Received January 25, 2018, accepted February 28, 2018, date of publication March 16, 2018, date of current version November 30, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2813334

Feedrate Scheduling of NURBS Interpolation Based on a Novel Jerk-Continuous ACC/DEC Algorithm

HEPENG NI[1, 2, 3], JIPENG YUAN[1, 2, 3], SHUAI JI[4], CHENGRUI ZHANG[1, 2, 3], AND TIANLIANG HU[1, 2, 3]

[1]School of Mechanical Engineering, Shandong University, Jinan 250061, China
[2]Key Laboratory of High Efficiency and Clean Mechanical Manufacture, Ministry of Education, Shandong University, Jinan 250061, China
[3]National Demonstration Center for Experimental Mechanical Engineering Education, Shandong University, Jinan 250061, China
[4]School of Mechanical and Electronic Engineering, Shandong Jianzhu University, Jinan 250101, China

Corresponding author: Chengrui Zhang (sduzcr@126.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 51405270 and in part by the Special Foundation for National Integrated Standardization and New Model of Intelligent Manufacturing, China, under Grant Z135060009002-132.

ABSTRACT In order to improve the motion smoothness while maintaining the machining efficiency of the non-uniform rational B-spline (NURBS) interpolation, a novel jerk-continuous acceleration/deceleration (ACC/DEC) algorithm and the corresponding feedrate scheduling method are proposed in this paper. The polynomial and trigonometric functions are combined to construct the jerk-continuous ACC/DEC profile. Meanwhile, to reduce the computational load of feedrate scheduling, a proportional control method is proposed to determine the form of the jerk profile, which also improves the flexibility of feedrate control. Therefore, the maximum jerk and acceleration can be maintained to achieve higher machining efficiency compared with the conventional trigonometric methods, while the feedrate scheduling process is far simpler than the polynomial methods with continuous jerk. In addition, to improve the machining accuracy and motion smoothness, the round-off error caused by cycle sampling is also considered in feedrate scheduling and real-time interpolation. The proposed ACC/DEC algorithm is simplified to achieve the round-off error compensation, which can guarantee the continuity of jerk profile. A series of simulations and experiments for two NURBS curves is conducted to verify the good performance and applicability of the proposed method.

INDEX TERMS NURBS interpolation, feedrate scheduling, jerk-continuous ACC/DEC algorithm, round-off error compensation.

I. INTRODUCTION

As a parametric curve model, non-uniform rational B-spline (NURBS) offers a common mathematical form for the precise representation of analytical shapes as well as free-form curves and surfaces [1]. Compared with conventional computer numerical control (CNC) machining which only supports straight line and circular interpolation, parametric interpolation for NURBS curves has a lot of advantages in surface quality, machining efficiency, memory consumption and motion smoothness [2]. Therefore, in order to achieve high-speed and high-accuracy machining, it is critical to develop an efficient and applicable NURBS interpolator for CNC machine tools.

The feedrate scheduling which determines the smoothness, accuracy and stability of the machining process plays an important role in NURBS interpolation. And it is the premise of trajectory tracking control for machine tools and other mechanisms [3]–[6]. A lot of feedrate scheduling methods based on various acceleration/deceleration (ACC/DEC) algorithms have been proposed. The linear ACC/DEC algorithm [7]–[9] is simple to implement. Nevertheless, the feedrate profile is not smooth, which induces the vibration of machine tools during movement. The constant feedrate profile with adaptive adjustment methods [10], [11] were employed for NURBS interpolation. Similarly, the acceleration profile is not continuous and the multiple constraints including chord error, centripetal acceleration and jerk cannot be satisfied.

Diverse jerk-limited ACC/DEC algorithms have been employed to improve the smoothness of feedrate profile.

Because of the efficiency and simplicity, S-shaped ACC/DEC algorithm [12] with seven sections is one of the most wildly used polynomial feedrate control method. Du *et al.* [13] and Dong *et al.* [14] proposed a similar adaptive feedrate scheduling method that the feedrate at the sensitive areas can be adjusted based on the various constrains and S-shaped algorithm. Du *et al.* [15] introduced an S-shaped feedrate scheduling method for NURBS curves while the round-off error caused by cycle sampling was also compensated. Lin *et al.* [16] proposed a real-time look-ahead algorithm to generate smooth and jerk-limited feedrate profile based on the curvature of the NURBS curves and the confined chord errors. Liu *et al.* [17] presented the complete procedures of NURBS interpolation including pre-processing and real-time interpolation. Sun *et al.* [18] presented a real-time and look-ahead interpolation methodology with dynamic B-spline transition scheme for short line segment machining and employed S-shaped ACC/DEC algorithm to schedule the feedrate. In order to simply the computing process of feedrate scheduling, Qiang *et al.* [19] and Leng *et al.* [20] proposed a similar cubic polynomial feedrate profile for NURBS interpolation. However, this ACC/DEC algorithm has low efficiency because the maximum acceleration and jerk cannot be maintained. In addition, the jerk profiles of the aforementioned methods are discontinuous and will bring flexible impulse to machine tools.

In order to simplify the expression and the computing process of polynomial profiles, some trigonometric ACC/DEC algorithms have been developed. Luo *et al.* [21] and Lee *et al.* [22] employed the single sine curve to construct the jerk profile based on the parametric method given in [23]. However, in each ACC/DEC process, only one point can reach the given maximum value of the acceleration or jerk profile, which leads to low machining efficiency. Furthermore, the jerk profile of this method is discontinuous with abrupt changes in the start and end points of each ACC/DEC section. Huang and Zhu [24] made some improvements and proposed a representation method of jerk profile using the sine series which combines the advantages of the polynomial and the single sine feedrate profile. Owing to the sine series, the maximum acceleration can be maintained and the efficiency is improved compared with single sine feedrate profile. However, the maximum jerk cannot be maintained, which still limits the machining efficiency. Meanwhile, as can be seen from the simulation results in [24], the jerk profile is also discontinuous.

To improve the smoothness of feedrate and acceleration profiles, various jerk-continuous ACC/DEC algorithms have been developed and implemented to parametric interpolation. Fan *et al.* [25] proposed a polynomial ACC/DEC algorithm under the specified jounce. Because of the constant jounce, the jerk profile is continuous and has the similar form as the acceleration profile of the typical S-shaped ACC/DEC algorithm. However, a complete jerk profile consists of 15 sections with 7 independent variables, which increases the computational load of feedrate scheduling significantly. Liu *et al.* [26] presented a feedrate scheduling method based on a jerk-continuous ACC/DEC algorithm. The jerk profile consists of several complete sine curves. Therefore, the representation and feedrate scheduling process of this method are simple. Jahanpour and Alizadeh [27] introduced an optimized S-shaped quantic feedrate scheduling scheme. The feedrate profile around the sharp corner can be modified using the exponential functions which guarantees that the jerk profile is continuous. However, the machining with these two methods is inefficiency for the same reason of [21] and [22] methods. Furthermore, none of these methods considered the round-off error which affects the machining accuracy and motion smoothness.

Following the idea of combination given in [24], this paper proposes a novel jerk-continuous ACC/DEC algorithm with high machining efficiency and low computational load. Correspondingly, the feedrate scheduling method is also presented. The polynomial and trigonometric functions are combined to construct the proposed ACC/DEC profile, which ensures the continuity of jerk profile. Meanwhile, in order to reduce the computational load of feedrate scheduling, a proportional control method is introduced to determine the form of the jerk profile. The ratio between the polynomial and trigonometric functions can be adjusted based on the requirements of motion smoothness and machining efficiency, which improves the flexibility of feedrate control. In addition, the round-off error is considered in feedrate scheduling and real-time interpolation to improve the machining accuracy and motion smoothness. The compensation scheme of round-off error is also presented with the simplified ACC/DEC algorithm which can maintain the continuity of the jerk profile. The comparison of this paper with previous works are summarized in Tab. 1.

The remainder of this paper is organized as follows. In section II, the definition and interpolation procedures of NURBS curves are introduces. Section III presents the proposed jerk-continuous ACC/DEC algorithm. Section IV develops the feedrate scheduling method of NURBS interpolation. The round-off error compensation is also introduced in this section. The Simulations and experiments are conducted in section V. The conclusions are given in section VI.

II. DEFINITION AND INTERPOLATION PROCEDURES OF NURBS CURVES

A. DEFINITION OF A NURBS CURVE

A NURBS curve $C(u)$ can be defined as follows [1]:

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u)\omega_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) P_i} \tag{1}$$

where $\{P_i\}$ are the control points which form a control polygon, $\{\omega_i\}$ are the corresponding weights of $\{P_i\}$, $(n+1)$ is the number of control points and p is the degree of a NURBS curve. $\{N_{i,p}(u)\}$ are the pth-degree B-spline basis

TABLE 1. Compensation of this paper with previous works.

Reference	Used ACC/DEC algorithm	Continuity of Jerk profile	Machining efficiency	Compensational load	Round-off error compensation
[7-9]	Linear algorithm constructed by polynomial function	Discontinuous	High	Low	No
[10-11]	Constant feedrate with adaptive adjustment	Discontinuous	High	Low	No
[13-14, 16-18]	S-shaped algorithm constructed by polynomial function	Discontinuous	High	Low	No
[15]	S-shaped algorithm constructed by polynomial function	Discontinuous	High	Low	Yes
[19]	Feedrate profile constructed by cubic polynomial function	Discontinuous	Low	Low	Yes
[20]	Feedrate profile constructed by cubic polynomial function	Discontinuous	Low	Low	No
[21-22, 24]	Jerk profile constructed by trigonometric function	Discontinuous	Low	Low	No
[25]	Jerk profile constructed by polynomial function	Continuous	High	High	No
[26]	Jerk profile constructed by trigonometric function	Continuous	Low	Low	No
[27]	Optimized S-shaped algorithm constructed by exponential function	Continuous	Low	Low	No
This paper	Jerk profile constructed by trigonometric and polynomial functions	Continuous	High	Low	Yes

functions defined on the non-uniform knot vector $U = \{u_0, u_1, \ldots, u_m\} = \{\underbrace{a, \ldots, a}_{p+1}, u_{p+1}, \ldots, u_{m-p-1}, \underbrace{b, \ldots, b}_{p+1}\}$ where $(m+1)$ is the number of knots. In general, we assume that $a = 0$,$b = 1$ and $\omega_i > 0$ for all i. The relation among the degree p, the number of control points $(n+1)$ and the number of knots $(m+1)$ can be expressed as follows:

$$m = n + p + 1 \tag{2}$$

The pth-degree B-spline basis functions $\{N_{i,p}(u)\}$ are defined as follows:

$$N_{i,0}(u) = \begin{cases} 1 & u_i \leq u \leq u_{i+1} \\ 0 & otherwise \end{cases} \tag{3}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \quad (i = 0, 1, \ldots, n) \tag{4}$$

B. INTERPOLATION PROCEDURES OF NURBS CURVES

The flowchart of NURBS interpolation is shown in Fig. 1. There are two main stages: the pre-processing stage for obtaining the feature data of NURBS curves and completing the feedrate scheduling and the real-time interpolation stage for calculating the interpolation point position in each interpolation period.

