IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received January 20, 2018, accepted February 14, 2018, date of publication February 19, 2018, date of current version March 16, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2807428

Global Stabilization for a Class of Genuinely Nonlinear Systems With a Time-Varying Power: An Interval Homogeneous Domination Approach

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This work was supported in part by the Ministry of Science and Technology, Taiwan, under Grant MOST 106-2218-E-006-026-.

ABSTRACT This paper addresses the problem of global state feedback stabilization for a class of genuinely nonlinear systems with a time-varying power. By revamping the so-called adding a power integrator technique and the homogeneous domination approach, a new design method called interval homogeneous domination approach is proposed to delicately design a state feedback control law that renders the nonlinear systems globally asymptotically stable. The novelty of the proposed scheme owes to the systematic fashion that provides a distinct perspective to solve the stabilization problem for the nonlinear systems with a time-varying power.

INDEX TERMS High-order nonlinear systems, adding a power integrator, homogeneous domination, global stabilization.

I. INTRODUCTION

The past decades have witnessed a rapid advance on research of efforts aimed at the development of systematic analysis and design approaches for nonlinear control systems. A large number of works published in the literature have been dedicated to the problem of global stabilization for various nonlinear systems (see, e.g., [1]–[14], and the references therein). In this paper, the primary objective is to investigate the problem of global state feedback stabilization for a class of genuinely nonlinear systems described by

$$\begin{aligned}\dot{x}_1 &= \lceil x_2 \rfloor^{p(t)} + \phi_1(\mathbf{x}, t, u)\\ &\vdots\\ \dot{x}_{n-1} &= \lceil x_n \rfloor^{p(t)} + \phi_{n-1}(\mathbf{x}, t, u)\\ \dot{x}_n &= \lceil u \rfloor^{p(t)} + \phi_n(\mathbf{x}, t, u)\end{aligned} \tag{1}$$

where $\mathbf{x} = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ and $u \in \mathbb{R}$ are the system states and control input, respectively. For $i = 1, \ldots, n$, the nonlinearity $\phi_i : \mathbb{R}^n \times \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ is a continuous function. The time-varying function $p : \mathbb{R}^+ \to [\underline{p}, \overline{p}] \subset \mathbb{R}$ with $1 \leq \underline{p} \leq \overline{p}$, which is called the power of the system (1), is a continuous bounded function, and the power sign function $\lceil \cdot \rfloor^{\alpha(t)}$ is defined as $\lceil \cdot \rfloor^{\alpha(t)} := |\cdot|^{\alpha(t)}\text{sign}(\cdot)$ for a continuous function $\alpha : \mathbb{R}^+ \to \mathbb{R}^+$.

When the power $p(t)$ is a fixed constant, the system (1) is called a high-order nonlinear system (i.e., nonlinear systems in p-normal form) in the literature [15]. Due to the existence of uncontrollable/unobservable linearization around the origin, the stabilization problem of high-order nonlinear systems has been recognized as a challenging problem in the field of nonlinear control. Fortunately, with the aid of the techniques of adding a power integrator [16] and homogeneous domination [17], many approaches have been proposed in the past two decades (see, e.g., [16]–[26]) to overcome the topological obstruction to the stabilization problem of high-order nonlinear systems, in which the power is assumed to be a fixed constant. However, in practice, the power $p(t)$ might vary for different operation conditions. For instance, the reduced-order dynamical model of a boiler-turbine unit in [27] can be expressed as

$$\begin{aligned}\dot{x}_1 &= a_1 \lceil x_2 \rfloor^{p(t)} + \phi_1(x_1, u)\\ \dot{x}_2 &= a_2 x_2 + \phi_2(u)\end{aligned} \tag{2}$$

where a_1 and a_2 are constant parameters, $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are continuous nonlinear functions, and the power $p(t)$ is varying in time since this power is usually estimated/identified from operational data obtained from a power plant; two typical values of $p(t)$ are $p(t) = 1.072$ [27] and $p(t) = 1.031$ [28].

Another example can be found in [29], in which an under-actuated weakly coupled mechanical system was considered and it may involve time-varying powers because of potential performance deterioration of a hardening spring.

Due to the existence of the time-varying power $p(t)$, it is not surprising that most of the existing results [16]–[26] dealing with the stabilization problem for systems with fixed (constant) powers are inapplicable to the system (1). Specifically, the main obstacles in constructing stabilizers for the system (1) lie in two aspects: (i) Based on the adding a power integrator technique [16]–[22] and/or homogeneous domination approach [23]–[26], the selected Lyapunov functions inevitably include the (time-varying) power of the system (1); thus, the associated design method is no longer workable for constructing stabilizing control laws, and the claimed stability conclusion is in general not satisfied. (ii) Because the power $p(t)$ is time-varying, the system (1) is intrinsically time-varying (non-autonomous) and more complicated. New perspectives and/or mathematical techniques should be developed for the control law design, as well as stability analysis. For these critical reasons, the global stabilization problem for the system (1) is exceptionally challenging and much more difficult than the case when the power $p(t)$ is fixed.

In this paper, we aim to tackle the obstacles mentioned above and shall provide a solution to the problem of global state feedback stabilization for the system (1). By extending our previous results [30], which considered the power $p(t)$ with $\underline{p} = 1$, we develop a new design strategy called *interval homogeneous domination approach* so that a state feedback globally stabilizing control law can be constructed for the system (1) with the power satisfying $\underline{p} \leq p(t) \leq \overline{p}$. Thus, the proposed scheme is applicable to a more general class of nonlinear systems. It should be pointed that the extension is nontrivial because in the case of $\underline{p} > 1$ the system (1) is a pure high-order system [16] and the design procedure together with stability analysis becomes more complicated and difficult. To overcome this difficulty, a change of coordinates with tunable parameters will be firstly constructed to acquire a transformed system. Based on the transformed system, a state feedback globally stabilizer will be organized for the transformed nominal system by revamping the technique of adding a power integrator [19]. Finally, the newly developed technique, i.e., the interval homogeneous domination approach, will be fulfilled by an exquisite selection of the parameters equipped in the coordinate transformation to construct a state feedback stabilizing control law for the transformed system; equivalently, in the original coordinate, the resultant control law will globally stabilize the system (1).

Notations: For easy reference, the notations used throughout this paper are summarized as follows. $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{R}^+$ the set of all nonnegative real numbers, and $\mathbb{R}^n$ represents the Euclidean space with dimension n. For a real vector $\mathbf{x} = (x_1, \dots, x_n)^T \in \mathbb{R}^n$, $\|\mathbf{x}\| := \sqrt{\sum_{i=1}^n x_i^2}$. For a scalar continuous function $\alpha : \mathbb{R}^+ \to \mathbb{R}^+$, $\lceil y \rfloor^{\alpha(t)} := |y|^{\alpha(t)}\text{sign}(y)$ where $\text{sign}(y) = 1$ if $y > 0$, $\text{sign}(y) = 0$ if $y = 0$ and $\text{sign}(y) = -1$ if $y < 0$. Finally, $p(t)$ is the time-varying power of the system (1), where $p(t)$ satisfies $\underline{p} \leq p(t) \leq \overline{p}$ with $1 \leq \underline{p} \leq \overline{p}$.

II. PRELIMINARIES AND TECHNICAL LEMMAS

To begin with, we recall the definition of interval homogeneity which is initially proposed in [30] by generalizing the idea of traditional weighting homogeneity [31]–[33].

