

Received January 9, 2018, accepted February 7, 2018, date of publication February 14, 2018, date of current version March 15, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2806178

# Distributed Resource Scheduling Based on Potential Game in Dense Cellular Network

ZHIQIANG QI<sup>1</sup> , (Student Member, IEEE), TAO PENG, (Member, IEEE), AND WENBO WANG, (Senior Member, IEEE)

<sup>1</sup>Wireless Signal Processing and Network Laboratory, Beijing 100876, China

<sup>2</sup>Key Laboratory of Universal Wireless Communication, Ministry of Education, Beijing 100876, China

<sup>3</sup>Beijing University of Posts and Telecommunications, Beijing 100876, China

Corresponding author: Zhiqiang Qi (zhiqiangqi.bupt@gmail.com)

This work was supported in part by the National Science and Technology Major Project of China under Grant 2016ZX03001017 and in part by the National Natural Science Foundation of China under Grant 61571054 and Grant 61631004.

**ABSTRACT** Based on a potential game model, a distributed resource scheduling scheme is proposed to coordinate severe interference in a dense cellular network. First, the subcarrier and power scheduling problem is modeled as a non-cooperative potential game. Then, individual utility and overall potential functions are proposed by considering attainable throughput and consumed power. According to the definition of the potential game, the difference in individual utilities caused by strategy changing has the same value as the difference in overall potential value, which guarantees that players can maximize respective utility without worsening system profit. Afterward, an iterative searching algorithm is designed to obtain optimal resource scheduling strategy. Theoretical analysis and simulation results both validate that the proposed scheduling scheme makes a performance improvement in spectral and resource efficiency. Meanwhile, the proposed scheme can also guarantee the quality of service by providing superior user signal to interference plus noise ratio.

**INDEX TERMS** Dense Deployment, distributed, interference coordination, nash equilibrium, potential game.

## I. INTRODUCTION

With the development of mobile internet and Internet of Things, a large amount of emerging services are provided to the users via wireless transmission, which means higher demanding on user rate and traffic density in mobile communication system. As an effective scheme to deploy huge amounts of communication nodes in the limited area, dense deployment can satisfy the increasing requirements of users' data services and provide powerful support for the blooming development of mobile communication network. Home eNodeB is a kind of micro-base station (BS) applied in small-coverage scenarios. Due to the low cost and self-organizational feature [1], Home eNodeB can provide high data rate and becomes an efficient component unit in the dense deployment of small cell network.

For dense cellular network, dynamism is one of the most salient features and exists during equipment on-off and user mobility [2]. As the state of the network changes all the time, it will cause severe fluctuations of channel condition and bring more challenges on interference management. It is already known to all that interference management has

always been a key problem in mobile network due to its influence on the service quality, coverage ability and system capacity of communication network. While dense deployment is introduced in future network, the system architecture and the user requirements become more complicated, which highlights the interference control problem and challenges the traditional interference management mechanism. Hence the existing interference mitigation technology is facing multiple difficulties. Moreover, managing interference with traditional centralized scheme in dense cellular network will generate excessive signaling overhead and time delay. As a result, it is in desperate need of distributed and effective interference coordination for dense cellular network.

With the growing needs for pervasive computing and communication, new analytical tools should be introduced to tackle the numerous technical challenges accompanying current and future wireless communication networks. As a branch of applied mathematics, game theory has been used not only in economics, but has also in a variety of other fields such as political science, biology, computer science and philosophy [3]. Due to the advantages in making cooperative

decisions and meeting competitive users' requirements, game theory is proposed as a key tool to coordinate interference for future wireless communication networks especially in dense deployment.

### A. RELATED WORK

In recent years, there are plenty of literatures concentrating on the resource allocation and interference management problems in wireless network. In the future communication system, how to allocate limited wireless resources is always a hot spot issue that researchers need to investigate. Reference [4] studies cluster-based wireless resource management issues in ultra-dense network and proposes a two-stage scheme to solve this problem. For interference mitigation in small cell network, Wang *et al.* [5] provides an effective subcarrier selection scheme, while Eraslan *et al.* [6] uses a novel auction-based algorithm for throughput maximizing scheduling in centralized cognitive radio network.

As a huge number of small cells exist in dense cellular network, distributed scheme is an efficient tool to allocate limited wireless resources. Application of a Bayesian game is discussed in [7] to solve the distributed resource allocation problem in wireless network under uncertainty. Semasinghe *et al.* [8] proposes an evolutionary game-based distributed resource allocation scheme and can assure fairness among users. References [9] and [10] both focus on distributed interference mitigation in LTE-Advanced network based on background interference matrix.

In dense cellular network, coloring algorithm [11], [12] is a frequently-used tool to cope with inter-cell interference. In addition to coloring algorithm, plenty of literatures focus on game theory to work out competitive resource distribution issues. Hew and White [13], Li and Han [14], and Fan and Tian [15] propose game-based models to deal with resource allocation in both cooperative and non-cooperative way. Specifically, potential game is good at solving distributed problems due to its non-decreasing feature of system performance while increasing individual utility, which will be applied in this paper. References [17] and [18] consider potential games for resource allocation in uplink and downlink system respectively. Based on potential game, a no-regret learning algorithm is designed in [19] for channel adaptation in a dynamic environment, which provides a reference for solving fully distributed channel selection problems. In addition to potential game, there are many other game models that are widely used to derive distributed resource allocation techniques. For example, stackelberg game [20], [21] and evolutionary game [22] have both been extensively practiced.

In dense and time-varying wireless system, throughput is not the only performance that should be focused on, energy efficiency is also a key indicator to assess the earning generated by consumed power [23]. References [24] and [25] both consider throughput and energy efficiency in ultra-dense network and achieve balance between them.

To realize distributed power-control, Zappone *et al.* [26], Yang *et al.* [27], and Zheng *et al.* [28] model the power allocation problem as game-based scheduling issue and propose energy-efficient schemes. Reference [29] introduces a self-organization rule based on minimizing cell transmit power and coordinated resource allocation algorithms are investigated.

### B. MAIN CONTRIBUTION

In this paper, resource scheduling in dense cellular network is investigated. Based on the dense deployment of small cells, the resource scheduling problem of the network is modeled as a potential game, where BSs are competitive players. In the network, BS locates in the center of each cell and the users are uniformly distributed under the coverage of BSs. The only interference for the user is coming from neighboring BS. For every BS, an individual utility function is proposed by considering attainable throughput and consumed power. Moreover a potential function is proposed to represent the system overall earning which is consisting of individual utilities. The two functions are one of the major contributions of this paper. The scheduling purpose of each BS is maximizing individual utility, and the overall potential value will increase accordingly. It is worth noting that all BSs are working in a distributed way and each BS is executing scheduling based on limited interference information, while cooperation among BSs is neglected.

Then a subcarrier and power scheduling scheme is proposed. In this algorithm, each BS calculates utilities for every subcarrier and decides if this subcarrier is worth using. For each subcarrier, BS will select a most probable transmit power level that maximizes individual utility. After stop criteria are satisfied, the proposed scheme will converge accordingly and current resource allocation method is the optimal scheduling strategy.

Finally, a grid-based simulation scenario is introduced to verify theoretical analysis. By selecting proper parameters, the simulation results prove that the proposed scheduling scheme makes improvements in spectral efficiency, power efficiency and user signal to interference plus noise ratio (SINR).

### C. PAPER ORGANIZATION

The main contributions of this paper can be summarized as follows. In section II, system model are described and the resource scheduling problem is modeled as a potential game. Then a distributed resource scheduling scheme based on potential game is proposed and analyzed in section III. In section IV, numerical results are presented to verify theoretical study. Section V concludes the whole paper.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. SYSTEM MODEL

In this paper, a downlink cellular network is considered. As shown in Fig.1, a basic application scenario of dense cellular network is presented.

