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A Short Linearly Homomorphic Proxy Signature Scheme

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ABSTRACT Linearly homomorphic signature schemes allow the performance of linear computations on authenticated data. They are important primitives for many applications, such as electronic voting, smart grids, electronic health records, and so on. Proxy signature schemes allow an original signer to delegate his/her signing power to a proxy signer, so that the proxy signer can sign on behalf of the original signer. Therefore, a signature scheme offering both of the above signatures' properties is very desirable. In this paper, we construct the first linearly homomorphic proxy signature scheme, so the proxy signer can produce a linearly homomorphic signature on behalf of the original signer. The scheme is provably secure in the random oracle model. Moreover, the length of signature is short and constant. Linearly homomorphic proxy signature scheme can be used in applications, such as electronic business and cloud computing.

INDEX TERMS Homomorphic signatures, proxy signature, bilinear pairings, random oracle.

I. INTRODUCTION

The conception of homomorphic signatures was originally proposed by Johnson *et al.* [1] in 2002. Homomorphic signature schemes are important primitives and allow to validate computation over authenticated data [9], [20]–[22], [28], [29], [32]–[34]. Informally, a signer holding a dataset $\{\mathbf{V}^{(i)}\}_{i=1}^{l}$ can produce corresponding signatures $\sigma_i = Sign(SK, \mathbf{V}^{(i)})$ for $i = 1, \ldots, l$ and store the signed dataset on a remote server. Later the server can publicly compute a succinct valid signature σ on $\mathbf{V} = \mathbf{f}(\mathbf{V}^{(1)}, \ldots, \mathbf{V}^{(l)})$. A keynote feature of homomorphic signatures is that the homomorphic signature σ can be computed without needing to know the original secret key. In the last years, various types of homomorphic signature schemes have been proposed. The first schemes proposed were only suitable for performing linear computations on authenticated data [2]–[7]. Then solutions have been developed to support polynomial functions [8], [9], [16]. Now, without any restrictions on the functions themselves, leveled fully homomorphic signature schemes have been designed [17], [18]. Homomorphic signature schemes can be employed in electronic business and cloud computing [10]–[15].

The concept of proxy signatures was first introduced by Mambo *et al.* [23] in 1996. Proxy signature schemes enable an original signer to delegate his/her signing capability to a proxy signer, and then the proxy signer can sign a message on behalf of the original signer. In 2012, Boldyreva *et al.* [24] gave the definition of proxy signatures in detail and formalized a model of security for proxy signature schemes. Furthermore, they specified the adversary's capabilities and goals. In their model, a public key infrastructure setting (PKI) is also assumed, where each entity holds a public and secret key pair. As usual, each user can sign messages using the signing algorithm of a standard digital signature scheme. A provably secure proxy signature scheme was also proposed in this model. Although the scheme is lack of efficiency, it can pay attention to the importance of security model [25]–[27], [30], [31], [35], [36]. According to the

delegation level, proxy signature schemes can be divided into full delegation, partial delegation and delegation by warrant. Proxy signature schemes have shown to be useful in many applications. Nowadays, various types of proxy signature schemes have been proposed, and they can offer the mixed natures of signatures, such as strong proxy signatures [37], proxy ring signatures [38], [39], and proxy blind signatures [40], [41]. But up to our knowledge, there are no schemes which combine the natures of linearly homomorphic signatures and proxy signatures.

In this paper, the concept of linearly homomorphic proxy signatures **(LHPS)** is proposed for the first time. It means that the proxy signer can produce linearly homomorphic signatures on behalf of the original signer. Suppose Alice wants to produce linearly homomorphic signatures, but she may be on vocation. So she can delegate Bob to generate signatures on behalf of her. Anyone can verify the validity of the signatures and perform linear computations on authenticated data. In a word, linearly homomorphic proxy signatures can combine the natures of linearly homomorphic signatures and proxy signatures.

A. OUR CONTRIBUTIONS

In this paper, we introduce the notion and security model of linearly homomorphic proxy signature schemes, and design a new LHPS scheme from bilinear pairings. We prove that our scheme is secure in the random oracle model. Moreover, our signature is an element of a circle group, so its length is very short.

B. ORGANIZATION

The rest of the paper is organized as follows. Section 2 contains some preliminaries about bilinear maps, the short signature scheme proposed by Boneh, Lynn, and Shacham (BLS), as well as the framework of linearly homomorphic proxy signature schemes and the security model. Section 3 gives a new linearly homomorphic proxy signature, and Section 4 gives the security and efficiency analysis of the scheme. Finally, Section 5 concludes this paper.

