IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received January 16, 2018, accepted February 12, 2018, date of publication February 23, 2018, date of current version March 15, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2808477

Disturbance Observer-Based Integral Sliding Mode Control for Singularly Perturbed Systems With Mismatched Disturbances

LINNA ZHOU, ZHIYUAN CHE, AND CHUNYU YANG, (Member, IEEE)
School of Information and Control Engineering, China University of Mining and Technology, Xuzhou 221116, China

Corresponding author: Zhiyuan Che (2319802431@qq.com)

This work was supported by the Fundamental Research Funds for the Central Universities under Grant 2017QNB16.

ABSTRACT This paper presents a disturbance observer-based integral sliding mode control (ISMC) for singularly perturbed systems (SPSs) with mismatched disturbances. First, a linear state feedback control law for the slow subsystem of the SPS is designed, and the fast subsystem is established. Second, a disturbance observer and an ISMC incorporating the disturbance estimate are constructed. The gain of ISMC is obtained by applying the H_∞ control theory, and the resulting ISMC can effectively attenuate the mismatched disturbances. Finally, the proposed method is applied to an electric system, which illustrates the effectiveness and feasibility. The proposed design methods are based on the reduced-order subsystems and thus free of the high dimensionality and possible numerical ill-conditioned problems.

INDEX TERMS Singularly perturbed systems (SPSs), mismatched disturbances, integral sliding mode control (ISMC), disturbance observer.

I. INTRODUCTION

Multi-time-scale systems exist in many industrial applications, such as chemical processes and power systems [1]–[3]. For these systems, the interaction between the slow and fast dynamics may lead to high dimensionality and numerical ill-conditioned issues in system analysis and controller design [2]. In order to deal with these problems, the systems are usually modeled as singularly perturbed systems (SPSs) with a singular perturbation parameter ε [4]. Singular perturbation theory as a powerful tool to analyze and design the SPSs has been widely investigated (see [1]–[5] and the references therein).

The lumped disturbances caused by model uncertainties, external disturbances, noises and parameter perturbations, may extremely deteriorate the system performance [6]. Sliding mode control (SMC) as a nonlinear control technique has been extensively investigated for a few decades because of its conceptual simplicity, fast response and powerful ability to suppress the disturbances satisfying the so-called matching condition [7]–[12], which means that the disturbances are implicit in the control input channels. Yang *et al.* [8] introduced the disturbance observer in the design of SMC. However, there are many disturbances may not satisfy the matching condition, which enter systems through the different channels from the control inputs or directly affect the system states [13]–[15]. Thus the integral sliding mode control (ISMC) is studied to guarantee the robustness and insensitivity to mismatched disturbances in [10] and [11]. It is worth mentioning that the SMC/ISMC for SPSs with mismatched disturbances have seldom been studied in the literature.

Many researchers have devoted themselves to investigating the SMC/ISMC for SPSs (see [16]–[24] and the references therein). Applying the routine design and analysis approaches for normal systems to SPSs usually leads to ill-conditioned numerical problems and high dimensionality [5]. The traditional methods to avoiding this problems are based on decomposing the original SPSs into fast and slow subsystems. Without considering the disturbances, two separate sliding surfaces were designed for subsystems and the synthetized composite control law can stabilize the full-order system in [16] and [17]. Ahmed *et al.* [18] designed a sliding mode controller via the slow component of SPSs, and the fast subsystem was considered as the unmodeled high frequency dynamics. On the contrary, Innocenti *et al.* [19] presented an approach that the control input only depends on the fast variables. Yue and Xu [20] took the external disturbances into account, and proposed a SMC law that could attenuate the disturbances to some extent. Nguyen *et al.* [21] proposed the composite control approaches comprised of a linear state feedback and a SMC for SPSs with matched disturbances, and different composite sliding surfaces were constructed in [22]. In [23], a SMC law was designed for spatial stabilization of the three-time-scale system: advanced heavy water reactor (AHWR). An alternative way to avoid the ill-conditioned numerical problems is independent of system decomposition. Gao *et al.* [24] presented an ISMC for the uncertain SPSs based on passivity theory. From the above analysis, we can conclude that the research on using ISMC to attenuate the adverse effects of mismatched disturbances on SPSs is of important significance and remains as an open area.

This paper will propose a novel ISMC for SPSs with mismatched disturbances. First of all, a linear state feedback control for the slow subsystem is designed, and the fast subsystem is obtained. After that, a disturbance observer that takes the singular perturbation parameter into account is constructed to estimate the disturbance, and the estimate is incorporated in the design of ISMC. Then, the mismatched components of the external disturbances extracted by the projection matrix theory are attenuated by the well-known H_∞ control theory. By combining with Lyapunov function candidate, a linear matrix inequality (LMI) is presented to determine the gain of ISMC. With the obtained ISMC, the closed-loop system is asymptotically stable. Finally, a circuit is studied to demonstrate the effectiveness and feasibility of the proposed method. The main contribution of this paper is to propose a disturbance observer-based ISMC, which can effectively attenuate the adverse effects of mismatched disturbances on SPSs.

The rest of this paper is organized as follows. In Section II, the problem under consideration is described and some preliminaries are presented. The main results are given in Section III. A disturbance observer is constructed to estimate the disturbances. After that, the ISMC law which incorporates the disturbance estimate is designed. An LMI is presented to determine the controller gain, and the system stability is analyzed. Simulation results are given in Section IV. Section V concludes the paper.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following linear time-invariant SPSs subject to external disturbances,

$$\begin{cases} \begin{bmatrix} \dot{x}(t) \\ \varepsilon \dot{z}(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \\ \qquad + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} f(t) \\ y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \end{cases} \tag{1}$$

where $\varepsilon > 0$ is the singular perturbation parameter, $x(t) \in R^{n_1}$ and $z(t) \in R^{n_2}$ are the slow- and fast-time state variables, $u(t) \in R^m$ is the control input, $y(t) \in R^p$ is the system output, $f(t) \in R^q$ represents the external disturbances that belong to $L_2[0, \infty)$, A_{ij}, B_i, C_i and D_i $(i, j = 1, 2)$ are constant matrices with appropriate dimensions.

As for system (1), we have the following assumptions.

Assumption 1: Matrix A_{22} is invertible, B_2 and D_2 are of full column, *i.e.*, $rank(A_{22}) = n_2$, $rank(B_2) = m$ and $rank(D_2) = q$. The disturbance vector $f(t)$ is assumed to satisfy the following conditions,

$$\|f(t)\| \le \alpha, \quad \left\|\dot{f}(t)\right\| \le \beta \tag{2}$$

where $\|\cdot\|$ denotes Euclidean norm, $\alpha > 0$ and $\beta > 0$ are scalars.

