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On the Use of Functional Additive Models for Electricity Demand and Price Prediction

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ABSTRACT This paper presents an application of functional additive models in the context of electricity demand and price prediction. Data from the Spanish electricity market are used to obtain the pointwise predictions. Also prediction intervals, based on a bootstrap procedure, are computed. This approach is compared with the use of other functional regression methods applied to the same data set by Aneiros *et al.* (2016).

INDEX TERMS Additive model, functional data, functional time series forecasting, load and price, prediction intervals.

I. INTRODUCTION

Prediction of electricity demand and price are significant problems for the agents and companies involved in the electricity markets. In particular, one day ahead hourly forecasts of demand and price has been extensively studied in the literature. Some methods are based on statistical models (dynamic regression, transfer functions, time series, exponential smoothing, etc.) whereas other ones are based on computational intelligence models (neuronal networks, support vector machines, etc.). See the book by Weron [1] for a nice monograph on electricity demand and price forecasting. See also [2] and [3] for reviews on electricity demand forecasting and [4] and [5] on electricity price forecasting. Most of the papers studying methods of electricity demand and price prediction take information from scalar variables, but in recent years, the use of functional data has been extended in this area. Considering the daily curves of electricity demand or price as functional data, the prediction problem in electric markets can be studied taking use of functional regression methods. The books [6] and [7] are comprehensive references for functional data analysis using a linear or non-parametric view, respectively.

Some papers that use functional data to predict electricity demand and price curves are the following: [8] used a parametric model to predict electricity consumption curves; functional time series methodology was applied in [9] to historical daily curves of load; [10] obtained probabilistic forecasts of electricity load, based on functional data analysis of generalized quantile curves; [11] used, among other methodology, two functional approaches to forecast the France's daily electricity load consumption; [12] proposed an adaptive functional autoregressive (AFAR) forecast model to predict electricity price curves; finally, [13] analysed the case of residual demand curves whereas [14] considered the prediction of both demand and price curves.

When the interest is to forecast scalar values (not curves) from functional data, the reader can see [15] or [16]. The case of forecasting scalar values (as well as curves) of demand and price from functional data within the Spanish Electricity Market are studied in [14]. In that paper, nonparametric and semi-functional partial linear models are employed. Non-parametric autoregressive models with functional data (FNP) provide good predictions, due to its flexibility, but the results can improve by adding exogenous variables to the model. In that way, when dealing with demand prediction, it is convenient to introduce the temperature and other weather variables as covariates. When the aim is the prediction of electricity price, the wind power production and the forecast daily demand can be considered instead. Due to the curse of the dimensionality, it is not recommendable to use nonparametric regression models with several covariates. References [13] and [14] solved that problem with the use of a Semi-Functional Partial Linear model (SFPL), with a nonparametric autoregressive functional component and introducing other scalar covariates in a linear way. SFPL models improve, in general, the results from the FNP models.

The aim of this paper is next-day forecasting of hourly values of electricity demand and price using functional additive models (FAM). These models combine flexibility and the control of the dimensionality effects. Additive models have been already used for prediction in the context of electricity demand (see [17], [18]) and price (see [19]). However, those references are not dealing with functional data, as in our proposal. Focusing now in the references named in the previous second and third paragraphs, it is worth being noted that, although they deal with functional data, they do not consider the models (FAM) used in this paper. Three approaches, taking information from functional covariates (one is endogenous), are considered. In the first case, the effect of the covariates on the response is linear (functional linear model), whereas in the other two proposals the regression is the sum of smoothing functions applied to the covariates. Also, algorithms for the construction of the prediction intervals (PI) and prediction density (PD) associated with the functional additive models are proposed. These algorithms use residual-based bootstrap methods.

This paper continues the following two studies: [14] in which nonparametric and semi-parametric functional regression methods were used to predict electricity demand and price; and [20] in which prediction intervals, using residual-based bootstrap algorithms, were obtained for the prediction methods proposed in [14]. It is worth being noted the main differences between the study in this paper and the ones presented in [14] and [20]. On the one hand, this paper considers models (FAM) that are more general than the ones considered in [14] and [20]. On the other hand, all the exogenous covariates in our models are of functional nature, while the exogenous covariates in [14] and [20] are scalar.

The remaining of the paper is organized as follows. The additive methods with functional covariates and scalar response are presented in Section II, together with the algorithms proposed to obtain PIs and PDs. Sections III and IV show numerical results concerning 1-day ahead forecasting of electricity demand and price, respectively, in mainland Spain during the year 2012. A comparative study of the proposed additive models and the results obtained in [14] and [20] is also included. Finally, Section V provides some relevant conclusions: basically, the proposed models (FAM) improve the results obtained in both [14] (pointwise prediction) and [20] (prediction intervals).

II. FORECASTING FROM FUNCTIONAL ADDITIVE MODELS

A. POINTWISE PREDICTION METHODS

We assume that the time series of interest (electricity demand and price) are continuous time stochastic processes, and we use the same notation $\{\zeta(t)\}_{t\in R}$ to refer to any of them (units for t are hours). As $\{\zeta(t)\}_{t\in R}$ is a seasonal process with seasonal length $\tau = 24$, and considering that such process is observed on the interval $(a, b]$ with $b = a + N\tau$, the observed daily curves (of electricity demand or price) can be written as $\{\zeta_i\}_{i=1}^{N}$, where $\zeta_i(t) = \zeta(a + (i-1)\tau + t)$, with $t \in (0, \tau]$.

In this paper, predictions and PIs are obtained for each hour, one day ahead, of electricity demand or price, $\zeta_{N+1}(t)$, with $t \in \{1, 2 \ldots, 24\}$, in year 2012 from information given by the previous 365 days. Three functional additive models are considered and, for each prediction method, 72 (3×24) models are computed according to the kind of day and the hour, considering that the dynamic of the curves depends on the type of day where are observed: weekdays, Saturdays or Sundays.

We wish to predict the variable $\zeta_{N+1}(t)$ using information given from $\left\{\vec{\chi}_i = \left(\chi_i^1, \ldots, \chi_i^d\right)\right\}_{i=N-364}^{N}$, where $\chi^j(t)$ are functional covariates. In both cases (demand or price) $\chi_i^1 = \zeta_i$ is the endogenous covariate. When the day $N+1$ corresponds to either Saturday or Sunday, information from the previous curve (i.e. from the curve observed on previous Friday or Saturday, respectively) will be used. If the day $N+1$ corresponds to a weekday, information will be taken from the curve observed on the previous weekday (note that the previous weekday to a Monday is a Friday). Let us assume that the day $N+1$ is Saturday (in the case of a weekday or Sunday, the procedure is analogous) and denote $\mathcal{I}_0 = \{N-364, N-363, \ldots, N-1, N\}$, $\mathcal{I}_{Sat} = \left\{j \in \mathcal{I}_0 \,/\, \zeta_j \text{ is a Saturday}\right\}$. The prediction of the electricity demand or price at the hour t, $\zeta_{N+1}(t)$, is obtained from the next general model:

$$\zeta_{i+1}(t) = r_t\left(\chi_i^1, \ldots, \chi_i^d\right) + \varepsilon_{t,i+1}, \qquad i+1 \in \mathcal{I}_{Sat}, \quad (1)$$

where $r_t(\cdot)$ is an unknown function of the covariates and $\varepsilon_{t,i+1}$ is an error term, with zero mean. Then, a prediction $\hat{\zeta}_{N+1}(t)$ for the variable $\zeta_{N+1}(t)$ can be obtained by estimating $r_t(\vec{\chi}_N)$ in (1), this is, $\hat{\zeta}_{N+1}(t) = \hat{r}_t(\vec{\chi}_N)$.

