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Robust MPC-Based Fault-Tolerant Control for Trajectory Tracking of Surface Vessel

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ABSTRACT A robust control method, based on a model predictive control, is developed for the trajectory tracking of a surface vessel with sensor failures. The kinematic model of a surface vessel is first obtained. The Lyapunov stability theory and linear matrix inequalities are then adopted to produce a closed-loop system that can have a fault-tolerant capability against sensor failures and to subsequently obtain the simulated results of a fault-tolerant controller through solving the linear matrix inequalities. The asymptotic tracking position, heading angle, and the velocities of the surface vessel can be obtained using the inputs of incomplete state information in the simulation. The feasibility and effectiveness of the proposed control method have been verified through simulation results.

INDEX TERMS Fault-tolerant control, predictive control, surface vessel, trajectory tracking.

I. INTRODUCTION

The objective of motion control of a surface vessel is to steer the vessel along a desired reference trajectory. After receiving the values of state variables from sensors on a surface vessel, a control system attempts to control actuators like rudders to reduce the difference between the desired trajectory and the actual trajectory. The purpose of fault-tolerant control is that the vessel has to stabilize with its component failures that cannot be artificially eliminated when the vessel cannot sail along the scheduled route [1]. The researches of fault-tolerant control for kinds of systems such as surface vessel [1]–[3], flexible spacecraft [4], [5], switched system [6], Markovian jump system [7] and some other unspecific systems [8], [9] have recently attracted increasing attention in control areas. A control system possessing redundant capabilities is known as a fault-tolerant control system. The fault-tolerant control system can remain stable and maintain good trajectory control of vessels when some components fail.

According to the different forms of redundancy, the design of a fault-tolerant controller has been classified into two types: hardware redundancy and software redundancy [10]. The purpose of hardware redundancy is to offer extra backups to those important or easily damaged components of the control system. Based on different backup modes of fault tolerance, hardware redundancy can be classified into static hardware redundancy and dynamic hardware redundancy. Static hardware redundancy analyzes multiple control results that all come from identical components and eventually confirm the correct structure decided by the results in the majority. The process of logical judgment is increased, and the fault diagnosis is thus no longer required in this way. In dynamic hardware redundancy, the backups remain off and start automatically to replace the faulty part once component failures appear. Usually, hardware redundancy has been rather effectively applied to the system with hardware failures. However, when the large cost and increased burden on the system are considered, only some crucial components of the system can be applied to hardware redundancy. Software redundancy can be divided into analytical redundancy, functional redundancy, parameter redundancy and etc. Software redundancy is achieved through the estimation methods or software algorithm such that the fault tolerance control of the control system can be achieved. The main idea of software redundancy is to fulfill the fault-tolerant control by taking advantage of the functional redundancy of the system components to extract and separate the compensation information, which underlies the whole control system. The extensive application, good performance, and low cost are the major merits of software redundancy. Software redundancy has been extensively used in various control systems [11]. The hot research topics in the area of controlling surface vessels are to make the vessel sail in the state of stability when component failures appear. For the stabilization problem of nonlinear Markovian jump systems which are common in practical systems, with the employed sliding mode observer design scheme used to eliminate the effects of actuator and sensor faults, the stabilization of the overall close loop system can be guaranteed based on the proposed fault tolerant control [7]. A difficult problem of velocity-free uncertain attenuation control for a class of nonlinear systems with external disturbance and multiple actuator faults has been addressed in [9], and a velocity-free controller is synthesized using the reconstructed state obtained from the only available output measurement. Damiano *et al.* [12] adopts a passive fault-tolerant control design based on software redundancy. This control requires no additional redundancies and has a rapid processing speed.

Due to changes in the water environment and the interference of sea wind and waves, real-time control of a surface vessel against the changes in the external environment is required. Different from the typical disturbance observer design scheme such as the nonlinear observer designed in [3], in addition to the advantages like rolling optimization and a strong anti-interference ability, the model predictive control (MPC) algorithm can also have soft constraints of input in the form of linear matrix inequalities (LMIs) to achieve the upper bound of the performance index at the cost of minimum input [13], [14]. Control accuracy is thus greatly improved, and the complex degree of manipulation is reduced. Therefore, one relatively accurate control can be obtained with a small cost of input through the above algorithm [15].

Based on the motion model of a surface vessel in four degrees of freedom under the SF coordinate system, this paper has established an uncertain failure model, with input constraints, of trajectory tracking of a surface vessel with additional heading error $\bar{\psi}$ and cross-track error e. Based on rolling constraint optimization, Lyapunov stability theory, and LMIs, the MPC is implemented to achieve the fault-tolerant control results after solving various LMIs under the sufficient conditions for the control system to have a fault-tolerant capability against sensor failures [16]. From the fault diagnosis diagram in [17], the sensor output can be regraded as the component input, thus sensor failures can absolutely lead to component failures.

The control system can give appropriate orders to the sensor device (like a gyroscope and GPS) and the actuator device (like a rudder and propeller) to reduce the heading error and the cross-track error. Consequently, the robust fault -tolerant control for asymptotic trajectory tracking of the surface vessel with incompletable state feedback is fulfilled.

Notation:The symbols used in this paper are illustrated as follows: R^n is n-dimensional real space; I is the identity matrix with corresponding dimensions; symbol * denotes symmetrical structure. If H and R are symmetrical matrices, then there will be $\begin{bmatrix} H+S+* & * \\ T & R \end{bmatrix} = \begin{bmatrix} H+S+S^T & T^T \\ T & R \end{bmatrix}$.

