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Potential-Game Based Optimally Rigid Topology Control in Wireless Sensor Networks

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ABSTRACT In this paper, the optimally rigid topology control problem in wireless sensor networks is considered to improve the algebraic rigidity properties. This problem is first formulated as a constrained optimization problem which can be solved by two stages. A minimally rigid network is constructed in the first stage, then the optimally rigid topology in the second stage. A potential game approach is proposed for solving the optimization problem by choosing a different performance metric as the potential function. It can be seen that the proposed algorithm can significantly improve the network performance, such as reducing communication complexity and transmit power, prolonging network lifetime, and so on. Finally, some simulations demonstrate the effectiveness of the proposed algorithms from multiple perspectives: topology complexity, average degree, consensus convergence speed, average radius, average link length, and network lifetime.

INDEX TERMS Wireless sensor network, topology control, rigid graph, game theory.

I. INTRODUCTION

Topology control is an effective technique for the best possible network performance to the chosen optimization criteria in wireless sensor networks (WSNs) [1]–[7]. By adjusting the radio transmission power of each individual sensor, many energy-efficient topology control algorithms have been proposed in WSNs [8]–[10]. Chu proposed a game-theoretic approach based distributed topology control algorithm in [11]. They considered the influences of many factors on the lifetime of WSNs. In order to achieve the minimum energy consumption, a distributed position-based network protocol named COMPOW is proposed in [12] which can support the peer-to-peer communications in mobile wireless networks. In [13], an adaptive localized minimum spanning tree (LMST) generation algorithm is proposed, by building LMST on individual node adaptively and independently and only keeps on tree nodes. Guo *et al.* [14] proposed a distributed selective diversity topology control approach to jointly optimize the energy efficiency, the network capacity and the energy consumption. In [15], Xu *et al.* proposed a lifetime-extension topology control algorithm which considered the selfish nodes in WSN. The topology has a low node degree and a better energy balance performance. However, the previous works are all concerned with the problem that how the power influence the topology structure, but not consider how the topology structure influence the network performance.

In this paper, we focus on the energy-efficient topology structure of the WSNs. In fact, the topology structure has a great influence on the network performance, e.g., the topology complexity determines the energy consumption; the node degree influences the robustness of network connection. Recently, as a special topology structure with low complexity and great robustness, the rigid network design problem has received much attention [16]–[24]. In [25], Rai *et al.* proposed inputs-based methods for localization judgement and topology control in WSNs by controlling the formation shape to deploy the sensor nodes according to the rigid graphs concepts. In Luo *et al.* [22] and Zhang *et al.* [23], have presented the rigid network optimization and control scheme in WSNs for node scheduling and energy-efficient topology control. However, very few papers considered designing the network structure with considering desirable algebraic rigidity properties. Shames and Summers [24] considered the topology design problem that exhibited desirable algebraic rigidity properties, e.g., the trace of a rigidity Gramian, via a submodular set function optimization approach. The designed rigid network generation algorithm can provide significant performance improvements for rigid topology construction, but they did not consider the optimally rigid graph and the limited sensing ability of sensors.

Since the sensor nodes can only communicate with their neighbor sensors, the local topology is determined by cooperation or competition between the nodes. Game theory is a powerful tool for the intelligent rational decision-makers to describe the phenomenon of competition and cooperation. The research efforts to address topology control as a game approach has been extensively studied [14], [15], [26]–[28]. In this paper, we firstly formulate topology design problem as a constrained optimization problem and divide the solution procedure into two stages. Then the optimization problem is transformed into a potential game [29]. By designing the algebraic rigidity properties of topology as a potential function, the existence of Nash Equilibrium (NE) is guaranteed. The contributions of this paper are summarized as follows.

1) The optimally rigid topology control problem is solved by a game approach with designed potential function. By formulating the topology control problem as a constrained optimization problem, the optimally rigid topology is constructed as the game converges to Nash equilibrium. The designed optimally rigid topology for WSNs has a superior network performance. Comparing with the geographical adaptive fidelity (GAF) topology [30], the rigid network has great energy efficiency and low complexity; comparing with D-Improvement Algorithm (DIA) in [13] and [26], the rigid network has a great robustness of connection.
2) The designed optimally rigid topology has great algebra rigidity properties. Few existing works considered the properties of the topology itself. By designing a utility function according to the properties of the network, which characterizes the willingness of a sensor in constructing a connected network, the minimally and optimally rigid topology is constructed.
3) The proposed algorithms can be used in different scenarios by choosing different utility weight. The utility weight of the potential function can be chosen according to the application scenarios. Then the algorithms iterate elegant solutions and the optimally rigid topology is constructed with great algebra rigidity properties.

The rest of the paper is organized as follows: the problem formulation and preliminaries including game theory and rigid graph theory are introduced in Section II. We present the potential game model for the rigid topology design and corresponding algorithms in Section III. Section IV gives characteristic analysis of the proposed algorithms. Some simulations are performed to illustrate the effectiveness of our proposed algorithms in Section V. Section VI concludes the paper.

II. PRELIMINARIES AND PROBLEM STATEMENT

In this section, we firstly introduce some preliminaries of rigid graph theory and game theory. The rigidity Gramian matrix constructed from rigidity matrix is used to quantify the algebraic rigidity properties of a network. Then the problem formulation as a constrained optimization problem is proposed.

A. RIGID GRAPH THEORY

The WSN can be expressed by an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ with the vertex set $\mathcal{V} = \{1, 2, \ldots, N\}$ and the edge set $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} : i \neq j\}$ in which (i, j) represents the interconnection edges among the vertices. Since the WSN is a typical terrestrial wireless network, we consider the graph framework and realization in 2-dimension case. Some standard definitions are given below.

Definition 1 (Framework and Realization [31]): A 2-dimensional framework is a pair $(\mathcal{G}, p)$, where $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a graph and $p : \mathcal{V} \mapsto \mathbb{R}^{2|\mathcal{V}|}$ denotes the coordinate vector associated with vertex $i \in \mathcal{V}$. p is called a 2-dimensional realization of $\mathcal{G}$.

Definition 2 (Equivalent and Congruent Frameworks [31]): Two frameworks $\mathcal{G}(\mathcal{V}, p)$ and $\mathcal{G}(\mathcal{V}, q)$ are equivalent if $\|p_i - p_j\| = \|q_i - q_j\|$ holds for every pair $i, j \in \mathcal{V}$ connected by an edge. Two frameworks $\mathcal{G}(\mathcal{V}, p)$ and $\mathcal{G}(\mathcal{V}, q)$ are congruent if $\|p_i - p_j\| = \|q_i - q_j\|$ holds for every pair $i, j \in \mathcal{V}$ no matter whether there is an edge between them.

The rigidity is an important notion of undirected graph. A framework $(\mathcal{G}, p)$ is rigid if there exists $\epsilon \geq 0$ such that if $(\mathcal{G}, q)$ is equivalent to $(\mathcal{G}, p)$ and $\|p(v) - q(v)\| \leq \epsilon$ for all $v \in \mathcal{V}$ then $(\mathcal{G}, q)$ is congruent to $(\mathcal{G}, p)$ [32], otherwise it is flexible [31]. More precisely, we have the following minimally rigid graph definition.

Definition 3 (Minimally Rigid Graph [33]): A rigid framework is minimally rigid if it becomes flexible after any one edge is removed.