Definition 1 [30]: A continuous vector-valued function $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}^n$ denoted by $\mathbf{f}(\mathbf{x}, t, u) = (f_1(\mathbf{x}, t, u), \dots, f_n(\mathbf{x}, t, u))^T$ with $\mathbf{x} = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ is said to be interval homogeneous of interval homogeneity degree $[\underline{\tau}, \overline{\tau}]$ with respect to the weights $(r_1, \dots, r_n, r_{n+1})$ and $r_i > 0$ for all $i = 1, \dots, n+1$ if there exists a continuous real-valued function $\tau : \mathbb{R} \to [\underline{\tau}, \overline{\tau}]$ such that

$$f_i(\varepsilon^{r_1}x_1, \dots, \varepsilon^{r_n}x_n, t, \varepsilon^{r_{n+1}}u) = \varepsilon^{r_i+\tau(t)}f_i(\mathbf{x}, t, u)$$

for all $\mathbf{x} \in \mathbb{R}^n$, $t \in \mathbb{R}^+$, $u \in \mathbb{R}$, $\varepsilon > 0$ and $i = 1, \dots, n$.

Definition 2 [30]: A nonlinear system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t, u)$ with $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}^n$ is said to be interval homogeneous of degree $[\underline{\tau}, \overline{\tau}]$ if $\mathbf{f}(\mathbf{x}, t, u)$ is continuous and interval homogeneous of interval homogeneity degree $[\underline{\tau}, \overline{\tau}]$.

Next, we list some useful lemmas that will be used frequently throughout this paper. The detailed proofs of the first three lemmas can be found in the literature, such as [34]–[36].

Lemma 1 [34]: Let $k(t)$ be a continuous real-valued function satisfying $k(t) \geq 1$ for all $t \in \mathbb{R}$. The following inequality holds for any $t, x_1, x_2 \in \mathbb{R}$:

$$\left| \lceil x_1 \rfloor^{k(t)} - \lceil x_2 \rfloor^{k(t)} \right| \leq k(t)\left(2^{k(t)-2}+2\right) \times \left(|x_1 - x_2|^{k(t)} + |x_1 - x_2||x_2|^{k(t)-1}\right)$$

where $|x_2|^{k(t)-1} := 0$ if $x_2 = 0$ and $k(t) = 1$.

Lemma 2 [35]: Let k_1, k_2 and g be continuous real-valued functions. The following inequality holds for any $t, x_1, x_2 \in \mathbb{R}$:

$$|x_1|^{k_1(t)}|x_2|^{k_2(t)} \leq \frac{k_1(t)g(x_1, x_2)}{k_1(t) + k_2(t)}|x_1|^{k_1(t)+k_2(t)} + \frac{k_2(t)g^{-\frac{k_1(t)}{k_2(t)}}(x_1, x_2)}{k_1(t) + k_2(t)}|x_2|^{k_1(t)+k_2(t)}.$$

Lemma 3 [36]: Let k be a continuous real-valued function. The following inequality holds for all $t \in \mathbb{R}$ and $x_i \in \mathbb{R}$ with $i = 1, \dots, n$:

$$(|x_1| + \cdots + |x_n|)^{k(t)} \leq \max\left(1, n^{k(t)-1}\right) \times \left(|x_1|^{k(t)} + \cdots + |x_n|^{k(t)}\right).$$

The lemma listed below contributes to a powerful tool for the stability analysis in this paper.

Lemma 4: Let g and k be real-valued functions. If the function $k(\cdot)$ satisfies that $\underline{k} \leq k(t) \leq \overline{k}$, then the following inequality holds for all $\mathbf{x} \in \mathbb{R}^n$ and $t \in \mathbb{R}$:

$$g^{k(t)}(\mathbf{x}) \geq \frac{g^{\overline{k}}(\mathbf{x})}{1 + g^{2\overline{k}-\underline{k}}(\mathbf{x})}.$$

Proof: Indeed, we only need to prove the case of $g(\mathbf{x}) > 0$. By direct calculation, one has

$$k(t) + \underline{k} \leq \overline{k} + \underline{k} \leq 2\overline{k} + k(t).$$

With this inequality in mind, one can verify that

$$\gamma^{\overline{k}+\underline{k}}(\mathbf{x}) \leq \gamma^{k(t)+\underline{k}}(\mathbf{x}) + \gamma^{2\overline{k}+k(t)}(\mathbf{x}).$$

This completes the proof. ■

Remark 1: Lemma 4 is crucial in the subsequent stability analysis due to the following two aspects. (i) It provides an effective lower bound of a power function, which is exploited to dominate time-varying properties of the power function. (ii) It gives an insight into the satisfactory solution for multiple nonidentical time-varying powers.

III. MAIN RESULTS

Without doubt the global stabilization problem of the system (1) is achievable only under certain conditions on the power $p(t)$ and nonlinearities $\phi_i(\mathbf{x}, t, u)$'s. Thus, we impose the following assumptions:

Assumption 1: For the bounds $\overline{p}$ and $\underline{p}$ in the system (1), the following inequality is satisfied:

$$(\overline{p} - \underline{p}) \leq \frac{(1-\kappa)}{\left(1 + \underline{p} + \underline{p}^2 + \cdots + \underline{p}^{n-2}\right)}$$

for a constant $0 < \kappa < 1$.

Assumption 2: There is a real constant $c \geq 0$ such that

$$|\phi_i(\mathbf{x}, t, u)| \leq c \times \sum_{i=1}^{n} |x_i|^{p(t)}$$

for all $\mathbf{x} \in \mathbb{R}^n$, $t \in \mathbb{R}^+$, $u \in \mathbb{R}$ and $i = 1, \ldots, n$, where $p(t)$ is the power of the system (1).

It is worth noting that Assumption 1 presents a restriction on the magnitude of the power $p(t)$ (i.e., the distance between $\overline{p}$ and $\underline{p}$) so that the existence of a stabilizing control law for the system (1) can be guaranteed; that is, we only consider the system (1) with a bounded time-varying power. Assumption 2 gives a growth condition of $\phi_i(\mathbf{x}, t, u)$ (i.e., the upper bounds of $\phi_i(\mathbf{x}, t, u)$) that is frequently used in the literature [16], [37] with a fixed power $p(t)$.

In the case when $\phi_i(\mathbf{x}, t, u) = 0$ for all $i = 1, \ldots, n$, one can find from Definition 2 that the system (1) is interval homogeneous of degree $[\underline{p} - 1, \overline{p} - 1]$ with respect to the weights

$$(r_1, r_2, \ldots, r_n, r_{n+1}) = (1, \ldots, 1, 1).$$

In addition, Assumption 2 implies that $\phi_i(\mathbf{x}, t, u)$'s are bounded by the interval homogeneous functions with interval homogeneity degree $[\underline{\tau}, \overline{\tau}]$ with respect to the weights $(1, \ldots, 1)$. Thus, in the case when $\phi_i(\mathbf{x}, t, u) \neq 0$ for some $i \in \{1, \ldots, n\}$, the nature of interval homogeneity of the system (1) can be appropriately utilized so that the influence of the nonlinearities $\phi_i(\mathbf{x}, t, u)$'s is dominated delicately by interval homogeneous parts of the system (1); this process is called interval homogeneous domination. To this end, consider the following change of coordinates:

$$z_i = \frac{x_i}{\mathfrak{L}_{i-1}}, \quad i = 1, \ldots, n, \text{ and } v = \frac{u}{\mathfrak{L}_n} \tag{3}$$

where

$$\mathfrak{L}_j = \mathfrak{L}_1^{\alpha(j)}, \quad \mathfrak{L}_0 \triangleq 1$$
$$\alpha(j) = \frac{\sum_{m=0}^{j-1} \underline{p}^m}{\underline{p}^{j-1}}, \quad \alpha(0) \triangleq 0, \; j = 1, \ldots, n \tag{4}$$