Assumption 2: The pairs (A_0, B_0) and (A_{22}, B_2) are controllable, where $A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21}$ and $B_0 = B_1 - A_{12}A_{22}^{-1}B_2$.

The objective of this paper is to design a control law for system (1), such that the adverse effect of mismatched disturbances on SPSs can be effectively attenuated.

According to the singular perturbation theory [2], the dynamics of the full-order SPSs can be approximated by the dynamics of quasi-steady-state model (slow subsystem) and boundary layer model (fast subsystem). Assumption 2 allows us to design the individual control laws for subsystems, separately.

With Assumption 2 and based on the eigenvalue placement technique, a linear slow-time state feedback control law for the slow subsystem can be designed as: $u_s(t) = K_s x(t)$, such that $A_0 + B_0 K_s$ is asymptotically stable.

Motivated by Nguyen *et al.* [21], we introduce the following transformation of variables,

$$\eta(t) = Lx(t) + z(t) \tag{3}$$

Substituting the composite control law $u(t) = u_s(t) + u_f(t)$ into system (1) leads to

$$\begin{cases} \begin{bmatrix} \dot{x}(t) \\ \varepsilon \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} A_{11} + B_1 K_s - A_{12} L & A_{12} \\ 0 & A_f \end{bmatrix} \\ \qquad \cdot \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_f \end{bmatrix} u_f(t) \\ \qquad + \begin{bmatrix} D_1 \\ D_f \end{bmatrix} f(t) \\ y(t) = \begin{bmatrix} C_1 - C_2 L & C_2 \end{bmatrix} \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} \end{cases} \tag{4}$$

where L is the solution to the following algebraic equation.

$$\varepsilon L(A_{11} + B_1 K_s) + A_{21} + B_2 K_s - A_{22} L - \varepsilon L A_{12} L = 0 \tag{5}$$

The equation (5) can be solved by the fixed-point recursive algorithm [25] with

$$L^{(i+1)} = A_{22}^{-1}\Big[\varepsilon L^{(i)}(A_{11}+B_1K_s) + A_{21}+B_2K_s-\varepsilon L^{(i)}A_{12}L^{(i)}\Big] \quad (6)$$

where $L^{(0)} = A_{22}^{-1}(A_{21}+B_2K_s)$.

Remark 1: From (6), we have $L = A_{22}^{-1}(A_{21}+B_2K_s)+O(\varepsilon)$, such that $A_{11}+B_1K_s-A_{12}L = A_0+B_0K_s+O(\varepsilon)$, which indicates that when ε is small enough, stability of the matrices $A_{11}+B_1K_s-A_{12}L$ implies that of $A_0+B_0K_s$.

The fast subsystem which can be extracted from an upper triangular-formed system (4) is as follows.

$$\begin{cases}\varepsilon\dot{\eta}(t) = A_f\eta(t)+B_fu_f(t)+D_ff(t)\\ y_\eta(t) = C_2\eta(t)\end{cases} \quad (7)$$

where $A_f = A_{22}+\varepsilon LA_{12}$, $B_f = B_2+\varepsilon LB_1$ and $D_f = D_2+\varepsilon LD_1$.

The following definition and lemma will be used in the sequel.

Definition 1 [4]: Given an H_∞ performance index $\gamma > 0$, the closed-loop system is said to be with an H_∞-norm less than or equal to γ if the following inequality holds for the zero initial condition.

$$\int_0^\infty \|y(t)\|^2\mathrm{d}t \le \gamma^2\int_0^\infty \|f(t)\|^2\mathrm{d}t \quad (8)$$

Lemma 1 [10]: For any matrix $B \in R^{n\times m}$ with full column rank, it holds that

$$I = BB^+ + B^\perp\left(B^\perp\right)^+ \quad (9)$$

where I is the identity matrix, $B^+ \in R^{m\times n}$ is the Moore-Penrose pseudoinverse matrix of B, that is $B^+ = \left(B^TB\right)^{-1}B^T$, and the columns of $B^\perp \in R^{n\times(n-m)}$ span the null space of B^T, *i.e.*, $B^TB^\perp = 0$.

III. MAIN RESULTS

In this section, we will design an ε-dependent disturbance observer and an ISMC law for the fast subsystem (7), and an LMI-based condition is given to determine the controller gain.

A. DISTURBANCE OBSERVER AND ISMC LAW DESIGN

For the sake of attenuating the adverse effect of disturbances on the system performances, a disturbance observer can be constructed for the fast subsystem (7).

$$\begin{cases}\dot{d}(t) = \Gamma\hat{f}(t)+\Gamma D_f^+(A_f\eta(t)+B_fu_f(t))\\ \hat{f}(t) = d(t)-\varepsilon\Gamma D_f^+\eta(t)\end{cases} \quad (10)$$

where $D_f^+ = (D_f^TD_f)^{-1}D_f^T$, $\hat{f}(t)$ is the disturbance estimate, $d(t)$ is the internal variable vector of the disturbance observer, with $d(0) = \varepsilon\Gamma D_f^+\eta(0)$, and Γ is chosen as a Hurwitz matrix.

Define the disturbance estimation error $\tilde{f}(t) = f(t)-\hat{f}(t)$. Then, the disturbance estimation error $\tilde{f}(t)$ satisfies $\left\|\tilde{f}(t)\right\| \le \chi$ under Assumption 1 [11], where $\chi = c\alpha - c\beta\frac{2}{\lambda_{\max}(\Gamma)} > 0$, $c > 0$ is a scalar satisfying $\left\|e^{\Gamma t}\right\| \le ce^{\frac{\lambda_{\max}(\Gamma)}{2}t}$, and $\lambda_{\max}(\Gamma) < 0$ represents the maximum eigenvalue of matrix Γ.

Motivated by the sliding manifold defined in [10], and incorporating the disturbance estimate, a novel disturbance observer-based integral sliding surface (DOB-ISS) can be designed as follows.

$$S_f(t) = B_f^+\Big[\varepsilon\eta(t)-\varepsilon\eta(0) - \int_0^t\Big(A_f\eta(t)+B_fu_{f0}(t)+D_f\hat{f}(t)\Big)dt\Big] \quad (11)$$

where $B_f^+ = (B_f^TB_f)^{-1}B_f^T$, $u_{f0}(t) = -K_f\eta(t)-K_d\hat{f}(t)$, $K_f \in R^{m\times n_2}$ is the controller gain, and $K_d \in R^{m\times q}$ is the disturbance rejection gain to be determined.

Then, an ISMC law can be designed as

$$u_f(t) = u_{f0}(t)+B_f^+D_fu_{f1}(t) \quad (12)$$

where $u_{f0}(t)$ and $u_{f1}(t)$ are nominal control and discontinuous control laws for ensuring the performance of the nominal system and attenuating the mismatched disturbances, respectively.