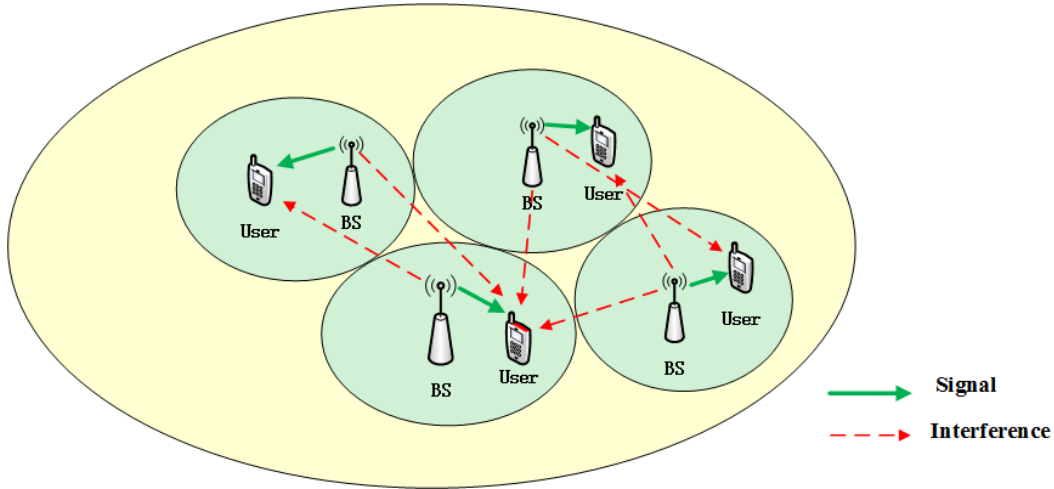


FIGURE 1. Architecture of dense cellular network.

Specifically, there are  $M$  BSs, denoted by  $\mathcal{M} = \{1, 2, \dots, m, \dots, M\}$ .  $P_{max}$  is the maximum transmit power for all BSs. In each small cell, one user is uniformly distributed under the coverage of each BS, interfered by neighboring BSs sharing same frequency bandwidth. As the single subcarrier scheduling among users of the same cell can be solved by time division multiplexing, multiple users attaching to one cell is not in the scope of this paper. Without loss of generality, it is assumed that the bandwidth per subcarrier is total bandwidth  $B$  divided by  $N$ , where  $N$  is the number of available subcarriers. The index set of all subcarriers is denoted by  $\mathcal{N} = \{1, 2, \dots, n, \dots, N\}$ . Let  $N_0$  represents the power spectral density of additive Gaussian noise. For simplicity, all subcarriers experience the same propagation conditions and only path loss is considered. To highlight the influence of inter-cell interference, the penetration loss of the walls between BSs is ignored.

Based on the above model, the instant SINR of user  $i$  of BS  $j$  could be expressed as Eq.(1):

$$SINR_{i,j} = \frac{P_j H_{j,i}}{N_0 B + \sum_{\substack{m=1 \\ m \neq j}}^M P_m H_{m,i}} \quad (1)$$

where  $P_j$  is the transmit power of BS  $j$  and  $H_{j,i}$  is the channel gain between BS  $j$  and user  $i$ . Similarly,  $P_m$  is the transmit power of interfering BS  $m$  and  $H_{m,i}$  is the channel gain between interfering BS  $m$  and user  $i$ .

For throughput model, Shannon capacity is used to represent the attainable throughput of a single user.

To be noticed, all subcarriers are divided into two categories: Primary Component Carrier (PCC) and Supplementary Component Carrier (SCC). PCC is used to guarantee basic service, transmitted with maximum power. Each BS can have only one PCC. While there is extra data requirement, BS could apply to SCC without limitation of number, only not to generate too much interference to

neighboring BSs, which means SCC may use lower transmit power than PCC.

To record the subcarriers allocation status of the network, Radio Resource Allocation Table (RRAT) is used to store the usage situation of subcarriers for each BS, which is a  $N \times M$  matrix, where element  $RRAT_{i,j} = 1$  means subcarrier  $i$  is occupied by BS  $j$ .

Due to the different power settings for PCC and SCC, a  $N \times M$  matrix called Component Carrier Power Allocation Table (CCPAT) is introduced to record transmit power for all BSs on each subcarriers [9], [10], where element  $CCPAT_{i,j}$  represents the transmit power of BS  $j$  for subcarrier  $i$ .

**B. PROBLEM FORMULATION**

Managing interference in centralized way is the most common and effective method to solve interference mitigation issue in traditional cellular network. But the centralized scheme has to pay unbearable overhead during feedback of interference information in order to perform global optimization. Meanwhile, excessive time delay is introduced in the message interaction. These defects will worsen system performance while dealing with highly dynamic interference especially in dense cellular network. To tackle these issues, the distributed scheme is proposed and focuses on solving problem with insufficient message interaction. However, the distributed scheme is not always a better solution than centralized scheme in all scenarios. Selecting proper interference management schemes according to application scenarios always provides optimal performances.

In dense cellular network, numerous communication nodes are competing for the same frequency resource and the inter-cell interference condition is extremely complicated. An effective scheduling scheme should make tradeoff among resource competitors, hence game theory is considered as a modeling tool due to its advantages in handling utility optimization among numerous players. Specifically, a non-cooperative game can reflect competitive situation where

each player needs to make its decision independently of the other players, which will be applied in the proposed model to reduce the feedback overhead. The term non-cooperative does not always imply that the players do not cooperate, but means any cooperation must be self-enforcing with no communication or coordination of strategic choices among the players.

In numerous game models, potential game is one of the most efficient game-theoretic models applied in wireless network [2]. In game theory, a game is said to be potential game if the incentive of all players to change their strategies can be expressed by using a single global function called the potential function.

As a typical potential game, the definition of exact potential game is presented as follows [3]:

$$\Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i}) = u(s_i, s_{-i}) - u(s'_i, s_{-i}) \quad (2)$$

where  $s_i$  is the strategy of player  $i$ , where  $s_{-i}$  is the strategy vectors of all players except  $i$ .  $\Phi(s_i, s_{-i})$  and  $u(s_i, s_{-i})$  are potential and utility functions respectively. The potential function represents the overall profit of the system, while the utility function denotes individual gains. It can be observed that in exact potential games, the difference in individual utilities achieved by each player when changing unilaterally its strategy has the same value as the difference in the overall potential value.

Consequently, the resource scheduling in dense cellular network is modeled as a non-cooperative exact potential game as shown in Eq.(2), where each BS is an independent player. As subcarriers are classified as PCC and SCC, the power strategy vector of each BS is express as  $\mathbf{P}_i = (P_{i,1}, P_{i,2}, \dots, P_{i,n}, \dots, P_{i,N})$ , where  $P_{i,n} \in [0, P_{\max}]$  denote the transmit power on subcarrier  $n$  for BS  $i$ . For all BSs in the network, the scheduling strategy is  $\mathbf{P} = (\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_m, \dots, \mathbf{P}_M) \in \mathcal{P}$ , where  $\mathcal{P}$  is the space of transmit power profile for all BSs.

As non-cooperative potential game has three components: the set of players, their strategies, and the payoffs or utilities [30], the proposed game model is denoted by  $\mathcal{G}_{M,N} = \{\mathcal{M}, \mathbf{P} = \{\mathbf{P}_i\}_{i \in \mathcal{M}}, \{U_i\}_{i \in \mathcal{M}}\}$ , where  $\mathcal{M}$  is the set of BSs,  $\mathbf{P}_i$  is the power strategy vector for BS  $i$  and  $U_i$  is the utility function.

As a result, the optimal resource allocation problem for BS  $i$  on subcarrier  $j$  can be formalized as:

$$P_{i,n}^* = \arg \max_{P_{i,n} \in [0, P_{\max}]} U_{i,n} \quad (3)$$

where  $U_{i,n}$  is utility function for BS  $i$  on subcarrier  $j$ .

From the system point of view, the optimal resource allocation problem can be formalized as:

$$\mathbf{P}^* = \arg \max_{\mathbf{P} \in \mathcal{P}} (\{U_i\}_{i \in \mathcal{M}}, \Phi(\mathbf{P})) \quad (4)$$

Based on the game definition, acquiring optimal scheduling strategy means finding an equilibrium point of the game, which is Nash Equilibrium (NE) where no player will benefit any gains by unilaterally changing its allocation strategy.

However, it is difficult to build proper functions which satisfies condition in Eq.(2) and achieves considerable system performances at the same time. Therefore constructing utility function and finding corresponding NE point to solve the resource allocation problem will be discussed in next section.

### III. GAME-THEORETIC SCHEDULING SCHEME FOR RESOURCE OPTIMIZATION

Based on the proposed game model, corresponding utility and potential functions are presented in this section. Meanwhile, the existence and uniqueness of NE for the proposed model are proved. Then a potential game-based resource scheduling algorithm is introduced. Finally the convergence and computational complexity analysis of the proposed algorithm is given.