Some details can also be found in [22] and [23]. An example is given in Fig.1 to show flexible, rigid and minimally rigid frameworks. The minimally rigid graph can be constructed by Henneberg sequences [34] via vertex addition operation and splitting edge operations. In this paper, the minimally rigid graph is firstly constructed and then the optimal rigid graph is also constructed. In $\mathbb{R}^2$, the rigidity test of a graph can be done by using the following lemma. According to this lemma, we construct the minimally rigid graph by using edge addition operation and rigidity test operation.

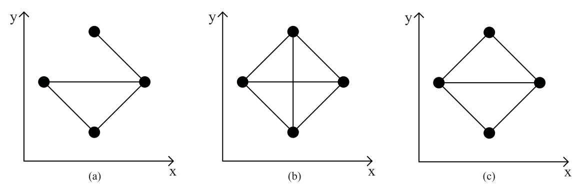

FIGURE 1. Flexible and rigid frameworks. (a) is flexible graph, (b) is rigid graph, and (c) is minimally rigid graph.

Lemma 1 (Laman's Theorem [31]): Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be a graph in $\mathbb{R}^2$, where $|\mathcal{V}| > 1$; then $\mathcal{G}$ is generically rigid if and only if there exists a subset $\mathcal{E}' \subseteq \mathcal{E}$ such that $\left|\mathcal{E}'\right| = 2|\mathcal{V}| - 3$, and for subset $\mathcal{E}'' \subseteq \mathcal{E}'$, $\left|\mathcal{E}''\right| \leq 2\left|\mathcal{V}(\mathcal{E}'')\right| - 3$.

To describe rigid graph accurately, the notion of rigidity matrix is introduced. The vertices coordinates are ordered as $\{p_1^1, p_1^2, \ldots, p_1^n, p_2^1, \ldots p_2^n, \ldots, p_N^1, \ldots, p_N^n\}$, then build a matrix $R_{(\mathcal{G},q)} \in \mathbb{R}^{|\mathcal{E}|\times 2N}$ whose rows and columns indexed by the edges and coordinates of the vertices, respectively. For example, the nonzero row entries of $R_{(\mathcal{G},q)}$ in columns $2i-1, 2i, 2j-1, 2j$ are $p_i^1 - p_j^1, p_i^2 - p_j^2, p_j^1 - p_i^1, p_j^2 - p_i^2$, respectively. The matrix $R_{(\mathcal{G},q)}$ is called rigidity matrix. Tay and Whitely [35] proved the following lemma.

Lemma 2 ([35]): $\mathcal{G}(\mathcal{V}, p)$ is a generic framework in $\mathbb{R}^2$ with N vertices. The framework $\mathcal{G}(\mathcal{V}, p)$ with $N \geq 2$ in $\mathbb{R}^2$ is infinitesimally rigid if and only if the rank of the rigidity matrix $R_{(\mathcal{G},p)}$ of $\mathcal{G}(\mathcal{V}, p)$ is equal to $2N-3$, i.e. $rank(R_{(\mathcal{G},p)}) = 2N-3$.

The rank of the rigidity matrix is an important quantitative metric to quantify the rigidity of the topology. Meanwhile, the singular values and the traces of the rigidity matrix are also used to quantify the algebraic quality of a network. In [24], Shames and Summers defined two symmetric matrices named vertex rigidity Gramian and edge rigidity Gramian according to rigidity matrix. The vertex rigidity Gramian is defined as

$$X_{(\mathcal{G},p)} = R^T_{(\mathcal{G},p)} R_{(\mathcal{G},p)} \in \mathbb{R}^{|2\mathcal{V}|\times|2\mathcal{V}|} \tag{1}$$

Similarly, the edge rigidity Gramian is defined as $X_{(\mathcal{G},p)} = R_{(\mathcal{G},p)} R^T_{(\mathcal{G},p)} \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{E}|}$ in [24]. It can be seen that the rigidity Gramian contains the complete information of rigidity matrix, it should be noted that the vertex and edge Gramians have the same spectrum, but have different eigenvalues. The optimally rigid graph is defined as follows.

Definition 4: A framework is said to be optimally rigid if the underlying graph satisfies the following conditions

1) The underlying graph is infinitesimally rigid.
2) The rigid graph satisfies the prescribed performance metrics.

B. GAME THEORY

In a pure strategic game, the choice of any one player always depends on the choice of others [36]. Similarly, by applying the game theory to the topology control in WSNs, every node expects to optimize the topology quality, the edges among the nodes are dependent on each other, hence topology control can be modeled as a pure strategic game. A strategy game generally consists of three parts: 1) player i; 2) strategic space S; 3) utility function u. In this paper, we model the topology control problem into a pure strategic game, in which by viewing the linked edges of the node i as its strategy space. The symbols used in game theory are listed in Table 1.

The pure strategy game is denoted as $\Gamma(\mathcal{V}, S, u)$, then S, S_{-i} can be expressed as

$$S = \prod_{i=1}^{N} S_i, \quad S_{-i} = \prod_{j\neq i} S_j. \tag{2}$$

Nash equilibrium is an important concept of the strategy game. When each involved player in the game choosing the best response strategy, if the other players do not change their strategies, no one will move away from the current strategy. The combination of the current strategy is called Nash equilibrium.

TABLE 1. Description of symbols in the game model.

Symbol	Definition
S	Strategy space
S_i	Optional strategies set of i
s	Strategy vector
S_{-i}	Strategies set of non-i nodes
s_i	Selected strategy of i
$U(s)$	Utility function vector
s_{-i}	Strategy vector of non-i nodes
$u_i(s)$	Utility function of i

Definition 5 (Nash equilibrium, NE): A strategy combination $s^* = (s_1^*, s_2^*, \ldots, s_N^*)$ is the Nash equilibrium of a game $\Gamma(\mathcal{V}, S, u)$, if $\forall i \in \mathcal{V}$ and $s_i \in S_i$ the following inequalities are satisfied

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*).$$

In [29], Monderer and Shapley presented a kind of special strategy game called potential game and proved that it has at least one Nash equilibrium.

Definition 6 Ordinal Potential Game (OPG), Ordinal Potential Function, (OPF) [36]): A game $\Gamma(\mathcal{V}, S, u)$ is an OPG, if there exists a function $U(s) : S \to \mathbb{R}$, for $\forall i \in \mathcal{V}, \forall s_{-i} \in S_{-i}$ and $\forall s_i^a, s_i^b \in S_i$, then

$$\begin{aligned} &U(s_i^a, s_{-i}) - U(s_i^b, s_{-i}) > 0 \\ \Leftrightarrow\ &u_i(s_i^a, s_{-i}) - u_i(s_i^b, s_{-i}) > 0. \end{aligned}$$

The function $\tilde{U}(s)$ is called ordinal potential function (OPF).

C. PROBLEM FORMULATION

The optimally rigid topology design problem in WSNs can be formulated as a constrained optimization problem, it is formulated as follows.