The discontinuous control law $u_{f1}(t)$ is selected as

$$u_{f1}(t) = -(\chi+\sigma)\frac{D_f^TB_f^{+T}S_f(t)}{\left\|D_f^TB_f^{+T}S_f(t)\right\|} \quad (13)$$

where $\sigma > 0$ is a scalar.

Remark 2: It is noticed that the singular perturbation parameter ε is taken into account in the disturbance observer (10) and the DOB-ISS (11), which is quite different from that of normal systems [8], [11]. Such a consideration can effectively avoid the ill-conditioned numerical problems.

Remark 3: Compared with the conventional SMC strategies [21], the proposed novel ISMC (12) combines the ISMC feedback with the disturbance estimation based-feedforward compensation straightforwardly to guarantee the nominal control performances, and the disturbances are actively attenuated. In addition, the ISMC employs a continuous approximation: the Euclidean norm of DOB-ISS (11), rather than a signum function, which can improve the chattering phenomenon.

The following theorem is proposed to show that the reachability condition can be guaranteed.

Theorem 1: With the ISMC (12), the reachability condition can be satisfied, *i.e.*, the system state trajectories will be globally driven onto the DOB-ISS (11) in a finite time satisfing

$$\frac{1}{(\sigma+2\chi)\delta}\sqrt{V_f(0)} \le \tau_f \le \frac{1}{\sigma\delta}\sqrt{V_f(0)} \quad (14)$$

where $\delta = \left\|\left(B_{\mathrm{f}}^{+} D_{\mathrm{f}}\right)^{\mathrm{T}}\right\|$, τ_{f} is the time needed to reach the DOB-ISS ($V_{\mathrm{f}}(\tau_{\mathrm{f}}) = 0$), and $V_{\mathrm{f}}(t) = S_{\mathrm{f}}^{\mathrm{T}}(t) S_{\mathrm{f}}(t)$.

Proof: Taking the time-derivative of the DOB-ISS (11) leads to

$$\begin{aligned}\dot{S}_{\mathrm{f}}(t) &= B_{\mathrm{f}}^{+}\left[\varepsilon \dot{\eta}(t) - \left(A_{\mathrm{f}}\eta(t) + B_{\mathrm{f}} u_{\mathrm{f}0}(t) + D_{\mathrm{f}} \hat{f}(t)\right)\right] \\ &= B_{\mathrm{f}}^{+} B_{\mathrm{f}} B_{\mathrm{f}}^{+} D_{\mathrm{f}} u_{\mathrm{f}1}(t) + B_{\mathrm{f}}^{+} D_{\mathrm{f}}\left(f(t) - \hat{f}(t)\right) \\ &= B_{\mathrm{f}}^{+} D_{\mathrm{f}}\left(u_{\mathrm{f}1}(t) + \tilde{f}(t)\right) \end{aligned} \tag{15}$$

Define the following Lyapunov function candidate.

$$V_{\mathrm{f}}(t) = S_{\mathrm{f}}^{\mathrm{T}}(t) S_{\mathrm{f}}(t) \tag{16}$$

Then,

$$\begin{aligned}\dot{V}_{\mathrm{f}}(t) &= 2 S_{\mathrm{f}}^{\mathrm{T}}(t) \dot{S}_{\mathrm{f}}(t) \\ &= 2 S_{\mathrm{f}}^{\mathrm{T}}(t)\left[B_{\mathrm{f}}^{+} D_{\mathrm{f}} \tilde{f}(t) - (\sigma+\chi) B_{\mathrm{f}}^{+} D_{\mathrm{f}} \frac{D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)}{\left\|D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)\right\|}\right] \\ &= -2(\sigma+\chi) \frac{\left(D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)\right)^{\mathrm{T}} \cdot D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)}{\left\|D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)\right\|} \\ &\quad + 2 S_{\mathrm{f}}^{\mathrm{T}}(t) B_{\mathrm{f}}^{+} D_{\mathrm{f}} \tilde{f}(t) \\ &= -2\sigma \left\|D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)\right\| \\ &\quad + 2\left(S_{\mathrm{f}}^{\mathrm{T}}(t) B_{\mathrm{f}}^{+} D_{\mathrm{f}} \tilde{f}(t) - \chi \left\|D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)\right\|\right) \\ &\leq -2\sigma \left\|D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)\right\| \end{aligned} \tag{17}$$

which implies that $\dot{V}_{\mathrm{f}}(t) = 2 S_{\mathrm{f}}^{\mathrm{T}}(t) \dot{S}_{\mathrm{f}}(t) \leq 0$. Thus, the DOB-ISS (11) can be attained in a finite time.

From (17) and the definition of χ, we have

$$\begin{aligned}-(2\sigma + 4\chi) &\left\|D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)\right\| \\ &\leq \dot{V}_{\mathrm{f}}(t) \leq -2\sigma \left\|D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)\right\| \end{aligned} \tag{18}$$

and

$$-(2\sigma + 4\chi)\delta \sqrt{V_{\mathrm{f}}(t)} \leq \frac{\mathrm{d} V_{\mathrm{f}}(t)}{\mathrm{d} t} \leq -2\sigma\delta \sqrt{V_{\mathrm{f}}(t)} \tag{19}$$

where $\delta = \left\|\left(B_{\mathrm{f}}^{+} D_{\mathrm{f}}\right)^{\mathrm{T}}\right\|$. The inequality (19) implies that

$$\frac{-\mathrm{d} V_{\mathrm{f}}(t)}{(2\sigma + 4\chi)\delta \sqrt{V_{\mathrm{f}}(t)}} \leq \mathrm{d} t \leq \frac{-\mathrm{d} V_{\mathrm{f}}(t)}{2\sigma\delta \sqrt{V_{\mathrm{f}}(t)}} \tag{20}$$

Let τ_{f} be the time needed to reach the DOB-ISS, then $V_{\mathrm{f}}(\tau_{\mathrm{f}}) = 0$.