1) FUNCTIONAL LINEAR MODELS

When the relation between the covariates and the response is linear, the regression function, $r_t(\vec{\chi})$, in (1) is the following

$$r_t(\vec{\chi}_i) = E(\zeta_{i+1}(t) \,/\, \vec{\chi}_i) = \beta_{t,0} + \sum_{j=1}^{d} \int_I \chi_i^j(s)\beta_{t,j}(s)ds. \quad (2)$$

The Functional Linear Model (FLM) has been widely studied in the literature (see, for instance, [6, Ch. 10]). Functional coefficients, $(\beta_{t,j})$, can be estimated by different ways. Reference [21] proposed an estimator using a functional principal component analysis and proved the convergence of this estimator. Reference [22] studied an estimator based on a B-splines expansion which, in some way, generalizes ridge regression. This one is the estimator that will be considered in Sections III and IV. Reference [23] proposed a similar estimator in the context of time series.

2) FUNCTIONAL ADDITIVE MODELS

In most of applied situations there is a lack of knowledge about the relationship between the response and the functional covariates and this leads naturally to consider nonparametric modelling. In this case, the regression function, $r_t(\vec{\chi})$

in (1) is

$$r_t\left(\vec{\chi}_i\right) = E\left(\zeta_{i+1}\left(t\right)/\vec{\chi}_i\right) = \beta_{t,0} + \sum_{j=1}^{d} r_{t,j}\left(\chi_i^j\right). \quad (3)$$

That model is called functional additive model (FAM) and here, the key point is the estimation of the partial functions $r_{t,j}$. Reference [24] proposed to estimate $r_{t,j}$ using one cyclic conditional algorithm. At each stage, the effect of a functional covariate is estimated, conditionally on previous estimation, using functional kernel estimates. We will refer to this model as the Functional Kernel Additive Model (FKAM). An alternative is proposed by [25], using in this case the functional principal component scores of χ^j, being $r_{t,j}\left(\chi_i^j\right) = \sum_{k=1}^{K} r_{t,j}^k(\xi_j^k)$ smooth functions of ξ_j^k, the $k-$principal score of variable j. We will refer to this approach as the Functional Spectral Additive Model (FSAM) because the use of spectral decomposition of the covariance operator of χ.

B. BOOTSTRAP PREDICTION INTERVALS

When dealing with forecasting, it is important to consider also prediction intervals and the prediction density, which help to understand the behaviour of the forecasts in a deeper way. A bootstrap algorithm is proposed to construct PIs and PDs in the problem of the next-day forecasting of electricity demand and price. This algorithm is similar to the proposal in [20], but adapting it to the case of the prediction with the additive models presented in Subsection. The algorithm is also adapted to deal with cases of homoscedasticity and heteroscedasticity.

We want to compute a PI for the variable $\zeta_{N+1}\left(t\right)$, where t is fixed and the day $N+1$ corresponds to a Saturday. In this case, the sample will be $\mathcal{S}' = \{(\vec{\chi}_i, \zeta_{i+1}(t)) : i+1 \in \mathcal{I}_{Sat}\}$, and we assume that the pair $(\vec{\chi}_i, \zeta_{i+1}(t))$ follows the additive model given in (2) or (3). Under the assumption that the model is heteroscedastic, the error of the model is $\varepsilon_{t,i+1} = \sigma_t\left(\zeta_i\right)\eta_{t,i+1}$, where $\eta_{t,i+1}$ are iid, $\mathbb{E}(\eta_{t,i+1}|\zeta_i) = 0$ and $Var(\eta_{t,i+1}|\zeta_i) = 1$. Then, $Var(\zeta_{i+1}\left(t\right)|\zeta_i) = Var(\varepsilon_{t,i+1}|\zeta_i) = \sigma_t^2(\zeta_i) = \nu_t(\zeta_i)$, where $\nu_t(\zeta_i)$ denotes the error conditional variance.

The predictor for $\zeta_{N+1}(t)|\vec{\chi}_N$ is $\hat{r}_t\left(\vec{\chi}_N\right)$ and one has the following decomposition:

$$\zeta_{N+1}(t)|\vec{\chi}_N = \hat{r}_t\left(\vec{\chi}_N\right) + \left(r_t\left(\vec{\chi}_N\right) - \hat{r}_t\left(\vec{\chi}_N\right)\right) + \left(\varepsilon_{t,N+1}|\vec{\chi}_N\right)$$

Hence, as the true regression function, $r_t\left(\vec{\chi}_N\right)$, is unknown in practice, one needs to approximate $r_t\left(\vec{\chi}_N\right) - \hat{r}_t\left(\vec{\chi}_N\right)$ and the error term, $\varepsilon_{t,N+1}|\vec{\chi}_N$, using bootstrap procedures. In our case, under heteroscedasticity, the proposed algorithm includes the estimation of the conditional variance, $\nu_t(\zeta_N)$. This estimation is made following the ideas of [26], but adapting them to functional data.

The bootstrap $(1-\alpha)$-prediction interval for $\zeta_{N+1}(t)|\vec{\chi}_N$ is constructed as:

$$I^*_{\chi_N,t,1-\alpha} = (\hat{r}_t\left(\vec{\chi}_N\right) + q^*_{t,\alpha/2}(\vec{\chi}_N),\ \hat{r}_t\left(\vec{\chi}_N\right) + q^*_{t,1-\alpha/2}(\vec{\chi}_N)),$$

where the bootstrap quantiles $q^*_{t,p}(\vec{\chi}_N)$ are computed in the following way:

1) Compute $\hat{r}_t\left(\vec{\chi}_i\right)$, $i+1 \in \mathcal{I}_{Sat}$, using one of the additive models proposed in Subsection II-A.
2) Compute the residuals $\widehat{\varepsilon}_{t,i+1} = \zeta_{i+1}\left(t\right) - \hat{r}_t\left(\vec{\chi}_i\right)$.
3) Based on the sample $\mathcal{S}_{\varepsilon,t} = \{(\zeta_i, \widehat{\varepsilon}^2_{t,i+1}) : i+1 \in \mathcal{I}_{Sat}\}$, using Nadaraya-Watson estimator for functional data (see [7]), the estimator for $\nu_t(\zeta)$ is obtained as:
$$\hat{\nu}_{t,g}(\zeta) = \sum\nolimits_{i+1\in\mathcal{I}_{Sat}} w_g(\zeta,\zeta_i)\widehat{\varepsilon}^2_{t,i+1}, \quad (4)$$
where g is the bandwidth. One obtains the estimators for $\nu_t(\zeta_i)$ and $\nu_t(\zeta_N)$, denoted as $\hat{\nu}_{t,i} = \hat{\sigma}^2_{t,i} = \hat{\nu}_{t,g}(\zeta_i)$, $i+1 \in \mathcal{I}_{Sat}$, and $\hat{\nu}_{t,g}(\zeta_N) = \hat{\sigma}^2_{t,g}(\zeta_N)$, respectively. The standardized residuals of the model can be obtained as:
$$\widehat{\eta}_{t,i+1} = \frac{\widehat{\varepsilon}_{t,i+1}}{\hat{\sigma}_{t,i}}, \quad i+1 \in \mathcal{I}_{Sat}.$$
4) Apply the naive bootstrap procedure to obtain the bootstrap errors: Draw $n_{Sat} = \#(\mathcal{I}_{Sat})$ iid random variables, $\eta^*_{t,i+1}$, $i+1 \in \mathcal{I}_{Sat}$, from the empirical distribution function of $\left\{(\widehat{\eta}_{t,i+1} - \overline{\widehat{\eta}}_t) : i+1 \in \mathcal{I}_{Sat}\right\}$, denoted by $F_{t,\eta}$, where $\overline{\widehat{\eta}}_t = n_{Sat}^{-1}\sum_{i+1\in\mathcal{I}_{Sat}}\widehat{\eta}_{t,i+1}$.
5) Denote $\varepsilon^*_{t,i+1} = \hat{\sigma}_{t,i}\eta^*_{t,i+1}$ and obtain $\zeta^*_{i+1}(t) = \hat{r}_t(\vec{\chi}_i) + \varepsilon^*_{t,i+1}$, $i+1 \in \mathcal{I}_{Sat}$, and from the sample $\mathcal{S}'^* = \{(\vec{\chi}_i, \zeta^*_{i+1}(t)) : i+1 \in \mathcal{I}_{Sat}\}$ we obtain $\hat{r}^*_t\left(\vec{\chi}_N\right)$.
6) Repeat B times Steps 4-5, giving the B estimates $\left\{\hat{r}_t^{*,j}\left(\vec{\chi}_N\right) : j = 1, \ldots, B\right\}$.
7) Draw B iid random variables $\widetilde{\eta}_{t,1}, \ldots, \widetilde{\eta}_{t,B}$ from the empirical distribution function, $F_{t,\eta}$, and compute $\widetilde{\varepsilon}_{t,j} = \hat{\sigma}_{t,g}(\zeta_N)\widetilde{\eta}_{t,j}$, $j = 1, \ldots, B$. $\widetilde{\varepsilon}_{t,j}$ approximates the error in the model.
8) Compute the set of bootstrap errors:
$$Errors.Boot = \left\{\hat{r}_t\left(\vec{\chi}_N\right) - \hat{r}_t^{*,j}\left(\vec{\chi}_N\right) + \widetilde{\varepsilon}_{t,j}\right\}_{j=1}^{B}.$$
9) Compute the bootstrap quantile, $q^*_{t,p}(\vec{\chi}_N)$, from the quantile of order p of $Errors.Boot$.

