}$ is connected, then the fault-tolerant *consensus problem of system (12) is solved by the distributed adaptive protocol (14). Moreover, $\hat{\delta}_i, i = 1, 2, \ldots, N$ are bounded and coupling weights $c_i(t), i = 1, 2, \ldots, N$ converge to finite values.*

Proof: Consider the following Lyapunov function candidate

$$
\begin{aligned}
V_1 &= \frac{1}{2}x^T(\mathcal{L} \otimes P^{-1})x + \sum_{i=1}^{N} \frac{\mu_i(c_i-\alpha)^2}{2\gamma_{i,1}} + \sum_{i=1}^{N} \frac{\mu_i\tilde{\delta}_i^2}{2\gamma_{i,2}} \\
&= \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i-x_j)^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N} \frac{\mu_i(c_i-\alpha)^2}{2\gamma_{i,1}} + \sum_{i=1}^{N} \frac{\mu_i\tilde{\delta}_i^2}{2\gamma_{i,2}}, \qquad (17)
\end{aligned}
$$

where $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$, $\mu_i, i = 1, 2, \ldots, N$ are unknown positive constants which are defined in Lemma 3. α is a positive constant to be determined later. $\tilde{\delta}_i = \hat{\delta}_i - \delta_i$, $i = 1, 2, \ldots, N$. The time derivative of V_1 along system (12) and with adaptive protocol (14) is given by

$$
\begin{aligned}
\dot{V}_1 &= \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\dot{x}_i^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N} \frac{\mu_i(c_i-\alpha)}{\gamma_{i,1}}\dot{c}_i + \sum_{i=1}^{N} \frac{\mu_i\tilde{\delta}_i}{\gamma_{i,2}}\dot{\hat{\delta}}_i \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(x_i^T A^T + f^T(x_i,t)\right) P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\Lambda_i v_i + \sigma_i u_i^S)^T B^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N} \mu_i(c_i-\alpha)\xi_i^T\Gamma\xi_i + \sum_{i=1}^{N} \mu_i\tilde{\delta}_i\left(\|K\xi_i\| - \sigma(t)\hat{\delta}_i\right) \\
&= \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i-x_j)^T(A^T P^{-1} + P^{-1}A)(x_i-x_j) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}f^T(x_i,t)P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}c_i\xi_i^T K^T\Lambda_i B^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\frac{\xi_i^T K^T\hat{\delta}_i^2\Lambda_i}{\|K\xi_i\|\hat{\delta}_i + \sigma(t)} B^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\sigma_i u_i^S)^T B^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N} \mu_i(c_i-\alpha)\xi_i^T\Gamma\xi_i + \sum_{i=1}^{N} \mu_i\tilde{\delta}_i\left(\|K\xi_i\| - \sigma(t)\hat{\delta}_i\right). \qquad (18)
\end{aligned}
$$

By using the Lipschitz condition (2), we can obtain

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}f^T(x_i,t)P^{-1}(x_i-x_j) \\
&= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left[f(x_i,t)-f(x_j,t)\right]^T P^{-1}(x_i-x_j) \\
&\le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}l\|x_i-x_j\|\|P^{-1}(x_i-x_j)\| \\
&\le \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}l(x_i-x_j)^T\left[(P^{-1})^2 + I\right](x_i-x_j) \\
&= \frac{1}{2}x^T\left[\mathcal{L} \otimes \left(l(P^{-1})^2 + lI\right)\right]x. \qquad (19)
\end{aligned}
$$

According to Lemma 3 and the definitions of K, Γ, ξ_i,

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}c_i\xi_i^T K^T\Lambda_i B^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N} \mu_i(c_i-\alpha)\xi_i^T\Gamma\xi_i \\
&= -\sum_{i=1}^{N} c_i\xi_i^T P^{-1}B\Lambda_i B^T P^{-1}\xi_i \\
&\quad + \sum_{i=1}^{N} \mu_i(c_i-\alpha)\xi_i^T P^{-1}BB^T P^{-1}\xi_i \\
&\le -\sum_{i=1}^{N} \alpha\mu_i\xi_i^T P^{-1}BB^T P^{-1}\xi_i. \qquad (20)
\end{aligned}
$$

Accompany with inequality (9), it is easy to get that

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\frac{\xi_i^T K^T\hat{\delta}_i^2}{\|K\xi_i\|\hat{\delta}_i + \sigma(t)}\Lambda_i B^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\sigma_i u_i^S)^T B^T P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N} \mu_i\tilde{\delta}_i\left(\|K\xi_i\| - \sigma(t)\hat{\delta}_i\right) \\
&\le -\sum_{i=1}^{N} \frac{\hat{\delta}_i^2\xi_i^T P^{-1}B\Lambda_i B^T P^{-1}\xi_i}{\|B^T P^{-1}\xi_i\|\hat{\delta}_i + \sigma(t)} + \sum_{i=1}^{N} \mu_i\delta_i\|B^T P^{-1}\xi_i\| \\
&\quad + \sum_{i=1}^{N} \mu_i\tilde{\delta}_i\left(\|B^T P^{-1}\xi_i\| - \sigma(t)\hat{\delta}_i\right) \\
&\le \sum_{i=1}^{N} \mu_i\left(-\frac{\hat{\delta}_i^2\xi_i^T P^{-1}BB^T P^{-1}\xi_i}{\|B^T P^{-1}\xi_i\|\hat{\delta}_i + \sigma(t)} + \hat{\delta}_i\|B^T P^{-1}\xi_i\|\right) \\
&\quad - \sum_{i=1}^{N} \mu_i\sigma(t)\tilde{\delta}_i\hat{\delta}_i
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{i=1}^{N}\mu_i\Big(\frac{\hat{\delta}_i\|B^TP^{-1}\xi_i\|\sigma(t)}{\|B^TP^{-1}\xi_i\|\hat{\delta}_i+\sigma(t)}-\sigma(t)(\tilde{\delta}_i^2+\tilde{\delta}_i\delta_i)\Big)\\
&\le \sum_{i=1}^{N}\mu_i\left(\sigma(t)-\sigma(t)\Big(\tilde{\delta}_i+\frac{1}{2}\delta_i\Big)^2+\frac{1}{4}\sigma(t)\delta_i^2\right)\\
&\le \sigma(t)\kappa, \qquad (21)
\end{aligned}
$$

where $\kappa = \sum_{i=1}^{N}\mu_i\big(1+\frac{1}{4}\delta_i^2\big)$.