In the stage of pre-processing, five modules are performed as follows:

1) CURVE SPLITTING AT BREAK POINTS

The breakpoints at where it is visually discontinuous should be found out to split the whole curve into several blocks. Correspondingly, the feedrate of each breakpoint is always assumed to be 0 mm/s. Thus, a series of intervals $\{[u_{si}^{blo}, u_{ei}^{blo}]\}$ can be obtained where u_{si}^{blo} and u_{ei}^{blo} are the start and end knots of i-th block, respectively.

2) ARC LENGTH AND CURVATURE CALCULATION

To achieve the feedrate scheduling, the arc length of each block needs to be calculated. A lot of methods have been proposed to obtain the arc length of NURBS curves. Considering the computational efficiency and accuracy, an adaptive quadrature method [28] based on Simpson rule, which divides each block into a series of subintervals, is employed in this paper. In addition, the curvature of each sampling point should be obtained during the arc length calculation. Therefore, a set of subintervals with the data buffer $\{[u_i, \kappa_i, S_i]\}$ of each block are generated where, μ_i is the parameter of end point, κ_i is the corresponding curvature and S_i is the cumulative arc length.

3) PARTITIONING SEGMENTS AT CRITICAL POINTS

In order to simplify the feedrate scheduling process and improve the interpolation accuracy, it is essential to further partition each block to several segments at the critical points which are the sharp corners. Based on the specified command feedrate F and the various constraints including chord error and centripetal acceleration, the critical points can be found by scanning the feature data buffer $\{[u_i, \kappa_i, S_i]\}$. Hence, the segment feature data buffer $\{[u_i^{seg}, v_i^{res}, S_i^{seg}]\}$ can be obtained with the end point parameter u_i^{seg}, the corresponding restricted feedrate v_i^{res} and the arc length S_i^{seg} of i-th segment.

4) FEEDRATE LOOK-AHEAD

After the features scanning, the feedrate look-ahead based on the proposed jerk-continuous ACC/DEC algorithm should be performed to obtain the endpoints feedrates of each segment. Meanwhile, the restricted feedrate v_i^{res} of each end point in $\{[u_i^{seg}, v_i^{res}, S_i^{seg}]\}$ should be updated by the look-ahead results.

5) FEEDRATE SCHEDULING

To prepare for the real-time interpolation, the feedrate scheduling should be conducted for each segment based on the proposed jerk-continuous ACC/DEC algorithm and

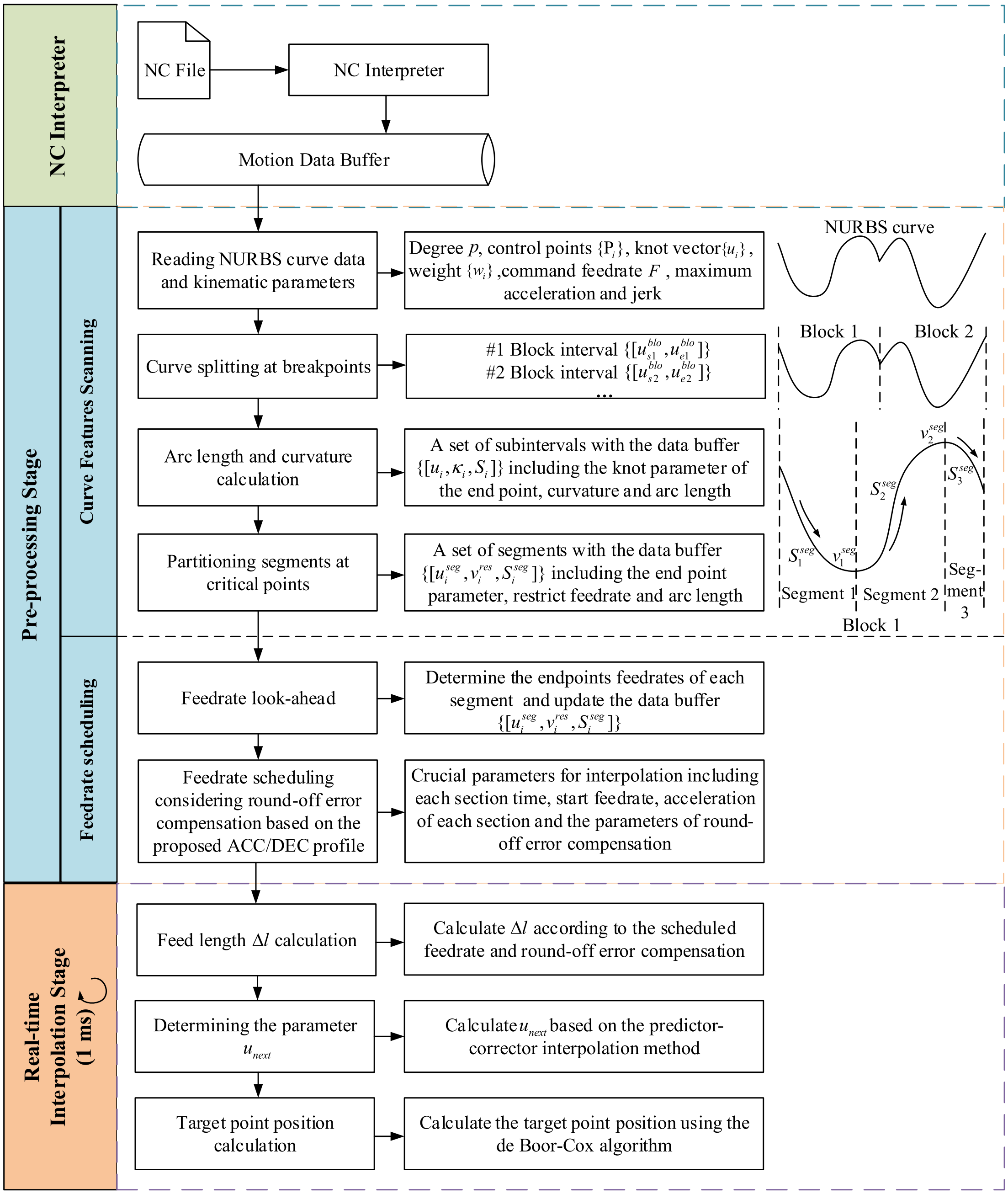


FIGURE 1. Flowchart of NURBS interpolation.

the look-ahead results. The time of each section and other crucial motion parameters can be obtained. Meanwhile, the round-off error compensation is considered is the feedrate scheduling and the corresponding parameters can be calculated.

In the stage of real-time interpolation, the main task is to calculate the position of target interpolation point in each interpolation period according to the feed length. A lot of methods including the first- and second-order approximations of Taylor series expansion [29], [30], the predictor-corrector interpolation (PCI) method [31] and the chord-tracking algorithm (CTA) [32] have been proposed to calculate the target parameter u_{next}. Considering the computational load and accuracy, the PCI method is applied in this paper. Then the position of target interpolation point $C(u_{next})$ can be obtained based on u_{next} and the de Boor-Cox algorithm [1].

III. THE PROPOSED JERK-CONTINUOUS ACC/DEC ALGORITHM

The polynomial ACC/DEC profile with continuous jerk has complicated feedrate scheduling process. In addition, the maximum jerk and acceleration cannot be maintained only with the trigonometric functions, which reduces the machining efficiency significantly. To cope with these problems, a novel jerk-continuous ACC/DEC algorithm is proposed where the trigonometric and polynomial functions are combined to construct the jerk profile. As shown in Fig. 2, each non-zero section of jerk profile has two trigonometric curves on both sides and the middle part is the horizontal line, which can guarantee the continuity of jerk and increase the motion efficiency.

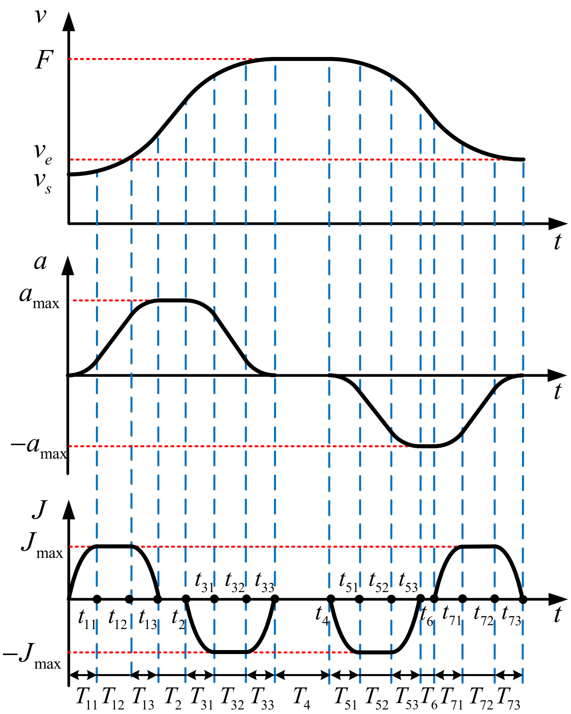


FIGURE 2. **The proposed jerk-continuous ACC/DEC profile.**

Since the deceleration phase is similar to the acceleration phase, only the acceleration and constant feedrate (CF) phases are discussed. The jerk equation in acceleration and CF phases can be expressed as Eq. (5) where $J_{\max}$ is the given maximum jerk. Integrating Eq. (5) yields the acceleration and feedrate equations expressed as Eq. (6)-(7) with the same time partitioning rule.