Problem 1: Consider the nodes set $\mathcal{V}$ and the position vector $p \in \mathbb{R}^{2|\mathcal{V}|}$, then the optimally rigid topology control problem in WSNs with underlying graph $\mathcal{G}$ is equivalent to solve the following optimization problem

$$\begin{aligned} &\text{maximize}\, f_S(\mathcal{E}), \\ &\text{sbuject to } (\mathcal{G}, p) \text{ is rigid graph.} \\ &\qquad \mathcal{G}(\mathcal{V}, \mathcal{E}), \quad |\mathcal{E}| \leq \kappa, \end{aligned} \tag{3}$$

where $\mathcal{E}$ is the variable edge set, $f_S(\mathcal{E})$ is an optimal objective function that quantifies the algebraic rigidity of the topology, it corresponds to the utility function in game theory, $\kappa \geq 2|\mathcal{V}| - 3$ is a given constant.

As stated in [24], this NP-hard combinatorial optimization problem is split into two stages by choosing different algebraic rigidity metrics. In the first stage, we construct a minimally rigid graph and an optimally rigid topology for WSNs is constructed in the second stage. The major differences between this paper and [24] are twofold. One is that we solve this combinatorial optimization problem by potential game theory. The other is that the limited wireless communicative capacity and sensing range of the sensors are considered in WSNs.

In the first stage, by setting $|\mathcal{E}| = 2|\mathcal{V}| - 3$, the minimally rigid graph can be constructed as a special case of Problem 1:

$$\begin{aligned}&\text{maximize}\, f_{S_1}(\mathcal{E})\\&\text{sbuject to rank}\,(R_{(\mathcal{G},p)}) = 2|\mathcal{V}| - 3\\&\qquad\qquad \mathcal{G}(\mathcal{V},\mathcal{E}),\quad |\mathcal{E}| = 2|\mathcal{V}| - 3.\end{aligned} \tag{4}$$

Then by adding the remainder $\kappa-(2N-3)$ edges in the second stage, the optimally rigid topology can be constructed.

$$\begin{aligned}&\text{maximize}\, f_{S_2}(\mathcal{E})\\&\text{sbuject to rank}\,(R_{(\mathcal{G},p)}) = 2|\mathcal{V}| - 3\\&\qquad\qquad \mathcal{G}(\mathcal{V},\mathcal{E}),\quad |\mathcal{E}| \le \kappa.\end{aligned} \tag{5}$$

Remark 1: Consider the WSNs in a two-dimensional plane, we focus on optimizing the algebraic rigidity performance of the topology structure. By adding different sets of edges, the underlying graph can achieve different rigidity performance. The algebraic rigidity performance is significant for consensus network [37], formation control [20] and localization [24], [38]. The cost functions associated with the selected edges can be chosen according to different scenarios.

III. THE OPTIMALLY RIGID NETWORK DESIGN

In this section, we consider the optimally rigid topology control problem. The optimally rigid network design problem for WSNs will be solved by OPG in the following two stages.

A. STAGE 1: THE GENERATION OF MINIMALLY RIGID GRAPH

According to different scenarios, the utility function for the game model $\Gamma_1(\mathcal{V}, S, u)$ is presented as

$$u_i(s_i, s_{-i}) = \alpha_1 f_i(s_i, s_{-i}) + \alpha_2 \text{trace}(X_{(\mathcal{G},S_i)}), \tag{6}$$

where α_1 is a large positive number, α_2 is a positive number. s_i denotes the edge set connected to node i, s_{-i} is the edge set connected to other nodes. In each round game, at least one edge changes for s_i. In other words, by adjusting the edges connected to node i, the utility function is maximized. $f_i(s_i, s_{-i})$ is called the rigidity indicator function. $f_i(s_i, s_{-i}) = 1$, if $\text{rank}(R_{(\mathcal{G},q)}) = 2N-3$, $|\mathcal{E}| = 2|\mathcal{V}| - 3$ and $f_i(s_i, s_{-i}) = 0$ otherwise. $f_i(s_i, s_{-i})$ is a monotone nondecreasing function. $X_{(\mathcal{G},S_i)} = R^T_{(\mathcal{G},S_i)} R_{(\mathcal{G},S_i)}$ is the vertex rigidity Gramian for a network $\mathcal{G}(\mathcal{V}, \mathcal{E})$ with the strategy S_i. This is the first main result by considering the trace of the rigidity Gramian.

Theorem 1: The game model $\Gamma_1(\mathcal{V}, S, u)$ for minimally rigid topology control is an OPG.

Proof 1: The OPF is defined as $\tilde{U}_1(s_i, s_{-i}) = \sum_{i\in\mathcal{V}} u_i(s_i, s_{-i})$ and let $\Delta u_i = u_i(s_i^b, s_{-i}) - u_i(s_i^a, s_{-i})$ which gives

$$\begin{aligned}\Delta\tilde{U} &= \tilde{U}_1(s_i^b, s_{-i}) - \tilde{U}_1(s_i^a, s_{-i})\\&= \sum_{i\in\mathcal{V}}(u_i(s_i^b, s_{-i}) - u_i(s_i^a, s_{-i}))\\&= \sum_{i\in\mathcal{V}}\{[\alpha_1 f_i(s_i^b, s_{-i}) + \alpha_2\text{trace}(X_{(\mathcal{G},s_i^b)})]\\&\quad - [\alpha_1 f_i(s_i^a, s_{-i}) + \alpha_2\text{trace}(X_{(\mathcal{G},s_i^a)})]\}\\&= \Delta u_i + \sum_{j\in\mathcal{V},j\neq i}([\alpha_1 f_j(s_i^b, s_{-i})\\&\quad + \alpha_2\text{trace}(R^T_{(\mathcal{G},q)}R_{(\mathcal{G},q)})] - [\alpha_1 f_j(s_i^a, s_{-i})\\&\quad + \alpha_2\text{trace}(R^T_{(\mathcal{G},q)}R_{(\mathcal{G},q)})])\\&= \Delta u_i + \sum_{j\in\mathcal{V},j\neq i}([\alpha_1 f_j(s_i^b, s_{-i}) + \alpha_2\text{trace}(r_j^T r_j)]\\&\quad - [\alpha_1 f_j(s_i^a, s_{-i}) + \alpha_2\text{trace}(r_j^T r_j)])\end{aligned} \tag{7}$$

where S_i^a, S_i^b are the strategy sets when the node i adopts s_i^a and s_i^b, respectively. Then the signs of $\Delta\tilde{U}$ and Δu_i are analyzed as follows

$$\Delta u_i \begin{cases} = 0 & \text{if}\, f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 0\\ < 0 & \text{if}\, f_i(s_i^a, s_{-i}) = 1,\ f_i(s_i^b, s_{-i}) = 0\\ > 0 & \text{if}\, f_i(s_i^a, s_{-i}) = 0,\ f_i(s_i^b, s_{-i}) = 1\\ = \Delta\tilde{U} & \text{if}\, f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 1,\ s_i^a > s_i^b\\ = \Delta\tilde{U} & \text{if}\, f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 1,\ s_i^a < s_i^b \end{cases} \tag{8}$$

and

$$\Delta\tilde{U} = \begin{cases} = 0 & \text{if} f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 0\\ < 0 & \text{if}\, f_i(s_i^a, s_{-i}) = 1,\ f_i(s_i^b, s_{-i}) = 0\\ > 0 & \text{if}\, f_i(s_i^a, s_{-i}) = 0,\ f_i(s_i^b, s_{-i}) = 1\\ = \Delta u_i & \text{if}\, f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 1. \end{cases} \tag{9}$$

It can be seen that the sign of $\Delta\tilde{U}$ is the same as the sign of Δu_i. Therefore, $\Gamma_1(\mathcal{V}, S, u)$ is an ordinal potential game. This completes the proof.

