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Rule Generation Based on Novel Kernel Intuitionistic Fuzzy Rough Set Model

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ABSTRACT This paper develops a novel kernel intuitionistic fuzzy rough set (KIFRS) model as a hybrid model to improve the effects of rule generation based on rough sets. The KIFRS model adopts new kernel intuitionistic fuzzy clustering (KIFCM) to enhance the performance of rough set theory (RST). To effectively improve the rule generation based on RST, the proposed hybrid method first adopts KIFCM to cluster raw data into similarity groups. Based on the KIFCM results, the RST can obtain superior performance in generating rules. Two benchmark machine learning data sets from the UCI machine learning repository are used to examine the performance of the developed model. The results show that the KIFRS model achieves superior performance to those of the traditional decision tree and rough set models.

INDEX TERMS Rule generation, rough set theory, kernel intuitionistic fuzzy clustering.

I. INTRODUCTION

Data mining involves clustering algorithms, classification, regression analysis and association rule learning [1]. Discovering domain knowledge requires accurately generating decision rules in data mining. Therefore, this study aims to develop a KIFRS model to improve the effect of rule generation based on KIFCM and rough set theory. Clustering analysis is first employed to group the pattern datasets in the novel model. This work develops a novel rule generation model that combines new KIFCM and RST and is a feasible and promising method of classification and rule generation. KIFCM can effectively reduce the complexity of pattern datasets that occurs because of effective clustering. RST is one of the most popular and important methods in rule generation. In this study, RST is utilized as the main rule generation technology together with the KIFCM results. The proposed KIFRS can improve the traditional RST performance. RST ([2], [3]) is an effective and popular model for analyzing inconsistent decision tables composed of attribute value data about many objects. RST is a mathematical tool for handling vagueness and uncertainty. RST can derive optimal attribute sets with less deterioration of the quality of approximation and can provide optimal decision rules based on lower/upper approximation. RST has been widely applied in many fields, such as knowledge acquisition, decision support systems, and medical information. The recent literature on rough set-based rule generation technology since 2000 is summarized in Table 1.

By examining the past RST research listed in Table 1, some phenomena can be observed: (1) RST can successfully be applied in areas such as manufacturing processes, machining operations, information systems, and power system stabilizers; (2) RST-based rule generation can obtain superior performance to those of other rule generation methods; and (3) hybrid methods can effectively enhance the performance of RST. The rest of this study can be organized as follows. The KIFRS for rule generation is introduced in Section II. Section III illustrates the experimental results and compares them with the performances of various other models in standard machine learning data sets. Finally, section IV presents the conclusions of this study.

II. PROPOSED KIFRS MODEL

It has been verified that hybrid models can improve traditional approaches, as in [36], when applied in forecasting problems. In this study, the novel KIFRS is a hybrid model that combines KIFCM and RST [37]. The goal is to improve the accuracy of traditional RST in data mining problems. A novel KIFCM was proposed by [38] and could effectively obtain superior performance in machining learning datasets. This KIFCM utilized the intuitionistic fuzzy sets [39] to

TABLE 1. Recent literature on rough set-based rule generation technology since 2000.

Author(s)	Technology	Applied problem	No. of condition attribute	Compared method(s)
Beynon et al. [4]	Variable precision rough set (VPRS)([5] [6])	Expenditure on elementary and secondary education	8	Linear discriminant analysis (LDT)
Khoo and Zhai [7]	Rough-set-based approach for classification (RClass)-Plus	State of machine	3	Iterative dichotomiser 3 (ID3) [8], learning from examples based on rough sets (LERS) [9], and RClass [10]
Mak and Munakata [11]	Rough set	New Product entry decision	3	ID3, and neural networks
Pal et al. [12]	Rough-fuzzy multilayer perceptron (MLP)	Vowel/ Pat/ Medical	6/ 2/ 11	Subset method [13], M of N method[14], X2R [15], and C4.5[16]
Hou and Huang [17]	Fuzzy rough set	Manufacturing process	8	Rough set
Tseng et al. [18]	Rough set	Machining operations	8	Multi-nominal regression, and general discriminant
Pattaraintakorn et al. [19]	Rough set with notion of ordinal prediction	Melanoma data set	7	—
Wang et al. [20]	Rough set	Brain glioma	14	Neural networks, decision trees and fuzzy min-max neural networks (FRE-FMMNN
Inuiguchi and Miyajima[21]	Rough set	Evaluation of television set	3	—
Hong et al. [22]	Rough set and fuzzy set theory	Fisher's Iris Data	4	—
Teoh et al. [23]	Rough set and cumulative probability distribution approach (CPDA)	Trading stock index/trading database	1/1	Chen's method [24] and Yu's method [25]
Qian et al. [26]	Rule extracting algorithm based on the converse approximation (REBCA)	Numerical example	4	—

TABLE 1. *(Continued.)* Recent literature on rough set-based rule generation technology since 2000.