II. MODEL DESCRIPTION

A. MATHEMATICAL MODEL OF A SURFACE VESSEL

Figure 1 shows the Serret-Frenet SF coordinate frame used for trajectory tracking control. The origin of the SF coordinate frame SF is defined by the point that is located on the curve Ω and closest to the origin of the body-fixed coordinate frame B [18]. Based on the SF equations, the error dynamics equations can be obtained as follows:

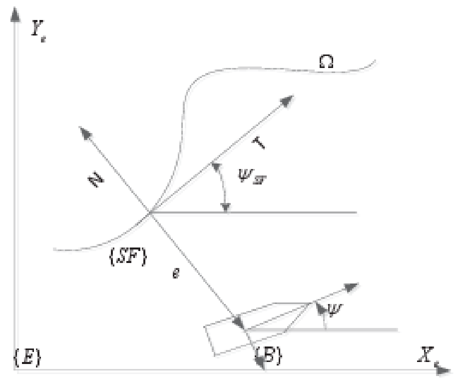

FIGURE 1. Illustration of the coordinates in the earth frame (inertial frame) (E), the surface vessel body-fixed frame (B) and the Serret-Frenet frame (SF).

$$\dot{\bar{\psi}} = \dot{\psi} - \dot{\psi}_{SF} = \frac{\mathrm{k}}{1-ek}\left(u\sin\bar{\psi} - v\cos\bar{\psi}\right) + r \quad (1)$$

$$\dot{e} = u\sin\bar{\psi} + v\cos\bar{\psi} \quad (2)$$

where e is defined as the cross-track error, the distance between SF and the origin B, and $\dot{\bar{\psi}} = \dot{\psi} - \psi_{SF}$ is defined as the heading error.

Figure 1 shows the earth-fixed coordinate frame, SF coordinate frame SF, and the ship body-fixed coordinate frame B. u, v and r are the surge, sway and yaw velocity, respectively. ψ is the heading angle of the vessel and ψ_{SF} is the tangential direction of the path, as is shown in the Figure 1. k is the curvature of the given path. T and N are the tangential and normal direction of the curve Ω at the origin {SF}. The control objective of trajectory tracking is to drive e and $\bar{\psi}$ to zero. Generally, the cross-track errors e and $\bar{\psi}$ cannot be eliminated simultaneously. In this circumstance, the primary objective is to maintain a small or near-zero cross-track error e, while maintaining a certain small but necessary heading error $\bar{\psi}$ to offset the disturbances.

For most trajectory tracking problems for surface vessels in the high seas, the trajectory is the straight line or route path consisting of piecewise lines with $k = 0$. If the desired trajectory has a non-zero curvature, the curve can be approximated as some piecewise lines. Thus, the heading error dynamics (1) can be simplified as follows:

$$\dot{\bar{\psi}} = r \quad (3)$$

Usually, three degrees of freedom (3-DoF) consisting of the surge, sway, and yaw are adopted to control the maneuverability of surface vessels. In this paper, to solve the trajectory-tracking problem with roll constraints, a 4-DoF model is proposed, including common 3-DoF (surge, sway, and yaw) and an additional DoF, namely the roll which has wave-resistant characteristics.

The nonlinear equations of motion (surge u, sway v, roll p and yaw r) are given by the following::

$$\left(m' + m'_x\right)\dot{u}' - \left(m' + m'_y\right)v'r' = X' \tag{4}$$

$$\left(m' + m'_y\right)\dot{v}' + \left(m' + m'_x\right)u'r' + m'_y\alpha'_y\dot{r}' - m'_y l'_y\dot{p}' = Y' \tag{5}$$

$$\left(I'_x + J'_x\right)\dot{p}' - m'_x l'_x u'r' + W'GM'\varphi' = K' \tag{6}$$

$$\left(I'_x + J'_z\right)\dot{r}' + m'_y\alpha'_y\dot{v}' = N' - Y'x'_G \tag{7}$$

where m' denotes the mass of the vessel; m'_x and m'_y denote the added mass in the x and y directions respectively. I'_X and I'_Z denote the moment of inertia, and J'_X and J'_Z denote the added moment of inertia about the x and z axes, respectively. In addition, α'_y denotes the x-coordinate of the center of m'_y; l'_X and l'_Y denote the z-coordinate of the center of m'_x and m'_y, respectively. W' is the displacement of the vessel. GM' is the metacentric height, and x'_G is the location of the center of gravity in the x-axis.

The movements of the surface vessels can be considered as the motion of a rigid body in the fluid, and are consisted of the movements of high frequency and low frequency. As the movements of high frequency can only result to the slight movements of the surface vessels without the position changes, the motions of low frequency are mainly considered [18]. After linearizing the curve around the equilibrium working point, assuming the surge speed to be constant, and neglecting the surge dynamics, the following dynamic equations that include surge, sway, roll, and yaw is established:

$$\dot{v} = a_{11}v + a_{12}r + a_{13}p + a_{14}\varphi + b_1\delta \tag{8}$$

$$\dot{r} = va_{21}v + a_{22}r + a_{23}p + a_{24}\varphi + b_2\delta \tag{9}$$

$$\dot{\bar{\psi}} = r \tag{10}$$

$$\dot{p} = a_{31}v + a_{32}r + a_{33}p + a_{34}\varphi + b_3\delta \tag{11}$$

$$\dot{\varphi} = p \tag{12}$$

When combined with formula (3), the following dynamic equations can be obtained:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\bar{\psi}} \\ \dot{p} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & a_{23} & a_{24} \\ 0 & 1 & 0 & 0 & 0 \\ a_{31} & a_{32} & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \bar{\psi} \\ p \\ \varphi \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ b_3 \\ 0 \end{bmatrix} \delta \tag{13}$$

where a_{11}, a_{12}, a_{21}, a_{22}, a_{31}, a_{32}, a_{33}, a_{34}, b_1, b_2 and b_3 are constant parameters, v is the sway velocity, r is the yaw velocity, $\bar{\psi}$ is the heading error, p is the roll velocity, φ is the heading angle and δ is the rudder angle.