It has been proven in [36] that there exists at least one NE in pure strategies potential games. Thus, the minimally rigid graph can be constructed. The OPG based minimally rigid graph design (OPG-MRGD) algorithm is listed in Algorithm 1 (A1).

Remark 2: The sensors implement game in accordance with the ID number. During every round of the game process, only one sensor is allowed to change its strategy, other nodes' strategies remain unchange. In order to coverage to NE, we use a better response strategy update scheme. It has been proven that a finite ordinal potential game will converge to NE in finite steps via a Better Response strategy update scheme [36]. The game process is shown in Fig.2. Every sensor firstly initialize its strategy space, in this paper, it means the sensor recognizing its neighbour links. Then by

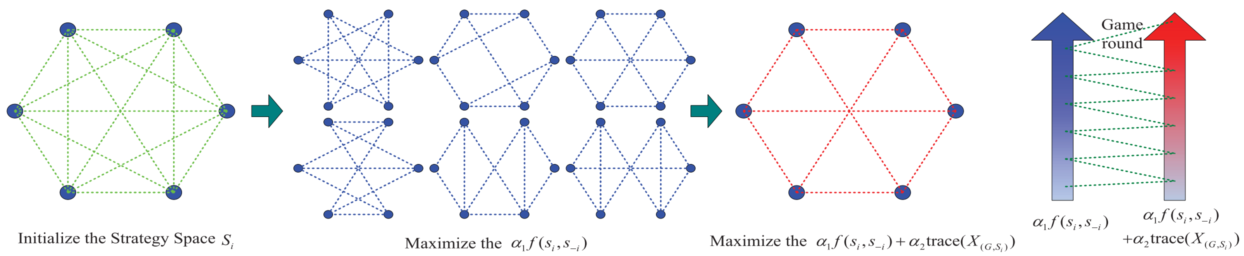

FIGURE 2. **Schematic diagram of the game process in Stage 1.**

Algorithm 1 OPG Based Minimally Rigid Topology Control Algorithm

1: Initialization: The node ID=$\{1, 2, \ldots, N\}$, the position of all sensors $p = \{p_1^1, p_1^2, \ldots, p_i^1, p_i^2, \ldots, p_N^1, p_N^2\}$, the optional strategies spaces S_i.
2: The game carries out according to the node ID, $\hat{s}_i = s_i^c$, $\forall i \in \mathcal{V}$ with s_1^c is optional strategies.
3: $\hat{S}_i = \{\hat{s_1}, \hat{s_2}, \ldots, \hat{s_N}\}$
4: For all $i \in \mathcal{V}$,
5: $\hat{s}_i = \arg\max_{s_i^b \in S_i} u_i(s_i^b, s_{-i})$
6: $|S_i| = |S_i| + 1$
7: End until $|S_i| = 2N - 3$, $\mathrm{rank}(R_{(\mathcal{G},q)}) = 2N - 3$, S_i does not change.
8: Output the minimally rigid graph with the strategy $s_i^* = \{s_1^*, s_2^*, \ldots, s_N^*\}$.

maximizing the utility function, the minimally rigid topology can be derived. The first term and the second term of the utility function increase alternately or synchronously as shown in Fig.2.

B. STAGE 2: THE GENERATION OF OPTIMALLY RIGID GRAPH

Due to the information flows through the network, the entire Gramian spectrum and trace related to Fisher Information affect the performance of control and communication task, i.e., formation shape control and localization estimation. It has been proven in [24] that the trace of the Gramian pseudo-inverse and the log product of non-zero eigenvalues can well quantify the rigidity of network. In this stage, in order to get great algebra rigidity properties, we will construct the utility function for the game model $\Gamma_2(\mathcal{V}, S, u)$ as follows

$$u_i(s_i, s_{-i}) = \alpha_1 f_i(s_i, s_{-i}) + \alpha_2 \log\Big(\prod_{i=1}^{rank(R_{(\mathcal{G},S_i)})} \lambda_i(X_{(\mathcal{G},S_i)})\Big) - \alpha_3 \mathrm{trace}(X^{\dagger}_{(\mathcal{G},S_i)}) \quad (10)$$

where α_1 is a large positive number used to guarantee the rank condition, α_2, α_3 are positive constants. $f_i(s_i, s_{-i}) = 1$, if $\mathrm{rank}(R_{(\mathcal{G},p)}) = 2N - 3$, otherwise $f_i(s_i, s_{-i}) = 0$. The *non-negative* parameters α_2, α_3 are weighted coefficients related to the importance of different objectives, therefore, this WSN topology control model can be applied for diverse objective by adjusting the weighted coefficients. $X^{\dagger}_{(\mathcal{G},S_i)}$ denotes the Moore Penrose pseudoinverse of $X_{(\mathcal{G},S_i)}$. $\lambda_i(X_{(\mathcal{G},S_i)})$ is the ith nonzreo eigenvalue of $X_{(\mathcal{G},S_i)}$ and $\lambda_1(X_{(\mathcal{G},S_i)}) < \lambda_2(X_{(\mathcal{G},S_i)}) < \cdots < \lambda_n(X_{(\mathcal{G},S_i)})$. The other parameters can be seen in Stage 1.

Next is the second result of this section.

Theorem 2: The game model $\Gamma_2(\mathcal{V}, S, u)$ for optimally rigid topology control is an OPG.

Proof 2: The OPF is defined as $\tilde{U}_2(s_i, s_{-i}) = \sum_{i \in \mathcal{V}} u_i(s_i, s_{-i})$ and let $\Delta u_i = u_i(s_i^b, s_{-i}) - u_i(s_i^a, s_{-i})$ which gives

$$\begin{aligned}
\Delta\tilde{U}_2 &= \tilde{U}_2(s_i^b, s_{-i}) - \tilde{U}_2(s_i^a, s_{-i}) \\
&= \sum_{i\in\mathcal{V}} (u_i(s_i^b, s_{-i}) - u_i(s_i^a, s_{-i})) \\
&= \Delta u_i + \sum_{j\in\mathcal{V}, j\neq i} \{[\alpha_1 f_j(s_i^b, s_{-i}) + \alpha_2 \log(\prod_{i=1}^{2N-3} \lambda_i \\
&\quad \times (X_{(\mathcal{G},s_i^b)})) - \alpha_3 \mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^b)})] - [\alpha_1 f_j(s_i^a, s_{-i}) \\
&\quad + \alpha_2 \log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^a)})) - \alpha_3 \mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^a)})]\}
\end{aligned} \quad (11)$$

Then the signs of $\Delta\tilde{U}_2$ and Δu_i are analyzed as follows

$$\Delta u_i \begin{cases} = 0 & \text{if } f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 0, \\ < 0 & \text{if } f_i(s_i^a, s_{-i}) = 1,\ f_i(s_i^b, s_{-i}) = 0, \\ > 0 & \text{if } f_i(s_i^a, s_{-i}) = 0,\ f_i(s_i^b, s_{-i}) = 1, \end{cases} \quad (12)$$

and

$$\Delta\tilde{U}_2 = \begin{cases} = 0 & \text{if } f_i(s_i^a s_{-i}) = f_i(s_i^b, s_{-i}) = 0, \\ < 0 & \text{if } f_i(s_i^a, s_{-i}) = 1,\ f_i(s_i^b, s_{-i}) = 0, \\ > 0 & \text{if } f_i(s_i^a, s_{-i}) = 0,\ f_i(s_i^b, s_{-i}) = 1, \end{cases} \quad (13)$$

