IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received December 19, 2017, accepted January 31, 2018, date of publication February 12, 2018, date of current version April 4, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2805334

A New Autonomous Adaptive MAC Protocol in Wireless Networks

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This work was supported by the Basic Science Research Program through the National Research Foundation of Korea, Ministry of Education, under Grant NRF-2017R1A2B4008581.

ABSTRACT Efficient sharing of the network spectrum is required to meet the exponentially growing demand of wireless communications, and the role of the medium access control (MAC) protocols has been becoming more important. In this regard, a novel MAC protocol called the renewal access protocol (RAP) was recently proposed. It was shown that the RAP achieves optimal throughput, high short-term fairness, and near-optimal delay performances when the number of nodes in the network is known to all nodes. However, it is not easy for a node to know the number of nodes in the network. So we propose an adaptive version of the RAP called the adaptive renewal access protocol (A-RAP) in this paper. A node equipped with the A-RAP estimates the number of nodes utilizing two dimensional backoff selection state, the estimated number of nodes and phase, based on its transmission results. We carefully determine two key parameters of the A-RAP, the adjustment probabilities and the numbers of phases. We also tackle the outlier problem in the A-RAP that is recently found and provide a solution to the problem. Numerical and simulation results are provided to verify that the proposed A-RAP achieves good throughput as the RAP. The results also show that the A-RAP achieves better short-term fairness than other MAC protocols proposed in the open literature.

INDEX TERMS Adaptive MAC protocol, backoff, distributed coordination function, estimated number of nodes, short-term fairness, throughput.

I. INTRODUCTION

According to recent studies, mobile data traffic will significantly increase in a few years [1]–[3]. Such growth in mobile data traffic is driven by the worldwide spreads of smart devices such as smartphones or the Internet of Thing (IoT) devices. One of the most severe adverse effects of the explosion of mobile data traffic is the spectrum shortage and hence a more efficient scheme in spectrum usage than existing wireless access schemes, should be provided to resolve it. As a promising solution, efficient sharing of the network spectrum has been more intensively considered in recent years. For instance, the use of 5GHz unlicensed band by LTE networks is being ahead of the commercialization [4].

The efficiency of a spectrum sharing scheme is more conspicuous when multiple users coexist in the same channel. In such a channel, sharing can be referred to as scheduling and MAC protocols have crucial roles for it. Moreover, recent trends for using unlicensed or free bands (due to the shortage of licensed spectrum bands) emphasize the necessity of efficient scheduling schemes in distributed manners. For instance, in unlicensed or free spectrum bands, any users can grab the channel without any regulations provided that a few technical standards are satisfied. Therefore, in such circumstances, the contentions of active users (users having data to send) usually occur in disorder and it is obvious that a distributed scheduling scheme is more suitable for scheduling them.

Among various distributed scheduling schemes *random access* schemes have been extensively studied from various perspectives. Whereas random access schemes have some advantages such as relatively easy implementation, they have a crucial and inevitable disadvantage of packet loss due to packet collision. Accordingly a random access scheme should be equipped with efficient collision resolution techniques. Among such techniques, random backoff, which randomly distributes the access instants of users, and Listen-Before-Talk, which enables the protection of ongoing transmissions, have been widely used and still get much attention for designing efficient random access schemes. A representative random access scheme using both techniques is Carrier Sense Multiple Access (CSMA) and it is usually implemented in the Wireless Local Area Network (WLAN). Therefore, analyzing the MAC protocol in the WLAN is a good starting point to design a new scheme for future random access networks.

The IEEE 802.11 distributed coordination function (DCF) is the *de facto* standard for the MAC protocol in today's WLAN. The key component of the IEEE 802.11 DCF is the binary exponential backoff (BEB) and it determines the network performance. In a seminal paper [5], a widely adopted model for the IEEE 802.11 DCF was provided. It was shown in [5] that the BEB can achieve quite favorable throughput performance if the initial backoff window is carefully selected. However, it has been also addressed in the literature that the BEB shows very poor performance in the fairness perspective [6]–[8]. This fact motivated a large number of research works that provided several modified backoff schemes improving the performance [9]–[11].

Recently a new MAC protocol called the *Renewal Access Protocol* (RAP) was proposed in [12]. In the RAP, each node selects a new backoff counter value based on a so-called selection distribution and transmits its packet whenever the backoff counter value becomes zero as in the IEEE 802.11 DCF. Note that the RAP allows any probability distribution as its selection distribution. It was shown in [12]–[14] that, if a Poisson distribution is used as the selection distribution, then the resulting RAP achieves optimal throughput, high short-term fairness, and near-optimal delay performances. However, deriving the optimal parameters in the RAP requires knowing the number of active nodes in the network and it is not easy for a node to know the exact number of active nodes in the network. Therefore, to accommodate the time varying number of nodes with the RAP, we need to design an adaptive version of the RAP.