and $\mathfrak{L}_1 \geq 1$ is a gain parameter to be determined later. Note that, the coordinate transformation together with the gain $\mathfrak{L}_1 \geq 1$ given by (3)–(4) plays a key role in constructing a global state feedback stabilizer for the system (1). From the relation (4), the following is easy to verify:

$$\begin{aligned}\frac{\mathfrak{L}_{i-1}\mathfrak{L}_{i+1}^{p(t)}}{\mathfrak{L}_i^{p(t)+1}} &= \frac{\mathfrak{L}_1^{\sum_{l=0}^{i-2}\underline{p}^l/\underline{p}^{i-2}}\,\mathfrak{L}_1^{p(t)\times\sum_{l=0}^{i}\underline{p}^l/\underline{p}^i}}{\mathfrak{L}_1^{(p(t)+1)\times\sum_{l=0}^{i-1}\underline{p}^l/\underline{p}^{i-1}}} \\ &= \mathfrak{L}_1^{(p(t)-\underline{p})/\underline{p}^i} \\ &\geq 1\end{aligned} \tag{5}$$

and

$$\begin{aligned}\frac{\mathfrak{L}_i^{p(t)}}{\mathfrak{L}_{i-1}} &\geq \frac{\mathfrak{L}_i^{\underline{p}}}{\mathfrak{L}_{i-1}} \\ &= \frac{\mathfrak{L}_1^{\sum_{l=0}^{i-1}\underline{p}^l/\underline{p}^{i-2}}}{\mathfrak{L}_1^{\sum_{l=0}^{i-2}\underline{p}^l\underline{p}^{i-2}}} \\ &= \mathfrak{L}_1^{\underline{p}}\end{aligned} \tag{6}$$

for $i = 1, \ldots, n-1$. Under the new coordinate (3)–(4), the system (1) can be equivalently described as

$$\begin{aligned}\dot{z}_1 &= \mathfrak{L}_1^{p(t)}\lceil z_2\rfloor^{p(t)} + \phi_1\left(t, z_1, \ldots, z_n\mathfrak{L}_{n-1}, v\mathfrak{L}_n\right) \\ &\vdots \\ \dot{z}_{n-1} &= \frac{\mathfrak{L}_{n-1}^{p(t)}}{\mathfrak{L}_{n-2}}\lceil z_n\rfloor^{p(t)} + \frac{\phi_{n-1}\left(t, z_1, \ldots, z_n\mathfrak{L}_{n-1}, v\mathfrak{L}_n\right)}{\mathfrak{L}_{n-2}} \\ \dot{z}_n &= \frac{\mathfrak{L}_n^{p(t)}}{\mathfrak{L}_{n-1}}\lceil v\rfloor^{p(t)} + \frac{\phi_n\left(t, z_1, \ldots, z_n\mathfrak{L}_{n-1}, v\mathfrak{L}_n\right)}{\mathfrak{L}_{n-1}}.\end{aligned} \tag{7}$$

With the transformed system (7) in mind, in what follows we first show that there is a state feedback control law that globally renders the nominal system of (7), i.e., the system (7) with $\phi_i(\mathbf{x}, t, u) = 0$ for all $i = 1, \ldots, n$, globally (uniformly) asymptotically stable. Then, by delicately choosing $\mathfrak{L}_i$ for $i = 1, \ldots, n$, a scaled globally stabilizing control law will be constructed for the system (7); that is, equivalently, in the original coordinate, a globally stabilizing control law can be obtained accordingly for the (original) system (1).

A. STABILIZING CONTROL LAW FOR THE NOMINAL SYSTEM

In this subsection, we consider the nominal system of (7) which takes the form of

$$\begin{aligned}\dot{z}_1 &= \mathfrak{L}_1^{p(t)}\lceil z_2\rfloor^{p(t)}\\ &\vdots\\ \dot{z}_{n-1} &= \frac{\mathfrak{L}_{n-1}^{p(t)}}{\mathfrak{L}_{n-2}}\lceil z_n\rfloor^{p(t)}\\ \dot{z}_n &= \frac{\mathfrak{L}_n^{p(t)}}{\mathfrak{L}_{n-1}}\lceil v\rfloor^{p(t)}. \end{aligned}\tag{8}$$

Clearly, (8) is interval homogeneous of degree $[\underline{p}-1, \overline{p}-1]$ with respect to the weights $(1, 1, \ldots, 1)$. The following theorem claims that the global stabilization problem of (8) is solvable by a state feedback control law.

Theorem 1: There always exists a state feedback control law that renders the nominal system (8) globally (uniformly) asymptotically stable.

Proof: This proof is based on an inductive argument that simultaneously constructs a positive definite and proper Lyapunov function and a globally stabilizing control law for the nominal system (8).

First Step: Select the following positive definite and proper[1] Lyapunov function

$$V_1(z_1) = \frac{1}{2}z_1^2$$

which is smooth. Then, denoting $\xi_1 := z_1$, we derive

$$\dot{V}_1(z_1) = \mathfrak{L}_1^{p(t)}\xi_1\lceil z_2^*\rfloor^{p(t)} + \mathfrak{L}_1^{p(t)}\xi_1\left(\lceil z_2\rfloor^{p(t)} - \lceil z_2^*\rfloor^{p(t)}\right)$$

where z_2^* is a virtual control law. Clearly, the virtual control law

$$z_2^* = -\beta_1 z_1 \quad \text{with } \beta_1 = n \geq 1$$

yields

$$\begin{aligned}\dot{V}_1(z_1) &\leq \mathfrak{L}_1^{p(t)}\xi_1\lceil -\beta_1 z_1\rfloor^{p(t)} + \mathfrak{L}_1^{p(t)}\xi_1\left(\lceil z_2\rfloor^{p(t)} - \lceil z_2^*\rfloor^{p(t)}\right)\\ &\leq -\mathfrak{L}_1^{p(t)} n|\xi_1|^{p(t)+1} + \mathfrak{L}_1^{p(t)}\xi_1\left(\lceil z_2\rfloor^{p(t)} - \lceil z_2^*\rfloor^{p(t)}\right).\end{aligned}\tag{9}$$

Inductive Step: Suppose that at step k with $k \in \{1, \ldots, n-1\}$, there exist a smooth positive definite and proper Lyapunov function $V_k(z_1, \ldots, z_k)$ and the virtual control laws $z_1^*, \ldots, z_k^*, z_{k+1}^*$, denoted by

$$\begin{aligned} z_1^* &= 0 & \xi_1 &= z_1 - z_1^*\\ z_2^* &= -\xi_1\beta_1 & \xi_2 &= z_2 - z_2^*\\ &\vdots & &\vdots\\ z_k^* &= -\xi_{k-1}\beta_{k-1} & \xi_k &= z_k - z_k^*\\ z_{k+1}^* &= -\xi_k\beta_k & \xi_{k+1} &= z_{k+1} - z_{k+1}^* \end{aligned}\tag{10}$$

[1] A nonnegative function $V : \mathbb{R}^n \to \mathbb{R}$ is said to be proper if for any $b > 0$ the set $V^{-1}([0, b]) \subset \mathbb{R}^n$ is compact in $\mathbb{R}^n$.