Integrating both sides of (20) from 0 to τ_{f} yields,

$$\begin{aligned}\int_{V_{\mathrm{f}}(\tau_{\mathrm{f}})}^{V_{\mathrm{f}}(0)} &\frac{\mathrm{d}\left(\sqrt{V_{\mathrm{f}}(t)}\right)^2}{(2\sigma + 4\chi)\delta \sqrt{V_{\mathrm{f}}(t)}} \\ &\leq \int_{0}^{\tau_{\mathrm{f}}} \mathrm{d} t \leq \int_{V_{\mathrm{f}}(\tau_{\mathrm{f}})}^{V_{\mathrm{f}}(0)} \frac{\mathrm{d}\left(\sqrt{V_{\mathrm{f}}(t)}\right)^2}{2\sigma\delta \sqrt{V_{\mathrm{f}}(t)}} \end{aligned} \tag{21}$$

As a result, the reaching time to the DOB-ISS (11) under the ISMC (12) lies in the interval given by

$$\frac{1}{(\sigma + 2\chi)\delta} \sqrt{V_{\mathrm{f}}(0)} \leq \tau_{\mathrm{f}} \leq \frac{1}{\sigma\delta} \sqrt{V_{\mathrm{f}}(0)} \tag{22}$$

This completes the proof. ■

Remark 4: From the proof of Theorem 1, we can conclude that σ guarantees the reachability condition can be satisfied. Meanwhile, from (14), it is easy to show that σ plays an important role in adjusting the reaching time. A big value of σ can generate a fast reaching time, but lead to a large magnitude of ISMC, which is not favored. Thus, the σ should be chosen in an appropriate way.

As a result, the whole block diagram of the proposed composite control for SPSs with mismatched disturbances is shown in Fig. 1.

B. ISMC GAINS DESIGN

This subsection will propose a method to determine the disturbance rejection gain K_{d} and the controller gain K_{f} for the ISMC (12), respectively.

Based on Lemma 1, which is known as projection matrix theory, we can project the mismatched disturbances into matched and mismatched spaces, *i.e.*,

$$\begin{aligned} D_{\mathrm{f}} f(t) &= B_{\mathrm{f}} B_{\mathrm{f}}^{+} D_{\mathrm{f}} f(t) + B_{\mathrm{f}}^{\perp}\left(B_{\mathrm{f}}^{\perp}\right)^{+} D_{\mathrm{f}} f(t) \\ &= f_1(t) + f_2(t) \end{aligned} \tag{23}$$

where $f_1(t) = B_{\mathrm{f}} B_{\mathrm{f}}^{+} D_{\mathrm{f}} f(t)$ and $f_2(t) = B_{\mathrm{f}}^{\perp}\left(B_{\mathrm{f}}^{\perp}\right)^{+} D_{\mathrm{f}} f(t)$ are the matched and mismatched components of $D_{\mathrm{f}} f(t)$, respectively.

When the sliding mode is achieved, the equivalent control method can be employed to determine the sliding motion equation [7], and the stability analysis can be performed.

Solving the equation $\dot{S}_{\mathrm{f}}(t) = 0$ yields $u_{\mathrm{f}1}^{\mathrm{eq}}(t) = -\tilde{f}(t)$. Then,

$$u_{\mathrm{f}}^{\mathrm{eq}}(t) = u_{\mathrm{f}0}(t) + B_{\mathrm{f}}^{+} D_{\mathrm{f}} u_{\mathrm{f}1}^{\mathrm{eq}}(t) \tag{24}$$

Substituting (24) into (7) leads to the following sliding motion equation,

$$\begin{aligned}\varepsilon \dot{\eta}(t) &= A_{\mathrm{f}}\eta(t) + B_{\mathrm{f}} u_{\mathrm{f}}^{\mathrm{eq}}(t) + D_{\mathrm{f}} f(t) \\ &= A_{\mathrm{f}}\eta(t) + B_{\mathrm{f}}\Big(-K_{\mathrm{f}}\eta(t) \\ &\qquad - K_{\mathrm{d}}\hat{f}(t) - B_{\mathrm{f}}^{+} D_{\mathrm{f}} \tilde{f}(t)\Big) + D_{\mathrm{f}} f(t) \\ &= (A_{\mathrm{f}} - B_{\mathrm{f}} K_{\mathrm{f}})\eta(t) - B_{\mathrm{f}} K_{\mathrm{d}} \hat{f}(t) \\ &\quad - B_{\mathrm{f}} B_{\mathrm{f}}^{+} D_{\mathrm{f}} \tilde{f}(t) + D_{\mathrm{f}} f(t) \end{aligned} \tag{25}$$

According to Lemma 1, we can design the disturbance rejection gain K_{d} as: $K_{\mathrm{d}} = B_{\mathrm{f}}^{+} D_{\mathrm{f}}$.

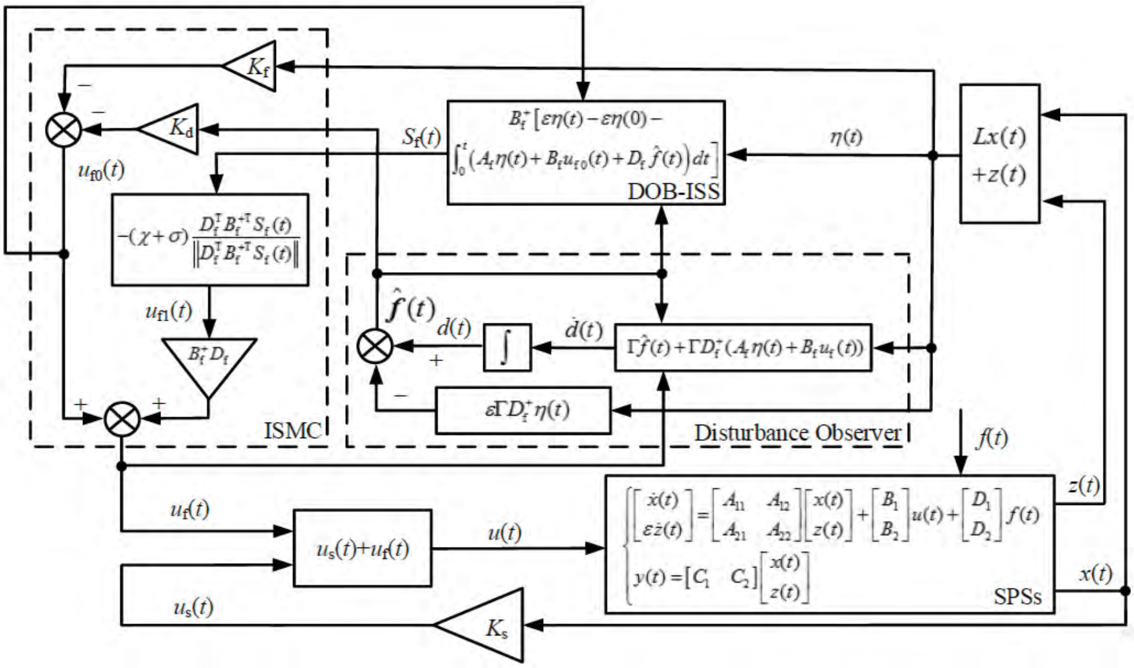


FIGURE 1. **The composite control for the SPSs with mismatched disturbances.**

Then, substituting K_{d} into equation (25) yields,

$$\begin{aligned}\varepsilon\dot{\eta}(t) &= (A_{\mathrm{f}} - B_{\mathrm{f}}K_{\mathrm{f}})\eta(t) \\ &\quad - B_{\mathrm{f}}B_{\mathrm{f}}^{+}D_{\mathrm{f}}\left(\hat{f}(t) + \tilde{f}(t)\right) + D_{\mathrm{f}}f(t) \\ &= (A_{\mathrm{f}} - B_{\mathrm{f}}K_{\mathrm{f}})\eta(t) \\ &\quad - \left[I - B_{\mathrm{f}}^{\perp}\left(B_{\mathrm{f}}^{\perp}\right)^{+}\right]D_{\mathrm{f}}f(t) + D_{\mathrm{f}}f(t) \\ &= (A_{\mathrm{f}} - B_{\mathrm{f}}K_{\mathrm{f}})\eta(t) + f_2(t) \end{aligned} \tag{26}$$

