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Non-Cooperative Game Theoretic Power Allocation Strategy for Distributed Multiple-Radar Architecture in a Spectrum Sharing Environment

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ABSTRACT This paper investigates the problem of non-cooperative game theoretic power allocation (NGTPA) for distributed multiple-radar architectures in a spectrum sharing environment, where multiple radars coexist with a communication system in the same frequency band. The primary objective of the multiple-radar system is to minimize the power consumption of each radar by optimizing the transmission power allocation, which is constrained by a predefined signal-to-interference-plus-noise ratio requirement for target detection and a maximum interference tolerant limit for communication system. Since each radar is rational and selfish to maximize its own utility, we utilize the non-cooperative game theoretic technique to tackle the distributed power allocation problem. Taking into consideration the target detection performance and received interference power at the communication receiver, a novel utility function is defined and employed as the optimization criterion for the NGTPA strategy. Furthermore, the existence and uniqueness of the proposed game's Nash equilibrium point are analytically proved. An iterative power allocation algorithm with low computational complexity and fast convergence is developed, where the optimal value of each radar's transmission power is simultaneously updated at the same time step. Numerical simulations are provided to verify the analysis and evaluate the performance of the proposed strategy as a function of the system parameters. It is shown that the distributed algorithm is effective for power allocation and could protect the communication system with limited implementation overhead.

INDEX TERMS Non-cooperative game theory, power allocation, spectrum sharing, Nash equilibrium, distributed multiple-radar architecture.

I. INTRODUCTION

A. BACKGROUND AND MOTIVATION

With the recent advances in large bandwidth wireless networks, multichannel electronically scanned antennas, high-speed low-cost processors and precise synchronization techniques, the implementation of distributed multiple-radar architecture has become feasible and is on a path from theory to practical use [1]. Due to the unique structure of the multiple-radar system, several diverse and independent waveforms can be simultaneously emitted by multiple transmitters [2]. It has been demonstrated that distributed multiple-radar system with multiple transmitters and multiple receivers at different sites has a number of performance advantages over monostatic radar owing to its advantage of spatial and waveform diversities, which has triggered a resurgence of interest in distributed multiple-radar architecture. Therefore, considerable research efforts have been devoted to the potential use of such system for achieving performance improvement in various contexts such as target detection [3], [4], target localization [5], target tracking [1], [6], parameter estimation [7], [8], radar waveform design [9], [10], sensor selection [11], [12], and information extraction [13].

Due to the services with high bandwidth requirements and rapid growing of mobile telecommunications, the radio frequency (RF) spectrum scarcity has become a very essential and changing problem that the whole world has to face. One of the feasible solutions is to improve spectrum efficiency by employing the potential of existing spectrum. In recent literature, the concept of spectrum sharing has been regarded as a promising solution to resolve the issue of spectrum congestion [14], which allows two or more users (radar or wireless communication systems) to share the RF spectrum as long as they do not generate any harmful interference to each other. In [15], a dynamic spectrum allocation scheme is proposed for the coexistence of a radar system with a communication system, where the transmitted waveform and power spectrum are jointly optimized under the constraint that a predefined signal-to-interference-plus-noise ratio (SINR) requirement is met. Bica and Koivunen [16] investigate the problem of time delay estimation for coexisting multicarrier radar and communication systems, and it is shown that the radar can enhance its target estimation performance by utilizing the communication signals scattered off the target in a passive way. More recently, recognizing that the exact knowledge of target spectra is impossible to capture in practice, the problem of power minimization based robust orthogonal frequency division multiplexing (OFDM) radar waveform design for the coexisting radar and communication systems in signal-dependent clutter and colored noise is addressed in [17], where the target spectra are assumed to lie in uncertainty sets bounded by known upper and lower bounds. It is demonstrated that exploiting the communication signals scattered off the target can significantly reduce the power consumption of radar system. Li and Petropulu [18] present a cooperative spectrum sharing approach, which improves the SINR of the radar system by optimizing the multiple-input multiple-output (MIMO) radar transmit precoder and the communication transmit covariance matrix with a given rate constraint for the communication system. Reference [19] also provides a novel framework for coexisting communication systems and pulsed radars. The problem of user association and power allocation in millimeter-wave-based ultra dense networks is investigated with the consideration of user quality of service (QoS) requirements, energy efficiency, and interference limits [20].

B. BRIEF SURVEY OF SIMILAR WORK

An established powerful tool for distributed optimization problems can be provided by the non-cooperative game theory [21]. Each player in such a game behaves in a selfish and rational fashion to maximize its own gain (utility) as a best response to the actions of the other players [22]. Game theoretic models are traditionally investigated and applied in areas of economics, politics science and biology, and has emerged in recent years as an effective and powerful tool for radar network and signal processing. Among the early contributions in this area, Gogineni and Nehorai [23] propose a polarimetric design algorithm for target detection in distributed MIMO radar system. In [24], the interaction between a smart target and a smart MIMO radar is modelled as a two-person zero-sum game. In the spirit of these studies, extensive literatures have since concentrated on developing power management techniques for radar network subject to system performance requirements and resource constraints. For example, Bacci *et al.* [25] develop a game theory based distributed algorithm for radar network, which optimally allocates the transmission power to each radar while improving the target detection performance in terms of probabilities of detection and false alarm. In [26], a non-cooperative game theory based power allocation approach is proposed for a multistatic MIMO radar network, whose main objective is to minimize the total power consumption of the system while maintaining a desired SINR threshold. Furthermore, [27] studies the problem of robust power allocation in the presence of estimation error. Deligiannis *et al.* [28] investigate the competitive power allocation game between a radar network and multiple jammers, and a Bayesian game theory based SINR maximization and power allocation algorithm is explored in [29]. Later, they revisit the power allocation problem of a multistatic MIMO radar network [2], which is formulated as a generalized Nash game. The objective of the radar network is to minimize the total transmission power while satisfying a given target detection criterion.

In view of the aforementioned works, the problem of radar and communication systems in spectral coexistence has been extensively investigated, whereas it is still at an early stage and there exist many aspects need to be further improved: (a) All the existing studies solely concentrate on the monostatic radar, which is not appropriate for the practical extension to the distributed multiple-radar system. In the latter case, the limitations and calculations are much more complicated; (b) Although the non-cooperative game model is employed to perform power allocation in multistatic radar, the analytical closed-form expressions for game theoretic power allocation have not yet been derived; (c) The non-cooperative game theoretic models have not been utilized to conduct spectrum sharing between multiple-radar and communication systems. On the other hand, although Labib *et al.* [14] propose an idea to solve the problem of radar and long term evolution (LTE) systems coexistence by employing non-cooperative game, both the detailed algorithm and numerical results are not given. In particular, [30] proposes an incomplete channel state information (CSI)-based power allocation and sub-channel assignment algorithm for heterogeneous networks, which is modeled as a non-cooperative game with the consideration of cross-tier/co-tier interference constraints. Our work builds on the non-cooperative game theoretic framework presented in [30]. Despite this similarity, the analysis of [30] does not account for the spectral coexistence between multiple-radar and communication systems, thus the resulting game theoretic model is not suitable for the problem scenario here.

Incorporating target detection requirements and aggregate interference changes the setting drastically, which is because the power allocation policy of distributed multiple-radar system depends not only on the maximum interference tolerant limit of communication system, but also on the target scattering characteristics and system geometry configuration. In this study, we will extend the analyses in [14] and [30] and the problem we will address is how to optimize transmit power allocation for a multistatic radar system coexisting with a communication system in the same frequency band. To the best of our knowledge, the problem of power allocation for distributed multiple-radar architecture in a spectrum sharing environment has not been well addressed in previous studies, and we will investigate this problem based on a non-cooperative game for the first time.

C. MAJOR CONTRIBUTIONS

In this paper, different from the existing algorithms, we investigate the power allocation problem of a distributed multiple-radar configuration, which is composed of multiple radars coexisting with a communication system in the same frequency band. We are primarily interested in a non-cooperative method due to the fact that in a future distributed multiple-radar system, there may be some implementation difficulties or the netted radars may not be controlled by the fusion center and these radars may not cooperate. Thus, it is preferred to consider autonomous distributed power allocation techniques [2], which also have an important advantage of avoiding the energy consumption associated with centralized policies requiring remarkable information exchange between radars and/or the system controller [31]. Note that the proposed strategy is particularly attractive for target tracking where the location and velocity of the target are approximately estimated, but fine detection performance is required to retrieve the exact target's position and characteristics. In this scenario, the primary goal of the multiple-radar system is to guarantee a predefined SINR requirement for target detection and secure a maximum aggregate interference tolerant limit for communication system, while minimizing the power consumption of each radar by optimizing the transmission power allocation.

