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Fixed-Time Synchronization of Memristive Fuzzy BAM Cellular Neural Networks With Time-Varying Delays Based on Feedback Controllers

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ABSTRACT This paper mainly investigates the fixed-time synchronization problem of memristive fuzzy bidirectional associative memory (BAM) cellular neural networks (MFBAMCNNs) with time-varying delays. MFBAMCNNs are formulated by virtue of differential inclusion and set-valued map theories. By utilizing the definition of fixed-time synchronization and some inequality techniques, some novel criteria for easy verification are derived to ensure the fixed-time synchronization of the drive-response MFBAMCNNs based on the Lyapunov stability theory and nonlinear feedback controllers. The results of the main theorem can be easily extended to the fuzzy BAM cellular neural networks without memristor and the memristive BAM cellular neural networks without fuzzy logic. In addition, the settling time of fixed-time synchronization, which does not depend on the initial values, can be simply calculated. At last, two numerical examples are presented to verify the effectiveness of main results.

INDEX TERMS Fixed-time synchronization, memristor, MFBAMCNNs, Lyapunov function, nonlinear feedback controller.

I. INTRODUCTION

In 1987, Kosok firstly proposed the bidirectional associative memory neural networks (BAMNNs), which are made up of two neuron layers, i.e. X-layer and Y-layer, and have the features of heteroassociative, content-addressable memory [1]. These two layers of neurons are completely interconnected, and there is no signal transmission between neurons on the same layer. In real life, BAMNNs have powerful information processing abilities and some good application fields, such as information associative memory, image processing, artificial intelligence, and so on. Thus, many scholars have studied various forms of BAMNNs models [2]–[6]. The memristor (memory resistor), which was proposed by Chua for the first time [7], is well-known as the forth basic passive nonlinear circuit element (please see Fig. 1).

Until the HP labs developed the real memristor in 2008 [8] (please see Fig. 2), it was widely recognized [2], [9]–[12]. The memristor has the characteristics of nanometer size, nonvolatile and hysteresis loops, which make it work like neuronal synapses. Based on the advantages of the memristor, it can replace the traditional resistors to simulate the biological synapses, and the synaptic density in the memristive neural network is further moved closer to the biological neural network. Naturally, the researchers combine the memristor with BAMNNs to form a class of neural networks named memristive bidirectional associative memory neural networks (MBAMNNs), which have been extensively investigated in the aspect of its dynamic behaviors [4], [5], [13]–[18]. For example, Ali *et al.* investigated a class of memristor-based neutral-type stochastic bidirectional associative memory

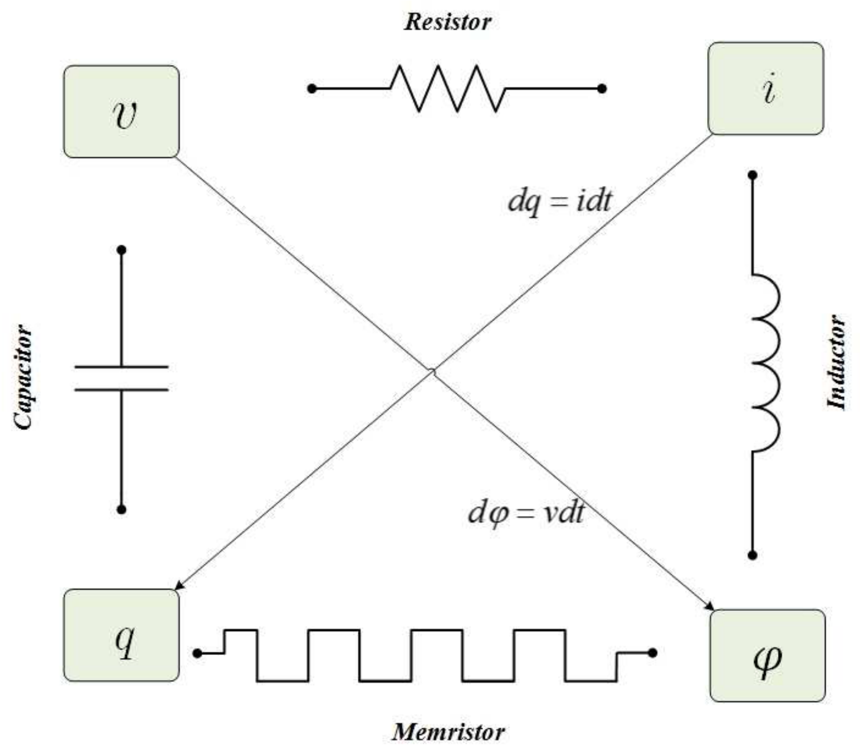

FIGURE 1. The position of memristor in four basic circuit elements.

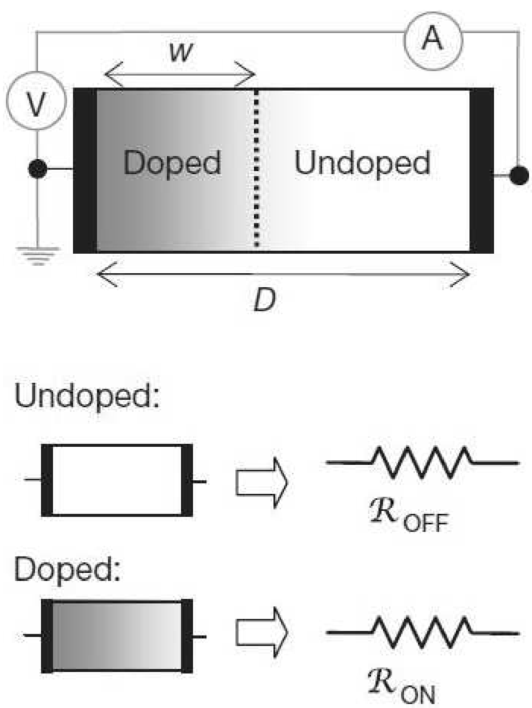

FIGURE 2. The schematic diagram of memristor developed by HP Laboratory [8].

neural networks; with the aid of a new Lyapunov-Krasovskii function, they obtained some delay-dependent passivity LMI inequality conditions [14]. Wang *et al*. discussed the global asymptotic stability of drive-response MBAMNNs with different delays based on a novel sampled-data feedback controller [5].

Since uncertainties or vagueness are unavoidable in the real world, Yang *et al*. combined fuzzy theory with cellular neural networks as a suitable method to deal with them, thus forming the fuzzy cellular neural networks (FCNNs) [19], [20]. The stability of FCNNs plays an important role in image processing and pattern recognition applications. Many scholars have done extensive researches on FCNNs [21]–[24]. As far as we know, few researchers have considered the combination of FCNNs and MBAMNNs to form a new class of memristive fuzzy BAM cellular neural networks (MFBAMCNNs). In [25], Balasubramaniam *et al*. studied the global asymptotic stability of a class of bidirectional associative memory fuzzy cellular neural networks with multiple types of delays by using the free-weighting matrix method. Xu and Li investigated the exponential stability of FBAMCNNs with delays and impulses [26]. However, the above research objects are not MFBAMCNNs. Compared to existing literature, MFBAMCNNs have more complex network structures and dynamic behaviors due to the addition of memristor and fuzzy logic. There are two main difficulties in the study of MFBAMCNNs. First, the addition of the memristor model, which makes MFBAMCNNs become a right-hand discontinuous switching differential equations, and traditional research methods do not suitable for such systems. Second, the addition of fuzzy logic makes MFBAMCNNs more complex, and how to deal with the fuzzy item effectively is a problem worth considering. This is one of the motivations of this paper.

On the other hand, since Pecora and Carrol first introduced the drive-response concept to synchronize two chaotic systems [27], the synchronization control of chaotic systems has been widely used in secure communication, image encryption, motion control, and so on. As a result, many synchronization control strategies and schemas are proposed for chaotic systems [4], [12], [13], [15], [23], [28]–[30]. In these synchronization studies of chaotic neural networks, the convergence rate (time) is a key consideration of synchronization performance indicator. According to the convergence time, synchronization forms mainly include the asymptotic (exponential) synchronization, finite-time synchronization and fixed-time synchronization. Among them, the asymptotic synchronization is based on the enough large convergence time, and the finite-time and fixed-time synchronization can synchronize the drive-response chaotic systems in a finite time, which is called the settling time. In practical applications, we sometimes expect the drive-response systems to achieve synchronization as quickly as possible. The critical difference between the finite-time and fixed-time synchronization is whether the settling time depends on the initial conditions of the drive-response systems. The finite-time synchronization depends on the initial conditions, while the fixed-time synchronization does not depend on it. In many practical cases, it is not easy to obtain the initial conditions of the chaotic systems, which make the settling time difficult to be determined. The fixed-time synchronization is derived from the definition of fixed-time stability, which was proposed by Parsegov *et al.* [31] and Polyakov [32] for the first time, can overcome this difficulty. In [28], Wan *et al.* investigated the fixed-time master-slave synchronization of Cohen-Grossberg neural networks with time-varying delays and parameter uncertainties by virtue of Filippov discontinuous theory and Lyapunov stability theory, and obtained some sufficient conditions to ensure fixed-time synchronization of master-slave systems. In [33], Liu and Chen discussed the finite-time and fixed-time cluster synchronization of complex networks under pinning control. However, there are few results on the fixed-time synchronization of MFBAMCNNs. In addition, how to design a suitable fixed-time synchronization controller for drive-response MFBAMCNNs systems is a challenging issue. This is another research motivation of this paper.

Motivated by the above discusses, we combine fuzzy cellular neural networks with MBAMNNs to form a new class of neural network model and study its fixed-time synchronization problem of the drive-response systems for the first time. With the help of the definition of fixed-time synchronization, differential inclusion theory, set-valued map, some inequalities techniques and Lyapunov stability theory, some easily verifiable sufficient conditions are obtained. In addition, it is worth noting that the settling time that does not depend on the initial conditions can be easily calculated directly. The main contributions of this paper are summarized as follows.

(1) Compared with the existing results on fixed-time synchronization of other neural network models [28], [34], the fixed-time synchronization problem of MFBAMCNNs with time-varying delays is studied for the first time. The existing BAMNNs models without the memristor or without the fuzzy logic can be regarded as a special case of the MFBAMCNNs model.

(2) By the construction of nonlinear feedback controllers and a simple Lyapunov function, some novel easily verifiable algebraic inequality conditions are obtained to ensure the fixed-time synchronization of the proposed model.

(3) The main results of this paper are more general. It can be easily extended to the fuzzy BAM cellular neural networks without memristor and the memristive BAM cellular neural networks without fuzzy logic.

The remainder of this paper is organized as follows. In Section II, the MFBAMCNNs mathematical model, the definition of the fixed-time synchronization, some assumptions and lemmas are presented as preliminaries. Through strict proof, some novel criteria to guarantee the drive-response MFBAMCNNs achieve the fixed-time synchronization are obtained in Section III. After two numerical simulations illustrate the validity of our results in Section IV, we sum up the whole paper in Section V.

Notations: In the whole paper, $\mathcal{R}, \mathcal{R}^m, \mathcal{R}^n$ denote real number set, m-dimensional Euclidean space and n-dimensional Euclidean space, respectively. $\mathcal{R}_+$ represents positive real number set. $\mathcal{C}([-\tau, 0], \mathcal{R}^n), \mathcal{C}([-\sigma, 0], \mathcal{R}^n)$ denotes a set of continuous function from interval $[-\tau, 0]$ and $[-\sigma, 0]$ to space $\mathcal{R}^n$. Let $\mathcal{I} \triangleq \{1, 2, \cdots, n\}$, $\mathcal{J} \triangleq \{1, 2, \cdots, m\}$ be a natural number set, respectively.

II. NETWORK MODEL AND PRELIMINARIES

Inspired by [14] and [35], we consider a class of the MFBAMCNNs model described by the following differential equations

$$\begin{aligned}
\dot{x}_i(t) &= -d_i^{(1)}(x_i(t))x_i(t) + \sum_{j=1}^{m} a_{ij}^{(1)}(x_i(t))f_j(y_j(t)) \\
&\quad + \sum_{j=1}^{m} b_{ij}^{(1)}(x_i(t-\sigma(t)))f_j(y_j(t-\tau(t))) \\
&\quad + \sum_{j=1}^{m} c_{ij}^{(1)}w_j^{(1)} + \bigwedge_{j=1}^{m} \alpha_{ij}^{(1)}f_j(y_j(t-\tau(t))) \\
&\quad + \bigvee_{j=1}^{m} \beta_{ij}^{(1)}f_j(y_j(t-\tau(t))) + \bigvee_{j=1}^{m} S_{ij}^{(1)}w_j^{(1)} \\
&\quad + \bigwedge_{j=1}^{m} T_{ij}^{(1)}w_j^{(1)} + I_i^{(1)}, \\
\dot{y}_j(t) &= -d_j^{(2)}(y_j(t))y_j(t) + \sum_{i=1}^{n} a_{ji}^{(2)}(y_j(t))g_i(x_i(t)) \\
&\quad + \sum_{i=1}^{n} b_{ji}^{(2)}(y_j(t-\tau(t)))g_i(x_i(t-\sigma(t)))
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{n} c_{ji}^{(2)} w_i^{(2)} + \bigwedge_{i=1}^{n} \alpha_{ji}^{(2)} g_i(x_i(t-\sigma(t)))\\
&+\bigvee_{i=1}^{n} \beta_{ji}^{(2)} g_i(x_i(t-\sigma(t))) + \bigvee_{i=1}^{n} S_{ji}^{(2)} w_i^{(2)}\\
&+\bigwedge_{i=1}^{n} T_{ji}^{(2)} w_i^{(2)} + I_j^{(2)}, \quad t \geq 0, \qquad (1)
\end{aligned}
$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$, $y(t) = (y_1(t), y_2(t), \cdots, y_m(t))^T$ denote the states of neurons in X-layer and Y-layer (the voltage of the capacitors $\mathcal{C}_i^{(1)}$ and $\mathcal{C}_j^{(2)}$ in the actual circuits), respectively. $i \in \mathcal{I}, j \in \mathcal{J}$. $f_j(\cdot), g_i(\cdot)$ represent the activation functions of the jth and ith neurons. $\bigvee$ and $\bigwedge$ represent fuzzy OR and fuzzy AND operations. $\alpha_{ij}^{(1)}(\alpha_{ji}^{(2)})$ and $\beta_{ij}^{(1)}(\beta_{ji}^{(2)})$ are respectively the connection weights of the fuzzy feedback MIN template and the fuzzy feedforward MAX template with delays. $c_{ij}^{(1)}(c_{ji}^{(2)})$, $S_{ij}^{(1)}(S_{ji}^{(2)})$ and $T_{ij}^{(1)}(T_{ji}^{(2)})$ denote the connection weights of the fuzzy feedforward template, the elements of the fuzzy feedforward MIN template and fuzzy feedforward MAX template without delays. $\tau(t), \sigma(t)$ are the time-varying delays, which meet $0 \leq \tau(t) \leq \tau$, $0 \leq \sigma(t) \leq \sigma$. $I_i^{(1)}$ and $I_j^{(2)}$ denote the bias values of the ith and jth neurons.