#### A. GAME FORMULATION AND NE ANALYSIS

Normally, individual utility function in game theory is normally defined as the received payoff minus the cost for using specific resources [2]. In this paper, the received payoff on subcarrier  $n$  for BS  $i$  is defined as the attainable maximum throughput while there is no interference, which is formulated as follows:

$$r_n(s_i, s_{-i}) = B_n \log_2 \left( 1 + \frac{P_{i,n}}{N_0 B_n} \right) \quad (5)$$

where  $B_n$  is the bandwidth of subcarrier  $n$  and  $P_{i,n}$  is the transmit power of BS  $i$  on subcarrier  $n$ .

The cost of BS for using specific resource is defined as average throughput decrease caused by and imposing to interfering BSs. Specifically, the estimated throughput decrease of BS  $i$  caused by incoming interfering BS  $j$  on subcarrier  $n$  is defined as follows:

$$C_{i,j,n} = B_n \log_2 \left( 1 + \frac{P_{\max}}{N_0 B_n} \right) - B_n \log_2 \left( 1 + \frac{P_{\max}}{N_0 B_n + P_{j,n} H_{j,i,n}} \right) \quad (6)$$

where  $P_{j,n}$  is the transmit power of interfering BS  $j$  and  $H_{j,i,n}$  is the channel gain from BS  $j$  to BS  $i$  on subcarrier  $n$ . To highlight the influence from BS  $j$ , the throughput decrease is defined as the maximum attainable throughput with no interference minus the throughput that considering BS  $j$  as the only interference source. To be noticed, the transmit power of BS  $i$  in Eq.(6) is  $P_{\max}$  instead of  $P_{i,n}$  for the reason of building valid potential game. Moreover, the purpose of Eq.(6) is estimating influence caused by interfering BS  $j$ , hence excluding  $P_{i,n}$  will not cause too much impact. Similarly, the estimated throughput decrease caused from BS  $i$  to BS  $j$  on subcarrier  $n$  is defined as:

$$C_{j,i,n} = B_n \log_2 \left( 1 + \frac{P_{\max}}{N_0 B_n} \right) - B_n \log_2 \left( 1 + \frac{P_{\max}}{N_0 B_n + P_{i,n} H_{i,j,n}} \right) \quad (7)$$

where  $H_{i,j,n}$  is the channel gain from BS  $i$  to BS  $j$  on subcarrier  $n$ . For the same reason, the transmit power of BS  $j$  in Eq.(7) is  $P_{\max}$  instead of  $P_{j,n}$ .



Thus the utility of each BS  $i$  on subcarrier  $n$  can be defined as follows:

$$u(s_i, s_{-i}, n) = B_n \log_2 \left( 1 + \frac{P_{i,n}}{N_0 B_n} \right) - \frac{\varepsilon \sum_{\substack{j \in \mathcal{M} \\ j \neq i}} (C_{i,j,n} + C_{j,i,n})}{M - 1} \quad (8)$$

where  $\varepsilon$  is weighted interference factor for balancing throughput and interference items. In Eq.(8), the first item is estimated throughput BS  $i$  can achieve while there is no interference. As there are  $M - 1$  interfering BSs for each BS, the last two items denote the average throughput decrement caused by and imposing to interfering BSs, which explains the  $M - 1$  in Eq.(8).

Naturally, the individual utility function of BS  $i$  for all subcarriers is formulated as follows:

$$u(s_i, s_{-i}) = \sum_{n \in \mathcal{N}} u(s_i, s_{-i}, n) \quad (9)$$

Based on the utility function, the overall potential function of the system is defined by summing up the weighted utility functions for all BSs:

$$\Phi(\mathbf{P}) = \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{M}} \left\{ \begin{array}{l} B_n \log_2 \left( 1 + \frac{P_{i,n}}{N_0 B_n} \right) \\ - \varepsilon \frac{b}{M - 1} \sum_{j \in \mathcal{M}, j \neq i} C_{i,j,n} \\ - \varepsilon \frac{1 - b}{M - 1} \sum_{j \in \mathcal{M}, j \neq i} C_{j,i,n} \end{array} \right\} \quad (10)$$

where  $b$  is an adjustment parameter for the constructing exact potential game. Parameter  $b$  is introduced in potential function to adjust proportion between different costs. In addition, introducing  $b$  is useful for the demonstration of constructing an exact potential game, detailed information can be found in Appendix A. To balance different cost items in the potential function,  $b$  is set to 0.5.

Firstly it will be proved that the proposed game is an exact potential game, which means the equation in Eq.(2) holds. Appendix A introduces the detailed process of proof.

Then the existence and uniqueness of NE for the proposed potential game will be proved. For potential game, an important corollary should be presented as followed [3], since it can be used to judge if a potential game has a pure strategy NE:

*Corollary 1:* Every finite potential game has at least one pure strategy NE.

According to Corollary 1, the proposed potential game model should have at least one pure strategy NE. Moreover, based on the definition of potential game, every NE is the maximizer of potential function. Therefore the best NE will definitely generate optimal potential value. In addition, if the strategy set  $\mathbf{P}$  of the proposed game model is compact and convex, meanwhile the potential function is a continuously differentiable function on the interior of  $\mathbf{P}$  and strictly concave on  $\mathbf{P}$ , then the NE is unique. First of all, it is easy to find that the strategy set  $\mathbf{P}$  of the proposed game model

is compact and convex. After investigating the concavity of the proposed potential function in Eq.(10), it is proved that the second derivative of the proposed potential function is constantly negative. The details of demonstration are given in Appendix B.

As a result, the proposed game model is an exact potential game and an unique NE can be obtained, which means an optimal strategy exists. How to find this optimal strategy will be discussed in the following section.

### B. DISTRIBUTED ITERATIVE ALGORITHM FOR OPTIMAL RESOURCE SCHEDULING

In order to obtain corresponding NE, it is required to develop an efficient scheduling algorithm. According to  $\gamma$ -logit based decentralized algorithm mentioned in [31], a distributed iterative algorithm is proposed to find the unique pure NE.

In this algorithm, all BSs are working in a distributed way, and only one BS is allowed to perform resource scheduling in a single iteration. Each BS is using  $K \geq 2$  power levels as  $\{\lambda_1 P_{\max}, \lambda_2 P_{\max}, \dots, \lambda_K P_{\max}\}$ , where  $0 = \lambda_1 < \lambda_2 < \dots < \lambda_K = 1$ . During each iteration, current BS executes a two-dimensional resource assessment for both subcarrier and transmit power perspective. At the end of each iteration, two stop criteria are checked: *i*) iteration times reaches a predefined maximum number, *ii*) the total system potential value remains the same for a certain duration or varies in a interval which is short enough to be treated as convergence.

Let  $p_{i,j}^*$  be the optimal transmit power of BS  $i$  on subcarrier  $j$ , and  $\mathbf{P}^* = \{p_{i,j}^*\}_{i \in \mathcal{M}, j \in \mathcal{N}} \in \mathcal{P}$  is the optimal transmit power strategy vector. The detailed procedure is described in Algorithm 1.

While BS  $m$  is updating power level for subcarrier  $n$ , BS  $m$  will transmit with most probable power level after calculation. If it is 0, BS  $m$  will not occupy subcarrier  $n$ . To be noticed, the transmit power level is introduced to quantify the transmit power of BS and simplify the resource scheduling procedure in Algorithm 1. Therefore it will not influence the continuity of power strategy  $\mathbf{P}$ .

### C. CONVERGENCE AND COMPUTATIONAL COMPLEXITY ANALYSIS

For power strategies in all iterations, they can be seen as a discrete time Markov process, which is irreducible and aperiodic. Obviously it has an unique stationary distribution. In this section, we will prove that Algorithm 1 converges to an unique stationary distribution  $\pi(\mathbf{P})$ .