Next, we analyze the signs of Δu_i for the case $f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 1$, $s_i^b > s_i^a$ or $s_i^b < s_i^a$, which means adding or removing an edge. Then we get

$$\Delta u_i = \alpha_2[\log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^b)})) - \log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^a)}))] + \alpha_3[\mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^a)}) - \mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^b)})]. \quad (14)$$

For the case $s_i^b > s_i^a$ which means an edge e is added to the topology by using the strategy s_i^b. $\mathcal{E}_a$ and $\mathcal{E}_b$ denote the edge sets after completing the strategy s_i^a and s_i^b, respectively. $\mathcal{E}_a \subseteq \mathcal{E}_b \subseteq \mathcal{E}\backslash\{e\}$, then according to the additivity property of the Gramian, one can see that $\mathcal{E}_a \subseteq \mathcal{E}_b \Rightarrow X_{(\mathcal{G},s_i^a)} \leq X_{(\mathcal{G},s_i^b)}$. Define

$$\begin{aligned}\mathcal{H}_{\mathcal{E}} &= \log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^b)})) - \log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^a)}))\\ &= \log\det(\tilde{X}_{\mathcal{E}_a\cup\{e\}}) - \log\det(\tilde{X}_{\mathcal{E}_a}) \qquad (15)\\ \mathcal{F}_{\mathcal{E}} &= \mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^a)}) - \mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^b)})\\ &= \mathrm{trace}(X^{\dagger}_{\mathcal{E}_a}) - \mathrm{trace}(X^{\dagger}_{\mathcal{E}_a\cup\{e\}}), \qquad (16)\end{aligned}$$

and $X_{\mathcal{E}}(t) = X_{(\mathcal{G},s_i^a)} + t(X_{(\mathcal{G},s_i^b)} - X_{(\mathcal{G},s_i^a)})$ for $t \in [0, 1]$. It has been proven by [24, Th. 5]that $\mathcal{H}_{\mathcal{E}}$ and $\mathcal{F}_{\mathcal{E}}$ are monotone increasing. This means that if $f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 1$ and $s_i^b > s_i^a$, then $\Delta u_i > 0$. The similar conclusion can be gotten for the case $s_i^b < s_i^a$, namely if $f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 1$ and $s_i^b < s_i^a$, then $\Delta u_i < 0$. Then for $\Delta\tilde{U}_2$, it yields

$$\begin{aligned}\Delta\tilde{U}_2 &= \Delta u_i + \sum_{j\in\mathcal{V},j\neq i} \{[\alpha_1\\ &\quad + \alpha_2 \log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^b)})) - \alpha_3 \mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^b)})]\\ &\quad - [\alpha_1 + \alpha_2 \log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^a)}))\\ &\quad - \alpha_3 \mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^a)})]\}\\ &= \Delta u_i + \sum_{j\in\mathcal{V},j\neq i} \{\alpha_2[\log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^b)}))\\ &\quad - \log(\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},s_i^a)}))]\\ &\quad + \alpha_3[\mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^a)}) - \mathrm{trace}(X^{\dagger}_{(\mathcal{G},s_i^b)})]\}. \qquad (17)\end{aligned}$$

According to the analysis of Δu_i and (17), we can see that the signs of $\Delta\tilde{U}_2(s_i, s_{-i})$ and Δu_i are the same for the case $f_i(s_i^a, s_{-i}) = f_i(s_i^b, s_{-i}) = 1$. In conclusion, the signs of $\Delta\tilde{U}_2(s_i, s_{-i})$ and Δu_i are always the same for any cases. Therefore, $\Gamma_2(\mathcal{V}, S, u)$ is an ordinal potential game. This completes the proof.

Theorem 2 shows that the optimally rigid topology control problem can be solved by a potential game approach. Then the ordinal potential game based optimally rigid graph design (OPG-ORGD) algorithm is listed in Algorithm 2 (A2). A2 begins with the strategy derived by A1. Then according to the new OPF, the optimally rigid topology is derived finally.

Remark 3: Note that the utility functions in the two stages of the game do not need to be the same, the both approaches can achieve the objective. In the Stage 1, we construct a minimally rigid graph according to the trace$(X_{(\mathcal{G},S_i)})$, it can also be done by using $\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},S_i)})$. It is just considered from different key points. The process of Stage 2 can be seen in Fig.3. Moreover, since the utility function is chosen as similar to the objective function in [24], the prove process can be seen in this paper.

Algorithm 2 OPG Based Optimally Rigid Topology Control Algorithm

1: Input: The node ID=$\{1, 2, \ldots, N\}$, the position of all sensors $\mathbf{p} = \{p_1^1, p_1^2, \ldots, p_i^1, p_i^2, \ldots, p_N^1, p_N^2\}$, the NE of Stage 1 $s_i^* = \{s_1^*, s_2^*, \ldots, s_N^*\}$. The optional strategies spaces S_i.
2: The game implements according to the node ID, $\hat{s}_i = s_i^c$, $\forall i \in \mathcal{V}$ with s_1^c is optional strategies.
3: $\hat{S}_i = \{\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_N\}$
4: For all $i \in \mathcal{V}$,
5: $\hat{s}_i = \arg\max_{s_i^b\in S_i} u_i(s_i^b, s_{-i})$
6: End until $|S_i| = \kappa$, $\mathrm{rank}(R_{(\mathcal{G},q)}) = 2N-3$, $\hat{S}_i$ does not change.
7: Output the optimally rigid graph with the strategy $s_i^* = \{s_1^*, s_2^*, \ldots, s_N^*\}$.

IV. ALGORITHM ANALYSIS

In this section, the performances of the proposed algorithm are analyzed from the following two perspectives.

A. CONVERGENCE

The two algorithms are theoretically proven to be convergent.

Theorem 3: OPG-based minimally rigid graph design (OPG-MRGD) algorithm converges to NE.

Proof 3: For all $i \in \mathcal{V}$, the strategy is denoted as $s(s_i, s_{-i})$ to achieve the minimally rigid graph, which is a monotonic and nonincreasing function of s_{-i}, that means if $s_{-i}^b \geq s_{-i}^a$, then $s(s_i, s_{-i}^b) \leq s(s_i, s_{-i}^a)$. Accordingly, the strategy space $S(s_{-i}) = \{s(s_i, s_{-i}), \ldots, s(s_i, s_{-i}^{\chi})\}$ with s_{-i}^{χ} denoting the maximal strategy vector. To adjust the strategy of i, $\hat{s}_i = \arg\max_{s_i^b\in S_i} u_i(s_i^b, s_{-i})$ can be rewritten as

$$s_i^* = \{\max u_i(s_i^*, s_{-i}), \quad s(s_i^b, s_{-i}) \geq s(s_i, s_{-i})\}. \qquad (18)$$

After strategy updating, utility function of the ith sensor will be maximized with the strategy vector $s(s_i^*, s_{-i})$; if j updates strategy to s_j^* as well, there must be $s_j^* \leq s_j^{\chi}$, $s(s_j^*, s_{-i-j}) \leq s(s_i^*, s_{-i})$ and $s_i^* \in S(s_j^*, s_{-i-j})$. In this case, the new strategy vector (s_i^*, s_j^*, s_{-i-j}) continues to maximize the utility of ith sensor. Therefore, the result of OPGMRGD algorithm execution $s = (s_1^*, s_2^*, \ldots, s_N^*)$ is determinately NE.