$$J(t)=\begin{cases} J_{\max}\sin\dfrac{\pi}{2T_{11}}t 0\leq t<t_{11} \\ J_{\max} t_{11}\leq t<t_{12} \\ J_{\max}\cos\dfrac{\pi}{2T_{13}}(t-t_{12}) t_{12}\leq t<t_{13} \\ 0 t_{13}\leq t<t_2 \\ -J_{\max}\sin\dfrac{\pi}{2T_{31}}(t-t_2) t_2\leq t<t_{31} \\ -J_{\max} t_{31}\leq t<t_{32} \\ -J_{\max}\cos\dfrac{\pi}{2T_{33}}(t-t_{32}) t_{32}\leq t<t_{33} \\ 0 t_{33}\leq t<t_4 \end{cases} \tag{5}$$

$$a(t)=\begin{cases} J_{\max}\dfrac{2T_{11}}{\pi}(1-\cos\dfrac{\pi}{2T_{11}}t) \\ a_{11}+J_{\max}(t-t_{11})a_{11}=J_{\max}\dfrac{2T_{11}}{\pi} \\ a_{12}+J_{\max}\dfrac{2T_{13}}{\pi}\sin\dfrac{\pi}{2T_{13}}(t-t_{12})a_{12}=a_{11}+J_{\max}T_{12} \\ a_2 a_2=a_{12}+J_{\max}\dfrac{2T_{13}}{\pi} \\ a_2-J_{\max}\dfrac{2T_{31}}{\pi}[1-\cos\dfrac{\pi}{2T_{31}}(t-t_2)] \\ a_{31}-J_{\max}ta_{31}=a_2-J_{\max}\dfrac{2T_{31}}{\pi} \\ a_{32}-J_{\max}\dfrac{2T_{33}}{\pi}\sin\dfrac{\pi}{2T_{33}}(t-t_{32})a_{32}=a_{31}-J_{\max}T_{32} \\ 0 \end{cases} \tag{6}$$

$$v(t)=\begin{cases} v_s+J_{\max}\dfrac{2T_{11}}{\pi}t-J_{\max}(\dfrac{2T_{11}}{\pi})^2\sin\dfrac{\pi}{2T_{11}}t \\ v_{11}+\dfrac{1}{2}J_{com}(t-t_{11})^2+a_{11}(t-t_{11})v_{11} \\ \quad =v_s+J_{\max}\dfrac{2T_{11}^2}{\pi}-J_{\max}(\dfrac{2T_{11}}{\pi})^2 \\ v_{12}+a_{12}(t-t_{12})+J_{\max}(\dfrac{2T_{13}}{\pi})^2[1-\cos\dfrac{\pi}{2T_{13}} \\ \quad (t-t_{12})]v_{12}=v_{11}+\dfrac{1}{2}J_{\max}T_{12}^2+a_{11}T_{12} \\ v_{13}+a_2(t-t_{13})v_{13}=v_{12}+a_{12}T_{13}+J_{\max}(\dfrac{2T_{13}}{\pi})^2 \\ v_2+a_2(t-t_2)-J_{\max}\dfrac{2T_{31}}{\pi}(t-t_2) \\ \quad +J_{\max}(\dfrac{2T_{31}}{\pi})^2\sin\dfrac{\pi}{2T_{31}}(t-t_2)v_2=v_{13}+a_2T_2 \\ v_{31}-\dfrac{1}{2}J_{\max}(t-t_{31})^2+a_{31}(t-t_{31})v_{31} \\ \quad =v_2+a_2T_{31}-J_{\max}\dfrac{2T_{31}^2}{\pi}+J_{\max}(\dfrac{2T_{31}}{\pi})^2 \\ v_{32}+a_{32}(t-t_{32})-J_{\max}(\dfrac{2T_{33}}{\pi})^2[1-\cos\dfrac{\pi}{2T_{33}} \\ \quad (t-t_{32})]v_{32}=v_{31}-\dfrac{1}{2}J_{\max}T_{32}^2+a_{31}T_{32} \\ v_4 v_4=v_{32}+a_{32}T_{33}-J(\dfrac{2T_{33}}{\pi})^2 \end{cases} \tag{7}$$

However, as shown in Fig.2, a complete ACC/DEC profile consists of 15 sections with complicated feedrate scheduling process similar to the algorithm given in [25]. Therefore, a proportional control method for jerk profile is proposed to reduce the computational load. Firstly, the two trigonometric curves corresponding to T_{11} and T_{13} in jerk profile are assumed to be symmetrical as well as T_{31} and T_{33}. Meanwhile, the curves of increasing acceleration sections and decreasing acceleration sections should be symmetrical to ensure that the acceleration can be reduced to zero at t_{33}. Therefore, it can be derived that $T_{12} = T_{32}$ and $T_{11} = T_{13} = T_{31} = T_{33}$. Correspondingly, the deceleration phase has the same relations. Secondly, a serious of coefficients $k_i(i = 1, 3, 5, 7)$ are introduced to determine the relation between the trigonometric and polynomial parts, which can be expressed as follows:

$$k_i = \frac{T_{i1}}{T_i} \tag{8}$$

where $T_i = T_{i1} + T_{i2} + T_{i3}$. Because of the symmetry, $k_1 = k_3$ and $k_5 = k_7$.Furthermore,k_i should be limited in $[0, 0.5]$. The larger the k_i, the smoother the feedrate and acceleration profiles, the lower the machining efficiency. If $k_i = 0$, the i-th section will degenerate into the typical S-shaped ACC/DEC profile; if $k_i = 0.5$, the jerk profile is same as the algorithm given in [26]. For a certain condition, the coefficients k_i should be selected in allowable range according to the requirements of motion smoothness and machining efficiency. In order to further reduce the computational load, k_i in acceleration and deceleration phases can be assumed to be equal and set to k.

Based on the proportional control method, the trigonometric and polynomial parts are always coexisting, which ensures that the jerk profile is continuous. Meanwhile, the maximum jerk and acceleration can be reached and maintained to improve the machining efficiency. Furthermore, the sections T_{i1},T_{i2} and T_{i3} can be packed up and processed as one section T_i based on the coefficients k_i. Therefore, the original 15 sections can be simplified to 7 sections with the similar feedrate scheduling procedures as the typical S-shaped ACC/DEC algorithm, which can simplify the computational load significantly. In addition, the proposed ACC/DEC algorithm is configurable according to the requirements of motion smoothness and machining efficiency, which increases the flexibility of feedrate control.

IV. FEEDRATE SCHEDULING BASED ON THE PROPOSED JERK-CONTINUOUS ACC/DEC ALGORITHM

In this section, the feedrate scheduling method for NURBS curves based on the proposed jerk-continuous ACC/DEC algorithm is introduced in detail. Meanwhile, in order to improve the machining accuracy and motion smoothness, the compensation method of round-off error is also discussed.

A. FEEDRATE SCHEDULING CONSIDERING ROUND-OFF ERROR COMPENSATION

The round-off error is defined as the difference between the desired arc length and the sum of the feed length obtained by scheduled feedrate and interpolation period T_s. Because the motion parameters such as the start feedrate v_s, end feedrate v_e, command feedrate F, maximum acceleration $a_{\max}$, maximum jerk $J_{\max}$, displacement S and the coefficients k_i are given arbitrarily, neither each section time T_j nor the total time $T_{total} = \sum T_j (j = 1, 2, \ldots, 7)$ obtained by feedrate scheduling can be discretized exactly in an integer multiple of T_s. Therefore, the round-off error is always exiting, which affects the machining accuracy and motion smoothness.

Some time rounding and compensation methods have been proposed in [15] and [19]. However, the round-off error is magnified because every section time is rounded in these methods. Meanwhile, the traditional compensation methods cannot maintain the continuity of jerk profile. As a matter of fact, it only needs to guarantee that the total scheduled time T_{total} is an integer multiple of T_s, which can reduce the round-off error significantly. Therefore, the total interpolation period number N_{total} can be calculated as follows:

$$N_{total} = [\frac{T_{total}}{T_s}] \tag{9}$$

where the operator '[]' denotes rounding down. The round-off time Δt can be expressed as follows:

$$\Delta t = T_{total} - N_{total} T_s < T_s \tag{10}$$

Hence, the round-off error is only the displacement ΔS corresponding to Δt. In order to maintain the motion smoothness, ΔS and Δt should be taken from the CF section. Therefore, the feedrate scheduling should make sure that the CF section is always existing with at least one interpolation period. According to the characteristics of the proposed jerk-continuous ACC/DEC algorithm given in section III, the procedures of feedrate scheduling are shown in Fig. 3 assuming $0 \leq v_s \leq v_e < F$. Firstly, the coefficient k should be selected according to the requirements of motion smoothness and machining efficiency. Then the feedrate can be scheduled with no more than three steps which are illustrated as follows:

1) STEP 1: ASSUMING $v_{\max}^{(1)} = F$

Firstly, $v_{\max}^{(1)}$ is the hypothetical maximum feedrate and equals to the command feedrate F. Meanwhile, the actual maximum acceleration $a_{a\max}^{act}$ in acceleration phase is assumed to be $a_{\max}$. Based on the selected k, the sections T_{i1},T_{i2} and T_{i3} can be processed as one section T_i. Therefore, T_1 can be calculated as follows:

$$T_1 = \frac{a_{a\max}^{act}\pi}{[(4-2\pi)k + \pi]J_{\max}} \tag{11}$$

According to the symmetry, the corresponding feedrate increment Δv_{acc} without constant acceleration section can be calculated as follows:

$$\Delta v_{acc} = a_{a\max}^{act} T_1 \tag{12}$$

Through comparing the relation between Δv_{acc} and $(v_{\max}^{(1)} - v_s)$, the constant acceleration section time T_2 can be obtained. If $\Delta v_{acc} \leq (v_{\max}^{(1)} - v_s)$, T_2 can be expressed as:

$$T_2 = \frac{v_{\max}^{(1)} - v_s}{a_{a\max}^{act}} - T_1 \tag{13}$$

On the contrary, if $\Delta v_{acc} > (v_{\max}^{(1)} - v_s)$, it means that the constant acceleration section is not existing and $T_2 = 0$. Meanwhile, T_1 and T_3 should be updated as follows:

$$T_1 = T_3 = \sqrt{\frac{(v_{\max}^{(1)} - v_s)\pi}{[(4-2\pi)k + \pi]J_{\max}}} \tag{14}$$