The major contributions of this work are listed as follows:

(1) *The problem of non-cooperative game theoretic power allocation (NGTPA) for the coexisting multiple-radar and communication systems is investigated.* Mathematically speaking, the NGTPA strategy is a problem of minimizing the power consumption of each radar subject to a desired SINR requirement for target detection and a maximum aggregate interference tolerant limit for communication system. Since each radar is rational and selfish to maximize its own utility, we employ the non-cooperative game theoretic technique to tackle the distributed power allocation problem. Previously, most of the power allocation works adopt the total transmission power as the utility function [2], [26], [27]. However, the received aggregate interference power at the communication system in a spectrum sharing environment is not considered, and thus it is reasonable for us to incorporate the transmission power of each radar, the specified SINR threshold and the maximum interference tolerant limit to define a novel utility function as the optimization criterion. As such, the basis of the NGTPA strategy is to optimally allocate the minimum transmission power to each radar, which can result in the maximization of the defined utility function.

(2) *The Nash equilibrium of the non-cooperative power allocation game is obtained based on the Lagrangian dual function and sub-gradient approach. The existence and uniqueness of the proposed game model to its Nash equilibrium points are also demonstrated.*

(3) *Iterative power allocation algorithm with low computational complexity and fast convergence is developed*, which determines the Nash equilibrium solutions to the NGTPA model starting from any initial feasible points. The proposed algorithm also ensures the distributed nature of the system with considerable reduction on the signaling overhead, confirming its potential for a practical scenario.

(4) *Numerical results are provide to demonstrate the superiority of the proposed NGTPA strategy compared to various state of the art algorithms.* It is shown that the NGTPA scheme not only guarantees the desired SINR requirement for target detection and secure the maximum interference tolerant limit for communication system, but also allocates the minimum transmission power to each radar. Additionally, we also reveal the relationships between the power allocation results and the following two factors: target's radar cross section (RCS) and the relative geometry between target and distributed multiple-radar architectures.

D. OUTLINE OF THE PAPER

The remainder of this paper is structured as follows. The distributed multiple-radar architecture coexisting with a communication system as well as the underlying assumptions needed in this paper are introduced in Section II. In Section III-A, the basis of the non-cooperative game theoretic power allocation strategy is introduced. Section III-B presents the game theoretic formulation of the problem. The existence and uniqueness of the Nash equilibrium are proved analytically in Section IV. In Section V, a non-cooperative distributed low-complexity and iterative algorithm is developed to determine the NGTPA model's solution. The performance of the proposed strategy is assessed in detail via modeling and simulation in Section VI, whose superiority compared to other existing methods is illustrated via detailed comparative numerical results. Finally, the concluding remarks of this paper are made in Section VII.

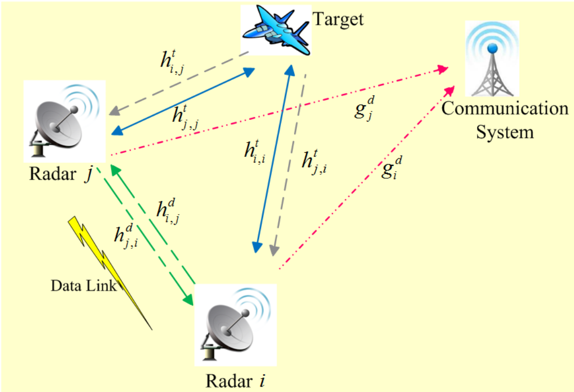

FIGURE 1. Illustration of the system model for spectrum sharing between distributed multiple-radar architecture and communication system with their corresponding channel gains.

II. SYSTEM MODEL

A. PROBLEM SCENARIO

Let us consider a scenario, where a distributed multiple-radar architecture consisting of M_T radars coexist with a communication system in the same frequency band. Such a multiple-radar system with a possible target is depicted in Fig.1. The main goal of the distributed multiple-radar configuration is to minimize the transmission power of each radar by optimizing the transmission power allocation, which is constrained by a predefined SINR requirement for target detection and a maximum interference tolerant limit for communication system. The ith radar receives the echoes from the target due to its transmitted signals as well as the signals from the other radars, both scattered off the target and through a direct path. The waveforms emitted from different radars may not be orthogonal because of various reasons, including the absence of radar transmission synchronization [32], which could induce considerable mutual interference. Assuming that successive interference cancellation (SIC) technique is employed at each radar receiver to remove both direct and target scattered communication signals from the observed signal [16]. At the communication system, it is also supposed that the radar transmitted signal scattered off the target is much weaker than that coming through the direct path from the radar transmitter, which is ignored for simplicity.

B. SIGNAL MODEL

This subsection describes signal model and presents system parameters utilized in the following. In the considered non-cooperative game theoretic framework, each radar performs target detection autonomously. It is assumed that each radar can determine the presence of a target by employing a binary hypothesis testing on the received signal based on the generalized likelihood ratio test (GLRT) [2]. Thus, the N time-domain samples of the received signals for radar i, with $\mathcal{H}_0$ corresponding to the target absence hypothesis and $\mathcal{H}_1$ corresponding to the target presence hypothesis, can be given by:

$$\begin{cases} \mathcal{H}_0 : \mathbf{s}_i = \sum_{j=1, j\neq i}^{M_T} \beta_{i,j}\sqrt{P_j}\mathbf{x}_j + w_i, \\ \mathcal{H}_1 : \mathbf{s}_i = \alpha_i\sqrt{P_i}\mathbf{x}_i + \sum_{j=1, j\neq i}^{M_T} \beta_{i,j}\sqrt{P_j}\mathbf{x}_j + w_i, \end{cases} \tag{1}$$

where $\mathbf{x}_i = \phi_i\mathbf{a}_i$ denotes the transmitted waveform from radar i, $\mathbf{a}_i = [1, e^{j2\pi f_{D,i}}, \cdots, e^{j2\pi(N-1)f_{D,i}}]$ denotes the Doppler steering vector of radar i with respect to the target, $f_{D,i}$ is the Doppler shift associated with the radar i, N is the number of received pulses in the time-on-target, and ϕ_i is the predesigned waveform transmitted from radar i. α_i represents the channel gain at the direction of the target, P_i is the transmit power of radar i, $\beta_{i,j}$ stands for the cross gain between radar i and j, and w_i denotes a zero-mean white Gaussian noise with variance σ_w^2. It is assumed that $\alpha_i \sim \mathcal{CN}(0, h^t_{i,i})$, $\beta_{i,j} \sim \mathcal{CN}(0, c_{i,j}(h^t_{i,j} + h^d_{i,j}))$ and $w_i \sim \mathcal{CN}(0, \sigma_w^2)$, where $h^t_{i,i}$ represents the variance of the channel gain for the radar i-target-radar i path, $c_{i,j}h^t_{i,j}$ represents the variance of the channel gain for the radar i-target-radar j path, $c_{i,j}h^d_{i,j}$ represents the variance of the channel gain for the direct radar i-radar j path, and $c_{i,j}$ denotes the cross correlation coefficient between the ith radar and jth radar.