Fan et al. [27]	Incremental rough set	Information system	4	—
Ma et al. [28]	Rule generation based on classification attribute (RGCA)	Lens/ Balloons/ Iris/ Wine (Machine learning databases)	6/4/4/4	—
Luo and Zhong [29]	Rough set	Medical expert diagnosis system (MEDS)	9	—
Othman et al. [30]	Rough set and genetic algorithm	Power protection system maintenance	30	Decision tree
Shi et al. [31]	Rough set	KANSEI Engineering	9	Orthogonal method
Huang et al. [32]	Rough set	Customer preferred estimation	4	—
Jia et al. [33]	Rough-set belief rule model using multinomial subjective logic	Numerical example	16	—
Meng and Lu[34]	Rough set	Service customization strategy	8	—
Fetouh and Zaky[35]	RST with genetic algorithm	Power system stabilizer	10	—

—: means none.

update the intuitionistic fuzzy membership and find better points in the class.

RST is one data mining tool that can effectively solve the problem of the vagueness of datasets. RST can form an approximate definition that includes lower and upper approximations for a target set in terms of some definable sets in an uncertain or imprecise target. RST can be found in ([2] and [3]). Therefore, an RST-based model for rule generation is adopted in KIFRS. Finally, based on the rough set decision rules for different groups, the system results can be obtained and analyzed for users. The illustration of the novel KIFRS model is shown in Figure 1. Moreover, this research adopts three measuring indexes. First, M_1 is the accuracy rate, and its formulation is as follows:

$$M_1 = 100 \times \frac{N_{correct}}{T}, \tag{1}$$

where $N_{correct}$ is the number of correct classifications and T represents the total amount of data. Second, M_2 is the coverage rate and can be formulated as follows:

$$M_2 = \frac{N_{coverage}}{T}, \tag{2}$$

where $N_{coverage}$ is the amount of data covered by generating rules.

The third measure index M is a comprehensive index that equals $M_1 + M_2$.

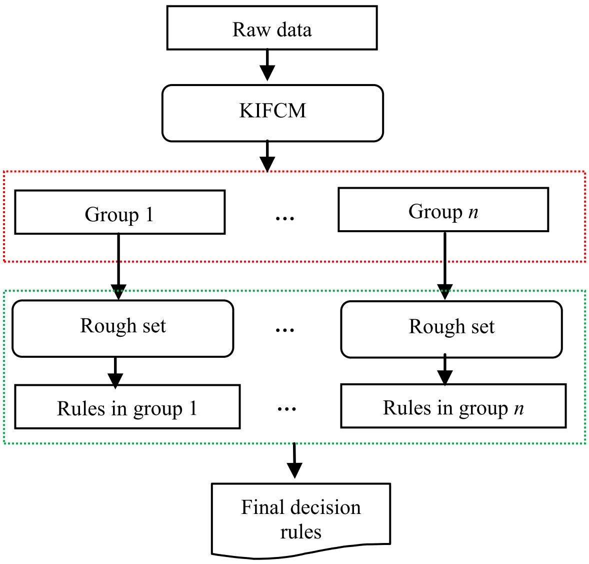

FIGURE 1. Structure of the KIFRS model for rule generation and extraction.

In this study, the KIFRS of rule generation is briefly introduced as follows:

Step 1 (Raw Data Clustering): First, assume that the decision table is $E =< \chi, A >$, where χ is a nonempty finite set and A is a nonempty finite set of attributes. F and $D = \{d_1, d_2, \ldots, d_l\}$ are condition attributes and decision attributes, respectively. An attribute is a function from χ to the value set V_a. Divide the decision table $E =< \chi, A >$ into l tables $E_i =< \chi_i, A_i >$, $i = 1, \ldots, l$, which correspond to the l decision attributes $d_1, \ldots, d_l$, where $\chi = \chi_1 \cup \ldots \cup \chi_l$ and $A_i = F \cup \{d_i\}$ and $d \notin F$ is the decision variable D.