Theorem 4: OPG-based optimally rigid topology control (OPG-ORGD) algorithm converges to NE.

Proof 4: The proof is similar to the proof of Theorem 3 and is omitted here.

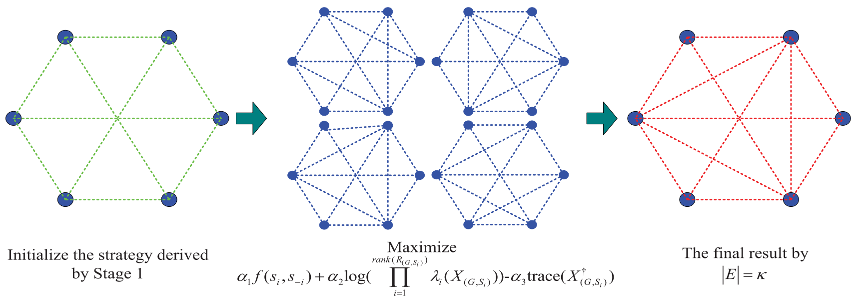

FIGURE 3. Schematic diagram of the game process in Stage 2 by $n = 6$, $\kappa = 12$.

B. COMPLEXITY

The algorithm complexity can represent the computing resource cost during the algorithm realization. The message complexity and time complexity are two important aspects to quantify the algorithm complexity [39]. The quantity of messages in OPG-MRGD or OPG-ORGD depends mostly on the calculation of trace($X_{(\mathcal{G},S_i)}$). According to Theorem 1 and Theorem 2, the strategy choice of every individual node may influence the global rigidity function of trace($X_{(\mathcal{G},S_i)}$) $\times N$.

Then, the total quantity of the communication messages is $\sum_{i=1}^{N}$(trace($X_{(\mathcal{G},S_i)}$)+rank($X_{(\mathcal{G},S_i)}$))$\cdot N$; therefore, the message complexity of OPG-MRGD and OPG-ORGD is $O(N^3)$. By executing N rounds of OPG-MRGD or OPG-ORGD, the time complexity of OPG-MRGD or OPG-ORGD is considered as $O(N)$. The polynomial complexity OPG-MRGD or OPG-ORGD means their realization has a relatively low implementation cost.

V. PERFORMANCE EVALUATION

In this section, some simulation results are provided to analyze the algorithm performance of A2 (A1 is a substage of A2). Since GAF in [30], XTC in [40], LMST in [13] come close to our work in this paper, we compare their performances with A2 proposed in this paper. As stated in [40], the topology derived by the maximal transmission power (MTP) is always used as a baseline in topology control problem of WSNs. In this paper, we consider that the WSNs nodes have limited communication ability and have the same unit circle communication range, which means the nodes can communicate with each other if and only if their Euclidean distance is less than their communication radius. The parameters are shown in Table 2. Assuming that there exist a rigid network in the neighboring topology of WSNs, this is easy to realize for the terrestrial WSNs. In order to reflect the superior performance of the proposed algorithm, we will analyse and compare the algorithm performance from the following aspects.

A. ALGORITHM PERFORMANCE

We firstly analyse the validity of the proposed algorithms by randomly deploying 100 nodes in the region. Since the sensor nodes can only exchange information with their neighbours,

TABLE 2. Parameters.

Symbol	Description	value
N	Number of sensor nodes	$100-1000$
R_e	Deployment region	300m$\times$300m
E_0	Initial energy of the sensor	8000J
d	The communication distance	100m
ε_{ele}	Radio dissipate	400nJ/byte
ε_{amp}	Transmitter amplifier	800pJ/byte/m^2
α_1	weight coefficient	100
α_2	weight coefficient	5
α_3	weight coefficient	1

the WSNs firstly connect with each other according to the neighbour rules. This is important to initial the strategy space for the nodes. Fig.4 shows the topology derived by MTP, GAF, A2 in this paper, XTC and A-LMST. The optimally rigid network is designed by A2 in Fig.4(c). By comparison, the topology structure generated by GAF algorithm ensures at least 4-connected network which has an excessive vertex connectivity. As shown in Fig.4(c-e), A2, XTC and A-LMST reduced the topology complexity significantly while the network is still connected. But some nodes contain only one neighbor in its communication radius in Fig.4(d), also in Fig.4(e), the A-LMST topology contains only one path. Once the only one connected path is destroyed, then the whole topology is destroyed. Therefore, the XTC and A-LMST topologies have a poor robustness. We also evaluate the change of the trace($X_{(\mathcal{G},S_i^b)}$) and log($\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},S_i^b)})$) in Fig.5. The $\kappa = 2N$. The log($\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},S_i^b)})$) is left out of consideration in the first stage. Then, the trace($X_{(\mathcal{G},S_i^b)}$) and log($\prod_{i=1}^{2N-3} \lambda_i(X_{(\mathcal{G},S_i^b)})$) are gradually increasing until $\kappa = 2N$. Obviously, the rigidity properties are getting better during the game process and they are consistent. In this paper, we do not compare the rigid topology derived by A2 and the one in [22], because we focus on different emphasis in these two papers.

It is widely recognized that network topology properties play a key role in consensus network. In order to further verify the superior algebraic properties of the topology derived by