II. PRELIMINARIES

A. BILINEAR GROUPS

In this section, we briefly review the facts about bilinear maps. Let (G_1,G_2) be bilinear groups which satisfy $|G_1|=|G_2|=q$ for some prime number q.

$e : G_1 \times G_1 \rightarrow G_2$ is a bilinear map with the following properties:

(1) Bilinear: $\forall g, h \in G_1, \forall a, b \in Z_q, e(g^a, h^b) = e(g, h)^{ab}$;

(2) Non-degenerate: If g is a generator of G_1, then $e(g, g)$ is a generator of G_2. In other words, $e(g, g) \neq 1$;

(3) Computable: For all $g, h \in G_1$, there exists an efficient algorithm to compute $e(g, h)$.

Now we introduce the Computational Diffie-Hellman assumption in G_1.

Definition 1: **(CDH).** Given a random generator $g \in G_1$, if there exists no probabilistic polynomial-time (PPT) algorithm A that on input (g, g^x, g^y) outputs g^{xy} with non-negligible probability, we say that the CDH assumption holds in G_1. Here the probability is taken over the uniform choices of $x, y \leftarrow Z_q{}^*$ and the internal coin tosses of A.

B. BLS SHORT SIGNATURE SCHEME

The BLS short signature scheme proposed in [19] consists of the following algorithms: a key generation algorithm **KeyGen**, a signature generation algorithm **Sign** and a signature verification algorithm **Verify**. And it uses a bilinear map $e : G_1 \times G_1 \rightarrow G_2$ and a full-domain hash function $H : \{0, 1\}^* \rightarrow G_1$, and g is a random generator of G_1.

KeyGen: The secret key is $x \in Z_q^*$, and the public key is $PK = g^x$.

Sign: Given a secret key x, and a message m, compute the signature $\sigma = H(m)^x$.

Verify: Given a public key PK, a message m and a signature σ, verify if the equation $e(\sigma, g) = e(H(m), PK)$ holds, this algorithm outputs 1; otherwise it outputs 0.

The security of the BLS short signature scheme is based on the CDH assumption. We refer to [19] for more details.

C. LINEARLY HOMOMORPHIC PROXY SIGNATURE

Definition 2: **(LHPS)**. A linearly homomorphic proxy signature (LHPS) scheme consists of six algorithms :(**Setup, KeyGen, Delegation, PSign, PVerify, Combine**). The algorithms are defined as follows:

- **Setup:** This algorithm takes a security parameter λ and an integer l as input, and returns the string params, which denotes the common scheme parameters. Notice that l denotes an upper bound for the number of messages signed in each file.
- **KeyGen:** This algorithm takes the system parameters as input and returns a secret/public key pair (SK, PK) for a user in the system.
- **Delegation:** The original signer A creates a warrant m_{ω_B} related to the proxy signer B, then interacts with B by a series of interactive algorithms forming the delegation protocol. As a result of the interaction, the final output of the protocol is a proxy key S_p that the proxy signer B uses to produce proxy signatures on behalf of the original signer A.
- **PSign:** On input a proxy key S_p, a file identifier $\tau \in \{0, 1\}^\lambda$, and a message vector $\mathbf{V}$, this algorithm outputs the proxy signature σ.
- **PVerify:** Given the public key PK_0 for the original signer A, the public key PK_B for the proxy signer B, a warrant m_{ω_B}, a file identifier τ, a message vector $\mathbf{V}$ and a proxy signature σ, it outputs 1 (accept) or 0 (reject).
- **Combine:** Given PK_0, PK_B, a warrant m_{ω_B}, a file identifier τ, and a set of tuple $\{(f_i, \sigma_i)\}_{i=1}^{l}$, this algorithm outputs a signature σ *(*Note that σ is intended to be a signature on $\sum_{i=1}^{l} f_i \mathbf{V}^{(i)}$, where $\mathbf{V}^{(i)}$ denotes the i-th vector in the list of vectors $\mathbf{V}^{(1)}, \mathbf{V}^{(2)}, \ldots, \mathbf{V}^{(l)}$*)*.