$X_c = \{x_{ci1}, \ldots, x_{cij}\}$ is a set generated by KIFCM in E_i, $i = 1, \ldots, l$, which has been divided into c clusters. The same group objects of X_c have more similar characteristics, which could effectively assist in RST rule generation.

Step 2 (Determining Indiscernibility Relation): Let $a \in A$ and $P \subseteq A$. I_P is the indiscernibility relation (a binary relation) and can be defined as follows:

$$I_P = \{(x_c, y) \in \chi : \text{for every } a \in P, a(x_c) = a(y)\}.$$

Then, $I_P = \cap_{a \in P} I_a$. If $X_c \subseteq \chi$, the sets $\{x_c \in \chi : [x]_P \subseteq X_c\}$ and $\{x_c \in \chi : [x]_P \cap X_c \neq \emptyset\}$, $[x_c]_P$ can be determined to be equivalence classes of the object with different groups $(c) x_c \in \chi$ relative to I_{Pc}. These sets are lower and upper (*Pc-lower* and *Pc-upper*) approximations of X_c in E and are determined by $\underline{I}X_c$ and $\overline{I}X_c$, respectively. If X_c is *Pc-definable*, then $\underline{I}X_c = \overline{I}X_c$; otherwise, X_c is *Pc-rough*.

Example 1: Consider the universe of discourse $\chi = \{x_{c1}, x_{c2}, x_{c3}, x_{c4}, x_{c5}, x_{c6}\}$, where I is any equivalence relation in I_P which partitions χ into $\{\{x_{c1}, x_{c2}, x_{c4}\}, \{x_{c3}, x_{c6}\}, \{x_{c5}\}\}$. Therefore, for any subset $X_c = \{x_{c1}, x_{c2}, x_{c3}, x_{c4}\}$ of U, $\underline{I}X_c = \{x_{c1}, x_{c2}, x_{c4}\}$ and $\overline{I}X_c = \{x_{c1}, x_{c2}, x_{c3}, x_{c4}, x_{c6}\}$. Figure 2 displays the lower and upper approximations of set X_c.

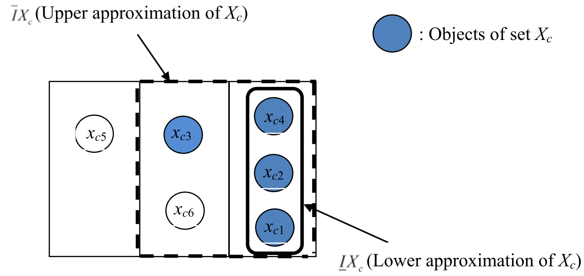

FIGURE 2. Illustration of the lower and upper approximations of set X_c [40].

The *Pc-positive* region of X_c can be defined as $\{x_{c1}, x_{c2}, x_{c4}\}$, and the *Pc-negative* region of X_c can be defined as $\{x_{c5}\}$. In contrast, consider a subset $Y_c = \{x_{c3}, x_{c5}, x_{c6}\}$ for which $\underline{I}Y_c = \{x_{c3}, x_{c5}, x_{c6}\}$ and $\underline{I}Y_c = \{x_{c3}, x_{c5}, x_{c6}\}$. Then, $\underline{I}Y_c = \overline{I}Y_c$, so Y_c is *Pc-definable*.

Step 3 (Dispensable and Indispensable Attributes): Let $f \in F$. An attribute f is dispensable in E if $POS_{(F-(f))}(D) = POS_F(D)$, where $POS_F(D)$ is $\bigcup_{X_c \in I_{d_i}} \underline{F}X_c$ and $\underline{F}X_c$ is the lower approximation with different groups. Otherwise, the attribute f is indispensable in E. If all $f \in F$ are indispensable, f is independent.

Step 4 (Reduct and CORE): Reduct represents the minimal attribute subset preserving the condition. A set of attributes $I \subseteq F$ is a *reduct* of F if the condition, E is independent and $POS_i(D)$ is satisfied. *CORE*(F) is defined as the set of all attributes with c groups that are indispensable in F.