Some remarks about this algorithm are the following.

- If the model is homoscedastic, the algorithm can be simplified, as Step 3 is deleted. In Step 4-5, n_{Sat} iid random variables $\varepsilon^*_{t,i}$, $i+1 \in \mathcal{I}_{Sat}$, must be drawn from the empirical distribution function of the centered residuals, denoted by $F_{t,\varepsilon}$. In Step 7, $\widetilde{\varepsilon}_{t,r}$ is obtained from $F_{t,\varepsilon}$. The rest of the algorithm remains the same.
- We assume that the conditional variance, $\nu(\zeta_i)$, only depends on the functional explanatory variable, $\zeta_i = \chi_i^1$, and not on the other covariates, as it happens in the most common situations. However, if one wants to assume that this conditional variance depends on all covariates, one may consider the expression $Var\left(\zeta_{i+1}(t)|\vec{\chi}_i\right) = Var\left(\varepsilon_{t,i+1}|\vec{\chi}_i\right) = \nu_t\left(\vec{\chi}_i\right)$. In that general case, the estimation of $\nu_t\left(\vec{\chi}_i\right)$ cannot be done by a nonparametric estimator. Alternatives as partial linear or additive models need to be employed instead.

- Note that one can consider, from the algorithm above:

$$\zeta^{*,j}_{N+1}(t)|\vec{\chi}_N = \hat{r}_t(\vec{\chi}_N) + \left(\hat{r}_t(\vec{\chi}_N) - \hat{r}^{*,j}_t(\vec{\chi}_N)\right) + \widetilde{\varepsilon}_{t,j},$$

with $j = 1, \ldots, B$. Now, using the bootstrap responses $\{\zeta^{*,j}_{N+1}(t)|\vec{\chi}_N\}_{j=1}^{B}$, one can obtain an estimation for the PD of $\zeta_{N+1}(t)|\vec{\chi}_N$ applying, for instance, the Rosenblatt Parzen kernel density estimator.

III. ANALYSIS OF ELECTRICITY DEMAND

Prediction methods presented in previous Section II will be applied to a real dataset coming from the Spanish Electricity Market. Specifically, within this section, an application to the electricity demand will be considered. Results of the next-day forecasting of hourly electricity demand, based on the functional additive models, are given. In addition, results from other prediction methods previously used in [14] will be shown with the aim of comparison. Finally, also the bootstrap procedures in Subsection II-B will be applied to compute the prediction intervals (PI) and prediction density (PD).

The electricity dataset involved in this application has been used before in [14] and [20]. In that case, functional nonparametric and semi-functional partial linear regression, among other prediction methods, were considered. The results in that paper will be compared to the functional additive models. Next paragraphs will contain a brief description of the dataset, both electrical data and additional covariates. See [14] for a detailed review of the dataset.

Electricity demand and price comes from OMIE (Operador del Mercado Ibérico de Energía), which is the Market Operator in Spain and which provides at its web page (www.omel.es) the hourly observations of the electricity demand, among other related variables. We consider each daily curve of electricity demand, computed from the 24 hourly observations, along the years 2011 and 2012. Then, each one of these daily curves is a functional datum composing a functional time series.

When dealing with electricity demand, one may take into account their particular features, summarized in the daily and weekly seasonality, the calendar effect on the weekend and the presence of outliers. Due to the different behaviour of the electricity demand between the weekdays and the weekend (and also between Saturdays and Sundays within the weekend), the procedures will be applied separately for three groups of days: Weekdays, Saturdays and Sundays. Prior to any statistical analysis of the data, outlier detection methods for functional time series presented in [27] and [28] are applied to our dataset and the selected outliers are replaced by weighted moving average.

Prediction methods are applied as autoregressive models, since the hourly electricity demand is predicted based on the previous daily curve of demand. However, one may consider additional information to be used as functional covariates within the prediction methods. It is known that temperature affects the energy consumption, and so does to the electricity demand, due to the use of heating or cooling systems when the temperature is low or high. Daily curves of the temperature in Spain, obtained from the 24 hourly observations, will be included in the models as functional covariate. That temperature information is given by AEMET (Agencia Estatal de Meteorología) for each province of Spain and so, by population weighted average, one can obtain the hourly temperature for Spain.

Once the dataset involved in the application is presented, the functional additive methods are applied to the next-day forecasting of the hourly electricity demand. Thus, the scalar response of the prediction model is the electricity demand for one hour, considering as functional explanatory variable the previous daily curve of demand, together with the daily curve of the temperature in the day to be predicted. Predictions for all the year 2012 are obtained.

FLM, FKAM and FSAM will be considered. Those methods are available in the R package *fda.usc*, whose routines were used in this application (see [29]). B-Splines basis and gaussian family was considered in the prediction methods. The external functional covariates in FSAM were smoothed and the L_2 norm was considered in the FKAM. In addition, due to the different behaviour between weekdays, Saturdays and Sundays, one may consider separate models for each kind of day and also for each one of the 24 hours in the day. Thus, one deals with 72 prediction models at the same time.

To compare the accuracy of each considered model and obtained forecast $\widehat{\zeta}_{N+1}$ from the different prediction methods, the mean absolute percentage error (MAPE) is used, which is defined as:

$$MAPE_{N+1} = \frac{1}{24}\sum_{j=1}^{24} APE_{N+1}(j),$$

where

$$APE_{N+1}(t) = 100 \times \left|\frac{\widehat{\zeta}_{N+1}(t) - \zeta_{N+1}(t)}{\zeta_{N+1}(t)}\right| \quad \text{for } t \in (0, 24].$$

Table 1 displays the MAPE error from the electricity demand predictions, when dealing with the additive models for functional data, divided by kind of day and quarter of the year. Also the Naive method is considered in order to compare the accuracy of the proposed procedures. This Naive method is a very simple procedure, working quite well in this context. It consists in just to assign, as the prediction for one day, the observed value in the previous one. Taking into account the different behaviour of the electricity between the days of the week, the prediction for a weekday will be the observed values in the day before (for example, the prediction for a Tuesday is obtained from previous Monday, taking into account that the previous weekday for a Monday is the previous Friday). Meanwhile the prediction for Saturday or Sunday will come from the Saturday or Sunday of the previous week.

In general, the best result is achieved by the FLM, which reaches the lowest prediction error for all the kind of days and almost all the quarters. FKAM and FSAM behave very similar in this case, being the FKAM slightly better in the

TABLE 1. Mean of the MAPE for the electricity demand curves using the functional additive and the Naive methods. Results are shown by type of day, week, quarter and year.