B. THE TRAJECTORY-TRACKING MODEL OF A SURFACE VESSEL

For most trajectory-tracking problems for surface vessels, the trajectory is the way-point path consisting of some piecewise lines with the curvature k being zero. If the non-zero curvature exists in the trajectory, it is feasible to make some piecewise lines approximately equal to the curve [19], [20]. According to the trajectory-tracking and heading error equation (2) under the SF coordinate frame, the $v\cos\bar{\psi}$ is neglected under the condition of surge velocity $u \gg 0$, sway velocity v near zero, and $\max(v\cos\bar{\psi}) = v$. The error increment of trajectory tracking is mainly influenced by the surge velocity component along the heading error direction. Formula (2) can be transformed into the following:

$$\dot{e} = u\sin\bar{\psi} \tag{14}$$

The heading error $\bar{\psi}$ error changes in a small range. Based on the Taylor expansion of the sine function $\sin\bar{\psi} = \bar{\psi} - \bar{\psi}^3/3! + o(\bar{\psi}^3)$ and the assumption of $\bar{\psi}$ being a rather small radian value, the above-mentioned sine function turns into $\sin\bar{\psi} \approx \bar{\psi}$, and formula (14) can be converted into the following:

$$\dot{e} = u\bar{\psi} \tag{15}$$

When the error dynamic equation (13)is considered, the design model of the trajectory tracking controller is shown as follows:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\bar{\psi}} \\ \dot{p} \\ \dot{\varphi} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & a_{13} & a_{14} & 0 \\ a_{21} & a_{22} & 0 & a_{23} & a_{24} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 0 & a_{33} & a_{34} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & u & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \bar{\psi} \\ p \\ \varphi \\ e \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ b_3 \\ 0 \\ 0 \end{bmatrix} \delta \tag{16}$$

For ease of calculation, the formula (16) can be described by the following state equation:

$$\dot{x} = Ax + Bu \tag{17}$$

where the state vector $x^T(t) = [v(t), r(t), \bar{\psi}(t), p(t), \varphi(t), e(t)]$, $\bar{x}(t) \in R^6$, A is the state matrix of the system, B is the control input matrix of the system, $u(t) = \delta(t)$ is the control input rudder angle and the input constraint is $|\delta(t)| \le 10$.

C. FAILURE SYSTEM FOR TRAJECTORY TRACKING OF SURFACE VESSELS

When the model (17) for trajectory tracking of surface vessels and the description of an uncertain system are combined, it is feasible to describe the uncertain model of a surface vessel as follows:

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) \tag{18}$$

where A and B are constant matrices belonging to the nominal model of the system, and ΔA and ΔB are the variables parameters belonging to the uncertain model of the system.

The variables are expressed as below to emphasize the range of parameter perturbations:

$$[\Delta A \quad \Delta B] = DG[E_a \quad E_b] \tag{19}$$

where D, E_a and E_b are the known constant matrices and are decided by the percentage of the nominal model, G is the time-varying function matrix satisfying the condition $G^T G \leq I$, and I is the identity matrix with the appropriate dimension. The above description can then be expressed as follows:

$$[\bar{A} \quad \bar{B}] = [A \quad B] + DG[E_a \quad E_b] \tag{20}$$

The uncertain system of underactuated surface vessels can be described as:

$$\dot{x}(t) = [\bar{A} \quad \bar{B}][x(t) \quad u(t)]^T \tag{21}$$

After discretization, the above formula can be expressed as

$$\frac{x(k+1) - x(k)}{T} = \bar{A}x(k) + \bar{B}u(k) \tag{22}$$

where T is the sampling time. The discrete system description of the underactuated surface vessel with parameter perturbations is obtained as follows:

$$x(k+1) = (T\bar{A} + I)x(k) + T\bar{B}u(k) \tag{23}$$

According to the system description (23), a state feedback is introduced:

$$u(t) = Kx(t) \tag{24}$$

Then the closed-loop system is

$$x(k+1) = (T\bar{A} + I)x(k) + T\bar{B}Kx(k) \tag{25}$$

When the possible sensor failures are considered, a switching matrix F is introduced between the feedback gain matrix K and the state $x(t)$. The form of the switching matrix can be expressed as

$$F = \text{diag}(f_1, f_2, \ldots, f_n)$$

f_i equals 1 if the i-th, the senor, is working; otherwise f_i equals 0.

The state feedback controller with sensor failures is

$$u(t) = KFx(t) \tag{26}$$

Then the closed-loop failure system is

$$x(k+1) = (T\bar{A} + I)x(k) + T\bar{B}KFx(k) \tag{27}$$

The fault-tolerant controller design under the sensor failures aims to confirm the feedback gain matrix K, which can make the system (17) asymptotically stable if all probabilities $F \in \Omega$ of sensor failures occur. Ω is a set consisting of all probable results of switching matrix F with sensor failures. When the uncertainty of the parameters is considered, the trajectory tracking model with sensor failures of surface vessels is obtained as

$$x(k+1) = (\hat{A} + \hat{B}KF)x(k) \tag{28}$$

where, $[\bar{A} \quad \bar{B}] = [A \quad B] + DG[E_a \quad E_b]$, $\hat{A} = T\bar{A} + I$, and $\hat{B} = T\bar{B}$.