where $f_2(t) = B_{\mathrm{f}}^{\perp}(B_{\mathrm{f}}^{\perp})^{+}D_{\mathrm{f}}f(t) = \tilde{D}_{\mathrm{f}}f(t)$ is the mismatched component of $D_{\mathrm{f}}f(t)$.

According to the sliding motion equation (26), it is clear that the matched component of disturbances $f_1(t)$ defined as (23) is completely rejected by the designed ISMC, but the mismatched component $f_2(t)$ still affects the closed-loop system performance. In the following, the well-known H_∞ control theory will be adopted to attenuate $f_2(t)$.

Theorem 2: Given an H_∞ performance index $\gamma > 0$, if there exist a positive define symmetric matrix $P_{\mathrm{f}} \in R^{n_2\times n_2}$ and a matrix $F_{\mathrm{f}} \in R^{m\times n_2}$ satisfying the following LMI,

$$\begin{bmatrix} A_{\mathrm{f}}P_{\mathrm{f}} + P_{\mathrm{f}}A_{\mathrm{f}}^{\mathrm{T}} - B_{\mathrm{f}}F_{\mathrm{f}} - F_{\mathrm{f}}^{\mathrm{T}}B_{\mathrm{f}}^{\mathrm{T}} & \tilde{D}_{\mathrm{f}} & P_{\mathrm{f}}C_2^{\mathrm{T}} \\ \tilde{D}_{\mathrm{f}}^{\mathrm{T}} & -\gamma^2 I & 0 \\ C_2P_{\mathrm{f}}^{\mathrm{T}} & 0 & -I \end{bmatrix} < 0 \tag{27}$$

then, the closed-loop fast subsystem (26) with the controller gain $K_{\mathrm{f}} = F_{\mathrm{f}}P_{\mathrm{f}}^{-1}$ is asymptotically stable, and the H_∞ performance index (8) can be guaranteed.

Proof: Suppose that the LMI (27) is feasible.

Substituting the controller gain $K_{\mathrm{f}} = F_{\mathrm{f}}P_{\mathrm{f}}^{-1}$ into (27) and applying the Schur complement [24] lead to

$$\begin{bmatrix} A_{\mathrm{f}}P_{\mathrm{f}} + P_{\mathrm{f}}A_{\mathrm{f}}^{\mathrm{T}} - B_{\mathrm{f}}K_{\mathrm{f}}P_{\mathrm{f}} - \\ P_{\mathrm{f}}K_{\mathrm{f}}^{\mathrm{T}}B_{\mathrm{f}}^{\mathrm{T}} + P_{\mathrm{f}}C_2^{\mathrm{T}}C_2P_{\mathrm{f}} & \tilde{D}_{\mathrm{f}} \\ \tilde{D}_{\mathrm{f}}^{\mathrm{T}} & -\gamma^2 I \end{bmatrix} < 0 \tag{28}$$

Pre- and post-multiplying (28) by $diag\left\{P_{\mathrm{f}}^{-1}, I\right\}$ and its transpose, respectively, yield

$$\Lambda_{\mathrm{f}} \triangleq \begin{bmatrix} P_{\mathrm{f}}^{-1}(A_{\mathrm{f}} - B_{\mathrm{f}}K_{\mathrm{f}}) + \\ \left(A_{\mathrm{f}}^{\mathrm{T}} - K_{\mathrm{f}}^{\mathrm{T}}B_{\mathrm{f}}^{\mathrm{T}}\right)P_{\mathrm{f}}^{-1} & P_{\mathrm{f}}^{-1}\tilde{D}_{\mathrm{f}} \\ \tilde{D}_{\mathrm{f}}^{\mathrm{T}}P_{\mathrm{f}}^{-1} & 0 \end{bmatrix} + \begin{bmatrix} C_2^{\mathrm{T}}C_2 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{29}$$

Then, for all $\eta(t) \in R^{n_2}$ and $f(t) \in R^q$, it holds that

$$\begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix}^{\mathrm{T}} \Lambda_{\mathrm{f}} \begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix} \le 0 \tag{30}$$

Define the following ε-dependent Lyapunov function candidate,

$$V_\eta(t) = \varepsilon\eta^{\mathrm{T}}(t)P_{\mathrm{f}}^{-1}\eta(t) \tag{31}$$

Taking the time-derivative of $V_\eta(t)$ along the sliding motion equation (26) leads to equation (32), as shown at the bottom of the next page.

From (29)-(32), we can get

$$\dot{V}_\eta(t) + \eta^{\mathrm{T}}(t)C_2^{\mathrm{T}}C_2\eta(t) - \gamma^2 f^{\mathrm{T}}(t)f(t) \le 0 \tag{33}$$

When $f(t) = 0$, we have the following inequality,

$$\dot{V}_\eta(t) < 0 \tag{34}$$

which indicates that the closed-loop fast subsystem (26) is asymptotically stable.

By (33), the following inequality holds

$$\dot{V}_\eta(t) + \eta^{\mathrm{T}}(t)C_2^{\mathrm{T}}C_2\eta(t) \le \gamma^2 f^{\mathrm{T}}(t)f(t) \tag{35}$$

Integrating both sides of (35) from 0 to ∞, with $y_\eta(t) = C_2\eta(t)$, yields

$$\int_0^\infty y_\eta^{\mathrm{T}}(t)y_\eta(t)\mathrm{d}t \le \gamma^2\int_0^\infty f^{\mathrm{T}}(t)f(t)\mathrm{d}t \tag{36}$$

which indicates that the H_∞ performance index (8) is guaranteed. From (36), it is easy to see that a smaller value of γ means that the disturbances have less influence on $y_\eta(t)$. This completes the proof. ■

Remark 5: Theorem 2 provides an alternative way to determine the controller gain K_f by the LMI (27) based on the H_∞ control theory. Such a combination with the ε-dependent Lyapunov function candidate enables us to find a set of values for K_f with the H_∞ performance index guaranteed. As a result, the closed-loop system is asymptotically stable, and the mismatched disturbances are effectively attenuated, which will be demonstrated by the simulation results.