Substituting (19), (20), and (21) into (18) yields

$$
\begin{aligned}
\dot{V}_1 &\le \frac{1}{2}x^T\Big[\mathcal{L}\otimes(A^TP^{-1}+P^{-1}A)+\mathcal{L}\otimes\Big(l(P^{-1})^2+lI\Big)\Big]x\\
&\quad-\sum_{i=1}^{N}\alpha\mu_i\xi_i^TP^{-1}BB^TP^{-1}\xi_i+\sigma(t)\kappa\\
&\le \frac{1}{2}x^T\Big[\mathcal{L}\otimes\Big(A^TP^{-1}+P^{-1}A+l(P^{-1})^2+lI\Big)\\
&\quad-2\alpha_0\mathcal{L}^2\otimes P^{-1}BB^TP^{-1}\Big]x+\sigma(t)\kappa,
\end{aligned}
$$

where $\alpha\mu_i \ge \alpha_0, i = 1, 2, \ldots, N$. $\alpha_0 > 0$ satisfies $2\alpha_0\lambda_i \ge \tau,\ i = 2, 3, \ldots, N$, $\tau > 0$ satisfies (16). $\lambda_2, \lambda_3, \ldots, \lambda_N$ are the eigenvalues of $\mathcal{L}$. According to Lemma 1, they are positive constants. Denote $\mathbf{1} = 1_N$, $e = [e_1^T, e_2^T, \ldots, e_N^T]^T$, where $e_i = x_i - \bar{x},\ \bar{x} = \frac{1}{N}\sum_{j=1}^{N}x_j$. Obviously,

$$
e = \Big[\Big(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\Big)\otimes I_n\Big]x.
$$

Therefore, in terms of $\mathcal{L}\mathbf{1} = 0$, for any matrix $M \in \mathbb{R}^{n\times n}$, we have

$$
\begin{aligned}
e^T(\mathcal{L}\otimes M)e &= x^T(\mathcal{L}\otimes M)x,\\
e^T(\mathcal{L}^2\otimes M)e &= x^T(\mathcal{L}^2\otimes M)x. \qquad (22)
\end{aligned}
$$

Thus

$$
\begin{aligned}
\dot{V}_1 &\le \frac{1}{2}e^T\Big[\mathcal{L}\otimes\Big(A^TP^{-1}+P^{-1}A+l(P^{-1})^2+lI\Big)\\
&\quad-2\alpha_0\mathcal{L}^2\otimes P^{-1}BB^TP^{-1}\Big]e+\sigma(t)\kappa. \qquad (23)
\end{aligned}
$$

Let $U \in \mathbb{R}^{N\times N}$ is an orthogonal matrix satisfies $U^T\mathcal{L}U = \mathrm{diag}\{0, \lambda_2, \lambda_3, \ldots, \lambda_N\}$. Because the right and left eigenvectors of $\mathcal{L}$ corresponding to the zero eigenvalue are $\mathbf{1}$ and $\mathbf{1}^T$ with dimension N, respectively, we can choose $U = \Big[\mathbf{1}\otimes\frac{1}{\sqrt{N}}\ \ Y_1\Big]$ and $U^T = \begin{bmatrix}\mathbf{1}^T\otimes\frac{1}{\sqrt{N}}\\ Y_2\end{bmatrix}$, with $Y_1 \in \mathbb{R}^{N\times(N-1)}$ and $Y_2 \in \mathbb{R}^{(N-1)\times N}$. Let $\tilde{e} = (U^T\otimes P^{-1})e$, then

$$
\begin{aligned}
\dot{V}_1 &\le \frac{1}{2}\sum_{i=2}^{N}\lambda_i\tilde{e}_i^T(AP+PA^T+lP^2+lI-2\alpha_0\lambda_iBB^T)\tilde{e}_i\\
&\quad+\sigma(t)\kappa\\
&\le \frac{1}{2}\sum_{i=2}^{N}\lambda_i\tilde{e}_i^T(AP+PA^T+lP^2+lI-\tau BB^T)\tilde{e}_i\\
&\quad+\sigma(t)\kappa, \qquad (24)
\end{aligned}
$$

where $2\alpha_0\lambda_i \ge \tau,\ i = 2, 3, \ldots, N$. From the definition of $\tilde{e}$,

$$
\tilde{e}_1 = \Big(\frac{1}{\sqrt{N}}\mathbf{1}^T\otimes P^{-1}\Big)\Big[\Big(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\Big)\otimes I_n\Big]x = 0. \qquad (25)
$$

Based on (15), (16), (17), (24), we have

$$
\begin{aligned}
0 &\le V_1(t) = V_1(0) + \int_0^t \dot{V}_1(s)ds\\
&\le V_1(0) + \frac{1}{2}\int_0^t\sum_{i=2}^{N}\lambda_i\tilde{e}_i^T(s)\Big(AP+PA^T+lP^2\\
&\quad+lI-\tau BB^T\Big)\tilde{e}_i(s)ds + \int_0^t\kappa\sigma(s)ds\\
&\le V_1(0)+\kappa\bar{\sigma}. \qquad (26)
\end{aligned}
$$

Denote $W(\tilde{e}_i(t)) = -\frac{1}{2}\sum_{i=1}^{N}\lambda_i\tilde{e}_i^T(t)(AP+PA^T+lP^2+lI-\tau BB^T)\tilde{e}_i(t)$, where $\lambda_1 = 0$, then

$$
\int_0^t W(\tilde{e}_i(s))ds \le V_1(0)+\kappa\bar{\sigma}. \qquad (27)
$$

Therefore $\int_0^t W(\tilde{e}_i(s))ds$ is bounded. Existence of $\lim_{t\to\infty}\int_0^t W(\tilde{e}_i(s))ds$ is guaranteed since the left-hand side of (27) is monotonically nondecreasing and bounded above. From (26), V_1 is bounded, which implies c_i and $\hat{\delta}_i$ are bounded. With (22) and $\tilde{e} = (U^T\otimes P^{-1})e$,

$$
V_1 = \frac{1}{2}\sum_{i=2}^{N}\lambda_i\tilde{e}_i^TP\tilde{e}_i + \sum_{i=1}^{N}\frac{\mu_i(c_i-\alpha)^2}{2\gamma_{i,1}} + \sum_{i=1}^{N}\frac{\mu_i\tilde{\delta}_i^2}{2\gamma_{i,2}}, \qquad (28)
$$

thus $\tilde{e}_i, i = 2, 3, \ldots, N$ are bounded, accompany with $\tilde{e}_1 = 0$, $\tilde{e}$ is bounded. Then $e = (U\otimes P)\tilde{e}$ is bounded, thus $v_i, i = 1, 2, \ldots, N$ are bounded. Because

$$
\begin{aligned}
\dot{e}_i &= Ae_i + B[\Lambda_iv_i+\sigma_iu_i^S] - B\sum_{j=1}^{N}\frac{1}{N}[\Lambda_jv_j+\sigma_ju_j^S]\\
&\quad+\frac{1}{N}\sum_{j=1}^{N}[f(x_i,t)-f(x_j,t)], \qquad (29)
\end{aligned}
$$

where $e_i = x_i - \bar{x}$ and

$$
\begin{aligned}
\Big\|\sum_{j=1}^{N}[f(x_i,t)-f(x_j,t)]\Big\| &\le \sum_{j=1}^{N}\|x_i-x_j\|\\
&= \sum_{j=1}^{N}\|e_i-e_j\|, \qquad (30)
\end{aligned}
$$

we have $\dot{e}_i$ are bounded for $i = 1, 2, \ldots, N$. With $\tilde{e} = (U^T\otimes P^{-1})e$, $\dot{\tilde{e}}$ is bounded. Therefore $W(\tilde{e}_i(t))$ is uniformly continuous. By Barbalat's Lemma, $W(\tilde{e}_i(t)) \to 0$ as $t \to 0$. Thus $\tilde{e}_i(t)$ converge to zero asymptotically. That is $e_i(t) = x_i - \bar{x}, i = 1, 2, \ldots, N$ converge to zero asymptotically. The states of all the agents reach a consensus state $\bar{x} = \frac{1}{N}\sum_{j=1}^{N}x_j$. Moreover, from (28), c_i and $\hat{\delta}_i, i = 1, 2, \ldots, N$ are bounded. Since $c_i(t)$ are nondecreasing, coupling weights $c_i(t), i = 1, 2, \ldots, N$ converge to finite values. □

Remark 2: The distributed protocol (14) consists of two parts, where the second part $v_{i,2}$ is used to deal with the effect of the additive actuator faults (bias faults) occurred in the multi-agent systems.