III. ROBUST FAULT-TOLERANT CONTROL BASED ON MPC

A. THE ROLLING OPTIMIZATION OF THE OBJECTIVE FUNCTION

According to the mathematical description of the fault (with parameter perturbations) of surface vessels, a set of control inputs to the MPC algorithm was found when the objective function is minimum and the control inputs satisfy the feasibility requirements.

Based on the equation (28), a discrete system with parameter perturbations of surface vessels can be obtained as

$$x(k+1) = \hat{A}x(k) + \hat{B}u(k) \tag{29}$$

The objective function of the infinite time domain of predictive control is

$$\begin{cases} \min\limits_{u(k+b|k), b=0,1,\ldots,\infty} J_\infty(k) \\ J_\infty(k) = \sum\limits_{b=0}^{\infty} [x^{\mathrm{T}}(k+b|k)Wx(k+b|k) \\ \qquad + u^{\mathrm{T}}(k+b|k)Ru(k+b|k)] \end{cases} \tag{30}$$

The minimum of the objective function represents the optimal performance of the system. $x(k) \in R^6$ is the system state vector, $\bar{A}$ and $\bar{B}$ are state matrix and input matrix with parameter perturbations respectively and $u(k) \in R^1$ is the control input, and $\bar{A}$ and $\bar{B}$ are the state matrix and input matrix with parameter perturbations, respectively. $W > 0$ is the state weighting matrix, $R > 0$ is the input weighting matrix of the system, and $x(k+b|k)$ is the state predictive value at time $k+b$ based on the model (29). $u(k+b|k)$ represents the value of control input sequence $\{u(k|k), u(k+1|k), \cdots, u(k+b|k)\}$ which enables the rolling optimization of the objective function (30) at time $k+b$. According to the characteristics of predictive control, the input $u(k+b|k)$ is applied to the system control. When the minimum of the objective function to the next moment is re-computed, a new input sequence is obtained, namely the rolling optimization.

According to the discrete uncertain mathematical description of surface vessels, a state feedback is introduced to the system with sensor failures

$$u(k) = KFx(k) \tag{31}$$

Then, the formula (31) is substituted into the formula (29) and the closed-loop fault system of the surface vessels can be obtained as

$$x(k+1) = (\hat{A} + \hat{B}KF)x(k) \tag{32}$$

The Lyapunov function of the system (29) at time k can be expressed in the form of $V(x(k|k)) = x^{\mathrm{T}}(k|k)Px(k|k)$ and $P > 0$. The following formula is then

$$\begin{aligned} &V(x(k+b+1|k)) - V(x(k+b|k)) \\ &\leq -(x(k+b|k)^T Wx(k+b|k) \\ &\quad - u(k+b|k)^T Ru(k+b|k)) \end{aligned} \tag{33}$$

The necessary stability condition $x(\infty|k) = 0$ can derive the formula $V(x(\infty|k)) = 0$. The equation (33) is added up from

$b = 0$ to $b = \infty$, and the formula (34) can be obtained:

$$-V\left(x\left(k|k\right)\right) \le -J_0^{\infty}\left(k\right) \quad (34)$$

The upper bound of the objective function in the infinite time domain can be expressed as follows:

$$J_0^{\infty}(k) \le V(x(k|k)) \quad (35)$$

Then the minimization of the objective function can be transformed into minimization of the Lyapunov function, which is

$$\min_{u(k+b|k),b=0,1,\ldots,\infty} V(x(k|k)) \quad (36)$$

Therefore, the objective of the predictive control is minimizing $V(x(k|k))$ by a set of control sequences, $u(k+b|k) = KFx(k+b|k)$, of which the first item, $u(k|k) = KFx(k|k)$, is added into the control system as inputs. Based on the current state $x(k+1|k+1)$, the future state $x(k+1+b|k+1+b)$ is re-predicted, and the minimum K which satisfies the condition $V(x(k+1|k+1))$ can be calculated to the next moment [21], [22].

B. FEASIBILITY ANALYSIS

The superiority of MPC lies in the optimization of the online solution. Predictive control cannot be conducted if the feasible solutions are not available. Feasibility depends on the hardware conditions.

The feasibility problem can be solved by relaxing constraints. Due to the structural limitation of the actuator, the input rudder angle is strictly constrained $\delta \in [-10°, 10°]$. When the control input sequence obtained by the predictive control algorithm satisfies the input constraints, the feasibility requirements are achieved.