The adaptivity of the backoff scheme in WLAN has been studied extensively in the literature. Existing adaptive backoff schemes can be categorized into two groups depending on whether it utilizes a metric to measure the occupancy of the channel: passively or actively. A node with a passive scheme adjusts its backoff parameters using transmission results without further extra works like calculating the collision probability or observing network status. The BEB is a representative example of a passive scheme. Due to the simplicity of passive schemes, there have been a large number of research works on diverse variations of the BEB which are still passive.

These passive schemes can be further classified into two subgroups based on whether or not each node uses the transmission results of other nodes: individually or cooperatively. In most of the passive schemes, when a packet transmission occurs, only the transmitting nodes adjust backoff parameters and the other nodes just keep their parameters. Such examples include polynomial backoff (PB) [15], exponential increase and exponential decrease (EIED) [16], and schemes provided in [10], [11], and [17]–[19]. On the contrary, there are some schemes where nodes overhear transmission results of other nodes and update their backoff parameters [9], [20], [21]. Generally cooperative update of backoff parameters shows better performance than individual update.

On the other hand, in active schemes, each node observes the network to gather data and actively employs them for measuring network status such as the number of active nodes in the network [22]–[28]. In general, protocols of this group introduce metrics to gauge the occupancy of the channel and use the matching backoff parameter. For example, in the idle sense [25], each node adjusts its contention window by counting the number of time slots in the idle period. Thus they provide more favorable performance, but have higher complexity compared to the first group (passive schemes). Filter algorithms were also proposed to tune the CW size based on the estimated number of active nodes [29], [30].

However, all these algorithms but a Bayesian approach [27] require that every node agrees to use the same adjusted backoff parameters. This involves a serious problem that a new node entering the network needs to know "the backoff parameters." Although the Bayesian work is an exceptional case since it includes a distributed scenario with the help of game theory, it requires additional tasks to detect the "rogue" terminal and adjusts only the minimum contention window size [27]. From this standpoint, the improved version of the idle sense proposed in [31] is a distinguishing example because each node has a different contention window based on its own estimate, and still achieves good network performance.

In this work, while cooperative updating or active backoff schemes accomplish good performance in general, we focus on individual updating passive backoff schemes. This is largely due to the recent trends of pursuing simple implementation. For instance, utilizing unlicensed or free spectrum bands is open yet to any kinds of devices. Thus, such bands can be used by some devices with the minimum and simplest technical standards. In this regard, designing a backoff scheme which can be implemented as simple as possible is desirable for future scalability or interoperability issues.

In this work, we design a new adaptive MAC protocol based on the RAP, called the *Adaptive Renewal Access Protocol (A-RAP)*. The A-RAP is a hybrid version of the above two groups (passive and active) that achieves the benefits of each group. It operates as a passive individual scheme, but is still able to estimate the number of active nodes as an active scheme. That is, each node with the A-RAP adjusts its backoff parameters individually only when it transmits a packet and adjusts the backoff counter selection distribution based on the estimated number of active nodes in the network.

Each node has a two dimensional '*Backoff Selection State* (BSS)' (m, i) to adjust its backoff behavior. To make use of the advantages of the RAP, each node with the A-RAP selects a backoff counter value according to the Poisson selection distribution whose parameter is determined by its BSS. The first element of the BSS, m, is called '*Estimated Number of Nodes* (ENN)'. The ENN indicates each node's estimation for the number of active nodes in the network. Each active node selects a backoff counter value according to its ENN value, i.e., the parameter of the Poisson selection distribution is determined by the ENN. The second element of the BSS, i, is called '*phase*'. The phase is used to control the transition speed between adjacent ENN values. In other words, the ENN is adjusted only when the phase moves beyond given boundary values that we will explain later.

The main contributions of this paper are summarized as follows.

- We develop a novel adaptive MAC protocol, the A-RAP, in which each node can estimate the number of active nodes in the network using its own transmission results without extra calculations such as the estimation of the collision probability.
- The A-RAP is an autonomous adaptive MAC protocol in which a node adjusts its backoff parameters independently without sharing information with other nodes.
- The way of adjustments of the BSS is elaborately designed based on the mathematical analysis. We model the adjustment process of the BSS as a lazy random walk and provide some conditions for a desirable adjustment.
- The A-RAP is built based on the RAP, thus it shows high short-term fairness and energy efficiency benefited from the RAP [12]–[14].