Remark 3: For Lipschitz nonlinear multi-agent systems (1), if the agents dynamics contain external disturbances, described as

$$\dot{x}_i = Ax_i + f(x_i, t) + B[u_i + \omega_i], \tag{31}$$

the distributed protocols (14) also can solve the fault-tolerant consensus problem of systems (31). ω_i is the bounded external disturbance for agent i, which satisfies matching condition. The proof process is similar to the proof of Theorem 1. The effect of the external disturbances is dealt by the second part $v_{i,2}$ of the protocols (14).

Theorem 1 investigates the leaderless fault-tolerant consensus problems for Lipschitz nonlinear multi-agent systems. The general linear multi-agent system

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, 2, \ldots, N \tag{32}$$

is a special case of Lipschitz nonlinear multi-agent system (1) with $f(x_i, t) = 0$, $i = 1, 2, \ldots, N$, which implies Lipschitz constant $l = 0$. Thus, we can present the following corollary.

Corollary 1: Suppose that Assumptions 3 and 4 hold. If the undirected graph $\mathcal{G}$ is connected, then the fault-tolerant consensus problem of system (32) is solved by the distributed adaptive protocol (14) with $K = -B^T P^{-1}$ and $\Gamma = P^{-1}BB^T P^{-1}$, where matrix $P > 0$ satisfies the LMI:

$$AP + PA^T - \tau BB^T < 0, \tag{33}$$

with $\tau > 0$. And $c_i(0) > 0$, $\hat{\delta}_i(0) > 0$, $\gamma_{i,1}$ and $\gamma_{i,2}$ are positive constants. $\sigma(t) \in \mathbb{R}^+$ is uniform continuous function and satisfies inequality (15). Moreover, $\hat{\delta}_i$, $i = 1, 2, \ldots, N$ are bounded and coupling weights $c_i(t)$, $i = 1, 2, \ldots, N$ converge to finite values.

Remark 4: As shown in [3], a sufficient condition for the existence of a matrix $P > 0$ and a scalar $\tau > 0$ to the LMI (33) is that (A, B) is stabilizable.

Remark 5: Compared with [33] which focuses on consensus problems of linear and Lipschitz nonlinear multi-agent systems without considering faults that occurred in the systems, this work studies the fault-tolerant consensus problems. Due to the complexity caused by the actuator faults, new Lyapunov functions are constructed to solve the fault-tolerant consensus problems with fully distributed protocols which only use the relative state information.

If the multi-agent system is failure-free as considered in [33], i.e. $\Lambda_i = I_m$ and $\sigma_i u_i^S = 0$ in (12), the δ_i which are defined in (9), are known constants and $\delta_i = 0$, $i = 1, 2, \ldots, N$. Since the adaptive gains $\hat{\delta}_i$ in protocol (14) are the estimations of δ_i. Thus $\hat{\delta}_i = 0$ and $v_{i,2} = 0$, in which case the adaptive protocol (14) will reduce to the adaptive protocol for failure-free consensus problems of linear and Lipschitz nonlinear multi-agent systems in [33].

IV. LEADER-FOLLOWER FAULT-TOLERANT CONSENSUS TRACKING

Consider a group of $N + 1$ agents consisting of N followers and a leader. The agent indexed by 0 is called the leader, and the rest agents indexed by $i = 1, 2, \ldots, N$ are referred as the followers. The dynamics of the followers and the leader are shown in (1) and (3), respectively. With actuator faults, the actual control input of follower i is given as (6).

Let ζ_i denote the local relative state information between ith agent and its neighboring agents that can be described by

$$\zeta_i = \sum_{j=0}^{N} a_{ij}(x_i - x_j). \tag{34}$$

For agent i, $i = 1, 2, \ldots, N$, based on ζ_i, we propose the following distributed adaptive protocol:

$$v_i = v_{i,1} + v_{i,2}, \quad v_{i,1} = c_i K\zeta_i, \quad v_{i,2} = \frac{K\zeta_i \hat{\delta}_i^2}{\|K\zeta_i\|\hat{\delta}_i + \sigma(t)},$$
$$\dot{c}_i = \gamma_{i,1}\zeta_i^T \Gamma\zeta_i, \quad \dot{\hat{\delta}}_i = \gamma_{i,2}\|K\zeta_i\| - \gamma_{i,2}\sigma(t)\hat{\delta}_i, \tag{35}$$

where $c_i(t)$, $\gamma_{i,1}$, $\gamma_{i,2}$, K, Γ, and $\sigma(t) \in \mathbb{R}^+$ are defined as in (14). $\hat{\delta}_i$ is the estimate of $\bar{\delta}_i$ with $\hat{\delta}_i(0) > 0$, where $\bar{\delta}_i$, $i = 1, 2, \ldots, N$ are unknown constants.

Theorem 2: Suppose that Assumptions 1-4 hold. The fault-tolerant consensus tracking problem of multi-agent systems with dynamics (1), (3), and actuator faults (6) is solved by the distributed adaptive protocol (35). Moreover, $\hat{\delta}_i, i = 1, 2, \ldots, N$ are bounded and coupling weights $c_i(t), i = 1, 2, \ldots, N$ converge to finite values.

Proof: The proof of Theorem 2 is similar with the steps in Theorem 1, see Appendix. □

Remark 6: Similar to Remark 3, the distributed protocol (35) also can solve the fault-tolerant consensus tracking problems of systems with dynamics

$$\dot{x}_i = Ax_i + f(x_i, t) + B[u_i + \omega_i], \quad i = 0, 1, \ldots, N, \tag{36}$$

For general linear multi-agent systems:

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 0, 1, \ldots, N, \tag{37}$$

we have the following corollary.

Corollary 2: Suppose that Assumptions 1-4 hold. The fault-tolerant consensus tracking problem of multi-agent systems with dynamics (37) is solved by the distributed adaptive protocol (35) with $K = -B^T P^{-1}$ and $\Gamma = P^{-1}BB^T P^{-1}$, where matrix $P > 0$ satisfies the LMI (33). And $c_i(0) > 0$, $\hat{\delta}_i(0) > 0$. $\gamma_{i,1}$ and $\gamma_{i,2}$ are positive constants. $\sigma(t) \in \mathbb{R}^+$ is uniform continuous function and satisfies inequality (15). Moreover, $\hat{\delta}_i, i = 1, 2, \ldots, N$ are bounded and coupling weights $c_i(t), i = 1, 2, \ldots, N$ converge to finite values.