*Theorem 1:* In the proposed potential game model in which all players adhere to Algorithm 1, the unique stationary distribution of the joint power and subcarrier strategy profiles is given as:

$$\pi(\mathbf{P}) = \frac{\exp\{\alpha \Phi(\mathbf{P})\}}{\sum_{\mathbf{P}' \in \mathcal{P}} \exp\{\alpha \Phi(\mathbf{P}')\}} \quad (12)$$

where  $\mathcal{P}$  is the space of player's strategy profile  $\mathbf{P}$ , which includes transmit power and subcarrier allocation

**Algorithm 1** Distributed Iterative Algorithm for Optimal Resource Allocation

- 1: **Initialization:** Each BS selects an initial transmit power randomly, then reset RRAT and CCPAT.
- 2: **for**  $t = 0, 1, 2, \dots$  **do**
- 3:   BS  $m$  is executing scheduling in iteration  $t$ :
- 4:   **for**  $n = 0, 1, 2, \dots, N$  **do**
- 5:     Based on latest RRAT and CCPAT, BS  $m$  calculates its utility  $U_{m,n}(t) = u^t(s_m, s_{-m}, n)$  on subcarrier  $n$  with every transmit power level, namely:  $\{U_{m,n,1}(t), U_{m,n,2}(t), \dots, U_{m,n,K}(t)\}$ .
- 6:     Then BS  $m$  updates its transmit power level on subcarrier  $n$  according to the following rule:
 
$$P_r(p_{m,n}(t+1) = p_{k^*}) = \frac{\exp\{\alpha U_{m,n,k^*}(t)\}}{\sum_{k \in [1,K], k \neq k^*} \exp\{\alpha U_{m,n,k}(t)\}}, \quad (11)$$
- 7:     Finally, BS  $m$  updates RRAT and CCPAT according to the result of power level selection.
- 8:     **if** Stop Criteria are Reached **then**
- 9:       break;(Convergence is achieved.)
- 10:    **end if**
- 11:   **end for**
- 12: **end for**
- 13: **Output:** System throughput of the proposed game  $\mathcal{G}_{M,N}$  and the optimal resource scheduling strategy.

vectors, and  $\Phi()$  is the potential function given in Eq.(10).

*Proof:* As  $\mathbf{P}$  denotes power allocation strategies for all sub-carriers, it can represent both power and subcarrier allocation status of the network. Firstly,  $P_x, P_y \in \mathcal{P}$  are two arbitrary transmit power allocation states of the network and  $Pr(P_y|P_x)$  is the transition probability from  $P_x$  to  $P_y$ . In order to prove Theorem 1, Eq.(13) should be proved in advance:

$$\pi(P_x)Pr(P_y|P_x) = \pi(P_y)Pr(P_x|P_y) \quad (13)$$

As power strategy  $\mathbf{P}$  is a  $M \times N$  matrix, it can also be expressed as an array whose length is  $M \times N$ . Denote  $P_x = \{p_1^x, p_2^x, \dots, p_{M \times N}^x\}$  and  $P_y = \{p_1^y, p_2^y, \dots, p_{M \times N}^y\}$ . If  $P_x = P_y$ , the equation in Eq.(13) is clearly satisfied. If not, there will be only one different element between state  $P_x$  and  $P_y$  since there is only one subcarrier changing its scheduling state between two successive iterations. Without loss of generality, supposing that the  $i$ th element from state  $P_x$  and  $P_y$  are different from each other, which means:  $p_{i^*}^x = p_{i^*}^y, \forall i^* \neq i$ . Then the left side of Eq.(13) can be written as:

$$\pi(P_x)Pr(P_y|P_x) = \frac{\exp\{\alpha \Phi(P_x)\}}{\sum_{\mathbf{P} \in \mathcal{P}} \exp\{\alpha \Phi(\mathbf{P})\}} \frac{1}{M \cdot K} \frac{\exp\{\alpha U_i(p_i^y)\}}{\sum_{\bar{p}_i^y \in \mathcal{K}} \exp\{\alpha U_i(\bar{p}_i^y)\}} \quad (14)$$

Similarly, the right side of Eq.(13) can be expressed as:

$$\pi(P_y)Pr(P_x|P_y) = \frac{\exp\{\alpha \Phi(P_y)\}}{\sum_{\mathbf{P} \in \mathcal{P}} \exp\{\alpha \Phi(\mathbf{P})\}} \frac{1}{M \cdot K} \frac{\exp\{\alpha U_i(p_i^x)\}}{\sum_{\bar{p}_i^x \in \mathcal{K}} \exp\{\alpha U_i(\bar{p}_i^x)\}} \quad (15)$$

As there is only one different element between state  $P_x$  and  $P_y$ , it is easy to conclude that:  $\bar{p}_i^x = \bar{p}_i^y, \forall \bar{p}_i^x, \bar{p}_i^y \in \mathcal{K}$ . Define  $\lambda$  as shown in Eq.(17), Eq.(14) can be rewritten as Eq.(17).

$$\lambda = \frac{1}{M \cdot K \cdot \sum_{\mathbf{P} \in \mathcal{P}} \exp\{\alpha \Phi(\mathbf{P})\} \cdot \sum_{\bar{p}_i^x \in \mathcal{K}} \exp\{\alpha U_i(\bar{p}_i^x)\}} \quad (16)$$

$$\pi(P_x)Pr(P_y|P_x) = \lambda \exp\{\alpha \Phi(P_x) + \alpha U_i(p_i^y)\} \quad (17)$$

Due to symmetry, Eq.(15) can be rewritten as Eq.(18).

$$\pi(P_y)Pr(P_x|P_y) = \lambda \exp\{\alpha \Phi(P_y) + \alpha U_i(p_i^x)\} \quad (18)$$

Based on Eq.(2), following equation can be obtained:

$$\Phi(P_y) - \Phi(P_x) = U_i(p_i^y) - U_i(p_i^x) \quad (19)$$

Subtract Eq.(17) from Eq.(18), it can be concluded that the equation in Eq.(13) is true. From the above analysis, the following equation can be obtained:

$$\begin{aligned} & \sum_{P_x \in \mathcal{P}} \pi(P_x)Pr(P_y|P_x) \\ &= \sum_{P_x \in \mathcal{P}} \pi(P_y)Pr(P_x|P_y) \\ &= \pi(P_y) \sum_{P_x \in \mathcal{P}} Pr(P_x|P_y) \\ &= \pi(P_y) \end{aligned} \quad (20)$$

Eq.(20) is the balanced stationary equation of the Markov process and Theorem 1 is proved.

For the computational complexity, it is assumed that each BS is executing resource scheduling iteration once. During each iteration, BS should go through all  $N$  subcarriers and calculates corresponding utility to decide if this subcarrier is worthy of using. For each subcarrier, BS should select the most probable transmit power level from  $K$  candidates based on respective utility function. Considering that utility calculation should include interference coming from  $M - 1$  neighboring BS, the computational complexity of Algorithm 1 is  $O(M^2 \cdot N \cdot K)$ .

**IV. SIMULATION RESULTS**

This section presents simulation results of the proposed resource scheduling scheme in predefined network architecture, in which a grid-based system limited in square area is considered. Then performance comparison is made among several resource scheduling schemes.

TABLE 1. Simulation parameters.

System Level Parameter	Value
Area	100m × 100m
Path Loss Model	$37 + 30 \log(d)$
Transmit Power of BS	20 dbm
Minimum distance from BS to User	1m
User Distribution	Uniform Distribution
Noise Power Spectrum Density	-204 dB/Hz
System Bandwidth	100MHz

A. SIMULATION SCENARIO

The simulation scenario is limited in a 100m × 100m square zone, where  $M$  BSs are distributed in  $M$  square grids and each BS locates in the center of each grid. Every test result is an average value based on multiple Monte Carlo trials and it is collected from a sequence of snapshots that evolves over time. Each snapshot has a mobile user distribution, which is different from the previous snapshot. After each snapshot, all users will randomly move to a new position without considering moving speed and direction. Assuming that the users will not move out of the coverage of current BS, handover to another cell is not considered. For each snapshot, it is assumed that all users in the system are active. Main simulation parameters are listed in TABLE 1.

B. RESULTS ANALYSIS

In order to provide convincing conclusion, several resource allocation algorithms are presented to compare with the proposed scheme on throughput, power efficiency and other measurements. The first one is an utility-based resource allocation scheme from [32]. For every available subcarrier, two BSs are selected based on strongest interference caused by and imposing to current BS. Then utility and potential function are calculated according to the estimation of selected BSs. Whether the subcarrier is occupied or not depends on the difference between utility and potential function. In the rest of this paper, it is referred as ACCS(Autonomous Component Carrier Selection). The second algorithm is based on potential game [33]. In [33], the author modeled distributed resource allocation problem in heterogeneous network by means of potential game.