with constants $\beta_1 \geq 1, \ldots, \beta_k \geq 1$, such that

$$\begin{aligned}\dot{V}_k(z_1, \ldots, z_k) \leq{}& -(n-k+1) \times \sum_{i=1}^{k}\frac{\mathfrak{L}_i^{p(t)}}{\mathfrak{L}_{i-1}}|\xi_i|^{p(t)+1}\\ &+ \frac{\mathfrak{L}_k^{p(t)}}{\mathfrak{L}_{k-1}}\xi_k\left(\lceil z_{k+1}\rfloor^{p(t)} - \lceil z_{k+1}^*\rfloor^{p(t)}\right).\end{aligned}\tag{11}$$

Clearly, (11) reduces to (9) when $k = 1$. We claim that (11) also holds at step $k+1$. To verify this, we set

$$V_{k+1}(z_1, \ldots, z_{k+1}) = V_k(z_1, \ldots, z_k) + \frac{\xi_{k+1}^2}{2} \tag{12}$$

which is smooth, positive definite, and proper. Then, it follows from (8) and (11) that

$$\begin{aligned}&\dot{V}_{k+1}(z_1, \ldots, z_{k+1})\\ &\leq -(n-k+1)\times\sum_{i=1}^{k}\frac{\mathfrak{L}_i^{p(t)}}{\mathfrak{L}_{i-1}}|\xi_i|^{p(t)+1}\\ &\quad+\frac{\mathfrak{L}_k^{p(t)}}{\mathfrak{L}_{k-1}}\xi_k\left(\lceil z_{k+1}\rfloor^{p(t)} - \lceil z_{k+1}^*\rfloor^{p(t)}\right)\\ &\quad+\sum_{j=1}^{k}\frac{\bar{\beta}_j^k L_j^{p(t)}}{\mathfrak{L}_{j-1}}\xi_{k+1}\lceil z_{j+1}\rfloor^{p(t)} + \frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\xi_{k+1}\lceil z_{k+2}^*\rfloor^{p(t)}\\ &\quad+\frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\xi_{k+1}\left(\lceil z_{k+2}\rfloor^{p(t)} - \lceil z_{k+2}^*\rfloor^{p(t)}\right)\end{aligned}\tag{13}$$

where $\bar{\beta}_j^k = \beta_k\beta_{k-1}\cdots\beta_j$ and z_{k+2}^* is a virtual control law. In order to proceed further, an estimate for each term in the right-hand side of (13) is necessary.

First, by Lemma 1, we have

$$\begin{aligned}&\xi_k\left(\lceil z_{k+1}\rfloor^{p(t)} - \lceil z_{k+1}^*\rfloor^{p(t)}\right)\\ &\quad\leq |\xi_k| \times \left|\lceil z_{k+1}\rfloor^{p(t)} - \lceil z_{k+1}^*\rfloor^{p(t)}\right|\\ &\quad\leq p(t)\left(2^{p(t)-2}+2\right)|\xi_k|\\ &\qquad\times\left(|\xi_{k+1}|^{p(t)} + |\xi_{k+1}|\,\beta_k^{p(t)-1}\,|\xi_k|^{p(t)-1}\right)\\ &\quad\leq c_{k+1}\left(|\xi_k|\,|\xi_{k+1}|^{p(t)} + |\xi_k|^{p(t)}\,|\xi_{k+1}|\right)\end{aligned}\tag{14}$$

for a constant $c_{k+1} > 0$. Using Lemma 2, one can further find from (14) that

$$\begin{aligned}&\xi_k\left(\lceil z_{k+1}\rfloor^{p(t)} - \lceil z_{k+1}^*\rfloor^{p(t)}\right)\\ &\leq \frac{1}{4}|\xi_k|^{p(t)+1} + \frac{4^{\frac{1}{p(t)}}c_{k+1}^{\frac{p(t)+1}{p(t)}}p(t)}{p(t)+1}\left(\frac{1}{p(t)+1}\right)^{\frac{1}{p(t)}}|\xi_{k+1}|^{p(t)}\\ &\quad+\frac{1}{4}|\xi_k|^{p(t)+1} + \frac{4^{p(t)}c_{k+1}^{p(t)+1}}{p(t)+1}\left(\frac{p(t)}{p(t)+1}\right)^{p(t)}|\xi_{k+1}|^{p(t)}\\ &\leq \frac{1}{2}\,|\xi_k|^{p(t)+1} + \tilde{c}_{k+1}\,|\xi_{k+1}|^{p(t)+1}\end{aligned}\tag{15}$$

for a constant $\tilde{c}_{k+1} \geq 1$.

Next, from (5) and (10), it is easy to verify

$$\begin{aligned}\sum_{j=1}^{k}\frac{\bar{\beta}_j^k\mathfrak{L}_j^{p(t)}}{\mathfrak{L}_{j-1}}\xi_{k+1}\lceil z_{j+1}\rfloor^{p(t)} &\le \sum_{j=1}^{k}\left|\frac{\bar{\beta}_j^k\mathfrak{L}_j^{p(t)}}{\mathfrak{L}_{j-1}}\xi_{k+1}\lceil z_{j+1}\rfloor^{p(t)}\right| \\ &\le \sum_{j=1}^{k}\frac{\bar{b}_{k+1}\mathfrak{L}_j^{p(t)}}{\mathfrak{L}_{j-1}}|\xi_{k+1}|\left(|\xi_j|^{p(t)}+|\xi_{j+1}|^{p(t)}\right)\end{aligned}$$

for a constant $\bar{b}_{k+1} \ge 1$. Using Lemma 2 and the fact described in (5), it is not difficult to find that

$$\begin{aligned}\sum_{j=1}^{k}\frac{\bar{\beta}_j^k\mathfrak{L}_j^{p(t)}}{\mathfrak{L}_{j-1}}\xi_{k+1}\left[z_{j+1}\right]^{p(t)} \le \frac{1}{2}\times\sum_{j=1}^{k}\frac{\mathfrak{L}_j^{p(t)}}{\mathfrak{L}_{j-1}}|\xi_j|^{p(t)+1}+\frac{\tilde{b}_{k+1}\mathfrak{L}_k^{p(t)}}{\mathfrak{L}_{k-1}}|\xi_{k+1}|^{p(t)+1} \quad (16)\end{aligned}$$

for a constant $\tilde{b}_{k+1} \ge 1$.

Substituting the above estimates into (13) and using the relation of (5), one has

$$\begin{aligned}\dot{V}_{k+1}(z_1,\ldots,z_{k+1}) \le& -(n-k)\times\sum_{i=1}^{k}\frac{\mathfrak{L}_i^{p(t)}}{\mathfrak{L}_{i-1}}|\xi_i|^{p(t)+1} \\ &+\frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\left(\tilde{c}_{k+1}+\tilde{b}_{k+1}\right)|\xi_{k+1}|^{p(t)+1} \\ &+\frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\xi_{k+1}\lceil z_{k+2}^*\rfloor^{p(t)} \\ &+\frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\xi_{k+1}\left(\lceil z_{k+2}\rfloor^{p(t)}-\lceil z_{k+2}^*\rfloor^{p(t)}\right). \quad (17)\end{aligned}$$