C. THE DESIGN OF ROBUST MPC-BASED FAULT-TOLERANT CONTROLLER

Theorem 1: Considering the input constraints, the failure system (21) with parameter perturbations and the parameter perturbation term satisfying formula (20), $x(k|k)$ is the state measurement at time k. A and B are the nominal matrices of the system after discretizing. The input is strictly constrained $u \in [-10°, 10°]$. According to a certain sensor failure condition $F \in \Omega$, if the matrix $Y = KFQ$, which satisfies the following LMIs with $Q \ge 0$ and $Q \ge 0$ exists, a set of state feedback sequences $u(k) = KFx(k)$ will be obtained. $YQ^{-1} = KF$ can make the closed-loop system (21) with parameter perturbations asymptotically stable.

$$\min_{\gamma,Q} \gamma \quad (37)$$

$$\begin{bmatrix} 1 & x(k|k)^T \\ x\left(k|k\right) & Q \end{bmatrix} \ge 0 \quad (38)$$

$$\begin{bmatrix} Q & * & * & * & * \\ T^{1/2}(E_aQ+E_bY) & \varepsilon & * & * & * \\ (TA+I)Q+TBY & 0 & Q-\varepsilon TDD^T & * & * \\ W^{1/2}Q & 0 & 0 & \gamma I & * \\ R^{1/2}Y & 0 & 0 & 0 & \gamma I \end{bmatrix} \ge 0 \quad (39)$$

$$\begin{bmatrix} u_{\max}^2 IY \\ (Y)^{\mathrm{T}} Q \end{bmatrix} \ge 0 \quad (40)$$

Proof: First, the problem of minimizing the objective function is solved. According to formula (35), the minimization of the objective function is equivalent to the minimization of the Lyapunov function. According to the condition $V(x(k|k)) = x^{\mathrm{T}}(k|k)Px(k|k)$, the formula (36) is equivalent to

$$\min_{\gamma,P} \gamma \quad (41)$$

$$x(k|k)^T Px\left(k|k\right) \le \gamma \quad (42)$$

We define the matrix $Q = \gamma P^{-1} > 0$. When the Schur complement is combined, the above formula is equivalent to

$$\min_{\gamma,Q} \gamma \quad (43)$$

$$\begin{bmatrix} 1 & x(k|k)^T \\ x\left(k|k\right) & Q \end{bmatrix} \ge 0 \quad (44)$$

The problem of minimizing the objective function has been solved.

According to the requirement of Lyapunov stability, the state feedback (31) with sensor failures is substituted into the equation (33)

$$\begin{aligned} &V(x(k+b+1|k)) - V(x(k+b|k)) \\ &\quad \le -(x(k+b|k)^T Wx(k+b|k) \\ &\qquad -x(k+b|k)^T (KF)^T R(KF)x(k+b|k) \end{aligned} \quad (45)$$

The closed loop system (28) of surface vessels is

$$V(x(k+b+1|k)) = V((\hat{A}+\hat{B}KF)x(k+b|k)) \quad (46)$$

which is equivalent to

$$\begin{aligned} &V(x(k+b+1|k)) \\ &\quad = ((\hat{A}+\hat{B}KF)x(k+b|k))^T P((\hat{A}+\hat{B}KF)x(k+b|k)) \end{aligned} \quad (47)$$

The following formula after transposition is obtained by substituting formulas (47) and (48) into formula (45):

$$\begin{aligned} &x(k+b|k))^T [(\hat{A}+\hat{B}KF)^T P(\hat{A}+\hat{B}KF) - P \\ &\qquad + W + (KF)^T R(KF)]x(k+b|k)) \le 0 \end{aligned} \quad (48)$$

Then

$$(\hat{A}+\hat{B}KF)^T P(\hat{A}+\hat{B}KF) - P + W + (KF)^T R(KF) \le 0 \quad (49)$$

Based on the Schur complement, formula (49) can be expressed in the form of an LMI:

$$\begin{bmatrix} P & * & * & * \\ \hat{A}+\hat{B}KF & P^{-1} & * & * \\ W^{\frac{1}{2}} & 0 & I & * \\ R^{\frac{1}{2}}KF & 0 & 0 & I \end{bmatrix} \ge 0 \quad (50)$$

The following can be obtained by multiplying the LMI (50) by a diagonal matrix diag(Q, I, I, I) on both sides, it is easy to obtain

$$\begin{bmatrix} Q & * & * & * \\ \hat{A}Q+\hat{B}Y & Q & * & * \\ W^{\frac{1}{2}}Q & 0 & \gamma I & * \\ R^{\frac{1}{2}}Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \tag{51}$$

where $Q = \gamma P^{-1}$ and $Y = KFQ$ are the positive definite matrices. The formula $\hat{A} = T\bar{A} + I$ and $\hat{B} = T\bar{B}$ are substituted into $\hat{A}$ and $\hat{B}$, respectively. Then the formula $[\bar{A} \quad \bar{B}] = [\tilde{A} \quad \tilde{B}] + DG[E_a \quad E_b]$ is obtained and the LMI (51) can be transformed into

$$\begin{bmatrix} Q & * & * & * \\ (TA+I)Q+TBY & Q & * & * \\ W^{\frac{1}{2}}Q & 0 & \gamma I & * \\ R^{\frac{1}{2}}Y & 0 & 0 & \gamma I \end{bmatrix} + T\begin{bmatrix} 0 \\ D \\ 0 \\ 0 \end{bmatrix} G\begin{bmatrix} E_aQ+E_bY & 0 & 0 & 0 \end{bmatrix} + T\begin{bmatrix} E_aQ+E_bY & 0 & 0 & 0 \end{bmatrix}^T G^T \begin{bmatrix} 0 \\ D \\ 0 \\ 0 \end{bmatrix}^T \geq 0 \tag{52}$$

Lemma 1 [16]: Considering any given matrices Y, D, F and H with certain dimensions, where Y is a symmetrical matrix and $F^TF \leq I$, if $Y + DFH + H^TF^TD^T > 0$, there must be a constant $\varepsilon > 0$ which makes the matrix inequality $Y - \varepsilon DD^T - \varepsilon^{-1}H^TH > 0$ workable.