$d_i^{(1)}(x_i(t))$, $d_j^{(2)}(x_j(t))$, $a_{ij}^{(1)}(x_i(t))$, $b_{ij}^{(1)}(x_i(t))$, $a_{ji}^{(2)}(x_j(t))$, $b_{ji}^{(2)}(x_j(t))$ denote the memristive weights. In the memristor-based neural network real circuits, they are given as follows

$$
\begin{aligned}
d_i^{(1)}(x_i(t)) &= \frac{1}{\mathcal{C}_i^{(1)}} \sum_{j=1}^{m} (\mathcal{W}_{ij}^{(1)} + \mathcal{M}_{ij}^{(1)}) \times sgn_{ij} + \frac{1}{\mathcal{R}_i^{(1)}},\\
d_j^{(2)}(x_j(t)) &= \frac{1}{\mathcal{C}_j^{(2)}} \sum_{i=1}^{n} (\mathcal{W}_{ji}^{(2)} + \mathcal{M}_{ji}^{(2)}) \times sgn_{ji} + \frac{1}{\mathcal{R}_j^{(2)}},\\
a_{ij}^{(1)}(x_i(t)) &= \frac{\mathcal{W}_{ij}^{(1)}}{\mathcal{C}_i^{(1)}} \times sgn_{ij},\\
b_{ij}^{(1)}(x_i(t)) &= \frac{\mathcal{M}_{ij}^{(1)}}{\mathcal{C}_i^{(1)}} \times sgn_{ij},\\
a_{ji}^{(2)}(y_j(t)) &= \frac{\mathcal{W}_{ji}^{(2)}}{\mathcal{C}_j^{(2)}} \times sgn_{ji},\\
b_{ji}^{(2)}(y_j(t)) &= \frac{\mathcal{M}_{ji}^{(2)}}{\mathcal{C}_j^{(2)}} \times sgn_{ji},
\end{aligned}
$$

where sgn_{ij} is the symbolic function with a value of 1 when $i = j$ and 0 otherwise. $\mathcal{C}_i^{(1)}$ and $\mathcal{C}_j^{(2)}$ are capacitors. $\mathcal{W}_{ij}^{(1)}, \mathcal{M}_{ij}^{(1)}, \mathcal{W}_{ji}^{(2)}, \mathcal{M}_{ji}^{(2)}$ denote the memductances of the resistors $\mathcal{R}_{ij}^{(11)}, \mathcal{R}_{ij}^{(12)}, \mathcal{R}_{ji}^{(21)}, \mathcal{R}_{ji}^{(22)}$, respectively. $\mathcal{R}_{ij}^{(11)}$ is the resistor between $x_i(t)$ and the activation function $f_j(y_j(t))$; $\mathcal{R}_{ij}^{(12)}$ is the resistor between $x_i(t)$ and the activation function $f_j(y_j(t-\tau_j(t)))$; $\mathcal{R}_{ji}^{(21)}$ is the resistor between $y_j(t)$ and the activation function $f_i(x_i(t))$; $\mathcal{R}_{ji}^{(22)}$ is the resistor between $y_j(t)$ and the activation function $g_i(y_i(t-\sigma(t)))$. Furthermore, according to the feature of memristor, we use the memristor simplified model, which is shown in Fig.3. Its memristive weights is presented as follows

$$
\begin{aligned}
d_i^{(1)}(x_i(t)) &= \begin{cases} \hat{d}_i^{(1)}, & |x_i(t)| \leq \Gamma_i^{(1)},\\ \check{d}_i^{(1)}, & |x_i(t)| > \Gamma_i^{(1)}, \end{cases}\\
d_j^{(2)}(y_j(t)) &= \begin{cases} \hat{d}_j^{(2)}, & |y_j(t)| \leq \Gamma_j^{(2)},\\ \check{d}_j^{(2)}, & |y_j(t)| > \Gamma_j^{(2)}, \end{cases}\\
a_{ij}^{(1)}(x_i(t)) &= \begin{cases} \hat{a}_{ij}^{(1)}, & |x_i(t)| \leq \Gamma_i^{(1)},\\ \check{a}_{ij}^{(1)}, & |x_i(t)| > \Gamma_i^{(1)}, \end{cases}\\
a_{ji}^{(2)}(y_j(t)) &= \begin{cases} \hat{a}_{ji}^{(2)}, & |y_j(t)| \leq \Gamma_j^{(2)},\\ \check{a}_{ji}^{(2)}, & |y_j(t)| > \Gamma_j^{(2)}, \end{cases}\\
b_{ij}^{(1)}(x_i(t-\sigma(t))) &= \begin{cases} \hat{b}_{ij}^{(1)}, & |x_i(t-\sigma(t))| \leq \Gamma_i^{(1)},\\ \check{b}_{ij}^{(1)}, & |x_i(t-\sigma(t))| > \Gamma_i^{(1)}, \end{cases}\\
b_{ji}^{(2)}(y_j(t-\tau(t))) &= \begin{cases} \hat{b}_{ji}^{(2)}, & |y_j(t-\tau(t))| \leq \Gamma_j^{(2)},\\ \check{b}_{ji}^{(2)}, & |y_j(t-\tau(t))| > \Gamma_j^{(2)}, \end{cases} \qquad (2)
\end{aligned}
$$

where the switching jumps $\Gamma_i^{(1)} > 0$, $\Gamma_j^{(2)} > 0$, $\hat{d}_i^{(1)}$, $\check{d}_i^{(1)}$, $\hat{d}_i^{(2)}$, $\check{d}_i^{(2)}$, $\hat{a}_{ij}^{(1)}$, $\check{a}_{ij}^{(1)}$, $\hat{a}_{ji}^{(2)}$, $\check{a}_{ji}^{(2)}$, $\hat{b}_{ij}^{(1)}$, $\check{b}_{ij}^{(1)}$, $\hat{b}_{ji}^{(2)}$, $\check{b}_{ji}^{(2)}$ are known constants, $i \in \mathcal{I}, j \in \mathcal{J}$. The initial values of MFBAMCNNs (1) are give by

$$
\begin{cases} x_i(s) = \varphi_i^{(1)}(s), & -\tau \leq s \leq 0,\\ y_j(s) = \psi_j^{(1)}(s), & -\sigma \leq s \leq 0, \end{cases} \qquad (3)
$$

where $\varphi_i^{(1)}(s) \in \mathcal{C}([-\tau, 0], \mathcal{R}^n)$, $\psi_j^{(1)}(s) \in \mathcal{C}([-\sigma, 0], \mathcal{R}^m)$, i.e. $\varphi_i^{(1)}(s)$ and $\psi_j^{(1)}(s)$ are continuous function on interval $[-\tau, 0]$ and $[-\sigma, 0]$.

Obviously, the memristive fuzzy BAM cellular neural networks (1) is a differential equation with discontinuous right-sides according the definitions of the memristive connection weights. It shows that the traditional processing method of dealing with differential equations does not apply to the model (1). By virtue of differential inclusion and set-valued map theories [36], [37], the MFBAMCNNs (1) can be regarded as a state-dependent switching system as shown below

$$
\begin{aligned}
\dot{x}_i(t) \in &-co[\underline{d}_i^{(1)}, \overline{d}_i^{(1)}]x_i(t)\\
&+\sum_{j=1}^{m} co[\underline{a}_{ij}^{(1)}, \overline{a}_{ij}^{(1)}] f_j(y_j(t))\\
&+\sum_{j=1}^{m} co[\underline{b}_{ij}^{(1)}, \overline{b}_{ij}^{(1)}] f_j(y_j(t-\tau(t)))\\
&+\sum_{j=1}^{m} c_{ij}^{(1)} w_j^{(1)} + \bigwedge_{j=1}^{m} \alpha_{ij}^{(1)} f_j(y_j(t-\tau(t)))\\
&+\bigvee_{j=1}^{m} \beta_{ij}^{(1)} f_j(y_j(t-\tau(t))) + \bigvee_{j=1}^{m} S_{ij}^{(1)} w_j^{(1)}\\
&+\bigwedge_{j=1}^{m} T_{ij}^{(1)} w_j^{(1)} + I_i^{(1)},
\end{aligned}
$$

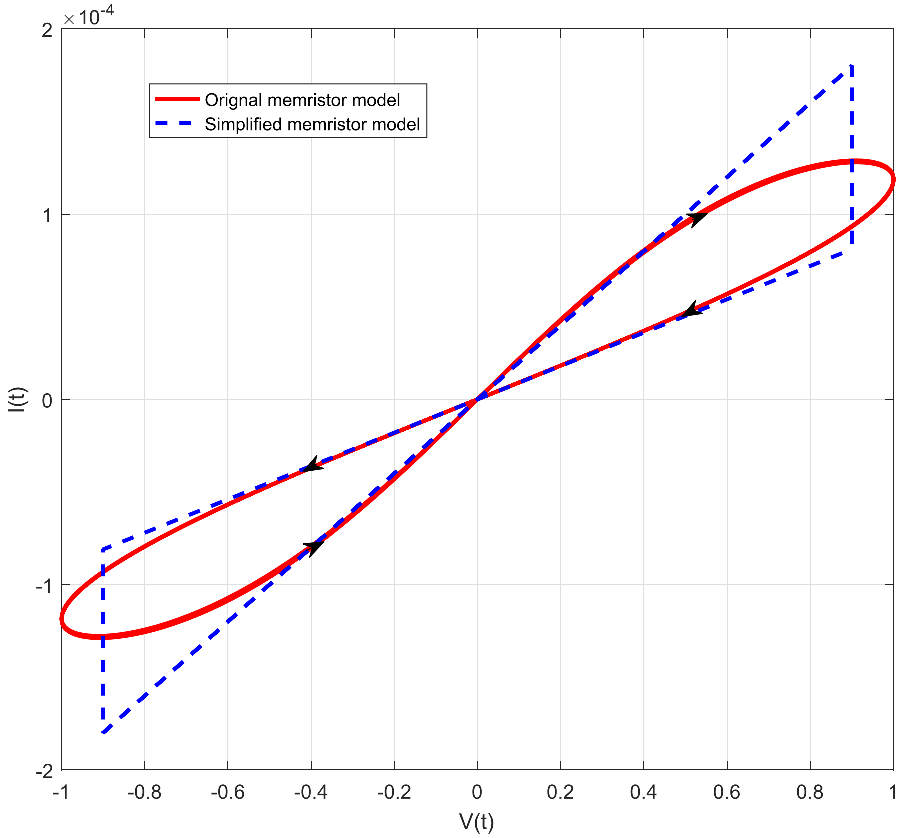

FIGURE 3. The hysteresis loop ($V(t) - I(t)$) of the original memristor model and the simplified memristor model (two-state).

$$
\begin{aligned}
\dot{y}_j(t) \in& -co[\underline{d}_j^{(2)}, \overline{d}_j^{(2)}]y_j(t) \\
&+ \sum_{i=1}^{n} co[\underline{a}_{ji}^{(2)}, \overline{a}_{ji}^{(2)}]g_i(x_i(t)) \\
&+ \sum_{i=1}^{n} co[\underline{b}_{ji}^{(2)}, \overline{b}_{ji}^{(2)}]g_i(x_i(t-\sigma(t))) \\
&+ \sum_{i=1}^{n} c_{ji}^{(2)} w_i^{(2)} + \bigwedge_{i=1}^{n} \alpha_{ji}^{(2)} g_i(x_i(t-\sigma(t))) \\
&+ \bigvee_{i=1}^{n} \beta_{ji}^{(2)} g_i(x_i(t-\sigma(t))) + \bigvee_{i=1}^{n} S_{ji}^{(2)} w_i^{(2)} \\
&+ \bigwedge_{i=1}^{n} T_{ji}^{(2)} w_i^{(2)} + I_j^{(2)}, \quad t \geq 0,
\end{aligned}
\tag{4}
$$

where $\underline{d}_i^{(1)} = \min\{\hat{d}_i^{(1)}, \check{d}_i^{(1)}\}$, $\overline{d}_i^{(1)} = \max\{\hat{d}_i^{(1)}, \check{d}_i^{(1)}\}$, $\underline{d}_j^{(2)} = \min\{\hat{d}_j^{(2)}, \check{d}_j^{(2)}\}$, $\overline{d}_j^{(2)} = \max\{\hat{d}_j^{(1)}, \check{d}_j^{(2)}\}$, $\underline{a}_{ij}^{(1)} = \min\{\hat{a}_{ij}^{(1)}, \check{a}_{ij}^{(1)}\}$, $\overline{a}_{ij}^{(1)} = \max\{\hat{a}_{ij}^{(1)}, \check{a}_{ij}^{(1)}\}$, $\underline{a}_{ji}^{(2)} = \min\{\hat{a}_{ji}^{(2)}, \check{a}_{ji}^{(2)}\}$, $\overline{a}_{ji}^{(2)} = \max\{\hat{a}_{ji}^{(2)}, \check{a}_{ji}^{(2)}\}$, $\underline{b}_{ij}^{(1)} = \min\{\hat{b}_{ij}^{(1)}, \check{b}_{ij}^{(1)}\}$, $\overline{b}_{ij}^{(1)} = \max\{\hat{b}_{ij}^{(1)}, \check{b}_{ij}^{(1)}\}$, $\underline{b}_{ji}^{(2)} = \min\{\hat{b}_{ji}^{(2)}, \check{b}_{ji}^{(2)}\}$, $\overline{b}_{ji}^{(2)} = \max\{\hat{b}_{ji}^{(2)}, \check{b}_{ji}^{(2)}\}$, $i \in \mathcal{I}, j \in \mathcal{J}$.