Because the fast subsystem is asymptotically stable, and the gain matrix K_s can be designed such that the eigenvalues of slow subsystem are placed on the desired position, according to Klimushchev-Krasovskii (*K-K*) lemma [26], there exists a bound $\bar{\varepsilon} > 0$ for the singular perturbation parameter ε, such that for all $\varepsilon \in (0, \bar{\varepsilon}]$, the full-order SPS is asymptotically stable.

Remark 6: In this paper, a linear feedback control law is firstly designed to maintain the slow subsystem asymptotically stable, while a disturbance observer-based ISMC law for the fast subsystem is proposed to attenuate the mismatched disturbances. As a result, the synthesized control law can guarantee the stability and robustness of the full-order SPSs.

IV. NUMERICAL EXAMPLE

In this section, an electric circuit system in the presence of the mismatched disturbances is studied to demonstrate the effectiveness and feasibility of the proposed approach. The state equations of the electric circuit system shown as Fig. 2 are obtained as follows.

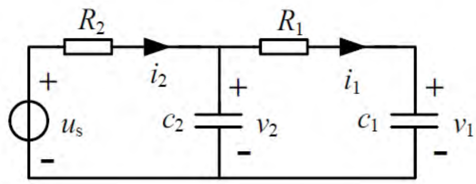


FIGURE 2. The electric circuit system.

$$\begin{cases} \dot{v}_1 = -\dfrac{1}{R_1c_1}v_1 + \dfrac{1}{R_1c_1}v_2 \\ c_2\dot{v}_2 = \dfrac{1}{R_1}v_1 - \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)v_2 + \frac{1}{R_2}u_\mathrm{s} \end{cases} \tag{37}$$

Let $R_1 = 1\Omega$, $R_2 = 0.5\Omega$, $c_1 = 1\mathrm{F}$ and $c_2 = \varepsilon = 0.001\mathrm{F}$. Define $x(t) = v_1$, $z(t) = v_2$ and $u(t) = u_\mathrm{s}$.

Then, the circuit system (37) affected by the external disturbance can be described by the SPS (1) with

$$\begin{aligned} &A_{11} = -1, \quad A_{12} = 1,\ A_{21} = 1, A_{22} = -3 \\ &B_1 = 0, \quad B_2 = 2,\ C_1 = 1 \\ &C_2 = 1, \quad D_1 = 1,\ D_2 = 2 \end{aligned} \tag{38}$$

The initial condition is chosen as $\left[x^{\mathrm{T}}(0)\ z^{\mathrm{T}}(0)\right]^{\mathrm{T}} = \left[6\ -5.5\right]^{\mathrm{T}}$, and the external disturbance is assumed to be

$$f(t) = \frac{1}{1+t^2} \tag{39}$$

The slow-time state feedback gain matrix K_s is selected as: $K_\mathrm{s} = -2$, such that the eigenvalue of $A_0 + B_0K_\mathrm{s}$ is -2. The calculation parameters are as follows:

$$\begin{aligned} &A_0 = -\frac{2}{3}, \quad B_0 = \frac{2}{3}, L = 1.0007 \\ &A_\mathrm{f} = -2.9990, \quad B_\mathrm{f} = 2, D_\mathrm{f} = 2.0010 \\ &B_\mathrm{f}^{+} = 0.5000, \quad D_\mathrm{f}^{+} = 0.4997, K_\mathrm{d} = 1.0005 \end{aligned} \tag{40}$$

By solving the LMI in Theorem 2 with $\gamma = 0.01$, the calculation results are as follows:

$$P_\mathrm{f} = 5.3317, \quad F_\mathrm{f} = 14.5592,\ K_\mathrm{f} = 2.7307 \tag{41}$$

$$\begin{aligned} \dot{V}_\eta(t) &= (\varepsilon\dot{\eta}(t))^{\mathrm{T}}P_\mathrm{f}^{-1}\eta(t) + \eta^{\mathrm{T}}(t)P_\mathrm{f}^{-1}(\varepsilon\dot{\eta}(t)) \\ &= \left(\begin{bmatrix} A_\mathrm{f} - B_\mathrm{f}K_\mathrm{f} & \tilde{D}_\mathrm{f} \end{bmatrix}\begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix}\right)^{\mathrm{T}}\begin{bmatrix} P_\mathrm{f}^{-1} & 0 \end{bmatrix}\begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} P_\mathrm{f}^{-1} & 0 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} A_\mathrm{f} - B_\mathrm{f}K_\mathrm{f} & \tilde{D}_\mathrm{f} \end{bmatrix}\begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix} \\ &= \begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix}^{\mathrm{T}}\left(\begin{bmatrix} (A_\mathrm{f} - B_\mathrm{f}K_\mathrm{f})^{\mathrm{T}} \\ \tilde{D}_\mathrm{f}^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} P_\mathrm{f}^{-1} & 0 \end{bmatrix} + \begin{bmatrix} P_\mathrm{f}^{-1} \\ 0 \end{bmatrix}\begin{bmatrix} A_\mathrm{f} - B_\mathrm{f}K_\mathrm{f} & \tilde{D}_\mathrm{f} \end{bmatrix}\right)\begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix} \\ &= \begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} (A_\mathrm{f} - B_\mathrm{f}K_\mathrm{f})^{\mathrm{T}}P_\mathrm{f}^{-1} + P_\mathrm{f}^{-1}(A_\mathrm{f} - B_\mathrm{f}K_\mathrm{f}) & P_\mathrm{f}^{-1}\tilde{D}_\mathrm{f} \\ \tilde{D}_\mathrm{f}^{\mathrm{T}}P_\mathrm{f}^{-1} & 0 \end{bmatrix}\begin{bmatrix} \eta(t) \\ f(t) \end{bmatrix} \end{aligned} \tag{32}$$

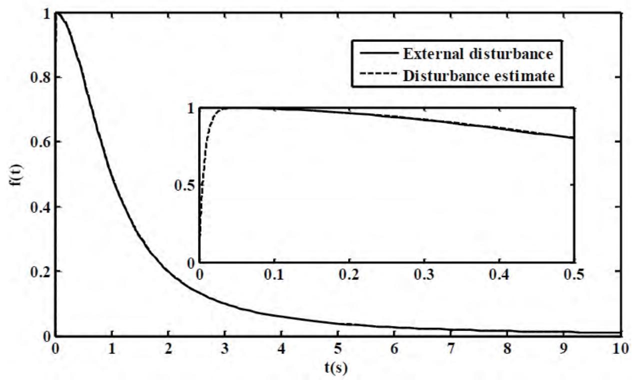


FIGURE 3. The estimate of the external disturbance.