Remark 7: In [20], fault-tolerant consensus tracking problems of linear and Lipschitz nonlinear multi-agent systems with actuator loss of effectiveness faults are studied. In contrast to the protocols in [20], which require the knowledge of eigenvalue information of the graph Laplacian matrix, the adaptive protocol (35) can be computed and implemented by each agent in a fully distributed fashion. Moreover, protocol (35) can also solve the consensus tracking problems of multi-agent systems with stuck and bias faults.

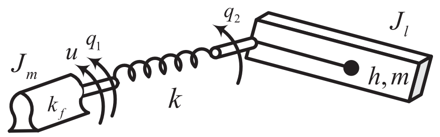


FIGURE 1. Schematic of flexible-joint manipulator modeled by torsional spring.

TABLE 1. Parameters.

System parameter	Value	Units
Motor inertia, J_m	3.7×10^{-3}	kg·m^2
Link inertia, J_l	9.3×10^{-3}	kg·m^2
Pointer mass, m	2.1×10^{-1}	kg
Link length, $2b$	3.0×10^{-1}	m
Torsional spring constant, k	1.8×10^{-1}	N/rad
Viscous friction coefficient, k_f	4.6×10^{-2}	N·s/m
Amplifier gain, K_T,	8.0×10^{-2}	N·s/m

V. SIMULATION

In this section, simulation examples are provided to validate the effectiveness of the theoretical results. Consider a network of single-link manipulators with revolute joints actuated by a DC motor [34] and model the elasticity of the joint as a linear torsional spring with stiffness k as shown in Fig. 1, which represents a class of industrial arms. The nonlinear dynamics for this system are

$$\begin{aligned}
\dot{x}_{i1} &= x_{i2}\\
\dot{x}_{i2} &= \frac{k}{J_m}(x_{i3}-x_{i1}) - \frac{k_f}{J_m}x_{i2} + \frac{K_T}{J_m}u_i\\
\dot{x}_{i3} &= x_{i4}\\
\dot{x}_{i4} &= -\frac{k}{J_l}(x_{i3}-x_{i1}) - \frac{mgh}{J_l}\sin(x_{i3}), \quad i = 1, 2, \ldots, 8,
\end{aligned}$$

where $x_{i1} = q_{i1}$, $x_{i2} = \dot{q}_{i1} = \dot{x}_{i1}$, $x_{i3} = q_{i2}$, $x_{i4} = 0.1\dot{q}_{i2} = 0.1\dot{x}_{i3}$, q_{i1}, q_{i2} are the motor and link angles, respectively. J_l is the link inertia, J_m being the inertia of motor, k is the spring stiffness, k_f represents the viscous friction coefficient, u_i is the input torque, and m, $h = 2b$ are the mass and length of link, respectively. The consensus problems of this multi-agent systems means that agents' motor and link angles and angular velocities reach consensus values, respectively. The system can be represented by the following equation

$$\dot{x}_i = Ax_i + f(x_i, t) + Bu_i, \tag{38}$$

with $x_i = \begin{bmatrix} x_{i1} & x_{i2} & x_{i3} & x_{i4}\end{bmatrix}^T$,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0\\ -48.6 & -1.25 & 48.6 & 0\\ 0 & 0 & 0 & 10\\ 1.95 & 0 & -1.95 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0\\ 21.6\\ 0\\ 0\end{bmatrix},$$
$$f(x_i, t) = [0 \quad 0 \quad 0 \quad -0.333\sin(x_{i3})]^T,$$

where the values of the parameters as taken from reference [35] and shown in Table 1. Clearly, the nonlinear function $f(x_i, t)$ satisfies (2) with a Lipschitz constant $l = 0.333$.

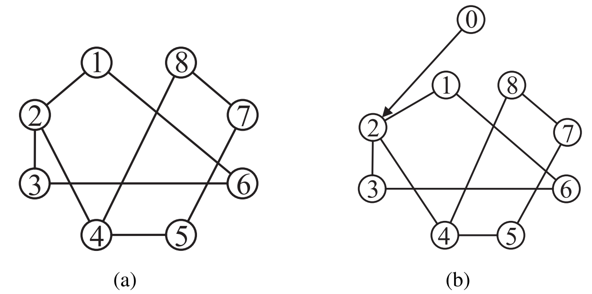


FIGURE 2. Communication topology. (a) Leaderless structure. (b) Leader-follower structure.

Solving the LMI (16) by using the LMI toolbox of Matlab gives the P and feedback gain matrices K, Γ as

$$P = \begin{bmatrix} 4.9285 & 0.1121 & -3.1606 & 7.1170\\ 0.1121 & 0.0111 & -0.0711 & 0.1635\\ -3.1606 & -0.0711 & 2.6384 & -3.6983\\ 7.1170 & 0.1635 & -3.6983 & 20.5406 \end{bmatrix},$$
$$\Gamma = \begin{bmatrix} 5.8629 & 0.5798 & -3.7186 & 8.5530\\ 0.5798 & 0.0573 & -0.3678 & 0.8459\\ -3.7186 & -0.3678 & 2.3586 & -5.4249\\ 8.5530 & 0.8459 & -5.4249 & 12.4775 \end{bmatrix},$$
$$K = \begin{bmatrix} -2.4213 & -0.2395 & 1.5358 & -3.5324\end{bmatrix}. \tag{39}$$

Example 1: We consider the fault-tolerant consensus problems for a leaderless multi-agent system with eight single-link manipulators under connected undirected topologies as shown in Fig. 2(a). The controller parameters in (14) and (16) are chosen as $\gamma_{i,1} = 10, \gamma_{i,2} = 10, \hat{\delta}_i(0) = 1$, $c_i(0) = 1$, $i = 1, 2, \ldots, 8$. P and feedback gain matrices K, Γ are given in (39). $\sigma(t) = 30e^{-0.5t}$. The initial states of the agents are chosen randomly in the interval $[-2, 2]$.

This simulation considers the fault model: before 1s, the actuator of the agent 1 lose 30% of its effectiveness, i.e. $\Lambda_1 = 0.7$, and the other agents are normal. After 1s, the actuator of the agent 1 lose 30% of its effectiveness, the agent 5 occurs actuator bias fault at 50, and the other agents are normal.

The trajectories of consensus errors under the adaptive controllers (14) and (16) are shown in Fig. 3, where $x_i = [x_{i1}, x_{i2}, x_{i3}, x_{i4}]^T, \bar{x} = \frac{1}{8}\sum_{j=1}^{8} x_j = [\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4]^T$. It can be seen that the consensus errors of agents converge to zero. Fig. 3(b) shown that the consensus errors increase suddenly at 1s. The time-varying coupling weights $\hat{\delta}_i$ are bounded and c_i converge to finite values, $i = 1, 2, \ldots, 8$ as shown in Fig. 4.