Therefore, the displacement of acceleration phase can be calculated as follows:

$$S_{acc} = \frac{v_{\max}^{(1)} + v_s}{2} \quad (T_1 + T_2 + T_3) \tag{15}$$

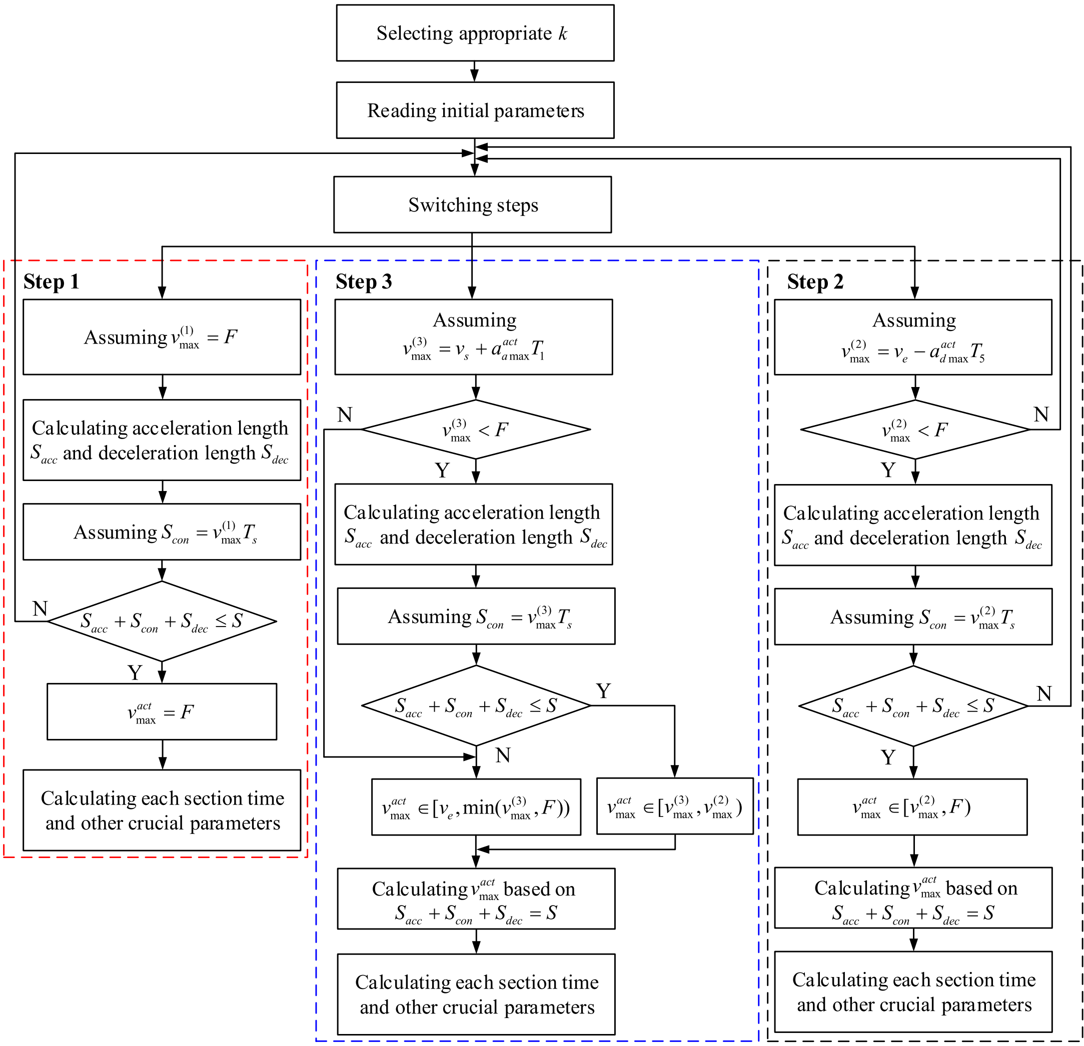


FIGURE 3. Feedrate scheduling based on the proposed ACC/DEC algorithm.

Similar to Eq. (11)-(15), each section time of deceleration phase can be obtained and the corresponding displacement is expressed as follows:

$$S_{dec} = \frac{v_{\max}^{(1)} + v_e}{2} \quad (T_5 + T_6 + T_7) \tag{16}$$

Then, the CF section time T_4 can be calculated based on S_{acc},S_{dec} and S_{con} where $S_{con} = v_{\max}^{(1)} T_s$.$S_{con}$ is of great significance for round-off error compensation and used to ensure that the CF section is always existing with at least one interpolation period. If $S \geq (S_{acc} + S_{dec} + S_{con})$, the actual maximum feedrate $v_{\max}^{act}$ is equal to $v_{\max}^{(1)}$ and T_4 can be calculated as follows:

$$T_4 = \frac{S - S_{acc} - S_{dec} - S_{con}}{v_{\max}^{(1)}} \tag{17}$$

Thus, each section time is obtained. Then, the start acceleration of each section and other crucial parameters can be calculated based on Eq. (5)-(7). Conversely, if $S < (S_{acc} + S_{dec} + S_{con})$, it means that the assumption for $v_{\max}^{(1)}$ is unreasonable and the feedrate scheduling process should go to Step 2.

2) STEP 2: ASSUMING $v_{\max}^{(2)} = v_e - a_{d\max}^{act} T_5$

In this case, $v_{\max}^{(2)}$ is the hypothetical maximum feedrate obtained by accelerating from v_e without constant acceleration section. The actual minimum acceleration $a_{d\min}^{act}$ in deceleration phase is assumed to be $-a_{\max}$. Therefore, T_5 and T_7 can be calculated as follows:

$$T_5 = T_7 = \frac{-a_{d\max}^{act} \pi}{[(4 - 2\pi)k + \pi] J_{\max}} \tag{18}$$

Meanwhile, CF section time T_4 and constant deceleration section time T_6 are equal to zero. If $v_{\max}^{(2)} \geq F$, it means that $v_{\max}^{(2)}$ is still larger and the scheduling process should go to Step 3. If $v_{\max}^{(2)} < F$, the time of each section can be obtained by Eq. (11)-(14). Furthermore, the displacements of acceleration and deceleration phases can be calculated as follows:

$$S_{acc} = \frac{v_{\max}^{(2)} + v_s}{2}(T_1 + T_2 + T_3) \tag{19}$$

$$S_{dec} = \frac{v_{\max}^{(2)} + v_e}{2}(T_5 + T_7) \tag{20}$$

$$S_{con} = v_{\max}^{(2)} T_s \tag{21}$$

If $S = (S_{acc} + S_{dec} + S_{con})$,$v_{\max}^{(2)}$ is the actual maximum feedrate exactly; if $S < (S_{acc} + S_{dec} + S_{con})$, the scheduling process should go to Step 3; if $S > (S_{acc} + S_{dec} + S_{con})$, it means that $v_{\max}^{act}$ belongs to $(v_{\max}^{(2)}, F)$. Therefore, an equation with respect to $v_{\max}^{act}$ can be derived according to Eq. (19)-(21) as follows:

$$f_1(v_{\max}^{act}) = A(v_{\max}^{act})^2 + Bv_{\max}^{act} + C = 0 \tag{22}$$

where $A = \frac{1}{a_{\max}}$,$B = T_5 + T_s$,$C = -\frac{v_s^2+v_e^2}{2a_{\max}} + \frac{v_s+v_e}{2}T_5 - S$. Since $B > 0$,$v_{\max}^{act}$ can be calculated using the quadratic formula as follows:

$$v_{\max}^{act} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \tag{23}$$

Therefore, the time of each section can be recalculated based on $v_{\max}^{act}$. Meanwhile, the start acceleration of each section and other parameters can be obtained based on Eq. (5)-(7).

3) STEP 3: ASSUMING $v_{\max}^{(3)} = v_s + a_{a\max}^{act} T_1$

In this situation, $v_{\max}^{(3)}$ is the hypothetical maximum feedrate obtained by accelerating from v_s without constant acceleration section and $a_{a\max}^{act}$ is assumed to be $a_{\max}$. Hence, T_1 and T_3 can be calculated using Eq. (11). Meanwhile, T_2, T_4 and T_6 are equal to zero. If $v_{\max}^{(3)} < F$,T_5 and T_7 can be calculated by the similar formula as Eq. (14). Then, the displacements of acceleration and deceleration phases can be obtained as follows:

$$S_{acc} = \frac{v_{\max}^{(3)} + v_s}{2}(T_1 + T_3) \tag{24}$$

$$S_{dec} = \frac{v_{\max}^{(3)} + v_e}{2}(T_5 + T_7) \tag{25}$$

$$S_{con} = v_{\max}^{(3)} T_s \tag{26}$$

If $S = (S_{acc} + S_{dec} + S_{con})$,$v_{\max}^{act}$ is equal to $v_{\max}^{(3)}$ exactly. If $S > (S_{acc} + S_{dec} + S_{con})$,$v_{\max}^{act}$ belongs to $(v_{\max}^{(3)}, v_{\max}^{(2)})$. For this situation, an equation with respect to $v_{\max}^{act}$ can be obtained as follows:

$$\begin{aligned} f_2(v_{\max}^{act}) = {} & \frac{v_{\max}^{act} + v_s}{2}\left(\frac{a_{a\max}^{act}\pi}{[(4-2\pi)k+\pi]J_{\max}} + \frac{v_{\max}^{act} - v_s}{a_{a\max}^{act}}\right) \\ & + (v_{\max}^{act} + v_e)\sqrt{\frac{(v_{\max}^{act} - v_e)\pi}{[(4-2\pi)k+\pi]J_{\max}}} \\ & + v_{\max}^{act} T_s - S = 0 \end{aligned} \tag{27}$$

If $S < (S_{acc} + S_{dec} + S_{con})$ or $v_{\max}^{(3)} \geq F$, $v_{\max}^{act}$ should belong to $[v_e, \min(v_{\max}^{(3)}, F))$. Another equation can be derived as follows:

$$\begin{aligned} f_3(v_{\max}^{act}) = {} & (v_{\max}^{act} + v_s)\sqrt{\frac{v_{\max}^{act} - v_s}{\frac{(4-2\pi)k+\pi}{\pi}J_{\max}}} \\ & + (v_{\max}^{act} + v_e)\sqrt{\frac{v_{\max}^{act} - v_e}{\frac{(4-2\pi)k+\pi}{\pi}J_{\max}}} \\ & + v_{\max}^{act} T_s - S = 0 \end{aligned} \tag{28}$$

As can be seen, $f_2(v_{\max}^{act})$ and $f_3(v_{\max}^{act})$ are the high order equations of $v_{\max}^{act}$ and difficult to solve by analytical method. Thus, the Newton-Raphson algorithm which is a widely used numerical method is employed to solve $v_{\max}^{act}$ with the following iterative format:

$$\begin{cases} do: v_m = v_{m-1} - \dfrac{f(v_{m-1})}{f'(v_{m-1})}, & m = 1, 2, \ldots \\ until: \left|\dfrac{v_m - v_{m-1}}{v_m}\right| \leq \delta_{new}, & then: v_{\max}^{act} = v_m \end{cases} \tag{29}$$

where δ_{new} denotes the maximum relative root error. Because $f_2(v_{\max}^{act})$ and $f_3(v_{\max}^{act})$ are monotonically increasing functions of $v_{\max}^{act}$, there must be a unique solution within the corresponding ranges. Hence, the initial value v_0 can be selected as $\frac{v_{\max}^{(2)}+v_{\max}^{(3)}}{2}$ in $(v_{\max}^{(3)}, v_{\max}^{(2)})$ or $\frac{v_e+\min(v_{\max}^{(3)},F)}{2}$ in $[v_e, \min(v_{\max}^{(3)}, F))$.