From the lemma 1, the constant $\varepsilon > 0$ exists and satisfies the following condition

$$\begin{bmatrix} Q & * & * & * \\ (TA+I)Q+TBY & Q & * & * \\ W^{\frac{1}{2}}Q & 0 & \gamma I & * \\ R^{\frac{1}{2}}Y & 0 & 0 & \gamma I \end{bmatrix} - \varepsilon T\begin{bmatrix} 0 \\ D \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ D \\ 0 \\ 0 \end{bmatrix}^T - \varepsilon^{-1}T\begin{bmatrix} E_aQ+E_bY \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} E_aQ+E_bY & 0 & 0 & 0 \end{bmatrix} \geq 0 \tag{53}$$

It is equivalent to

$$\begin{bmatrix} \Theta & * & * & * \\ (T\tilde{A}+I)Q+T\tilde{B}Y & \gamma^{-1}Q-\varepsilon T^2DD^T & * & * \\ W^{\frac{1}{2}}Q & 0 & I & * \\ R^{\frac{1}{2}}Y & 0 & 0 & I \end{bmatrix} \geq 0 \tag{54}$$

where $\Theta = \gamma Q - \varepsilon^{-1}(E_aQ + E_bY)^T(E_aQ + E_bY)$.

According to the Schur complement, the formula (53) is equivalent to

$$\begin{bmatrix} Q & * & * & * & * \\ T^{1/2}(E_aQ+E_bY) & \varepsilon & * & * & * \\ (TA+I)Q+TBY & 0 & Q-\varepsilon TDD^T & * & * \\ W^{1/2}Q & 0 & 0 & \gamma I & * \\ R^{1/2}Y & 0 & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \tag{55}$$

Formula (39) has thus been proved.

Now consider the feasibility of predictive control. Predictive control that satisfies the input constraints is an effective method of solving the problem of input constraints. According to the matrix, equation (42) can be transformed into the following state transition condition:

$$x^{\mathrm{T}}(k+b|k)Q^{-1}x(k+b|k) \leq 1 \tag{56}$$

$u_{\max} = \delta_{\max}$ represents the maximum of the control input, which is the rudder angle of the surface vessel. Then, there is

$$\|u(k+b|k)\|_2 \leq u_{\max} \tag{57}$$

$$\max_{b\geq 0} \|u(k+b|k)\|_2^2 = \max_{b\geq 0} \left\|YQ^{-1}x(k+b|k)\right\|_2^2 \leq (Q^{-1/2}Y^{\mathrm{T}}YQ^{-1/2}) \leq u_{\max}^2 \tag{58}$$

According to the Schur complement, the formula (58) can be expressed in the form of an LMI:

$$\begin{bmatrix} u_{\max}^2 IY \\ Y^{\mathrm{T}}Q \end{bmatrix} \geq 0 \tag{59}$$

The LMI (40) has been proved and the problem of control input has been solved.

IV. SIMULATION ANALYSES

The mathematical description of trajectory tracking with parameter perturbations is given as follows

$$\dot{x} = \hat{A}x + \hat{B}u \quad -10^\circ \leq u(t) \leq 10^\circ$$

where, $u(t) = \delta(t)$ is the control input, the rudder angle input. The state vector can be shown as $x(t) \in R^6, x^T(t) = [v(t), r(t), \bar{\psi}(t), p(t), \varphi(t), e(t)]$, $\hat{A} = T\bar{A} + I$, $\hat{B} = T\bar{B}$, $[\bar{A} \quad \bar{B}] = [A \quad B] + DG[E_a \quad E_b]$ąč

In [20], a surface vessel, whose model parameters are as follows, is introduced:

$$A = \begin{bmatrix} 1.0683 & -0.8373 & 0 & 0.0276 & -0.0351 & 0 \\ -0.0003 & 0.9680 & 0 & 0.003 & 0 & 0 \\ 0 & 0.3 & 1 & 0 & 0 & 0 \\ 0.0006 & -0.0917 & 0 & 0.9941 & -0.0135 & 0 \\ 0 & 0 & 0 & 0.3 & 1 & 0 \\ 0 & 0 & s & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.0171 & 0.0009 & 0 & 0.0009 & 0 & 0 \end{bmatrix}^T$$

$$E_a = \begin{bmatrix} -0.11 & -1.39 & 0 & -0.04 & -0.06 & 0 \\ 0.002 & -0.05 & 0 & 0.005 & -0.0002 & 0 \\ 0 & 0.5 & 0.003 & 0 & 0 & 0 \\ 0.002 & -0.15 & 0 & 0.01 & -0.224 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5s & 0 & 0 & 0 \end{bmatrix}$$

$$E_b = \begin{bmatrix} -0.0057 & 0.0004 & 0 & 0.0003 & 0 & 0 \end{bmatrix}^T$$

$$D = 0.1 * \mathrm{diag}(1, 1, 1, 1, 1, 1)$$

$$G = \sin(3.14 * \frac{k}{180}) * \mathrm{diag}(1, 1, 1, 1, 1, 1)$$

The weighting matrix of the state vectors is $W = \mathrm{diag}(1, 10, 1, 1, 1, 1)$ and the weighting matrix of the control input is $R = 1$. The initial values of the state variables are set as follows:

$$x^T(0) = (v(0), r(0), \bar{\psi}(0), p(0), \varphi(0), e(0)) = (0, 0, 5/57.3, 0, 0, 50)$$

where the sampling time is $T = 0.2$, the initial input of the rudder angle is $\delta(0) = 0°$ and the input constraint is $|\delta(t)| \leq 10°$. The following fault matrices simulate sensor failures in which the sensor cannot deliver status information such as sway velocity v, yaw angular velocity r, trajectory error $\bar{\psi}$, roll angular velocity p, roll angle φ and cross-track error e, respectively. Suppose that, in the initial situation, the trajectory error is 5°, the cross-track error is 50m, and the surge velocity is $s = 5m/s$.