Define the propagation gains of the corresponding paths as:

$$\begin{cases} h^t_{i,i} = \frac{G_t G_r \sigma^{\text{RCS}}_{i,i}\lambda^2}{(4\pi)^3 R_i^4}, \\ h^t_{i,j} = \frac{G_t G_r \sigma^{\text{RCS}}_{i,j}\lambda^2}{(4\pi)^3 R_i^2 R_j^2}, \\ h^d_{i,j} = \frac{G'_t G'_r \lambda^2}{(4\pi)^2 d^2_{i,j}}, \\ g^d_i = \frac{G'_t G_c \lambda^2}{(4\pi)^2 d_i^2}, \end{cases} \tag{2}$$

where $h^t_{i,i}$ represents the propagation gain for the radar i-target-radar i path, $h^t_{i,j}$ represents the propagation gain for the radar i-target-radar j path, $h^d_{i,j}$ represents the direct radar i-radar j path, g^d_i represents the direct radar i-communication system path. G_t is the radar main-lobe transmitting antenna gain, G_r is the radar main-lobe receiving antenna gain, G'_t is the radar side-lobe transmitting antenna gain, G'_r is the radar side-lobe receiving antenna gain, and G_c is the communication receiving antenna gain. $\sigma^{\text{RCS}}_{i,i}$ is the RCS of the target with respect to the ith radar, $\sigma^{\text{RCS}}_{i,j}$ is the RCS of the target from radar i to radar j, λ denotes the wavelength, R_i denotes the distance from radar i to the target, R_j denotes the distance from radar j to the target, $d_{i,j}$ denotes the distance between radar i and radar j, d_i denotes the distance between radar i and communication system. All the channel gains are assumed to be fixed during observation.

Here, the generalized likelihood ratio test (GLRT) is used to determine the appropriate detector [2], [26].

The probabilities of detection $p_{\mathrm{D},i}(\delta_i, \gamma_i)$ and false alarm $p_{\mathrm{FA},i}(\delta_i)$ are:

$$\begin{cases} p_{\mathrm{D},i}(\delta_i, \gamma_i) = \left(1 + \frac{\delta_i}{1-\delta_i} \cdot \frac{1}{1+N\gamma_i}\right)^{1-N}, \\ p_{\mathrm{FA},i}(\delta_i) = (1-\delta_i)^{N-1}, \end{cases} \tag{3}$$

where δ_i is the detection threshold, N is the number of received pulses in the time-on-target. γ_i denotes the SINR received at the ith radar, which can be given by:

$$\gamma_i = \frac{h_{i,i}^t P_i}{\sum_{j=1, j\neq i}^{M_T} c_{i,j}\left(h_{i,j}^d P_j + h_{i,j}^t P_j\right) + \sigma_w^2}, \tag{4}$$

It can be seen from Equation (4) that the numerator of the SINR describes the return signal scattered off the target, while the dominator consists of the interference and noise [33]. Thus, Equation (4) can equivalently be rewritten as:

$$\gamma_i = \frac{h_{i,i}^t P_i}{I_{-i}}, \tag{5}$$

where the total interference and noise received at the ith radar is defined as:

$$I_{-i} = \sum_{j=1, j\neq i}^{M_T} c_{i,j}\left(h_{i,j}^d P_j + h_{i,j}^t P_j\right) + \sigma_w^2. \tag{6}$$

To guarantee its target detection performance, the received SINR of radar i should be no smaller than a predefined minimum value denoted by $\gamma_{\min}$. Then, we obtain a target detection condition as:

$$\gamma_i \geq \gamma_{\min}. \tag{7}$$

Finally, it is also assumed that the transmit power of each radar is individually limited by $P_i^{\max}$, that is:

$$0 \leq P_i \leq P_i^{\max}. \tag{8}$$

C. INTERFERENCE POWER CONSTRAINT

This subsection presents the transmission interference regulation between the distributed multiple-radar and communication systems accounted in the system model.

In this work, the distributed multiple-radar architecture is allowed to coexist with a wireless communication system in the same frequency band provided that the degradation induced on the QoS of the communication system is tolerable. Thus, it is crucial to impose interference power constraint to control the harmful interference generated by the multiple radars.

The interference power constraint is utilized to prevent the total aggregate interference generated by all radars to the communication user from exceeding a predetermined threshold $T_{\max}$, which can be expressed as:

$$\sum_{i=1}^{M_T} g_i^d P_i \leq T_{\max}, \tag{9}$$

where $T_{\max}$ represents the maximum interference tolerant limit prescribed by the communication system. In this way, the communication user's transmission can be protected when the transmit power of distributed multiple-radar system is constrained by the maximum interference tolerant threshold $T_{\max}$.

Remark 1: Technically speaking, the target detection performance can be evaluated in terms of the probabilities of detection $p_{\mathrm{D},i}(\delta_i, \gamma_i)$ and false alarm $p_{\mathrm{FA},i}(\delta_i)$ for each radar [2]. The GLRT utilized here is similar to the Neyman-Pearson detector as the number of samples approaches infinity. Then, the detection threshold δ_i can be calculated from the predefined probability of false alarm $p_{\mathrm{FA},i}(\delta_i)$, whereas the probability of detection $p_{\mathrm{D},i}(\delta_i, \gamma_i)$ depends on the threshold δ_i and the SINR associated with the received signal. Therefore, given the probabilities of detection and false alarm, the specified SINR value $\gamma_{\min}$ can be determined. In order to examine the interaction among radars and determine the best strategy for each radar, we propose to optimize transmission power allocation for the distributed multiple-radar architecture coexisting with a communication system by exploiting a non-cooperative game model, as presented in the next section.

III. GAME THEORETIC STRATEGY FOR POWER ALLOCATION

A. BASIS OF THE TECHNIQUE

Mathematically, the non-cooperative game theoretic power allocation strategy for spectrum sharing can be described as a problem of minimizing the power consumption of each radar subject to a predefined SINR requirement for target detection and a maximum interference tolerant limit for communication system. Consider that radars in the system are selfish and greedy to maximize their own utilities, the non-cooperative game theory is exploited to model the interactions between different radars as a Nash game. Then, the existence and uniqueness of Nash equilibrium are proved analytically. Finally, an iterative power allocation algorithm with low computational complexity and fast convergence is proposed to play the game among different radars.

We are then in a position to optimize the transmission power allocation for multiple-radar system in a spectrum sharing environment. The general power allocation strategy can be detailed as follows.

B. GAME THEORETIC FORMULATION

As previously stated, the main objective is to minimize the power consumption of each radar by optimizing the transmission power allocation, which is constrained by a predefined SINR requirement for target detection and a maximum interference tolerant limit for communication system. It can be observed from (4) and (9) that increased power allocation can improve the target detection performance, which in turn induces higher interference to the communication system and consequently to the remaining radars of the architecture. Hence, in order to consider players' rational and self-interested behavior, game theory arises as an efficient mathematical tool. To be specific, the radars that act as players compete with each other and choose a strategy space of transmission power and subsequently achieve a payoff, which is expressed by their utility functions.

The characteristics of the interaction of the players in the non-cooperative power allocation game in strategic form can be expressed by:

$$\mathcal{G} = \langle \mathcal{K}, \{\mathcal{P}_i\}_{i\in\mathcal{K}}, \{U_i\}_{i\in\mathcal{K}} \rangle, \tag{10}$$

where $\mathcal{K} = \{1, \cdots, M_T\}$ denotes the finite set of players, $\mathcal{P}_i$ denotes the ith player's strategy space, where $\mathcal{P}_i = [0, P_i^{\max}]$, and U_i denotes the player's utility function. The strategy space of the NGTPA model $\mathcal{P}$ depends not only on the strategy of the ith player $\mathcal{P}_i$ but also on the strategies of all other players $\mathcal{P}_{-i}$, that is, $\mathcal{P} = \mathcal{P}_1 \times \mathcal{P}_2 \times \cdots \times \mathcal{P}_{M_T}$, where the subscript $-i$ represents all players except player i.

It is very important to select an ideal utility function when utilizing non-cooperative game theory. For spectrum sharing between multiple-radar system and communication system, the target detection performance and received aggregate interference power at the communication receiver should be taken into account, which should be reflected in the utility function. Utility function is the foundation of game theory, which will deduce the iterative algorithm. Moreover, as indicated in [22], whether the utility function is better or not depends on the iterative algorithm deduced by the utility function. If the algorithm does not converge or cannot get ideal transmission power and SINR value, the utility function will be discarded. Here, the primary objective of the multiple-radar architecture is to minimize the transmission power of each radar while guaranteeing a predefined SINR requirement for target detection and a maximum interference tolerant limit for communication system. Therefore, a novel utility function can be defined as:

$$U_i(P_i, \mathbf{P}_{-i}) = \ln(\gamma_i - \gamma_{\min}) - \mu_i h_{i,i}^t P_i - \vartheta_i \sum_{i=1}^{M_T} g_i^d P_i, \tag{11}$$

where $\mathbf{P}_{-i}$ is the power allocation adopted by all radars apart from radar i, μ_i and ϑ_i denote the time-varying pricing variables corresponding to the radar transmit power and the aggregate interference caused by the radar transmission, respectively. In (11), one can observe that the utility function is composed of three components. The first component $\ln(\gamma_i - \gamma_{\min})$ measures the utility of a radar when certain target detection performance is required. The second component $\mu_i h_{i,i}^t P_i$ is a penalty term corresponding to the radar transmission power pricing cost, while the third one $\vartheta_i \sum_{i=1}^{M_T} g_i^d P_i$ is also a penalty term representing the aggregate interference pricing cost.