In summary, the key of feedrate scheduling is to calculate the actual maximum feedrate $v_{\max}^{act}$. Through three assumptions,$v_{\max}^{act}$ can be calculated by analytic or numerical methods. Then, the time of each section and other crucial parameters can be obtained.

B. ROUND-OFF ERROR COMPENSATION WITH CONTINUOUS JERK PROFILE

In order to improve machining accuracy and motion smoothness, the round-off error needs to be compensated with continuous jerk profile. As given in subsection IV.A, the round-off error ΔS can be calculated after the feedrate scheduling as follows:

$$\Delta S = v_{\max}^{act} \Delta t \tag{30}$$

To reduce the effect of round-off error compensation to the scheduled feedrate,ΔS should be compensated throughout the interpolation process of current NURBS segment. Thus, the compensation time T_{total}^{com} should be equal to the total interpolation time $N_{total}T_s$. In addition, the start and end compensation feedrates v_s^{com} and v_e^{com} should be set to zero. Therefore, different from the conditions of feedrate scheduling, the round-off error compensation is conducted based on ΔS,T_{total}^{com},v_s^{com} and v_e^{com}.

In order to achieve the error compensation with continuous jerk profile, the proposed ACC/DEC algorithm is simplified according to the given conditions. It can be assumed that all sections are existing with the following relation:

$$T_{\sec}^{com} = T_i^{com} = \frac{N_{total}T_s}{7} \quad (i = 1, 2, \ldots, 7) \tag{31}$$

TABLE 2. Interpolator parameters.

Parameters	Butterfly-shaped curve	∞-shaped curve
Interpolation period T_s	1 ms	1 ms
Arc length tolerance δ_{arc}	10^{-6} mm	10^{-6} mm
Maximum chord error δ_{cho}	0.5 μm	0.5 μm
Feedrate fluctuation tolerance δ_{flu}	10^{-8}	10^{-8}
Newton-Raphson method tolerance δ_{new}	10^{-4}	10^{-4}
Command feedrate F	200 mm/s	600 mm/s
Maximum centripetal acceleration $a_{\max c}$	1000 mm/s^2	2500 mm/s^2
Maximum tangential acceleration $a_{\max t}$	1000 mm/s^2	2500 mm/s^2
Maximum jerk $J_{\max}$	40000 mm/s^3	50000 mm/s^3
Coefficient k	0.3	0.2

where T_i^{com} is i-th section time of error compensation. Hence, the relation among ΔS,$T_{\sec}^{com}$ and the maximum feedrate of error compensation $v_{\max}^{com}$ can be derived as follows:

$$v_{\max}^{com} = \frac{\Delta S}{4T_{\sec}^{com}} \tag{32}$$

Then, the corresponding maximum acceleration $a_{\max}^{com}$ and jerk $J_{\max}^{com}$ can be obtained as follows:

$$a_{\max}^{com} = \frac{v_{\max}^{com}}{2T_{\sec}^{com}} \tag{33}$$

$$J_{\max}^{com} = \frac{a_{\max}^{com}\pi}{[(4-2\pi)k+\pi]T_{\sec}^{com}} \tag{34}$$

Therefore, the parameters of error compensation can be calculated based on Eq. (5)-(7). During the real-time interpolation, the feed length of i-th interpolation period Δl_i can be calculated through the procedures shown in Fig. 4 where Δl_i^1 is calculated by the scheduled feedrate and Δl_i^2 is the corresponding compensation length.

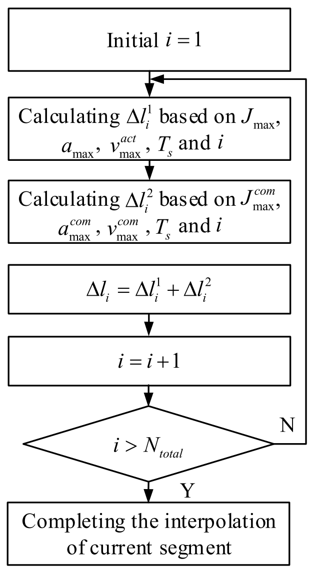


FIGURE 4. Calculation procedures of the feed length in each interpolation period.

V. SIMULATION AND EXPERIMENT RESULTS

In this section, analytical simulations of two NURBS curves are conducted to evaluate the performance of the proposed jerk-continuous ACC/DEC algorithm and the corresponding feedrate scheduling method. In V.A, analysis and comparisons are performed with the representative method given in [26]. In addition, the experimental results of the test NURBS curves are shown in V.B to validate the applicability and feasibility of the proposed method.

A. SIMULATION RESULTS AND ANALYSIS

1) TEST CASE AND SIMULATION ENVIRONMENT

As shown in Fig. 5, the butterfly-shaped curve and the ∞-shaped curve are selected as the case studies. the degree, control points, knot vectors and weight vectors of the test curves are given in appendix A and B, respectively. The interpolator parameters used for simulations are listed in Tab. 2. In addition, the simulations are conducted on a personal computer with Intel(R) Core(TM) i7-6700HQ 2.60GHz CPU, 8.00GB SDRAM and Windows 7 operating system. And all the algorithms for simulations are developed and implemented on Microsoft Visual Studio 2008 by C++ language.

2) ANALYSIS AND COMPARISONS OF BUTTERFLY-SHAPED CURVE

The simulation results of butterfly-shaped curve obtained by the proposed ACC/DEC algorithm and feedrate scheduling method are shown in Fig. 6(a)-(d). As can be seen, the jerk profile is continuous while the acceleration and feedrate profiles are smooth. However, the jerk profile in some interpolation points exceeds the confined range due to the round-off error compensation but still keeps in a certain extent with small effect to the machining accuracy and motion smoothness. The simulation results obtained by the method given in [26] are shown in Fig. 7(a)-(d). However, the

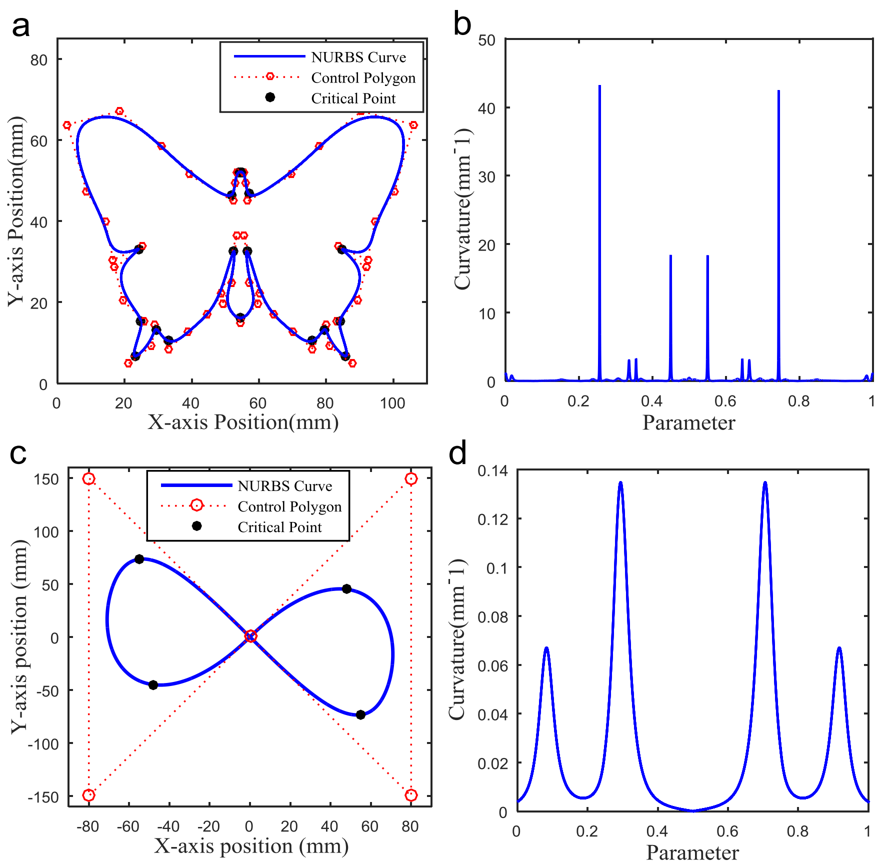


FIGURE 5. Test curves and their curvature curves. (a) Butterfly-shaped curve. (b) Curvature of butterfly-shaped curve. (c) ∞-shaped curve. (d) Curvature of ∞-shaped curve.

TABLE 3. Static comparison of butterfly-shaped curve simulation results.

Parameters	Butterfly-shaped curve	∞-shaped curve
Interpolation period T_s	1 ms	1 ms
Arc length tolerance δ_{arc}	10^{-6} mm	10^{-6} mm
Maximum chord error δ_{cho}	0.5 μm	0.5 μm
Feedrate fluctuation tolerance δ_{flu}	10^{-8}	10^{-8}
Newton-Raphson method tolerance δ_{new}	10^{-4}	10^{-4}
Command feedrate F	200 mm/s	600 mm/s
Maximum centripetal acceleration $a_{\max c}$	1000 mm/s^2	2500 mm/s^2
Maximum tangential acceleration $a_{\max t}$	1000 mm/s^2	2500 mm/s^2
Maximum jerk $J_{\max}$	40000 mm/s^3	50000 mm/s^3
Coefficient k	0.3	0.2

round-off error compensation method is not given. Thus the total interpolation time in Fig. 7 is not an integer multiple of T_s. But the jerk profile might also exceed the given range if the round-off error is compensated because of the superposition of two jerk profiles. As shown in Fig 7(d), the maximum jerk cannot be maintained, which leads to low machining efficiency. The statistic comparison of the interpolation time and the maximum chord error are shown in Tab. 3. As can be seen, the total interpolation time decreases from 5393.2 ms obtained by the method given in [26] to 4828 ms obtained by the proposed method wit $k = 0.3$, decreasing by 10.47%. Furthermore, the maximum chord errors obtained by these two methods can satisfy the specified tolerance.