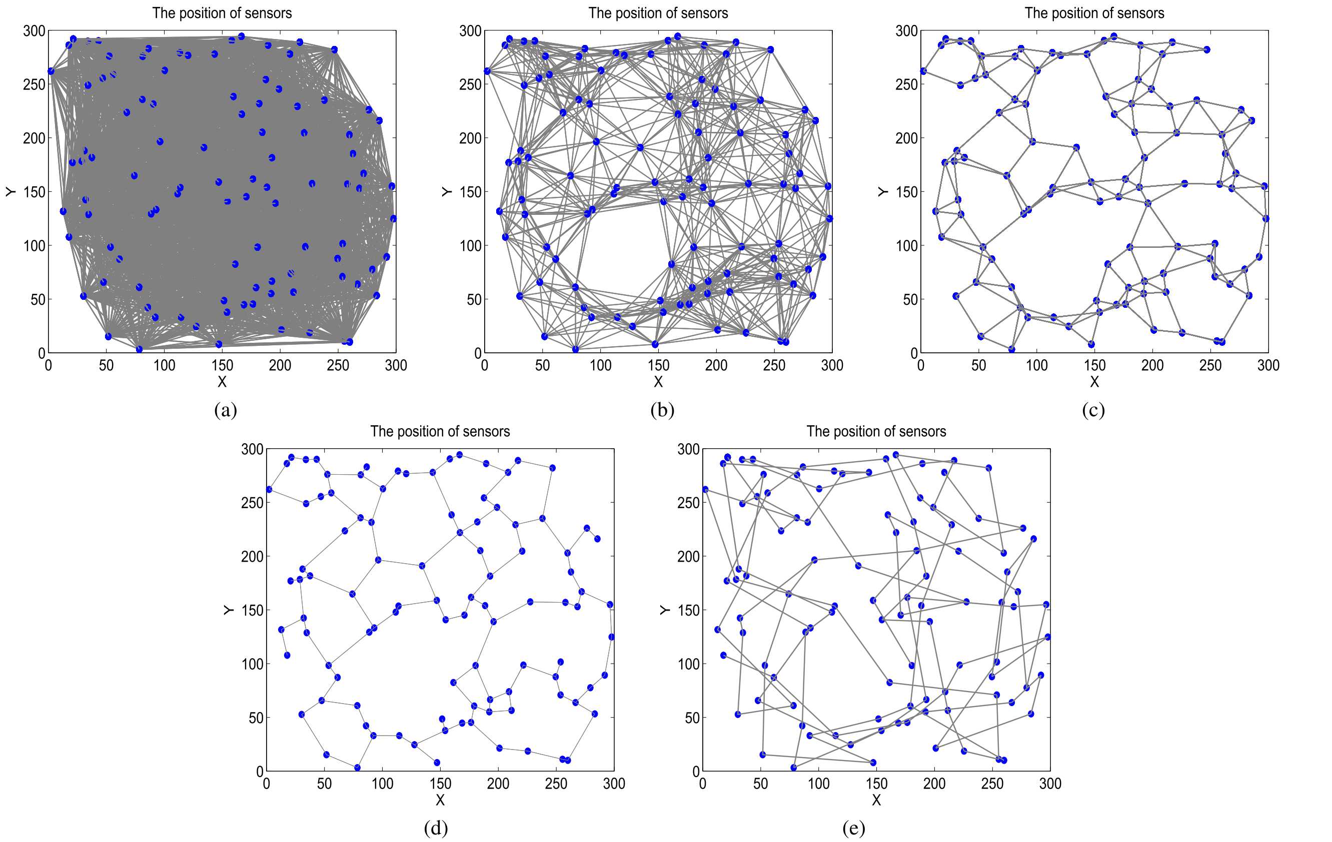

FIGURE 4. Topologies derived by Unit Disk, GAF, A2, XTC and LMST and the algebra properties of A2 topology. (a) MTP topology. (b) GAF topology. (c) Optimal rigid topology by A2. (d) XTC topology. (e) A-LMST topology.

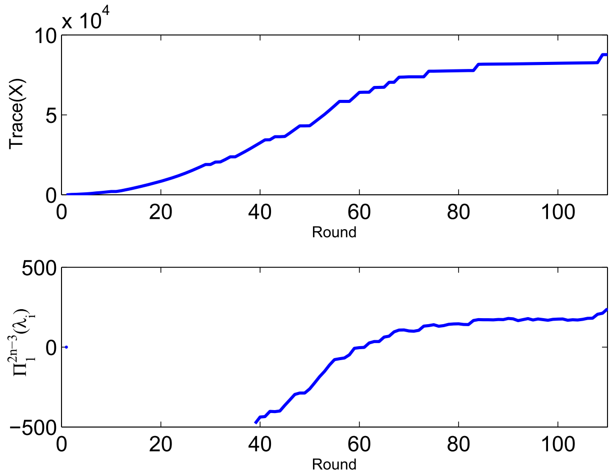

FIGURE 5. Algebra properties of the optimal rigid topology.

A2 in this paper, we consider the following consensus network dynamics

$$\dot{x}_i = \sum_{j \in N_i} (x_j - x_i)$$

where x_i is the state of the ith network node, N_i is a set of neighbours of node i. Consensus is an important dynamical process in a variety of networks. Fig.6 shows the average consensus in different network topology derived by GAF, A2, XTC and A-LMST. For convenience, we only apply 10 nodes to derived the topology and GAF topology is similar to MTP topology with 10 nodes. It can be seen that the consensus performance of A2 topology is better only behind GAF. But A2 topology has less edges, less energy consumption than GAF. The consensus convergence speed in A2 topology is much faster than XTC topology and A-LMST topology.

B. AVERAGE NODE DEGREE

Average node degree (AND) is another important criterion of network topology control. In the this subsection, we vary the number of nodes in the region from 100 to 1000 to calculate the AND of the four algorithms. AND can not only reflect the topology complexity, but also the great significance of energy balance. The energy balance in WSNs will seriously affect the sensor lifetime even the whole network lifetime. In [41], Tel have theoretically proved that the optimal average degree of WSNs is approximately 6. We can see in Fig.7 that the optimally rigid graph has an approximate 4 AND, which means it has a better network performance than other three algorithms. The average radius and the average link length for the topologies derived using the four algorithms is shown, respectively, in Fig.8(a) and (b). The average radius

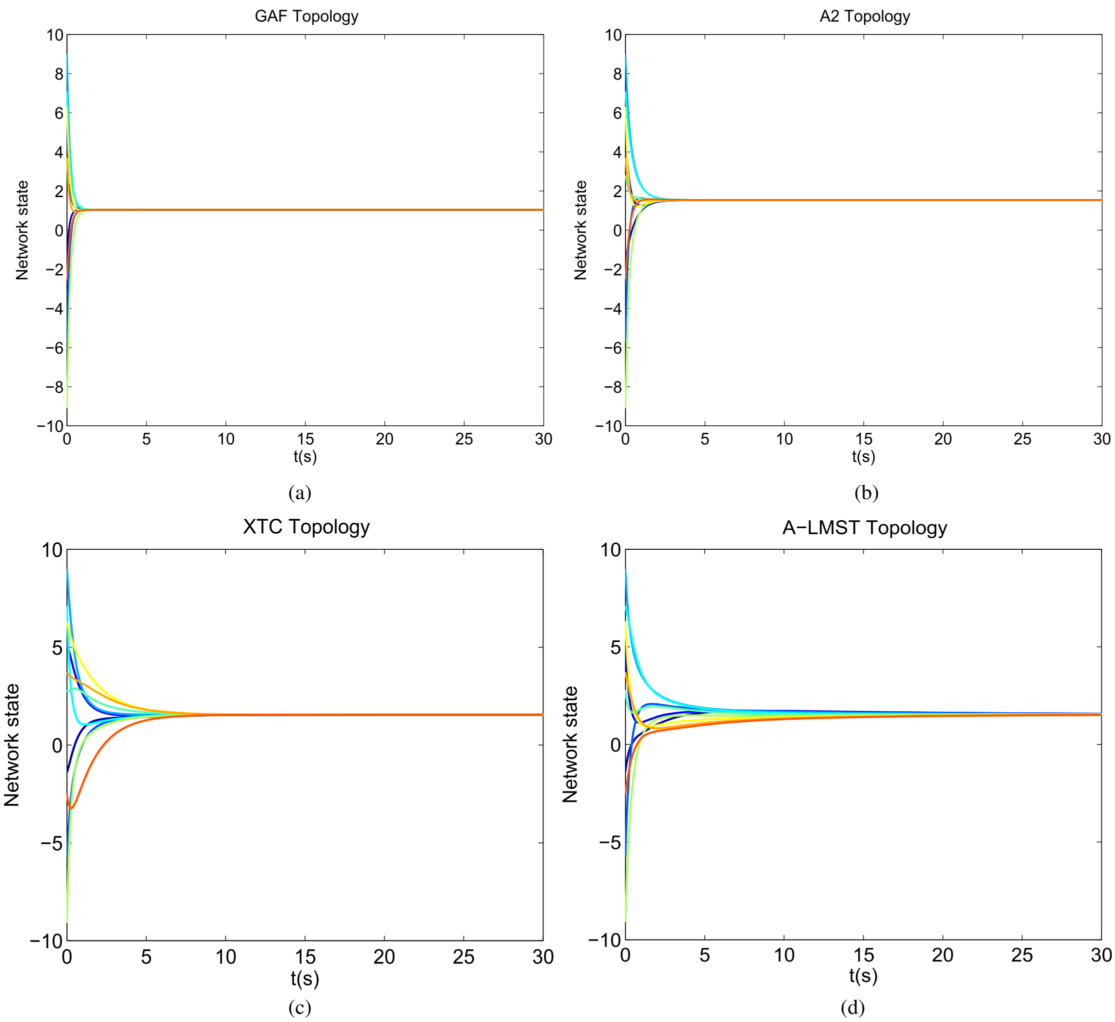

FIGURE 6. **Consensus performance of the GAF, A2, XTC, A-LMST topology. (a) Consensus in GAF topology. (b) Consensus in A2 topology. (c) Consensus in XTC topology. (d) Consensus in A-LMST topology.**