1) POTENTIAL FUNCTION AND SYSTEM THROUGHPUT

For exact potential game, the difference in the individual utilities achieved by each player when changing its strategy unilaterally has the same value as the difference in the overall potential value. Thus the overall potential value will keep increasing as long as the individual utility of a single BS grows when updating its scheduling strategy. The trend of potential function and system throughput varying along with the scheduling procedure for the proposed algorithm is presented in Fig.2. The simulation is performed in 100 grids located in 100m × 100m square area. It is obvious that the potential function value increases monotonously with the iteration times. When the algorithm converges, the resource scheduling procedure ends and the overall potential value

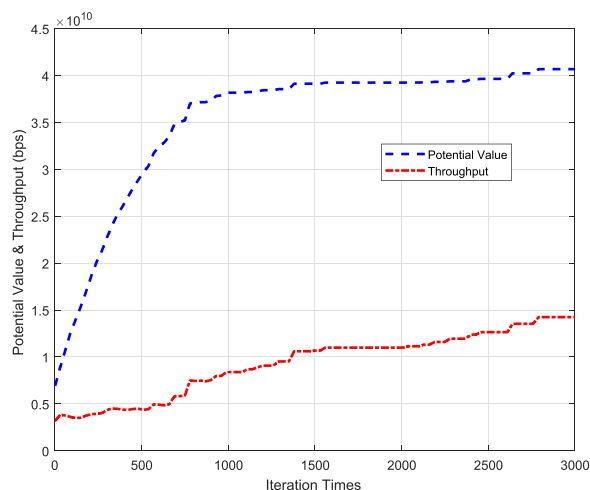


FIGURE 2. Potential value and system throughput of the proposed algorithm vs iterations times.

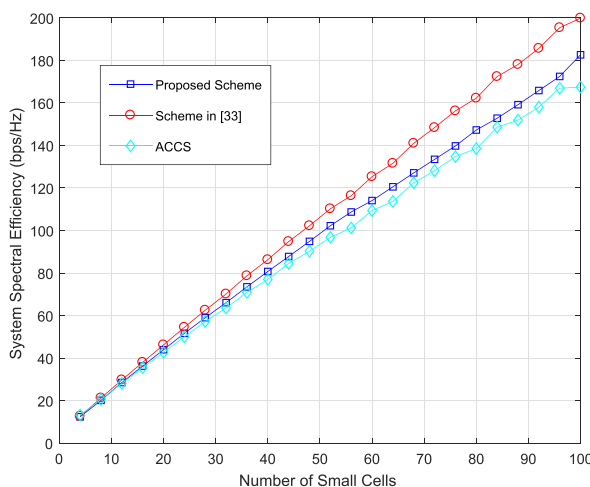


FIGURE 3. System throughput of different algorithms vs number of BSs.

remains constant. For the system throughput, the overall trend is growing in most cases. However, in certain sections, the system throughput trend appears to decrease slightly. That's because although potential function shows undiminished feature and reflects system throughput to some extent, there is still deviation between the potential function value and actual system throughput. Hence, sometimes system throughput may decrease while the potential function value increases. But it will not influence the general trend. To clearly illustrate the detailed process of convergence, it can be observed that the maximum convergent iterations is almost 3000. Actually, the convergent speed can be improved by modifying the number of available power levels and other parameters if it is applied into practical system.

2) SPECTRAL EFFICIENCY

For a cellular communication system, throughput is always an important measurement that should be considered to assess the performance of a resource scheduling scheme. As in Fig.3, the throughput performance of several resource

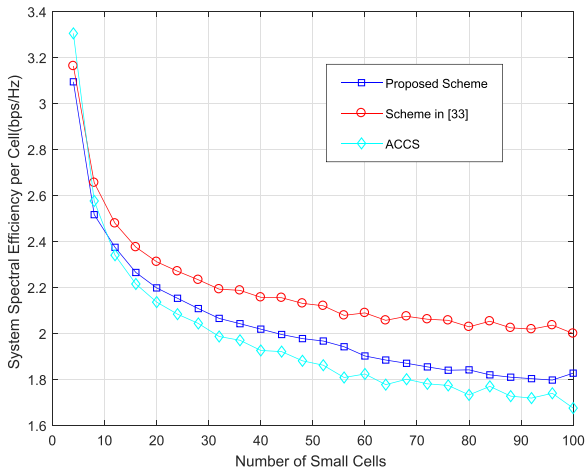


FIGURE 4. Average throughput per cell of different algorithms vs number of BSs.

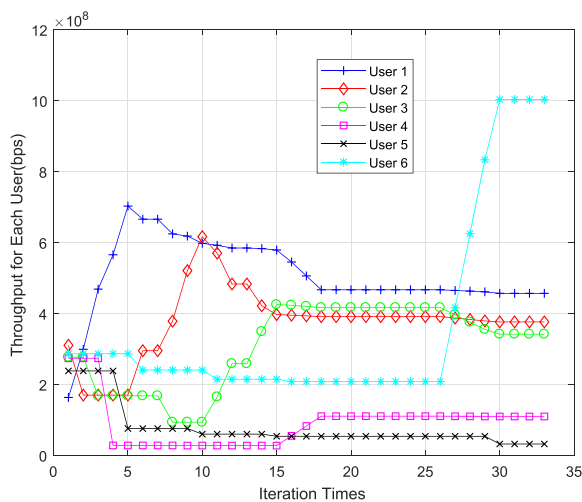


FIGURE 5. Individual throughput for each user vs iteration times.

allocation schemes are presented. The proposed scheme outperforms ACCS as the interference estimation in ACCS is not accurate enough to describe current interference environment. For scheme in [33], higher throughput is achieved than the proposed scheme. Actually, the proposed scheme can also obtain higher throughput by selecting proper interference weighting factor and judgment threshold. But the cost for pursuing higher throughput will jeopardize the power efficiency and user SINR, which will be discussed later. While the number of BSs is increasing in the limited area, the throughput for a single cell decreases rapidly due to the increased inter-cell interference as shown in Fig.4.

Specifically, Fig.5 presents the individual throughput of different users in a specified scenario, where six BSs locate in  $100\text{m} \times 100\text{m}$  grid-based area and there is only one user associated to one BS. At first each user only occupies one primary subcarrier and their throughputs are in the same level. As the iteration time increases, each BS is trying to occupy or give up subcarriers by assessing corresponding

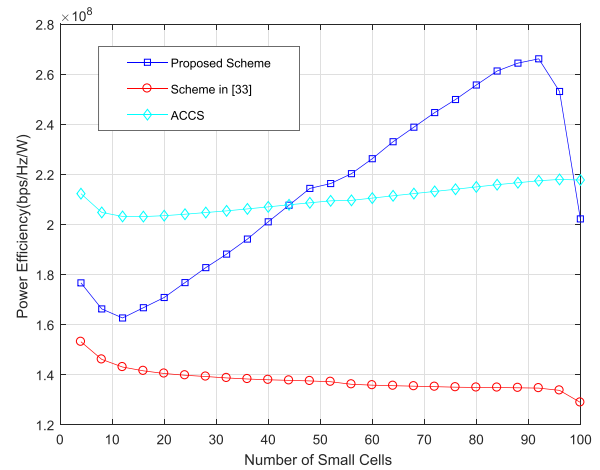


FIGURE 6. Power efficiency of different algorithms vs number of cells.

utilities. Consequently the individual throughput is going through rapid changes and finally converges to a stable value. With the proposed scheduling scheme, every BS can obtain an optimal subcarrier and power allocation strategy based on respective interference situation.

### 3) RESOURCE EFFICIENCY

In addition to system throughput, power efficiency is also significant as investigating energy consumption plays a great part in reducing system cost, especially in dense network. The power efficiency for BS in this paper is defined as the attainable throughput divided by the consumed power. Although throughput performance of the proposed scheme is not as good as scheme in [33], the proposed scheme outperforms scheme in [33] in power efficiency due to lower subcarriers occupation as shown in Fig.6. Since the efficient use of energy resource becomes more important, this advantage is considered more often by operators, especially in dense deployment where huge energy consumption occurs all the time. Compared with ACCS, the proposed scheme has lower power efficiency when the number of BSs is smaller than 45. But its power efficiency is monotonously increasing after the number of BSs surpasses 45, which highlights the ability of the proposed scheme to deal with interference in dense deployment. Based on the above analysis, it can be concluded that the proposed scheme achieves high power efficiency while guaranteeing considerable spectral efficiency due to its power-oriented utility function, especially in dense deployment. As shown in Fig.7, it can also be noticed that the proposed scheme uses less subcarrier resource than scheme in [33], which verifies its high power efficiency.