Correctness. For correctness, we require:
(1) For all message vector $\mathbf{V}$ and all file identifier $\tau \in \{0,1\}^\lambda$, if $\sigma \leftarrow$ **PSign**$(S_p, \tau, \mathbf{V})$, then

$$\textbf{PVerify}(PK_0, PK_B, m_{\omega_B}, \tau, \mathbf{V}, \sigma) = 1.$$

(2) For all $\tau \in \{0,1\}^\lambda$, and all sets of triples $\{(f_i, \sigma_i, \mathbf{V}^{(i)})\}_{i=1}^{l}$, if it holds that

$$\textbf{PVerify}(PK_0, PK_B, m_{\omega_B}, \tau, \mathbf{V}^{(i)}, \sigma_i) = 1$$

for all $i = 1, \ldots, l$, then

$$\textbf{PVerify}(PK_0, PK_B, m_{\omega_B}, \tau, \sum_{i=1}^{l} f_i \mathbf{V}^{(i)},$$
$$\textbf{Combine}(PK_0, PK_B, m_\omega, \tau, \{(f_i, \sigma_i)\}_{i=1}^{l})) = 1.$$

Security Model: We should consider two types of unforgeability in LHPS: **Delegation unforgeability** and **Linearly homomorphic proxy signature unforgeability.** Delegation unforgeability means that if the adversary does not obtain the targeted delegation from the original signer, it is hard to output a forgery of the targeted proxy signature. Linearly homomorphic proxy signature unforgeability means that, except the proxy signer, anyone else including the origin signer cannot generate a valid linearly homomorphic proxy signature on behalf of the proxy signer. So we can divide the adversaries into the following two types: **Type I**, the adversary A has the key pair (SK_j, PK_j) for the proxy signer j, but it can not obtain the delegation from the original signer; **Type II**, the adversary obtains the delegation from the original signer, but it has no secret key of the challenged proxy signer.

Now we introduce the security models in detail as follows:

Definition 3 (Delegation Unforgeability): A LHPS = (**Setup, KeyGen, Delegation, PSign, PVerify, Combine**) has delegation unforgeability if the advantage of any PPT Type I adversary A in the following security game is negligible in the security parameter λ (Note that in this model, the adversary A does not obtain the targeted delegation from the original signer).

The challenger C sets $(SK_0, PK_0) \leftarrow$ **KeyGen** (1^λ) as the secret/public key for the original signer, then gives PK_0 and the system parameter *params* to A. The *params* define a message space and a signature space. Note that l denotes an upper bound for the number of messages signed in each file.
Queries: The adversary's attack capabilities are modelled by providing it access to a series of oracles, so A can ask a polynomial number of queries as follows:

1) **KR queries.** Given a key pair (SK_i, PK_i), C first checks if (SK_i, PK_i) is a valid key pair. If it is true, then (SK_i, PK_i) is stored in a list. Otherwise, C rejects and outputs a special symbol $\perp$.
2) **DE queries.** Given PK_0 and any registered public key PK_i, C returns a warrant m_{ω_i} and a delegation key S_{w_i} corresponding to the warrant m_{ω_i}.
3) **Signing queries.** Given any registered public key PK_i, PK_0, a file identifier $\tau \in \{0,1\}^\lambda$, and a message vector $\mathbf{V}$, C outputs the signature σ including the corresponding warrant m_{ω_i}.

Output: A outputs a public key PK_j, PK_0, a warrant m_{ω_j}, a file identifier $\tau^* \in \{0,1\}^\lambda$, a message vector $\mathbf{V}^*$ and a signature σ^*.

The adversary wins if **PVerify**$(PK_0, PK_j, m_{w_j}, \tau^*, \mathbf{V}^*, \sigma^*) = 1$, and it must satisfy that the public key PK_j does not appear in DE queries and signing queries.

The advantage of the adversary is the probability that he wins the above game.

Definition 4 (Linearly Homomorphic Proxy Signature Unforgeability): A LHPS = (**Setup, KeyGen, Delegation, PSign, PVerify, Combine**) has linearly homomorphic proxy signature unforgeability if the advantage of any PPT Type II adversary A in the following security game is negligible in the security parameter λ (Note that in this model, the adversary A has no secret key of the challenged proxy signer).