Method	Q1	Q2	Q3	Q4	Year
			Weekday		
Naive	4.43	5.91	4.46	6.36	5.24
FKAM	4.25	5.44	4.18	5.87	4.90
FSAM	4.30	5.11	4.05	5.80	4.78
FLM	4.27	5.07	4.00	5.78	4.75
			Saturday		
Naive	7.29	8.06	5.21	13.62	8.41
FKAM	4.33	5.32	3.64	10.17	5.74
FSAM	5.52	5.15	4.05	10.09	6.09
FLM	4.55	5.16	4.01	9.18	5.62
			Sunday		
Naive	9.08	5.62	8.94	16.50	10.10
FKAM	6.21	5.71	5.19	8.83	6.48
FSAM	6.07	5.49	6.41	9.13	6.79
FLM	5.64	5.08	6.29	8.41	6.38
			Week		
Naive	5.50	6.18	5.21	8.84	6.39
FKAM	4.54	5.47	4.25	6.90	5.25
FSAM	4.73	5.17	4.39	6.89	5.26
FLM	4.50	5.08	4.33	6.64	5.11

global result. By kind of days, the FSAM is better on the weekdays, while FKAM improves their results during the weekend. All the three procedures turn out to be better than the Naive method, reducing its MAPE error in a 20%. That improvement is more remarkable in the weekend, reaching a reduction about 30% of the prediction error.

Graphical comparison of the prediction errors reported in Table 1 is given in Figure 1. In that plot, one can see the daily errors from the four considered methods divided by kind of day. It is easy to distinguish that the higher errors are committed during the Sundays. However, weekdays and Saturdays are closer and the prediction errors are more concentrated along the same values. Comparing each one of the procedures, one can distinguish that Naive method generates the worst errors, specially during the weekend. FLM seems to be the best method in terms of global error, with small differences among the three functional additive models.

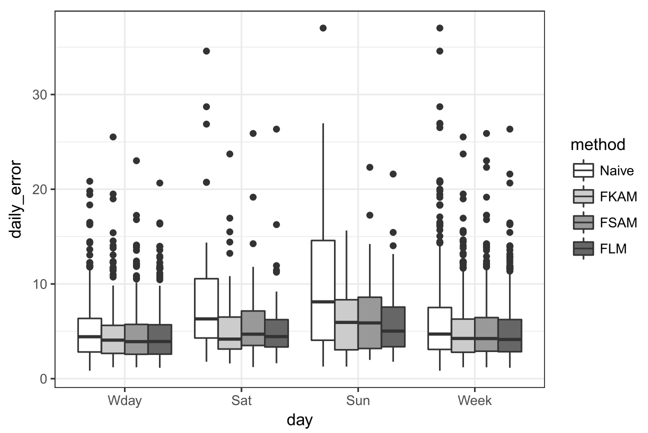

FIGURE 1. Daily errors (MAPE) for electricity demand curves corresponding to the functional additive and the Naive methods.

Real versus predicted demand along one entire week is represented in Figure 2, considering the Naive, FKAM and

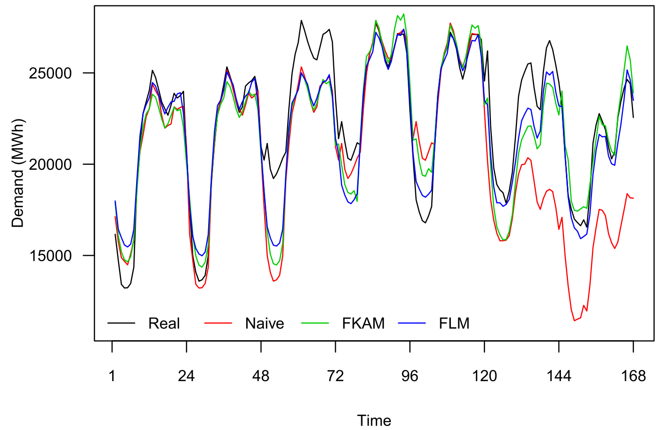

FIGURE 2. Observed and predicted demand curves for the week November 26–December 2.

FLM method. One can easily distinguish the poor accuracy of the Naive method, motivated by its poor behaviour during the weekend. Even if no major differences are seen between FKAM and FLM, the FKAM is slightly worse than FLM. In general, the accuracy is better during the weekdays than in the weekend, when the variability is more remarkable.

Some other prediction methods can be considered in order to compare the behaviour of the proposed procedures. For that reason, one may consider the results in [14], which were obtained based on the same dataset as the current application. The methods used for the comparison will be the Naive (N) (which has been presented in the previous paragraphs), ARIMA (A), ARIMAX (Ax), Functional Nonparametric regression (FNP) and Semi-functional partial linear (SFPL).

ARIMA models are used to predict hourly electricity demand, fitting one model for the univariate time series coming from each hour of the day, considering all the week together. ARIMAX follows the same procedure but including as external covariates the temperature information. Functional Nonparametric regression, based on Nadaraya-Watson estimators, is applied in the same way as the additive models. That is, to consider scalar response and functional covariate in the 72 models (one for each kind of day and hour). Finally, Semi-functional partial linear adds a linear component with the temperature information as external and scalar covariates. One may take into account that the proposal in [14] includes the two functional regression methods (FNP and SFPL) using both functional or scalar response, with similar results among them. In order to compare with the functional additive models, which work with scalar response, only the FNP and SFPL with scalar response will be considered.

The application in [14] presented also two combined forecasting methods (CF1 and CF2), which are also included in the comparison. CF1 is obtained by simple average of the individual predictions computed by the models indicated above. CF2 consists in the average of the two best individual predictors, separately for each kind of days.

The comparative study with the methods in [14] will be carried out using the relative prediction errors (RPE). Those RPE are computed as the MAPE for each one of the considered procedures in the comparison, divided by the MAPE of a reference method. For that purpose the FLM, which gets the best individual predictions, will be considered as the reference method. It will be compared with the other procedures applied in [14], including the two combined prediction models (CF1 and CF2), and the two functional additive models FKAM and FSAM. Figure 3 summarizes this relative error analysis, divided by kind of day and for all the week.

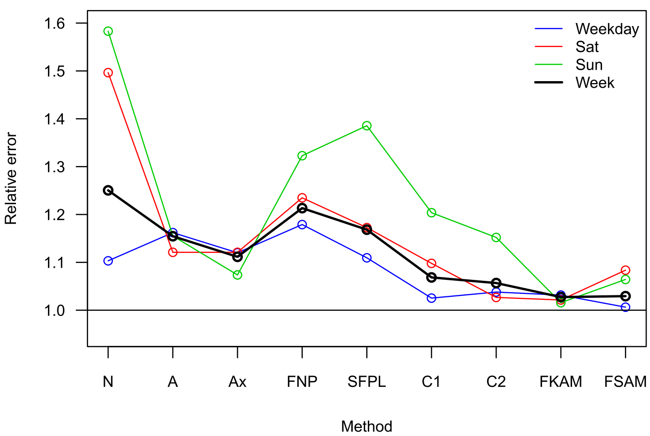

FIGURE 3. Relative prediction errors for electricity demand curves, comparing to the FLM method.

Combined and additive prediction methods seem to be closer to the FLM during the weekdays. Results get worse during the weekend for Naive, among others, and also for the FNP and SFPL (especially on Sunday). In general, as for the weekdays, combined and additive prediction methods are the closest to the FLM, which is the benchmark in this study.

TABLE 2. *p*-values from Diebold-Mariano test for demand predictions.