The goal of each player is to maximize its utility by selecting an appropriate strategy of transmission power. Hence, consider a specified SINR requirement for target detection and a maximum interference tolerant limit for communication system, the NGTPA model can be formulated mathematically as a distributed utility maximization problem, as follows:

$$\mathcal{P}_1: \max_{\{P_i\in\mathcal{P}_i\}_{i\in\mathcal{K}}} U_i(P_i, \mathbf{P}_{-i}), \tag{12a}$$

$$\text{s.t.}: \begin{cases} C1: 0 \le P_i \le P_i^{\max}, \forall i \in \mathcal{K}, \\ C2: \gamma_i \ge \gamma_{\min}, \forall i \in \mathcal{K}, \\ C3: \sum_{i=1}^{M_T} g_i^d P_i \le T_{\max}. \end{cases} \tag{12b}$$

The first constraint $C1$ limits the transmit power of each radar to be below $P_i^{\max}$, the second constraint $C2$ implies that the power allocation results should be no less than the predetermined SINR threshold $\gamma_{\min}$, while the last constraint $C3$ stands that the total received interference power at the communication receiver cannot exceed the maximum interference tolerant limit $T_{\max}$.

Given the decisions made by the rest of the players, the individual decision of each player can be concluded. Thus, the solution of the non-cooperative power allocation game should determine the optimal equilibrium for the multiple-radar system in a spectrum sharing environment. In the following, the Nash equilibrium point of the proposed NGTPA model can be defined as:

Definition 1 (Nash Equilibrium): A power vector $\mathbf{P}_i^* = (P_i^*, \mathbf{P}_{-i}^*)$ in the strategy set $\mathbf{P}_i^* \in \mathcal{P}$ is a Nash equilibrium of the NGTPA model $\mathcal{G}$ if for every player $i \in \mathcal{K}$ the following condition holds true:

$$U(P_i^*, \mathbf{P}_{-i}^*) \ge U(P_i, \mathbf{P}_{-i}^*), \tag{13}$$

for all $P_i \in \mathcal{P}_i$.

The objective of a non-cooperative game is to find a Nash equilibrium point at which none of the players has the incentive to change its strategy. This is because the player cannot unilaterally improve its personal utility by making any change to its own strategy, given the strategies of the rest of the players. Therefore, it is worth to point out that the existence of Nash equilibrium solution guarantees a stable outcome of the NGTPA model, while on the contrary, the non-existence of such a Nash equilibrium solution is translated to an unstable and unsteady situation of the game model.

Subsequently, taking the first derivative of $U_i(P_i, \mathbf{P}_{-i})$ with respect to P_i, we can obtain:

$$\frac{\partial U_i(P_i, \mathbf{P}_{-i})}{\partial P_i} = \frac{1}{\gamma_i - \gamma_{\min}} \frac{h_{i,i}^t}{I_{-i}} - \mu_i h_{i,i}^t - \vartheta_i g_i^d, \tag{14}$$

Then, we set the first derivative $\frac{\partial U_i(P_i, \mathbf{P}_{-i})}{\partial P_i} = 0$. Rearranging terms yields:

$$\frac{1}{\gamma_i - \gamma_{\min}} \frac{h_{i,i}^t}{I_{-i}} = \mu_i h_{i,i}^t + \vartheta_i g_i^d. \tag{15}$$

After basic algebraic manipulations, we have:

$$\gamma_i = \gamma_{\min} + \frac{h_{i,i}^t}{I_{-i}} \frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d}. \tag{16}$$

Substituting $\gamma_i = \frac{h_{i,i}^t P_i}{I_{-i}}$ into Equation (16), we can obtain the ith radar transmit power optimum point as follows:

$$P_i = \frac{I_{-i}}{h_{i,i}^t}\gamma_{\min} + \frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d}. \tag{17}$$

Obviously, according to Equations (5) and (17), (18) can be used to obtain the Nash equilibrium solutions $\mathbf{P}_i^*$ through iterations as:

$$P_i^{(ite+1)} = \frac{P_i^{(ite)}}{\gamma_i^{(ite)}}\gamma_{\min} + \frac{1}{\mu_i^{(ite)} h_{i,i}^t + \vartheta_i^{(ite)} g_i^d}, \tag{18}$$

where ite denotes the iteration index.

On the basis of Equation (18), the corresponding ith radar transmit power iteration function can be expressed by:

$$P_i^{(ite+1)} = \left[\frac{P_i^{(ite)}}{\gamma_i^{(ite)}}\gamma_{\min} + \frac{1}{\mu_i^{(ite)} h_{i,i}^t + \vartheta_i^{(ite)} g_i^d}\right]_0^{P_i^{\max}}, \tag{19}$$

where $[x]_a^b = \max\{\min(x,b), a\}$.

In this paper, the dynamic pricing method is considered, where the pricing variables will be updated in the proposed iterative power allocation algorithm in Section V. It should be noted that the selection of pricing variable μ_i is of high importance. More precisely, if $\gamma_i^{(ite)} \leq \gamma_{\min}$, $\mu_i^{(ite+1)}$ remains unchanged; whereas if $\gamma_i^{(ite)} > \gamma_{\min}$, we adjust the level of $\mu_i^{(ite+1)} = \mu_i^{(ite)} \left(\frac{\gamma_i^{(ite)}}{\gamma_{\min}}\right)^2$ adaptively, which decreases the transmit power by increasing the punishment for player i. In addition, to guarantee the proposed iterative algorithm can achieve Nash equilibrium, the sub-gradient approach [34] is adopted to update the pricing variable $\vartheta_i^{(ite)}$ as follows:

$$\vartheta_i^{(ite+1)} = \left[\vartheta_i^{(ite)} - \alpha^{(ite)}\left(T_{\max} - \sum_{i=1}^{M_T} g_i^d P_i^{(ite+1)}\right)\right]^+, \tag{20}$$

where $(x)^+ = \max(0, x)$, $\alpha^{(ite)}$ is the step size of iteration ite ($ite \in (1, 2, \ldots, L_{\max})$), $L_{\max}$ is the maximum number of iterations. It is noted that $\alpha^{(ite)}$ is locally updated, which should satisfy the following conditions:

$$\begin{cases} \sum_{l=1}^{\infty} \alpha^{(l)} = \infty, \\ \lim_{l\to\infty} \alpha^{(l)} = 0. \end{cases} \tag{21}$$

C. POTENTIAL EXTENSION

Without loss of generality, we concentrate on a single communication system case in this work. However, the derivations and results can be extended to the multiple communication systems scenario, in which the interference limits are imposed to protect each communication user's QoS. For Q communication users case, the resulting optimization problem can be reformulated as:

$$\mathcal{P}_2: \max_{\{P_i \in \mathcal{P}_i\}_{i\in\mathcal{K}}} U_i(P_i, \mathbf{P}_{-i}), \tag{22a}$$

$$\text{s.t.}: \begin{cases} C1: 0 \leq P_i \leq P_i^{\max}, \forall i \in \mathcal{K}, \\ C2: \gamma_i \geq \gamma_{\min}, \forall i \in \mathcal{K}, \\ C3: \sum_{i=1}^{M_T} g_{i,q}^d P_i \leq T_{q,\max}, \forall q \in \mathcal{Q}. \end{cases} \tag{22b}$$

where the parameters with subscript q denote the corresponding ones of communication user q ($q \in \mathcal{Q} = \{1, 2, \ldots, Q\}$). Then, we can also employ the following iterative power allocation algorithm to search for the Nash equilibrium solutions for $\mathcal{P}_2$. In this scenario, we can conclude that the proposed NGTPA strategy can straightforward be extended to multiple communication users case by adding the interference limits for each user.