The challenger C sets $(SK_0, PK_0) \leftarrow$ **KeyGen** (1^λ) as the secret/public key for the original signer and $(SK^*, PK^*) \leftarrow$ **KeyGen** (1^λ) as the secret/public key for the targeted proxy signer. Then C gives (SK_0, PK_0), PK^* and the system parameter *params* to A. The *params* define a message space and a signature space. Similarly, l denotes an upper bound for the number of messages signed in each file.
Queries: The adversary's attack capabilities are modelled by providing it access to a series of oracles, so A can ask a polynomial number of queries as follows:

1) **KR queries.** Given a key pair (SK_i, PK_i), C first checks if (SK_i, PK_i) is a valid key pair. If it is true, then (SK_i, PK_i) is stored in a list. Otherwise, C rejects and outputs a special symbol $\perp$.
2) **Signing queries.** Given the targeted public key PK^*, PK_0, a warrant m_{ω^*}, a file identifier $\tau \in \{0,1\}^\lambda$, and a message vector $\mathbf{V}$, C outputs the signature σ.

Output: A outputs the targeted public key PK^*, PK_0, a warrant m_{ω^*}, a file identifier $\tau^* \in \{0,1\}^\lambda$, a message vector $\mathbf{V}^* \neq \mathbf{0}$ and a signature σ^*.

The adversary wins if **PVerify**$(PK_0, PK^*, m_{\omega^*}, \tau^*, \mathbf{V}^*, \sigma^*) = 1$, and the file identifier τ^* does not appear in signing queries.

The advantage of the adversary is the probability that he wins the above game.

III. THE PROPOSED SCHEME

In this section, we propose a provably secure linearly homomorphic proxy signature from bilinear pairings. And our proxy signature scheme belongs to delegation by warrant. So the original signer makes a warrant m_ω before delegation. The warrant m_ω contains some explicit information including the description of the delegation relation.

Now we construct the new scheme as follows:

1) **Setup:** Let (G_1, G_2) be bilinear groups satisfying $|G_1| = |G_2| = q$ for some prime number q and g be the generator of G_1. The bilinear map is given by e: $G_1 \times G_1 \rightarrow G_2$. Define two hash functions

$H_1 : \{0,1\}^* \to G_1$, $H_2 : \{0,1\}^* \times Z \to G_1$. H_1 and H_2 will be viewed as random oracles in our security proof. Let $[l] = \{1, \ldots, l\}$, $[N] = \{1, \ldots, N\}$. The security parameter is λ. The system parameter $params = (G_1, G_2, q, g, e, \lambda, H_1, H_2)$. Assume that $params$ defines the file identifier space $\mathcal{ID}$, where $|\mathcal{ID}| = poly(\lambda)$.

2) **KeyGen:** The original signer chooses a secret key $x_0 \in Z_q^*$ and the public key is $PK_0 = g^{x_0}$. The secret key of the proxy signer B is $x_B \in Z_q^*$ and the corresponding public key is $PK_B = g^{x_B}$.
3) **Delegation:** The original signer generates a standard warrant m_ω related to the proxy signer B, computes $S_\omega = H_1(m_w)^{x_0}$, and sends S_ω to B. Then B verifies the equation $e(S_\omega, g) = e(H_1(m_w), PK_0)$. If the equation holds, then the proxy signer B gets the proxy key $S_p = (x_B, S_\omega)$.
4) **PSign:** Given a message vector $\mathbf{V} = (v_1, \ldots, v_N) \in Z_q^N$, a proxy key S_p, and a file identifier $\tau \in \mathcal{ID}$, it returns $\perp$ if $\sum_{j\in[N]} v_j = 0$. Otherwise, the proxy signer B can compute

$$\sigma = S_\omega^{\sum_{j\in[N]} v_j} \cdot (\prod_{j\in[N]} H_2(\tau, j)^{v_j})^{x_B}$$

5) **PVerify:** Given PK_0, PK_B, a warrant m_ω, a file identifier τ, a message vector $\mathbf{V} = (v_1, \ldots, v_N) \in Z_q^N$, and a proxy signature σ, return 0 if $\sum_{j\in[N]} v_j = 0$. Otherwise, the verifier checks if

$$e(\sigma, g) = e(H_1(m_w), PK_0)^{\sum_{j\in[N]} v_j} \cdot e(\prod_{j\in[N]} H_2(\tau, j)^{v_j}, PK_B)$$

If the equation holds, output 1; otherwise, output 0.
6) **Combine:** Given PK_0, PK_B, a warrant m_ω, a file identifier τ and a set of tuple $\{(f_i, \sigma_i)\}_{i=1}^l$, this algorithm outputs a signature $\sigma \leftarrow \prod_{i\in[l]} \sigma_i^{f_i}$.