Method	A	Ax	FNP	SFPL	CF1	CF2	FKAM	FSAM	FLM
N	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A		0.039	0.004	0.763	0.000	0.000	0.000	0.000	0.000
Ax			0.000	0.019	0.001	0.000	0.000	0.000	0.000
FNP				0.000	0.000	0.000	0.000	0.000	0.000
SFPL					0.000	0.000	0.000	0.000	0.000
CF1						0.018	0.007	0.082	0.000
CF2							0.261	0.713	0.000
FKAM								0.573	0.000
FSAM									0.000

Previous paragraphs were based on the pointwise forecast accuracy, measured by means of the MAPE. However, one can also test the statistical significance of the differences observed among the considered methods using the Diebold-Mariano test [30]. Table 2 reports the p-values from Diebold-Mariano test, taking into account that the null hypothesis of the mentioned test is that no difference is found between the accuracy of the methods. Focusing on the comparative and additive models, one can see that both combined predictions and also FKAM and FSAM predictions are similar. However, FLM cannot be assimilate to any of the considered prediction methods.

The predictions obtained for the hourly electricity demand can be complemented through the bootstrap procedures developed in Subsection II-B to build prediction intervals and also prediction density. For that purpose, only FLM and FSAM will be considered, due to its best performance in terms of prediction errors and also in computation times. Both procedures are faster than the FKAM, which is about 10 times slower than the FSAM, resulting in almost unreachable computational cost when dealing with iterative algorithms as the bootstrap. Comparing FLM and FSAM, the FLM is the fastest one being nearly 10 times faster than the FSAM.

PIs are analysed in terms of coverage and Winkler score which allows to assess, jointly, the unconditional coverage and interval width (for details about this Winkler or interval score, see [31] and [32]). Better PIs are those with lower score. Table 3 displays the results of the PIs for each group of days, together with the global outcome for all the week, comparing both additive models and also different confidence levels. Coverages are computed as the proportion of times that each PI covers the corresponding observed demand value, while the length and Winkler score are computed as the mean length and mean Winkler score of the PIs for the corresponding kind of day. In general, coverages are closer to the nominal level in the weekdays, while Saturdays attain the worst behaviour. No major differences are appreciated between the different confidence levels, except for an expected decrease in the length and the corresponding coverage. Comparing the PIs from FLM and FSAM, one can see slightly lower coverages in the FSAM and also shorter intervals, even if both results are close.

TABLE 3. Coverage, mean length and mean Winkler score of the PIs for electricity demand using FLM and FSAM, by kind of day.

	Weekdays	Saturday	Sunday	Week
FLM				
$100 \times \alpha = 5$				
Cov. (Length)	93.4 (5337)	87.3 (4383)	90.1 (5387)	92.1 (5209)
Winkler score	7691.11	9670.88	10096.73	8320.74
$100 \times \alpha = 10$				
Cov. (Length)	88.5 (4131)	81.3 (3546)	85.1 (4310)	87.0 (4074)
Winkler score	6235.79	7318.54	8033.97	6650.02
$100 \times \alpha = 20$				
Cov. (Length)	78.3 (2974)	69.6 (2676)	76.2 (3228)	76.8 (2968)
Winkler score	4863.18	5570.13	6078.33	5139.59
FSAM				
$100 \times \alpha = 5$				
Cov. (Length)	93.3 (5203)	84.7 (4221)	88.5 (5257)	91.4 (5071)
Winkler score	7721.34	11795.41	10366.42	8683.20
$100 \times \alpha = 10$				
Cov. (Length)	88.3 (4042)	78.6 (3415)	83.4 (4261)	86.2 (3985)
Winkler score	6227.46	8574.59	8166.11	6841.67
$100 \times \alpha = 20$				
Cov. (Length)	77.9 (2931)	68.3 (2593)	73.3 (3239)	75.9 (2927)
Winkler score	4868.42	6228.71	6244.84	5261.00

In [20] PIs for electricity demand and price, based on a similar bootstrap procedure as in II-B, but considering FNP and SFPL regression models, were obtained. Comparing the results for electricity demand (see [20, Tables 1, 3, and 4]), one can see that Winkler score is lower for the additive models.

Figure 4 represents the PIs obtained from FLM for each hourly demand along four consecutive days, corresponding to Saturday, Sunday, Monday and Tuesday. This election allows to see also the different pattern of the electricity demand between the group of days, being lower during the weekend, especially on Sundays. In general, the intervals are quite similar among the four days.

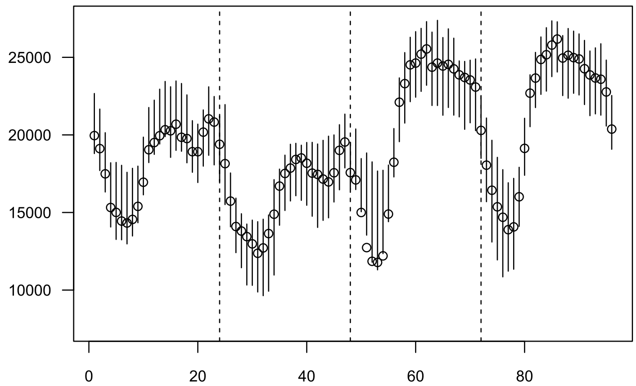

FIGURE 4. Prediction intervals, using FLM from Saturday to Tuesday (2-5 June 2012) in demand, $\alpha = 0.05$.

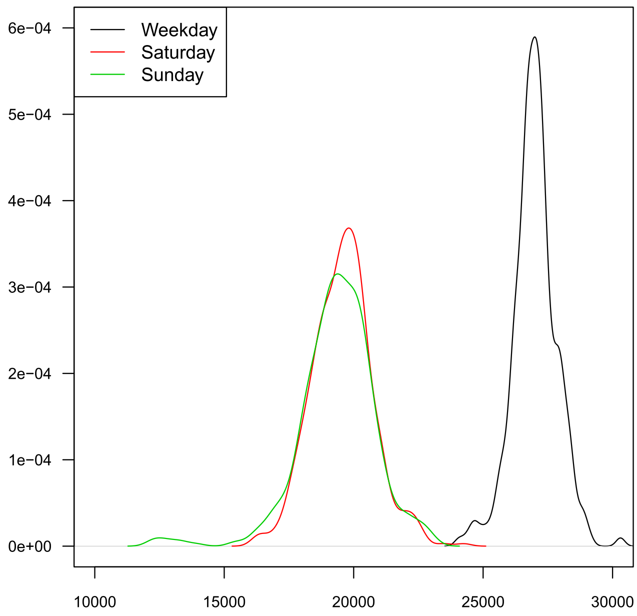

FIGURE 5. Prediction density for the electricity demand, using FLM, for different kind of days.

Finally, to conclude this application, the PD obtained from the FLM is computed under distinct situations. Figures 5 and 6 represent the PD for the three kind of days at the fixed hour 12:00, and also for different hours in the same day, respectively. One can clearly distinguish the behaviour of the electricity demand between weekdays and the weekend, being more similar between Saturday and Sunday. Related to

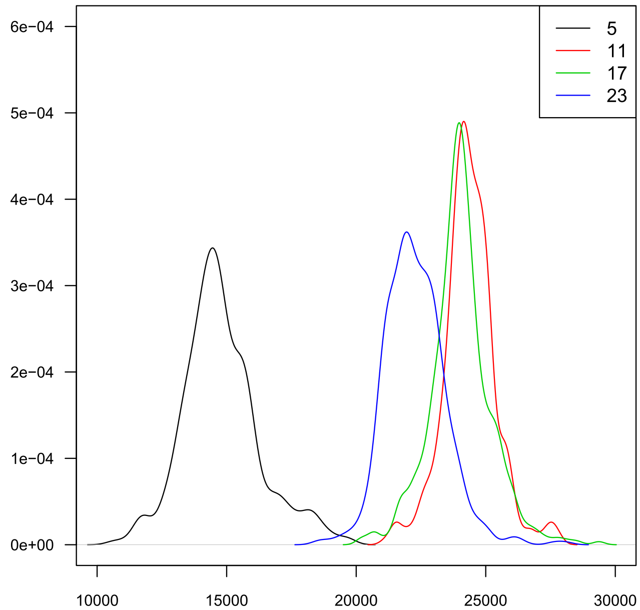

FIGURE 6. Prediction density for the electricity demand, using FLM, for different hours along the day.

the hour of the day, one can recognize the contrast between day and night.