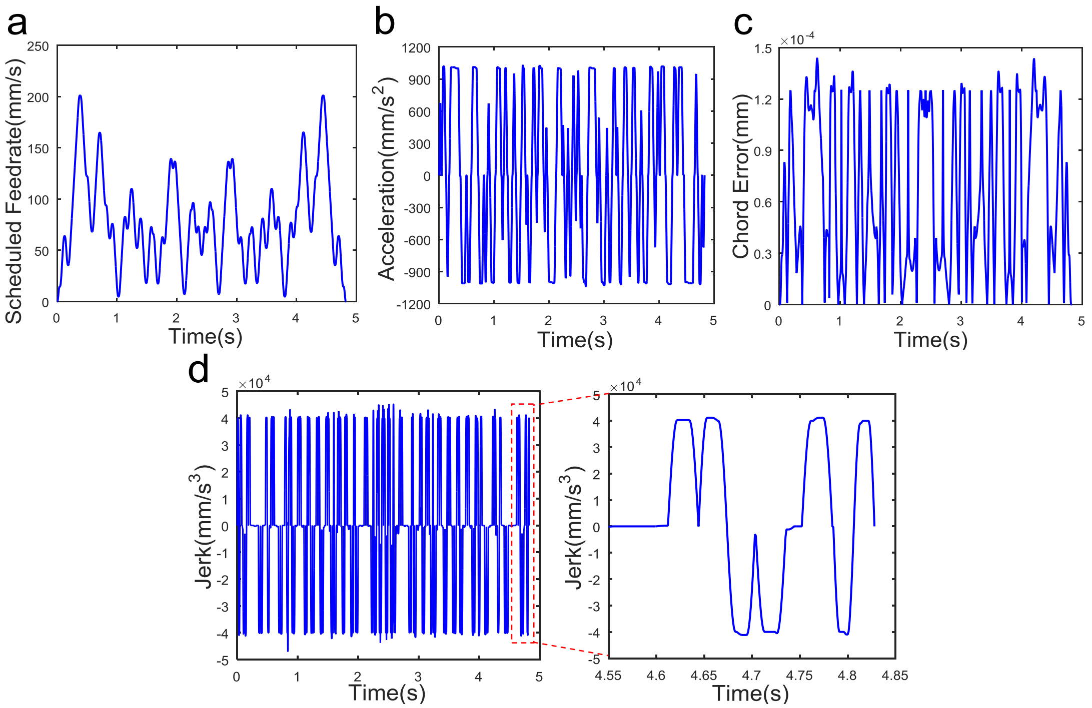


FIGURE 6. **Simulation results of butterfly-shaped curve by the proposed ACC/DEC algorithm and feedrate scheduling method. (a) Scheduled feedrate. (b) Acceleration. (c) Chord error. (d) Jerk.**

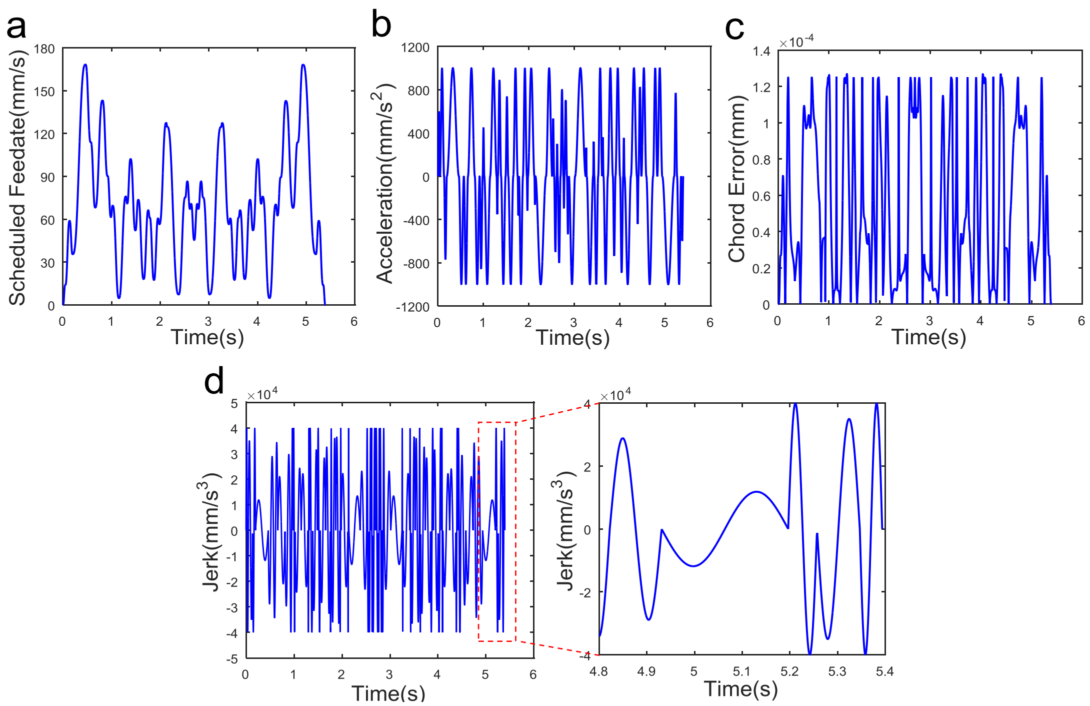


FIGURE 7. **Simulation results of butterfly-shaped curve by the method given in [26]. (a) Scheduled feedrate. (b) Acceleration. (c) Chord error (d) Jerk.**

3) ANALYSIS AND COMPARISONS OF ∞-SHAPED CURVE

The simulation results of ∞-shaped curve obtained by the proposed method are shown in Fig. 8(a)-(d) with continuous jerk and smooth acceleration profiles. As can be seen form Fig. 8(b) and 8(d), the maximum acceleration and jerk can be maintained to obtain high machining efficiency. Meanwhile, the jerk profile keeps in a certain extent although it exceeds the confined range in some areas resulted from the round-off

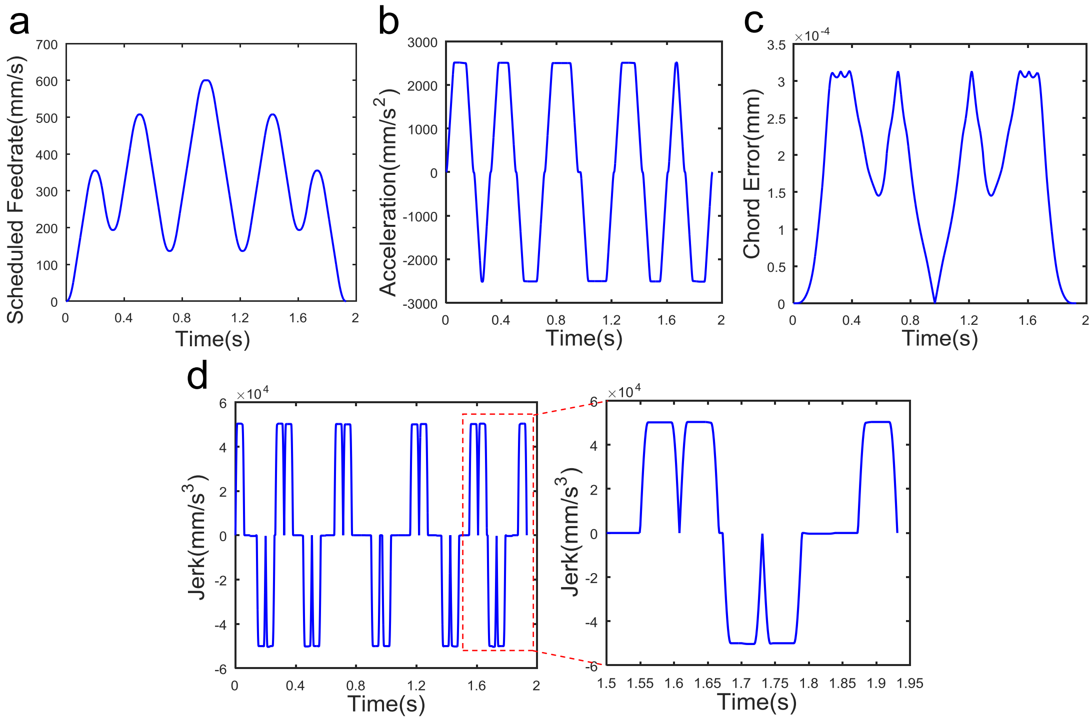


FIGURE 8. Simulation results of ∞-shaped curve by the proposed ACC/DEC algorithm and feedrate scheduling method. (a) Scheduled feedrate. (b) Acceleration. (c) Chord error. (d) Jerk.

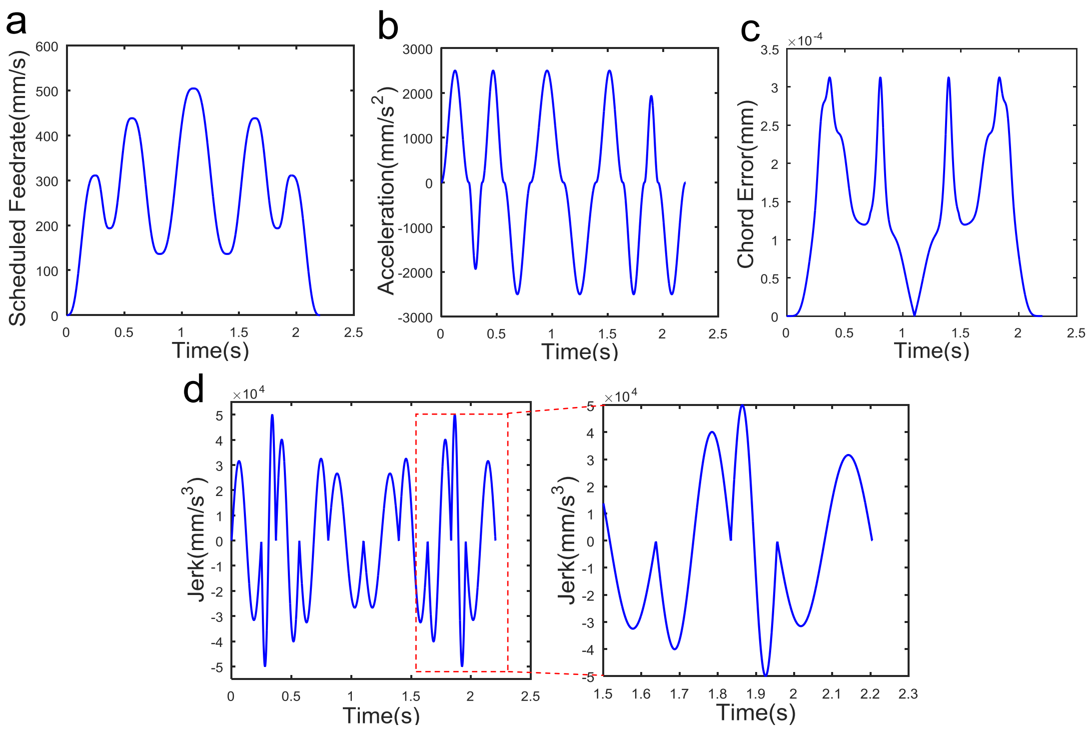


FIGURE 9. Simulation results of ∞-shaped curve by the method given in [26]. (a) Scheduled feedrate. (b) Acceleration. (c) Chord error (d) Jerk.

error compensation. The simulation results obtained by the method given in [26] are shown in Fig. 9(a)-(d). The interpolation time is not rounded either. As shown in Fig. 9(b) and 9(d), in each ACC/DEC process, only one point can reach the given maximum value of acceleration or jerk, which limits the machining efficiency.