### 4) DISTRIBUTION OF USER SINR

In order to investigate the SINR performance of individual user, the cumulative distribution function of each scheduling scheme is investigated and the result is shown in Fig.8. For scheme in [33], the portion of user SINR under 1 is still too large to guarantee good communication quality even though it has high total throughput. Combined with the previous



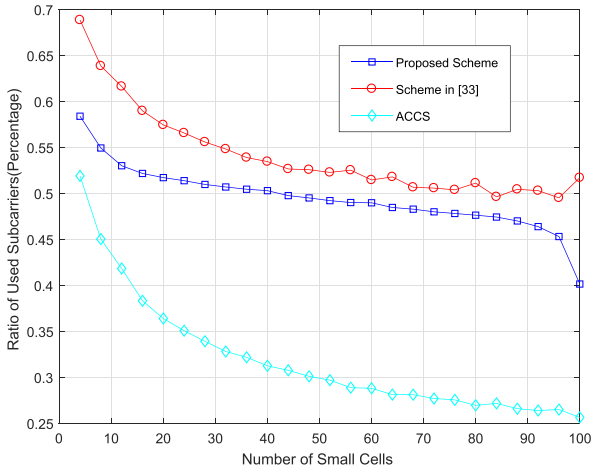


FIGURE 7. Ratio of used subcarriers of different algorithms vs number of cells.

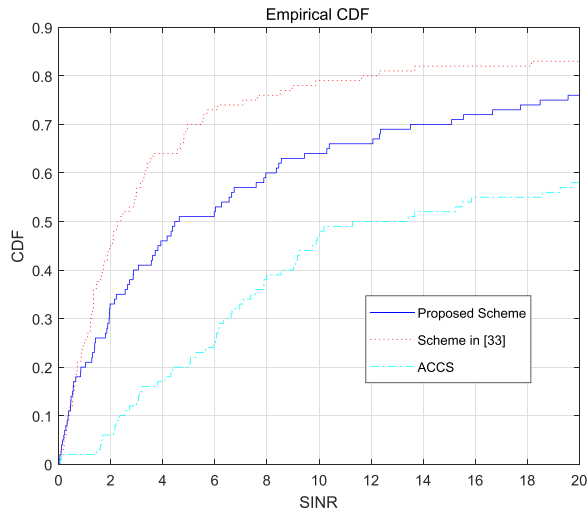


FIGURE 8. CDF of user SINR of different algorithms.

analysis, it means the high throughput brought by scheme in [33] is based on over-use of subcarriers, which generates a number of low-SINR users. Conversely, the proposed scheme achieves better SINR distribution and maintains certain quantity of high-SINR users.

V. CONCLUSION

In this paper, the subcarrier and power scheduling problem in dense cellular network is modeled as a potential game. Based on the model, this paper proposes the utility and potential functions, which consider attainable throughput and consumed power. Then a distributed iterative algorithm for optimal resource scheduling is introduced. In the simulation part, a grid-based scenario is introduced and the results show that the proposed scheme can always converge to an equilibrium. Meanwhile, high resource efficiency is achieved on the basis of guaranteeing considerable spectral efficiency. In addition, the proposed scheme can ensure communication quality by satisfactory user SINR distribution. In conclusion, this paper provides guidelines for resource scheduling in future dense

deployments of small cells. Based on the results, interference mitigation in other network architecture can also be modeled as a game theoretic problem. Moreover, to coordinate interference in a multi-level way, adding physical features of the BS like antenna tilt into consideration is also a worthwhile topic to enhance traditional scheduling solutions.

APPENDIX A DEMONSTRATION OF VALIDITY FOR THE PROPOSED POTENTIAL GAME MODEL

To prove that the proposed game model is an exact potential game, it means proving the equation in Eq.(2) holds. To simplify the verification, only one subcarrier is considered and it will not affect the correctness of the verification. Therefore index  $k$  is excluded in the following demonstration. Firstly, the proposed potential function in Eq.(10) will be decomposed:

$$\begin{aligned}
 &\Phi(s_i, s_{-i}) \\
 &= F(u(s_k, s_{-k})) \\
 &= \sum_M \left\{ \begin{aligned} &B\log_2\left(1 + \frac{P_i}{N_0B}\right) \\ &- \varepsilon \frac{b}{M-1} \sum_{j \in M, j \neq i} \left[ \begin{aligned} &B\log_2\left(1 + \frac{P_{\max}}{N_0B}\right) \\ &- B\log_2\left(1 + \frac{P_{\max}}{N_0B + P_j H_{j,i}}\right) \end{aligned} \right] \\ &- \varepsilon \frac{1-b}{M-1} \sum_{j \in M, j \neq i} \left[ \begin{aligned} &B\log_2\left(1 + \frac{P_{\max}}{N_0B}\right) \\ &- B\log_2\left(1 + \frac{P_{\max}}{N_0B + P_i H_{i,j}}\right) \end{aligned} \right] \end{aligned} \right\} \\
 &= B\log_2\left(1 + \frac{P_i}{N_0B}\right) \\
 &+ \varepsilon \frac{b}{M-1} \sum_{j \in M, j \neq i} B\log_2\left(1 + \frac{P_{\max}}{N_0B + P_j H_{j,i}}\right) \\
 &+ \varepsilon \frac{1-b}{M-1} \sum_{j \in M, j \neq i} B\log_2\left(1 + \frac{P_{\max}}{N_0B + P_i H_{i,j}}\right) \\
 &- \varepsilon \frac{1}{M-1} \sum_{j \in M, j \neq i} B\log_2\left(1 + \frac{P_{\max}}{N_0B}\right) \\
 &+ \sum_{n \in M, n \neq i} \left\{ \begin{aligned} &B\log_2\left(1 + \frac{P_n}{N_0B}\right) - \varepsilon \frac{b}{M-1} \\ &\times \sum_{j \in M, j \neq n} \left[ \begin{aligned} &B\log_2\left(1 + \frac{P_{\max}}{N_0B}\right) \\ &- B\log_2\left(1 + \frac{P_{\max}}{N_0B + P_j H_{j,n}}\right) \end{aligned} \right] \\ &- \varepsilon \frac{1-b}{M-1} \\ &\times \sum_{j \in M, j \neq n} \left[ \begin{aligned} &B\log_2\left(1 + \frac{P_{\max}}{N_0B}\right) \\ &- B\log_2\left(1 + \frac{P_{\max}}{N_0B + P_n H_{n,j}}\right) \end{aligned} \right] \end{aligned} \right\} \\
 &= B\log_2\left(1 + \frac{P_i}{N_0B}\right) \\
 &+ \varepsilon \frac{b}{M-1} \sum_{j \in M, j \neq i} B\log_2\left(1 + \frac{P_{\max}}{N_0B + P_j H_{j,i}}\right)
 \end{aligned}$$

$$\begin{aligned}
 & + \varepsilon \frac{1-b}{M-1} \sum_{j \in M, j \neq i} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_i H_{i,j}} \right) \\
 & + \sum_{n \in M, n \neq i} \left[ \begin{aligned} & \varepsilon \frac{b}{M-1} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_i H_{i,n}} \right) \\ & + \varepsilon \frac{1-b}{M-1} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_n H_{n,i}} \right) \end{aligned} \right] \\
 & - \varepsilon \frac{1}{M-1} \sum_{j \in M, j \neq i} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B} \right) \\
 & + \sum_{\substack{n \in M \\ n \neq i}} \left\{ \begin{aligned} & \text{B} \log_2 \left( 1 + \frac{P_n}{N_0 B} \right) \\ & - \varepsilon \frac{b}{M-1} \sum_{\substack{j \in M \\ j \neq n}} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B} \right) \\ & + \varepsilon \frac{b}{M-1} \sum_{\substack{j \in M \\ j \neq n}} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_j H_{j,n}} \right) \\ & - \varepsilon \frac{1-b}{M-1} \sum_{\substack{j \in M \\ j \neq n}} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B} \right) \\ & + \varepsilon \frac{1-b}{M-1} \sum_{\substack{j \in M \\ j \neq n}} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_n H_{n,j}} \right) \end{aligned} \right\}
 \end{aligned}$$

where first four items are *i*-related and the rests are *i*-free.

