

Received January 25, 2018, accepted February 19, 2018, date of publication March 5, 2018, date of current version May 16, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2810891

# An Intelligent Adaptive Algorithm for Environment Parameter Estimation in Smart Cities

MOU WU<sup>1</sup>, NEAL N. XIONG<sup>2</sup>, (Senior Member, IEEE), AND LIANSHENG TAN<sup>3</sup>

<sup>1</sup>School of Computer Science and Technology, Hubei University of Science and Technology, Xianning 437100, China

<sup>2</sup>Department of Mathematics and Computer Science, Northeastern State University, Tahlequah, OK 73096, USA

<sup>3</sup>Department of Computer Science, Central China Normal University, Wuhan 430079, China

Corresponding author: Neal N. Xiong (xiong31@nsuok.edu).

This work was supported in part by the National Natural Science Foundation of China under Grant 61672258 and in part by the Scientific Research Project of Education Department of Hubei Province under Grant B2017181.

**ABSTRACT** Least mean squares (LMS) adaptive algorithms are attractive for distributed environment parameter estimation problems in a smart city due to the benefits of cooperation, adaptation, and rapid convergence. To obtain a reliable estimate of the network-wide parameter vector, local results can be further fused by intermediate agents in a distributed incremental way. In this paper, we propose an intelligent variable step size incremental LMS (VSS-ILMS) algorithm to solve the dilemma between fast convergence rate and low mean-square deviation (MSD) in conventional incremental LMS (ILMS) algorithms. The main idea behind our proposal is that the local step-size is adaptively updated by minimizing the MSD in every iteration, where Tikhonov regularization and time-averaging estimation methods are adopted. A theoretical analysis of proposed algorithm is presented in terms of mean square performance and mean step size in a closed form. Simulation results show that VSS-ILMS algorithm outperforms the constant step size ILMS algorithm and several classical variable step-size LMS algorithms. The derived theoretical results shows good agreement with those based on simulated data. For a practical consideration, the proposed algorithm is also verified by the model of target localization in sensor networks.

**INDEX TERMS** Smart city, distributed estimation, LMS adaptive algorithm, variable step-size.

## I. INTRODUCTION

A typical application in smart cities is to sense and collect the global environmental parameters such as local temperature, humidity or PM<sub>2.5</sub> over the observed area [1], [2]. Due to the impact of the geographical position and ambient noise, however, it is inherently difficult for different agents in the network to reach consensus on a unknown estimated parameter vector [3]. It has become a problem that needs to be urgently solved for improving the performance of the network. One of the main challenges is that each agent access only local data instead of the network-wide information due to the restriction on the range of communication. In the traditional centralized algorithms, the fusion agent performs a parameter estimate task by collecting global observations from other agents and broadcasts the result back to them. The centralized fusion has the advantages of simple implementation and high-performance because the global knowledge is available, but it leads to more communication cost and suffers from the failure

of fusion center due to the excessive concentration of data processing.

As a better solution to achieving the data consistency, distributed processing receives much attention due to the benefits of cooperation and interaction between neighboring agents. In future, it may be a primary mode of data acquisition, control, and information processing. By distributing a specific stochastic gradient method into different mode of cooperation, two distributed estimation algorithms referred to as incremental algorithm [4]–[7] and diffusion algorithm [8], [9], have recently been proposed. In the diffusion mode, each agent of the network share information with its neighboring agents to estimate the unknown parameter by implementing two phases: an adaptation stage in which the estimate is updated by using a LMS-type replacement for the second-order moments and a combination stage in which the information from the neighbors is aggregated. According the implementation order of the two stages, new versions

of diffusion algorithm called as Adapt-then-Combine (ATC) diffusion LMS algorithm and Combine-then-Adapt (CTA) diffusion LMS algorithm are proposed in [8]. In the incremental mode, the network is organized into a Hamiltonian cycle, where each agent is visited only once in any iteration such that local information from agent is only sent to one of its immediate neighbor agents. Under such a cooperation mode, at any time step the estimate of current agent can be achieved based on the local measurements and the results from previous agent on cycle path, such that the real time information of the entire network can be passed to every agent and used to obtain an accurate estimate. Several variants of ILMS algorithm are proposed, for example, incremental RLS (recursive least-squares) [10], incremental APA (affine projection algorithm) [5] and incremental parallel projection techniques [11].

In this work, we focus on the incremental strategy based on the following two considerations. First, it is well-known that ILMS algorithm can achieve the performance of the centralized-like solution, which cannot be achieved by diffusion-based algorithms. Although the diffusion strategies by adopting the optimized combination rules can outperform incremental strategies [12], they rely heavily on prior knowledge of the noise statistics at the different agents. Second, the incremental cooperation requires less communication cost than centralized strategies and diffusion strategies as well.

The single stand-alone LMS adaptive filters have been widely studied by the researchers in the field of signal processing. It is generally known that the step size plays a vital role to improve the performance of standard LMS algorithms. To solve the conflict of fast early convergence and low steady state deviation, numerous variable step-size algorithms have been proposed in [13]–[22]. Although they provide good performances under various scenarios (e.g., traditional adaptive filtering and acoustic echo cancellation), the main problem in directly applying them on distributed estimation is that spatial diversity is not being considered because of stand-alone filters. In other words, the conventional LMS algorithms have not been adequately allowed for distributed estimation in the context of multi-agent networks. In this case, our variable step-size ILMS algorithm is designed for incremental adaptive networks, in which unknown parameter vector is estimated in a distributed and cooperative way with improved robustness against the variation of statistics information on different agent.

The researches for ILMS algorithm are carried out from different perspectives to improve its performances. In [23], the estimated parameters are classified into three categories: local interest, global interest to the whole network and common interest to a subset of agents. Thus, standard LMS algorithm is implemented individually by three kinds of network agents based on above classification. Since the obvious difference of observation quality between agents will result in the performance degradation of ILMS algorithm, the step-sizes are allotted on the base of the quality of measurement

information by solving a constrained optimization problem [24]. Thus, small step-sizes are allocated for agents presenting high noise level and vice versa. However, a prerequisite for the optimum step-size assignment is that observation noise variance for each agent is available. Performance analysis of ILMS algorithm considering noisy links and finite precision arithmetic are presented in [25] and [26], respectively.

In this paper, we propose a new variable step size ILMS algorithm which overcome the tradeoff between fast convergence rate and low steady state error for constant step size by tracking the network profile resulting from the statistical variation of measurements and noise levels. An optimal step size expression for ILMS algorithm is derived by minimizing the MSD in every iteration. For practical implementation, we estimate the unknown quantities in derived expression by using time-averaging method. Moreover, we analyze the mean square performance of the proposed algorithm and confirm the theoretical results by simulations.

The rest of this paper is organized as follows. In Section II, we introduce distributed estimation problem for incremental network and LMS solution. In Section III, we derive the optimal variable step size for ILMS and propose the VSS-ILMS algorithm in detail for practical usage. In Section IV, we provide the theoretical analysis of the mean square performance and steady state step size. Simulation results are presented in Section V. We conclude this paper and points out future work in Section VI.

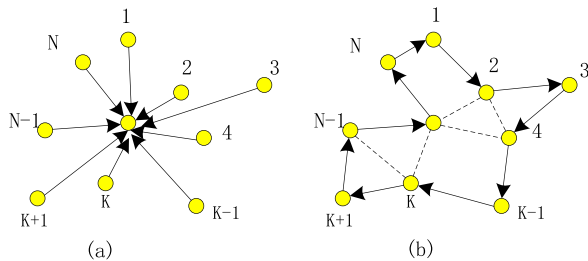
*Notations:* Let us follow the idiomatic symbol adopted in [4], [6]. That is, boldface letters and normal font are used to refer to random quantities and nonrandom quantities, respectively. Matrices and vectors are denoted by Capital letters and small letters, respectively.  $E(\cdot)$  represents mathematical expectation. The complex-conjugate transposition for matrices are denoted with the notation  $(\cdot)^*$ . The Euclidean norm of a vector and the trace of a matrix are denoted by  $\|\cdot\|$  and  $Tr(\cdot)$ , respectively.

## II. DISTRIBUTED PARAMETER ESTIMATION AND INCREMENTAL LMS ALGORITHM

Consider a distributed network with  $N$  agents deployed in a sensing area of smart city via a predefined topology. At each time instant  $i$ , each agent  $k$  obtains a time realization  $\{d_k(i), u_{k,i}\}$  of zero-mean spatial data  $\{\mathbf{d}_k, \mathbf{u}_k\}$ , where  $\mathbf{d}_k$  is a noisy measurement and  $\mathbf{u}_k$  is a  $1 \times M$  row regressor vector. The time realization  $\{d_k(i), u_{k,i}\}$  follow the customary model given by:

$$d_k(i) = u_{k,i}w^o + v_k(i), \quad (1)$$

where  $v_k(i)$  represents background noise which is zero mean with variance  $\sigma_{v,k}^2$  and independent of regression data spatially and temporally, and  $w^o$  is the estimated vector parameter with size  $M \times 1$ . Then, a global cost function is described



**FIGURE 1.** An example for centralized and distributed incremental cooperative networks.

as follow:

$$J^{\text{glob}}(w) = \sum_{k=1}^N J_k(w), \quad (2)$$

where  $J_k(w) = E|\mathbf{d}_k - \mathbf{u}_k w|^2$  denotes the mean-square error for individual agent  $k$ .

Thus, the original problem is converted to the following optimization problem

$$\arg \min_w \sum_{k=1}^N E|\mathbf{d}_k - \mathbf{u}_k w|^2. \quad (3)$$

The optimal solution  $w^o$  of (3) is given in [27] by

$$w^o = \left( \sum_{k=1}^N R_{u,k} \right)^{-1} \left( \sum_{k=1}^N R_{du,k} \right), \quad (4)$$

where  $R_{u,k} = E\mathbf{u}_k^* \mathbf{u}_k$  and  $R_{du,k} = E\mathbf{d}_k \mathbf{u}_k^*$ . It is known that the local data  $\{d_k(i), u_{k,i}\}$  from any agent in the network can be used to produce a local instantaneous approximation of  $w^o$  since the exact second-order moments  $\{R_{u,k}, R_{du,k}\}$  are unavailable.

In the LMS adaptive algorithms based on the traditional iterative steepest-descent method, the update equation for determining the solution  $w^o$  is given by [6], [7]

$$w_i = w_{i-1} + \mu \sum_{k=1}^N u_{k,i}^* (d_k(i) - u_{k,i} w_{i-1}), \quad (5)$$

where  $w_i$  is an estimate of  $w^o$  at the time  $i$ , a positive step-size parameter  $\mu$  is used to guarantee the convergence of LMS algorithm. The centralized or distributed scheme can be considered in the implementation of Equation (5).