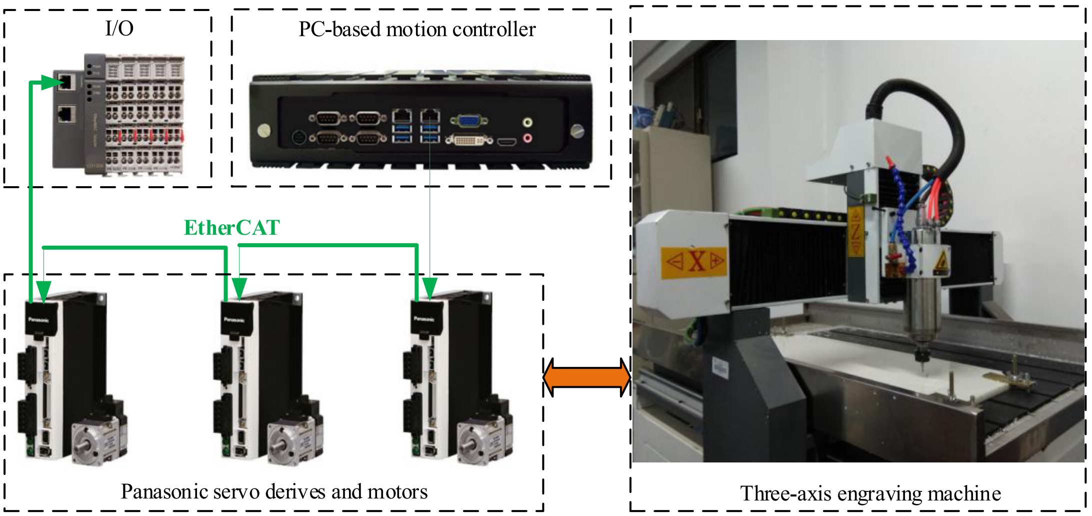


FIGURE 10. Layout of the experimental system.

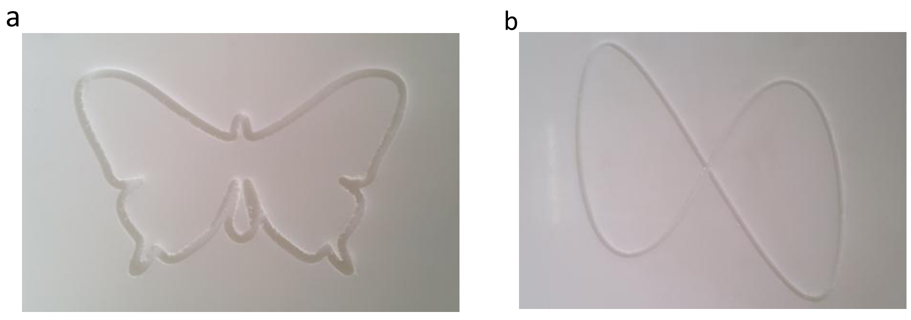


FIGURE 11. Machining results of the two NURBS curves by the proposed method. (a) Butterfly-shaped curve. (b) ∞-shaped curve.

TABLE 4. Static comparison of ∞-shaped curve simulation results.

Test methods	Total interpolation time	Maximum chord error
Proposed method	4828 ms	0.144 μm
Method in [26]	5393.2 ms	0.127 μm

Tab. 4 illustrates that the chord errors of these two methods are both within allowable range. However, the total interpolation time obtained by the proposed method with $k = 0.2$ is 1931 ms which is 12.31% shorter than 2202.1 ms obtained by the method given in [26].

B. EXPERIMENT RESULTS

In this paper, the experiments are conducted on a three-axis engraving machine with Panasonic MBDH series servo derives and MHMD series motors. The layout of the experimental system is shown in Fig. 10. The proposed ACC/DEC algorithm and the corresponding feedrate scheduling method are implemented on a PC-based motion controller developed by our team with Intel(R) Core(TM) i5-4460 3.2GHz CPU, 4.00GB SDRAM and Windows 7 operating system (OS). Meanwhile, the Kithara real-time suite (KRTS) [33] which is a modular real-time extension software for Windows OS is installed to the controller to improve the real-time performance of the control system. Therefore, the pre-processing task and the real-time interpolation task can be performed in one controller. The control data of each axis can be sent to the servo derives by EtherCAT.

Corresponding to the simulation, the butterfly-shaped curve and ∞-shaped curve are machined based on the proposed method. The machining results is shown in Fig. 11(a)-(b). As can be seen, smooth trajectories can be obtained. Meanwhile, the real-time performance of the proposed method is also tested. The computational time in each interpolation period is measured in real-time and the statistical data is summarized in Tab. 5. As can be seen, both

TABLE 5. Statistical data of the computational time by the proposed method.

Test methods	Total interpolation time	Maximum chord error
Proposed method	1931 ms	0.313μm
Method in [26]	2205.1 ms	0.313μm

the maximum and average computational times of butterfly-shaped curve and ∞-shaped curve are smaller than 15 μs Therefore, the real-time requirements can always be satisfied with 1 ms interpolation period. And the applicability and feasibility of the proposed method can be validated.

VI. CONCLUSION

A novel jerk-continuous ACC/DEC algorithm and the corresponding feedrate scheduling method for NURBS interpolation are proposed in this paper. The polynomial and trigonometric functions are combined to construct the ACC/DEC profile. Meanwhile, a proportional control method is proposed to determine the form of the jerk profile, which reduces the computational load of feedrate scheduling and improves the flexibility of feedrate control. Therefore, the generated feedrate profile is more efficient than the traditional trigonometric methods and has simpler scheduling process compared with the polynomial methods with continuous jerk. To improve the machining accuracy and motion smoothness, the round-off error is also considered and compensated based on the simplified ACC/DEC algorithm. Finally, simulations of two NURBS curves are performed to illustrate that the proposed method can generate a jerk-continuous profile with higher efficiency and simpler feedrate scheduling process similar to the typical S-shaped ACC/DEC algorithm. The practical experiments based on a self-developed motion controller and a three-axis engraving machine are also conducted to validate the feasibility and applicability of the proposed algorithm.

APPENDIX

A. PARAMETERS OF BUTTERFLY-SHAPED CURVE

The degree: $p = 3$.

The control point (mm): P = [(54.493, 52.139), (55.507, 52.139), (56.082, 49.615), (56.780, 44.971), (69.575, 51.358), (77.786, 58.573), (90.526, 67.081), (105.973, 63.801), (100.400, 47.326), (94.567, 39.913), (92.369, 30.485), (83.440, 33.757), (91.892, 28.509), (89.444, 20.393), (83.218, 15.446), (87.621, 4.830), (80.945, 9.267), (79.834, 14.535), (76.074, 8.522), (70.183, 12.550), (64.171, 16.865), (59.993, 22.122), (55.680, 36.359), (56.925, 24.995), (59.765, 19.828), (54.493, 14.940), (49.220, 19.828), (52.060, 24.994), (53.305, 36.359), (48.992, 22.122), (44.814, 16.865), (38.802, 12.551), (32.911, 8.521), (29.152, 14.535), (28.040, 9.267), (21.364, 4.830), (25.768, 15.447), (19.539, 20.391), (17.097, 28.512), (25.537, 33.750), (16.602, 30.496), (14.199, 39.803), (8.668, 47.408), (3.000, 63.794), (18.465, 67.084), (31.197, 58.572), (39.411, 51.358), (52.204, 44.971), (52.904, 49.614), (53.478, 52.139), (54.492, 52.139)].

The knot vector: U = [0, 0, 0, 0, 0.0083, 0.015, 0.0361, 0.0855, 0.1293, 0.1509, 0.1931, 0.2273, 0.2435, 0.2561, 0.2692, 0.2889, 0.3170, 0.3316, 0.3482, 0.3553, 0.3649, 0.3837, 0.4005, 0.4269, 0.4510, 0.4660, 0.4891, 0.5000, 0.5109, 0.5340, 0.5489, 0.5731, 0.5994, 0.6163, 0.6351, 0.6447, 0.6518, 0.6683, 0.6830, 0.7111, 0.7307, 0.7439, 0.7565, 0.7729, 0.8069, 0.8491, 0.8707, 0.9145, 0.9639, 0.9850, 0.9917, 1.0, 1.0, 1.0, 1.0].

The weight vector: W = [1.0, 1.0, 1.0, 1.2, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1, 2, 1.0, 1.0, 5.0, 3.0, 1.0, 1.1, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.1, 1.0, 3.0, 5.0, 1.0, 1.0, 2.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.2, 1.0, 1.0, 1.0].

B. PARAMETERS OF ∞-SHAPED CURVE

The degree: $p = 3$.

The control point (mm): P = [(0, 0), (−80, −150), (−80, 150), (0, 0), (80, −150), (80, 150), (0, 0)].

The knot vector: U = [0, 0, 0, 0, 0.25, 0.5, 0.75, 1.0, 1.0, 1.0, 1.0].

The weight vector: W = [1.0, 0.6, 0.85, 1, 0.85, 0.6, 1.0].