Let measurable functions $d_i^x(t) \in co[\underline{d}_i^{(1)}, \overline{d}_i^{(1)}]$, $d_j^y(t) \in co[\underline{d}_j^{(2)}, \overline{d}_j^{(2)}]$, $a_{ij}^x(t) \in co[\underline{a}_{ij}^{(1)}, \overline{a}_{ij}^{(1)}]$, $a_{ji}^y(t) \in co[\underline{a}_{ji}^{(2)}, \overline{a}_{ji}^{(2)}]$, $b_{ij}^x(t) \in co[\underline{b}_{ij}^{(1)}, \overline{b}_{ij}^{(1)}]$, $b_{ji}^y(t) \in co[\underline{b}_{ji}^{(2)}, \overline{b}_{ji}^{(2)}]$, then MFBAMCNNs (4) can be equivalently to the following form

$$
\begin{aligned}
\dot{x}_i(t) =& -d_i^x(t)x_i(t) + \sum_{j=1}^{m} a_{ij}^x(t) f_j(y_j(t)) \\
&+ \sum_{j=1}^{m} b_{ij}^x(t) f_j(y_j(t-\tau(t))) + \sum_{j=1}^{m} c_{ij}^{(1)} w_j^{(1)} \\
&+ \bigwedge_{j=1}^{m} \alpha_{ij}^{(1)} f_j(y_j(t-\tau(t))) \\
&+ \bigvee_{j=1}^{m} \beta_{ij}^{(1)} f_j(y_j(t-\tau(t))) + \bigvee_{j=1}^{m} S_{ij}^{(1)} w_j^{(1)} \\
&+ \bigwedge_{j=1}^{m} T_{ij}^{(1)} w_j^{(1)} + I_i^{(1)}, \\
\dot{y}_j(t) =& -d_j^y(t)y_j(t) + \sum_{i=1}^{n} a_{ji}^y(t) g_i(x_i(t)) \\
&+ \sum_{i=1}^{n} b_{ji}^y(t) g_i(x_i(t-\sigma(t))) + \sum_{i=1}^{n} c_{ji}^{(2)} w_i^{(2)} \\
&+ \bigwedge_{i=1}^{n} \alpha_{ji}^{(2)} g_i(x_i(t-\sigma(t)))
\end{aligned}
$$

$$+\bigvee_{i=1}^{n}\beta_{ji}^{(2)}g_i(x_i(t-\sigma(t)))+\bigvee_{i=1}^{n}S_{ji}^{(2)}w_i^{(2)}$$
$$+\bigwedge_{i=1}^{n}T_{ji}^{(2)}w_i^{(2)}+I_j^{(2)},\quad t\geq 0. \tag{5}$$

if the MFBAMCNNs (1) are assumed to be the drive (master) system, then the response (slave) MFBAMCNNs are designed as follows

$$\begin{aligned}\dot{u}_i(t) &= -d_i^{(1)}(u_i(t))u_i(t)+\sum_{j=1}^{m}a_{ij}^{(1)}(u_i(t))f_j(v_j(t))\\&\quad+\sum_{j=1}^{m}b_{ij}^{(1)}(u_i(t))f_j(v_j(t-\tau(t)))+\sum_{j=1}^{m}c_{ij}^{(1)}w_j^{(1)}\\&\quad+\bigwedge_{j=1}^{m}\alpha_{ij}^{(1)}f_j(v_j(t-\tau(t)))\\&\quad+\bigvee_{j=1}^{m}\beta_{ij}^{(1)}f_j(v_j(t-\tau(t)))+\bigvee_{j=1}^{m}S_{ij}^{(1)}w_j^{(1)}\\&\quad+\bigwedge_{j=1}^{m}T_{ij}^{(1)}w_j^{(1)}+I_i^{(1)}+r_i^u(t),\\\dot{v}_j(t) &= -d_j^{(2)}(v_j(t))v_j(t)+\sum_{i=1}^{n}a_{ji}^{(2)}(v_j(t))g_i(u_i(t))\\&\quad+\sum_{i=1}^{n}b_{ji}^{(2)}(v_j(t))g_i(u_i(t-\sigma(t)))+\sum_{i=1}^{n}c_{ji}^{(2)}w_i^{(2)}\\&\quad+\bigwedge_{i=1}^{n}\alpha_{ji}^{(2)}g_i(u_i(t-\sigma(t)))\\&\quad+\bigvee_{i=1}^{n}\beta_{ji}^{(2)}g_i(u_i(t-\sigma(t)))+\bigvee_{i=1}^{n}S_{ji}^{(2)}w_i^{(2)}\\&\quad+\bigwedge_{i=1}^{n}T_{ji}^{(2)}w_i^{(2)}+I_j^{(2)}+r_j^v(t),\quad t\geq 0,\end{aligned} \tag{6}$$

where $r_i^u(t)$ and $r_j^v(t)$ are the fixed-time synchronization controllers to be designed. The initial values of the MFBAMCNNs (6) are defined as

$$\begin{cases}u_i(s)=\varphi_i^{(2)}(s), & -\tau\leq s\leq 0,\\ v_j(s)=\psi_j^{(2)}(s), & -\sigma\leq s\leq 0,\end{cases} \tag{7}$$

where $\varphi_i^{(2)}(s)\in\mathcal{C}([-\tau,0],\mathcal{R}^n)$, $\psi_j^{(2)}(s)\in\mathcal{C}([-\sigma,0],\mathcal{R}^m)$, i.e. $\varphi_i^{(2)}(s)$ and $\psi_j^{(2)}(s)$ are continuous functions on interval $[-\tau,0]$ and $[-\sigma,0]$, respectively.

Using the same processing method as the model (1) by applying the set-valued map and differential inclusion theories, we have

$$\begin{aligned}\dot{u}_i(t) &= -d_i^u(t)u_i(t)+\sum_{j=1}^{m}a_{ij}^u(t)f_j(v_j(t))\\&\quad+\sum_{j=1}^{m}b_{ij}^u(t)f_j(v_j(t-\tau(t)))+\sum_{j=1}^{m}c_{ij}^{(1)}w_j^{(1)}\\&\quad+\bigwedge_{j=1}^{m}\alpha_{ij}^{(1)}f_j(v_j(t-\tau(t)))\\&\quad+\bigvee_{j=1}^{m}\beta_{ij}^{(1)}f_j(v_j(t-\tau(t)))+\bigvee_{j=1}^{m}S_{ij}^{(1)}w_j^{(1)}\\&\quad+\bigwedge_{j=1}^{m}T_{ij}^{(1)}w_j^{(1)}+I_i^{(1)}+r_i^u(t),\\\dot{v}_j(t) &= -d_j^v(t)v_j(t)+\sum_{i=1}^{n}a_{ji}^v(t)g_i(u_i(t))\\&\quad+\sum_{i=1}^{n}b_{ji}^v(t)g_i(u_i(t-\sigma(t)))+\sum_{i=1}^{n}c_{ji}^{(2)}w_i^{(2)}\\&\quad+\bigwedge_{i=1}^{n}\alpha_{ji}^{(2)}g_i(u_i(t-\sigma(t)))\\&\quad+\bigvee_{i=1}^{n}\beta_{ji}^{(2)}g_i(u_i(t-\sigma(t)))+\bigvee_{i=1}^{n}S_{ji}^{(2)}w_i^{(2)}\\&\quad+\bigwedge_{i=1}^{n}T_{ji}^{(2)}w_i^{(2)}+I_j^{(2)}+r_j^v(t),\quad t\geq 0,\end{aligned} \tag{8}$$

where $d_i^u(t)\in co[\underline{d}_i^{(1)},\overline{d}_i^{(1)}]$, $d_j^v(t)\in co[\underline{d}_j^{(2)},\overline{d}_j^{(2)}]$, $a_{ij}^u(t)\in co[\underline{a}_{ij}^{(1)},\overline{a}_{ij}^{(1)}]$, $a_{ji}^v(t)\in co[\underline{a}_{ji}^{(2)},\overline{a}_{ji}^{(2)}]$, $b_{ij}^u(t)\in co[\underline{b}_{ij}^{(1)},\overline{b}_{ij}^{(1)}]$, $b_{ji}^v(t)\in co[\underline{b}_{ji}^{(2)},\overline{b}_{ji}^{(2)}]$.

A. THE DEFINITION OF FIXED-TIME SYNCHRONIZATION

In this subsection, we give the definition of fixed-time synchronization between drive-response (master-slave) systems (1) and (6). We define the error systems between (1) and (6) as $e_i^x(t)=u_i(t)-x_i(t)$, $e_j^y(t)=v_j(t)-y_j(t)$, $i\in\mathcal{I}, j\in\mathcal{J}$. Then, we have

$$\begin{cases}\dot{e}_i^x(t)=D_i^x(t)+F_i^x(t)+G_i^x(t)+r_i^u(t),\\ \dot{e}_j^y(t)=D_j^x(t)+F_j^y(t)+G_j^y(t)+r_j^y(t),\end{cases} \tag{9}$$

where

$$\begin{aligned}D_i^x(t) &= -(d_i^u(t)u_i(t)-d_i^x(t)x_i(t)),\\D_j^y(t) &= -(d_j^v(t)v_j(t)-d_j^y(t)y_j(t)),\\F_i^x(t) &= \sum_{j=1}^{m}a_{ij}^u(t)f_j(v_j(t))-\sum_{j=1}^{m}a_{ij}^x(t)f_j(y_j(t))\\&\quad+\sum_{j=1}^{m}b_{ij}^u(t)f_j(v_j(t-\tau(t)))\\&\quad-\sum_{j=1}^{m}b_{ij}^x(t)f_j(y_j(t-\tau(t))),\\G_i^x(t) &= \bigwedge_{j=1}^{m}\alpha_{ij}^{(1)}f_j(v_j(t-\tau(t)))\\&\quad-\bigwedge_{j=1}^{m}\alpha_{ij}^{(1)}f_j(y_j(t-\tau(t)))\end{aligned}$$

$$
\begin{aligned}
&+\bigvee_{j=1}^{m}\beta_{ij}^{(1)}f_j(v_j(t-\tau(t)))\\
&-\bigvee_{j=1}^{m}\beta_{ij}^{(1)}f_j(y_j(t-\tau(t))),\\
F_j^y(t)=&\sum_{i=1}^{n}a_{ji}^{v}(t)g_i(u_i(t))-\sum_{i=1}^{n}a_{ji}^{y}(t)g_i(x_i(t))\\
&+\sum_{i=1}^{n}b_{ji}^{v}(t)g_i(u_i(t-\sigma(t)))\\
&-\sum_{i=1}^{n}b_{ji}^{y}(t)g_i(x_i(t-\sigma(t))),\\
G_j^y(t)=&\bigwedge_{i=1}^{n}\alpha_{ji}^{(2)}g_i(u_i(t-\sigma(t)))\\
&-\bigwedge_{i=1}^{n}\alpha_{ji}^{(2)}g_i(x_i(t-\sigma(t)))\\
&+\bigvee_{i=1}^{n}\beta_{ji}^{(2)}g_i(u_i(t-\sigma(t)))\\
&-\bigvee_{i=1}^{n}\beta_{ji}^{(2)}g_i(x_i(t-\sigma(t))).
\end{aligned}
$$

The initial condition of the error system (9) can be expressed as

$$
\begin{cases} e_i^x(s)=\varphi_i^{(2)}(s)-\varphi_i^{(1)}(s),\\ e_j^y(s)=\psi_j^{(2)}(s)-\psi_j^{(1)}(s). \end{cases} \tag{10}
$$

Definition 1: The systems (1) and (6) are said to achieve the finite-time synchronization, for $i\in\mathcal{I}, j\in\mathcal{J}$, if there exists a locally bounded function $T(e_i^x(0),e_j^y(0)):\mathcal{R}^n\rightarrow\mathcal{R}_+\bigcup\{0\}$ that depends on $e_i^x(0),e_j^y(0)$, such that $e_i^x(t,e_i^x(0))=0, e_j^y(t,e_j^y(0))=0$ for all $t\geq T(e_i^x(0),e_j^y(0))$, where $e_i^x(t,e_i^x(0)), e_j^y(t,e_j^y(0))$ are the solutions of the Cauchy problem (9). The function $T(e_i^x(0),e_j^y(0))$ is called the settling-time function.