In a centralized scheme, a fusion center collects the data  $\{d_k(i), u_{k,i}\}$  of all agents at time  $i$  to run iteration (5) as is implied by the summation notation in (5). The updated estimate  $w_i$  is obtained by the fusion center and sent back to every agent by using broadcast method. In a distributed manner, on the other hand, the summation notation is implemented in a cooperation manner by passing the data between agents. Especially in incremental mode, the estimate of current agent is passed to its only immediate neighboring agent when the network is organized into a Hamiltonian cycle, where each agent is visited exactly once per iteration as shown in Figure 1. This implementation leads to the known

distributed ILMS algorithm. Then, a set of coupled  $N$  equalities implemented on  $N$  agents can be obtained as

$$\begin{aligned} \psi_1^{(i)} &= \psi_N^{(i-1)} + \mu_1 u_{1,i}^* (d_1(i) - u_{1,i} \psi_N^{(i-1)}), \\ &\vdots \\ \psi_k^{(i)} &= \psi_{k-1}^{(i)} + \mu_k u_{k,i}^* (d_k(i) - u_{k,i} \psi_{k-1}^{(i)}), \\ &\vdots \\ \psi_N^{(i)} &= \psi_{N-1}^{(i)} + \mu_N u_{N,i}^* (d_N(i) - u_{N,i} \psi_{N-1}^{(i)}), \end{aligned} \quad (6)$$

where  $\psi_k^{(i)}$  is a local estimate of  $w^o$  for agent  $k$  at time  $i$ ,  $\mu_k$  is the step size of agent  $k$ . After all agents on the cycle are visited once, the local estimate  $\psi_N^{(i)}$  of last agent  $N$  is regarded as the global estimate  $w_i$  at iteration  $i$  and the input of first agent for the next iteration  $i + 1$ .

Although the unknown vector  $w^o$  can be estimated by using above two schemes, it is a well-known fact that ILMS adopts the cooperative scheme since each individual agent shares the results with its predefined neighbor. In the centralized algorithm, only temporal dimension within an individual agent indicated by variable  $i$  is used to obtain the local estimate. Instead, ILMS exploits the spatio-temporal diversity as is indicated by the variables  $i$  and  $k$  in (6).

Moreover, taking a simple one-hop network with  $N$  agents as an example, a total of  $NM$  communicated scalars is required for ILMS per iteration while the total number of communications for traditional fusion-based implementations is  $2NM + N$ . Thus, ILMS is noticeably more advantageous since it requires less communication cost. It is useful for reducing energy consumption to prolong the network lifetime, particularly those with limited energy supply.

### III. PROPOSED VSS-ILMS ALGORITHM

#### A. OPTIMAL VARIABLE STEP SIZE FOR ILMS

It is expected that LMS-type adaptive algorithms can obtain rapid early convergence and low mean squares error at steady state. In this senses, the step size has a significant impact on both sides. In this section we tried to derive optimum step-size  $\mu_k^o(i)$  for every agent  $k$  per iteration  $i$  based on incremental adaptive network, such that the expecting effect can be attained.

A good indicator for measuring parameter estimation performance is the mean-square deviation (MSD) which for each agent  $k$  is defined as follows

$$MSD_k \triangleq E\|\tilde{\psi}_k^{(i)}\|^2, \quad (7)$$

where

$$\tilde{\psi}_k^{(i)} \triangleq w^o - \psi_k^{(i)} \quad (8)$$

is the weight error vector at time  $i$  and used to measure the deviation between the estimate of agent  $k$  at time  $i$  and the optimal solution  $w^o$ .

In order to obtain the minimum MSD for agent  $k$ , the optimal step size  $\mu_k^o(i)$  can be regarded as a solution to the

following minimization problem in a form of Tikhonov regularization

$$\min_{\mu_k(i)} \{E \|\tilde{\psi}_k^{(i)}\|^2 + \delta \mu_k^2(i)\}, \quad (9)$$

where  $\delta$  is a nonnegative regularization parameter. It is known that above minimization problem will be much better conditioned than the original least MSD problem when  $\delta$  has a good choice [28].

Let the output error  $e_k(i)$  for agent  $k$  at time  $i$  be denoted by

$$e_k(i) = d_k(i) - u_{k,i} \psi_{k-1}^{(i)}. \quad (10)$$

Substituting (10) into (6), we get

$$\psi_k^{(i)} = \psi_{k-1}^{(i)} + \mu_k(i) u_{k,i}^* e_k(i), \quad (11)$$

where  $\mu_k(i)$  is the variable step size for agent  $k$  at time  $i$ . Substituting (11) into (8), we get

$$\tilde{\psi}_k^{(i)} = \tilde{\psi}_{k-1}^{(i)} - \mu_k(i) u_{k,i}^* e_k(i). \quad (12)$$

Squaring both sides of (12) and taking expectations, we get

$$E \|\tilde{\psi}_k^{(i)}\|^2 = E \|\tilde{\psi}_{k-1}^{(i)}\|^2 - 2\mu_k(i) E[u_{k,i} \tilde{\psi}_{k-1}^{(i)} e_k(i)] + \mu_k^2(i) E[u_{k,i} u_{k,i}^* e_k^2(i)] \quad (13)$$

Based on (9), we takes the derivative of the right side of (13) and Tikhonov regularization cost with respect to  $\mu_k(i)$ , the optimal variable step size is obtained in the form

$$\mu_k^o(i) = \frac{E[u_{k,i} \tilde{\psi}_{k-1}^{(i)} e_k(i)]}{E[u_{k,i} u_{k,i}^* e_k^2(i)] + \delta}. \quad (14)$$

Combining (1) and (10), we get

$$e_k(i) = u_{k,i} \tilde{\psi}_{k-1}^{(i)} + v_k(i). \quad (15)$$

Considering the noise  $v_k(i)$  which is zero mean and independent of the input regressors, (14) becomes

$$\mu_k^o(i) = \frac{E \|u_{k,i} \tilde{\psi}_{k-1}^{(i)}\|^2}{E \|u_{k,i} e_k(i)\|^2 + \delta}. \quad (16)$$

It can be seen that  $\mu_k^o(i)$  can be obtained theoretically from (16), however, the major obstacle is that the weight error vector  $\tilde{\psi}_k^{(i)}$  is not available during the iterations, since  $w^o$  is unknown.

### B. PROPOSED VSS-ILMS ALGORITHM

We notice from (15) that

$$u_{k,i} \tilde{\psi}_{k-1}^{(i)} = e_k(i) - v_k(i), \quad (17)$$

and consider the independence of the background noise  $v_k(i)$ . Thus, (16) can be rewritten by

$$\mu_k^o(i) = \frac{\sigma_{e_k,i}^2 - \sigma_{v_k,i}^2}{E \|u_{k,i} e_k(i)\|^2 + \delta}, \quad (18)$$

where  $\sigma_{e_k,i}^2 = E \|e_k(i)\|^2$  denotes the power of error.

It can be seen from (18) that, at the beginning,  $\sigma_{e_k,i}^2$  is considerably larger than  $\sigma_{v_k,i}^2$ . In this situation, a large step size is used to quicken the convergence speed due to the excessive system mismatch. Both  $\sigma_{e_k,i}^2$  and  $\mu_k^o(i)$  become smaller as the algorithm starts to converge. When in the steady-state,  $\sigma_{e_k,\infty}^2 \approx \sigma_{v_k,i}^2$  leads to  $\mu_k^o(\infty) \approx 0$ . Our algorithm can effectively adjust step-size to match actual system behaviour. More importantly, this algorithm obtains the minimum MSD per iteration, resulting in the better performance compared to other variable step size algorithms.

In practice, we adopt time averaging method to estimate error variance and noise variance. First, the estimation  $\hat{\sigma}_{e_k,i}^2$  and  $\hat{\sigma}_{v_k,i}^2$  of error variance  $\sigma_{e_k,i}^2$  and noise variance  $\sigma_{v_k,i}^2$  can be obtained by time averaging as below [19], [29]:

$$\hat{\sigma}_{e_k,i}^2 = \alpha_1 \hat{\sigma}_{e_k,i-1}^2 + (1 - \alpha_1) e_k^2(i) \quad (19)$$

with a smoothing factor  $\alpha_1 (0 < \alpha_1 < 1)$ , and

$$\hat{\sigma}_{v_k,i}^2 = \hat{\sigma}_{e_k,i}^2 - \frac{1}{\hat{\sigma}_{u_k,i}^2} \hat{r}_{u,e}(i)^* \hat{r}_{u,e}(i), \quad (20)$$

where  $\hat{\sigma}_{u_k,i}^2$  and  $\hat{r}_{u,e}(i)$  are the estimation of input power and the cross-correlation between the input regressor  $u_{k,i}$  and the error  $e_k(i)$ . And they can be obtained in the same manner

$$\hat{\sigma}_{u_k,i}^2 = \alpha_2 \hat{\sigma}_{u_k,i-1}^2 + (1 - \alpha_2) u_{k,i} u_{k,i}^*, \quad (21)$$

$$\hat{r}_{u,e}(i) = \alpha_3 \hat{r}_{u,e}(i-1) + (1 - \alpha_3) u_{k,i} e_k(i), \quad (22)$$

where  $0 < \alpha_2, \alpha_3 < 1$ .

Using  $\|\hat{r}_{u,e}(i)\|^2$ ,  $\hat{\sigma}_{e_k,i}^2$  and  $\hat{\sigma}_{v_k,i}^2$  instead of  $E \|u_{k,i} e_k(i)\|^2$ ,  $\sigma_{e_k,i}^2$  and  $\sigma_{v_k,i}^2$  in (18), the proposed variable step-size  $\mu_k(i)$  for ILMS algorithm becomes

$$\mu_k(i) = \frac{\hat{\sigma}_{e_k,i}^2 - \hat{\sigma}_{v_k,i}^2}{\|\hat{r}_{u,e}(i)\|^2 + \delta}. \quad (23)$$

where  $\hat{\sigma}_{e_k,i}^2$ ,  $\hat{\sigma}_{v_k,i}^2$  and  $\hat{r}_{u,e}(i)$  are given by (19)-(22). Our analysis in following section shows that only  $\alpha_3$  has a great effect on the steady state performance of the proposed algorithm because steady state step-size is independent of  $\alpha_1$  and  $\alpha_2$ . Thus, a single smoothing factor  $\alpha$  can be used to replace  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$ , and set to  $1 - \frac{1}{kM}$  where the value of  $k$  ranges from 2-6.

### C. CONVERGENCE OF MEAN WEIGHT VECTOR

To guarantee algorithm stability, another important consideration is in determining the upper bound of  $\mu_k(i)$ , which will be considered from the perspective of the convergence of the mean weight vector. For tractable analysis, we first introduce the following assumption:

*Assumption 1:* Step-sizes are independent of input regressors  $u_{k,i}$ , error component  $e_k(i)$ .

This assumption cannot really hold for the proposed algorithm as we can see in (23) that the values of step-size per iteration are affected by the instantaneous input regressors,

error and noise. However, in steady state, step size  $\mu_k(i)$  will be very close to its mean value. By writing

$$E[\mu_k(i)u_{k,i}^*e_k(i)] = E[\mu_k(i)]E[u_{k,i}^*e_k(i)] + E\{[\mu_k(i) - E[\mu_k(i)]]u_{k,i}^*e_k(i)\}, \quad (24)$$

it can be seen that the second term on the right-hand side of (24) is much smaller than the first. Thus, we think the following equation is approximately true.

$$E[\mu_k(i)u_{k,i}^*e_k(i)] = E[\mu_k(i)]E[u_{k,i}^*e_k(i)]. \quad (25)$$

By (25), Assumption 1 is considered valid. Also, such an assumption is adopted commonly in adaptive filtering [13] and distributed estimation [29], in which the derived theoretical results match empirical results well.