IV. ANALYSIS OF ELECTRICITY PRICE

This section contains a prediction study of electricity price, which follows a similar structure as the one conducted in Section III for the electricity demand.

The data source for the hourly electricity price along the years 2011 and 2012 was again OMIE. In that case, price shares the main features of the electricity demand, with the particularity of the zero-price days. The price of the electricity depends, among other factors, on the energy source and there is a significant influence of the wind power production: when this production increases, the price decreases following a linear relation, reaching even the value zero. Due to this feature, wind power production will be included in the prediction models.

Hourly electricity price will be predicted based on the previous daily price curve, following an autoregressive model as in Section III. Together with the price, also information about the electricity demand (due to the influence over the price, following the market rules) and the wind power production will be added to the models. Both covariates are functional, as they represent the daily curves of demand and wind power production in the day to be predicted. Electricity demand curves will be constructed from the hourly forecasts obtained with FLM model in Section III, whereas observed wind power production will be obtained from Red Eléctrica de España (System Operator in the Spanish Electricity Market).

FLM, FKAM and FSAM will be applied again to predict the hourly electricity price along the year 2012, following the same indications as Section III and considering also different models for each kind of day (weekdays, Saturday and Sunday). The measure of the prediction error will be the weighted mean absolute errors (WMAE). This election is motivated by the zero price days, which could disturb the results given by MAPE. WMAE is computed as:

$$WMAE_{N+1} = \frac{1}{7}\sum_{i=1}^{7} MARE_{N+i},$$

where $N + i$ indicates the ith day in the week to forecast, and the mean absolute relative error (MARE) quantifies the accuracy of the daily forecasts with respect to the weekly mean (WM) of the values to forecast:

$$MARE_{N+i} = \frac{1}{24}\sum_{j=1}^{24} \frac{\left|\widehat{\zeta}_{N+i}(j) - \zeta_{N+i}(j)\right|}{WM_{N+1}},$$

with $WM_{N+1} = \frac{1}{168}\sum_{i=1}^{7}\sum_{j=1}^{24}\zeta_{N+i}(j)$.

Table 4 displays the prediction errors (MARE), comparing the functional additive models with the Naive method. Results are analysed again by kind of day and quarter of the year. As in the demand case, FLM is the best global predictor, being also the best one for each kind of day. FSAM is not far away in this case, being always the second best predictor. FKAM is relatively close to those two models, while the entire three additive models are better than the reference Naive method.

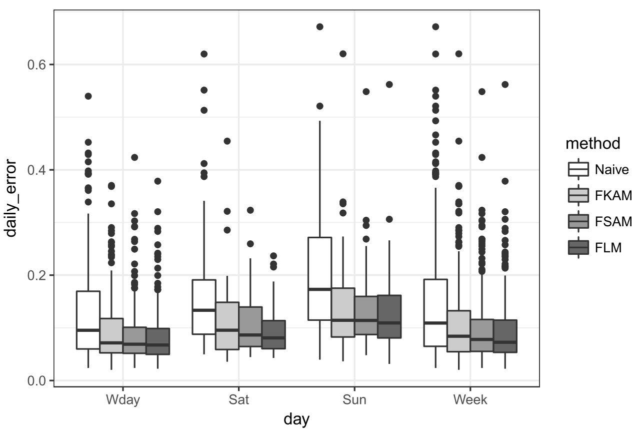

FIGURE 7. Daily errors (MARE) for electricity price curves corresponding to the functional additive and the Naive methods.

Graphically, in Figure 7, one can see the daily error for each kind of day, comparing the same prediction methods as in Table 4. Sundays, as in the demand case, concentrate the worse prediction errors. Weekdays and Saturdays seem to be more stable and the results are quite close among the additive models. Naive is clearly the worst predictor, while FLM seems to be the most accurate.

Figure 8 represents the observed versus predicted price curves along an entire week. Comparing this graph with the analogous Figure 2 in Section III, one can see the different patter of the electricity price curves. Now, the differences between weekdays and weekend are not so remarkable. Naive method is clearly the worst predictor in this example, due to the instability of its predictions. FLM, followed closely by the FSAM, seems to attain the best performance.

TABLE 4. Mean of the MARE for the electricity price curves using the functional additive and the Naive methods. Results are shown by type of day, week, quarter and year.

Method	Q1	Q2	Q3	Q4	Year
			Weekday		
Naive	0.099	0.169	0.092	0.160	0.128
FKAM	0.086	0.122	0.071	0.108	0.096
FSAM	0.073	0.104	0.073	0.097	0.086
FLM	0.072	0.101	0.069	0.092	0.083
			Saturday		
Naive	0.111	0.171	0.161	0.273	0.176
FKAM	0.092	0.105	0.095	0.178	0.115
FSAM	0.087	0.116	0.082	0.148	0.107
FLM	0.075	0.100	0.078	0.140	0.097
			Sunday		
Naive	0.172	0.145	0.220	0.310	0.212
FKAM	0.132	0.124	0.120	0.217	0.147
FSAM	0.124	0.118	0.127	0.196	0.141
FLM	0.113	0.110	0.114	0.184	0.130
			Week		
Naive	0.111	0.166	0.120	0.197	0.147
FKAM	0.093	0.120	0.081	0.133	0.106
FSAM	0.083	0.108	0.082	0.119	0.097
FLM	0.078	0.102	0.076	0.112	0.092

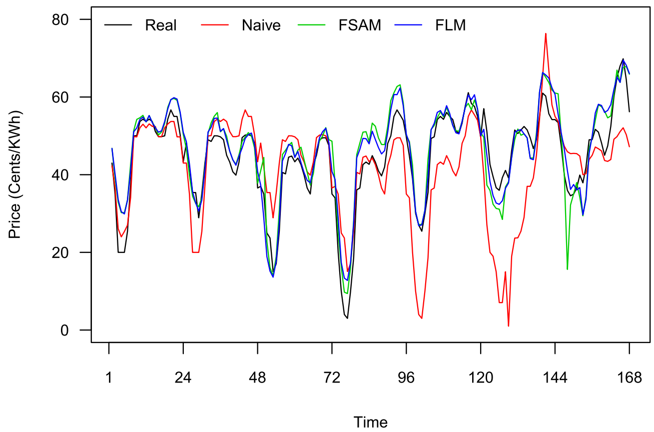

FIGURE 8. Observed and predicted price curves for the week November 26–December 2.

A comparison with the prediction methods presented in [14] will be carried out also in this electricity price study. Relative prediction errors, considering as reference the FLM method, of the different procedures applied in this study are represented in Figure 9. In general, Naive, ARIMA and FNP models are the farthest from the FLM, while FSAM is always very close to it. Comparing to the previous demand case in Figure 3, the differences among the kind of days are now attenuated.

Diebold-Mariano test is also applied to the electricity price predictions, in order to analyse if the results are similar among the different procedures. Table 5 displays the p-values for all the considered methods. Even if one can find some similarities between the predictions, both the two best predictors (FLM and FSAM) are always unique, they do not look like any other.

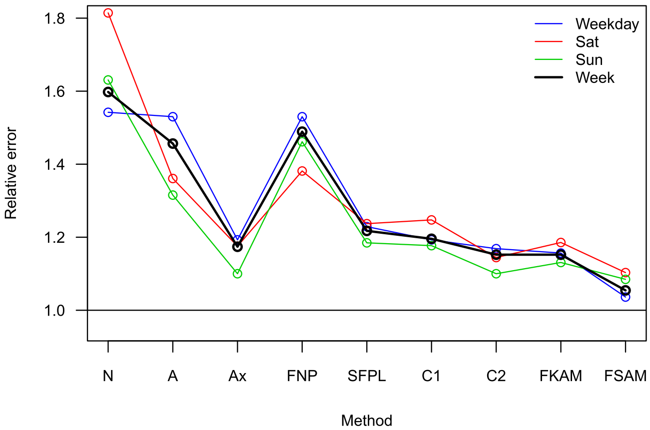

FIGURE 9. Relative prediction errors for electricity price curves, comparing to the FLM method.