From Remark 2, the second part $v_{i,2}$ of the controller (14) is used to deal with the effect of the additive actuator faults (bias faults). And $\hat{\delta}_i$ is the estimation of δ_i, which is related to the amplitude of the additive actuator faults. In this simulation, we assume there is no agents occur additive actuator faults before 1s. Thus in Fig. 4(a), we can see that $\hat{\delta}_i(0) = 1$, $i = 1, 2, \ldots, 8$ at the beginning. Since there is no additive actuator faults, $\hat{\delta}_i$ quickly descend and close to zero. After 1s, agent 5 occurs actuator bias fault at 50, all $\hat{\delta}_i$ begin to rising and $\hat{\delta}_5$ is lager than other $\hat{\delta}_i$, $i = 1, 2, \ldots, 8$, $i \neq 5$.

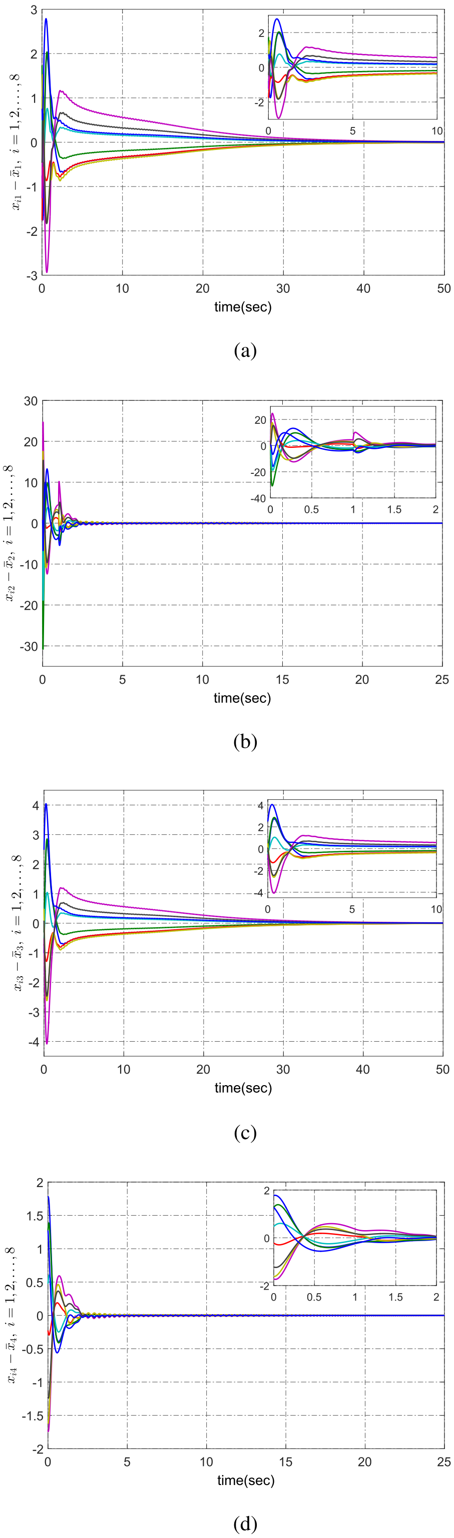


FIGURE 3. Consensus errors for leaderless multi-agent system. (a) First coordinate of consensus errors. (b) Second coordinate of consensus errors. (c) Third coordinate of consensus errors. (d) Forth coordinate of consensus errors.

Example 2: Considering a leader-follower multi-agent system consisting of eight followers and a leader under directed topology Fig. 2(b). The dynamics of ith agent are

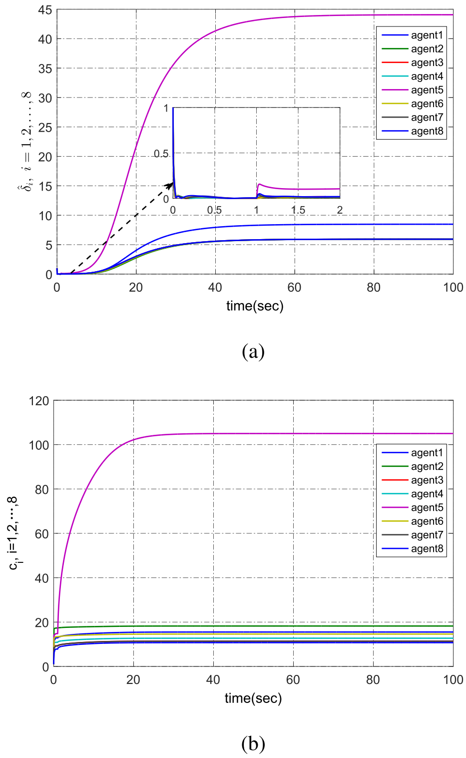


FIGURE 4. Coupling weights for leaderless multi-agent system. (a) Coupling weights $\hat{\delta}_i$. (b) Coupling weights c_i.

described by (38), $i = 0, 1, \ldots, 8$, where the agent indexed by 0 is the leader, $u_0 = 1$. The controller parameters in (35) and (16) are chosen as Example 1. The initial states of the agents are chosen randomly in the interval $[-2, 2]$.

This simulation considers the fault model: before 2s, the actuator of the agent 1 lose 30% of its effectiveness, i.e. $\Lambda_1 = 0.7$, and the other agents are normal. After 2s, the actuator of the agent 1 lose 30% of its effectiveness, the agent 5 occurs actuator bias fault at -10, and the other agents are normal.

The trajectories of tracking errors under the adaptive controllers (35) and (16) are shown in Fig. 5, where $x_i = [x_{i1}, x_{i2}, x_{i3}, x_{i4}]^T$, $i = 0, 1, \ldots, 8$. It can be seen that the tracking errors converge to zero. Fig. 5(d) shows that the tracking errors increase suddenly at 2s. The time-varying coupling weights $\hat{\delta}_i$ are bounded and c_i converge to finite values, $i = 1, 2, \ldots, 8$ as shown in Fig. 6.

Example 3: In this example, we consider the fault-tolerant consensus tracking problems for a linear multi-agent system with leader-follower structure as shown in Fig. 2(b). The dynamics of ith agent are described by (37) with

$$x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$u_0 = -3x_{01} - 2x_{02}$. This simulation considers the fault model: before 5s, all agents are normal. After 5s, the actuator of the agent 5 occurs actuator bias fault at 10, and the other agents are normal.