$$F_1 = \mathrm{diag}(0, 1, 1, 1, 1, 1), \quad F_2 = \mathrm{diag}(1, 0, 1, 1, 1, 1),$$
$$F_3 = \mathrm{diag}(1, 1, 0, 1, 1, 1), \quad F_4 = \mathrm{diag}(1, 1, 1, 0, 1, 1),$$
$$F_5 = \mathrm{diag}(1, 1, 1, 1, 0, 1), \quad F_6 = \mathrm{diag}(1, 1, 1, 1, 1, 0).$$

From Fig.2, the state feedback control makes all these state variables converge gradually, the cross-track error e and the heading error $\bar{\psi}$ converge to zero, resulting in an asymptotic trajectory tracking control of the surface vessel. The yaw angle error $\bar{\psi}$ is small enough to satisfy $\sin\bar{\psi} \approx \bar{\psi}$. Therefore, the expression (15) of cross-track error e is workable. The control method is feasible and effective so that the roll angle φ, which is produced by the acceleration during the whole control process, is small enough. Fig.3 shows that the path tracking of the experimental vessel. The asymptotic trajectory tracking has been fulfilled in a normal working condition. Fig.4 and Fig.6 show the control results in the condition that the sensor cannot deliver the sway velocity and yaw angular velocity, respectively.

The simulation results of path tracking shown in the Fig.5 and Fig.7 correspond to Fig.4 and Fig.6, respectively. Moreover, the fault-tolerant control of path tracking is fulfilled in the conditions above. Fig.8 shows the control result when there is no feedback information of regarding the heading

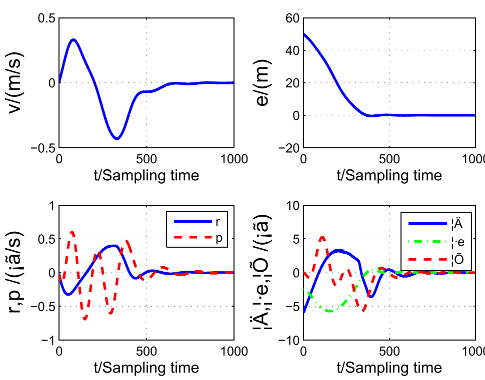

FIGURE 2. Control results when the sensor is failure-free.

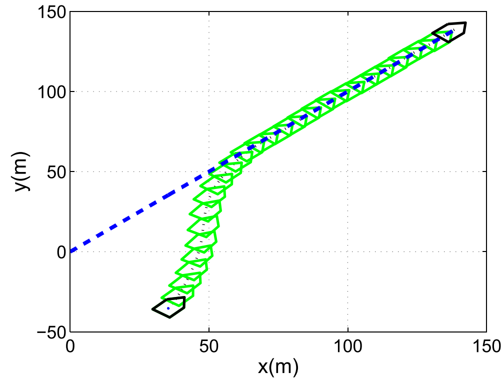

FIGURE 3. The trajectory tracking result when the sensor is failure-free.

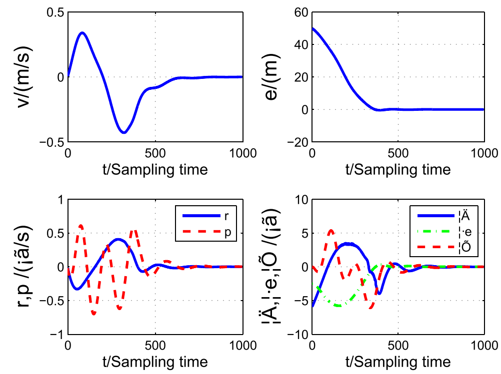

FIGURE 4. Control results when the sensor cannot feedback sway velocity.

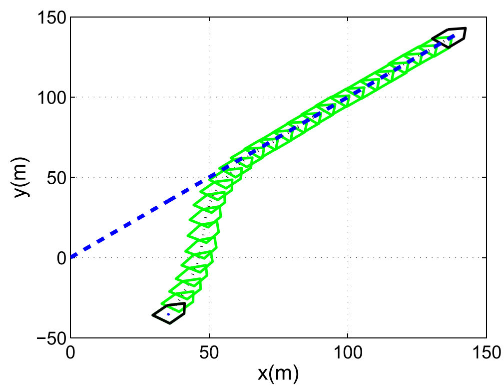

FIGURE 5. The trajectory tracking result when the sensor cannot feedback sway velocity.

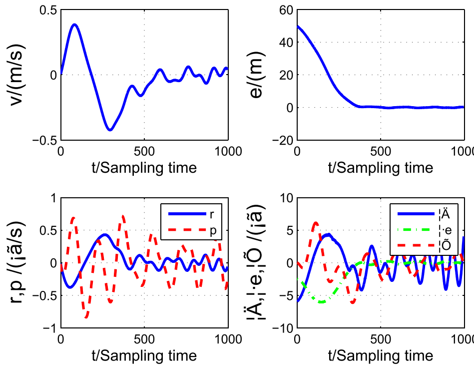

FIGURE 6. Control results when the sensor cannot feedback yaw angular velocity.