TABLE 5. *p*-values from Diebold-Mariano test for price predictions.

Method	A	Ax	FNP	SFPL	CF1	CF2	FKAM	FSAM	FLM
N	0.888	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A		0.002	0.302	0.005	0.005	0.002	0.005	0.001	0.000
Ax			0.000	0.000	0.000	0.108	0.002	0.000	0.000
FNP				0.000	0.000	0.000	0.000	0.000	0.000
SFPL					0.492	0.000	0.386	0.000	0.000
CF1						0.000	0.638	0.000	0.000
CF2							0.000	0.000	0.000
FKAM								0.000	0.000
FSAM									0.000

Finally, this application concludes with the prediction intervals and prediction density obtained from the bootstrap procedures developed in Subsection II-B. Both tools allow to complement the pointwise predictions with additional information which is very useful in the practice.

TABLE 6. Coverage, mean length and mean Winkler score of the PIs for electricity price using FLM and FSAM by kind of day.

	Weekdays	Saturday	Sunday	Week
FLM				
$100 \times \alpha = 5$				
Cov. (Length)	91.5 (20.0)	85.3 (17.9)	85.4 (22.1)	89.7 (20.0)
Winkler score	32.42	36.64	52.38	35.91
$100 \times \alpha = 10$				
Cov. (Length)	85.6 (15.7)	77.7 (14.7)	77.4 (18.3)	83.3 (15.9)
Winkler score	25.69	29.29	39.79	28.24
$100 \times \alpha = 20$				
Cov. (Length)	74.7 (11.6)	66.5 (11.2)	67.1 (14.1)	72.4 (11.9)
Winkler score	20.02	22.91	30.10	21.89
FSAM				
$100 \times \alpha = 5$				
Cov. (Length)	90.0 (18.8)	82.1 (17.8)	77.2 (20.1)	87.0 (18.8)
Winkler score	33.00	43.82	63.30	38.92
$100 \times \alpha = 10$				
Cov. (Length)	83.3 (15.0)	75.8 (14.7)	69.4 (17.0)	80.2 (15.2)
Winkler score	26.14	33.44	45.87	30.03
$100 \times \alpha = 20$				
Cov. (Length)	72.0 (11.2)	65.2 (11.4)	58.6 (13.4)	69.1 (11.6)
Winkler score	20.57	25.36	34.01	23.19

Table 6 summarizes the coverage, length mean and Winkler score mean of the PI from the FLM and FSAM, which combines a good performance in terms of accuracy and computational cost. In general, coverages are slightly lower than those obtained for the electricity demand. Saturdays and Sundays are very similar in terms of coverage for the FLM, while Sundays perform worse in the case of FSAM. As in the case of the demand, those PIs can be compared with the ones obtained in [20] for the FNP and SFPL, based on a similar bootstrap procedure. Again, Winkler score is lower for the additive models, even if the coverages are higher for the two functional regression models (but also with higher lengths).

Figure 10 represents the FLM PIs obtained in four consecutive days, from Saturday to Tuesday, which are quite similar among them. Even if one can see some differences between Saturday, Sunday and the weekdays, those are not so remarkable as in the demand case.

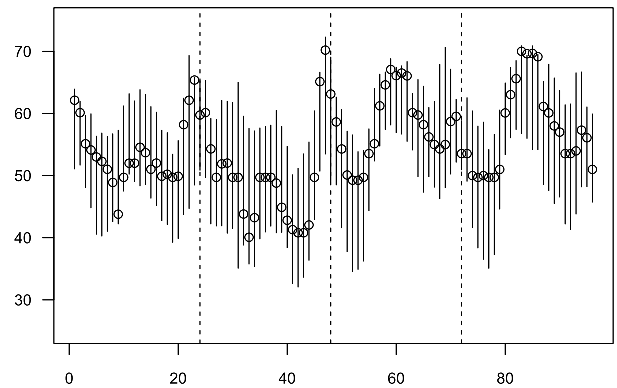

FIGURE 10. Prediction intervals, using FLM from Saturday to Tuesday (2-5 June 2012) in price, $\alpha = 0.05$.

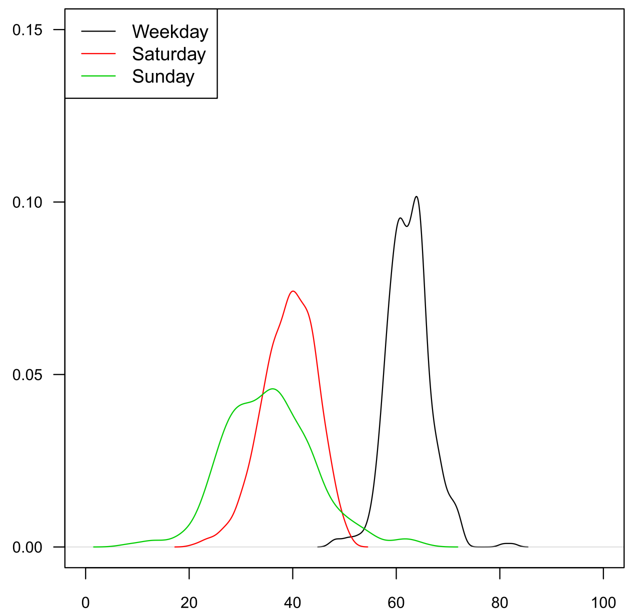

FIGURE 11. Prediction density for the electricity price, using FLM, for different kind of days.

To conclude this application, Figures 11 and 12 represent the PD for the FLM comparing their behaviours by group of days and hours of the day, respectively. Left panel in this

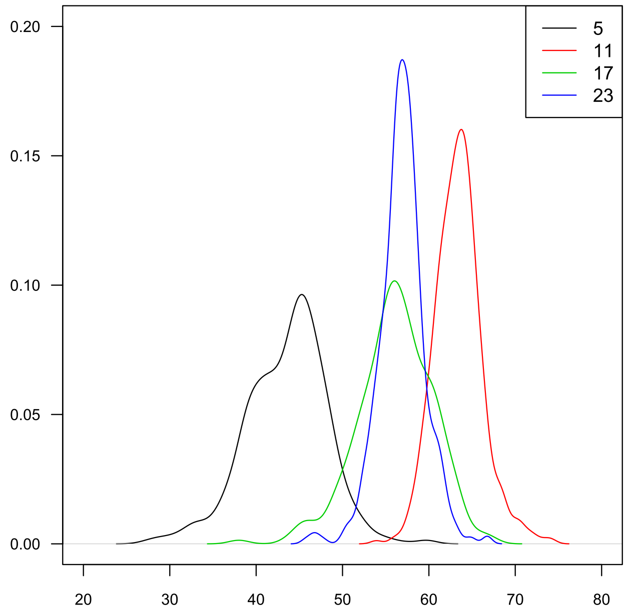

FIGURE 12. Prediction density for the electricity price, using FLM, for different hours along the day.

Figure allows to recognize the particular pattern of the weekdays versus weekend, also with slight differences between Saturday and Sunday. Concerning the hour of the day, one can distinguish the contrast between the early morning and the rest of the day, while afternoon and evening hours are alike.

V. CONCLUSIONS AND PERSPECTIVES

Additive regression models with scalar response and functional covariates have been successfully used for electricity demand and price forecast in the Spanish Electricity Market. FLM attains the best results, indicating a linear relation between the response and the covariates introduced in the model. Both additive models (FKAM and FSAM) have greater flexibility and also give accuracy results. The three considered models improve the results obtained by parametric models (ARIMA and ARIMAX), functional nonparametric regression, semi-functional partial linear regression models and also combined predictions methods used in [14].