Taking expectations on both sides of (12) and using Assumption 1, we obtain

$$\begin{aligned} E[\tilde{\psi}_k^{(i)}] &= E[\tilde{\psi}_{k-1}^{(i)}] - E[\mu_k(i)]E[u_{k,i}^*e_k(i)] \\ &= E[\tilde{\psi}_{k-1}^{(i)}] - E[\mu_k(i)]E[u_{k,i}^*(u_{k,i}\tilde{\psi}_{k-1}^{(i)} + v_k(i))] \\ &= E[\tilde{\psi}_{k-1}^{(i)}] - E[\mu_k(i)]R_{u,k}E[\tilde{\psi}_{k-1}^{(i)}] \\ &= (I - E[\mu_k(i)]R_{u,k})E[\tilde{\psi}_{k-1}^{(i)}]. \end{aligned} \quad (26)$$

The mean weight vector is convergent if and only if

$$\prod_{i=1}^{\infty} \prod_{k=1}^N (I - E[\mu_k(i)]R_{u,k}) \rightarrow 0. \quad (27)$$

Under (27), the change of weights between two neighbouring agents become small, thus ensuring the convergence of  $E[\psi_k^{(i)}]$  to  $w^o$ .

A sufficient condition for (27) to hold is

$$E[\mu_k(i)] < \frac{2}{\lambda_{\max}(R_{u,k})}, \quad (28)$$

where  $\lambda_{\max}(R_{u,k})$  is the maximum eigenvalue of the covariance matrix  $R_{u,k}$ . This result is consistent with findings reported in the constant step size LMS algorithm [13].

A stronger but simpler sufficient condition for satisfying (28) is

$$\mu_{k,\max}(i) < \frac{2}{\lambda_{\max}(R_{u,k})}, \quad (29)$$

where  $\mu_{k,\max}(i)$  is the upper bound for the step-size for agent  $k$  at time  $i$ .

Furthermore, for practical usage, we use instantaneous approximations to replace the actual second-order moments  $R_{u,k}$ . As a result,  $\mu_{k,\max}(i)$  is set to

$$\mu_{k,\max}(i) = \frac{2}{\hat{\sigma}_{u_{k,i}}^2}. \quad (30)$$

Finally, the proposed VSS-ILMS algorithm is summarized in Algorithm 1.

It can be seen from Algorithm 1 that each agent only needs to communicate with its immediate neighbor agent, and the total number of communications per iteration for each agent is  $2M$  (i.e., the size of received  $\psi_{k-1}^{(i)}$  and transmitted  $\psi_k^{(i)}$ ).

### Algorithm 1 Variable Step Size Incremental LMS Algorithm

---

```

1: Start with  $w_0 = \psi_N^{(0)} = 0$  and  $\mu_N(0) = 0$ 
2: for every time  $i \geq 1$  do
3:   for agents  $k = 1$  to  $N$  do
4:     if  $k = 1$  then
5:        $\psi_{k-1}^{(i)} = \psi_N^{(i-1)}$ 
6:     end if
7:     receive  $\psi_{k-1}^{(i)}$  from agent  $k - 1$ 
8:      $e_k(i) = d_k(i) - u_{k,i}\psi_{k-1}^{(i)}$ 
9:      $\hat{\sigma}_{e_{k,i}}^2 = \alpha\hat{\sigma}_{e_{k,i-1}}^2 + (1 - \alpha)e_{k,i}^2$ 
10:     $\hat{\sigma}_{u_{k,i}}^2 = \alpha\hat{\sigma}_{u_{k,i-1}}^2 + (1 - \alpha)u_{k,i}u_{k,i}^*$ 
11:     $\hat{r}_{u,e}(i) = \alpha\hat{r}_{u,e}(i-1) + (1 - \alpha)u_{k,i}e_k(i)$ 
12:     $\hat{\sigma}_{v_{k,i}}^2 = \hat{\sigma}_{e_{k,i}}^2 - \frac{1}{\hat{\sigma}_{u_{k,i}}^2}\hat{r}_{u,e}(i)^*\hat{r}_{u,e}(i)$ 
13:    step size update  $\mu_k(i) = \min\{\frac{\hat{\sigma}_{e_{k,i}}^2 - \hat{\sigma}_{v_{k,i}}^2}{\|\hat{r}_{u,e}(i)\|^2 + \delta}, \frac{2}{\hat{\sigma}_{u_{k,i}}^2}\}$ 
14:    weight update  $\psi_k^{(i)} = \psi_{k-1}^{(i)} + \mu_k(i)u_{k,i}^*e_k(i)$ 
15:  end for
16:   $w_i = \psi_N^{(i)}$ 
17:  send  $\psi_N^{(i)}$  to agent 1
18: end for

```

---

In other word, there are no growing communication cost compared with the traditional algorithm. The additional requirement for our algorithm is computational cost for variable step size. It is known that the energy consumed on communication is far more than that on computing for a cyber system typically deployed with sensing, computing and wireless communicating [30]. Various methods [31]–[33] have been proposed to sacrifice reasonable amount of computational cost for a longer network lifetime and the improvement of performance. Therefore, our algorithm is acceptable in terms of cost consumption.

Moreover, determining a Hamiltonian path in a connected network is known to be NP-complete [34], [35]. However, this problem can be solved by constructing an approximate Hamiltonian path in a distributed way. Several distributed methods have been proposed in [36] and [37] and perform very well in practice. The idea behind them is that every agent makes local decisions based on a heuristic approach, which might cause a defect that not all agents are included in the final path. This situation does not influence the performance of algorithm since a typical sensor network is deployed with much greater density than is needed [38], [39], mainly to satisfy full network coverage requirements [40] and compensate for the impact of agents failure [41]. Here, we assume that the network can be organized in a Hamiltonian cycle in this paper.

## IV. STEADY-STATE PERFORMANCE ANALYSIS

In order to pursue the steady-state performance analysis for the proposed VSS-ILMS LMS algorithm, we first consider the following additional assumptions, which are commonly done in distributed estimation algorithms [42], [43]:

Assumption 2:  $u_{k,i}$  is independent of  $u_{l,i}$  for  $k \neq l$ .

Assumption 3:  $u_{k,i}$  is independent of  $u_{k,j}$  for  $i \neq j$ .

Above assumptions suggest that all regressors are spatially and temporally independent. With these the assumptions, the performance analysis can be simplified without loss of generality.

### A. STEADY-STATE MSD AND EMSE

In this subsection, we are interested in evaluating key performance indicators like the MSD, the MSE, the excess mean-square error (EMSE) in steady-state for every agent  $k$ , which are defined as follows:

$$\eta_k = E \|\tilde{\psi}_{k-1}^{(\infty)}\|^2 \quad (MSD), \quad (31)$$

$$\zeta_k = E \|u_{k,\infty} \tilde{\psi}_{k-1}^{(\infty)}\|^2 \quad (EMSE), \quad (32)$$

$$\begin{aligned} \xi_k &= E \|e_k(\infty)\|^2 = E \|u_{k,\infty} \tilde{\psi}_{k-1}^{(\infty)} + v_{k,\infty}\|^2 \\ &= \zeta_k + \sigma_{v,k}^2 \quad (MSE). \end{aligned} \quad (33)$$

The weighted norm notation  $\|x\|_{\Sigma}^2 \triangleq x^* \Sigma x$  is introduced to obtain the expressions for these quantities, and  $\Sigma$  is a Hermitian positive definite matrix that we are free to choose. Under the assumed data conditions, we have

$$\eta_k = E \|\tilde{\psi}_{k-1}^{(\infty)}\|_I^2, \quad \zeta_k = E \|\tilde{\psi}_{k-1}^{(\infty)}\|_{R_{u,k}}^2. \quad (34)$$

Therefore, the problems of steady state MSD and EMSE are transformed to evaluate two weighted norms of  $\psi_{k-1}^{(\infty)}$  in (34).

To do this, we take the squared weighted  $l_2$ -norm of (12) as follows:

$$\begin{aligned} \|\tilde{\psi}_k^{(i)}\|_{\Sigma}^2 &= \|\tilde{\psi}_{k-1}^{(i)} - \mu_k(i) u_{k,i}^* e_k(i)\|_{\Sigma}^2 \\ &= \|\tilde{\psi}_{k-1}^{(i)}\|_{\Sigma}^2 - \tilde{\psi}_{k-1}^{(i)*} \Sigma \mu_k(i) u_{k,i}^* e_k(i) \\ &\quad - e_k^*(i) u_{k,i} \mu_k^*(i) \Sigma \tilde{\psi}_{k-1}^{(i)} \\ &\quad + e_k^*(i) u_{k,i} \mu_k^*(i) \Sigma \mu_k(i) u_{k,i}^* e_k(i) \end{aligned} \quad (35)$$

Substituting (15) into (35) and taking the expectation of both sides yields

$$\begin{aligned} E \|\tilde{\psi}_k^{(i)}\|_{\Sigma}^2 &= E \|\tilde{\psi}_{k-1}^{(i)}\|_{\Sigma}^2 - E[\tilde{\psi}_{k-1}^{(i)*} \Sigma \mu_k(i) u_{k,i}^* u_{k,i} \tilde{\psi}_{k-1}^{(i)}] \\ &\quad - E[\tilde{\psi}_{k-1}^{(i)*} u_{k,i}^* u_{k,i} \mu_k^*(i) \Sigma \tilde{\psi}_{k-1}^{(i)}] \\ &\quad + E[\tilde{\psi}_{k-1}^{(i)*} u_{k,i}^* u_{k,i} \mu_k^*(i) \Sigma \mu_k(i) u_{k,i}^* u_{k,i} \tilde{\psi}_{k-1}^{(i)}] \\ &\quad + E[v_k^*(i) u_{k,i} \mu_k^*(i) \Sigma \mu_k(i) u_{k,i}^* v_k(i)] \\ &= E \|\tilde{\psi}_{k-1}^{(i)}\|_{\Sigma}^2 - E \|\tilde{\psi}_{k-1}^{(i)}\|_{\Sigma \mu_k(i) u_{k,i}^* u_{k,i}}^2 \\ &\quad - E \|\tilde{\psi}_{k-1}^{(i)}\|_{u_{k,i}^* u_{k,i} \mu_k^*(i) \Sigma}^2 \\ &\quad + E \|\tilde{\psi}_{k-1}^{(i)}\|_{u_{k,i}^* u_{k,i} \mu_k^*(i) \Sigma \mu_k(i) u_{k,i}^* u_{k,i}}^2 \\ &\quad + \sigma_{v,k}^2 E \|\mu_k(i) u_{k,i}^*\|_{\Sigma}^2. \end{aligned} \quad (36)$$

Given  $\|x\|_A^2 + \|x\|_B^2 = \|x\|_{A+B}^2$ , Equation (36) can be rewritten as a comprehensive form

$$E \|\tilde{\psi}_k^{(i)}\|_{\Sigma}^2 = E \|\tilde{\psi}_{k-1}^{(i)}\|_{\Sigma'}^2 + \sigma_{v,k}^2 E \|\mu_k(i) u_{k,i}^*\|_{\Sigma}^2, \quad (37)$$

where

$$\begin{aligned} \Sigma' &= \Sigma - \Sigma \mu_k(i) u_{k,i}^* u_{k,i} - u_{k,i}^* u_{k,i} \mu_k^*(i) \Sigma \\ &\quad + \|\mu_k(i) u_{k,i}^*\|_{\Sigma}^2 u_{k,i}^* u_{k,i}. \end{aligned} \quad (38)$$

On the basis of the assumed independence of the regression data  $u_{k,i}$  and step size  $\mu_k(i)$ , we have

$$E \|\tilde{\psi}_{k-1}^{(i)}\|_{\Sigma'}^2 = E \|\tilde{\psi}_{k-1}^{(i)}\|_{E \Sigma'}^2, \quad (39)$$

so that (37) and (38) become

$$E \|\tilde{\psi}_k^{(i)}\|_{\Sigma}^2 = E \|\tilde{\psi}_{k-1}^{(i)}\|_{E \Sigma'}^2 + \sigma_{v,k}^2 E \|\mu_k(i) u_{k,i}^*\|_{\Sigma}^2, \quad (40)$$

where

$$\begin{aligned} E \Sigma' &= \Sigma - E[\mu_k(i)] E[\Sigma u_{k,i}^* u_{k,i}] - E[u_{k,i}^* u_{k,i} \Sigma] E[\mu_k^*(i)] \\ &\quad + E \|\mu_k(i) u_{k,i}^*\|_{\Sigma}^2 u_{k,i}^* u_{k,i}. \end{aligned} \quad (41)$$

Recursion (40) is a spatial variance relation by which steady-state performance measures of every agent can be evaluated.