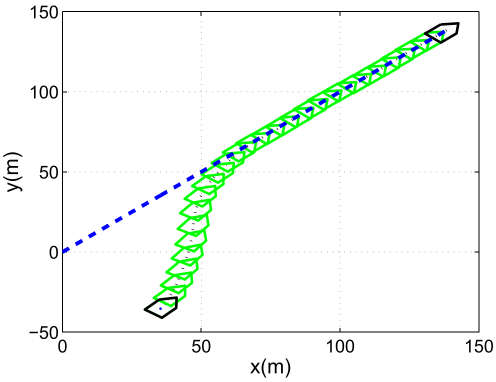

FIGURE 7. The trajectory tracking result when the sensor cannot feedback yaw velocity.

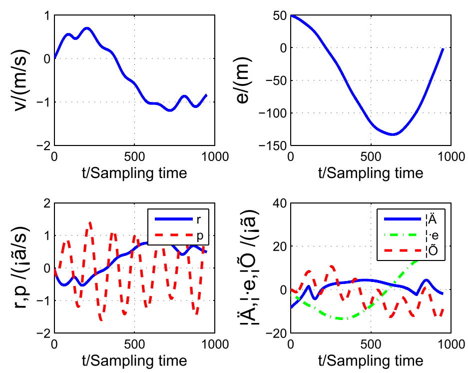

FIGURE 8. Control results when the sensor cannot feedback heading angle.

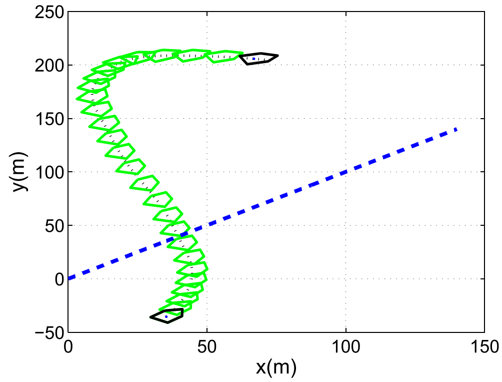

FIGURE 9. The trajectory tracking result when the sensor cannot feedback heading angle.

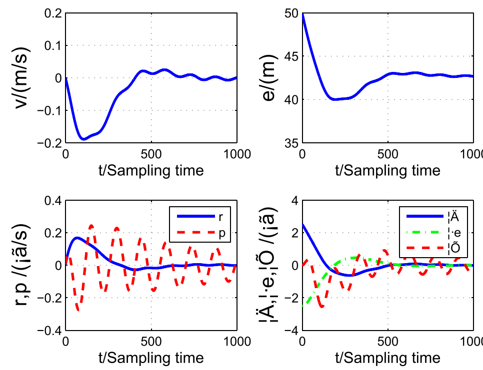

FIGURE 10. Control results when the sensor cannot feedback cross-track error.

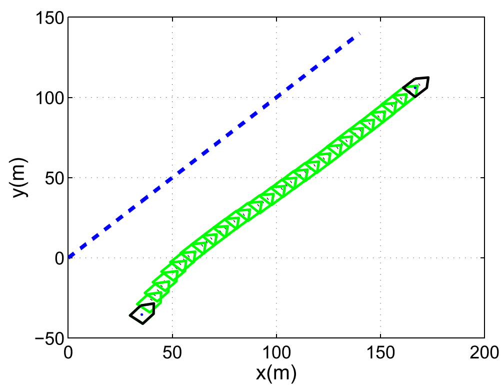

FIGURE 11. The trajectory tracking result when the sensor cannot feedback cross-track error.

angle. Fig.9 shows the path tracking with no heading angle information. Fig.8 and Fig.9 illustrate that the heading angle sensor is critical for a surface vessel. If the heading angle sensor does not work, it is unable to obtain effective deviation so that the control results cannot converge to zero. Fig. 10 and 11 show the control result and the simulation result of path tracking when there is no feedback information regarding the cross-track error. Fig.10 shows that the control system cannot track the path effectively when the valid cross-track error is unavailable. However, the convergence of $\bar{\psi}$ is achieved, and asymptotic course tracking is fulfilled. Moreover, the inputs of Fig.2 to Fig.11 satisfy the constraints showing that MPC is an effective way of solving the constraint problem.

Remark: Due to the space limitations in this paper, the analyses of fault scenarios of roll angular velocity and the roll angle are not given.

V. CONCLUSION

Based on the motion characteristics of a surface vessel and its mechanical parameters, and the hydrodynamic analysis and moment analysis in 4-DoF of surface vessels, this paper has analyzed a kinematic model of a surface vessel including the heading error under the SF frame. Furthermore, considering the uncertainty of the model, a mathematical description with parameter perturbations is established. The constraints of the control input are given at the same time. Given the parameter uncertainty of surface vessel model, after considering the possibilities of sensor failure, the LMI method has been applied to the mathematical model with parameter perturbations to develop the robust fault-tolerant control of trajectory tracking of surface vessels based on the MPC. In this paper, to achieve control constraints, the problem of input constraints has been transformed into the feasibility problem through solving a model predictive problem in term of LMIs. The algorithm has been proved effective by means of a series of simulation experiments using the data from a real vessel. With the effectiveness analyses of fault-tolerant control with various possible sensor failures, the effectiveness of the developed algorithm has been verified through simulation results.

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