Our proposed A-RAP is shown to achieve good network performances including throughput and short-term fairness as the RAP. However, we recently observed that the A-RAP has so-called the outlier problem. That is, nodes that hold its ENN larger than the ENNs of other nodes, called outliers, appear sometimes especially when the number of nodes in the network is large. To resolve this outlier problem in the A-RAP, we introduce a method of forcing a node that keeps its ENN unchanged during a certain time period to decrease its ENN. We analyze the trade-off between the beneficial effects and the side effects to determine a proper length of the time period.

The rest of the paper is organized as follows. We provide a general description of the A-RAP in Section II. In Section III we elaborate on rigorous analysis to determine the parameters for the A-RAP. We also explain the outlier problem in the A-RAP and offer a solution. We compare the performance of the A-RAP with that of other backoff schemes and validate our analysis through numerical and simulation studies in Section IV. Finally, we provide our conclusions in Section V.

II. DESCRIPTION OF THE A-RAP

In this section we first describe the adaptive MAC protocol based on the RAP, called the A-RAP, that is adaptive to the time varying network environment such as the number of nodes. As said before, the RAP in [12] requires knowing the number of active nodes in the network to achieve the optimal throughput. This is because the optimal access probability (or the optimal expectation of the Poisson selection distribution) is shown to be a function of the number of nodes. To tackle this issue, we combine the RAP in [12] with an adaptive scheme while still providing good performance as the RAP.

The main characteristics and operations of the A-RAP are summarized as follows.

1) Each node has a two dimensional '*Backoff Selection State* (BSS)' (m, i). The first element of the BSS, m, is called '*Estimated Number of Nodes* (ENN)' and it describes each node's estimation for the number of active nodes in the network. The second element of the BSS, i, is called '*phase*' and it is used to control the transition between adjacent ENN values.
2) The ENN determines the backoff counter selection distribution. That is, when a node has m as its ENN, it selects a backoff counter value according to the Poisson selection distribution with mean μ_m where μ_m is the optimal expectation given in [14] for a network consisting of m nodes. If the ENN values of some nodes are the same, their backoff behaviors are the same regardless of their phase values.
3) Each ENN value has a number of phases and upper and lower boundaries for phases. For each node the transition between adjacent phases is determined by its packet transmission result, and the ENN is adjusted only if the phase moves beyond one of the two boundaries of the ENN.

In what follows, we provide a detailed description of the A-RAP.

A. GENERAL DESCRIPTION

With the A-RAP, each node maintains BSS (m, i). We assume $m \geq 2$ since we are interested in a contending network. We denote ENN-m to indicate that the value of the ENN is $m(\geq 2)$. A node with ENN-m can take values on $\{-\lfloor L_m/2 \rfloor, \ldots, -1, 0, 1, \ldots, \lfloor (L_m-1)/2 \rfloor\}$ for its phase where $L_m \geq 1$. The node accesses the channel with a backoff counter selection distribution with the expectation μ_m regardless of its phase value. In this paper we use the Poisson distribution as the selection distribution to exploit the results in [12] and μ_m is the optimal expectation given in [14] for a network consisting of m nodes.

We now explain the motivation of the A-RAP and describe the transition of the BSS in the A-RAP. Ideally, the A-RAP is optimized when all nodes have the same ENN values exactly equal to the *actual* number of nodes in the network. In this case, the nodes experience more successful packet transmissions than collided packet transmissions. This implies that the current value of the ENN might provide a good estimate for the number of nodes in the network when a node has a successful packet transmission and hence there might be no need to adjust the current ENN value. On the other hand, when a node has a collided packet transmission, it is likely that the current ENN value might not provide a good estimate for the number of nodes in the network and hence the node should adjust its ENN value. From this viewpoint, the A-RAP