Definition 2: The systems (1) and (6) are said to obtain the fixed-time synchronization, if they can achieve the finite-time synchronization and the settling-time function $T(e_i^x(0),e_j^y(0))$ is globally bounded, i.e. there exists a fixed constant $T_{max}\in\mathcal{R}_+$ such that $T(e_i^x(0),e_j^y(0))\leq T_{max}$, for any $e_i^x(0),e_j^y(0)\in\mathcal{R}$.

Remark 1: According to **Definition 2**, let $e(t)\triangleq(e_1^x(t),e_2^x(t),\cdots,e_n^x(t),e_1^y(t),\cdots,e_m^y(t))^T$ and $T(e(0))\triangleq T(e_i^x(0),e_j^y(0))$, we have the following equivalent form

$$
\begin{cases} \lim\limits_{t\rightarrow T(e(0))}\|e(t)\|=0,\\ e(t)\equiv 0, & \forall t\geq T(e(0))),\\ T(e(0))\leq T_{max}, & \forall e(0)\in\mathcal{C}([-\tau,0],\mathcal{R}^n). \end{cases}
$$

where $\|\cdot\|$ denotes the Euclidean norm.

Remark 2: Compared the **Definition 1** and **Definition 2**, the main difference between the finite-time synchronization and the fixed-time synchronization is whether the settling-time function T and T_{max} depend on the initial conditions.

B. SOME ASSUMPTIONS AND LEMMAS

In this subsection, we will give some assumptions and lemmas in order to obtain our main results.

Assumption 1: For $i\in\mathcal{I}, j\in\mathcal{J}$, the activation functions $f_j(\cdot), g_i(\cdot)$ meet the Lipschitz conditions, i.e. for any $x_1,x_2\in\mathcal{R}, x_1\neq x_2$, there exist positive constants k_i, l_j satisfying the following conditions

$$
\begin{aligned}
|f_j(x_1)-f_j(x_2)|&\leq k_j|x_1-x_2|,\\
|g_i(x_1)-g_i(x_2)|&\leq l_i|x_1-x_2|.
\end{aligned}
$$

Lemma 1 [38]: Suppose x_1,x_2 are any two states of model (1), then we have the following inequalities

$$
\begin{aligned}
\left|\bigwedge_{j=1}^{m}\alpha_{ij}^{(1)}f_j(x_2)-\bigwedge_{j=1}^{m}\alpha_{ij}^{(1)}f_j(x_1)\right|&\leq\sum_{j=1}^{m}|\alpha_{ij}^{(1)}||f_j(x_2)-f_j(x_1)|,\\
\left|\bigvee_{j=1}^{m}\beta_{ij}^{(1)}f_j(x_2)-\bigvee_{j=1}^{m}\beta_{ij}^{(1)}f_j(x_1)\right|&\leq\sum_{j=1}^{m}|\beta_{ij}^{(1)}||f_j(x_2)-f_j(x_1)|,\\
\left|\bigwedge_{i=1}^{n}\alpha_{ji}^{(2)}g_i(x_2)-\bigwedge_{i=1}^{n}\alpha_{ji}^{(2)}g_i(x_1)\right|&\leq\sum_{i=1}^{n}|\alpha_{ji}^{(2)}||g_i(x_2)-g_i(x_1)|,\\
\left|\bigvee_{i=1}^{m}\beta_{ji}^{(2)}g_i(x_2)-\bigvee_{i=1}^{m}\beta_{ji}^{(2)}g_i(x_1)\right|&\leq\sum_{j=1}^{m}|\beta_{ji}^{(2)}||g_i(x_2)-g_i(x_1)|.
\end{aligned}
$$

Lemma 2: If **Assumption 1** and $f_j(\pm\Gamma_j^{(2)})=0$, $g_i(\pm\Gamma_i^{(1)})=0$ hold, we have the following inequalities

$$
\begin{aligned}
|a_{ij}^{u}(t)f_j(v_j(t))-a_{ij}^{x}f_j(y_j(t))|&\leq a_{ij}^{*}k_j|v_j(t)-y_j(t)|,\\
|b_{ij}^{u}(t)f_j(v_j(t))-b_{ij}^{x}f_j(y_j(t))|&\leq b_{ij}^{*}k_j|v_j(t)-y_j(t)|,\\
|a_{ji}^{v}(t)g_i(u_i(t))-a_{ji}^{y}(t)g_i(x_i(t))|&\leq a_{ji}^{**}l_i|u_i(t)-x_i(t)|,\\
|b_{ji}^{v}(t)g_i(u_i(t))-b_{ji}^{y}(t)g_i(x_i(t))|&\leq b_{ji}^{**}l_i|u_i(t)-x_i(t)|,
\end{aligned}
$$

where $a_{ij}^{*}=\max\{|\hat{a}_{ij}^{(1)}|,|\check{a}_{ij}^{(1)}|\}$, $b_{ij}^{*}=\max\{|\hat{b}_{ij}^{(1)}|,|\check{b}_{ij}^{(1)}|\}$, $a_{ji}^{**}=\max\{|\hat{a}_{ji}^{(2)}|,|\check{a}_{ji}^{(2)}|\}$, $b_{ji}^{**}=\max\{|\hat{b}_{ji}^{(2)}|,|\check{b}_{ji}^{(2)}|\}$, m_j, n_i are the same as those in **Assumption 1**.

Proof: Please see [9] for its proof process. ■

Lemma 3: For Eq.(9), the following inequalities hold

$$
\begin{cases} sgn(e_i^x(t))D_i^x(t)\leq-\underline{d}_i^{(1)}|e_i^x(t)|+\Gamma_i^{(1)}|\hat{d}_i^{(1)}-\check{d}_i^{(1)}|,\\ sgn(e_j^y(t))D_j^y(t)\leq-\underline{d}_j^{(2)}|e_j^y(t)|+\Gamma_j^{(2)}|\hat{d}_j^{(2)}-\check{d}_j^{(2)}|. \end{cases} \tag{11}
$$

Lemma 4 [39]: Let the real numbers $z_1,z_2,\cdots,z_N\geq 0$, $p>1, 0<q\leq 1$, then the following inequalities hold

$$
\sum_{i=1}^{N}z_i^p\geq N^{1-p}\Big(\sum_{i=1}^{N}z_i\Big)^p,\quad \sum_{i=1}^{N}z_i^q\geq\Big(\sum_{i=1}^{N}z_i\Big)^q. \tag{12}
$$

Lemma 5 [32]: Suppose there exists a continuous radically unbounded function $V:\mathcal{R}^n\rightarrow\mathcal{R}_+\bigcup\{0\}$ such that 1) $V(\chi)=0\Rightarrow\chi=0$; 2) any solution $e(t)$ of system (9) satisfies

$$
\dot{V}(e(t))\leq -aV^s(e(t))-bV^r(e(t))
$$

for $a, b > 0$, $s > 1$, $0 < r < 1$. Then the system (9) can be fixed-time stable, and the settling time can be calculated as follows

$$T(e_0(t)) \le T_{max} = \frac{1}{a(s-1)} + \frac{1}{b(1-r)}. \tag{13}$$

III. MAIN RESULTS

Here, we will design the fixed-time synchronization controllers and derive the sufficient conditions to guarantee the synchronization between the drive MFBAMCNN (1) and the response MFBAMCNN (6).

To obtain the fixed-time synchronization, we design the following controllers $r_i^u(t), r_j^y(t)$, which are added to the response MFBAMCNN (6)

$$\begin{aligned} r_i^u(t) &= -\lambda_i^x e_i^x(t) - sgn(e_i^x(t))(\iota_i^x + \omega_i^x|e_i^x(t-\sigma(t))| \\ &\quad + \mu_i^x|e_i^x(t)|^{\gamma_1} + \nu_i^x|e_i^x(t)|^{\gamma_2}), \\ r_j^y(t) &= -\lambda_j^y e_j^y(t) - sgn(e_j^y(t))(\iota_j^y + \omega_j^y|e_j^y(t-\tau(t))| \\ &\quad + \mu_j^y|e_j^y(t)|^{\gamma_1} + \nu_j^y|e_j^y(t)|^{\gamma_2}), \end{aligned} \tag{14}$$

where λ_i^x, λ_j^y, ω_i^x, ω_j^y, μ_i^x, ν_i^x, μ_j^y, ν_j^y are nonnegative real numbers to be determined, $\gamma_1 > 1$, $0 < \gamma_2 < 1$ are constants.

For simplicity, we will transform the Eq. (9) and Eq. (10) to the following inequalities according to **Lemma 1** and **Lemma 2**. Obviously, we have

$$\begin{aligned} F_i^x(t) &\le \sum_{j=1}^m a_{ij}^* k_j e_j^y(t) + \sum_{j=1}^m b_{ij}^* k_j e_j^y(t-\tau(t)), \\ G_i^x(t) &\le \sum_{j=1}^m |\alpha_{ij}^{(1)}| k_j |e_j^y(t-\tau(t)) \\ &\quad + \sum_{j=1}^m |\beta_{ij}^{(1)}| k_j |e_j^y(t-\tau(t)); \\ F_j^y(t) &\le \sum_{i=1}^n a_{ji}^{**} l_i e_i^x(t) + \sum_{i=1}^n b_{ji}^{**} l_i e_i^x(t-\sigma(t)) \\ G_j^y(t) &\le \sum_{i=1}^n |\alpha_{ji}^{(2)}| l_i |e_i^x(t-\sigma(t)) \\ &\quad + \sum_{i=1}^n |\beta_{ji}^{(2)}| l_i |e_i^x(t-\tau(t)). \end{aligned} \tag{15}$$

According to Eq. (14) and Eq. (15), the error system (9) can be rewritten as

$$\begin{aligned} \dot e_i^x(t) &\le D_i^x(t) + \sum_{j=1}^m k_j a_{ij}^* e_j^y(t) \\ &\quad + \sum_{j=1}^m k_j(b_{ij}^* + |\alpha_{ij}^{(1)}| + |\beta_{ij}^{(1)}|) e_j^y(t-\tau) \\ &\quad - \lambda_i^x e_i^x(t) - sgn(e_i^x(t))(\iota_i^x + \omega_i^x|e_i^x(t-\sigma(t))| \\ &\quad + \mu_i^x|e_i^x(t)|^{\gamma_1} + \nu_i^x|e_i^x(t)|^{\gamma_2}), \\ \dot e_j^y(t) &\le D_j^y(t) + \sum_{i=1}^n l_i a_{ji}^{**} e_i^x(t) \\ &\quad + \sum_{i=1}^n l_i(b_{ji}^{**} + |\alpha_{ji}^{(2)}| + |\beta_{ji}^{(2)}|) e_i^x(t-\sigma) \\ &\quad - \lambda_j^y e_j^y(t) - sgn(e_j^y(t))(\iota_j^y + \omega_j^y|e_j^y(t-\tau(t))| \\ &\quad + \mu_j^y|e_j^y(t)|^{\gamma_1} + \nu_j^y|e_j^y(t)|^{\gamma_2}). \end{aligned} \tag{16}$$

Theorem 1: Under **Assumption 1** and the controllers (14), if the following algebraic conditions hold

$$\begin{cases} \sum_{j=1}^m a_{ji}^{**} l_i - (\underline{d}_i^{(1)} + \lambda_i^x) < 0, \\ \sum_{i=1}^n a_{ij}^* k_j - (\underline{d}_j^{(2)} + \lambda_j^y) < 0, \\ \sum_{i=1}^n (\Gamma_i^{(1)} |\hat d_i^{(1)} - \check d_i^{(1)}| - \iota_i^x) \\ + \sum_{j=1}^m (\Gamma_j^{(2)} |\hat d_j^{(2)} - \check d_j^{(2)}| - \iota_j^y) < 0, \\ \sum_{i=1}^n k_j (b_{ij}^* + |\alpha_{ij}^{(1)}| + |\beta_{ij}^{(1)}|) - \omega_j^y < 0, \\ \sum_{j=1}^m l_i (b_{ji}^{**} + |\alpha_{ji}^{(2)}| + |\beta_{ji}^{(2)}|) - \omega_i^x < 0. \end{cases} \tag{17}$$

then the drive MFBAMCNN (1) and the response MFBAMCNN (6) can achieve the fixed-time synchronization. Furthermore, the settling time can be calculated by

$$T_{max} = \frac{1}{\min_{i,j}\{n^{1-\gamma_1}\mu_i^x, m^{1-\gamma_1}\mu_j^y\} \cdot 2^{1-\gamma_1}(\gamma_1 - 1)} + \frac{1}{\min_{i,j}\{\nu_i^x, \nu_j^y\}(1-\gamma_2)}. \tag{18}$$

Proof: Define the Lyapunov function as follows

$$V(t) = V_1(t) + V_2(t), \tag{19}$$

where

$$V_1(t) = \sum_{i=1}^n |e_i^x(t)|,\ V_2(t) = \sum_{j=1}^m |e_j^y(t)|. \tag{20}$$