Step 5 (Discernibility Matrix): Now, for each *reductP* $= \{p_1, \ldots, p_k\}$, a discernibility matrix $M_{d_i}(P)$ can be defined as follows [41]:

$$f_{ij} = \{a \in P : a(x_{ci}) \neq a(x_{cj})\}, \tag{3}$$

for $i, j = 1, \ldots, n$.

For each object $x_{cj} \in x_{ci1}, \ldots, x_{cib}$, the discernibility function $k_{d_i}^{x_{cj}}$ can be defined as follows:

$$k_{d_i}^{x_{cj}} = \bigcap \{\bigcup (f_{ij}) : 1 \leq i, j \leq n, j < i, f_{ij} \neq \emptyset\}, \tag{4}$$

where $k_{d_i}^{x_{cj}}$ is converted into an equivalent formula (Boolean logic) and $\bigcup (f_{ij})$ is the disjunction of all members of c_{ij}.

Example 2 [42]: Consider a knowledge representation system. Let $F = \{a, b, c, d\}$ and $D = \{E\}$ be condition and decision attributes, respectively. $\{\{x_{11}, x_{12}, x_{13}, x_{14}\}, \{x_{25}, x_{26}, x_{27}\}\}$ is the clustering result of objects. Figure 3 shows the discernibility matrix.

The element $\{b, c, d\}$ can be shown as $b \vee c \vee d$ if the elements of the discernibility matrix employ "OR". Furthermore, the connective AND can be employed in the entire matrix.

III. EXPERIMENTAL RESULTS AND DICUSSIONS

In this section, this study compares ID3 and RST with the proposed novel two-stage model in various numerical cases.

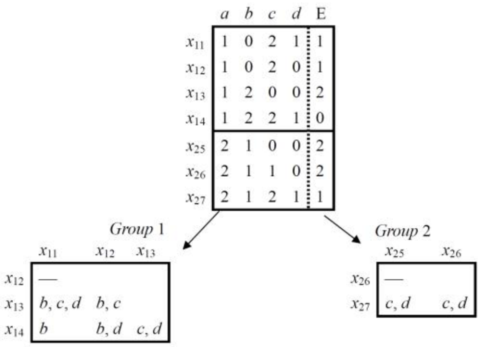

FIGURE 3. Illustration of the discernibility matrix based on KIFCM clustering technique [37].

TABLE 2. Summary statistics and attribute information of selected machine learning data sets.

Title	No. of instances	Predictive attributes	Attributes Statistics				No. of class	Reference
			Minimum	Maximum	Mean	Standard deviation		
IRIS	150	Sepal length in cm	4.3	7.9	5.84	0.83	3	[44]
		Sepal width in cm	2.0	4.4	3.05	0.43		
		Petal length in cm	1.0	6.9	3.76	1.76		
		Petal width in cm	0.1	2.5	1.20	0.76		
Haberman's Survival Data	306	Age of patient at time of operation	30	83	52.45	10.80	2	[45]
		Patient's year of operation	58	69	62.85	3.24		
		No. of positive axillary nodes detected	0	52	4.02	7.18		

This study coded the above models of rule generation with MATLAB 2010. The machine learning data sets IRIS and Haberman's Survival Data were examined. The machine learning data sets are popular datasets from [43].

A series of experiments was executed with machine learning data sets using standard ID3, RST and the proposed novel KIFRS model. UCI Machine learning data sets were tested, including IRIS and Haberman's Survival Data. These data sets are popular in the machine learning field. Table 2 shows summary statistics and attribute information for the selected machine learning data sets. The IRIS data set is a standard data set in experimental cases. Haberman's Survival Data Set has more instances, a higher standard deviation of attributes and fewer classes. This study adopts these two benchmark datasets to test whether the KIFRS rule generation model can obtain better performance than standard and popular models can.

TABLE 3. Testing measurement indexes of the KM+RST, FCM+RST and KIFCM+RST models with different Cluster values in the Machine learning datasets.