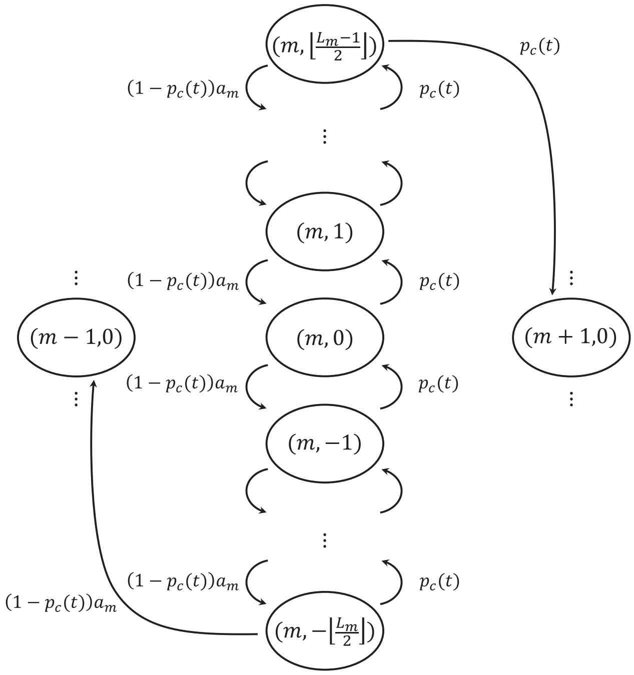


FIGURE 1. Transition diagram of the BSS at time t. The probability that a node with the BSS $(m, 0)$ increases its phase is equal to $p_c(t)$; The probability that it decreases its phase is equal to $a_m(1 - p_c(t))$.

is designed to adjust its ENN value *slowly* when a node has a successful packet transmission but to adjust its ENN value *relatively fast* when a node has a collided packet transmission. To this end, we introduce the adjustment probabilities $\{a_m\}_{m\geq 2}$ to adjust the BSS slowly in the A-RAP when a node has a successful packet transmission. On the other hand, a node adjusts its BSS immediately when it has a collided packet transmission.

We tag an arbitrary node, called the tagged node. Let the tagged node be in BSS (m, i). The tagged node can adjust its BSS (m, i) only after its packet transmission and the adjustment depends on the transmission result. The following describe the detailed operations of the tagged node with the A-RAP.

1) After a successful packet transmission
   a) When $-\lfloor L_m/2 \rfloor < i \leq \lfloor (L_m - 1)/2 \rfloor$, the tagged node decreases its *phase* value by one with probability a_m.
   b) When $i = -\lfloor L_m/2 \rfloor$, the tagged node decreases its *ENN* value by one and sets its phase value to zero with probability a_m.
2) After a collided packet transmission
   a) When $-\lfloor L_m/2 \rfloor \leq i < \lfloor (L_m - 1)/2 \rfloor$, the tagged node increases its *phase* value by one.
   b) When $i = \lfloor (L_m - 1)/2 \rfloor$, the tagged node increases its *ENN* value by one and sets its phase value to zero.

The transition diagram of the BSS of the tagged node at time t is illustrated in Fig. 1 where $p_c(t)$ denotes the collision probability at time t.

B. BACKOFF COUNTER SELECTION DISTRIBUTION

As mentioned above, to exploit the results in [12], the A-RAP uses the Poisson selection distribution to select a *backoff counter* value. That is, when a node is in BSS (m, i), the node selects a backoff counter value X that satisfies $X = Z + 1$ where Z is a Poisson random variable with parameter $\mu_m - 1$.

To determine the value of μ_m, we consider a wireless network consisting of N homogeneous nodes with the RAP and let $\tau^*(N)$ be the optimal access probability which maximizes the throughput. Here, the throughput is defined by the fraction of time that the channel is used to successfully transmit payload bits as given in [12]. We also define c^* by the unique solution of the following equation [14]:

$$(1-c)e^c - \frac{E[B_c]}{1+E[B_c]} = 0 \tag{1}$$

where $E[B_c]$ denotes the average duration of a time period for a collided packet transmission. Since it can be easily verified that $0 < c^* < 1$ for all practical values of $E[B_c]$, we assume $0 < c^* < 1$ from now on. The following theorem provides the relation between $\tau^*(N)$ and c^*.

Theorem 1 [14]: $\lim_{N\to\infty} N\tau^*(N) = c^*$.

It is shown in [14] that $\tau^*(N)$ and c^*/N are almost the same for a large set of different values of N and $E[B_c]$. Moreover, it is also shown that the expectation μ of the Poisson selection distribution and the corresponding access probability τ have the relation of $\tau = 1/\mu$.[1] Since the A-RAP is an adaptive version of the RAP that mimics the RAP by trying to accurately estimate the number of nodes in the network, for a node with ENN-m, it is reasonable to select its backoff counter value according to the Poisson selection distribution with expectation $\mu_m := m/c^*$.