Obviously, $V(t) \ge 0$. Calculating the upper right-hand Dini derivative of $V_1(t)$ with respect to t along the solution of (10), we have

$$\begin{aligned} D^+V_1(t) &= \sum_{i=1}^n sgn(e_i^x(t)) \dot e_i^x(t) \\ &\le \sum_{i=1}^n sgn(e_i^x(t)) \Big\{ D_i^x(t) + \sum_{j=1}^m k_j a_{ij}^* e_j^y(t) \\ &\quad + \sum_{j=1}^m k_j(b_{ij}^* + |\alpha_{ij}^{(1)}| + |\beta_{ij}^{(1)}|) e_j^y(t-\tau(t)) \\ &\quad - \lambda_i^x e_i^x(t) - sgn(e_i^x(t))(\iota_i^x + \omega_i^x|e_i^x(t-\sigma(t))| \\ &\quad + \mu_i^x|e_i^x(t)|^{\gamma_1} + \nu_i^x|e_i^x(t)|^{\gamma_2}) \Big\} \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{i=1}^{n}\Big\{-\underline{d}_i^{(1)}|e_i^x(t)|+\Gamma_i^{(1)}|\hat{d}_i^{(1)}-\check{d}_i^{(1)}|\Big\} \\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{m}a_{ij}^{*}k_j|e_j^y(t)| \\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{m}b_{ij}^{*}k_j|e_j^y(t-\tau(t))| \\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{m}k_j(|\alpha_{ij}^{(1)}|+\beta_{ij}^{(1)})|e_j^y(t-\tau(t))| \\
&\quad-\sum_{i=1}^{n}\lambda_i^x|e_i^x(t)|-\sum_{i=1}^{n}\iota_i^x-\sum_{i=1}^{n}\omega_i^x|e_i^x(t-\sigma(t))| \\
&\quad-\sum_{i=1}^{n}\mu_i^x|e_i^x(t)|^{\gamma_1}-\sum_{i=1}^{n}\nu_i^x|e_i^x(t)|^{\gamma_2} \\
&=\sum_{i=1}^{n}(-\underline{d}_i^{(1)}-\lambda_i^x)|e_i^x(t)| \\
&\quad+\sum_{i=1}^{n}(\Gamma_i^{(1)}|\hat{d}_i^{(1)}-\check{d}_i^{(1)}|-\iota_i^x) \\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{m}a_{ij}^{*}k_j|e_j^y(t)|-\sum_{i=1}^{n}\omega_i^x|e_i^x(t-\sigma(t))| \\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{m}k_j(b_{ij}^{*}+|\alpha_{ij}^{(1)}|+\beta_{ij}^{(1)})|e_j^y(t-\tau(t))| \\
&\quad-\sum_{i=1}^{n}\mu_i^x|e_i^x(t)|^{\gamma_1}-\sum_{i=1}^{n}\nu_i^x|e_i^x(t)|^{\gamma_2}. \qquad (21)
\end{aligned}$$

Similarly, the upper right-hand Dini derivative of $V_2(t)$ can be calculated as follows

$$\begin{aligned}
D^+V_2(t) &= \sum_{j=1}^{n} sgn(e_j^y(t))\dot{e}_j^y(t) \\
&\leq \sum_{j=1}^{m} sgn(e_j^y(t))\Big\{D_j^y(t)+\sum_{i=1}^{n}l_ja_{ji}^{**}e_i^x(t) \\
&\quad+\sum_{i=1}^{n}l_i(b_{ji}^{**}+|\alpha_{ji}^{(2)}|+|\beta_{ji}^{(2)}|)e_i^x(t-\sigma(t)) \\
&\quad-\lambda_j^ye_j^y(t)-sgn(e_j^y(t))(\iota_j^y+\omega_j^y|e_j^y(t-\tau(t))| \\
&\quad+\mu_j^y|e_j^y(t)|^{\gamma_1}+\nu_j^y|e_j^y(t)|^{\gamma_2})\Big\} \\
&\leq \sum_{j=1}^{m}\Big\{-\underline{d}_j^{(2)}|e_j^y(t)|+\Gamma_j^{(2)}|\hat{d}_j^{(2)}-\check{d}_j^{(2)}|\Big\} \\
&\quad+\sum_{j=1}^{m}\sum_{i=1}^{n}a_{ji}^{**}l_i|e_i^x(t)| \\
&\quad+\sum_{j=1}^{m}\sum_{i=1}^{n}b_{ji}^{**}l_i|e_i^x(t-\sigma(t))| \\
&\quad+\sum_{j=1}^{m}\sum_{i=1}^{n}l_i(|\alpha_{ji}^{(2)}|+\beta_{ji}^{(2)})|e_i^x(t-\sigma(t))| \\
&\quad-\sum_{j=1}^{m}\lambda_j^x|e_j^y(t)|-\sum_{j=1}^{m}\iota_j^y \\
&\quad-\sum_{j=1}^{m}\omega_j^y|e_j^y(t-\tau(t))| \\
&\quad-\sum_{j=1}^{m}\mu_j^y|e_j^y(t)|^{\gamma_1}-\sum_{j=1}^{m}\nu_j^y|e_j^y(t)|^{\gamma_2} \\
&=\sum_{j=1}^{m}(-\underline{d}_j^{(2)}-\lambda_j^y)|e_j^y(t)| \\
&\quad+\sum_{j=1}^{m}(\Gamma_j^{(2)}|\hat{d}_j^{(2)}-\check{d}_j^{(2)}|-\iota_j^y) \\
&\quad+\sum_{j=1}^{m}\sum_{i=1}^{n}a_{ji}^{**}l_i|e_i^x(t)|-\sum_{j=1}^{m}\omega_j^y|e_j^y(t-\tau(t))| \\
&\quad+\sum_{j=1}^{m}\sum_{i=1}^{n}l_i(b_{ji}^{**}+|\alpha_{ji}^{(2)}|+\beta_{ji}^{(2)})|e_j^y(t-\tau(t))| \\
&\quad-\sum_{j=1}^{m}\mu_j^y|e_j^y(t)|^{\gamma_1}-\sum_{j=1}^{m}\nu_j^y|e_j^y(t)|^{\gamma_2}. \qquad (22)
\end{aligned}$$

Merging Eq.(21) and Eq.(22) into Eq.(18), we have

$$\begin{aligned}
D^+V(t) &= D^+V_1(t)+D^+V_2(t) \\
&\leq \sum_{i=1}^{n}\Big\{\sum_{j=1}^{m}a_{ji}^{**}l_i-(\underline{d}_i^{(1)}+\lambda_i^x)\Big\}|e_i^x(t)| \\
&\quad+\sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}a_{ij}^{*}k_j-(\underline{d}_j^{(2)}+\lambda_j^y)\Big\}|e_j^y(t)| \\
&\quad+\sum_{i=1}^{n}(\Gamma_i^{(1)}|\hat{d}_i^{(1)}-\check{d}_i^{(1)}|-\iota_i^x) \\
&\quad+\sum_{j=1}^{m}(\Gamma_j^{(2)}|\hat{d}_j^{(2)}-\check{d}_j^{(2)}|-\iota_j^y) \\
&\quad+\sum_{j=1}^{m}\Big\{\sum_{i=1}^{n}k_j(b_{ij}^{*}+|\alpha_{ij}^{(1)}|+|\beta_{ij}^{(1)}|) \\
&\quad-\omega_j^y\Big\}|e_j(t-\tau(t))| \\
&\quad+\sum_{i=1}^{n}\Big\{\sum_{j=1}^{m}l_i(b_{ji}^{**}+|\alpha_{ji}^{(2)}|+|\beta_{ji}^{(2)}|) \\
&\quad-\omega_i^x\Big\}|e_j(t-\tau(t))| \\
&\quad-\sum_{i=1}^{n}\mu_i^x|e_i^x(t)|^{\gamma_1}-\sum_{i=1}^{n}\nu_i^x|e_i^x(t)|^{\gamma_2}
\end{aligned}$$

$$-\sum_{j=1}^{m}\mu_j^y|e_j^y(t)|^{\gamma_1}-\sum_{j=1}^{m}\nu_j^y|e_j^y(t)|^{\gamma_2}.$$

According to the condition (16) and **Lemma 4**, we get

$$\begin{aligned}D^+V(t)\le&-\sum_{i=1}^{n}\mu_i^x|e_i^x(t)|^{\gamma_1}-\sum_{i=1}^{n}\nu_i^x|e_i^x(t)|^{\gamma_2}\\&-\sum_{j=1}^{m}\mu_j^y|e_j^y(t)|^{\gamma_1}-\sum_{j=1}^{m}\nu_j^y|e_j^y(t)|^{\gamma_2}\\\le&-\min_i\{\mu_i^x\}\sum_{i=1}^{n}|e_i^x(t)|^{\gamma_1}\\&-\min_i\{\nu_i^x\}\sum_{i=1}^{n}|e_i^x(t)|^{\gamma_2}\\&-\min_j\{\mu_j^y\}\sum_{j=1}^{m}|e_j^y(t)|^{\gamma_1}\\&-\min_j\{\nu_j^y\}\sum_{j=1}^{m}|e_j^y(t)|^{\gamma_2}\\\le&-\min_i\{n^{1-\gamma_1}\mu_i^x\}\Big(\sum_{i=1}^{n}|e_i^x(t)|\Big)^{\gamma_1}\\&-\min_i\{\nu_i^x\}\Big(\sum_{i=1}^{n}|e_i^x(t)|\Big)^{\gamma_2}\\&-\min_j\{m^{1-\gamma_1}\mu_j^y\}\Big(\sum_{j=1}^{m}|e_j^y(t)|\Big)^{\gamma_1}\\&-\min_j\{\nu_j^y\}\Big(\sum_{j=1}^{m}|e_j^x(t)|\Big)^{\gamma_2}.\end{aligned}$$

Let $a=\min_{i,j}\{n^{1-\gamma_1}\mu_i^x,m^{1-\gamma_1}\mu_j^y\}$, $b=\min_{i,j}\{\nu_i^x,\nu_j^y\}$, according to the **Lemma 4**, we have

$$\begin{aligned}D^+V(t)&\le-a(V_1^{\gamma_1}(t)+V_2^{\gamma_1}(t))-b(V_1^{\gamma_2}(t)+V_2^{\gamma_2}(t))\\&\le-a\cdot 2^{1-\gamma_1}V^{\gamma_1}(t)-bV^{\gamma_2}(t).\end{aligned}$$

By **Lemma 5**, the origin of error system (9) is fixed-time stable. Equivalently, the drive MFBAMCNNs (1) and the response system MFBAMACNNs (6) can achieve the fixed-time synchronization. Furthermore, the settling time can be calculated by

$$\begin{aligned}T_{max}&=\frac{1}{a\cdot 2^{1-\gamma_1}(\gamma_1-1)}+\frac{1}{b(1-\gamma_2)}\\&=\frac{1}{\min_{i,j}\{n^{1-\gamma_1}\mu_i^x,m^{1-\gamma_1}\mu_j^y\}\cdot 2^{1-\gamma_1}(\gamma_1-1)}\\&\quad+\frac{1}{\min_{i,j}\{\nu_i^x,\nu_j^y\}(1-\gamma_2)}.\end{aligned}$$

The proof is finished. ■

Remark 3: From the conclusions of **Theorem 1**, we can see that the controller parameters μ_i^x, μ_j^y, ν_i^x, ν_j^y, γ_1, γ_2 do not appear in the condition (17), but directly determine the result of the settling time T_{max}. That is to say, we do not need to consider them when we select the control parameters that meet the condition (17).

Remark 4: According to the settling time formula (18), we know that T_{max} is inversely proportional to μ_i^x, μ_j^y, ν_i^x, ν_j^y when γ_1 and γ_2 are fixed.

The conclusions of **Theorem 1** can be easily extended to common BAM cellular neural networks without the memristor. Considering the following drive-response FBAMCNNs

$$\begin{aligned}\dot{x}_i(t)=&-d_i^{(1)}x_i(t)+\sum_{j=1}^{m}a_{ij}^{(1)}f_j(y_j(t))\\&+\sum_{j=1}^{m}b_{ij}^{(1)}f_j(y_j(t-\tau(t)))+\sum_{j=1}^{m}c_{ij}^{(1)}w_j^{(1)}\\&+\bigwedge_{j=1}^{m}\alpha_{ij}^{(1)}f_j(y_j(t-\tau(t)))\\&+\bigvee_{j=1}^{m}\beta_{ij}^{(1)}f_j(y_j(t-\tau(t)))+\bigvee_{j=1}^{m}S_{ij}^{(1)}w_j^{(1)}\\&+\bigwedge_{j=1}^{m}T_{ij}^{(1)}w_j^{(1)}+I_i^{(1)},\\\dot{y}_j(t)=&-d_j^{(2)}y_j(t)+\sum_{i=1}^{n}a_{ji}^{(2)}g_i(x_i(t))\\&+\sum_{i=1}^{n}b_{ji}^{(2)}g_i(x_i(t-\sigma(t)))+\sum_{i=1}^{n}c_{ji}^{(2)}w_i^{(2)}\\&+\bigwedge_{i=1}^{n}\alpha_{ji}^{(2)}g_i(x_i(t-\sigma(t)))\\&+\bigvee_{i=1}^{n}\beta_{ji}^{(2)}g_i(x_i(t-\sigma(t)))+\bigvee_{i=1}^{n}S_{ji}^{(2)}w_i^{(2)}\\&+\bigwedge_{i=1}^{n}T_{ji}^{(2)}w_i^{(2)}+I_j^{(2)},\quad t\le 0.\end{aligned}\tag{23}$$

$$\begin{aligned}\dot{u}_i(t)=&-d_i^{(1)}u_i(t)+\sum_{j=1}^{m}a_{ij}^{(1)}f_j(v_j(t))\\&+\sum_{j=1}^{m}b_{ij}^{(1)}f_j(v_j(t-\tau(t)))+\sum_{j=1}^{m}c_{ij}^{(1)}w_j^{(1)}\\&+\bigwedge_{j=1}^{m}\alpha_{ij}^{(1)}f_j(v_j(t-\tau(t)))\\&+\bigvee_{j=1}^{m}\beta_{ij}^{(1)}f_j(v_j(t-\tau(t)))+\bigvee_{j=1}^{m}S_{ij}^{(1)}w_j^{(1)}\\&+\bigwedge_{j=1}^{m}T_{ij}^{(1)}w_j^{(1)}+I_i^{(1)}+s_i^u(t),\\\dot{v}_j(t)=&-d_j^{(2)}v_j(t)+\sum_{i=1}^{n}a_{ji}^{(2)}g_i(u_i(t))\end{aligned}$$

$$+\sum_{i=1}^{n} b_{ji}^{(2)} g_i(u_i(t-\sigma(t)))+\sum_{i=1}^{n} c_{ji}^{(2)} w_i^{(2)}$$
$$+\bigwedge_{i=1}^{n} \alpha_{ji}^{(2)} g_i(u_i(t-\sigma(t)))$$
$$+\bigvee_{i=1}^{n} \beta_{ji}^{(2)} g_i(u_i(t-\sigma(t)))+\bigvee_{i=1}^{n} S_{ji}^{(2)} w_i^{(2)}$$
$$+\bigwedge_{i=1}^{n} T_{ji}^{(2)} w_i^{(2)}+I_j^{(2)}+s_j^y(t), \quad t \leq 0, \quad (24)$$

where $s_i^u(t)$, $s_j^y(t)$ are designed as follows

$$\begin{aligned} s_i^u(t) &= -\lambda_i^x e_i^x(t) - sgn(e_i^x(t))(\omega_i^x |e_i^x(t-\sigma(t))| \\ &\quad + \mu_i^x |e_i^x(t)|^{\gamma_1} + \nu_i^x |e_i^x(t)|^{\gamma_2}), \\ s_j^y(t) &= -\lambda_j^y e_j^y(t) - sgn(e_j^y(t))(\omega_j^y |e_j^y(t-\tau(t))| \\ &\quad + \mu_j^y |e_j^y(t)|^{\gamma_1} + \nu_j^y |e_j^y(t)|^{\gamma_2}), \end{aligned} \quad (25)$$