In order to simplify the analysis, we assume that the regressors  $u_k$  arise from a source with circular Gaussian distribution and introduce the eigendecomposition  $R_{u,k} = U_k \Lambda_k U_k^*$ , where  $U_k$  is unitary and  $\Lambda_k$  is a diagonal matrix with the eigenvalues of  $R_{u,k}$ . We define the following relations

$$\bar{\psi}_k^{(i)} = U_k^* \tilde{\psi}_k^{(i)}, \quad \bar{\psi}_{k-1}^{(i)} = U_k^* \tilde{\psi}_{k-1}^{(i)}, \quad \bar{u}_{k,i} = u_{k,i} U_k,$$

$$\bar{\Sigma} = U_k^* \Sigma U_k, \quad \bar{\Sigma}' = U_k^* E[\Sigma'] U_k,$$

$$E[\mu_k(i)] = \bar{\mu}_k(i), \quad E[\mu_k^2(i)] = \bar{\mu}_k^2(i).$$

Thus, we have that  $\|\tilde{\psi}_{k-1}^{(i)}\|_{\Sigma}^2 = \|\bar{\psi}_{k-1}^{(i)}\|_{\bar{\Sigma}}^2$  and  $\|u_{k,i}\|_{\Sigma}^2 = \|\bar{u}_{k,i}\|_{\bar{\Sigma}}^2$ . Consequently, (40) and (41) can be rewritten as

$$E \|\bar{\psi}_k^{(i)}\|_{\bar{\Sigma}}^2 = E \|\bar{\psi}_{k-1}^{(i)}\|_{\bar{\Sigma}'}^2 + \sigma_{v,k}^2 E \|\mu_k(i) \bar{u}_{k,i}^*\|_{\bar{\Sigma}}^2, \quad (42)$$

where

$$\begin{aligned} \bar{\Sigma}' &= \bar{\Sigma} - \bar{\mu}_k(i) E(\bar{\Sigma} \bar{u}_{k,i}^* \bar{u}_{k,i} + \bar{u}_{k,i}^* \bar{u}_{k,i} \bar{\Sigma}) \\ &\quad + E \|\mu_k(i) \bar{u}_{k,i}^*\|_{\bar{\Sigma}}^2 \bar{u}_{k,i}^* \bar{u}_{k,i}. \end{aligned} \quad (43)$$

Since

$$E[\bar{u}_{k,i}^* \bar{u}_{k,i}] = E[U_k^* u_{k,i}^* u_{k,i} U_k] = \Lambda_k, \quad (44)$$

$$\begin{aligned} E \|\mu_k(i) \bar{u}_{k,i}^*\|_{\bar{\Sigma}}^2 &= E[\mu_k(i) u_{k,i}^* U_k \bar{\Sigma} U_k^* u_{k,i} \mu_k(i)] \\ &= \bar{\mu}_k^2(i) \text{Tr}(\Lambda_k \bar{\Sigma}), \end{aligned} \quad (45)$$

and

$$E \|\mu_k(i) \bar{u}_{k,i}^*\|_{\bar{\Sigma} \bar{u}_{k,i}^* \bar{u}_{k,i}}^2 = \bar{\mu}_k^2(i) (\Lambda_k \text{Tr}(\bar{\Sigma} \Lambda_k) + 2 \Lambda_k \bar{\Sigma} \Lambda_k), \quad (46)$$

substituting (45) and (46) into (42) and (43), we have

$$E \|\bar{\psi}_k^{(i)}\|_{\bar{\Sigma}}^2 = E \|\bar{\psi}_{k-1}^{(i)}\|_{\bar{\Sigma}'}^2 + \bar{\mu}_k^2(i) \text{Tr}(\Lambda_k \bar{\Sigma}), \quad (47)$$

and

$$\begin{aligned} \bar{\Sigma}' &= \bar{\Sigma} - \bar{\mu}_k(i) (\Lambda_k \bar{\Sigma} + \bar{\Sigma} \Lambda_k) + \bar{\mu}_k^2(i) (\Lambda_k \text{Tr}(\bar{\Sigma} \Lambda_k) \\ &\quad + 2 \Lambda_k \bar{\Sigma} \Lambda_k). \end{aligned} \quad (48)$$

Note that Equations (47) and (48) are similar to the results obtained in [4], however, the process to reach them is different and there is a noticeable difference between variable step size and static step size. This allows us to use the same diagonalization method adopted in [4] for obtaining steady-state ( $i \rightarrow \infty$ ) MSD and EMSE for agent  $k$ , which can be described as follows:

$$\eta_k = E\|\psi_{k-1}^{(\infty)}\|^2 = a_k(I - \Pi_{k,1})^{-1}q \quad (MSD), \quad (49)$$

and

$$\zeta_k = E\|u_{k,\infty}\tilde{\psi}_{k-1}^{(\infty)}\|^2 = a_k(I - \Pi_{k,1})^{-1}\lambda_k \quad (EMSE), \quad (50)$$

where

$$\Pi_{k,l} = F_{k+l-1}F_{k+l}\cdots F_N F_1 \cdots F_{k-1}, \quad (51)$$

$$F_k = I - 2\overline{\mu_k}\Lambda_k + 2\overline{\mu_k}^2\Lambda_k^2 + \overline{\mu_k}^2\lambda_k\lambda_k^T, \quad (52)$$

$$a_k = g_k\Pi_{k,2} + g_{k+1}\Pi_{k,3} + \cdots + g_{k-2}\Pi_{k,N} + g_{k-1}, \quad (53)$$

$$g_k = \overline{\mu_k}^2\sigma_{v,k}^2\lambda_k^T, \quad (54)$$

$\overline{\mu_k}(\infty) \triangleq \overline{\mu_k}$ ,  $\overline{\mu_k}^2(\infty) \triangleq \overline{\mu_k}^2$ ,  $q$  and  $\lambda_k$  are two column vectors containing the main diagonal of matrices  $I$  and  $\Lambda_k$ , respectively.

In steady state,  $\sigma_{e_k,\infty}^2$  will tend to  $\sigma_{v_k,\infty}^2$  as the algorithm starts to converge (i.e.,  $\psi_k^\infty \rightarrow w^o$ ). This leads to a very small steady state step size by analyzing (23). Hence, the approximation  $F_k \approx I - 2\overline{\mu_k}\Lambda_k$  is acceptable. From this, we can rewrite  $\Pi_{k,1}$  as a diagonal matrix and approximate it as follows:

$$\begin{aligned} \Pi_{k,1} &= \Pi = F_1 F_2 \cdots F_N, \\ &= (I - 2\overline{\mu_1}\Lambda_1)(I - 2\overline{\mu_2}\Lambda_2) \cdots (I - 2\overline{\mu_N}\Lambda_N) \\ &\approx I - 2(\overline{\mu_1}\Lambda_1 + \overline{\mu_2}\Lambda_2 + \cdots + \overline{\mu_N}\Lambda_N), \end{aligned} \quad (55)$$

we have

$$I - \Pi \approx 2(\overline{\mu_1}\Lambda_1 + \overline{\mu_2}\Lambda_2 + \cdots + \overline{\mu_N}\Lambda_N), \quad (56)$$

and  $a_k \approx \sum_{k=1}^N g_k$ , in which  $\Pi$  is further approximated by  $I$ .

Finally, the steady-state MSD, EMSE and MSE of each agent are given by

$$\begin{aligned} \eta_k &\approx (\overline{\mu_1}^2\sigma_{v,1}^2\lambda_1^T + \cdots + \overline{\mu_N}^2\sigma_{v,N}^2\lambda_N^T) \\ &\quad \times (2\overline{\mu_1}\Lambda_1 + \cdots + 2\overline{\mu_N}\Lambda_N)^{-1}q, \end{aligned} \quad (57)$$

$$\begin{aligned} \zeta_k &\approx (\overline{\mu_1}^2\sigma_{v,1}^2\lambda_1^T + \cdots + \overline{\mu_N}^2\sigma_{v,N}^2\lambda_N^T) \\ &\quad \times (2\overline{\mu_1}\Lambda_1 + \cdots + 2\overline{\mu_N}\Lambda_N)^{-1}\lambda_k, \end{aligned} \quad (58)$$

$$\xi_k \approx \zeta_k + \sigma_{v,k}^2. \quad (59)$$

### B. STEADY-STATE MEAN STEP SIZE

In this subsection, we will evaluate the steady state mean step size that is required for  $\eta_k$  and  $\zeta_k$ . Taking expectations on both sides of Equation (23), and giving the following reasonable approximation used widely in [29], [44], [45]:

$$E[\mu_k(i)] \approx \frac{E[\hat{\sigma}_{e_k,i}^2] - E[\hat{\sigma}_{v_k,i}^2]}{E\|\hat{r}_{u,e}(i)\|^2 + \delta}. \quad (60)$$

From (19) and (21), the expectations of  $\hat{\sigma}_{e_k,i}^2$  and  $\hat{\sigma}_{u_k,i}^2$  are calculated as follows:

$$\begin{aligned} E[\hat{\sigma}_{e_k,i}^2] &= \alpha_1 E[\hat{\sigma}_{e_k,i-1}^2] + (1 - \alpha_1)E[e_k^2(i)] \\ &= \alpha_1 E[\hat{\sigma}_{e_k,i-1}^2] + (1 - \alpha_1)E\|\tilde{\psi}_{k-1}^i\|_{R_{u,k}}^2 + (1 - \alpha_1)\sigma_{v,k}^2, \end{aligned} \quad (61)$$

$$E[\hat{\sigma}_{u_k,i}^2] = \alpha_2 E[\hat{\sigma}_{u_k,i-1}^2] + (1 - \alpha_2)\text{Tr}(R_{u,k}). \quad (62)$$

From (22), we have

$$\begin{aligned} \|\hat{r}_{u,e}(i)\|^2 &= \alpha_3^2\|\hat{r}_{u,e}(i-1)\|^2 + 2\alpha_3(1 - \alpha_3) \\ &\quad \times \hat{r}_{u,e}^*(i-1)u_{k,i}e_k(i) + (1 - \alpha_3)^2 e_k^*(i)u_{k,i}^*u_{k,i}e_k(i). \end{aligned} \quad (63)$$

As  $i \rightarrow \infty$ , we get

$$\begin{aligned} \lim_{i \rightarrow \infty} E[\hat{\sigma}_{e_k,i}^2] &= \alpha_1 \lim_{i \rightarrow \infty} E[\hat{\sigma}_{e_k,i-1}^2] \\ &\quad + (1 - \alpha_1) \lim_{i \rightarrow \infty} (E\|\tilde{\psi}_{k-1}^i\|_{R_{u,k}}^2 + \sigma_{v,k}^2) \end{aligned} \quad (64)$$

and

$$\lim_{i \rightarrow \infty} E[\hat{\sigma}_{u_k,i}^2] = \alpha_2 \lim_{i \rightarrow \infty} E[\hat{\sigma}_{u_k,i-1}^2] + (1 - \alpha_2)\text{Tr}(R_{u,k}). \quad (65)$$

From (64) and (65), we find that two smoothing factors  $\alpha_1$  and  $\alpha_2$  are removed. As a result,

$$E[\hat{\sigma}_{e_k,\infty}^2] = E\|\tilde{\psi}_{k-1}^\infty\|_{R_{u,k}}^2 + \sigma_{v,k}^2, \quad (66)$$

and

$$E[\hat{\sigma}_{u_k,\infty}^2] = \text{Tr}(R_{u,k}). \quad (67)$$

In order to evaluate  $E\|\hat{r}_{u,e}(\infty)\|^2$ , we need to deal with the expectations in (63). For this purpose, we shall rely on the following additional assumption used widely in [46] and [29].