Before we go further, we have to discuss how to compute c^* because it is necessary to get $E[B_c]$ for it. In a practical wireless network, due to the adaptive coding scheme based on the channel conditions it might be difficult to get *a priori* the expected length of $E[B_c]$. Therefore, each node should first observe some collided packet transmission results in the network and estimate $E[B_c]$ based on the observation results when it is initially connected to the wireless network. Obviously, the estimation contains some error and hence it is important to investigate the impact of the error in $E[B_c]$ on throughput performance. Fortunately, we see that the impact of the error in $E[B_c]$ is not significant. To see in detail, we first consider $rE[B_c](r > 0)$ instead of $E[B_c]$ in eq. (1) and solve the equation to get a solution, denoted by c_r^*. We then use the solution c_r^* for τ^*, i.e., $\tau^* = c_r^*/m$ when the ENN is m and simulate the wireless network to get throughput performance. The results when $N = 30$ is plotted in Fig. 2. In the figure we use $0.4 \leq r \leq 1.6$. Note that we use the exact values of

[1] Embedded epochs in [12] are revised. We consider the embedded epochs where the backoff counter value is decremented by one and exclude time points where the backoff counter value is newly selected. With the new embedded epochs, if we follow the same derivation as in [12], we can easily obtain that $\tau = 1/\mu$.

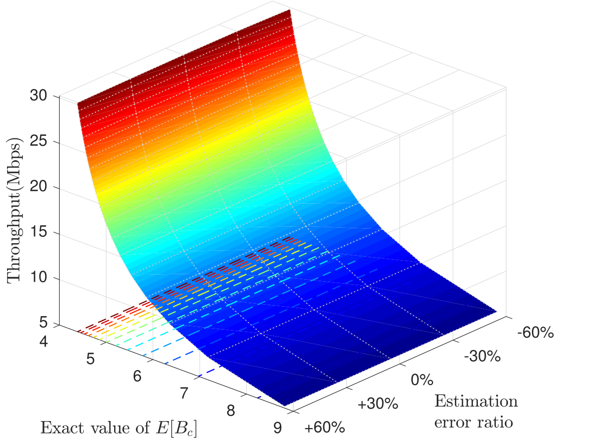


FIGURE 2. The impact of the errors in the value of $E[B_c]$ on throughput.

$E[B_c]$ and τ^* when $r = 1$. As seen in the figure, the impact of the error in $E[B_c]$ on throughput is not significant and we can conclude that the estimation of $E[B_c]$ is not an issue and some initial estimation on $E[B_c]$ for each node is enough to get c^*.

III. DETERMINATION OF PARAMETERS OF THE A-RAP

A. ADJUSTMENT PROBABILITIES

To achieve the optimal throughput and high short-term fairness as the RAP, we need to carefully determine the adjustment probabilities $\{a_m\}_{m\geq 2}$. To this end, let N be the actual number of nodes in the network, and (m_k, i_k) denote the BSS of node k, $1 \leq k \leq N$. In this case, node k uses the Poisson selection distribution with expectation $m_k/c^* - 1$, and hence its access probability τ_k is c^*/m_k.

For our purpose, we consider the embedded time epochs where packet transmissions occur in the network and the embedded time epochs are indexed as $t = 1, 2, \cdots$. In this section we simply use the term *time* to indicate an embedded time epoch. When node j with BSS (m_j, i_j) transmits a packet at time t, the collision probability, denoted by $p_{c,j}(t)$, is given by

$$p_{c,j}(t) = 1 - \frac{\prod_{k=1}^{N}(1 - c^*/m_k)}{1 - c^*/m_j}.$$

So, when node j transmits a packet at time t, $a_m(1 - p_{c,j}(t))$ is the probability that node j decreases its BSS, i.e., decreases either its ENN value or its phase value. On the other hand, $p_{c,j}(t)$ is the probability that node j increases its BSS, i.e., increases either its ENN value or its phase value. It is worth noting that, when $a_m(1 - p_{c,j}(t)) \geq p_{c,j}(t)$, node j is more likely to decrease its BSS. Otherwise, node j is more likely to increase its BSS. For later use, we say that node j has a negative net drift if $a_m(1 - p_{c,j}(t)) \geq p_{c,j}(t)$ is satisfied and has a positive net drift if $a_m(1 - p_{c,j}(t)) < p_{c,j}(t)$ is satisfied.

The key idea to determine the adjustment probabilities $\{a_m\}_{m\geq 2}$ is that the adjustment in the ENN values of nodes should be performed to guarantee the convergence of the

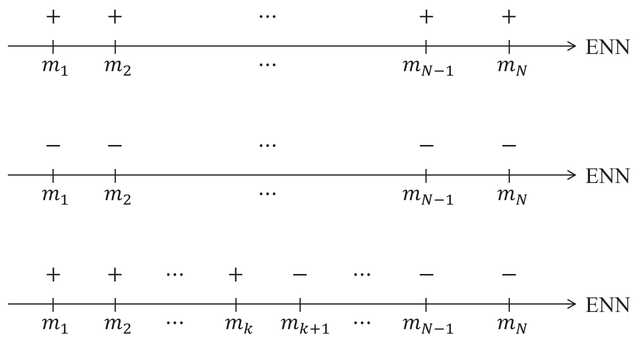


FIGURE 3. Three possible scenarios on the net drifts of the nodes in the network where '+' and '-' denote a positive net drift and a negative net drift, respectively. The ENN values are denoted by m_k for $1 \leq k \leq N$ and m_k's are arranged in the ascending order.