Define the error systems $e_i^x(t)=u_i(t)-v_i(t)$, $e_j^y(t)=v_j(t)-y_j(t)$, $i \in \mathcal{I}, j \in \mathcal{J}$ between the drive system (23) and the response system (24), and according to **Assumption 1** and **Lemma 1**, we have

$$\begin{aligned} \dot{e}_i^x &\leq -d_i^{(1)} e_i^x(t) + \sum_{j=1}^{m} a_{ij}^{(1)} k_j e_j^y(t) + s_i^u(t) \\ &\quad + \sum_{j=1}^{m} k_j \big(b_{ij}^{(1)} + |\alpha_{ij}^{(1)}| + |\beta_{ij}^{(1)}|\big) e_j^y(t-\tau(t)), \\ \dot{e}_j^y &\leq -d_j^{(2)} e_j^y(t) + \sum_{i=1}^{n} a_{ji}^{(2)} l_i e_i^x(t) + s_j^y(t) \\ &\quad + \sum_{i=1}^{n} l_i \big(b_{ji}^{(2)} + |\alpha_{ji}^{(2)}| + |\beta_{ji}^{(2)}|\big) e_i^x(t-\sigma(t)). \end{aligned} \quad (26)$$

For Eq. (23) and Eq. (24), we have the following corollary.

Corollary 1: Under the controllers (25), if the following algebraic conditions hold

$$\begin{cases} \sum_{j=1}^{m} a_{ji}^{(2)} l_i - d_i^{(1)} - \lambda_i^x) < 0, \\ \sum_{i=1}^{n} a_{ij}^{(1)} k_j - d_j^{(2)} - \lambda_j^y) < 0, \\ \sum_{i=1}^{n} k_j (b_{ij}^{(1)} + |\alpha_{ij}^{(1)}| + |\beta_{ij}^{(1)}|) - \omega_j^y < 0, \\ \sum_{j=1}^{m} l_i (b_{ji}^{(2)} + |\alpha_{ji}^{(2)}| + |\beta_{ji}^{(2)}|) - \omega_i^x < 0. \end{cases} \quad (27)$$

then the drive-response FBAMCNNs (23) and (24) can achieve the fixed-time synchronization. Furthermore, the settling time can be calculated by Eq. (18).

Proof: The result can be obtained directly by the proof of **Theorem 1**. ■

We can also easily extend the results to the case without fuzzy logic. Consider the following drive-response memristive BAM cellular neural networks

$$\begin{aligned} \dot{x}_i(t) &= -d_i^{(1)}(x_i(t))x_i(t) + \sum_{j=1}^{m} a_{ij}^{(1)}(x_i(t)) f_j(y_j(t)) \\ &\quad + \sum_{j=1}^{m} b_{ij}^{(1)}(x_i(t)) f_j(y_j(t-\tau(t))) \\ &\quad + \sum_{j=1}^{m} c_{ij}^{(1)} w_j^{(1)} + I_i^{(1)}, \\ \dot{y}_j(t) &= -d_j^{(2)}(y_j(t))y_j(t) + \sum_{i=1}^{n} a_{ji}^{(2)}(y_j(t)) g_i(x_i(t)) \\ &\quad + \sum_{i=1}^{n} b_{ji}^{(2)}(y_j(t)) g_i(x_i(t-\sigma(t))) \\ &\quad + \sum_{i=1}^{n} c_{ji}^{(1)} w_i^{(2)} + I_j^{(2)}. \end{aligned} \quad (28)$$

$$\begin{aligned} \dot{u}_i(t) &= -d_i^{(1)}(u_i(t))u_i(t) + \sum_{j=1}^{m} a_{ij}^{(1)}(u_i(t)) f_j(v_j(t)) \\ &\quad + \sum_{j=1}^{m} b_{ij}^{(1)}(u_i(t)) f_j(v_j(t-\tau(t))) \\ &\quad + \sum_{j=1}^{m} c_{ij}^{(1)} w_j^{(1)} + I_i^{(1)} + r_i^u(t), \\ \dot{v}_j(t) &= -d_j^{(2)}(v_j(t))v_j(t) + \sum_{i=1}^{n} a_{ji}^{(2)}(v_j(t)) g_i(u_i(t)) \\ &\quad + \sum_{i=1}^{n} b_{ji}^{(2)}(v_j(t)) g_i(u_i(t-\sigma(t))) \\ &\quad + \sum_{i=1}^{n} c_{ji}^{(1)} w_i^{(2)} + I_j^{(2)} + r_j^v(t), t \geq 0. \end{aligned} \quad (29)$$

For Eq.(28) and Eq.(29), we can obtain the following corollary.

Corollary 2: Under the controllers (14), if the following algebraic inequalities hold

$$\begin{cases} \sum_{j=1}^{m} a_{ji}^{**} l_i - (\underline{d}_i^{(1)} + \lambda_i^x) < 0, \\ \sum_{i=1}^{n} a_{ij}^{*} k_j - (\underline{d}_j^{(2)} + \lambda_j^y) < 0, \\ \sum_{i=1}^{n} (\Gamma_i^{(1)} |\hat{d}_i^{(1)} - \check{d}_i^{(1)}| - \iota_i^x) \\ + \sum_{j=1}^{m} (\Gamma_j^{(2)} |\hat{d}_j^{(2)} - \check{d}_j^{(2)}| - \iota_j^y) < 0, \\ \sum_{i=1}^{n} k_j b_{ij}^{*} - \omega_j^y < 0, \\ \sum_{j=1}^{m} l_i b_{ji}^{**} - \omega_i^x < 0, \end{cases} \quad (30)$$

then the drive MBAMCNNs (28) and response MBAMCNNs(29) can achieve the fixed-time synchronization. Meanwhile, the settling time can be calculated by Eq. (18).

Proof: The result can be obtained directly by the proof of **Theorem 1**. ■

Remark 5: The research on fixed-time synchronization of neural networks is still in the initial stage, and the related research results are still relatively few. We first studied the fixed-time synchronization problem of drive-response MFBAMCNNs. The nonlinear feedback controller (14) we designed is simple in form and contains more freely adjustable parameters, which can provide convenience for the design of the actual controller.

IV. NUMERICAL EXAMPLES

Now, we will validate the effectiveness of **Theorem 1** and **Corollary 2** by two numerical simulations.

Example 1: Consider the following MFBAMCNNs with time-varying delays as the drive system

$$\begin{aligned}\dot{x}_i(t) &= -d_i^{(1)}(x_i(t))x_i(t) + \sum_{j=1}^{2} a_{ij}^{(1)}(x_i(t))f_j(y_j(t)) \\ &\quad + \sum_{j=1}^{2} b_{ij}^{(1)}(x_i(t))f_j(y_j(t-\tau(t))) \\ &\quad + \sum_{j=1}^{2} c_{ij}^{(1)} w_j^{(1)} + \bigwedge_{j=1}^{2} \alpha_{ij}^{(1)} f_j(y_j(t-\tau(t))) \\ &\quad + \bigvee_{j=1}^{2} \beta_{ij}^{(1)} f_j(y_j(t-\tau(t))) \\ &\quad + \bigvee_{j=1}^{2} S_{ij}^{(1)} w_j^{(1)} + \bigwedge_{j=1}^{2} T_{ij}^{(1)} w_j^{(1)} + I_i^{(1)}, \\ \dot{y}_j(t) &= -d_j^{(2)}(y_j(t))y_j(t) + \sum_{i=1}^{2} a_{ji}^{(2)}(x_i(t))g_i(x_i(t)) \\ &\quad + \sum_{i=1}^{2} b_{ji}^{(2)}(y_j(t))g_i(x_i(t-\sigma(t))) \\ &\quad + \sum_{i=1}^{2} c_{ji}^{(2)} w_i^{(2)} + \bigwedge_{i=1}^{n} \alpha_{ji}^{(2)} g_i(x_i(t-\sigma(t))) \\ &\quad + \bigvee_{i=1}^{2} \beta_{ji}^{(2)} g_i(x_i(t-\sigma(t))) + \bigvee_{i=1}^{2} S_{ji}^{(2)} w_i^{(2)} \\ &\quad + \bigwedge_{i=1}^{2} T_{ji}^{(2)} w_i^{(2)} + I_j^{(2)}, \ t \geq 0. \end{aligned} \tag{31}$$

The corresponding response MFBAMCNNs are defined as follows

$$\begin{aligned}\dot{u}_i(t) &= -d_i^{(1)}(u_i(t))u_i(t) + \sum_{j=1}^{2} a_{ij}^{(1)}(u_i(t))f_j(v_j(t)) \\ &\quad + \sum_{j=1}^{2} b_{ij}^{(1)}(u_i(t))f_j(v_j(t-\tau(t))) \\ &\quad + \sum_{j=1}^{2} c_{ij}^{(1)} w_j^{(1)} + \bigwedge_{j=1}^{2} \alpha_{ij}^{(1)} f_j(v_j(t-\tau(t))) \\ &\quad + \bigvee_{j=1}^{2} \beta_{ij}^{(1)} f_j(v_j(t-\tau(t))) + \bigvee_{j=1}^{2} S_{ij}^{(1)} w_j^{(1)} \\ &\quad + \bigwedge_{j=1}^{2} T_{ij}^{(1)} w_j^{(1)} + I_i^{(1)} + r_i^u(t), \\ \dot{v}_j(t) &= -d_j^{(2)}(v_j(t))v_j(t) + \sum_{i=1}^{2} a_{ji}^{(2)}(u_i(t))g_i(u_i(t)) \\ &\quad + \sum_{i=1}^{2} b_{ji}^{(2)}(v_j(t))g_i(u_i(t-\sigma(t))) \\ &\quad + \sum_{i=1}^{2} c_{ji}^{(2)} w_i^{(2)} + \bigwedge_{i=1}^{2} \alpha_{ji}^{(2)} g_i(u_i(t-\sigma(t))) \\ &\quad + \bigvee_{i=1}^{2} \beta_{ji}^{(2)} g_i(u_i(t-\sigma(t))) + \bigvee_{i=1}^{2} S_{ji}^{(2)} w_i^{(2)} \\ &\quad + \bigwedge_{i=1}^{2} T_{ji}^{(2)} w_i^{(2)} + I_j^{(2)} + r_j^y(t), \ t \geq 0. \end{aligned} \tag{32}$$