*Assumption 4:* At steady-state,  $u_{k,i}$  is independent of  $e_{k,i}$  for all  $k$ .

Under Assumption 4 and (63), we have

$$\begin{aligned} E[\|\hat{r}_{u,e}(i)\|^2] &= \alpha_3^2 E[\|\hat{r}_{u,e}(i-1)\|^2] + (1 - \alpha_3)^2 E[e_k^*(i)u_{k,i}^*u_{k,i}e_k(i)] \\ &= \alpha_3^2 E[\|\hat{r}_{u,e}(i-1)\|^2] + (1 - \alpha_3)^2 \text{Tr}(R_{u,k})E\|e_k(i)\|^2. \end{aligned} \quad (68)$$

In a similar way as  $i \rightarrow \infty$ ,  $E\|\hat{r}_{u,e}(\infty)\|^2$  can be obtained by

$$E[\|\hat{r}_{u,e}(\infty)\|^2] = \frac{1 - \alpha_3}{1 + \alpha_3} \text{Tr}(R_{u,k})(E\|\tilde{\psi}_{k-1}^\infty\|_{R_{u,k}}^2 + \sigma_{v,k}^2). \quad (69)$$

From (20), by using the following approximation

$$E[\hat{\sigma}_{v_k,i}^2] \approx E[\hat{\sigma}_{e_k,i}^2] - \frac{1}{E[\hat{\sigma}_{u_k,i}^2]} E\|\hat{r}_{u,e}(i)\|^2, \quad (70)$$

and substituting (66) (67) and (69) into (70) when  $i \rightarrow \infty$ , we get

$$E[\hat{\sigma}_{v_k,\infty}^2] = E[\hat{\sigma}_{e_k,\infty}^2] - \frac{1}{E[\hat{\sigma}_{u_k,\infty}^2]} E\|\hat{r}_{u,e}(\infty)\|^2$$

$$= \frac{2\alpha_3}{1 + \alpha_3} (E\|\tilde{\psi}_{k-1}^\infty\|_{R_{u,k}}^2 + \sigma_{v,k}^2). \quad (71)$$

Substituting (66) (69) and (71) into (60) when  $i \rightarrow \infty$  yields

$$E[\mu_k(\infty)]$$

$$= \frac{E[\hat{\sigma}_{e_k,\infty}^2] - E[\hat{\sigma}_{v_k,\infty}^2]}{E\|\hat{r}_{u,e}(\infty)\|^2 + \delta}$$

$$= \frac{(1 - \alpha_3)(E\|\tilde{\psi}_{k-1}^\infty\|_{R_{u,k}}^2 + \sigma_{v,k}^2)}{(1 - \alpha_3)\text{Tr}(R_{u,k})(E\|\tilde{\psi}_{k-1}^\infty\|_{R_{u,k}}^2 + \sigma_{v,k}^2) + (1 + \alpha_3)\delta}.$$

(72)

Note that only one parameter  $\alpha_3$  is shown in (72), while other parameters  $\alpha_1$  and  $\alpha_2$  are removed. This is the reason that we use a single smoothing factor  $\alpha$  to replace all the others. Equation (72) gives an expression of the mean steady-state step-size. However, it can be seen that  $E\|\tilde{\psi}_{k-1}^\infty\|_{R_{u,k}}^2$  is unavailable since it is a function of  $E[\mu_k(\infty)]$ . In steady state, it is known that  $\psi_{k-1}^{(\infty)}$  converges to  $w^o$ . By the definition (8), we expect that the weight error vector  $\tilde{\psi}_{k-1}^\infty$  is very small for deriving a closed-form solution for Equation (72). Therefore,  $E[\hat{\sigma}_{e_k,\infty}^2]$ ,  $E\|\hat{r}_{u,e}(\infty)\|^2$  and  $E[\hat{\sigma}_{v_k,\infty}^2]$  can be approximated in a reasonable way

$$E[\hat{\sigma}_{e_k,\infty}^2] = \sigma_{v,k}^2, \quad (73)$$

$$E\|\hat{r}_{u,e}(\infty)\|^2 = \frac{1 - \alpha}{1 + \alpha} \text{Tr}(R_{u,k})\sigma_{v,k}^2, \quad (74)$$

$$E[\hat{\sigma}_{v_k,\infty}^2] = \frac{2\alpha}{1 + \alpha} \sigma_{v,k}^2. \quad (75)$$

As a consequence, the mean step size in steady state can be obtained as

$$E[\mu_k(\infty)] = \bar{\mu}_k = \frac{(1 - \alpha)\sigma_{v,k}^2}{(1 - \alpha)\text{Tr}(R_{u,k})\sigma_{v,k}^2 + (1 + \alpha)\delta}. \quad (76)$$

Because the step size in (23) is derived by the time-averaging method, it holds true that  $\mu_k(\infty) \approx E[\mu_k(\infty)]$ . Thus, the variance of steady state step size can be assumed to be equal to zero. Then, we can obtain  $E[\mu_k^2(\infty)]$  as follows:

$$E[\mu_k^2(\infty)] = \bar{\mu}_k^2 = (E[\mu_k(\infty)])^2$$

$$= \frac{(1 - \alpha)^2 \sigma_{v,k}^4}{[(1 - \alpha)\text{Tr}(R_{u,k})\sigma_{v,k}^2 + (1 + \alpha)\delta]^2}. \quad (77)$$

By substituting (76) (77) into (57) and (58), the steady-state MSD and EMSE of each agent can be

rewritten as

$$\eta_k = \sum_{j=1}^M \left( \frac{\sum_{k=1}^N \overline{\mu_k^2} \sigma_{v,k}^2 \lambda_{k,j}}{2 \sum_{k=1}^N \overline{\mu_k} \lambda_{k,j}} \right), \quad (78)$$

$$\zeta_k = \sum_{j=1}^M \left( \frac{\lambda_{k,j} \sum_{k=1}^N \overline{\mu_k^2} \sigma_{v,k}^2 \lambda_{k,j}}{2 \sum_{k=1}^N \overline{\mu_k} \lambda_{k,j}} \right), \quad (79)$$

where  $\lambda_{k,j}$  is the  $j$ th ( $1 \leq j \leq M$ ) element of column vector  $\lambda_k$ .

## V. SIMULATION RESULTS

To evaluate the proposed VSS-ILMS algorithm, in this section, we provide two computer simulation results. One is the numerical comparison between the proposed algorithm and the traditional LMS adaptive filtering algorithms. Meanwhile, we verify the theoretical expressions derived in Section IV. The other one is based on the target localization application in sensor networks to illustrate the practical usage of the algorithm presented here.

### A. NUMERICAL SIMULATIONS

In this subsection, all simulations were carried out using the following parameters selection. In our scenario, the network with 20 agents are connected in a way of Hamiltonian cycle as shown in Figure 1. The independent Gaussian regressors are generated as the measurements  $\{u_{k,i}\}$  with power  $\sigma_{u,k}^2 \in (0, 1]$  (Figure 2(a)). The tracked optimal weight vector  $w^o = \text{col}\{1, 1, \dots, 1\}/\sqrt{M}$  with  $M = 10$  is known for us but unknown for evaluated algorithms. The noise variances  $\sigma_{v,k}^2 \in (0, 0.1]$  following Gaussian distributions for all agents is plotted in Figure 2(b), and the corresponding signal-to-noise ratio (SNR) is shown in Figure 2(c). we assume the measurements are acquired via  $d_k(i) = u_{k,i}w^o + v_k(i)$ . The smoothing factor and regularization parameter are selected as  $\alpha = 0.985$  and  $\delta = 3$ . The learning curves of transient MSD and EMSE are obtained by performing 5000 iterations and averaging them for 200 independent experiments. The steady-state performance of step size, MSD and EMSE for each agent are then generated by averaging 200 samples at time 5000.

First, we provide the MSD curves for agent  $N = 20$  by running proposed VSS-ILMS algorithm and ILMS algorithm with three different step sizes from small to large. On the basis of the results presented in Figure 3, we have the following observations: for the conventional ILMS algorithm, the one with a relatively large step-size has a fast convergence speed at the initial state while the other one with a relatively small step-size has a low MSD at steady state. Unlike the ILMS algorithm, our VSS-ILMS algorithm shows a great effect both on convergence speed at initial state and low MSD for at steady state. In Figure 4, one can also see the same results on



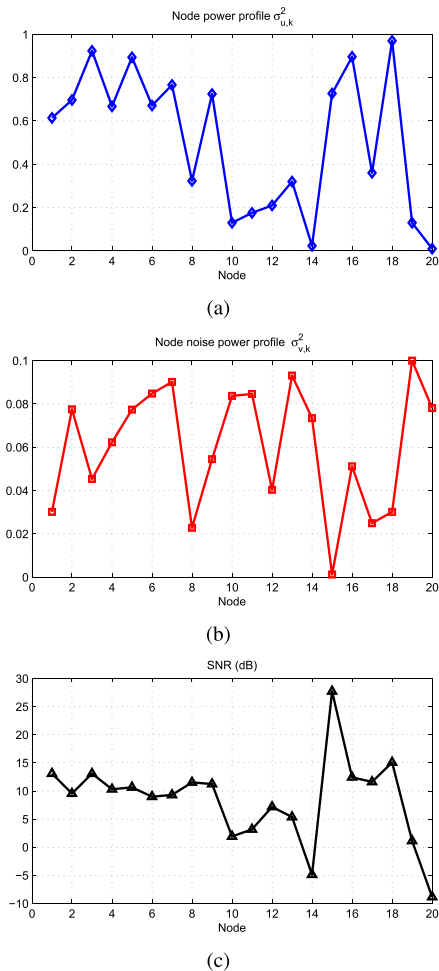


FIGURE 2. (a) Power profile. (b) Noise power profile. (c) Signal-to-noise ratio profile.

the EMSE learning curves for agent  $N$ . These facts indicate that the derived optimal variable step size shows the very good tracking performance when the error changes in the system occur.