Then it will be proved that the proposed potential function satisfies Eq.(2). As only the first four items are *i*-related, the rests will not be considered as follows:

$$\begin{aligned}
 & \Phi(s_{i'}, s_{-i}) - \Phi(s_i, s_{-i}) \\
 & = \text{B} \log_2 \left( 1 + \frac{P_{i'}}{N_0 B} \right) \\
 & + \varepsilon \frac{b}{M-1} \sum_{j \in M, j \neq i'} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_j H_{j,i'}} \right) \\
 & + \varepsilon \frac{1-b}{M-1} \sum_{j \in M, j \neq i'} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_{i'} H_{i',j}} \right) \\
 & + \sum_{n \in M, n \neq i'} \left[ \begin{aligned} & \varepsilon \frac{b}{M-1} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_{i'} H_{i',n}} \right) \\ & + \varepsilon \frac{1-b}{M-1} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_n H_{n,i'}} \right) \end{aligned} \right] \\
 & - \left\{ \begin{aligned} & \text{B} \log_2 \left( 1 + \frac{P_i}{N_0 B} \right) \\ & + \varepsilon \frac{b}{M-1} \sum_{j \in M, j \neq i} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_j H_{j,i}} \right) \\ & + \varepsilon \frac{1-b}{M-1} \sum_{j \in M, j \neq i} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_i H_{i,j}} \right) \\ & + \sum_{n \in M, n \neq i} \left[ \begin{aligned} & \varepsilon \frac{b}{M-1} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_i H_{i,n}} \right) \\ & + \varepsilon \frac{1-b}{M-1} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_n H_{n,i}} \right) \end{aligned} \right] \end{aligned} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & = \text{B} \log_2 \left( 1 + \frac{P_{i'}}{N_0 B} \right) \\
 & + \varepsilon \frac{1}{M-1} \sum_{j \in M, j \neq i'} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_j H_{j,i'}} \right) \\
 & + \varepsilon \frac{1}{M-1} \sum_{j \in M, j \neq i'} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_{i'} H_{i',j}} \right) \\
 & + \left\{ \begin{aligned} & \text{B} \log_2 \left( 1 + \frac{P_i}{N_0 B} \right) \\ & + \varepsilon \frac{1}{M-1} \sum_{j \in M, j \neq i} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_j H_{j,i}} \right) \\ & + \varepsilon \frac{1}{M-1} \sum_{j \in M, j \neq i} \text{B} \log_2 \left( 1 + \frac{P_{\max}}{N_0 B + P_i H_{i,j}} \right) \end{aligned} \right\} \\
 & = u(s_{i'}, s_{-i}) - u(s_i, s_{-i})
 \end{aligned}$$

**APPENDIX B  
DEMONSTRATION OF MONOTONICITY FOR THE  
PROPOSED POTENTIAL FUNCTION**

To prove that the second derivative of the potential function is constantly negative, the first derivative is obtained as follows:

$$\begin{aligned}
 & \Phi'(s_i, s_{-i}) \\
 & = B \frac{1}{\ln 2 \cdot (N_0 B + P_i)} \\
 & + \varepsilon \frac{b}{M-1} \frac{B}{\ln 2} \sum_{j \in M, j \neq i} \left[ \frac{1}{1 + \frac{P_{\max}}{N_0 B + P_i H_{i,j}}} \frac{-P_{\max} H_{i,j}}{(N_0 B + P_i H_{i,j})^2} \right] \\
 & + \varepsilon \frac{1-b}{M-1} \frac{B}{\ln 2} \sum_{j \in M, j \neq i} \left[ \frac{1}{1 + \frac{P_{\max}}{N_0 B + P_i H_{i,j}}} \frac{-P_{\max} H_{i,j}}{(N_0 B + P_i H_{i,j})^2} \right] \\
 & = B \frac{1}{\ln 2 \cdot (N_0 B + P_i)} \\
 & + \varepsilon \frac{1}{M-1} \frac{B}{\ln 2} \sum_{j \in M, j \neq i} \left[ \frac{-P_{\max} H_{i,j}}{(N_0 B + P_i H_{i,j})^2 + P_{\max} (N_0 B + P_i H_{i,j})} \right]
 \end{aligned}$$

The second derivative of the potential function is:

$$\begin{aligned}
 & \Phi''(s_i, s_{-i}) \\
 & = \frac{-B}{\ln 2 \cdot (N_0 B + P_i)^2} + \varepsilon \frac{1}{M-1} \frac{B}{\ln 2} \\
 & \quad \times \sum_{j \in M, j \neq i} \frac{P_{\max} H_{i,j} (2H_{i,j}^2 P_i + 2N_0 B H_{i,j} + P_{\max} H_{i,j})}{[(N_0 B + P_i H_{i,j})^2 + P_{\max} (N_0 B + P_i H_{i,j})]^2}
 \end{aligned}$$

Proving above expression is constantly less than 0 equals proving the following formula:

$$\begin{aligned}
 & \varepsilon \frac{1}{M-1} \sum_{\substack{j \in M \\ j \neq i}} \frac{P_{\max} H_{i,j} (2H_{i,j}^2 P_i + 2N_0 B H_{i,j} + P_{\max} H_{i,j})}{[(N_0 B + P_i H_{i,j})^2 + P_{\max} (N_0 B + P_i H_{i,j})]^2} \\
 & < \frac{1}{(N_0 B + P_i)^2}
 \end{aligned}$$

which means:

$$\begin{aligned} \frac{P_{\max} H_{i,j} (2H_{i,j}^2 P_i + 2N_0 B H_{i,j} + P_{\max} H_{i,j}) (N_0 B + P_i)^2}{\varepsilon \left[ (N_0 B + P_i H_{i,j})^2 + P_{\max} (N_0 B + P_i H_{i,j}) \right]^2} &< 1 \\ \frac{(2H_{i,j} P_i P_{\max} + 2N_0 B P_{\max} + P_{\max}^2) (N_0 B + P_i)^2}{\varepsilon \left[ \frac{(N_0 B + P_i H_{i,j})^2}{H_{i,j}} + P_{\max} \left( \frac{N_0 B}{H_{i,j}} + P_i \right) \right]^2} &< 1 \\ \frac{(2H_{i,j} P_i P_{\max} + 2N_0 B P_{\max} + P_{\max}^2) (N_0 B + P_i)^2}{\varepsilon \left[ \frac{N_0^2 B^2}{H_{i,j}} + 2N_0 B P_i + P_i^2 H_{i,j} + \frac{P_{\max} N_0 B}{H_{i,j}} + P_{\max} P_i \right]^2} &< 1 \end{aligned}$$

After expanding the numerator and denominator in the above formulation, there is always a bigger item existing in the denominator for every component of the numerator. Moreover  $\varepsilon$  is less than 1. Hence the above inequation is true.