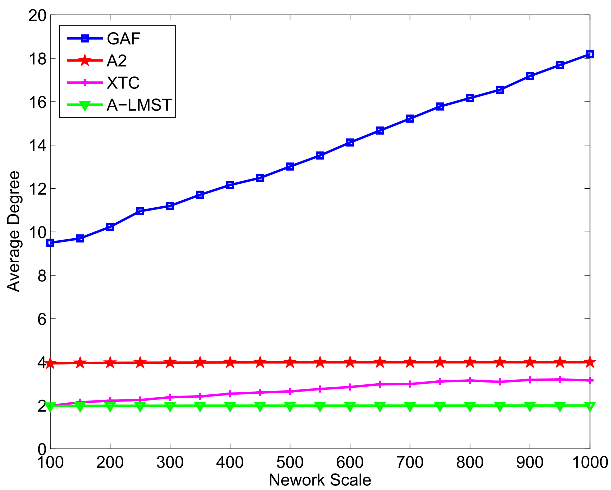

FIGURE 7. **Comparison of the average degree.**

of A2 topology outperforms the GAF and A-LMST topology, the reason is that A2 topology has a lower complexity. The average link length in Fig.8(b) also means the A2 topology is a better tradeoff between the algebraic properties and energy consumption.

C. AVERAGE ENERGY CONSUMPTION

In order to further analysis the energy efficiency of the topology under GAF, A2, XTC and A-LMST, the following definition of the network lifetime is given.

Definition 7: The network lifetime represents the time when one of the nodes run out of energy.

The following model used in [42] is usually used to model the energy consumption of the WSN.

$$E_{Tx}(l, d_{ij}) = \varepsilon_{elec}(l) + \varepsilon_{amp}(l, d_{ij}) \\ = \varepsilon_{elec} l + \varepsilon_{amp} l d_{ij}^2 \tag{19}$$

$$E_{Rx}(l) = \varepsilon_{elec} l \tag{20}$$

where $E_{Tx}(l, d_{ij})$ means the energy consumed in transmitting l units of data from i to j and $E_{Rx}(l)$ is the energy consumed by sensor i in receiving l units of data, with d_{ij} is the transmit distance. It can be seen that the energy consumption is proportional to the communication distance. Fig.9 shows

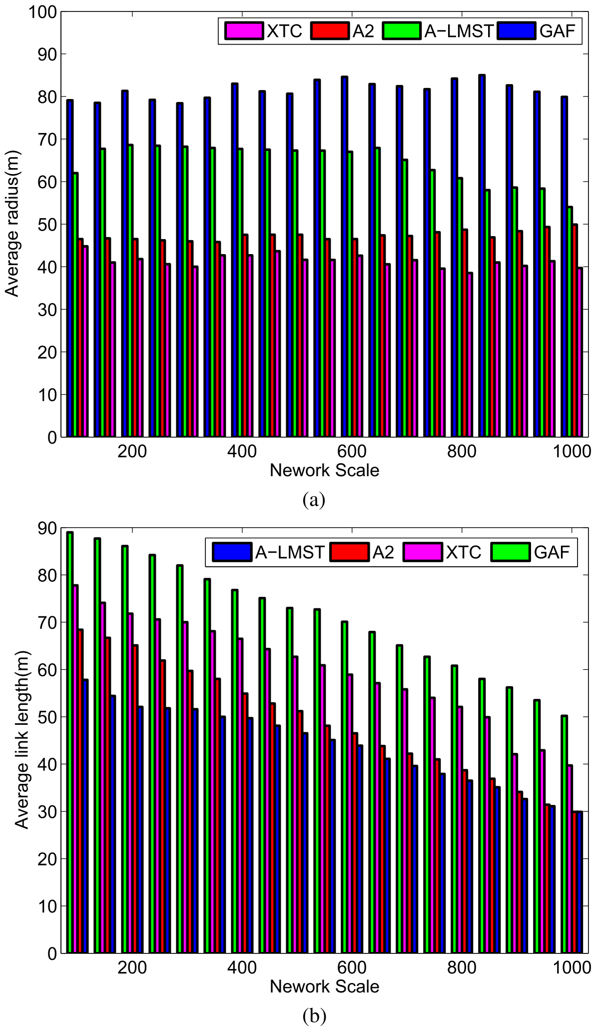

FIGURE 8. **Average radius and average link length comparisons among different algorithms. (a) Average radius. (b) Average link length.**

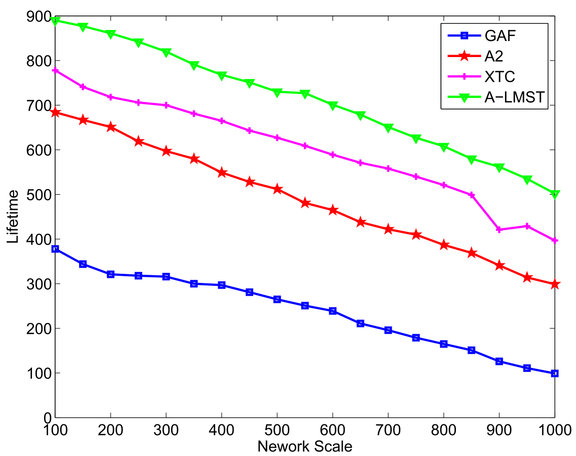

FIGURE 9. **Comparisons of network lifetime.**

the comparisons of energy efficiency among GAF, A2, XTC and A-LMST. We can see that the A-LIST topology has the longest lifetime. This is in accordance with the average radius and link length. On the other hand, the optimal rigid topology is a better tradeoff network structure.

VI. CONCLUSIONS AND FUTURE WORK

This paper addresses the optimally rigid network design problem of WSNs via potential game approach. We consider this problem as a constrained optimization problem and can be solved by two stages. By choosing the algebra rigid properties as the OPF, both stages are proved to be OPG. In this paper, we innovatively consider the topology properties in WSNs. Some examples are given to illustrate the effectiveness of the proposed algorithms. However, there are many open problems in topology control of WSNs. Next, we will study a fully distributed way to generated the optimal rigid network in WSNs. In addition, other indexes used to appraise the algebra rigid properties will be investigated.

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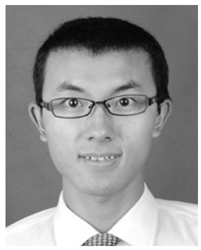

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