For comparison purposes, the following variable step-size LMS adaptive filtering algorithms are applied to incremental networks for performing parameter estimation. These stand-alone LMS algorithms can be modified to fit the measurement exchange case in network. The parameters used in simulations are also shown as follows.

- 1) Kwong's VSS-LMS algorithm [13]:

$$\mu_k(i) = \alpha\mu_k(i-1) + \gamma e_k^2(i)$$

Parameters:  $\alpha = 0.95, \gamma = 0.065$

- 2) Abounasr's RVS-LMS algorithm [14]:

$$\begin{aligned} \mu_k(i) &= \alpha\mu_k(i-1) + \gamma p_k^2(i) \\ p_k(i) &= \beta p_k(i-1) + (1-\beta)e_k(i)e_k^*(i-1) \end{aligned}$$

Parameters:  $\alpha = 0.95, \beta = 0.98, \gamma = 0.015$

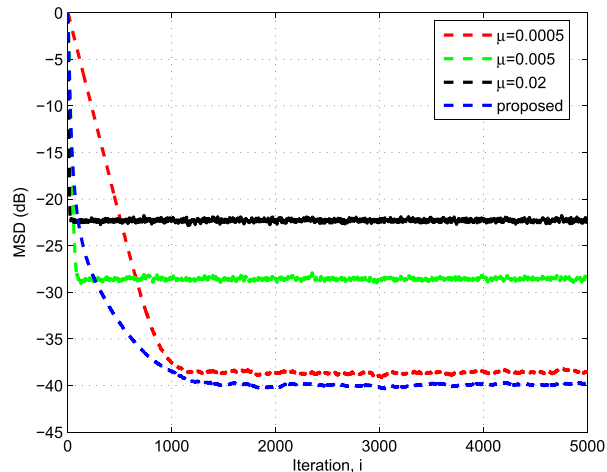


FIGURE 3. Transient MSD performance at agent  $N$  for proposed VSS-ILMS and conventional ILMS algorithm with different constant step size  $\mu$ .

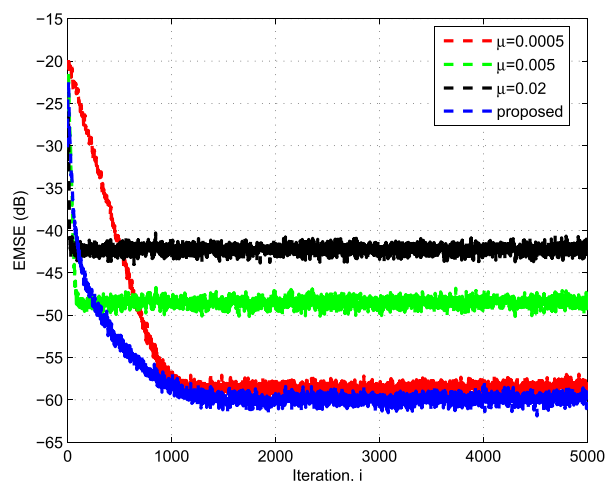


FIGURE 4. Transient EMSE performance at agent  $N$  for proposed VSS-ILMS and conventional ILMS algorithm with different constant step size  $\mu$ .

- 3) Huang's VSS-NLMS algorithm [19]:

$$\begin{aligned} \mu_k(i) &= \alpha\mu_k(i-1) + (1-\alpha)\frac{\hat{\sigma}_{e_k,i}^2}{\beta\hat{\sigma}_{v_k,i}^2} \\ \hat{\sigma}_{e_k,i}^2 &= \alpha\hat{\sigma}_{e_k,i-1}^2 + (1-\alpha)e_k^2(i) \\ \hat{\sigma}_{u_k,i}^2 &= \alpha\hat{\sigma}_{u_k,i-1}^2 + (1-\alpha)u_{k,i}u_{k,i}^* \\ \hat{r}_{u,e}(i) &= \alpha\hat{r}_{u,e}(i-1) + (1-\alpha)u_{k,i}e_k(i) \\ \hat{\sigma}_{v_k,i}^2 &= \hat{\sigma}_{e_k,i}^2 - \frac{1}{\hat{\sigma}_{u_k,i}^2}\hat{r}_{u,e}(i)^*\hat{r}_{u,e}(i) \end{aligned}$$

Parameters:  $\alpha = 0.995, \beta = 50, \mu_{max} = 0.01, \mu_{min} = 0.0005$

It should be noted that VSS-LMS and RVS-LMS algorithms have always been regarded as good comparison for LMS algorithm, and Huang's VSS-NLMS algorithm is recently proposed variable step-size normalized LMS algorithm that uses a similar time-averaging estimation method with good performance. Based on the implementations of

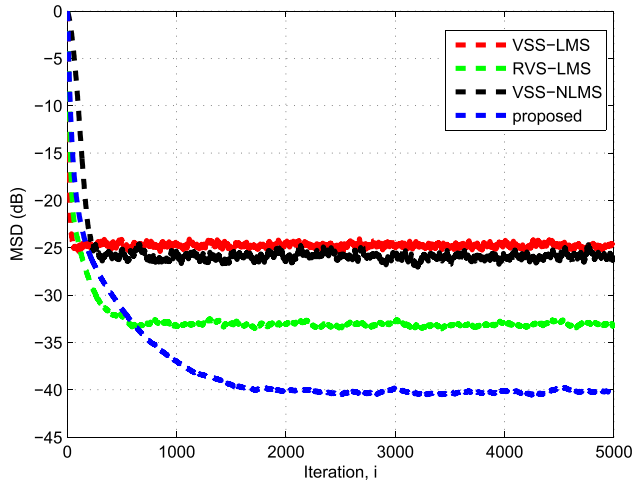


FIGURE 5. Transient MSD performance for VSS-ILMS and three variable step-size LMS algorithms.

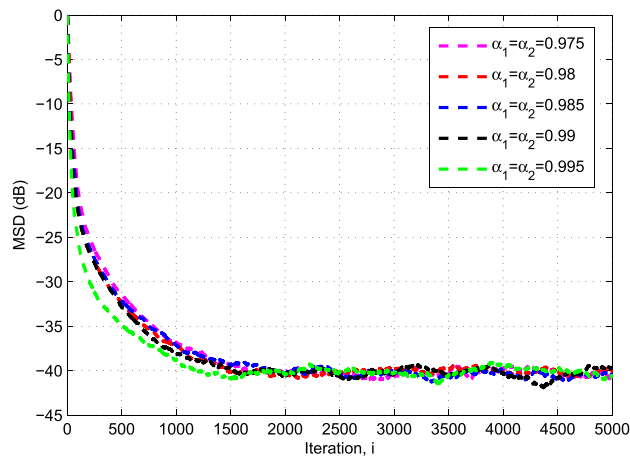


FIGURE 6. Effect of  $\alpha_1, \alpha_2$  on the MSD for agent  $N$ .

above algorithms, in which parameters are selected to achieve average-case performance for fair comparison, we can see in Figure 5 that these variable step size algorithms have similar initial convergence rate. But as the process reaches steady state, the proposed VSS-ILMS algorithm provides the lowest squared error of the four algorithms. According this simulation, it can be known that with the same convergence rate, the proposed algorithm is clearly superior for final steady-state error. As mentioned, the main reason is that our algorithm adopts the approximated optimal step size that assure the minimum MSD at each iteration.

To evaluate experimentally the effect of three parameters  $\alpha_1, \alpha_2, \alpha_3$ , we perform two types of simulations: varying  $\alpha_1, \alpha_2$  under fixed  $\alpha_3 = 0.985$  (Figure 6) and varying  $\alpha_3$  under fixed  $\alpha_1 = \alpha_2 = 0.985$  (Figure 7). One can see from Figure 6 that variation in  $\alpha_1, \alpha_2$  for Equation (19) and (21) has a very small impact on the MSD learning curve. On the other hand, one can also see from Figure 7 that variation in  $\alpha_3$  for Equation (22) greatly affects the steady state performance of the proposed algorithm. By analyzing (72), we know that the steady state mean step size is not related to  $\alpha_1, \alpha_2$  and

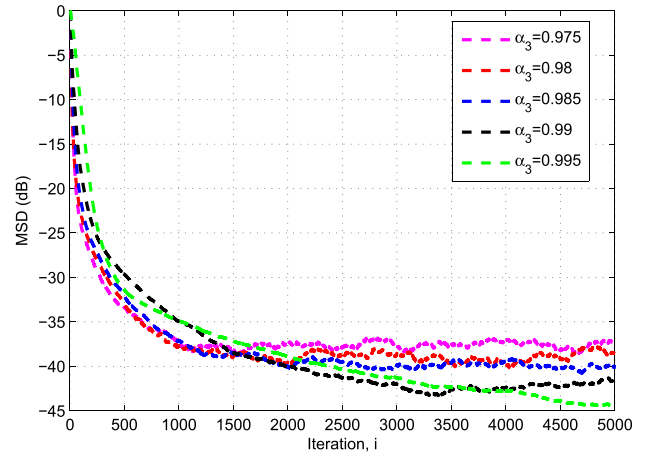


FIGURE 7. Effect of  $\alpha_3$  on the MSD for agent  $N$ .

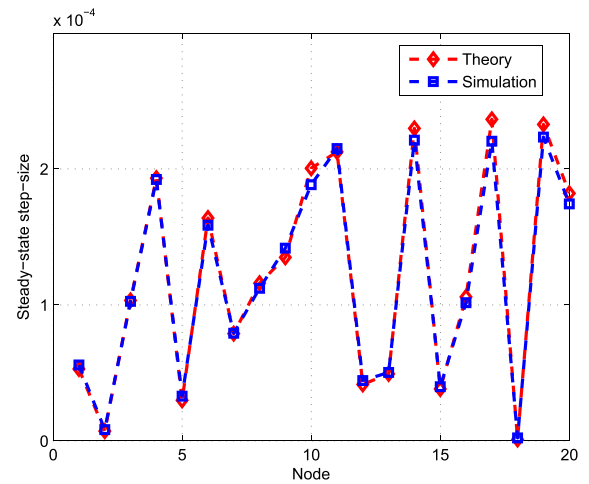


FIGURE 8. Simulation and theory of steady state step size at each agent.

is inversely proportional to  $\alpha_3$ . Therefore, a reasonable large  $\alpha_3$  leads to a low steady state error and vice versa. Simulated results also match the steady-state analysis. From this, we recommend that  $\alpha_3$  (i.e.,  $\alpha$  in our algorithm) should be large when high error accuracy is required for network application.