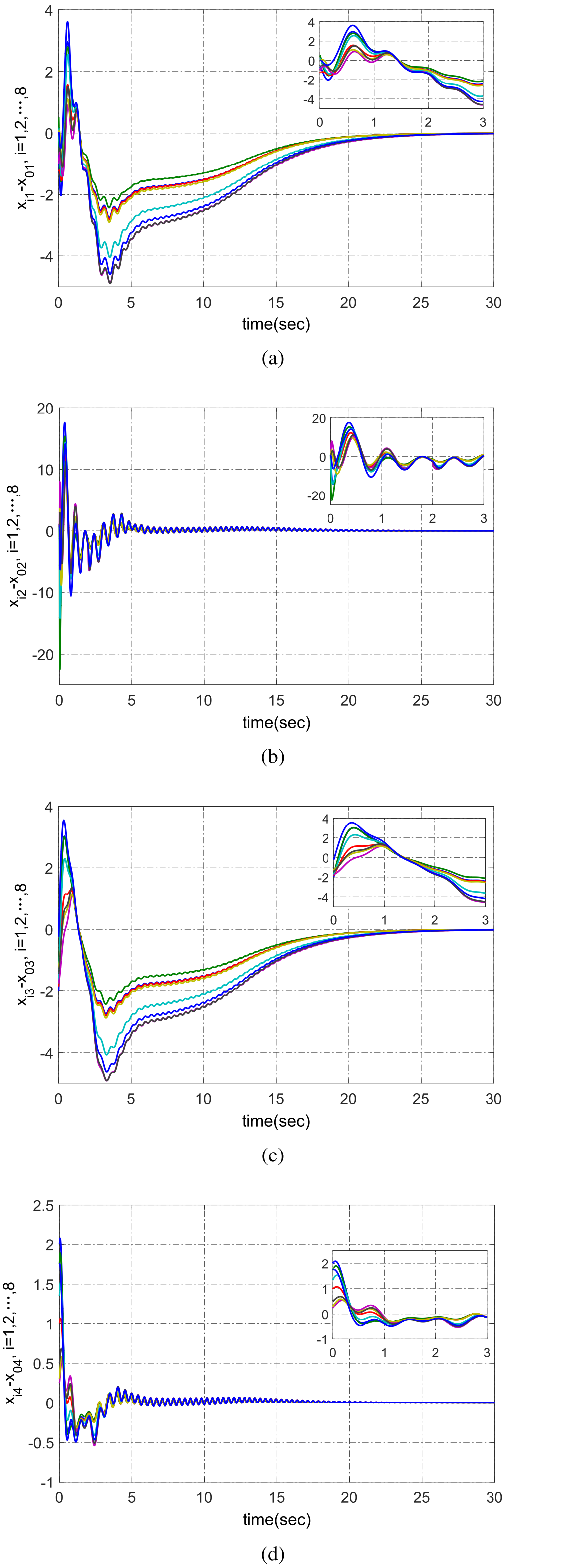


FIGURE 5. **Tracking errors of the followers for leader-follower multi-agent system. (a) First coordinate of tracking errors. (b) Second coordinate of tracking errors. (c) Third coordinate of tracking errors. (d) Forth coordinate of tracking errors.**

In [32], Wang and Yang study fault-tolerant tracking control for multi-agent systems with mismatched parameter uncertainties. The protocol is also applicable to linear multi-agent systems (37). However, the protocol needs the information of $\bar{\lambda}_1$ which is the minimum eigenvalue of $\mathcal{H} = \mathcal{L} + \mathcal{B}$, is global information and thus is unknown. According to the number of agents and the communication topology, $\bar{\lambda}_1$ may be very small. Therefore, we need to choose control gain c sufficiently large, such that $c > \frac{1}{2\bar{\lambda}_1}$, and thus the amplitude of the control inputs may be large.

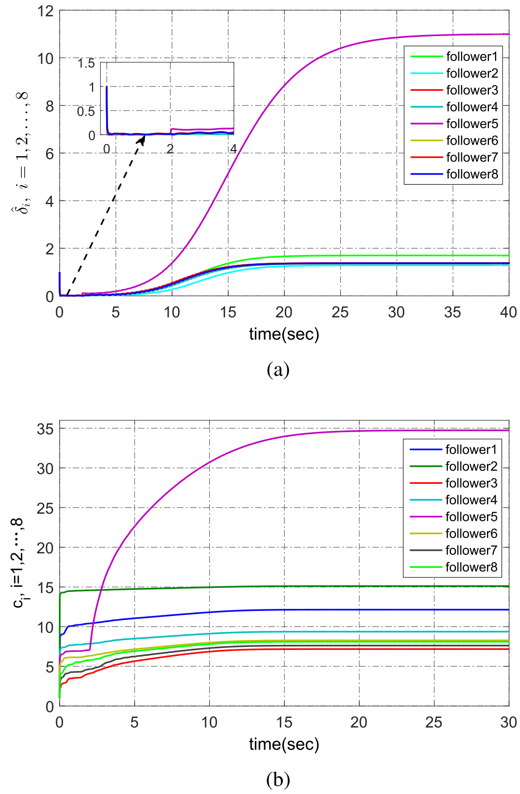


FIGURE 6. **Coupling weights of the followers for leader-follower multi-agent system. (a) Coupling weights $\hat{\delta}_i$. (b) Coupling weights c_i.**

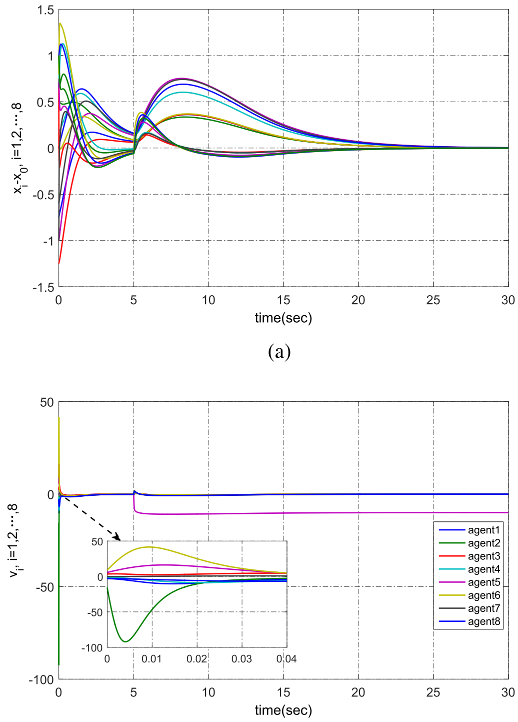
In Fig. 7 and Fig. 8, we compare the protocol proposed in this paper with the protocol in [32]. The initial states of the agents are chosen randomly in the interval $[-2, 2]$.

The controller parameters in (35) are chosen as Example 1 and matrix $P > 0$ satisfies LMI (33).