IV. EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM

To analyse the outcome of the presented non-cooperative game theoretic power allocation model, we first introduce the basic theorems to prove the existence and uniqueness of a Nash equilibrium.

A. EXISTENCE

Theorem 1 (Existence): The proposed NGTPA model $\mathcal{G} = \langle \mathcal{K}, \{\mathcal{P}_i\}_{i\in\mathcal{K}}, \{U_i\}_{i\in\mathcal{K}}\rangle$ has at least one Nash equilibrium.

Proof of Theorem 1: At least one Nash equilibrium exists in the proposed NGTPA model $\mathcal{G} = \langle \mathcal{K}, \{\mathcal{P}_i\}_{i\in\mathcal{K}}, \{U_i\}_{i\in\mathcal{K}}\rangle$. If for $\forall i \in \mathcal{K}$:

(a) The transmission power P_i is a non-empty, convex and compact subset of some Euclidean space.

(b) The utility function $U_i(P_i, \mathbf{P}_{-i})$ is continuous in power domain $\mathbf{P}$ and quasi-concave in P_i.

It is evident that our proposed NGTPA strategy satisfies the first Condition (a), which is because each radar's transmission power P_i ranges from 0 to $P_i^{\max}$.

One can notice from Equation (11) that the utility functions $U_i(P_i, \mathbf{P}_{-i})(\forall i \in \mathcal{K})$ are continuous with respect to P_i. Now take the first derivative of $U_i(P_i, \mathbf{P}_{-i})$ with respect to P_i, we can obtain:

$$\frac{\partial U_i(P_i, \mathbf{P}_{-i})}{\partial P_i} = \frac{1}{\gamma_i - \gamma_{\min}}\frac{h_{i,i}^t}{I_{-i}} - \mu_i h_{i,i}^t - \vartheta_i g_i^d, \tag{23}$$

then we take the second order derivative of $U_i(P_i, \mathbf{P}_{-i})$ with respect to P_i and obtain:

$$\frac{\partial^2 U_i(P_i, \mathbf{P}_{-i})}{\partial P_i^2} = -\frac{(h_{i,i}^t)^2}{I_{-i}^2(\gamma_i - \gamma_{\min})^2}. \tag{24}$$

Obviously, the second order derivative of $U_i(P_i, \mathbf{P}_{-i})$ with respect to P_i is less than 0, thus,

$$\frac{\partial^2 U_i(P_i, \mathbf{P}_{-i})}{\partial P_i^2} < 0, \tag{25}$$

$U_i(P_i, \mathbf{P}_{-i})$ is concave with respect to P_i. As a result, the utility functions are continuous and quasi-concave. This proves the existence of Nash equilibrium in the proposed NGTPA model. ■

B. UNIQUENESS

It is worth to point out that **Theorem 1** guarantees the existence of at least one Nash equilibrium point of the proposed NGTPA model, while this point is not necessarily unique.

In this subsection, the uniqueness of the Nash equilibrium for the non-cooperative game $\mathcal{G}$ is demonstrated.

Theorem 2 (Uniqueness): The Nash equilibrium of the proposed NGTPA model $\mathcal{G} = \langle \mathcal{K}, \{\mathcal{P}_i\}_{i\in\mathcal{K}}, \{U_i\}_{i\in\mathcal{K}} \rangle$ is unique.

Proof of Theorem 2: Aiming at showing that the Nash equilibrium of the proposed game model $\mathcal{G}$ is unique, we have to prove that radar's best response strategy function:

$$f(P_i) = \frac{P_i}{\gamma_i}\gamma_{\min} + \frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d}$$

should be standard, which satisfies the following conditions:

(a) *Positivity:* For $\forall i \in \mathcal{K}, f(P_i) > 0$.

(b) *Monotonicity:* If $P_i^1 > P_i^2, f(P_i^1) > f(P_i^2)$.

(c) *Scalability:* For all $a > 1$, $af(P_i) > f(aP_i)$.

For Condition (a), it is apparent that:

$$f(P_i) = \frac{P_i}{\gamma_i}\gamma_{\min} + \frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d} > 0.$$

Hence, the positivity property is satisfied.

For Condition (b), if $P_i^1 > P_i^2$,

$$\begin{aligned} f(P_i^1) - f(P_i^2) &= \frac{P_i^1 - P_i^2}{\gamma_i}\gamma_{\min} \\ &\quad + \left(\frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d} - \frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d}\right) \\ &= \frac{P_i^1 - P_i^2}{\gamma_i}\gamma_{\min} > 0. \end{aligned} \tag{26}$$

Then,

$$f(P_i^1) - f(P_i^2) > 0. \tag{27}$$

Hence, the monotonicity property is satisfied.

For Condition (c), $\forall i \in \mathcal{K}$,

$$\begin{aligned} &af(P_i) - f(aP_i) \\ &= a\left(\frac{P_i}{\gamma_i}\gamma_{\min} + \frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d}\right) \\ &\quad - \left(\frac{aP_i}{\gamma_i}\gamma_{\min} + \frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d}\right) \\ &= \frac{a}{\mu_i h_{i,i}^t + \vartheta_i g_i^d} - \frac{1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d} \\ &= \frac{a-1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d}. \end{aligned} \tag{28}$$

Owing to $a > 1$, we can obtain:

$$\frac{a-1}{\mu_i h_{i,i}^t + \vartheta_i g_i^d} > 0. \tag{29}$$

Then,

$$af(P_i) - f(aP_i) > 0. \tag{30}$$

Hence, the scalability property is satisfied.

In conclusion, the best response strategy function $f(P_i)$ is standard. Therefore, our proposed non-cooperative power allocation model has only one unique Nash equilibrium solution, which completes the Nash equilibrium uniqueness proof. ■

V. ITERATIVE POWER ALLOCATION ALGORITHM

Herein, a distributed iterative and low complexity power allocation algorithm is developed, which determines the Nash equilibrium point of NGTPA model starting from any initial feasible point.

The presented NGTPA strategy is executed by each radar at each time step in a distributed manner so that the Nash equilibrium point of NGTPA model can be determined, that is, each radar determines its optimal transmission power and achieved SINR value. For this reason, **Algorithm 1** is suitable for asynchronous implementation in dynamic network architectures, where each radar only requires the transmit strategies of all the other radars, without any further information on the system [31]. Thus, the iteration power allocation algorithm is a fully distributed process and its pseudo-code is summarized in **Algorithm 1**, which is based on the existence of a unique Nash equilibrium for the proposed NGTPA model.

Algorithm 1 : Iterative Power Allocation Algorithm

set iteration index $ite = 0$;

initialize $P_i^{(ite=0)}$ with a random feasible power allocation among all radars; $\gamma_{\min}$, $T_{\max}$, μ_i, ϑ_i, and $\varepsilon > 0$ (ε is a small positive constant); Obtain the corresponding channel gains.

repeat

  for $i = 1$ to M_T **do**

    a) update $P_i^{(ite)}$ by solving (19), and broadcast those values to all the other radars via data link;

    b) **if** $\gamma_i^{(ite)} > \gamma_{\min}$

      $\mu_i^{(ite+1)} \leftarrow \mu_i^{(ite)} \left(\frac{\gamma_i^{(ite)}}{\gamma_{\min}}\right)^2$;

    else

      $\mu_i^{(ite+1)} \leftarrow \mu_i^{(ite)}$;

    end if

    c) update $\vartheta_i^{(ite+1)}$ by solving (20);

  end for

  set $ite \leftarrow ite + 1$;

until $\left|P_i^{(ite+1)} - P_i^{(ite)}\right| < \varepsilon$ for all $i \in \mathcal{K}$ or $ite = L_{\max}$.

Remark 2 (Implementation Overhead): Each radar updates its action at each iteration step such that the utility function in problem $\mathcal{P}_1$ is maximized. In the foregoing procedure, the transmission power iteration function $P_i^{(ite+1)}$ can be updated according to (19), where the optimum power allocation results can be determined locally.

In order to implement **Algorithm 1** in a distributed manner, each radar has to compute and estimate the channel gains $\{h_{i,j}^t\}_{j=1,j\neq i}^{M_T}$, $\{h_{i,j}^d\}_{j=1,j\neq i}^{M_T}$, $\{h_{i,i}^t\}_{i=1}^{M_T}$, and $\{g_i^d\}_{i=1}^{M_T}$. This can be done by having each radar measures the channel and feed back to its transmitter. At each radar receiver, the interference caused by all the other radars is treated as noise. Here, the best response of the ith radar P_i^* depends on the strategies of all the other radars, that is, $\mathbf{P}_{-i}^*$. During the iteration, each radar only needs to broadcast its transmit strategy to the other radars via data link, and thus the signaling overhead for the convergence are quite low [35].