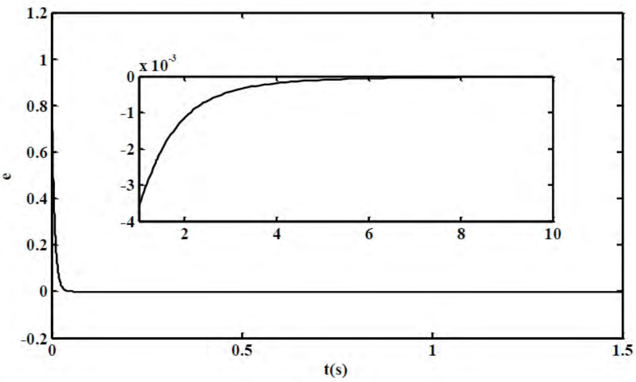


FIGURE 4. The estimation error of the external disturbance.

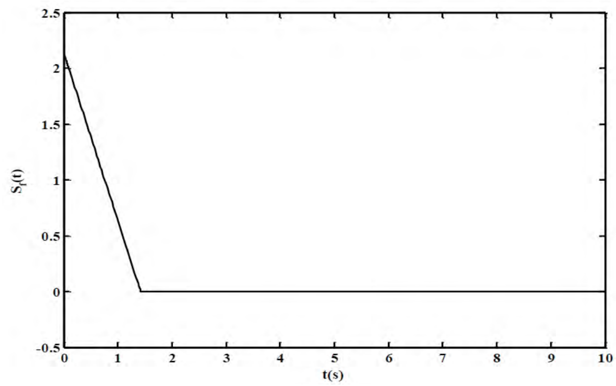


FIGURE 5. The DOB-ISS $S_{\mathrm{f}}(t)$.

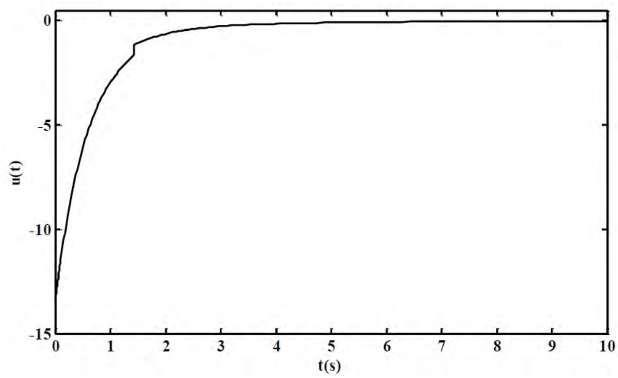


FIGURE 6. The control input $u(t)$.

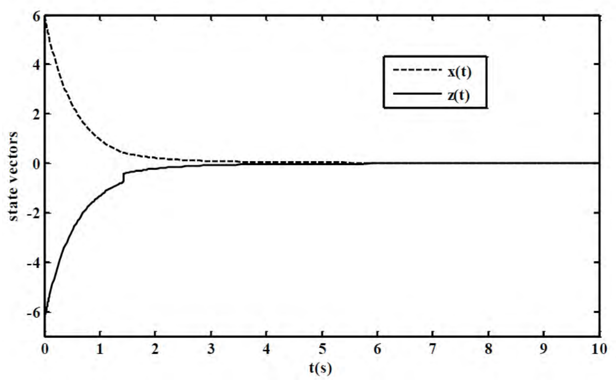


FIGURE 7. The evolutions of slow- and fast-time state vectors $x(t)$ and $z(t)$.

The disturbance observer can be constructed by selecting $\Gamma = -140$. According to Assumption 1 and the analysis of disturbance observer (10), we can select: $\alpha = 1$, $\beta = 0.7$ and $c = 1.1$, thus χ and σ can be set as: $\chi = 1.2$ and $\sigma = 0.3$. The disturbance estimate and disturbance estimation error are shown in Fig. 3 and Fig. 4. From Fig. 3 and Fig. 4, it can be concluded that the estimation error of disturbance rapidly converges to zero within the error bound, which indicate that the disturbance observer can estimate the external disturbance well.

The reaching time satisfies

$$0.7895 \leq \tau \leq 7.1057 \tag{42}$$

In the simulation study, the discontinuous control term $u_{\mathrm{f}1}(t)$ (13) is replaced by the following form (43) for an actual implementation [11].

$$u_{\mathrm{f}1}(t) = -(\chi + \sigma)\frac{D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t)}{\left\| D_{\mathrm{f}}^{\mathrm{T}} B_{\mathrm{f}}^{+\mathrm{T}} S_{\mathrm{f}}(t) \right\| + 0.001} \tag{43}$$

The DOB-ISS (11) with the reaching time (42) is shown in Fig. 5. Correspondingly, the control input $u(t)$ with $u_{\mathrm{f}1}(t)$ (43) is presented in Fig. 6. From Fig. 5 and Fig. 6, we can conclude that the sliding surface is attained at the time τ. In addition, it should be emphasized that the chattering phenomenon of ISMC is improved and even eliminated.

Finally, the evolutions of slow- and fast-time state vectors are presented in Fig. 7. With the proposed ISMC law (12), the closed-loop SPS is asymptotically stable, and robust to the mismatched disturbances. It is easy to see that when the sliding surface is attained at the time τ, a small jump occurs in Fig. 6 and Fig. 7. The reason is that the change in value of S_{f} has an influence on $u_{\mathrm{f}1}(t)$ according to (43).

V. CONCLUSION

This paper has investigated the problem of ISMC for SPSs with mismatched disturbances. The ε-dependent disturbance observer and the LMI-based approach to design the ISMC law has been presented. By virtue of this method, the obtained ISMC law can ensure that, for any singular perturbation parameter within the upper bound, the closed-loop SPSs are asymptotically stable, and the mismatched disturbances can be effectively attenuated. Our future work will extend the proposed method to deal with the mismatched disturbances and nonlinear uncertainties in SPSs, simultaneously.

REFERENCES

[1] C. Yang, L. Zhang, and J. Sun, "Anti-windup controller design for singularly perturbed systems subject to actuator saturation," *IET Control Theory Appl.*, vol. 10, no. 4, pp. 469–476, 2016.

[2] C. Yang, Q. Zhang, J. Sun, and T. Chai, "Lur'e Lyapunov function and absolute stability criterion for lur'e singularly perturbed systems," *IEEE Trans. Autom. Control*, vol. 56, no. 11, pp. 2666–2671, Nov. 2011.

[3] Y. Gao, G. Lu, and Z. Wang, "Passivity analysis of uncertain singularly perturbed systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 57, no. 6, pp. 486–490, Jun. 2010.