## REFERENCES

- [1] O. G. Aliu, A. Imran, M. A. Imran, and B. Evans, "A survey of self organisation in future cellular networks," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 1, pp. 336–361, 1st Quart., 2013.
- [2] Y. Xu, J. Wang, Q. Wu, Z. Du, L. Shen, and A. Anpalagan, "A game-theoretic perspective on self-organizing optimization for cognitive small cells," *IEEE Commun. Mag.*, vol. 53, no. 7, pp. 100–108, Jul. 2015.
- [3] Z. Han, *Game Theory in Wireless and Communication Networks: Theory, Models, and Applications*. Cambridge, U.K.: Cambridge Univ. Press, 2012.
- [4] R. Wei, Y. Wang, and Y. Zhang, "A two-stage cluster-based resource management scheme in ultra-dense networks," in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC)*, Oct. 2014, pp. 738–742.
- [5] W. Wang, Y. Zhou, Y. Huang, and J. Shi, "A pseudo-random subchannel selection scheme for downlink interference mitigation in two-tier cellular networks," in *Proc. 7th Int. ICST Conf. Commun. Netw. China (CHINA-COM)*, Aug. 2012, pp. 524–529.
- [6] B. Eraslan, D. Gozuek, and F. Alagoz, "An auction theory based algorithm for throughput maximizing scheduling in centralized cognitive radio networks," *IEEE Commun. Lett.*, vol. 15, no. 7, pp. 734–736, Jul. 2011.
- [7] K. Akkarajitsakul, E. Hossain, and D. Niyato, "Distributed resource allocation in wireless networks under uncertainty and application of Bayesian game," *IEEE Commun. Mag.*, vol. 49, no. 8, pp. 120–127, Aug. 2011.
- [8] P. Semasinghe, E. Hossain, and K. Zhu, "An evolutionary game for distributed resource allocation in self-organizing small cells," *IEEE Trans. Mobile Comput.*, vol. 14, no. 2, pp. 274–287, Feb. 2015.
- [9] L. G. U. Garcia, K. I. Pedersen, and P. E. Mogensen, "Autonomous component carrier selection: Interference management in local area environments for LTE-Advanced," *IEEE Commun. Mag.*, vol. 47, no. 9, pp. 110–116, Sep. 2009.
- [10] L. G. U. Garcia, K. I. Pedersen, and P. E. Mogensen, "Autonomous component carrier selection for local area uncoordinated deployment of LTE-advanced," in *Proc. IEEE 70th Veh. Technol. Conf. Fall*, Sep. 2009, pp. 1–5.
- [11] H. Mei, J. Bigham, P. Jiang, and E. Bodanese, "Distributed dynamic frequency allocation in fractional frequency reused relay based cellular networks," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1327–1336, Apr. 2013.
- [12] H. Xia, Y. Zhao, and Z. Zeng, "Best effort spectrum allocation scheme for femtocell networks in dense deployment," *China Commun.*, vol. 11, no. 8, pp. 109–116, Aug. 2014.
- [13] S. L. Hew and L. B. White, "Cooperative resource allocation games in shared networks: Symmetric and asymmetric fair bargaining models," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4166–4175, Nov. 2008.
- [14] H. Li and Z. Han, "Competitive spectrum access in cognitive radio networks: Graphical game and learning," in *Proc. IEEE Wireless Commun. Netw. Conf.*, Apr. 2010, pp. 1–6.
- [15] S. Fan and H. Tian, "Cooperative resource allocation for self-healing in small cell networks," *IEEE Commun. Lett.*, vol. 19, no. 7, pp. 1221–1224, Jul. 2015.
- [16] Z. Su, Q. Xu, M. Fei, and M. Dong, "Game theoretic resource allocation in media cloud with mobile social users," *IEEE Trans. Multimedia*, vol. 18, no. 8, pp. 1650–1660, Aug. 2016.
- [17] S. Buzzi, G. Colavolpe, D. Saturnino, and A. Zappone, "Potential games for energy-efficient power control and subcarrier allocation in uplink multicell OFDMA systems," *IEEE J. Sel. Topics Signal Process.*, vol. 6, no. 2, pp. 89–103, Apr. 2012.
- [18] Q. D. La, Y. H. Chew, and B. H. Soong, "Performance analysis of downlink multi-cell OFDMA systems based on potential game," *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3358–3367, Sep. 2012.
- [19] J. Zheng, Y. Cai, Y. Xu, and A. Anpalagan, "Distributed channel selection for interference mitigation in dynamic environment: A game-theoretic stochastic learning solution," *IEEE Trans. Veh. Technol.*, vol. 63, no. 9, pp. 4757–4762, Nov. 2014.
- [20] X. Kang, Y.-C. Liang, and H. K. Garg, "Distributed power control for spectrum-sharing femtocell networks using Stackelberg game," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2011, pp. 1–5.
- [21] K. Zhu, E. Hossain, and D. Niyato, "Pricing, spectrum sharing, and service selection in two-tier small cell networks: A hierarchical dynamic game approach," *IEEE Trans. Mobile Comput.*, vol. 13, no. 8, pp. 1843–1856, Aug. 2014.
- [22] S. Yan, M. Peng, M. A. Abana, and W. Wang, "An evolutionary game for user access mode selection in fog radio access networks," *IEEE Access*, vol. 5, pp. 2200–2210, 2017.
- [23] Z. Li, W. Wang, W. Xie, M. Wang, and Y. Li, "An efficient two-step power allocation method for OFDM-based two-way full-duplex links," *IEEE Commun. Lett.*, vol. 20, no. 7, pp. 1445–1448, Jul. 2016.
- [24] Q. Ren, J. Fan, X. Luo, Z. Xu, and Y. Chen, "Analysis of spectral and energy efficiency in ultra-dense network," in *Proc. IEEE Int. Conf. Commun. Workshop (ICCW)*, Jun. 2015, pp. 2812–2817.
- [25] N. Li, W. Sun, Y. Li, M. Peng, and W. Wang, "Joint power allocation and subcarrier pairing for dual-hop OFDM links with full-duplex relaying," in *Proc. 8th Int. Conf. Wireless Commun. Signal Process. (WCSP)*, Oct. 2016, pp. 1–6.
- [26] A. Zappone, L. Sanguinetti, G. Bacci, E. Jorswieck, and M. Debbah, "Energy-efficient power control: A look at 5G wireless technologies," *IEEE Trans. Signal Process.*, vol. 64, no. 7, pp. 1668–1683, Apr. 2016.
- [27] C. Yang, J. Li, P. Semasinghe, E. Hossain, S. M. Perlaza, and Z. Han, "Distributed interference and energy-aware power control for ultra-dense D2D networks: A mean field game," *IEEE Trans. Wireless Commun.*, vol. 16, no. 2, pp. 1205–1217, Feb. 2017.
- [28] J. Zheng, Y. Wu, N. Zhang, H. Zhou, Y. Cai, and X. Shen, "Optimal power control in ultra-dense small cell networks: A game-theoretic approach," *IEEE Trans. Wireless Commun.*, vol. 16, no. 7, pp. 4139–4150, Jul. 2017.
- [29] D. López-Pérez, X. Chu, A. V. Vasilakos, and H. Claussen, "Minimising cell transmit power: Towards self-organized resource allocation in OFDMA femtocells," in *Proc. ACM SIGCOMM Conf.*, 2011, pp. 410–411.
- [30] Z. Han and K. J. R. Liu, "Noncooperative power-control game and throughput game over wireless networks," *IEEE Trans. Commun.*, vol. 53, no. 10, pp. 1625–1629, Oct. 2005.
- [31] J. Zheng, Y. Cai, Y. Liu, Y. Xu, B. Duan, and X. Shen, "Optimal power allocation and user scheduling in multicell networks: Base station cooperation using a game-theoretic approach," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 6928–6942, Dec. 2014.
- [32] L. G. U. Garcia, I. Z. Kovacs, K. I. Pedersen, G. W. O. Costa, and P. E. Mogensen, "Autonomous component carrier selection for 4G femtocells—A fresh look at an old problem," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 525–537, Apr. 2012.
- [33] L. Giupponi and C. Ibars, "Distributed interference control in OFDMA-based femtocells," in *Proc. 21st Annu. IEEE Int. Symp. Pers., Indoor Mobile Radio Commun.*, Sep. 2010, pp. 1201–1206.



**ZHIQIANG QI** (S'16) received the B.S. and M.S. degrees from the Beijing University of Posts and Telecommunications in 2008 and 2011, respectively, where he is currently pursuing the Ph.D. degree in information and communication engineering. His research interests include interference coordination, game theory, and optimization techniques.



**TAO PENG** (M'04) received the B.S., M.S., and Ph.D. degrees from the Beijing University of Posts and Telecommunications (BUPT) in 1999, 2002, and 2010, respectively. He is currently an Associate Professor with BUPT. He has been the Chair of Device-to-Device (D2D) Technical Discussion Group, IMT-A/IMT-2020 Propulsion Group, CCSA. He has authored over 70 academic papers with 16 SCI indexed. He is the inventor of 25 international patents. His current research interests include cognitive radio and software-defined radio, D2D communication, CRAN and ultradense network, and mobile ad-hoc network.



**WENBO WANG** (M'95–SM'15) received the B.S., M.S., and Ph.D. degrees from the Beijing University of Posts and Telecommunications (BUPT) in 1986, 1989, and 1992, respectively. He is currently a Professor and the Vice President of BUPT. His current research interests include radio transmission technology, wireless network theory, and wireless signal processing.

...