Remark 3 (Complexity Analysis): The computational complexity of **Algorithm 1** is dominated by the size of multiple-radar system and the procedure of sub-gradient iteration steps. In **Algorithm 1**, the calculation of (19) for each radar in the distributed multiple-radar system entails M_T operations in each iteration. Assume the sub-gradient method employed in **Algorithm 1** requires Δ iterations to converge, the update of ϑ_i needs $\mathcal{O}(M_T)$ operations. Hence, Δ is a polynomial function of M_T. The total complexity of **Algorithm 1** is $\mathcal{O}(M_T\Delta)$.

Additionally, the convergence performance of NGTPA strategy, in terms of necessary iterations, is thoroughly evaluated in the following section, demonstrating the fast convergence property of the proposed scheme.

VI. NUMERICAL EXAMPLES AND ANALYSIS

In this section, simulation results are provided to verify the accuracy of the theoretical derivations as well as access the performance of the proposed non-cooperative game theoretic power allocation strategy. Besides, the effects of several factors on the equilibrium power allocation results are studied.

A. NUMERICAL SETUP

For numerical simulations, it is assumed a distributed multiple-radar system with $M_T = 4$ radars. The positions of multiple radars, communication system and target are illustrated in Fig.2. To investigate the dependence of the power allocation strategy on the relative geometry between the target and the multiple-radar configuration, two different

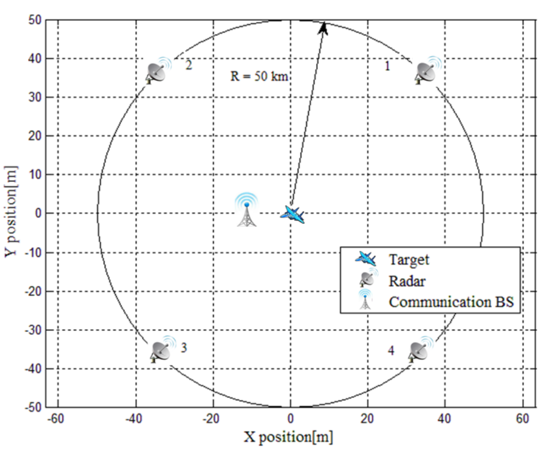

FIGURE 2. Simulated 2D scenario with locations of multiple radars, communication system and target.

target locations with respect to the multiple-radar system are chosen. In the first case, it is supposed that the target is located at [0, 0]km. In the second case, we simulate a scenario where the target is located at $[-\frac{25}{\sqrt{2}}, \frac{25}{\sqrt{2}}]$km. In every time slot, each radar receives $N = 512$ pulses. The maximum number of iterations is set to be $L_{\max} = 25$ to study the convergence of the proposed non-cooperative game model. We set the system parameters as given in TABLE 1.

TABLE 1. System parameters.

Parameter	Value	Parameter	Value
G_t	27 dB	G_r	27 dB
$G_t^{'}$	-30 dB	$G_r^{'}$	-30 dB
G_c	0 dB	σ_w^2	10^{-18} W
$T_{\max}$	-105 dBmW	$c_{i,j}$	0.01
λ	0.10 m	$P_i^{\max}(\forall i)$	1000 W
ε	10^{-16}	$\mu_i(\forall i)$	10^{10}

Here, two target RCS models are adopted. The first RCS model is uniform reflectivity, where $\boldsymbol{\sigma}_1^{\text{RCS}} = [1, 1, 1, 1]\text{m}^2$. On the other hand, in order to evaluate the effect of the target's RCS on the power allocation results, we also utilize the second RCS model $\boldsymbol{\sigma}_2^{\text{RCS}} = [1, 0.2, 3, 5]\text{m}^2$.

Before the initialization of the non-cooperative game theoretic model, the target detection performance, namely the desired SINR threshold $\gamma_{\min}$, should be first determined. In the considered scenario, we set the desired probabilities of false alarm and detection at $p_{\text{FA}} = 10^{-6}$ and $p_{\text{D}} = 0.9973$, respectively. Then, we can obtain the detection threshold $\delta_i = 0.0267$ for each radar, and the corresponding SINR $\gamma_{\min} = 10$ dB.

B. SIMULATION RESULTS

In order to study the convergence performance of the proposed NGTPA model, Fig.3 plots the transmission power allocation iterations of all the radars in the system for different initial power allocations, where the game is initialized with $\mathbf{P} = [60, 10, 160, 200]$W, $\mathbf{P} = [100, 100, 100, 100]$W, $\mathbf{P} = [280, 570, 120, 60]$W, and $\mathbf{P} = [210, 100, 50, 350]$W, respectively. One can observe from Fig.3 that the efficiency of the NGTPA model is evident, which only needs about 9-12 iterations to reach the optimal power allocation strategy. The results highlight that regardless of the initial strategy of the players, the NGTPA model converges to the unique Nash equilibrium. In addition, as the number of iterations increases, the interference power to the communication user tends to approach the predefined interference limits. Therefore, the proposed **Algorithm 1** converges.

Furthermore, the transmit power ratio results in different cases employing the proposed NGTPA strategy are highlighted in Fig.4, where the transmit power ratio is defined as:

$$\pi_i = \frac{P_i}{\sum_{i=1}^{M_T} P_i}. \tag{31}$$

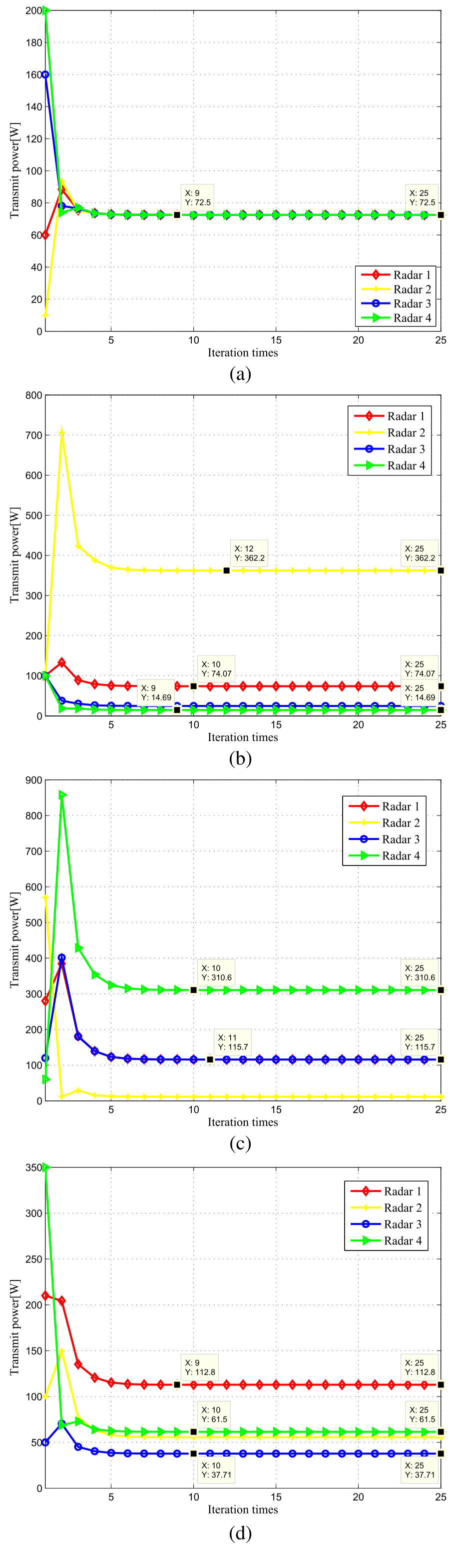

FIGURE 3. **Convergence of power allocation results in each Case: (a) Case 1 with σ_1^{RCS}, (b) Case 1 with σ_2^{RCS}, (c) Case 2 with σ_1^{RCS}, (d) Case 2 with σ_2^{RCS}.**