Thus, the virtual control law of the form

$$\begin{aligned} z_{k+2}^* &= -\beta_{k+1}\xi_{k+1} \\ \text{with}\quad \beta_{k+1} &= (\tilde{c}_{k+1}+\tilde{b}_{k+1}+n-k) \ge 1\end{aligned}$$

renders that

$$\begin{aligned}&\dot{V}_{k+1}(z_1,\ldots,z_{k+1}) \\ &\le -(n-k)\times\sum_{i=1}^{k}\frac{\mathfrak{L}_i^{p(t)}}{\mathfrak{L}_{i-1}}|\xi_i|^{p(t)+1} \\ &\quad+\frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\left(\tilde{c}_{k+1}+\tilde{b}_{k+1}\right)|\xi_{k+1}|^{p(t)+1} \\ &\quad+\frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\xi_{k+1}\lceil -\beta_{k+1}\xi_{k+1}\rfloor^{p(t)} \\ &\quad+\frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\xi_{k+1}\left(\lceil z_{k+2}\rfloor^{p(t)}-\lceil z_{k+2}^*\rfloor^{p(t)}\right) \\ &\le -(n-k)\times\sum_{i=1}^{k+1}\frac{\mathfrak{L}_i^{p(t)}}{\mathfrak{L}_{i-1}}|\xi_i|^{p(t)+1} \\ &\quad+\frac{\mathfrak{L}_{k+1}^{p(t)}}{\mathfrak{L}_k}\xi_{k+1}\left(\lceil z_{k+2}\rfloor^{p(t)}-\lceil z_{k+2}^*\rfloor^{p(t)}\right) \quad (18)\end{aligned}$$

in which we have used the fact of $\beta_{k+1}^{p(t)} \ge \beta_{k+1} \ge 1$. As a result, (18) completes the inductive proof and (11) holds at step $k+1$. From **Inductive Step**, one can conclude that (18) holds for all $k = 1,\ldots,n-1$, with a set of virtual control laws (10).

Last Step: At step $k = n$ with $\lceil z_{n+1}\rfloor^{p(t)} := v$, one can find that there exist a virtual control law

$$z_{n+1}^* = -\beta_n\xi_n \quad \text{with } \beta_n \ge 1$$

and a smooth positive definite and proper Lyapunov function

$$V_n(\mathbf{z}) = V_{n-1}(z_1,\ldots,z_{n-1})+\frac{\xi_n^2}{2} \quad (19)$$

such that

$$\begin{aligned}\dot{V}_n(\mathbf{z}) \le& -\sum_{i=1}^{n}\frac{\mathfrak{L}_i^{p(t)}}{\mathfrak{L}_{i-1}}|\xi_i|^{p(t)+1} \\ &+\frac{\mathfrak{L}_n^{p(t)}}{\mathfrak{L}_{n-1}}\xi_n\left(v-\lceil z_{n+1}^*\rfloor^{p(t)}\right) \quad (20)\end{aligned}$$

where $\mathbf{z} = (z_1,\ldots,z_n)^T \in \mathbb{R}^n$. Simply choosing

$$\begin{aligned} v &= z_{n+1}^* \\ &= -\beta_n\xi_n \\ &= -\beta_n\left(z_n+\beta_{n-1}z_{n-1}+\beta_{n-1}\beta_{n-2}z_{n-2}+\cdots\right. \\ &\qquad\left.+\beta_{n-1}\beta_{n-2}\cdots\beta_1 z_1\right) \quad (21)\end{aligned}$$

it is easy to deduce from (6) and (20) that

$$\begin{aligned}\dot{V}_n(\mathbf{z}) &\le -\sum_{i=1}^{n}\frac{\mathfrak{L}_i^{p(t)}}{\mathfrak{L}_{i-1}}|\xi_i|^{p(t)+1} \\ &\le -\mathfrak{L}_1^{\underline{p}}\left(|\xi_1|^{p(t)+1}+|\xi_2|^{p(t)+1}+\cdots+|\xi_n|^{p(t)+1}\right). \quad (22)\end{aligned}$$

Now, applying Lemmas 3 and 4 to the inequality (22) yields

$$\dot{V}_n(\mathbf{z}) \le -\frac{\mathfrak{L}_1^{\underline{p}}}{n^{\bar{p}}}\left(\frac{(|\xi_1|+|\xi_2|+\cdots+|\xi_n|)^{\bar{p}+1}}{1+(|\xi_1|+|\xi_2|+\cdots+|\xi_n|)^{2\bar{p}-1}}\right).$$

It is obvious that the right-hand side of the above inequality is continuous and positive definite (with respect to $\mathbf{x} \in \mathbb{R}^n$). Therefore, one can conclude that the control law (21) is a globally (uniformly) stabilizing control law for the nominal system (8). ■

B. INTERVAL HOMOGENEOUS DOMINATION APPROACH

In this section, we shall show that under Assumptions 1 and 2, the problem of global state feedback stabilization for the system (7) is solvable. Specifically, by fulfilling the interval homogeneous domination, i.e., the process of suitably scaling the gain $\mathfrak{L}_1$, a state feedback control law will be constructed to globally stabilize the perturbed system (7). To see this, we set

$$\begin{aligned}\mathbf{f}(\mathbf{z},t) &= (f_1(\mathbf{z},t), f_2(\mathbf{z},t),\ldots,f_n(\mathbf{z},t))^T \\ &:= \left(\mathfrak{L}_1^{p(t)}\lceil z_2\rfloor^{p(t)}, \frac{\mathfrak{L}_2^{p(t)}}{\mathfrak{L}_1}\lceil z_3\rfloor^{p(t)},\ldots,\frac{\mathfrak{L}_n^{p(t)}}{\mathfrak{L}_{n-1}}v\right)^T\end{aligned}$$

and

$$\begin{aligned}\mathbf{\Phi}(\mathbf{z},t,v) &= (\Phi_1(\mathbf{z},t,v),\Phi_2(\mathbf{z},t,v),\ldots,\Phi_n(\mathbf{z},t,v))^T\\ &:= \left(\frac{\phi_1(\cdot)}{\mathfrak{L}_1^{\underline{p}}},\frac{\phi_2(\cdot)}{\mathfrak{L}_2^{\underline{p}}},\ldots,\frac{\phi_n(\cdot)}{\mathfrak{L}_n^{\underline{p}}}\right)^T.\end{aligned}$$

It is straightforward to verify that the closed-loop system (7) and (21) can be expressed in the following compact form

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z},t) + \mathfrak{L}_1^{\underline{p}}\mathbf{\Phi}(\mathbf{z},t,v). \tag{23}$$

In addition, from Assumption 1, it is not difficult to show that

$$\begin{aligned}\bar{p}\alpha(j-1)-\underline{p}\alpha(i) &= \bar{p}\frac{\sum_{m=0}^{j-2}\underline{p}^m}{\underline{p}^{j-2}} - \underline{p}\frac{\sum_{m=0}^{i-1}\underline{p}^m}{\underline{p}^{i-1}}\\ &\le \frac{\underline{p}^{i-2}\left(\bar{p}-\underline{p}\right)+\left(\bar{p}-\underline{p}\right)\left(\sum_{m=0}^{i-3}\underline{p}^m\right)-1}{\underline{p}^{i-2}}\\ &\le \frac{\left(\bar{p}-\underline{p}\right)\left(\sum_{m=0}^{i-2}\underline{p}^m\right)-1}{\underline{p}^{i-2}}\\ &\le -\frac{\kappa}{\underline{p}^n}\end{aligned} \tag{24}$$

for all $j = 2,\ldots,i$ and $i = 2,\ldots,n$. Before proceeding, we first introduce a proposition whose proof is included in the Appendix.