Let the switching jumps $\Gamma_i^{(1)} = 1$, $\Gamma_j^{(2)} = 2$, then the memristor weights parameters of the drive-response MFBAMCNNs (31) and (32) are selected as follows

$$d_1^{(1)}(x_1) = \begin{cases} 1.2, & |x_1| \leq 1, \\ 1.0, & |x_1| > 1, \end{cases} d_2^{(1)}(x_2) = \begin{cases} 1.5, & |x_2| \leq 1, \\ 1.2, & |x_2| > 1, \end{cases}$$

$$d_1^{(2)}(y_1) = \begin{cases} 1.0, & |y_1| \leq 2, \\ 1.2, & |y_1| > 2, \end{cases} d_2^{(2)}(y_2) = \begin{cases} 1.5, & |y_2| \leq 2, \\ 1.1, & |y_2| > 2, \end{cases}$$

$$a_{11}^{(1)}(x_1) = \begin{cases} -1.5, & |x_1| \leq 1, \\ -1.8, & |x_1| > 1, \end{cases} a_{12}^{(1)}(x_1) = \begin{cases} -0.8, & |x_1| \leq 1, \\ -0.5, & |x_1| > 1, \end{cases}$$

$$a_{21}^{(1)}(x_2) = \begin{cases} -0.4, & |x_2| \leq 1, \\ 0.4, & |x_2| > 1, \end{cases} a_{22}^{(1)}(x_2) = \begin{cases} -1.8, & |x_2| \leq 1, \\ -1.5, & |x_2| > 1, \end{cases}$$

$$a_{11}^{(2)}(y_1) = \begin{cases} -0.5, & |y_1| \leq 2, \\ 0.5, & |y_1| > 2, \end{cases} a_{12}^{(2)}(y_1) = \begin{cases} -0.5, & |y_1| \leq 2, \\ 0.5, & |y_1| > 2, \end{cases}$$

$$a_{21}^{(2)}(y_2) = \begin{cases} -0.6, & |y_2| \leq 2, \\ 0.6, & |y_2| > 2, \end{cases} a_{22}^{(2)}(y_2) = \begin{cases} -0.8, & |y_2| \leq 2, \\ 0.8, & |y_2| > 2, \end{cases}$$

$$b_{11}^{(1)}(x_1) = \begin{cases} -1.2, & |x_1| \leq 1, \\ -1.5, & |x_1| > 1, \end{cases} b_{12}^{(1)}(x_1) = \begin{cases} -1.2, & |x_1| \leq 1, \\ -0.8, & |x_1| > 1, \end{cases}$$

$$b_{21}^{(1)}(x_2) = \begin{cases} -0.1, & |x_2| \leq 1, \\ 0.2, & |x_2| > 1, \end{cases} b_{22}^{(1)}(x_2) = \begin{cases} -1.6, & |x_2| \leq 1, \\ -1.2, & |x_2| > 1, \end{cases}$$

$$b_{11}^{(2)}(y_1) = \begin{cases} 1.5, & |y_1| \leq 2, \\ 1.2, & |y_1| > 2, \end{cases} b_{12}^{(2)}(y_1) = \begin{cases} -1.2, & |y_1| \leq 2, \\ 1.5, & |y_1| > 2, \end{cases}$$

$$b_{21}^{(2)}(y_2) = \begin{cases} 1.1, & |y_2| \leq 2, \\ 1.5, & |y_2| > 2, \end{cases} b_{22}^{(2)}(y_2) = \begin{cases} 2.8, & |y_2| \leq 2, \\ 2.2, & |y_2| > 2, \end{cases}$$

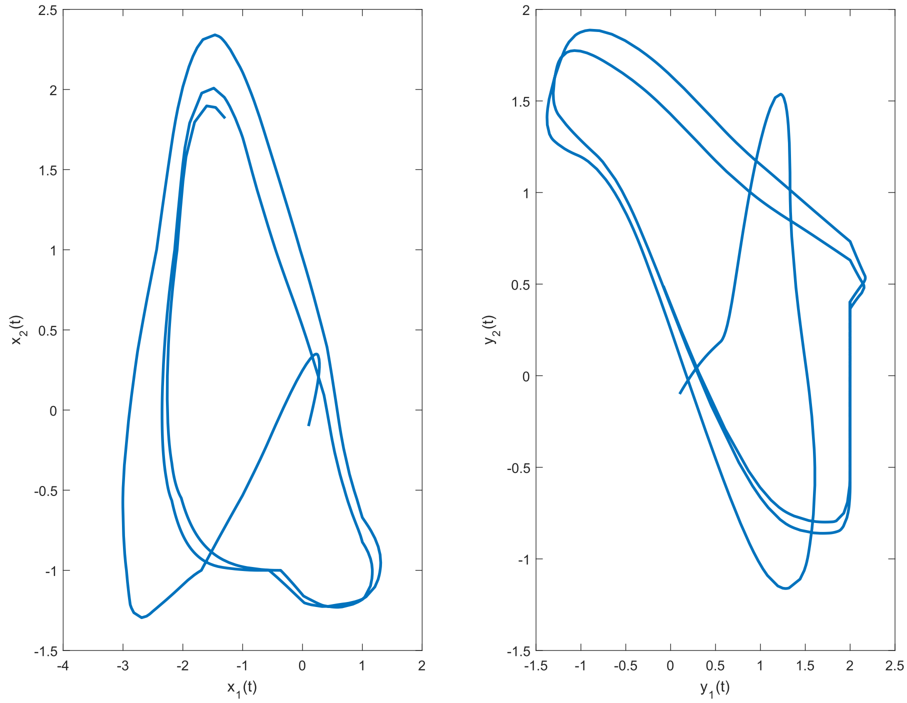

FIGURE 4. The phase diagrams of the drive MFBAMCNNs (31).

Note: In the definitions of $b_{ji}^{(1)}(y_j)$ and $b_{ji}^{(2)}(x_i)$, x_i and y_j are denoted as $x_i(t-\sigma(t))$ and $y_j(t-\tau(t))$ in order to simplify the expressions.

Other parameters are shown below.

$$\tau(t)=\sigma(t)=\frac{e^t}{1+e^t};$$
$$f_j(x)=tanh(|x|-1);\quad g_i(x)=tanh(|x|-2);$$
$$C^{(1)}=C^{(2)}=\begin{bmatrix}0.4 & 0.2\\ 0.2 & 0.4\end{bmatrix};$$
$$\alpha^{(1)}=\alpha^{(2)}=\begin{bmatrix}-0.2 & -0.02\\ -0.01 & -0.1\end{bmatrix};$$
$$\beta^{(1)}=\beta^{(2)}=\begin{bmatrix}-0.1 & -0.01\\ -0.1 & -0.1\end{bmatrix};$$
$$S^{(1)}=S^{(2)}=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix};$$
$$T^{(1)}=T^{(2)}=\begin{bmatrix}0.2 & 0.1\\ 0.1 & 0.2\end{bmatrix};\quad I^{(1)}=I^{(2)}=(0,0)^T;$$
$$w^{(1)}=w^{(2)}=(1.2,1.2)^T;$$
$$\varphi_1^{(1)}(s)=\psi_1^{(1)}(s)=0.1+0.2sin(t);$$
$$\varphi_2^{(1)}(s)=\psi_2^{(1)}(s)=-0.2+0.1cos(t);$$
$$\varphi_1^{(2)}(s)=\psi_1^{(2)}(s)=-0.4+0.1sin(t);$$
$$\varphi_2^{(2)}(s)=\psi_2^{(2)}(s)=0.6-0.2cos(t);$$
$$\Gamma_i^{(1)}=1;\quad \Gamma_j^2=2;$$

In the controller (14), to ensure the conditions (17) hold, we choose the control parameters $\lambda^x=(2,2)^T$, $\iota^x=(2,2)^T$, $\omega^x=(2.5,2.5)^T$, $\mu^x=(6,6)^T$, $\nu^x=(2,2)^T$, $\lambda^y=(2,2)^T$, $\iota^y=(2,2)^T$, $\omega^y=(2.5,2.5)^T$, $\mu^y=(6,6)^T$, $\nu^y=(2,2)^T$, $\gamma_1=1.2$, $\gamma_2=0.6$, $l_i=k_j=0.5$. Next, we will verify the conditions (17) of **Theorem 1**. According to the above parameter values, we have

$$(a_{ji}^*)_{2\times 2}=\begin{bmatrix}1.8 & 0.8\\ 0.4 & 1.8\end{bmatrix},\quad (a_{ij}^{**})_{2\times 2}=\begin{bmatrix}0.5 & 0.5\\ 0.6 & 0.8\end{bmatrix},$$
$$(b_{ji}^*)_{2\times 2}=\begin{bmatrix}1.5 & 1.2\\ 0.2 & 1.6\end{bmatrix},\quad (b_{ij}^{**})_{2\times 2}=\begin{bmatrix}1.5 & 1.5\\ 1.5 & 2.8\end{bmatrix}.$$

By simple calculation, these parameters make the conditions (17) of **Theorem 1** hold. With the help of Matlab, we can solve the numerical solutions of the drive-response MFBAMCNNs (31) and (32). Fig. 4 shows the phase diagrams of the drive MFBAMCNNs (31).

Fig. 5 presents the state trajectories between the drive MFBAMCNNs (31) and the response MFBAMCNNs (32).

Fig. 6 illustrates the error curves of the drive MFBAMCNNs (31) and the response MFBAMCNNs (32).

We calculate the settling time $T_{max}=3.7154$ based on Eq. (18) in **Theorem 1**.

Example 2: Consider the following common drive-response MBAMCNNs without fuzzy logic

$$\dot{x}_i(t)=-d_i^{(1)}(x_i(t))x_i(t)+\sum_{j=1}^{2}a_{ij}^{(1)}(x_i(t))f_j(y_j(t))+\sum_{j=1}^{2}b_{ij}^{(1)}(x_i(t))f_j(y_j(t-\tau(t)))+I_i^{(1)},$$
$$\dot{y}_j(t)=-d_j^{(2)}(y_j(t))y_j(t)+\sum_{i=1}^{2}a_{ji}^{(2)}(y_j(t))g_i(x_i(t))+\sum_{i=1}^{2}b_{ji}^{(2)}(y_j(t))g_i(x_i(t-\sigma(t)))+I_j^{(2)}. \quad (33)$$

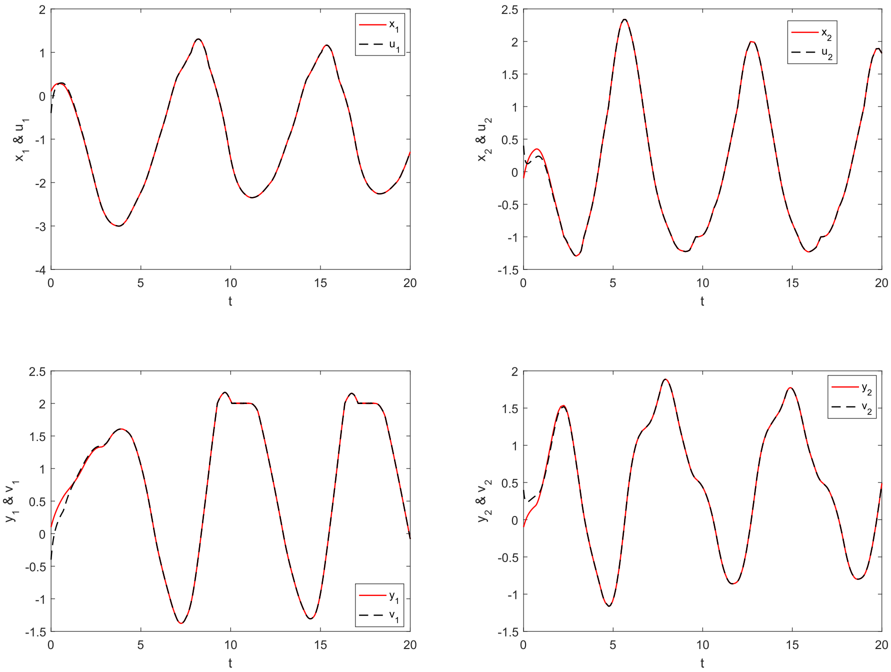

FIGURE 5. The state trajectories between the drive MFBAMCNNs (31) and the response MFBAMCNNs (32) under the controllers (14).

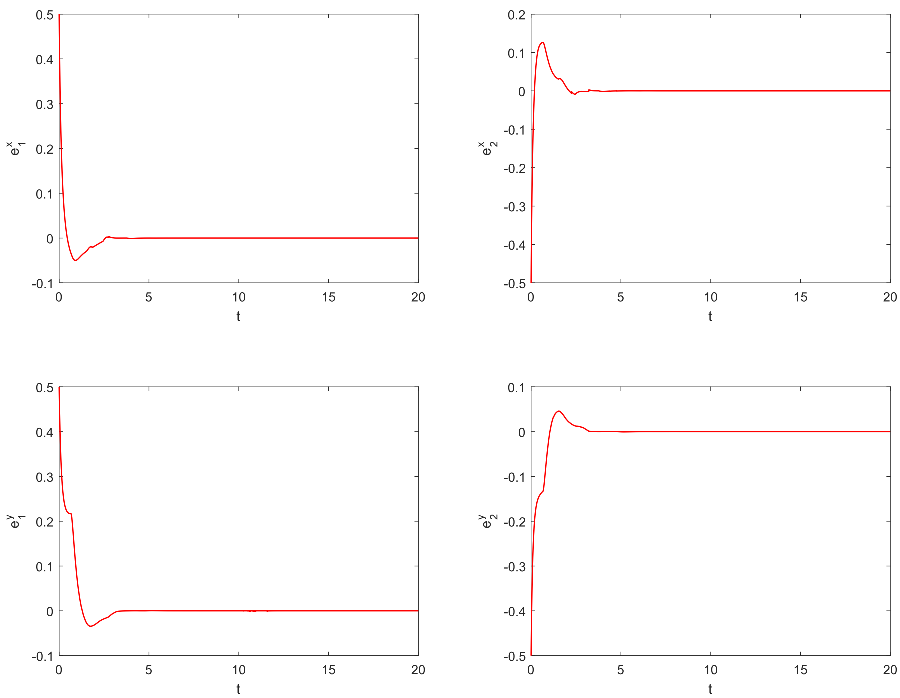

FIGURE 6. The state error curves of the drive MFBAMCNNs (31) and the response MFBAMCNNs (32) under the controllers (14).