In Figure 8, we evaluate the derived expression (76), where the mean steady state step-size is obtained theoretically. We can easily see that there is a good match between simulation and theory. The experimental values for MSD and EMSE in steady state are plotted in Figure 9 and Figure 10, respectively. It can be seen that theoretical results differ slightly from simulated results because we assume that the step sizes are independent of input regressors and errors in the derivation of MSD and EMSE (i.e., Assumption 1). More precisely, the theoretical MSD and EMSE are smaller than the simulation ones. By analyzing (78), steady state MSD is achieved in an equalization way (i.e.,  $\eta_k = \eta_l$  for  $k \neq l$ ), which is also confirmed by both the simulation and theory results shown in Figure 9. By analyzing (79), the EMSE is more sensitive to the level of input data as depicted in Figure 10. Moreover, we can see that two empirical results

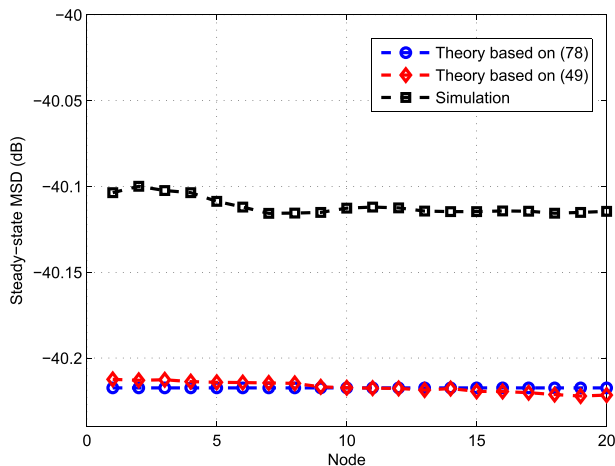


FIGURE 9. Simulation and theory of MSD at each agent.

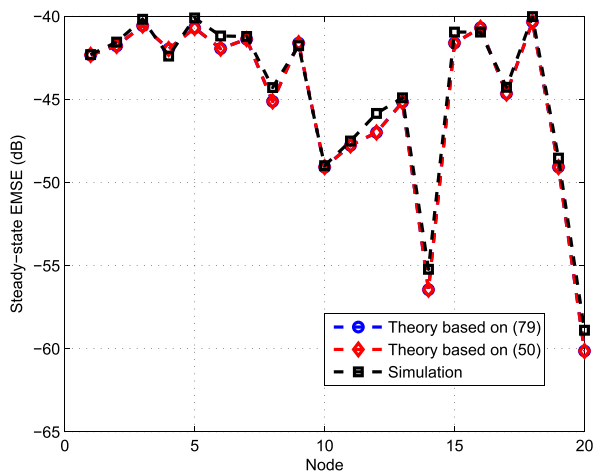


FIGURE 10. Simulation and theory of MSD at each agent.

based on (78) and (49) are very close. Thus, the approximation for  $F_k$  is reasonable because the square of steady state step size is very small as shown in Figure 8. From Figure 10, furthermore, we can even see the experimental values based on (79) and (50) are overlapped.

**B. APPLICATION TO TARGET LOCALIZATION**

The adaptive estimation model (1)-(3) is useful for some practical applications in smart cities, for example, target localization and collaborative spectral sensing in wireless sensor networks [47], complex behavior in biological and social networks [48]. In this subsection, the proposed algorithm is applied to the parameter estimation for a target position in a wireless sensor network, where the agents can be referred as sensor nodes. It is well known that target localization is a significant determinant of success for some techniques in the realm of sensor networks, such as position-based routing, coverage and connectivity protocols.

Given the target localization model [7], [47] by using the LMS estimation method, we introduce the following notations that are consistent with those presented in the previous sections. The estimated position column vector  $w^o$  with size of 2 is denoted as the actual location of target in the

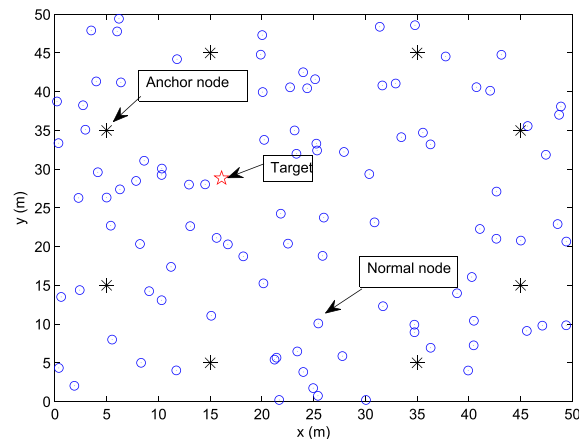


FIGURE 11. An illustration of the simulated sensor network deployed for target localization.

two-dimensional coordinate. The  $N$  anchor nodes that know their own positions are deployed in the monitored area. The collected measurements  $u_{k,i}$  denote the noisy direction vector pointing from the anchor node  $k = 1, \dots, N$  towards the target and are modeled as

$$u_{k,i} = u_k^o + a_k(i)u_k^{o\perp} + b_k(i)u_k^o, \tag{80}$$

where  $u_k^o$  and  $u_k^{o\perp}$  are the unit direction vector and the perpendicular unit direction vector of  $u_{k,i}$ , respectively,  $a_k(i)$  and  $b_k(i)$  are zero-mean spatially independent random noises, each of which has the variance  $\sigma_{a,k}^2$  and  $\sigma_{b,k}^2$ . The other measurements  $d_k(i)$  are defined as

$$d_k(i) \triangleq u_{k,i}w^o + v_k(i), \tag{81}$$

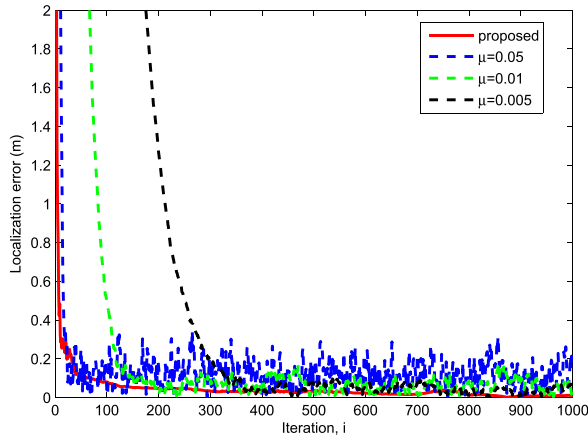
where  $v_k(i)$  denotes the noise on distance from anchor  $k$  to target at time  $i$ , which is zero-mean and spatially independent with variance  $\sigma_{v,k}^2$ . In our simulation,  $d_k(i)$  can be obtained by

$$d_k(i) = r_k^i + u_{k,i}p_k, \tag{82}$$

where  $p_k$  is the known position vector of anchor node  $k$ ,  $r_k^i$  is the the noisy distance measurement between anchor node  $k$  and target.

As a result, the position parameter vector  $w^o$  of a random target can be tracked by using the proposed VSS-ILMS algorithm with the desired convergence rate and localization accuracy. Figure 11 is an illustration of the simulated sensor network that consists of 100 normal nodes that are randomly distributed in a square area of  $50m \times 50m$  and 8 anchor nodes that know their positions exactly and communicate in a way of Hamiltonian cycle. The noise variances  $\sigma_{v,k}^2 \in (0, 2]$ ,  $\sigma_{a,k}^2$  and  $\sigma_{b,k}^2 \in (0, 0.1]$  are chosen randomly.

Figure 12 shows the changes on localization error of VSS-ILMS compared to ILMS algorithm with different fixed step-size. From Figure 12, we see that the convergence performance is similar to those in Figure 3. It is important to note that the huge oscillation appears in ILMS algorithm when the step-size is large, which can also be found in Figure 3. The steady-state oscillation is defined as the MSD deviation



**FIGURE 12.** Localization errors of ILMS algorithm with different fixed step-size and proposed algorithm.

between the successive iterations after the convergence. That happens because the amplitude of oscillation is proportional to the step-size in ILMS algorithm [7]. On the other hand, the small step-size can reduce the amplitude of oscillation but result in the slow convergence. As shown in Figure 12, both high convergence speed and low oscillation are achieved by our algorithm since the proposed variable step-size follow adaptively the underlying data changes.

## VI. CONCLUSIONS AND FUTURE WORK

In this paper, we presented a novel variable step-size incremental LMS algorithm, in which a local optimal step size is estimated at each node in order to achieve the minimum MSD per iteration. The proposed algorithm solve the problem of performance degradation resulting from the constant step size when difference of noise level appears between nodes. The other benefit of our step size update scheme is that the tradeoff between fast convergence rate and low MSD for conventional ILMS algorithm is overcome. The steady-state performance of proposed algorithm is analyzed by deriving the expressions of the steady-state step-size, MSD and EMSE in a closed form. The advantages of proposed algorithm are also showed in simulation results by comparing it with ILMS algorithms with different constant step size and several classical stand-alone LMS filtering algorithms applied in incremental cooperative network. The theoretical results are verified by simulations one by one. Moreover, our algorithm is also verified by the model of target localization in sensor networks. In future work, we will consider the effective combination of a lightweight incremental construction mode and distributed estimation to achieve better performance based on the sensing data in smart cities.

## REFERENCES

- [1] L.-J. Chen et al. "An open framework for participatory PM2.5 Monitoring in smart cities," *IEEE Access*, vol. 5, pp. 14441–14454, 2017.
- [2] T. S. Brisimi, C. G. Cassandras, C. Osgood, I. C. Paschalidis, and Y. Zhang, "Sensing and classifying roadway obstacles in smart cities: The street bump system," *IEEE Access*, vol. 4, pp. 1301–1312, 2016.