Correctness:

Given PK_0, PK_B, a warrant m_ω, a file identifier τ, a message vector $\mathbf{V} = (v_1, \ldots, v_N) \in Z_q^N$ and a proxy signature σ, if $\sigma \leftarrow$ **PSign**$(S_p, \tau, \mathbf{V})$, the correctness of the scheme can be verified by the following equations:

$$\begin{aligned} e(\sigma, g) &= e(S_\omega, g)^{\sum_{j\in[N]} v_j} \cdot e((\prod_{j\in[N]} H_2(\tau, j)^{v_j})^{x_B}, g) \\ &= e(H_1(m_w), PK_0)^{\sum_{j\in[N]} v_j} \\ &\quad \cdot e(\prod_{j\in[N]} H_2(\tau, j)^{v_j}, PK_B) \end{aligned}$$

Furthermore, given $\tau \in \mathcal{ID}$ and all sets of triples $\{(f_i, \sigma_i, \mathbf{V}_i)\}_{i=1}^l$, if $\sigma_i \leftarrow$ **PSign**$(S_p, \tau, \mathbf{V}^{(i)})$, then by our definition of **Combine**, we have $\sigma \leftarrow \prod_{i\in[l]} \sigma_i^{f_i}$.

Now, we only need to check that σ is a signature on the $\mathbf{V} = (v_1, \ldots, v_N) = \sum_{i\in[l]} f_i \mathbf{V}^{(i)}$, where $\mathbf{V}^{(i)}$ denotes the i-th vector in the list of vectors $\mathbf{V}^{(1)}, \mathbf{V}^{(2)}, \ldots, \mathbf{V}^{(l)}$. Suppose $\mathbf{V}^{(i)} = (v_1^{(i)}, \ldots, v_N^{(i)})$, by correctness of individual signature, we have

$$\begin{aligned} e(\sigma_i, g) = e(H_1(m_w), PK_0)^{\sum_{j\in[N]} v_j^{(i)}} \\ \cdot e(\prod_{j\in[N]} H_2(\tau, j)^{v_j^{(i)}}, PK_B). \end{aligned}$$

So by the bilinear property, we have

$$\begin{aligned} e(\sigma, g) &= \prod_{i\in[l]} e(\sigma_i, g)^{f_i} \\ &= e(H_1(m_w), PK_0)^{\sum_{i\in[l]}\sum_{j\in[N]} f_i v_j^{(i)}} \\ &\quad \cdot e(\prod_{j\in[N]} H_2(\tau, j)^{\sum_{i\in[l]} f_i v_j^{(i)}}, PK_B) \\ &= e(H_1(m_w), PK_0)^{\sum_{j\in[N]}\sum_{i\in[l]} f_i v_j^{(i)}} \\ &\quad \cdot e(\prod_{j\in[N]} H_2(\tau, j)^{\sum_{i\in[l]} f_i v_j^{(i)}}, PK_B) \\ &= e(H_1(m_w), PK_0)^{\sum_{j\in[N]} v_j} \\ &\quad \cdot e(\prod_{j\in[N]} H_2(\tau, j)^{v_j}, PK_B) \end{aligned}$$

This completes the proof.

IV. PROPOSED SCHEME ANALYSIS

Theorem 1: Assuming that the type I adversary makes at most q_{H_1}, q_{H_2}, q_D and q_S queries to the H_1, H_2, Delegation and Signing oracles, respectively, the signature scheme has delegation unforgeability on adaptively chosen-message attacks in the random oracle model if the CDH assumption holds in G_1.

Proof: Supposing C is a challenger and A is an adversary, C is given (g, g^x, g^y) in order to output g^{xy}. In this model, the adversary A does not obtain the delegation from the original signer.

First, C runs A on input $PK_0 = g^x$ as the public key of the original signer, then sends the system $params$=$(G_1, G_2, q, g, e, \lambda, H_1, H_2, PK_0)$ to the adversary A and responses as follows:

Key registration queries: We assume that the number of users in the game is q_{H_1}. When A requests to register a new user i by outputting pair (x_i, PK_i), C verifies if they are valid key pairs, then adds (x_i, PK_i) to the **Key-List**.

H_1-**queries:** Assuming w.l.o.g A makes q_{H_1} times to H_1-queries and gets the warrant m_{ω_i} for $1 \le i \le q_{H_1}$ from C before these queries, C randomly chooses $s \in [1, q_{H_1}]$ and $t_i \in Z_q^*$ for $1 \le i \le q_{H_1}$, where s is the targeted proxy signer's number. When A queries m_{ω_i} to H_1-oracle, C answers $H_1(m_{w_i}) = g^{t_i}$ if $i \ne s$; Otherwise, $H_1(m_{w_i}) = g^y$ if

$i = s$. Then C adds $(m_{\omega_i}, g^{t_i}, t_i)_{i \neq s}$ to the H_1-**List**; If $i = s$, C adds $(m_{\omega_s}, g^y, *)$ to the H_1-**List** (Note that $*$ means the corresponding value is unknown).