Consider the protocol proposed in [32]. In this example, $\Delta A = 0$. According to the proof of Theorem 1 in [32], $\hat{k}_{i2}(t) = 0$ and inequality (14) in [32] reduce to inequality:

$$PA + A^T P - 2c\bar{\lambda}_1 PBB^T P < 0. \tag{40}$$

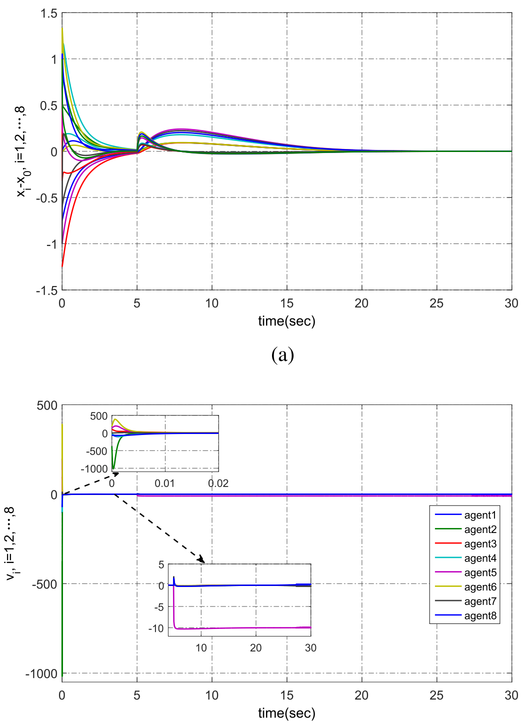
$\bar{\lambda}_1 = 0.0849$ in this paper. Choosing $c = 6$, $\hat{\alpha}_{i,j}(0) = 0$, $\varphi_{i,j} = 1$, and $\hat{k}_{i3}(0) = 1$, $\hat{k}_{i4}(0) = 1$, $i = 1, 2, \ldots, 8$. Fig. 7(a), Fig. 7(b) show the trajectories of tracking errors and control inputs of agents under the adaptive controllers (35) and (33). Fig. 8(a), Fig. 8(b) show the trajectories of tracking errors and control inputs of agents under the protocols proposed in [32]. From Fig. 7 and Fig. 8, we can see that the amplitude of the control inputs with protocol in [32] is larger than the protocol proposed in this paper.



(a)

(b)

FIGURE 7. Leader-follower consensus tracking under controller (35). (a) Tracking errors. (b) Control inputs of the followers.



(a)

(b)

FIGURE 8. Leader-follower consensus tracking under controller in [32]. (a) Tracking errors. (b) Control inputs of the followers.

VI. CONCLUSION

In this paper, fault-tolerant consensus problems for linear and Lipschitz nonlinear multi-agent systems with actuator faults are studied. The adaptive protocols which based on the relative state information, are independent of any global information of the communication graph, and thereby are fully distributed. For the leaderless case, the states of all the agents reach a consensus state with connected undirected communication graph. For the leader-follower case, the state of each follower converges to the state of the leader with undirected connections between followers and directed connections between the leader and the followers. Further work include considering the fault-tolerant consensus problems of more general nonlinear systems and considering a more general graph case where the communication graph is directed or switching graph.

APPENDIX

PROOF OF THEOREM 2

Proof: Define the tracking error for agent i, $i = 1, \cdots, N$ as $\bar{e}_i = x_i - x_0$. Let $\bar{e} = [\bar{e}_1^T, \bar{e}_2^T, \ldots, \bar{e}_N^T]^T$. Consider the following Lyapunov function candidate

$$\begin{aligned}
V_2 &= \frac{1}{2}\bar{e}^T(\mathcal{H}\otimes P^{-1})\bar{e} + \sum_{i=1}^{N}\frac{\mu_i(c_i-\bar{\alpha})^2}{2\gamma_{i,1}} + \sum_{i=1}^{N}\frac{\mu_i\tilde{\delta}_i^2}{2\gamma_{i,2}} \\
&= \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_i-x_j)^T P^{-1}(x_i-x_j) \\
&\quad + \frac{1}{2}\sum_{i=1}^{N}a_{i0}(x_i-x_0)^T P^{-1}(x_i-x_0) \\
&\quad + \sum_{i=1}^{N}\frac{\mu_i(c_i-\bar{\alpha})^2}{2\gamma_{i,1}} + \sum_{i=1}^{N}\frac{\mu_i\tilde{\delta}_i^2}{2\gamma_{i,2}},
\end{aligned} \tag{41}$$

where μ_i are unknown positive constants which are defined in Lemma 3. $\bar{\alpha}$ is a positive constant to be determined later. $\tilde{\delta}_i = \hat{\delta}_i - \bar{\delta}_i$, where $\bar{\delta}_i$ are positive constants to be determined later. According to Assumption 1 and Lemma 2, $\mathcal{H}$ is symmetric positive definite. The time derivative of V_2 along dynamics (1), (3), (6) and with adaptive protocol (35) is given by

$$\begin{aligned}
\dot{V}_2 &= \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_i-x_j)^T(A^TP^{-1}+P^{-1}A)(x_i-x_j) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}f^T(x_i,t)P^{-1}(x_i-x_j) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(\Lambda_i v_i + \sigma_i u_i^S)^T B^T P^{-1}(x_i-x_j) \\
&\quad + \frac{1}{2}\sum_{i=1}^{N}a_{i0}(x_i-x_0)^T(A^TP^{-1}+P^{-1}A)(x_i-x_0) \\
&\quad + \sum_{i=1}^{N}a_{i0}\left(f(x_i,t)-f(x_0,t)\right)^T P^{-1}(x_i-x_0)
\end{aligned}$$

$$+\sum_{i=1}^{N} a_{i0}(\Lambda_i v_i + \sigma_i u_i^S - u_0)^T B^T P^{-1}(x_i - x_0)$$
$$+\sum_{i=1}^{N} \mu_i(c_i - \bar{\alpha})\zeta_i^T \Gamma \zeta_i + \sum_{i=1}^{N} \mu_i \tilde{\delta}_i \left(\|K\zeta_i\| - \sigma(t)\hat{\delta}_i\right). \tag{42}$$

Thus

$$\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i - x_j)^T (A^T P^{-1} + P^{-1}A)(x_i - x_j)$$
$$+\frac{1}{2}\sum_{i=1}^{N} a_{i0}(x_i - x_0)^T (A^T P^{-1} + P^{-1}A)(x_i - x_0)$$
$$= \frac{1}{2}\bar{e}^T[\mathcal{H} \otimes (P^{-1}A + A^T P^{-1})]\bar{e}. \tag{43}$$

By using the Lipschitz condition (2) and inequality (19), we have the following inequalities

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} f^T(x_i, t) P^{-1}(x_i - x_j)$$
$$+\sum_{i=1}^{N} a_{i0}\left(f(x_i, t) - f(x_0, t)\right)^T P^{-1}(x_i - x_0)$$
$$\leq \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i - x_j)^T \left[l(P^{-1})^2 + lI\right](x_i - x_j)$$
$$+\frac{1}{2}\sum_{i=1}^{N} a_{i0}\bar{e}_i^T \left[l(P^{-1})^2 + lI\right]\bar{e}_i$$
$$= \frac{1}{2}\bar{e}^T\left[\mathcal{H} \otimes \left(l(P^{-1})^2 + lI\right)\right]\bar{e}. \tag{44}$$