Based on the predictions from the functional additive models, a residual-based bootstrap algorithm has been proposed to obtain prediction intervals and to estimate the prediction density. The proposed bootstrap algorithm is able to capture both sources of variability: first one due to errors in model estimation and the second one caused by the innovation error, with the advantage of not assuming hypothesis about the distribution of the data. Computed PIs attain true (unconditional) coverages close to the nominal coverages and a Winkler score lower than the ones in [20], which are based on prediction methods as FNP and SFPL. In general, best results are achieved during the weekdays while they get worse on the weekend. This behaviour is motivated by a twofold cause: on the one hand, the lower sample size of the weekends and, on the other hand, their greater variability. It is also known that bootstrap PIs in regression are often characterized by finite-sample undercoverage (see [33] for details).

Future research related to the additive models in this context includes the extension to various topics: to consider higher orders in the autoregression with the endogenous variable, to seek for new informative covariates that can entry in the additive model and also to adapt the proposed methods to consider functional response (that is, to predict all the daily curve of electricity demand or price as a functional datum) and, as a consequence, to obtain functional prediction intervals (prediction region). All these features make this approach appealing and with plenty of potential for improving.

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REFERENCES

[1] R. Weron, *Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach*. Hoboken, NJ, USA: Wiley, 2006.
[2] L. Suganthi and A. A. Samuel, "Energy models for demand forecasting—A review," *Renew. Sustain. Energy Rev.*, vol. 16, no. 2, pp. 1223–1240, 2012.
[3] T. Hong and S. Fan, "Probabilistic electric load forecasting: A tutorial review," *Int. J. Forecasting*, vol. 32, no. 3, pp. 914–938, 2016.
[4] R. Weron, "Electricity price forecasting: A review of the state-of-the-art with a look into the future," *Int. J. Forecasting*, vol. 30, no. 4, pp. 1030–1081, 2014.
[5] J. Nowotarski and R. Weron, "Recent advances in electricity price forecasting: A review of probabilistic forecasting," *Renew. Sustain. Energ Rev.*, vol. 81, no. 1, pp. 1548–1568, 2018.
[6] J. O. Ramsay and B. W. Silverman, *Functional Data Analysis*. New York, NY, USA: Springer-Verlag, 2005.
[7] F. Ferraty and P. Vieu, *Nonparametric Functional Data Analysis* (Springer Series in Statistics). New York, NY, USA: Springer-Verlag, 2006.
[8] J. Antoch, L. Prchal, M. R. De Rosa, and P. Sarda, "Electricity consumption prediction with functional linear regression using spline estimators," *J. Appl. Stat.*, vol. 37, no. 12, pp. 2027–2041, 2010.
[9] E. Paparoditis and T. Sapatinas, "Short-term load forecasting: The similar shape functional time-series predictor," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3818–3825, Nov. 2013.
[10] B. López-Cabrera and F. Schulz, "Forecasting generalized quantiles of electricity demand: A functional data approach," *J. Amer. Stat. Assoc.*, vol. 112, no. 517, pp. 127–136, 2017.
[11] P. Gaillard and Y. Goude, "Forecasting electricity consumption by aggregating experts; how to design a good set of experts," in *Modeling and Stochastic Learning for Forecasting in High Dimensions*. Cham, Switzerland: Springer, 2015, pp. 95–115.
[12] Y. Chen and B. Li, "An adaptive functional autoregressive forecast model to predict electricity price curves," *J. Bus. Econ. Stat.*, vol. 35, no. 3, pp. 371–388, 2017.
[13] G. Aneiros, J. M. Vilar, R. Cao, and A. Muñoz-San-Roque, "Functional prediction for the residual demand in electricity spot markets," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4201–4208, Nov. 2013.
[14] G. Aneiros, J. Vilar, and P. Raña, "Short-term forecast of daily curves of electricity demand and price," *Int. J. Elect. Power Energy Syst.*, vol. 80, pp. 96–108, Sep. 2016.
[15] D. Liebl, "Modeling and forecasting electricity spot prices: A functional data perspective," *Ann. Appl. Stat.*, vol. 7, no. 3, pp. 1562–1592, 2013.
[16] J. M. Vilar, R. Cao, and G. Aneiros, "Forecasting next-day electricity demand and price using nonparametric functional methods," *Int. J. Elect. Power Energy Syst.*, vol. 39, no. 1, pp. 48–55, 2012.
[17] S. Fan and R. J. Hyndman, "Short-term load forecasting based on a semi-parametric additive model," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 134–141, Feb. 2012.

[18] Y. Goude, R. Nedellec, and N. Kong, "Local short and middle term electricity load forecasting with semi-parametric additive models," *IEEE Trans. Smart Grid*, vol. 5, no. 1, pp. 440–446, Jan. 2014.

[19] F. Serinaldi, "Distributional modeling and short-term forecasting of electricity prices by generalized additive models for location, scale and shape," *Energy Econ.*, vol. 33, no. 6, pp. 1216–1226, 2011.

[20] J. Vilar, P. Raña, and G. Aneiros, "Prediction intervals for electricity demand and price using functional data," *Int. J. Elect. Power Energy Syst.*, vol. 96, pp. 457–472, Mar. 2018.

[21] H. Cardot, F. Ferraty, and P. Sarda, "Functional linear model," *Stat. Probab. Lett.*, vol. 45, no. 1, pp. 11–22, 1999.

[22] H. Cardot, F. Ferraty, and P. Sarda, "Spline estimators for the functional linear model," *Stat. Sinica*, vol. 13, pp. 571–591, 2003.

[23] A. Goia, "A functional linear model for time series prediction with exogenous variables," *Stat. Probab. Lett.*, vol. 82, no. 5, pp. 1005–1011, 2012.

[24] F. Ferraty and P. Vieu, "Additive prediction and boosting for functional data," *Comput. Stat. Data Anal.*, vol. 53, no. 4, pp. 1400–1413, 2009.

[25] H.-G. Müller and F. Yao, "Functional additive models," *J. Amer. Stat. Assoc.*, vol. 103, no. 484, pp. 1534–1544, 2008.

[26] J. Fan and Q. Yao, "Efficient estimation of conditional variance functions in stochastic regression," *Biometrika*, vol. 85, no. 3, pp. 645–660, 1998.

[27] P. Raña, G. Aneiros, and J. M. Vilar, "Detection of outliers in functional time series," *Environmetrics*, vol. 26, no. 3, pp. 178–191, 2015.

[28] J. M. Vilar, P. Raña, and G. Aneiros, "Using robust FPCA to identify outliers in functional time series, with applications to the electricity market," *Stat. Oper. Res. Trans.*, vol. 40, no. 2, pp. 321–348, 2016.

[29] M. Febrero-Bande and M. O. de la Fuente, "Statistical computing in functional data analysis: The R package fda. usc," *J. Stat. Softw.*, vol. 51, no. 4, pp. 1–28, 2012.

[30] F. X. Diebold and R. S. Mariano, "Comparing predictive accuracy," *J. Bus. Econ. Stat.*, vol. 13, no. 3, pp. 253–263, 1995.

[31] R. L. Winkler, "A decision-theoretic approach to interval estimation," *J. Amer. Stat. Assoc.*, vol. 67, no. 337, pp. 187–191, 1972.

[32] T. Gneiting and A. E. Raftery, "Strictly proper scoring rules, prediction and estimation," *J. Amer. Stat. Assoc.*, vol. 102, no. 477, pp. 359–378, 2007.

[33] L. Pan and D. N. Politis, "Bootstrap prediction intervals for linear, nonlinear and nonparametric autoregressions," *J. Stat. Planning Inference*, vol. 177, pp. 1–27, Oct. 2016.

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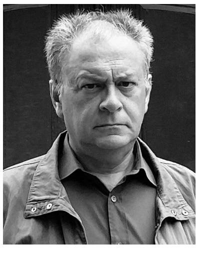

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