Proposition 1: For any $j = 2,\ldots,k,k+1$ with $k \in \{1,\ldots,n\}$, there exists a constant $\pi_j \ge 1$, which is independent of $p(t)$, such that

$$|z_1|^{p(t)}+\cdots+|z_j|^{p(t)} \le \pi_j \times \left(|\xi_1|^{p(t)}+\cdots+|\xi_j|^{p(t)}\right).$$

Now, by Assumption 2, Proposition 1, (6), (10) and (24), one can verify that

$$\begin{aligned}\left|\frac{\phi_i(\cdot)}{\mathfrak{L}_i^{\underline{p}}}\right| &\le \gamma\left(\frac{1}{\mathfrak{L}_i^{\underline{p}}}|z_1|^{p(t)}+\frac{1}{\mathfrak{L}_i^{\underline{p}}}\times\sum_{j=2}^{i}|\mathfrak{L}_{j-1}z_j|^{p(t)}\right)\\ &\le \frac{\gamma}{\mathfrak{L}_1^{\frac{\kappa}{\underline{p}^n}}}\left(|z_1|^{p(t)}+\cdots+|z_i|^{p(t)}\right)\\ &\le \frac{\tilde{\gamma}}{\mathfrak{L}_1^{\frac{\kappa}{\underline{p}^n}}}\left(|\xi_1|^{p(t)}+\cdots+|\xi_i|^{p(t)}\right)\end{aligned} \tag{25}$$

for all $i = 1,\ldots,n$ and for a constant $\tilde{\gamma} \ge 0$.

With the aid of (23) and (25), we are now ready to present our main results.

Theorem 2: Suppose that Assumptions 1 and 2 are satisfied. There exists a state feedback control law that renders the system (7) globally (uniformly) asymptotically stable.

Proof: Choose the same Lyapunov function $V_n(\mathbf{z})$ given by (19). One can deduce from (22) and (23) that

$$\begin{aligned}\dot{V}_n(\mathbf{z}) &= \frac{\partial V_n(\mathbf{z})}{\partial \mathbf{z}}\mathbf{f}(\mathbf{z},t)+\mathfrak{L}_1^{\underline{p}}\frac{\partial V_n(\mathbf{z})}{\partial \mathbf{z}}\mathbf{\Phi}(\mathbf{z},t,v)\\ &\le -\mathfrak{L}_1^{\underline{p}}\left(|\xi_1|^{p(t)+1}+\cdots+|\xi_n|^{p(t)+1}\right)\\ &\quad+\mathfrak{L}_1^{\underline{p}}\frac{\partial V_n(\mathbf{z})}{\partial \mathbf{z}}\mathbf{\Phi}(\mathbf{z},t,v).\end{aligned} \tag{26}$$

We further analyze the right-hand side of the inequality (26). By a straightforward calculation, one has

$$\begin{aligned}\left\|\frac{\partial V_n(\mathbf{z})}{\partial \mathbf{z}}\right\| &\le |\xi_1|+\beta_1|\xi_2|+\cdots+(\beta_{n-1}\beta_{n-2}\cdots\beta_1)|\xi_n|\\ &\quad+|\xi_2|+\beta_2|\xi_3|+\cdots+(\beta_{n-1}\beta_{n-2}\cdots\beta_2)|\xi_n|\\ &\quad+\cdots+|\xi_{n-1}|+\beta_{n-1}|\xi_n|+|\xi_n|\\ &\le \sigma\left(|\xi_1|+\cdots+|\xi_n|\right)\end{aligned} \tag{27}$$

for a constant $\sigma > 0$. Besides, it is also easy to deduce from (25) that

$$\begin{aligned}\|\mathbf{\Phi}(\mathbf{z},t,v)\| &\le \left|\frac{\phi_1(\cdot)}{\mathfrak{L}_1^{\underline{p}}}\right|+\left|\frac{\phi_2(\cdot)}{\mathfrak{L}_2^{\underline{p}}}\right|+\cdots+\left|\frac{\phi_n(\cdot)}{\mathfrak{L}_n^{\underline{p}}}\right|\\ &\le \frac{\tilde{\gamma}}{\mathfrak{L}_1^{\frac{\kappa}{\underline{p}^n}}}|\xi_1|^{p(t)}+\frac{\tilde{\gamma}}{\mathfrak{L}_1^{\frac{\kappa}{\underline{p}^n}}}\left(|\xi_1|^{p(t)}+|\xi_2|^{p(t)}\right)+\cdots\\ &\quad+\frac{\tilde{\gamma}}{\mathfrak{L}_1^{\frac{\kappa}{\underline{p}^n}}}\left(|\xi_1|^{p(t)}+|\xi_2|^{p(t)}+\cdots+|\xi_n|^{p(t)}\right)\\ &\le \frac{\bar{\gamma}}{\mathfrak{L}_1^{\frac{\kappa}{\underline{p}^n}}}\left(|\xi_1|^{p(t)}+\cdots+|\xi_n|^{p(t)}\right)\end{aligned} \tag{28}$$

for a constant $\bar{\gamma} \ge 0$. With the facts from (27) and (28) in mind, it follows from (26) and Lemma 2 that

$$\begin{aligned}\dot{V}_n(\mathbf{z}) &\le -\mathfrak{L}_1^{\underline{p}}\left(|\xi_1|^{p(t)+1}+\cdots+|\xi_n|^{p(t)+1}\right)\\ &\quad+\mathfrak{L}_1^{\underline{p}}\left\|\frac{\partial V_n(\mathbf{z})}{\partial \mathbf{z}}\right\|\times\|\mathbf{\Phi}(\mathbf{z},t,v)\|\\ &\le -\mathfrak{L}_1^{\underline{p}}\left(1-\frac{\rho}{\mathfrak{L}_1^{\frac{\kappa}{\underline{p}^n}}}\right)\left(|\xi_1|^{p(t)+1}+\cdots+|\xi_n|^{p(t)+1}\right)\end{aligned} \tag{29}$$

for a constant $\rho \ge 0$ (noting that $\rho > 0$ when $\gamma > 0$). Now, by choosing

$$\mathfrak{L}_1 \ge \max\left\{\left(\frac{\rho}{1-\lambda}\right)^{\frac{\underline{p}^n}{\kappa}},1\right\} \tag{30}$$

with an arbitrary real constant $0 < \lambda < 1$, we have

$$\dot{V}_n(\mathbf{z}) \le -\lambda\mathfrak{L}_1^{\underline{p}}\left(|\xi_1|^{p(t)+1}+|\xi_2|^{p(t)+1}+\cdots+|\xi_n|^{p(t)+1}\right). \tag{31}$$

Using Lemmas 3 and 4, it can be deduced from (31) that

$$\dot{V}_n(\mathbf{z}) \le -\frac{\lambda\mathfrak{L}_1^{\underline{p}}}{n^{\bar{p}}}\left(\frac{(|\xi_1|+|\xi_2|+\cdots+|\xi_n|)^{\bar{p}+1}}{1+(|\xi_1|+|\xi_2|+\cdots+|\xi_n|)^{2\bar{p}-1}}\right)$$

where the right-hand side of the above inequality is continuous and positive definite (with respect to $\mathbf{x} \in \mathbb{R}^n$). On the basis of the derivations given above, one can conclude that the closed-loop system (7) and (21) is globally (uniformly) asymptotically stable if $\mathfrak{L}_1$ and $\mathfrak{L}_i$ for all $i = 2,\ldots,n$ are selected according to the relations (4) and (30). ■

Remark 2: It is worth pointing out that, in the existing results [16]–[26], a common strategy for global stabilization design is to select a Lyapunov function that involves the power; however, such a design is inapplicable to the system (1) since the resultant Lyapunov function including the time-varying power $p(t)$ will produce obstacles preventing us from constructing a global stabilizer. Instead of using Lyapunov functions with the power $p(t)$, the interval homogeneous domination approach developed in this paper guides us to adopt a Lyapunov function, as shown in (19), which is independent of the time-varying power. Besides, by the interval homogeneous domination approach, the tedious process in estimating the nonlinearities during the stabilizing control law design can be avoided skillfully. Compared to [16]–[26], our approach not only avoids the usage of Lyapunov functions with the power $p(t)$ but also provides an innovative way to achieve the global stabilization for the systems with a time-vary power.