The corresponding response system is shown below

$$\dot{u}_i(t) = -d_i^{(1)}(u_i(t))u_i(t) + \sum_{j=1}^{2} a_{ij}^{(1)}(u_i(t))f_j(v_j(t)) + \sum_{j=1}^{2} b_{ij}^{(1)}(u_i(t))f_j(v_j(t-\tau(t))) + I_i^{(2)} + r_i^u(t),$$

$$\dot{v}_j(t) = -d_j^{(2)}(v_j(t))v_j(t) + \sum_{i=1}^{2} a_{ji}^{(2)}(v_j(t))g_i(u_i(t)) + \sum_{i=1}^{2} b_{ji}^{(2)}(v_j(t))g_i(u_i(t-\sigma(t))) + I_j^{(2)} + r_j^v(t), \quad (34)$$

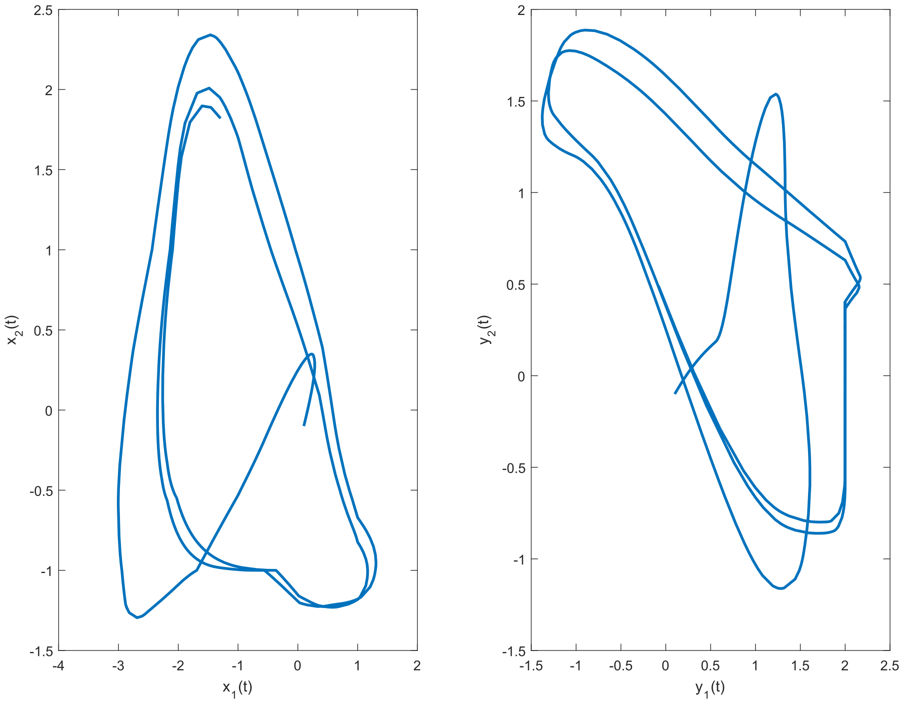

FIGURE 7. The phase diagrams of the drive system (33).

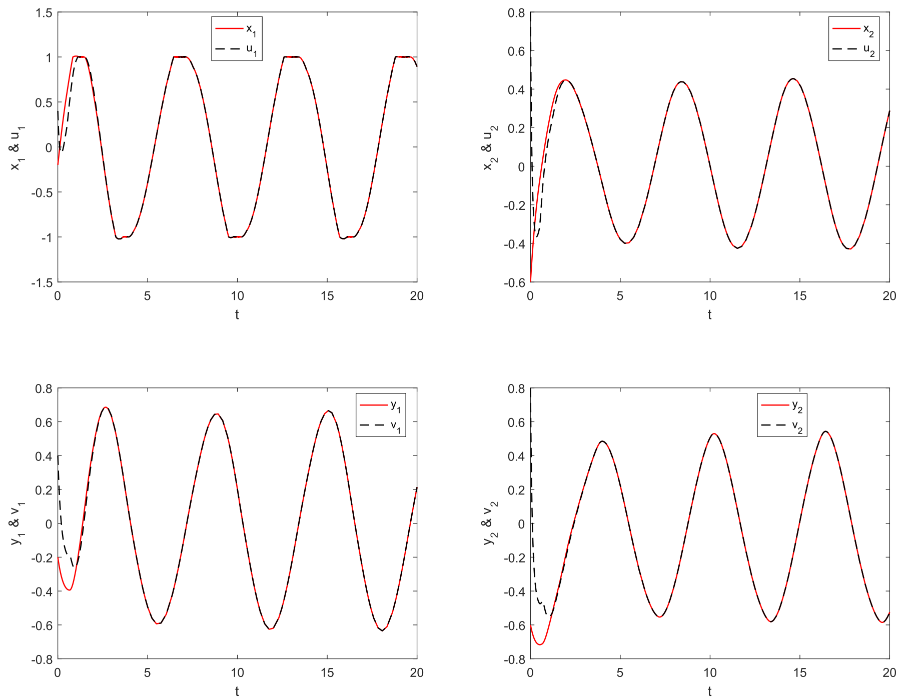

FIGURE 8. The state trajectories of the drive-response MBAMCNNs (33) and (34) under the controllers (14).

where the switching jumps $\Gamma_i^{(1)} = 1; \Gamma_j^{(2)} = 2$ and the memristor weights are given as follows

$$d_1^{(1)}(x_1) = \begin{cases} 1.2, & |x_1| \le 1, \\ 1.5, & |x_1| > 1, \end{cases} \quad d_2^{(1)}(x_2) = \begin{cases} 1.5, & |x_2| \le 1, \\ 2.0, & |x_2| > 1, \end{cases}$$

$$d_1^{(2)}(y_1) = \begin{cases} 1.1, & |y_1| \le 2, \\ 1.3, & |y_1| > 2, \end{cases} \quad d_2^{(2)}(y_2) = \begin{cases} 1.05, & |y_2| \le 2, \\ 1.25, & |y_2| > 2, \end{cases}$$

$$a_{11}^{(1)}(x_1) = \begin{cases} -1.5, & |x_1| \le 1, \\ -1.6, & |x_1| > 1, \end{cases} \quad a_{12}^{(1)}(x_1) = \begin{cases} -0.6, & |x_1| \le 1, \\ 0.4, & |x_1| > 1, \end{cases}$$

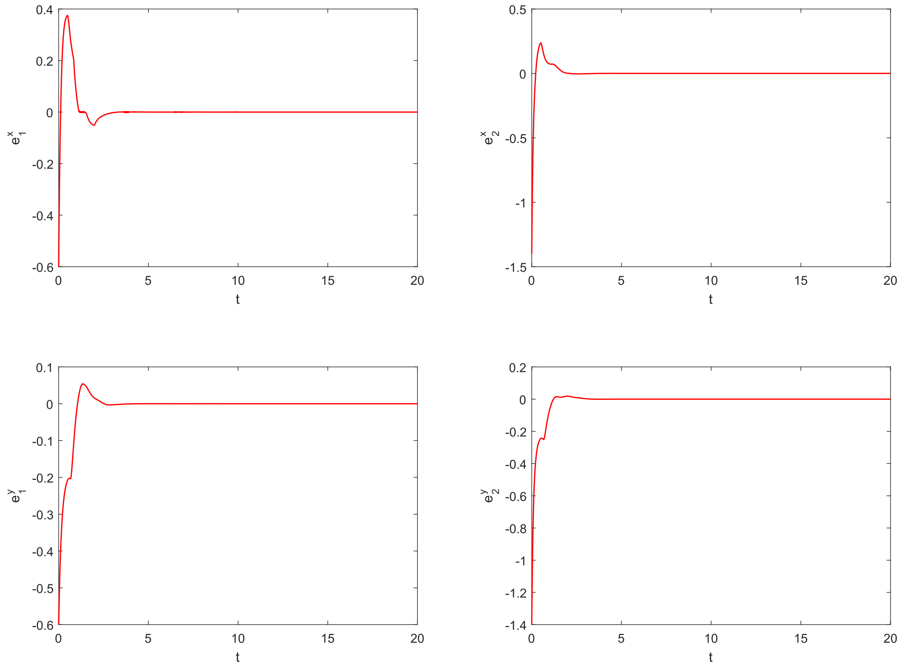

FIGURE 9. **The state error curves between the drive-response MBAMCNNs(31) and (32) under the controllers (14).**

$$a_{21}^{(1)}(x_2) = \begin{cases} 0.4, & |x_2| \le 1, \\ 0.35, & |x_2| > 1, \end{cases} a_{22}^{(1)}(x_2) = \begin{cases} 0.4, & |x_2| \le 1, \\ 0.42, & |x_2| > 1, \end{cases}$$

$$a_{11}^{(2)}(y_1) = \begin{cases} 0.2, & |y_1| \le 2, \\ 0.25, & |y_1| > 2, \end{cases} a_{12}^{(2)}(y_1) = \begin{cases} 0.56, & |y_1| \le 2, \\ 0.61, & |y_1| > 2, \end{cases}$$

$$a_{21}^{(2)}(y_2) = \begin{cases} 0.2, & |y_2| \le 2, \\ 0.25, & |y_2| > 2, \end{cases} a_{22}^{(2)}(y_2) = \begin{cases} 0.5, & |y_2| \le 2, \\ 0.56, & |y_2| > 2, \end{cases}$$

$$b_{11}^{(1)}(x_1) = \begin{cases} -1.1, & |x_1| \le 1, \\ -1.3, & |x_1| > 1, \end{cases} b_{12}^{(1)}(x_1) = \begin{cases} -1.2, & |x_1| \le 1, \\ -1.05, & |x_1| > 1, \end{cases}$$

$$b_{21}^{(1)}(x_2) = \begin{cases} -1.5, & |x_2| \le 1, \\ -1.3, & |x_2| > 1, \end{cases} b_{22}^{(1)}(x_2) = \begin{cases} -0.8, & |x_2| \le 1, \\ -0.9, & |x_2| > 1, \end{cases}$$

$$b_{11}^{(2)}(y_1) = \begin{cases} 0.45, & |y_1| \le 2, \\ 0.38, & |y_1| > 2, \end{cases} b_{12}^{(2)}(y_1) = \begin{cases} 0.85, & |y_1| \le 2, \\ 1.0, & |y_1| > 2, \end{cases}$$

$$b_{21}^{(2)}(y_2) = \begin{cases} -0.60, & |y_2| \le 2, \\ 0.65, & |y_2| > 2, \end{cases} b_{22}^{(2)}(y_2) = \begin{cases} 1.6, & |y_2| \le 2, \\ 1.4, & |y_2| > 2, \end{cases}$$

We assume that the memristor weights of the response system (34) are the same as that of the drive system (33). Other parameters are shown below

$$\tau(t) = 0.4 + 0.1cos(t); \quad \sigma(t) = 0.6 + 0.1sin(t);$$
$$f_j(x) = tanh(|x| - 1); \quad g_i(x) = tanh(|x| - 2);$$
$$\varphi_1^{(1)}(s) = \psi_1^{(1)}(s) = -0.2; \quad \varphi_2^{(1)}(s) = \psi_2^{(1)}(s) = 0.6;$$
$$\varphi_1^{(2)}(s) = \psi_1^{(2)}(s) = 0.4; \quad \varphi_2^{(2)}(s) = \psi_2^{(2)}(s) = -0.8;$$

Similarly, we choose the control parameters $\lambda^x = (2,2)^T$, $\iota^x = (2,2)^T$, $\omega^x = (1,1)^T$, $\mu^x = (4,4)^T$, $\nu^x = (2,2)^T$, $\lambda^y = (2,2)^T$, $\iota^y = (2,2)^T$, $\omega^y = (1,1)^T$, $\mu^y = (4,4)^T$, $\nu^y = (2,2)^T$, $\gamma_1 = 1.5$, $\gamma_2 = 0.8$. Next, we will verify the conditions (30) of **Corollary 2**. According to the above parameter values, we have

$$(a_{ji}^*)_{2\times2} = \begin{bmatrix} 1.6 & 0.6 \\ 0.4 & 0.42 \end{bmatrix}, \quad (a_{ij}^{**})_{2\times2} = \begin{bmatrix} 0.45 & 1.0 \\ 0.65 & 1.6 \end{bmatrix},$$
$$(b_{ji}^*)_{2\times2} = \begin{bmatrix} 1.3 & 1.2 \\ 1.5 & 0.9 \end{bmatrix}, \quad (b_{ij}^{**})_{2\times2} = \begin{bmatrix} 0.45 & 1.0 \\ 0.65 & 1.6 \end{bmatrix}.$$

The above parameters make the conditions (30) of **Corollary 2** hold. Here is our simulation results.

Fig.8 presents the state trajectories of the drive MBAMCNNs (33) and the response MBAMCNNs (34) under the controllers (14).

Fig. 9 illustrates the state error curves between the drive-response MBAMCNNs (33) and (34) under the controllers (14).

We can also calculate the settling time $T_{max} = 3.2500$ based on Eq. (18).

Remark 6: In the existing literature on fixed-time synchronization of neural networks [28], [33], [34], the model and the fixed-time synchronization controllers proposed in this paper are more general. The proof process of this paper is easier to understand. Meanwhile, it can be seen from the two numerical examples that the settling-time can be easily calculated.

V. CONCLUSION

In this paper, we address the fixed-time synchronization for the drive-response MFBAMCNNs with time-varying delays. MFBAMCNNs are a class of neural networks that combine FCNN with MBAMNN, and have more complex structure and dynamic behavior. Most of previous work did not involved such neural networks. Since the discontinuous right-sides of MFBAMCNNs, we deal with the memristive connection weights by using differential inclusion and set-valued map theories. By the definition of fixed-time synchronization, inequality techniques, and Lyapunov stability theory, some novel criteria are derived to ensure the fixed-time synchronization of the drive-response MFBAMCNNs based on nonlinear feedback controllers. In addition, the settling time can be obtained by simple calculation. At last, the effectiveness of our results is validated by two numerical examples.

So far, there are still many worthwhile studies on the fixed-time stability or synchronization of the MFBAM-CNN model. For example, some uncertainty, stochastic phenomena or various impulsive is inevitable in practical applications [40]–[44]. Therefore, it may be an interesting problem to study the MFBAMCNN system with uncertainty or stochastic phenomena. Moreover, how to design a continuous fixed-time synchronization controller, or an adaptive controller similar to that in [45] and [46] is also a problem worth studying in the future.

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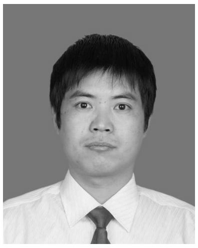

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