With protocol (35), Lemma 3 and the definitions of K, Γ, ζ_i,

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\Lambda_i v_{i,1})^T B^T P^{-1}(x_i - x_j)$$
$$+\sum_{i=1}^{N} a_{i0}(\Lambda_i v_{i,1})^T B^T P^{-1}(x_i - x_0) + \sum_{i=1}^{N} \mu_i(c_i - \bar{\alpha})\zeta_i^T \Gamma \zeta_i$$
$$= -\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij} c_i \zeta_i^T P^{-1} B \Lambda_i B^T P^{-1}(x_i - x_j)$$
$$+\sum_{i=1}^{N} \mu_i(c_i - \bar{\alpha})\zeta_i^T \Gamma \zeta_i$$
$$\leq -\sum_{i=1}^{N} \bar{\alpha}\mu_i \zeta_i^T P^{-1} B B^T P^{-1}\zeta_i. \tag{45}$$

According to Assumption 1 and the definition of $\bar{e}$ and ζ_i, $\bar{e} = (\mathcal{H}^{-1} \otimes I_n)\zeta$, where $\bar{e} = [\bar{e}_1^T, \bar{e}_2^T, \ldots, \bar{e}_N^T]^T$, $\zeta = [\zeta_1^T, \zeta_2^T, \ldots, \zeta_N^T]^T$, $\mathcal{H}$ is symmetric positive definite. Define $\mathcal{H}^{-1} = [h_{ij}]_{N\times N}$, $h = \max_{1\leq i,j\leq N}\{|h_{ij}|\}$, thus $\bar{e}_i = \sum_{j=1}^{N} h_{ij}\zeta_j$ and

$$\|B^T P^{-1}\bar{e}_i\| = \left\|B^T P^{-1}\sum_{j=1}^{N} h_{ij}\zeta_j\right\| \leq h\sum_{j=1}^{N}\|B^T P^{-1}\zeta_j\|.$$

With Assumption 2, $-\sum_{i=1}^{N} a_{i0}u_0^T B^T P^{-1}(x_i - x_0) \leq \sum_{i=1}^{N}\|a_{i0}u_0^T\|\|B^T P^{-1}\bar{e}_i\| \leq \max_{1\leq i\leq N}\{a_{i0}\}D\sum_{i=1}^{N} h\sum_{j=1}^{N}\|B^T P^{-1}\zeta_j\| = \max_{1\leq i\leq N}\{a_{i0}\}DhN\sum_{j=1}^{N}\|B^T P^{-1}\zeta_j\|$. Accompany with Assumption 3, there exists positive constants $\bar{\delta}_i$, such that

$$\|\sigma_i u_i^S\| + \max_{1\leq i\leq N}\{a_{i0}\}DhN \leq \mu_i\bar{\delta}_i, \quad i = 1, 2, \ldots, N, \tag{46}$$

where μ_i are defined in Lemma 3. Then

$$\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}(\sigma_i u_i^S)^T B^T P^{-1}(x_i - x_j)$$
$$-\sum_{i=1}^{N} a_{i0}u_0^T B^T P^{-1}(x_i - x_0)$$
$$\leq \sum_{i=1}^{N}\|\sigma_i u_i^S\|\|B^T P^{-1}\zeta_i\|$$
$$+\max_{1\leq i\leq N}\{a_{i0}\}DhN\sum_{i=1}^{N}\|B^T P^{-1}\zeta_i\|$$
$$\leq \sum_{i=1}^{N}\mu_i\bar{\delta}_i\|B^T P^{-1}\zeta_i\|. \tag{47}$$

Thus following similar step as (21), we have

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\Lambda_i v_{i,2} + \sigma_i u_i^S)^T B^T P^{-1}(x_i - x_j)$$
$$+\sum_{i=1}^{N} a_{i0}(\Lambda_i v_{i,2} + \sigma_i u_i^S - u_0)^T B^T P^{-1}(x_i - x_0)$$
$$+\sum_{i=1}^{N}\mu_i\tilde{\delta}_i\left(\|K\zeta_i\| - \sigma(t)\hat{\delta}_i\right)$$
$$= -\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\frac{\hat{\delta}_i^2\zeta_i^T P^{-1}B\Lambda_i B^T P^{-1}(x_i - x_j)}{\|K\zeta_i\|\hat{\delta}_i + \sigma(t)}$$
$$+\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}(\sigma_i u_i^S)^T B^T P^{-1}(x_i - x_j)$$
$$-\sum_{i=1}^{N} a_{i0}u_0^T B^T P^{-1}(x_i - x_0)$$
$$+\sum_{i=1}^{N}\mu_i\tilde{\delta}_i\left(\|K\zeta_i\| - \sigma(t)\hat{\delta}_i\right)$$

$$\leq \sum_{i=1}^{N}\mu_i\Big(-\frac{\hat{\delta}_i^2\zeta_i^TP^{-1}BB^TP^{-1}\zeta_i}{\|B^TP^{-1}\xi_i\|\hat{\delta}_i+\sigma(t)}+\hat{\delta}_i\|B^TP^{-1}\zeta_i\|\Big)$$
$$-\sum_{i=1}^{N}\mu_i\sigma(t)\tilde{\delta}_i\hat{\delta}_i\leq\sigma(t)\bar{\kappa}, \quad (48)$$

where $\bar{\kappa}=\sum_{i=1}^{N}\mu_i\big(1+\frac{1}{4}\bar{\delta}_i^2\big)$.

Substituting (43), (44), (45), and (48) into (42) yields

$$\dot{V}_2\leq\frac{1}{2}\bar{e}^T[\mathcal{H}\otimes(A^TP^{-1}+P^{-1}A+l(P^{-1})^2+lI)]\bar{e}$$
$$-\sum_{i=1}^{N}\mu_i\bar{\alpha}\zeta_i^TP^{-1}BB^TP^{-1}\zeta_i+\sigma(t)\bar{\kappa}. \quad (49)$$

Since $\mathcal{H}$ is symmetric positive definite. There exists an orthogonal matrix $\overline{U}\in\mathbb{R}^{N\times N}$, such that $\overline{U}^T\mathcal{H}\overline{U}=\text{diag}\{\bar{\lambda}_1,\bar{\lambda}_2,\ldots,\bar{\lambda}_N\}$. $\bar{\lambda}_1,\bar{\lambda}_2,\ldots,\bar{\lambda}_N$ are the eigenvalues of $\mathcal{H}$. Choose $\bar{\alpha}\mu_i\geq\bar{\alpha}_0, i=1,2,\ldots,N$, $\bar{\alpha}_0>0$ satisfies $2\bar{\alpha}_0\bar{\lambda}_i\geq\tau, i=1,2,\ldots,N$. Let $\tilde{\bar{e}}=[\tilde{\bar{e}}_1^T,\tilde{\bar{e}}_2^T,\ldots,\tilde{\bar{e}}_N^T]^T=(\overline{U}^T\otimes P^{-1})\bar{e}$, then

$$\dot{V}_2\leq\frac{1}{2}\sum_{i=1}^{N}\bar{\lambda}_i\tilde{\bar{e}}_i^T(AP+PA^T+lP^2+lI-\tau BB^T)\tilde{\bar{e}}_i+\sigma(t)\bar{\kappa}, \quad (50)$$

which is similar to (24). Thus the rest of the proof follows similar steps to those in Theorem 1. □

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