H_2-**queries:** Assume that A makes q_{H_2} times to H_2-queries. When A queries (τ, j) to H_2- oracle, C randomly chooses $\alpha_{\tau,j} \in Z_q^*$ for $1 \leq j \leq N$ and answers $H_2(\tau, j) = g^{\alpha_{\tau,j}}$. Then C adds $((\tau, j), g^{\alpha_{\tau,j}}, \alpha_{\tau,j})$ to the H_2-**List**.

Delegation queries: When A requests delegation for user i, assuming w.l.o.g A has requested H_1-**queries** on m_{ω_i}, C checks the H_1-**List** and computes $S_{\omega_i} = (g^{t_i})^x = PK_0^{t_i}$ if $i \neq s$; Otherwise, C aborts if $i = s$. Then C adds (i, S_{ω_i}) to the **DG-List**.

Signing queries: Given PK_0, PK_i, a warrant m_{ω_i}, a file identifier $\tau \in \mathcal{ID}$, and a message vector $\mathbf{V} = (v_1, \ldots, v_N) \in Z_q^N$ such that $\sum_{j\in[N]} v_j \neq 0$, assuming w.l.o.g A has requested the above corresponding queries for user i, C checks the **Key-List**, H_2-**List**, **DG-List** and responses as follows:

- if $i \neq s$, C answers the signature

$$\sigma = S_{\omega_i}^{\sum_{j\in[N]} v_j} \cdot \Big(\prod_{j\in[N]} H_2(\tau, j)^{v_j}\Big)^{x_i} = PK_0^{t_i \sum_{j\in[N]} v_j} \cdot \Big(\prod_{j\in[N]} PK_i^{\alpha_{\tau,j} v_j}\Big)$$

- if $i = s$, C aborts.

Output: A outputs a a signature σ^* on a message vector $\mathbf{V}^* = (v_1^*, \ldots, v_N^*) \in Z_q^N$ with respect to PK_0, PK_i, a warrant m_{ω_i}, and a file identifier τ^* such that $\sum_{j\in[N]} v_j^* \neq 0$ and

$$\mathbf{PVerify}(PK_0, PK_i, m_{\omega_i}, \tau^*, \mathbf{V}^*, \sigma^*) = 1$$

- If $i \neq s$, C aborts.
- Otherwise, it holds that σ^* can satisfy the verification equation

$$e(\sigma^*, g) = e(H_1(m_{w_s}), PK_0)^{\sum_{j\in[N]} v_j^*} \cdot e\Big(\prod_{j\in[N]} H_2(\tau^*, j)^{v_j^*}, PK_s\Big)$$

Assuming w.l.o.g A has requested H_1-queries on m_{ω_s} and H_2-queries on τ^*, C checks H_1-**List**, and gets $H_1(m_{\omega_s}) = g^y$. Furthermore, C gets $H_2(\tau^*, j) = g^{\alpha_{\tau^*,j}}$ for $j \in [N]$ from H_2-**List**. Then we have

$$\begin{aligned} e(\sigma^*, g) &= e(g^y, g^x)^{\sum_{j\in[N]} v_j^*} \cdot e(g^{\sum_{j\in[N]} \alpha_{\tau^*,j} v_j^*}, PK_s) \\ &= e(S_{\omega_s}^{\sum_{j\in[N]} v_j^*}, g) \cdot e(PK_s^{\sum_{j\in[N]} \alpha_{\tau^*,j} v_j^*}, g) \\ &= e(S_{\omega_s}^{\sum_{j\in[N]} v_j^*} \cdot PK_s^{\sum_{j\in[N]} \alpha_{\tau^*,j} v_j^*}, g) \end{aligned}$$

So by the non-degenerate property, we have

$$\sigma^* = S_{\omega_s}^{\sum_{j\in[N]} v_j^*} \cdot PK_s^{\sum_{j\in[N]} \alpha_{\tau^*,j} v_j^*}$$

C can compute $g^{xy} = S_{\omega_s} = (\sigma^* / \prod_{j\in[N]} PK_s^{\alpha_{\tau^*,j} v_j^*})^{1/\sum_{j\in[N]} v_j^*}$, then the CDH problem is solved.