- [3] A. Bertrand and M. Moonen, "Consensus-based distributed total least squares estimation in ad hoc wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 59, no. 5, pp. 2320–2330, May 2011.
- [4] C. G. Lopes and A. H. Sayed, "Incremental adaptive strategies over distributed networks," *IEEE Trans. Signal Process.*, vol. 55, no. 8, pp. 4064–4077, Aug. 2007.
- [5] L. Li, J. A. Chambers, C. G. Lopes, and A. H. Sayed, "Distributed estimation over an adaptive incremental network based on the affine projection algorithm," *IEEE Trans. Signal Process.*, vol. 58, no. 1, pp. 151–164, Jan. 2010.
- [6] F. S. Cattivelli and A. H. Sayed, "Analysis of spatial and incremental LMS processing for distributed estimation," *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1465–1480, Apr. 2011.
- [7] M. Wu, L. Tan, R. Yang, and R. Wan, "An improved incremental least-mean-squares algorithm for distributed estimation over wireless sensor networks," *Int. J. Distrib. Sensor Netw.*, vol. 13, no. 4, 2017, doi: 10.1177/1550147717703967.
- [8] J. Chen, C. Richard, and A. H. Sayed, "Diffusion LMS over multitask networks," *IEEE Trans. Signal Process.*, vol. 63, no. 11, pp. 2733–2748, Jun. 2015.
- [9] W. Wang and H. Zhao, "Diffusion signed LMS algorithms and their performance analyses for cyclostationary white gaussian inputs," *IEEE Access*, vol. 5, pp. 18876–18894, 2017.
- [10] A. H. Sayed and C. G. Lopes, "Adaptive processing over distributed networks," *IEICE Trans. Fundam. Electron., Commun. Comput. Sci.*, vol. 90, no. 8, pp. 1504–1510, 2007.
- [11] N. Takahashi and I. Yamada, "Incremental adaptive filtering over distributed networks using parallel projection onto hyperslabs," *Tech. Rep. IEICE*, vol. 108, no. 108, pp. 17–22, 2008.
- [12] X. Zhao and A. H. Sayed, "Performance limits for distributed estimation over LMS adaptive networks," *IEEE Trans. Signal Process.*, vol. 60, no. 10, pp. 5107–5124, Oct. 2012.
- [13] R. H. Kwong and E. W. Johnston, "A variable step size LMS algorithm," *IEEE Trans. Signal Process.*, vol. 40, no. 7, pp. 1633–1642, Jul. 1992.
- [14] T. Aboulnasr and K. Mayyas, "A robust variable step-size LMS-type algorithm: Analysis and simulations," *IEEE Trans. Signal Process.*, vol. 45, no. 3, pp. 631–639, Mar. 1997.
- [15] S. Koike, "A class of adaptive step-size control algorithms for adaptive filters," *IEEE Trans. Signal Process.*, vol. 50, no. 6, pp. 1315–1326, Jun. 2002.
- [16] H.-C. Shin, A. H. Sayed, and W.-J. Song, "Variable step-size NLMS and affine projection algorithms," *IEEE Signal Process. Lett.*, vol. 11, no. 2, pp. 132–135, Feb. 2004.
- [17] L. R. Vega, H. Rey, J. Benesty, and S. Tressens, "A new robust variable step-size NLMS algorithm," *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 1878–1893, May 2008.
- [18] J. Benesty, H. Rey, L. R. Vega, and S. Tressens, "A nonparametric vss nLMS algorithm," *IEEE Signal Process. Lett.*, vol. 13, no. 10, pp. 581–584, Oct. 2006.
- [19] H. C. Huang and J. Lee, "A new variable step-size nLMS algorithm and its performance analysis," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 2055–2060, Apr. 2012.
- [20] J. Liu, X. Yu, and H. Li, "A nonparametric variable step-size nLMS algorithm for transversal filters," *Appl. Math. Comput.*, vol. 217, no. 17, pp. 7365–7371, 2011.
- [21] G. Gui, W. Peng, L. Xu, B. Liu, and F. Adachi, "Variable-step-size based sparse adaptive filtering algorithm for channel estimation in broadband wireless communication systems," *EURASIP J. Wireless Commun. Netw.*, vol. 2014, no. 1, p. 195, 2014.
- [22] G. Gui, A. Mehdodniya, and F. Adachi, "Sparse lms/f algorithms with application to adaptive system identification," *Wireless Commun. Mobile Comput.*, vol. 15, no. 12, pp. 1649–1658, 2015.
- [23] N. Bogdanovic, J. Plata-Chaves, and K. Berberidis, "Distributed incremental-based LMS for node-specific adaptive parameter estimation," *IEEE Trans. Signal Process.*, vol. 62, no. 20, pp. 5382–5397, Oct. 2014.
- [24] A. Khalili, A. Rastegarnia, J. A. Chambers, and W. M. Bazzi, "An optimum step-size assignment for incremental LMS adaptive networks based on average convergence rate constraint," *AEU-Int. J. Electron. Commun.*, vol. 67, no. 3, pp. 263–268, 2013.
- [25] A. Khalili, M. A. Tinati, and A. Rastegarnia, "Performance analysis of distributed incremental LMS algorithm with noisy links," *Int. J. Distrib. Sensor Netw.*, vol. 7, no. 1, p. 756067, 2011.
- [26] A. Rastegarnia, M. A. Tinati, and A. Khalili, "Performance analysis of quantized incremental LMS algorithm for distributed adaptive estimation," *Signal Process.*, vol. 90, no. 8, pp. 2621–2627, 2010.

- [27] A. H. Sayed, *Adaptive Filters*. Hoboken, NJ, USA: Wiley, 2011.
- [28] P. C. Hansen, "Analysis of discrete ill-posed problems by means of the L-curve," *SIAM Rev.*, vol. 34, no. 4, pp. 561–580, Dec. 1992.
- [29] H. S. Lee, S. E. Kim, J. W. Lee, and W. J. Song, "A variable step-size diffusion LMS algorithm for distributed estimation," *IEEE Trans. Signal Process.*, vol. 63, no. 7, pp. 1808–1820, Apr. 2015.
- [30] M. Vinyals, J. A. Rodriguez-Aguilar, and J. Cerquides, "A survey on sensor networks from a multiagent perspective," *Comput. J.*, vol. 54, no. 3, pp. 455–470, 2010.
- [31] T. Rault, A. Bouabdallah, and Y. Challal, "Energy efficiency in wireless sensor networks: A top-down survey," *Comput. Netw.*, vol. 67, pp. 104–122, Jul. 2014.
- [32] J. Zheng, P. Wang, and C. Li, "Distributed data aggregation using Slepian-Wolf coding in cluster-based wireless sensor networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 5, pp. 2564–2574, Jun. 2010.
- [33] H. Jiang, S. Jin, and C. Wang, "Prediction or not? An energy-efficient framework for clustering-based data collection in wireless sensor networks," *IEEE Trans. Parallel Distrib. Syst.*, vol. 22, no. 6, pp. 1064–1071, Jun. 2011.
- [34] E. M. Arkin, S. P. Fekete, and J. S. Mitchell, "Approximation algorithms for lawn mowing and milling," *Comput. Geometry*, vol. 17, no. 1, pp. 25–50, 2000.
- [35] V. G. Deri et al., "Exact algorithms for the hamiltonian cycle problem in planar graphs," *Operat. Res. Lett.*, vol. 34, no. 3, pp. 269–274, 2006.
- [36] A. Gasparri, B. Krishnamachari, and G. S. Sukhatme, "A framework for multi-robot node coverage in sensor networks," *Ann. Math. Artif. Intell.*, vol. 52, nos. 2–4, pp. 281–305, 2008.
- [37] S. Anbuudayasankar, K. Ganesh, and S. Mohapatra, "Survey of methodologies for TSP and VRP," in *Models for Practical Routing Problems Logistics*. Springer, 2014, pp. 11–42.
- [38] A. Liu, X. Jin, G. Cui, and Z. Chen, "Deployment guidelines for achieving maximum lifetime and avoiding energy holes in sensor network," *Inf. Sci.*, vol. 230, pp. 197–226, May 2013.
- [39] M. Wu, L. Tan, and N. Xiong, "A structure fidelity approach for big data collection in wireless sensor networks," *Sensors*, vol. 15, no. 1, pp. 248–273, Jan. 2015.
- [40] C. Zhu, C. Zheng, L. Shu, and G. Han, "A survey on coverage and connectivity issues in wireless sensor networks," *J. Netw. Comput. Appl.*, vol. 35, no. 2, pp. 619–632, 2012.
- [41] P. Jiang, "A new method for node fault detection in wireless sensor networks," *Sensors*, vol. 9, no. 2, pp. 1282–1294, 2009.
- [42] F. S. Cattivelli and A. H. Sayed, "Diffusion LMS strategies for distributed estimation," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1035–1048, Mar. 2010.
- [43] C. G. Lopes and A. H. Sayed, "Distributed adaptive incremental strategies: Formulation and performance analysis," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, vol. 3, May 2006, pp. 584–587.
- [44] K. Mayyas and F. Momani, "An LMS adaptive algorithm with a new step-size control equation," *J. Franklin Inst.*, vol. 348, no. 4, pp. 589–605, 2011.
- [45] L. Ning, Z. Yonggang, and H. Yanling, "Steady state performance analysis of an approximately optimal variable step size LMS algorithm," in *Proc. 3rd IEEE Conf. Ind. Electron. Appl. (ICIEA)*, Jun. 2008, pp. 1379–1382.
- [46] H.-C. Shin and A. H. Sayed, "Mean-square performance of a family of affine projection algorithms," *IEEE Trans. Signal Process.*, vol. 52, no. 1, pp. 90–102, Jan. 2004.
- [47] A. H. Sayed, "Diffusion adaptation over networks," in *Academic Press Library in Signal Processing*, vol. 3. Amsterdam, The Netherlands: Elsevier, 2014.
- [48] A. H. Sayed, "Adaptation, learning, and optimization over networks," *Found. Trends Mach. Learn.*, vol. 7, nos. 4–5, pp. 311–801, 2014.



**NEAL N. XIONG** received the Ph.D. degrees in sensor system engineering from Wuhan University and in dependable sensor networks from the Japan Advanced Institute of Science and Technology, respectively. He was with Georgia State University, Wentworth Technology Institution, and Colorado Technical University (as a Full Professor for five years) about 10 years. He is currently an Associate Professor (with three-year credits) with the Department of Mathematics and Computer

Science, Northeastern State University, Tahlequah, OK, USA. His research interests include cloud computing, security and dependability, parallel and distributed computing, networks, and optimization theory.

Dr. Xiong has published over 300 international journal papers and over 100 international conference papers. Some of his papers were published in the IEEE JSAC, the IEEE transactions, or the ACM transactions, the ACM Sigcomm Workshop, the IEEE INFOCOM, ICDCS, and IPDPS. He has received the Best Paper Award at the 10th IEEE International Conference on High Performance Computing and Communications (HPCC-08) and the Best Student Paper Award at the 28th North American Fuzzy Information Processing Society Annual Conference (NAFIPS2009). He has been the General Chair, the Program Chair, the Publicity Chair, a PC Member, and a OC Member of over 100 international conferences, and as a reviewer of about 100 international journals, including the IEEE JSAC, the IEEE SMC (Park: A/B/C), the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE TRANSACTIONS ON MOBILE COMPUTING, the IEEE TRANSACTIONS ON PARALLEL AND DISTRIBUTED SYSTEMS. He is serving as the Editor-in-Chief, an Associate Editor or Editor Member for over 10 international journals (including an Associate Editor for the IEEE TRANSACTIONS ON SYSTEMS, MAN CYBERNETICS: SYSTEMS, an Associate Editor for the *Information Science*, the Editor-in-Chief of the *Journal of Internet Technology*, and the Editor-in-Chief of the *Journal of Parallel Cloud Computing*, and a Guest Editor for over 10 international journals, including the *Sensor Journal*, WINET, and MONET.

Dr. Xiong is a Senior Member of IEEE Computer Society. He is the Chair of Trusted Cloud Computing Task Force, the IEEE Computational Intelligence Society, and the Industry System Applications Technical Committee.



**LIANSHENG TAN** received the Ph.D. degree from Loughborough University, U.K., in 1999. He is currently a Professor with the Department of Computer Science, Central China Normal University. He was a Research Fellow with the Research School of Information Sciences and Engineering, The Australian National University, Australia, from 2006 to 2009, and a Post-Doctoral Research Fellow with the School of Information Technology and Engineering, University of Ottawa, Canada,

in 2001. He held a number of visiting research positions at Oxford University, Loughborough University, University of Tsukuba, City University of Hong Kong, and the University of Melbourne. He has published two monographs with the Elsevier and Taylor Francis and over 130 referred papers in top venues. His research interests include modeling, congestion control analysis and performance evaluation of computer communication networks, resource allocation and management of wireless and wireline networks and routing and transmission control protocols. He was an Editor of the *Dynamics of Continuous, Discrete Impulsive Systems* (Series B: Applications Algorithms) from 2006 to 2008 and an Editor of the *International Journal of Communication Systems* from 2003 to 2008. He is currently the Editor-in-Chief of the *Journal of Computers* and an Editor of the *International Journal of Computer Networks and Communications*.

• • •



**MOU WU** received the Ph.D. degree in radio physics from Central China Normal University in 2015. Since 2015, he has been with the School of Computer Science and Technology, Hubei University of Science and Technology. His research interests include wireless sensor networks, distributed algorithms for performance optimization over network, and computer communication.