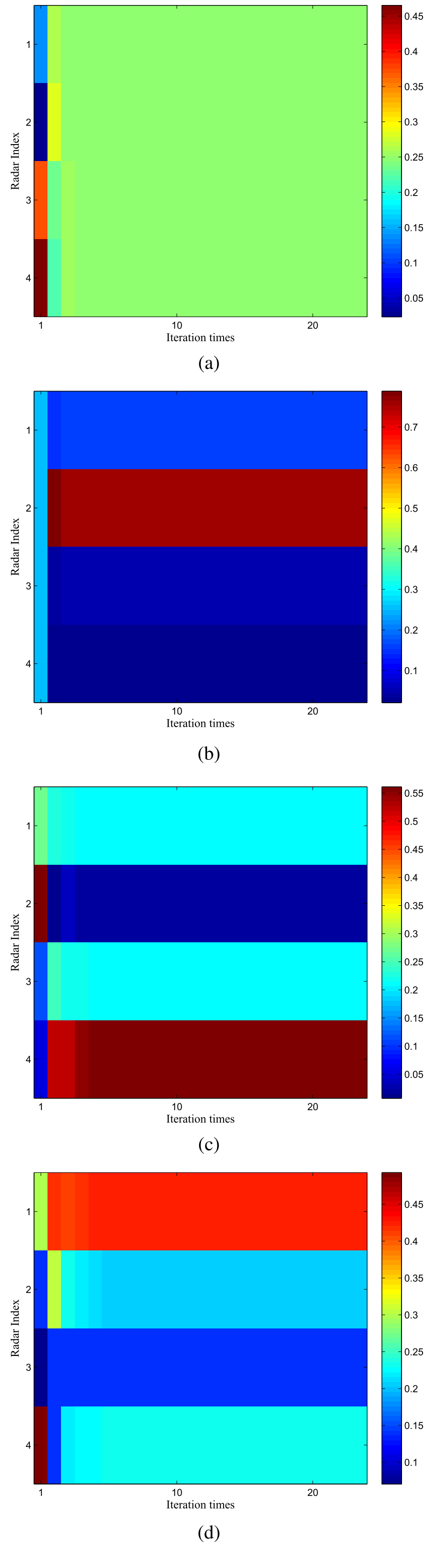

FIGURE 4. **The transmit power ratio results in each Case: (a) Case 1 with σ_1^{RCS}, (b) Case 1 with σ_2^{RCS}, (c) Case 2 with σ_1^{RCS}, (d) Case 2 with σ_2^{RCS}.**

In Fig.4 (a) & (b), it can be seen that more transmission power is allocated to Radar 1 and Radar 2 to guarantee the predetermined SINR requirement, as the target's RCSs with respect to these two radars are smaller than others. Now, to show the importance of the relative geometry between the target and distributed multiple-radar architectures, we change the target

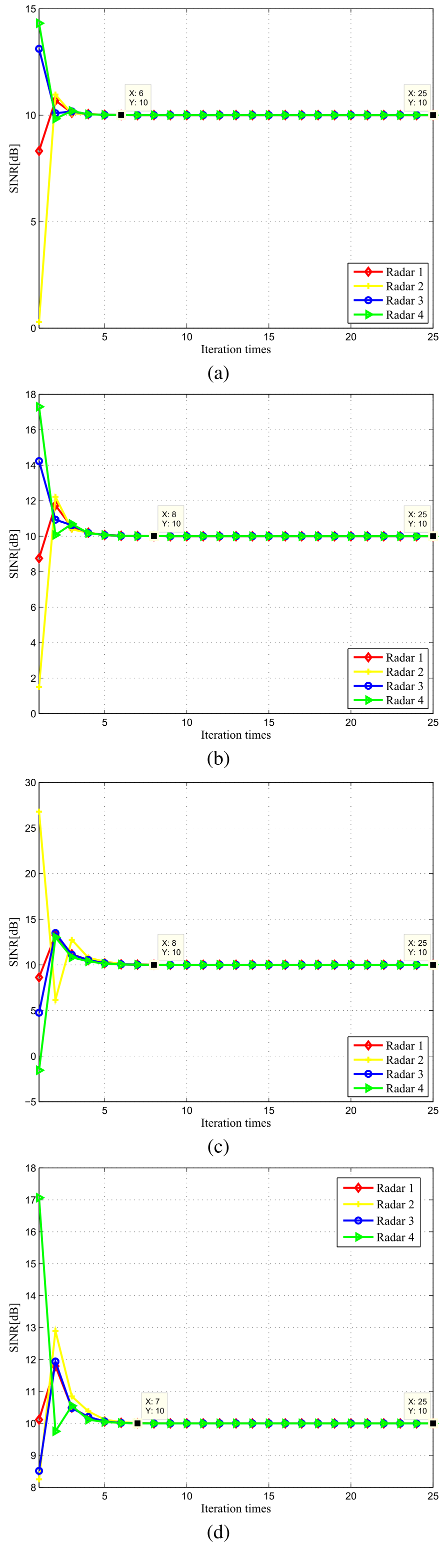

FIGURE 5. Convergence of SINR in each Case: (a) Case 1 with σ_1^{RCS}, (b) Case 1 with σ_2^{RCS}, (c) Case 2 with σ_1^{RCS}, (d) Case 2 with σ_2^{RCS}.

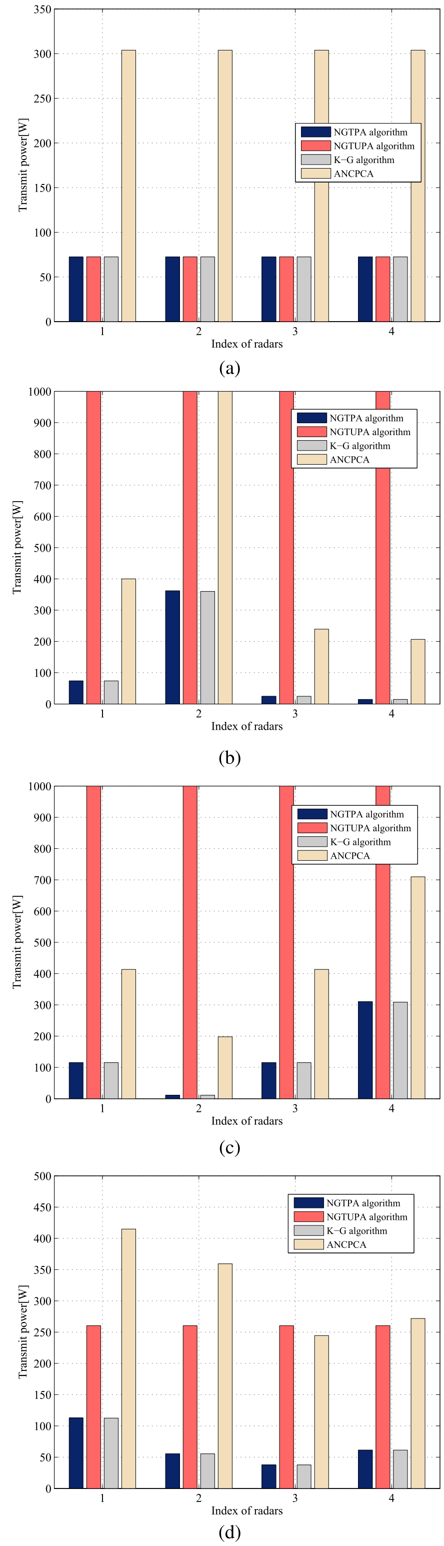

FIGURE 6. Comparisons of equilibrium power consumption in each Case employing different algorithms: (a) Case 1 with σ_1^{RCS}, (b) Case 1 with σ_2^{RCS}, (c) Case 2 with σ_1^{RCS}, (d) Case 2 with σ_2^{RCS}.

position for which we are calculating the power allocation strategy to $[-\frac{25}{\sqrt{2}}, \frac{25}{\sqrt{2}}]$km, and provide the power allocation update of all the radars in Fig.4 (c) & (d). In Fig.4 (c), less transmission power is assigned to Radar 1, Radar 2, and Radar 3, as they are closer to the target. That is to say, the radar farther from the target tends to be allocated more power.