ENN values toward the actual number of nodes in the network. To this end, we consider two arbitrary nodes in the network. Without loss of generality we consider node 1 and node 2 with ENN-m_1 and ENN-m_2, repectively, and assume $m_1 \leq m_2$. For the convergence of the ENN values of nodes in the network, we first provide a sufficient condition under which it is impossible to occur the event where node 1 (having a smaller ENN value) has a negative net drift while node 2 (having a larger ENN value) has a positive net drift. Note that if such an event occurs, the ENN values of two nodes are likely to diverge.

Theorem 2: Suppose that $\{(a_m + 1)/(1 - c^*/m)\}_{m\geq 2}$ is an increasing sequence. Consider node 1 and node 2 with ENN-m_1 and ENN-m_2, respectively, and assume $m_1 \leq m_2$. Assume further that the packet transmissions in the network do not change the ENN values m_1 and m_2 (but change the phase values of two nodes). Then the following statements hold.

(i) *If node 1 has a negative net drift, then node 2 also has a negative net drift.*

(ii) *If node 2 has a positive net drift, then node 1 also has a positive net drift.*

Proof: The proof is given in Appendix A. ■

From now on, we assume that $\{(a_m + 1)/(1 - c^*/m)\}_{m\geq 2}$ is an increasing sequence. To illustrate Theorem 2 we provide Fig. 3 that covers all the possible scenarios on the net drifts of the nodes in the network.

We next consider the following observation. It is reasonable that, if the ENN values of all nodes are greater than the actual number of nodes N, then the values of $\{a_m\}_{m\geq 2}$ should be determined to guarantee negative net drifts for all nodes. Similarly, if the ENN values of all nodes are smaller than N, then the values of $\{a_m\}_{m\geq 2}$ should be determined to guarantee positive net drifts for all nodes. The following theorem is inspired by this important observation and provides upper and lower bounds for $\{a_m\}_{m\geq 2}$.

Theorem 3: Let N be the actual number of nodes in the network. If $\{a_m\}_{m\geq 2}$ satisfies

$$\frac{1}{(1 - c^*/m)^{m-2}} - 1 \leq a_m < \frac{1}{(1 - c^*/m)^m} - 1, \quad m \geq 2, \tag{2}$$

then the following statements hold.

(i) *Every node has a negative net drift if* $N < m_k$ *for* $1 \leq k \leq N$.

(ii) *Every node has a positive net drift if* $m_k < N$ *for* $1 \leq k \leq N$.

Proof: The proof is given in Appendix B. ■

The following theorem provides the values of $\{a_m\}_{m \geq 2}$ for which the sequence $\{(a_m + 1)/(1 - c^*/m)\}_{m \geq 1}$ becomes increasing, which is crucial in our analysis.

Theorem 4: Suppose

$$a_m = \frac{1}{(1 - c^*/m)^{m-r-1}} - 1, \quad m \geq 2. \tag{3}$$

If $r \in [c^*, 1]$, *then* $\{(a_m + 1)/(1 - c^*/m)\}_{m \geq 2}$ *is an increasing sequence.*

Proof: The proof is given in Appendix C. ■

Note that the main purpose of backoff adjustments is the contention resolution. In this regard, lowering access aggressiveness (by taking the minimum possible value of a_m) is more desirable for the network. Therefore, we propose to use the values of $\{a_m\}_{m \geq 2}$ in (3) when $r = 1$, that is,

$$a_m = \frac{1}{(1 - c^*/m)^{m-2}} - 1, \quad m \geq 2. \tag{4}$$

The importance of Theorem 2 and Theorem 3 is explained in the following from the viewpoint of the convergence in the ENNs of all nodes. Let $\widetilde{m}$ and $\widehat{m}$ denote the minimum and the maximum values among the ENN values of all nodes in the network, respectively. Then we consider two nodes, say, node 1 and node 2 with respective ENN values $\widetilde{m}$ and $\widehat{m}$ and the following three cases.