REFERENCES

[1] L. Piegl and W. Tiller, *The NURBS Book*, 2nd ed. New York, NY, USA: Springer-Verlag, 1997.

[2] H. Liu, Q. Liu, P. Sun, Q. Liu, and S. Yuan, "A polynomial equation-based interpolation method of NURBS tool path with minimal feed fluctuation for high-quality machining," *Int. J. Adv. Manuf. Technol.*, vol. 90, pp. 2751–2759, Jun. 2017.

[3] L. Jin and S. Li, "Distributed task allocation of multiple robots: A control perspective," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 5, pp. 693–701, May 2018.

[4] L. Jin, S. Li, H. M. La, and X. Luo, "Manipulability optimization of redundant manipulators using dynamic neural networks," *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 4710–4720, Jun. 2017.

[5] S. Li, M. Zhou, and X. Luo, "Modified primal-dual neural networks for motion control of redundant manipulators with dynamic rejection of harmonic noises," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 10, pp. 4791–4801, Oct. 2018.

[6] L. Jin, S. Li, X. Luo, Y. Li, and B. Qin, "Neural dynamics for cooperative control of redundant robot manipulators," *IEEE Trans. Ind. Informat.*, vol. 14, no. 9, pp. 3812–3821, Sep. 2018.

[7] J. W. Jeon and Y. Y. Ha, "A generalized approach for the acceleration and deceleration of industrial robots and CNC machine tools," *IEEE Trans. Ind. Electron.*, vol. 47, no. 1, pp. 133–139, Feb. 2000.

[8] J. Hu, L. Xiao, Y. Wang, and Z. Wu, "An optimal feedrate model and solution algorithm for a high-speed machine of small line blocks with look-ahead," *Int. J. Adv. Manuf. Technol.*, vol. 28, nos. 9–10, pp. 930–935, 2006.

[9] L. X. Zhang, R. Y. Sun, X. S. Gao, and H. B. Li, "High speed interpolation for micro-line trajectory and adaptive real-time look-ahead scheme in CNC machining," *Sci. China Technol. Sci.*, vol. 54, no. 6, pp. 1481–1495, 2011.

[10] S.-S. Yeh and P.-L. Hsu, "Adaptive-feedrate interpolation for parametric curves with a confined chord error," *Comput. Aided Des.*, vol. 34, no. 3, pp. 229–237, 2002.

[11] M.-Y. Cheng, M.-C. Tsai, and J.-C. Kuo, "Real-time NURBS command generators for CNC servo controllers," *Int. J. Mach. Tools Manuf.*, vol. 42, no. 7, pp. 801–813, 2002.

[12] K. Erkorkmaz and Y. Altintas, "High speed CNC system design. Part I: Jerk limited trajectory generation and quintic spline interpolation," *Int. J. Mach. Tools Manuf.*, vol. 41, no. 9, pp. 1323–1345, 2001.

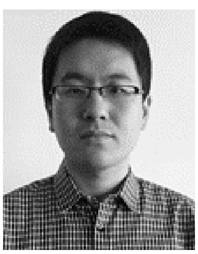
[13] D. Du, Y. Liu, X. Guo, K. Yamazaki, and M. Fujishima, "An accurate adaptive NURBS curve interpolator with real-time flexible acceleration/deceleration control," *Robot. Comput.-Integr. Manuf.*, vol. 26, no. 4, pp. 273–281, 2010.

[14] H. Dong, B. Chen, Y. Chen, J. Xie, and Z. Zhou, "An accurate NURBS curve interpolation algorithm with short spline interpolation capacity," *Int. J. Adv. Manuf. Technol.*, vol. 63, nos. 9–12, pp. 1257–1270, 2012.

[15] X. Du, J. Huang, and L.-M. Zhu, "A complete S-shape feed rate scheduling approach for NURBS interpolator," *J. Comput. Des. Eng.*, vol. 2, no. 4, pp. 206–217, 2015.

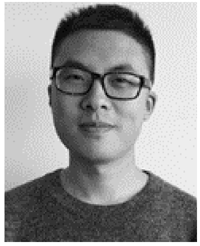
[16] M. T. Lin, M.-S. Tsai, and H.-T. Yau, "Development of a dynamics-based NURBS interpolator with real-time look-ahead algorithm," *Int. J. Mach. Tools Manuf.*, vol. 47, no. 15, pp. 2246–2262, 2007.

[17] M. Liu, Y. Huang, L. Yin, J. W. Guo, X. Y. Shao, and G. J. Zhang, "Development and implementation of a NURBS interpolator with smooth feedrate scheduling for CNC machine tools," *Int. J. Mach. Tools Manuf.*, vol. 87, pp. 1–15, Dec. 2014.
[18] S. Sun, H. Lin, L. Zheng, J. Yu, and Y. Hu, "A real-time and look-ahead interpolation methodology with dynamic B-spline transition scheme for CNC machining of short line segments," *Int. J. Adv. Manuf. Technol.*, vol. 84, nos. 5–8, pp. 1359–1370, 2016.
[19] Q. Liu, H. Liu, and S. Yuan, "High accurate interpolation of NURBS tool path for CNC machine tools," *Chin. J. Mech. Eng.*, vol. 29, no. 5, pp. 911–920, 2016.
[20] H.-B. Leng, Y.-J. Wu, and X.-H. Pan, "Research on cubic polynomial acceleration and deceleration control model for high speed NC machining," *J. Zhejiang Univ.-Sci. A*, vol. 9, no. 3, pp. 358–365, 2008.
[21] F.-Y. Luo, Y.-F. Zhou, and J. Yin, "A universal velocity profile generation approach for high-speed machining of small line segments with look-ahead," *Int. J. Adv. Manuf. Technol.*, vol. 35, nos. 5–6, pp. 505–518, 2007.
[22] A.-C. Lee, M.-T. Lin, Y.-R. Pan, and W.-Y. Lin, "The feedrate scheduling of NURBS interpolator for CNC machine tools," *Comput.-Aided Des.*, vol. 43, no. 6, pp. 612–628, 2011.
[23] J. W. Jeon, "Efficient acceleration and deceleration technique for short distance movement in industrial robots and CNC machine tools," *Electron. Lett.*, vol. 36, no. 8, pp. 766–768, Apr. 2000.
[24] J. Huang and L.-M. Zhu, "Feedrate scheduling for interpolation of parametric tool path using the sine series representation of jerk profile," *Proc. Inst. Mech. Eng. B, J. Eng. Manuf.*, vol. 231, no. 13, pp. 2359–2371, 2017.
[25] W. Fan, X.-S. Gao, W. Yan, and C.-M. Yuan, "Interpolation of parametric CNC machining path under confined jounce," *Int. J. Adv. Manuf. Technol.*, vol. 62, nos. 5–8, pp. 719–739, 2012.
[26] X. Liu, J. Peng, L. Si, and Z. Wang, "A novel approach for NURBS interpolation through the integration of acc-jerk-continuous-based control method and look-ahead algorithm," *Int. J. Adv. Manuf. Technol.*, vol. 88, nos. 1–4, pp. 961–969, 2017.
[27] J. Jahanpour and M. R. Alizadeh, "A novel acc-jerk-limited NURBS interpolation enhanced with an optimized S-shaped quintic feedrate scheduling scheme," *Int. J. Adv. Manuf. Technol.*, vol. 77, nos. 9–12, pp. 1889–1905, 2015.
[28] W. T. Lei, M. P. Sung, L. Y. Lin, and J. J. Huang, "Fast real-time NURBS path interpolation for CNC machine tools," *Int. J. Mach. Tools Manuf.*, vol. 47, no. 10, pp. 1530–1541, 2007.
[29] J.-T. Huang and D. C. H. Yang, "Precision command generation for computer controlled machines," Precis. Mach., Technol. Mach. Develop. Improvement, ASME-PED, Tech. Rep., 1992, pp. 89–104, vol. 58.
[30] Y. Koren, C. C. Lo, and M. Shpitalni, "CNC interpolators: Algorithms and analysis," in *ASME Prod. Eng. Div. Publ. Ped.*, vol. 64, NewYork, NY, USA: ASME, 1993, pp. 83–92.
[31] M.-C. Tsai and C.-W. Cheng, "A real-time predictor-corrector interpolator for CNC machining," *J. Manuf. Sci. Eng.*, vol. 125, no. 3, pp. 449–460, 2003.
[32] H. Zhao, L. M. Zhu, and H. Ding, "A parametric interpolator with minimal feed fluctuation for CNC machine tools using arc-length compensation and feedback correction," *Int. J. Mach. Tools Manuf.*, vol. 75, pp. 1–8, Dec. 2013.
[33] *Kithara Real-Time Suite*. Accessed: Nov. 9, 2017. [Online]. Available: http://kithara.com/en/products/realtime-suite



HEPENG NI received the B.S. degree in mechanical design, manufacturing and automation from Shandong University, Jinan, China, in 2014, where he is currently pursuing the Ph.D. degree in mechatronics engineering.

His research interests include motion control of computer numerical control machine tools and robot, chatter analysis, and suppression of robot machining.



JIPENG YUAN received the B.S. degree in mechanical design, manufacturing and automation from Shandong University, Jinan, China, in 2016, where he is currently pursuing the M.S. degree in mechanical manufacturing and automation.

His research interests include motion control of computer numerical control machine tools and robot, chatter analysis, and suppression of robot machining.

SHUAI JI received the B.S. degree in mechanical design, manufacturing and automation and the Ph.D. degree in mechatronics engineering from Shandong University, Jinan, China, in 2008 and 2014, respectively.

Since 2014, he has been an Assistant Professor with Shandong Jianzhu University. His research interests include robotics computer numerical controller, real-time Ethernet, and networked control systems.

CHENGRUI ZHANG received the B.S. degree in mechanical engineering from the Shandong Institute of Mining Technology, Tai'an, China, in 1982, and the M.S. and Ph.D. degrees in mechanical engineering from the Shandong University of Technology, Jinan, China, in 1988 and 1995, respectively.

He is currently a Professor with the School of Mechanical Engineering, Shandong University, Jinan. His research interests include intelligent manufacture, numerical control technology, robot control, and Ethernet-based distributed control systems.

TIANLIANG HU received the B.S. degree in mechanical design, manufacturing and automation and the Ph.D. degree in mechatronics engineering from Shandong University, Jinan, China, in 2003 and 2008, respectively.

He is currently an Associate Professor with the School of Mechanical Engineering, Shandong University. His research interests include intelligent manufacture, numerical control technology, and robot.