[4] C. Yang and Q. Zhang, "Multiobjective control for T–S fuzzy singularly perturbed systems," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 1, pp. 104–115, Feb. 2009.

[5] C. Yang, J. Sun, and X. Ma, "Stabilization bound of singularly perturbed systems subject to actuator saturation," *Automatica*, vol. 49, no. 2, pp. 457–462, 2013.

[6] L. Zhang, D. Cong, Z. Yang, Y. Zhang, and J. Han, "Robust tracking and synchronization of double shaking tables based on adaptive sliding mode control with novel reaching law," *IEEE Access*, vol. 4, pp. 8686–8702, 2016.

[7] V. Utkin, "Variable structure systems with sliding modes," *IEEE Trans. Autom. Control*, vol. AC-22, no. 2, pp. 212–222, Apr. 1977.

[8] J. Yang, S. Li, and X. Yu, "Sliding-mode control for systems with mismatched uncertainties via a disturbance observer," *IEEE Trans. Ind. Electron.*, vol. 60, no. 1, pp. 160–169, Jan. 2013.

[9] S. U. Din, Q. Khan, U.-R. Fazal, and R. Akmeliawanti, "A comparative experimental study of robust sliding mode control strategies for underactuated systems," *IEEE Access*, vol. 5, pp. 10068–10080, 2017.

[10] F. Castanos and L. Fridman, "Analysis and design of integral sliding manifolds for systems with unmatched perturbations," *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 853–858, May 2006.

[11] J. Zhang, X. Liu, Y. Xia, Z. Zuo, and Y. Wang, "Disturbance observer-based integral sliding-mode control for systems with mismatched disturbances," *IEEE Trans. Ind. Electron.*, vol. 63, no. 11, pp. 7041–7047, Nov. 2016.

[12] M. I. Ghiasi, M. A. Golkar, and A. Hajizadeh, "Lyapunov based-distributed fuzzy-sliding mode control for building integrated-DC microgrid with plug-in electric vehicle," *IEEE Access*, vol. 5, pp. 7746–7752, 2017.

[13] W.-H. Chen, "Nonlinear disturbance observer-enhanced dynamic inversion control of missiles," *J. Guid., Control, Dyn.*, vol. 26, no. 1, pp. 161–166, Jan. 2003.

[14] S. Li, J. Yang, W.-C. Chen, and X. Chen, "Generalized extended state observer based control for systems with mismatched uncertainties," *IEEE Trans. Ind. Electron.*, vol. 59, no. 12, pp. 4792–4802, Dec. 2012.

[15] J. Yang, A. Zolotas, W.-H. Chen, K. Michail, and S. Li, "Robust control of nonlinear MAGLEV suspension system with mismatched uncertainties via DOBC approach," *ISA Trans.*, vol. 50, no. 3, pp. 389–396, 2011.

[16] B. S. Heck, "Sliding-mode control for singularly perturbed systems," *Int. J. Syst. Sci.*, vol. 53, no. 4, pp. 985–1001, 1991.

[17] T.-H. S. Li, J.-L. Lin, and F.-C. Kung, "Composite sliding-mode control of singular perturbation systems," in *Proc. Amer. Control Conf.*, Seattle, WA, USA, 1995, pp. 2248–2249.

[18] A. E. Ahmed, H. M. Schwartz, and V. C. Aitken, "Sliding mode control for singularly perturbed system," in *Proc. 5th Asian Control Conf.*, Melbourne, VIC, Australia, 2004, pp. 1946–1950.

[19] M. Innocenti, L. Greco, and L. Pollini, "Sliding mode control for two-time scale systems: Stability issues," *Automatica*, vol. 39, no. 2, pp. 273–280, 2003.

[20] Y. Dong and X. Shifan, "Sliding mode control of singular perturbation systems," in *Proc. IEEE Int. Conf. Syst. Man, Cybern.*, Beijing, China, Oct. 1996, pp. 113–116.

[21] T. Nguyen, W.-C. Su, and Z. Gajic, "Sliding mode control for singularly perturbed linear continuous time systems: Composite control approaches," in *Proc. IEEE Int. Symp. Comput.-Aided Control Syst. Design*, Yokohama, Japan, Sep. 2010, pp. 2011–2016.

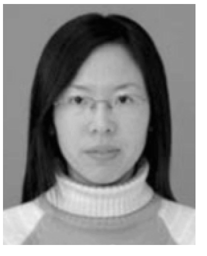
[22] T. Nguyen, W.-C. Su, and Z. Gajic, "Variable structure control for singularly perturbed linear continuous systems with matched disturbances," *IEEE Trans. Autom. Control*, vol. 57, no. 3, pp. 777–783, Mar. 2012.

[23] R. K. Munje, B. M. Patre, S. R. Shimjith, and A. P. Tiwari, "Sliding mode control for spatial stabilization of advanced heavy water reactor," *IEEE Trans. Nucl. Sci.*, vol. 60, no. 4, pp. 3040–3050, Aug. 2013.

[24] Y. Gao, B. Sun, and G. Lu, "Passivity-based integral sliding-mode control of uncertain singularly perturbed systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 58, no. 6, pp. 386–390, Jun. 2011.

[25] H. Yoo, "Design of observers for systems with slow and fast modes," M.S. thesis, Rutgers Univ.–New Brunswick, New Brunswick, NJ, USA, 2014, pp. 15–47.

[26] P. V. Kokotovic, H. K. Khalil, and J. O'Reilly, *Singular Perturbation Methods in Control: Analysis and Design*. New York, NY, USA: Academic, 1986, pp. 157–247.



LINNA ZHOU received the B.S. degree from the Department of Water Conservancy Engineering, North China Institute of Water Conservancy and Hydroelectric Power, Zhengzhou, China, in 2001, and the Ph.D. degree from Northeastern University, Shenyang, China, in 2007. She is currently an Associate Professor with the China University of Mining and Technology, Xuzhou, China.

Her research interests include analysis and design for nonlinear singular systems, descriptor systems, robust control, and fuzzy control.

ZHIYUAN CHE received the B.Eng. degree from the School of Automation and Electrical Engineering, Lanzhou Jiaotong University, Lanzhou, China, in 2015. He is currently pursuing the M.Eng. degree with the School of Information and Control Engineering, China University of Mining and Technology, Xuzhou, China.

His current research interest is sliding mode control.

CHUNYU YANG received the B.S. degree in applied mathematics and the Ph.D. degree in control theory and control engineering from Northeastern University, China, in 2002 and 2009, respectively. He is currently a Professor with the China University of Mining and Technology, Xuzhou, China.

His research interests include analysis and design of singularly perturbed systems.