We analyze the probability of success for C. There are three sceneries in which C will abort. Assume **E1** means that $i = s$ in Delegation queries; **E2** means that $i = s$ in Signature queries; **E3** means that $i \neq s$ in Output period.

We have $\Pr[\mathbf{E1}] = \frac{q_D}{q_{H_1}}$, $\Pr[\mathbf{E2}] = \frac{q_S}{q_{H_1}}$, $\Pr[\mathbf{E3}] = 1 - \frac{1}{q_{H_1}}$. So if A is an adversary with success probability ε, C can solve the CDH problem with probability $(1 - \frac{q_D}{q_{H_1}})(1 - \frac{q_S}{q_{H_1}})\frac{1}{q_{H_1}}\varepsilon$.

This completes the proof. ■

Theorem 2: Assuming that the type II adversary makes at most q_{H_1}, q_{H_2} and q_S queries to the H_1, H_2 and Signing oracles, respectively, the scheme has linearly homomorphic proxy signature unforgeability on adaptively chosen-message attacks in the random oracle model if the CDH assumption holds in G_1.

Proof: Assuming C is a challenger and A is an adversary, C is given (g, g^x, g^y) in order to output g^{xy}. In this model, the adversary A does not obtain the secret key of the challenged proxy signer.

First, C runs A on input $PK^* = g^x$ as the public key of the targeted proxy signer. Moreover, C chooses a random integer $x_0 \in Z_q^*$ and sets $PK_0 = g^{x_0}$, then sends the system parameter *params*=$(G_1, G_2, q, g, e, \lambda, H_1, H_2, x_0, PK_0, PK^*)$ to A and responses as follows:

Key registration queries: When A requests to register a new user i by outputting pair (x_i, PK_i), C verifies if they are valid key pairs, and then adds (x_i, PK_i) to the **Key-List**.

H_1-**queries:** Assuming A makes q_{H_1} times to H_1-queries, C randomly chooses $t_i \in Z_q^*$ for $1 \leq i \leq q_{H_1}$. When A queries m_{ω_i} to H_1-oracle, C answers $H_1(m_{w_i}) = g^{t_i}$. Then C adds (m_{w_i}, g^{t_i}, t_i) to the H_1-**List**.

H_2-**queries:**Assume that A makes q_{H_2} times to H_2-queries. C randomly chooses $s \in [N]$ and a file identifier $\tau^* \in \{0, 1\}^\lambda$ as the target. When A queries (τ, j) to H_2- oracle, C randomly chooses $\alpha_{\tau,j} \in Z_q^*$ for $1 \leq j \leq N$ and answers $H_2(\tau, j) = g^{\alpha_{\tau,j}}$ if $(\tau, j) \neq (\tau^*, s)$; Otherwise, $H_2(\tau^*, s) = g^y$. Then C adds $((\tau, j), g^{\alpha_{\tau,j}}, \alpha_{\tau,j})_{(\tau,j)\neq(\tau^*,s)}$ to the H_2-**List**. If $(\tau, j) = (\tau^*, s)$, then C adds $((\tau^*, s), g^y, *)$ to the H_2-**List** (Note that $*$ means the corresponding value is unknown).

Signing queries: Given PK_0, PK^*, a warrant m_{ω^*}, a file identifier $\tau \in \mathcal{ID}$, and a message vector $\mathbf{V} = (v_1, \ldots, v_N) \in Z_q^N$, assuming w.l.o.g A has requested H_1-queries on m_{ω^*} and H_2-queries on τ, C checks and gets the corresponding t^* from H_1-**List**. Furthermore, if $\tau \neq \tau^*$, C gets the corresponding $\alpha_{\tau,j}$ for $j \in [N]$ from H_2-**List**. Then C responses as follows:

- if $\tau \neq \tau^*$, C answers the signature

$$\sigma = S_{\omega^*}^{\sum_{j\in[N]} v_j} \cdot \Big(\prod_{j\in[N]} H_2(\tau, j)^{v_j}\Big)^x = PK_0^{t^* \sum_{j\in[N]} v_j} \cdot PK^{*\sum_{j\in[N]} \alpha_{\tau,j} v_j}$$

- if $\tau = \tau^*$, C aborts.