Remark 3: It should be also emphasized that the design parameters β_i and $\mathfrak{L}_i$ can be determined by a systematic manner. With the recursively selected β_i and $\mathfrak{L}_i$, the resultant control law will globally stabilize the system (1). Specifically, the gain (parameter) $\mathfrak{L}_1$, which leads to a recursive selection of $\mathfrak{L}_i$ for all $i = 2, \ldots, n$, is intentionally introduced to enlarge the influence of the interval homogeneous parts of the system (1) and reduce the effects of nonlinearities. By suitably selecting the gain $\mathfrak{L}_1 \geq 1$ (i.e., performing interval homogeneous domination), system nonlinearities can be dominated/compensated perfectly by the interval homogeneous parts.

IV. EXTENSION

In this section, we will show that the technique presented previously can be further extended to fulfill the global state feedback stabilization problem for a more general class of nonlinear systems. To be more specific, the presented scheme can be used to cope with the system (1) whose nonlinearities are not necessarily bounded in the sense of Assumption 2; i.e., the upper bounds the nonlinearities need not be triangular functions. To this end, the following assumption is imposed.

Assumption 3: There are two real constants $\gamma \geq 0$ and $0 < \omega < 1$ such that

$$\left|\frac{\phi_i(\mathbf{x}, t, u)}{\mathfrak{L}^{\alpha(i)\underline{p}}}\right| \leq \frac{\gamma}{\mathfrak{L}^{\frac{\omega}{\underline{p}^n}}} \times \sum_{i=1}^{n} |z_i|^{p(t)}$$

for all $x \in \mathbb{R}^n$, $t \in \mathbb{R}^+$, $u \in \mathbb{R}$ and $i = 1, \ldots, n$, where $\mathfrak{L} \geq 1$ is an arbitrary real number, $z_j = x_j/\mathfrak{L}^{\alpha(j-1)}$ for all $j = 1, \ldots, n$, and $\alpha(i)$ is given by (4).

Note that, by the relation (25), one can find that Assumption 2 is a special case of Assumption 3. In addition, since Assumption 3 also includes the information of the power $p(t)$ (i.e., the lower bound $\underline{p}$), Assumption 1 is also unnecessary for control law design. In fact, using only Assumption 3, one can verify that

$$\begin{aligned}
&\|\boldsymbol{\Phi}(\mathbf{z}, t, v)\| \\
&\leq \left|\frac{\phi_1(\cdot)}{\mathfrak{L}_1^{\underline{p}}}\right| + \left|\frac{\phi_2(\cdot)}{\mathfrak{L}_2^{\underline{p}}}\right| + \cdots + \left|\frac{\phi_n(\cdot)}{\mathfrak{L}_n^{\underline{p}}}\right| \\
&\leq \frac{\tau}{\mathfrak{L}_1^{\frac{\omega}{\underline{p}^n}}}|\xi_1|^{p(t)} + \frac{\tau}{\mathfrak{L}_1^{\frac{\omega}{\underline{p}^n}}}\left(|\xi_1|^{p(t)} + \cdots + |\xi_n|^{p(t)}\right) + \cdots \\
&\quad + \frac{\tau}{\mathfrak{L}_1^{\frac{\omega}{\underline{p}^n}}}\left(|\xi_1|^{p(t)} + \cdots + |\xi_n|^{p(t)}\right) \\
&\leq \frac{\tilde{\tau}}{\mathfrak{L}_1^{\frac{\omega}{\underline{p}^n}}}\left(|\xi_1|^{p(t)} + \cdots + |\xi_n|^{p(t)}\right) \qquad (32)
\end{aligned}$$

for a constant $\tilde{\tau} \geq 0$, which is similar to (28) and is crucial for achieving interval homogeneous domination. With this fact in mind, we are ready to present the following theorem claiming a more general result on the problem of global stabilization for the system (1).

Theorem 3: Under Assumption 3, there exists a state feedback control law that renders the system (7) globally (uniformly) asymptotically stable.

Proof: Consider exactly the exactly same Lyapunov function $V_n(\mathbf{z})$ and control law given by (19) and (21). By a fashion similar to the arguments in (27)–(31), it can be deduced from (32) that

$$\dot{V}_n(\mathbf{z}) \leq -\frac{\tilde{\lambda}\mathfrak{L}_1^{\underline{p}}}{n^{\overline{p}}}\left(\frac{(|\xi_1| + |\xi_2| + \cdots + |\xi_n|)^{\overline{p}+1}}{1 + (|\xi_1| + |\xi_2| + \cdots + |\xi_n|)^{2\overline{p}-1}}\right)$$

for a constant $0 < \tilde{\lambda} < 1$. As a result, the control law (21) with $\mathfrak{L}_1$ and $\mathfrak{L}_i$ for all $i = 2, \ldots, n$ being chosen according to (4) and (30) can globally (uniformly) asymptotically stabilize the system (7) ■

V. ILLUSTRATIVE EXAMPLE

Here, we present an example to illustrate how our main results can be applied.

Consider a planar system of the form

$$\begin{aligned}
\dot{x}_1 &= [x_2]^{2+\cos(0.5t)} + x_2^{\frac{1}{3}} \sin(5x_2)[x_1]^{\cos(0.5t)+\frac{5}{3}} \\
\dot{x}_2 &= [u]^{2+\cos(0.5t)}
\end{aligned} \qquad (33)$$

which has the same structure as (1) with $p(t) = 2+\cos(0.5t)$, $\underline{p} = 1$, $\overline{p} = 3$, $\phi_1(\mathbf{x}, t, u) = x_2^{1/3} \sin(5x_2)[x_1]^{\cos(0.5t)+5/3}$ and $\phi_2(\mathbf{x}, t, u) = 0$. It is worth pointing out that the global stabilization problem of the system (33) cannot be solved by the approaches in existing works, such as [16]–[26], because the schemes developed in [16]–[26] are only applicable to the systems with fixed powers. By Lemma 2, one has

$$\left|\frac{\phi_1(\mathbf{x}, t, u)}{\mathfrak{L}_1}\right| \leq \frac{1}{\mathfrak{L}_1^{\frac{2}{3}}}\left(|z_1|^{p(t)} + |z_2|^{p(t)}\right)$$

which implies that Assumption 3 is satisfied with $\gamma = 1$ and $\omega = 2/3$. Following the proofs given previously, the globally (uniformly) stabilizing control law can be constructed as

$$u = -\mathfrak{L}_1^2 \beta_2 \left(\beta_1 x_1 + \frac{1}{\mathfrak{L}_1} x_2 \right) \tag{34}$$

with the gains $\beta_1 \geq 0$, $\beta_2 \geq 0$ and $\mathfrak{L}_1 \geq 1$. The simulation results depicted in Figs. 1 and 2 are conducted for the case when $(x_1(0), x_2(0))^T = (-0.5, 2.5)^T$, $\beta_1 = 2$, $\beta_2 = 3$, and $\mathfrak{L}_1 = 2.8$. The responses of the system (33) with no control (i.e., open-loop system) are also shown in Figs. 3 and 4 with the same initial condition. Clearly, the control law (34) renders the system (33) globally (uniformly) asymptotically stable but the open-loop system of (33) exhibits unstable behavior; this reveals the effectiveness of the proposed scheme.