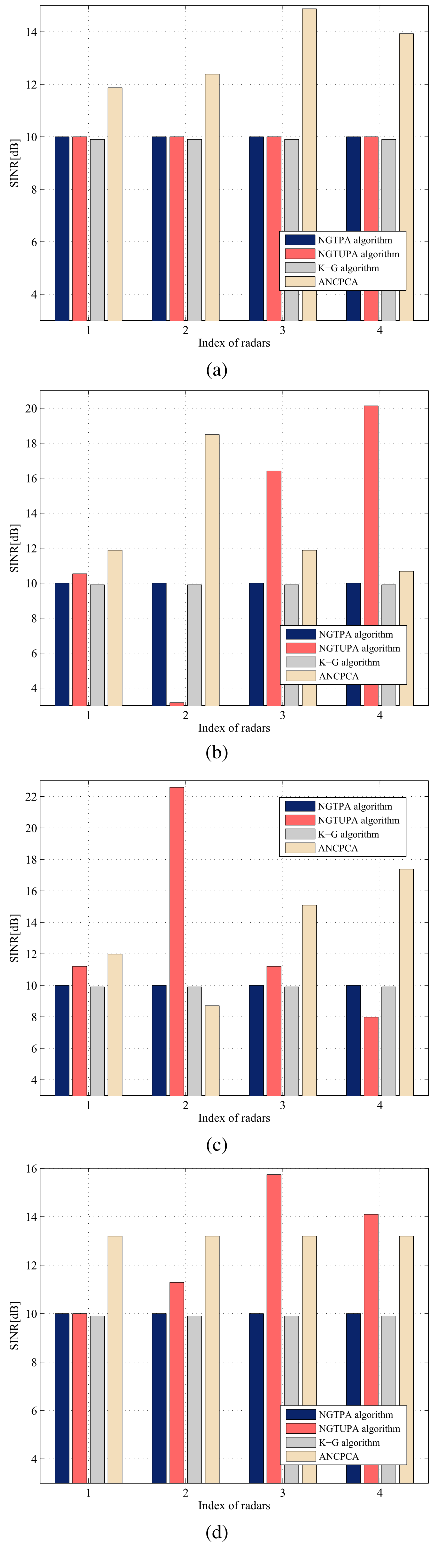

FIGURE 7. Comparisons of equilibrium SINR in each Case employing different algorithms: (a) Case 1 with σ_1^{RCS}, (b) Case 1 with σ_2^{RCS}, (c) Case 2 with σ_1^{RCS}, (d) Case 2 with σ_2^{RCS}.

Hence, we can conclude from these subfigures that higher power is allocated to the radars with relative weaker propagation channel gains in the iterative process. The transmission

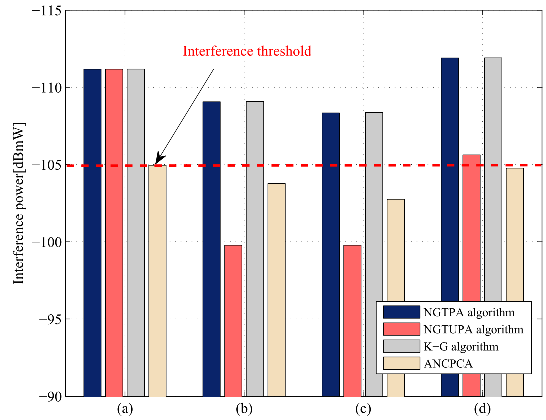

FIGURE 8. Comparisons of interference power levels in each Case employing different algorithms: (a) Case 1 with σ_1^{RCS}, (b) Case 1 with σ_2^{RCS}, (c) Case 2 with σ_1^{RCS}, (d) Case 2 with σ_2^{RCS}.

power allocation strategy is determined by the following two factors: the target's RCS and the relative geometry between target and distributed multiple-radar architectures.

In Fig.5, the SINR convergence curves of the proposed NGTPA algorithm are depicted. It is apparent that the achieved SINR tends to converge to the specified SINR threshold $\gamma_{\min} = 10$ dB when the number of iterations approaches 8. Therefore, we can notice that the proposed power allocation strategy can meet the SINR requirement of each radar, confirming that the NGTPA model can maintain fairness among all the radars in the system. From Figs.4 & 5, it should be the proposed NGTPA strategy is attractive for target tracking application, where the fine target detection performance is required to obtain the exact location of the target. In this scenario, the aim is to optimize the transmit power allocation to guarantee a specified SINR value hence probability of target detection.

In order to assess the efficiency of the presented power allocation algorithm, we compare the results of the proposed method with other three algorithms for power allocation: the uniform power allocation algorithm, the Koskie and Gajic's (K-G) algorithm in [21], and the adaptive non-cooperative power control algorithm (ANCPCA) in [33]. By imposing an additional constraint in the $\mathcal{P}_1$, which allocates uniformly the transmission power among the radars in the system, we can obtain the non-cooperative game theory based uniform power allocation (NGTUPA) algorithm. From Figs.6 & 7, it is worth to point out that the proposed NGTPA strategy not only minimizes the transmission power but also maintains the desired SINR threshold of each radar. This is because the radars in the system perceive the interference environment well and accordingly make the most appropriate transmission power adjustment decision. It is obvious that the presented game theoretic technique outperforms the uniform transmission power allocation in all cases, in terms of the power consumption and the achieved SINR value of each radar. Although the K-G algorithm consumes the least power, the target detection requirements of all the radars cannot be met, where the SINRs are below the specified SINR threshold. In particular, the ANCPCA transmits the most power due to the radars' self-interested non-cooperative behaviour in the game process, which is consistent with the results in [33]. Specifically, if one of the radars in the multiple-radar system cannot reach or guarantee its predefined SINR threshold, it resorts to the only means of increasing the transmission power to maintain the desired SINR performance, as do other radars in a similar situation. Consequently, the power-saving performance of the distributed multiple-radar architectures degrades. The results further reveal the superiority of the proposed NGTPA strategy compared to other existing approaches.

In order to illustrate the effects of the distributed multiple-radar configurations on the coexisting communication system, Fig.8 presents a histogram of the interference power level received at the communication system, comparing the four algorithms for the different cases. As we can observe, the interference power levels for the proposed strategy and K-G algorithm are much lower than the NGTUPA method and ANCPCA, which are below the maximum interference tolerant limit $T_{\max}$ for communication system in all scenarios. Thus, the QoS can be guaranteed by ensuring the multiple radars do not generate high interference to the communication system. However, as previously stated, the K-G algorithm is not ideal because the SINR requirement of each radar cannot be satisfied. Finally, it should be noted that the proposed NGTPA strategy outperforms other state of the art techniques in terms of power saving, target detection performance, and spectrum coexistence performance between multistatic radar and communication system in the same frequency band.

VII. CONCLUSION AND FUTURE WORK

In this paper, we have investigated a non-cooperative game theoretic power allocation strategy for distributed multiple-radar configurations in a spectrum sharing environment. The primary goal is to minimize the power consumption of each radar by optimizing the transmission power allocation, which are constrained by a predefined SINR requirement for target detection and a maximum interference tolerant limit for communication system. Then, the Nash equilibrium of the NGTPA model was obtained based on the Lagrangian dual function and sub-gradient method, and the existence and uniqueness of Nash equilibrium were proved. To attain the Nash equilibrium in a distributed manner, we also proposed an iterative power allocation algorithm with low computational complexity and fast convergence. Finally, the convergence and performance of the proposed NGTPA strategy were further evaluated by numerical simulations. It was shown that the optimal power allocation strategy of a multiple-radar system is dependent on the target's RCS and the relative geometry between target and distributed multiple-radar architectures. In particular, the received interference power at the communication system is below the maximum interference tolerant limit in various scenarios. The presented scheme also strengthens the distributed nature of the system with significant reduction on the signaling overhead, illustrating its potential for a practical application.

In future work, we will extend the non-cooperative game theoretic power allocation to a multiple-target case and concentrate on the other game theoretic power allocation for spectrum sharing in distributed multiple-radar architecture. As previously stated, with the fine detection performance to retrieve the exact target's position and characteristics, the proposed NGTPA strategy can be extended to target tracking application by adding target kinematic model, which will also be part of our future work.

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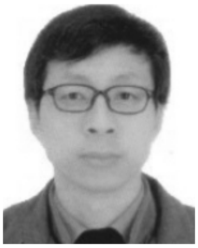

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