1) $\widetilde{m} \leq \widehat{m} < N$: Since both nodes have a positive net drift by Theorem 3, the ENN values are likely to move toward N.

2) $N < \widetilde{m} \leq \widehat{m}$: Since both nodes have negative net drifts by Theorem 3, the ENN values move toward N.

3) $\widetilde{m} \leq N \leq \widehat{m}$: In this case we first focus on a node with $\widetilde{m}$ and suppose that the node has a negative net drift. Then, by Theorem 2 all nodes have negative net drifts. This implies that, for $1 \leq j \leq N$,

$$(a_{m_j} + 1) \prod_{k=1, k \neq j}^{N} \left(1 - \frac{c^*}{m_k}\right) \geq 1 \tag{5}$$

and all m_j's are likely to decrease. Then, for any j, since $\{a_m\}_{m \geq 2}$ is an increasing sequence, the left hand side of (5) is likely to decrease. Since all nodes cannot have negative net drifts in the long run (by Theorem 3, statement (ii)), the node with $\widetilde{m}$ should have a positive net drift at some time point.

We next focus on a node with $\widehat{m}$ and suppose that the node has a positive net drift. Then, by Theorem 2 all nodes have positive net drifts. This implies that, for $1 \geq j \geq N$

$$(a_{m_j} + 1) \prod_{k=1, k \neq j}^{N} \left(1 - \frac{c^*}{m_k}\right) < 1 \tag{6}$$

and all m_j's are likely to increase. Then, for any j, since $\{a_m\}_{m \geq 2}$ is an increasing sequence, the left hand side of (6) is likely to increase. Since all nodes cannot have positive net drifts in the long run (by Theorem 3, statement (i)), the node with $\widehat{m}$ should have a negative net drift at some time point.

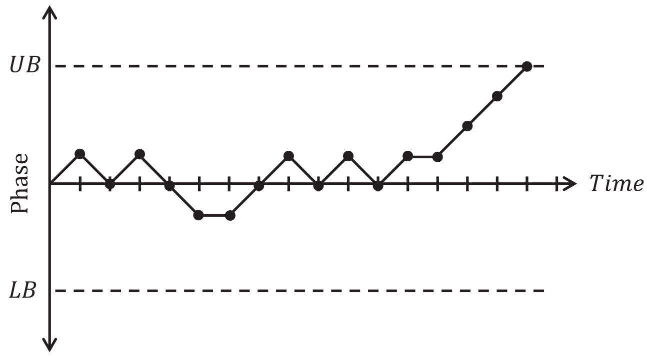


FIGURE 4. **A path in a lazy random walk with two absorbing barriers.**

From the above observation, we expect that the ENN values of all nodes are likely to converge to the actual number of nodes in the network. We will verify the convergence through simulation.

B. THE NUMBER OF THE PHASES FOR EACH ENN

In this section we discuss how to determine the number of phases, i.e., $\{L_m\}_{m \geq 2}$. This issue is important because it significantly affects the convergence speed of the ENN value and hence the stable and effective operation of the A-RAP. That is, if there are too many phases for each ENN, it might take too much time for all ENN values of nodes to move toward N. On the other hand, if there are very few phases for each ENN, the changes in the ENN value might occur too frequently, which is not desirable.

To go further in our discussion and for simplicity we limit ourselves to the scenario where there are N nodes in the network and the ENN values of nodes are all equal to m. We tag an arbitrary node, called the tagged node, and observe its behavior. Let the phase of the tagged node be 0. For a technical purpose, we assume that the other nodes' ENN values are remained *unchanged until the tagged node adjusts its ENN*. With this assumption, the collision probability of the tagged node also remains unchanged because it depends only on the ENN values of the other nodes. Then the collision probability $p_{c,N,m}$ is given by

$$p_{c,N,m} = 1 - \left(1 - \frac{c^*}{m}\right)^{N-1}.$$

We mathematically model the behavior of the phase of the tagged node as a lazy random walk with two absorbing barriers. Refer to Fig. 4. In other words, each step adds up 0, +1 or −1 to the current phase value until the phase reaches either of the two barriers. In what follows, we investigate the lazy random walk in detail. It can take values on

$$\{LB, -\lfloor L_m/2\rfloor, \ldots, -1, 0, 1, \ldots, \lfloor (L_m-1)/2\rfloor, UB\}$$

where LB and UB denote the lower barrier and the upper barrier. The barriers LB and UB mean that the tagged node changes its ENN value to $m-1$ and $m+1$, respectively. When the packet transmission of the tagged node is successful, the phase goes down by one with probability a_m and remains unchanged with probability $1-a_m$. Thus, the probability α that the phase goes down is given by