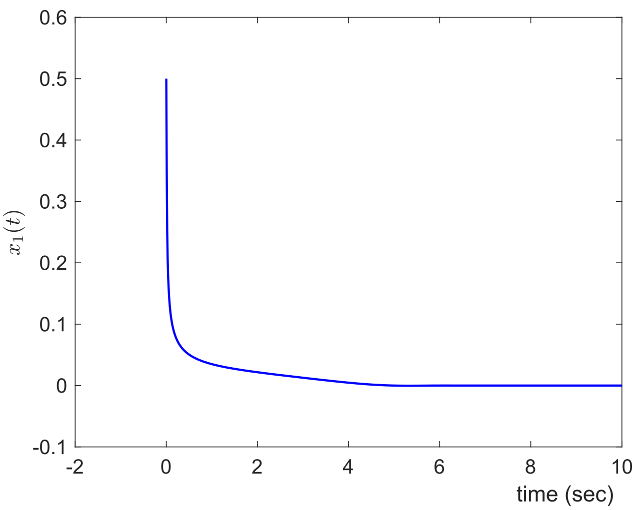

FIGURE 1. Trajectories of $x_1(t)$ of the system (33)–(34) with $(x_1(0), x_2(0)) = (0.5, -2.5)$.

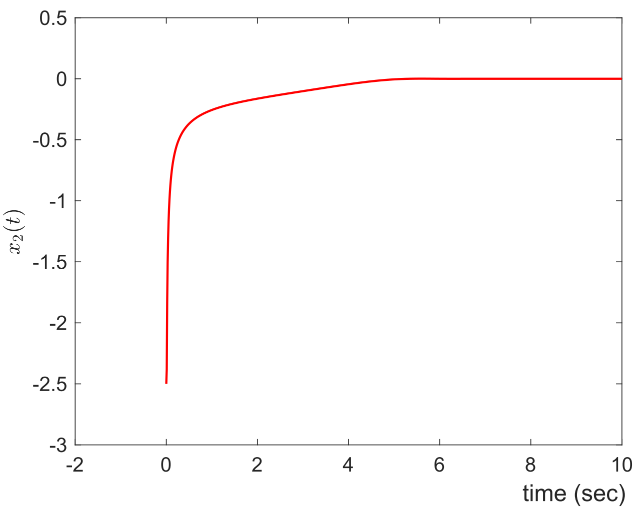

FIGURE 2. Trajectories of $x_2(t)$ of the system (33)–(34) with $(x_1(0), x_2(0)) = (0.5, -2.5)$.

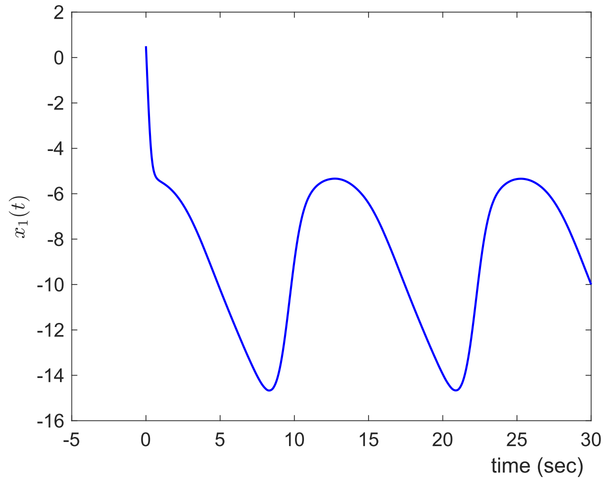

FIGURE 3. Trajectories of $x_1(t)$ of the system (33) with $u = 0$ and $(x_1(0), x_2(0)) = (0.5, -2.5)$.

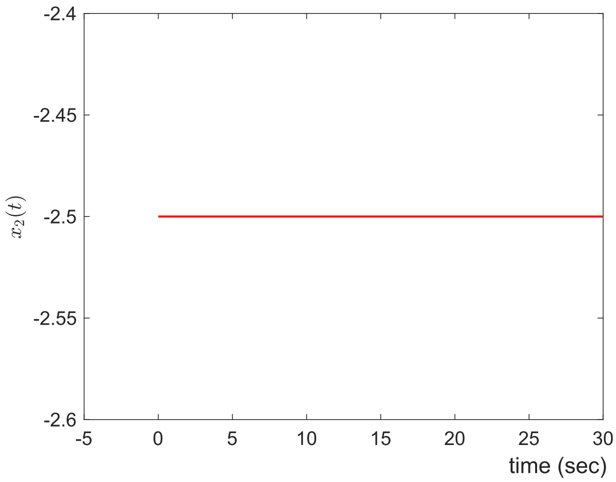

FIGURE 4. Trajectories of $x_2(t)$ of the system (33) with $u = 0$ and $(x_1(0), x_2(0)) = (0.5, -2.5)$.

VI. CONCLUSION

This paper has presented a new systematic scheme for designing a state feedback globally (uniformly) stabilizing control law for a class of nonlinear systems with a time-varying power. The proposed scheme is accomplished by combining the revamped adding a power integrator technique and the newly developed interval homogeneous domination approach. A distinctive feature of the presented technique is the ability to cope with more general systems that have not only the time-varying power but also the nonlinearities bounded by non-triangular functions.

APPENDIX
PROOF OF PROPOSITION 1

According to (10), it is clear that $|z_1|^{p(t)} = |\xi_1|^{p(t)}$. In addition, by Lemma 3 one can deduce that

$$\begin{aligned} |x_i|^{p(t)} &= \left|\xi_i + x_i^*\right|^{p(t)} \\ &= |\xi_i - \xi_{i-1}\beta_{i-1}|^{p(t)} \\ &\leq 2^{\overline{p}-1} |\xi_i|^{p(t)} + 2^{\overline{p}-1} \beta_{i-1}^{\overline{p}} |\xi_{i-1}|^{p(t)} \\ &= \left(2^{\overline{p}-1} + 2^{\overline{p}-1}\beta_{i-1}^{\overline{p}}\right) \cdot \left(|\xi_{i-1}|^{p(t)} + |\xi_i|^{p(t)}\right). \end{aligned} \tag{35}$$

Clearly, $2^{\bar{p}-1} + 2^{\bar{p}-1}\beta_{i-1}^{\bar{p}} \geq 1$. Define

$$\rho_i := 2^{\bar{p}-1} + 2^{\bar{p}-1} \cdot \beta_{i-1} \geq 1 \quad \text{for all } i = 2, \ldots, j. \tag{36}$$

It follows from (35) and (36) that

$$\begin{aligned}
|z_1| &= |\xi_1|^{p(t)} \\
|z_2| &\leq \rho_2 \left(|\xi_1|^{p(t)} + |\xi_2|^{p(t)}\right) \\
|z_3| &\leq \rho_3 \left(|\xi_2|^{p(t)} + |\xi_3|^{p(t)}\right) \\
&\vdots \\
|z_j| &\leq \rho_j \left(\left|\xi_{j-1}\right|^{p(t)} + \left|\xi_j\right|^{p(t)}\right).
\end{aligned}$$

This implies that

$$\begin{aligned}
&|z_1|^{p(t)} + |z_2|^{p(t)} + \cdots + |z_j|^{p(t)} \\
&\quad \leq |\xi_1|^{p(t)} + \rho_2 \left(|\xi_1|^{p(t)} + |\xi_2|^{p(t)}\right) + \cdots \\
&\qquad + \rho_j \left(\left|\xi_{j-1}\right|^{p(t)} + \left|\xi_j\right|^{p(t)}\right) \\
&\quad = \pi_j \left(|\xi_1|^{p(t)} + |\xi_2|^{p(t)} + \cdots + |\xi_j|^{p(t)}\right)
\end{aligned}$$

for a constant $\pi_j \geq 1$. This completes the proof. ■

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