Data set	KIFRS with various clustering algorithm		M_1	M_2	M	Average of M	No. of rules	CPU time (Minute)
IRIS	KM	c=2	1.00	1.00	2	1.78	42	1.96
		c=3	0.67	1.00	1.67		37	1.35
		c=4	0.67	1.00	1.67		40	1.30
	FCM	c=2, m=2	1.00	1.00	2	1.96	42	2.07
		c=3, m=2	0.89	1.00	1.89		37	1.58
		c=4, m=2	1.00	1.00	2		42	1.82
	KIFCM	c=2, m=2, σ=150,α=2	1.00	1.00	2	2	42	2.05
		c=3, m=2, σ=150,α=2	1.00	1.00	2		33	1.38
		c=4, m=2, σ=150,α=2	1.00	1.00	2		38	1.79
Haberman's Survival	KM	c=2	0.70	0.79	1.49	1.45	166	2.30
		c=3	0.61	0.79	1.4		174	2.14
		c=4	0.61	0.86	1.47		162	1.96
	FCM	c=2, m=2	0.79	0.78	1.57	1.63	167	2.50
		c=3, m=2	0.82	0.79	1.61		162	2.34
		c=4, m=2	0.85	0.87	1.72		162	2.74
	KIFCM	c=2, m=2, σ=150,α=10	0.85	0.80	1.65	1.65	161	9.93
		c=3, m=2, σ=150,α=8	0.84	0.83	1.67		166	12.40
		c=4, m=2, σ=150,α=7	0.76	0.88	1.64		168	24.06

TABLE 4. Comparison of the testing measurement indexes with various methods in Machine learning datasets.

Data set	Method	M_1	M_2	M	No. of rules	CPU time (Minutes)
IRIS	ID3	0.67	0.45	1.12	63	0.28
	Rough set	1.00	1.00	2.00	42	3.61
	KIFRS (c=2, m=2, σ=150,α=2)	1.00	1.00	2.00	38	2.05
Haberman's Survival	ID3	0.15	0.19	0.34	52	0.50
	Rough set	0.79	0.79	1.58	175	2.93
	KIFRS (c=3, m=2, σ=150,α=8)	0.84	0.83	1.67	166	12.4

The novel KIFRS model effectively combines fuzzy clustering technologies with RST rule generation. In clustering technologies, the k-means (KM), FCM and novel KIFCM models were examined with *Cluster* values of 2, 3 and 4, and the standard RST was employed for rule generation.

Table 3 displays the measurement indexes, which are "Accuracy rate", "Cover rate", "Comprehensive index", "Average of comprehensive index", "Number of rules" and "CPU time", of the different *Cluster* methods with RST rule generation in the two machine learning datasets. The results show that the KIFRS model with $c = 2$ can obtain higher comprehensive index values of 2 in the IRIS dataset and that the KIFRS model with $c = 3$ can achieve higher comprehensive index values of 1.67 in Haberman's Survival dataset.

Table 3 shows that the KIFRS model obtains the minimum number of rules while testing different *Cluster* values in the two machine learning datasets and that it again demonstrates good, stable performance. Therefore, the KIFRS model can also be recommended as an alternative rule generation model in machine learning data sets.

Table 4 shows the performance of the ID3, RST, and KIFRS models in the two machine learning datasets. The proposed KIFRS model demonstrates better performance than do ID3 and standard RST in terms of the measurement indexes in the two machine learning datasets. Therefore, these results verify that our proposed approach can help traditional models achieve better performance in machine learning datasets. Based on Tables 3 and 4, the following conclusions were drawn: (1) the proposed KIFRS model outperformed the other models in machine learning datasets, and (2) KIFRS demonstrated superior performance compared to the common decision tree approach (ID3) and standard RST because KIFCM effectively divides raw data into similarity groups.

IV. CONCLUSIONS

A KIFRS model for enhancing the effect of rule generation was developed in this study. The performance of the KIFRS model was examined in two machine learning datasets. The results indicated that the KIFRS model offers a promising alternative for rule generation and can achieve superior and stable performance. It can be concluded that due to KIFCM, the KIFRS model may obtain superior similar groups. This will improve the traditional RST-based rule generation.

Several issues remain for further research. In this paper, the KIFRS model only examined machine learning datasets. The KIFRS model may extend its realizable application in industry ([46], [47]). Recent fuzzy set methodologies ([48]–[52]) may be used to improve the performance. Moreover, artificial intelligence/new clustering algorithms [53] may be considered in KIFRS.

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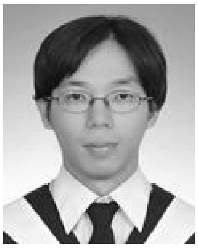

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