Output: A outputs a a signature σ^* on a message vector $\mathbf{V}^* = (v_1^*, \ldots, v_N^*) \in Z_q^N$ with respect to PK_0, PK^*, a warrant m_{ω^*}, and a file identifier $\overline{\tau}$ such that $\mathbf{V}^* \neq \mathbf{0}$ and

$$\mathbf{PVerify}(PK_0, PK^*, m_{\omega^*}, \overline{\tau}, \mathbf{V}^*, \sigma^*) = 1$$

- If $\overline{\tau} \neq \tau^*$ or $v_s^* = 0$, C aborts.
- Otherwise, it holds that σ^* can satisfy the verification equation

$$e(\sigma^*, g) = e(H_1(m_{\omega^*}), PK_0)^{\sum_{j\in[N]} v_j^*} \cdot e(\prod_{j\in[N]} H_2(\tau^*, j)^{v_j^*}, PK^*)$$

Assuming w.l.o.g A has requested H_1-queries on m_{ω^*} and H_2-queries on τ^*, C checks H_1-**List**, H_2-**List** and gets the corresponding values. Then we have

$$\begin{aligned} e(\sigma^*, g) &= e(H_1(m_{\omega^*}), g^{x_0})^{\sum_{j\in[N]} v_j^*} \cdot e(g^{\sum_{j\neq s} \alpha_{\tau^*,j} v_j^*} g^{y v_s^*}, g^x) \\ &= e(S_{\omega^*}^{\sum_{j\in[N]} v_j^*}, g) \cdot e(PK^{*\sum_{j\neq s} \alpha_{\tau^*,j} v_j^*} (g^{xy})^{v_s^*}, g) \\ &= e(S_{\omega^*}^{\sum_{j\in[N]} v_j^*} \cdot PK^{*\sum_{j\neq s} \alpha_{\tau^*,j} v_j^*} (g^{xy})^{v_s^*}, g) \end{aligned}$$

So by the non-degenerate property, we have

$$\sigma^* = S_{\omega^*}^{\sum_{j\in[N]} v_j^*} \cdot PK^{*\sum_{j\neq s} \alpha_{\tau^*,j} v_j^*} (g^{xy})^{v_s^*}$$

C can compute $g^{xy} = (\sigma^* / (S_{\omega^*}^{\sum_{j\in[N]} v_j^*} \cdot PK^{*\sum_{j\neq s} \alpha_{\tau^*,j} v_j^*}))^{\frac{1}{v_s^*}}$, then the CDH problem is solved.

We analyze the probability of success for C. There are three sceneries in which C will abort. Assume **E1** means that $\tau = \tau^*$ in Signature queries; **E2** means that $\overline{\tau} \neq \tau^*$ in Output period, **E3** means that $v_s^* = 0$ in Output period.

We have $\Pr[\mathbf{E1}] = \frac{q_S}{poly(\lambda)}$, $\Pr[\mathbf{E2}] = 1 - \frac{1}{poly(\lambda)}$, $\Pr[\mathbf{E3}] = \frac{1}{q}$. So if A is an adversary with success probability ε, C can solve the CDH problem with probability $(1 - \frac{q_S}{poly(\lambda)})\frac{1}{poly(\lambda)}(1 - \frac{1}{q})\varepsilon$.

This completes the proof. ■

Efficiency Analysis: Our scheme has a series of advantages. Firstly, the delegation uses the BLS short signature scheme, so the certification is an element in group G_1. Secondly, our linearly homomorphic proxy signature is also an element in group G_1, so the length of the signature is very short. Our scheme is more efficient and suitable for low-bandwidth communication environments. Finally, the verification of the new proxy signature only requires three pair computations, so it is efficient and practical.

V. CONCLUSION

In this paper, we formally introduce the concept of linearly homomorphic proxy signatures, which allows a proxy signer to produce linearly homomorphic signatures on behalf of the original signer. Moreover, we give the formal security definition and design a linearly homomorphic proxy signature. Then we prove the signature is secure against existentially forgery on adaptively chosen-message attacks in the random oracle model based on the CDH assumption. The length of our signature scheme is very short, so our scheme is suitable for low-bandwidth communication environments. Linearly homomorphic proxy signature schemes can be used in applications such as electronic business and cloud computing.

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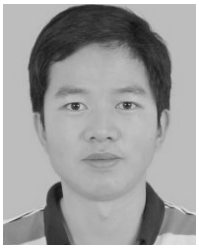

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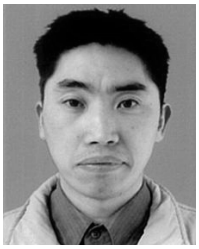

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