$$\alpha = (1-p_{c,N,m})a_m = \left(1-\frac{c^*}{m}\right)^{N-m+1} - \left(1-\frac{c^*}{m}\right)^{N-1}$$

where a_m in (4) is used. On the other hand, the phase goes up by one when the packet transmission of the tagged node is collided. This occurs with probability β given by

$$\beta = p_{c,N,m} = 1-\left(1-\frac{c^*}{m}\right)^{N-1}.$$

If we consider $-\lfloor L_m/2\rfloor - 1$ and $\lfloor (L_m-1)/2\rfloor + 1$ as LB and UB, respectively, then the transition probabilities are given as follows:

$$\begin{aligned} p_{i,i-1} &= \alpha, \\ p_{i,i} &= 1-\alpha-\beta, \\ p_{i,i+1} &= \beta \end{aligned}$$

for $i \in \{-\lfloor L_m/2\rfloor, \ldots, -1, 0, 1, \ldots, \lfloor (L_m-1)/2\rfloor\}$ and 0 otherwise.

We let $\widehat{P}_{N,m}$ denote the probability that the tagged node eventually changes its ENN value to $m-1$, i.e., the random walk hits the LB first. Then $1-\widehat{P}_{N,m}$ denotes the probability that the tagged node eventually changes its ENN value to $m+1$. These probabilities $\widehat{P}_{N,m}$ and $1-\widehat{P}_{N,m}$ have a key role in determining $\{L_m\}_{m\geq 2}$. For simplicity, we assume that L_m is an odd number. Using the gambler's ruin probability, we derive the following formulas. The proof is given in Appendix D.

$$\begin{aligned} \widehat{P}_{N,m} &= \frac{1}{1+(\beta/\alpha)^{(L_m+1)/2}}, \\ 1-\widehat{P}_{N,m} &= \frac{1}{1+(\alpha/\beta)^{(L_m+1)/2}}. \end{aligned} \tag{7}$$

Now, we are ready to determine $\{L_m\}_{m\geq 1}$. We first derive an asymptotic condition for L_m under which the tagged node adjusts its ENN value in the desirable direction. For given $m \geq 2$, to derive the condition we introduce the following inequalities

$$\begin{aligned} \widehat{P}_{N,m} &\geq \gamma_1 \quad \text{for all } k_1 \geq 1 \quad \text{if } N = m-1-k_1, \\ 1-\widehat{P}_{N,m} &\geq \gamma_2 \quad \text{for all } k_2 \geq 1 \quad \text{if } N = m-1+k_2, \end{aligned} \tag{8}$$

where γ_1 and γ_2 are two constants in $(1/2, 1)$. The reason for the introduction of two inequalities is obvious. That is, when $N = m-1-k_1$, it is reasonable that $\widehat{P}_{N,m}$, the probability that the ENN value decreases, should be at least as large as $1/2$. Similarly, when $N = m-1+k_2$, it is also reasonable that $1-\widehat{P}_{N,m}$, the probability that the ENN value increases, should be at least as large as $1/2$. We begin with the first inequality in (8). Suppose $N = m-1-k_1$. Then we have $\alpha > \beta$. We rewrite β/α as follows:

$$\begin{aligned} \frac{\beta}{\alpha} &= \frac{1-(1-c^*/m)^{N-1}}{a_m(1-c^*/m)^{N-1}} \\ &= \frac{1-(1-c^*/m)^{N-1}}{((1-c^*/m)^{-m+2}-1)(1-c^*/m)^{N-1}} \\ &= \frac{(1-c^*/m)^{-N+1}-1}{(1-c^*/m)^{-m+2}-1} \\ &= \frac{(1-c^*/m)^{-m+2+k_1}-1}{(1-c^*/m)^{-m+2}-1} \\ &= \left(1-\frac{c^*}{m}\right)^{k_1} + \frac{(1-c^*/m)^{k_1}-1}{(1-c^*/m)^{-m+2}-1} \\ &\simeq 1-\frac{k_1c^*}{m} - \frac{k_1c^*}{m}\left(\frac{1}{e^{c^*}-1}\right) \\ &= 1-\frac{1}{m}\left(\frac{k_1c^*}{1-e^{-c^*}}\right). \end{aligned}$$

Thus

$$\left(\frac{\beta}{\alpha}\right)^m \simeq e^{-k_1\rho} \quad \text{where } \rho = \frac{c^*}{1-e^{-c^*}} > 0.$$

Taking log on both sides yields

$$\log\left(\frac{\beta}{\alpha}\right) \simeq -\frac{k_1\rho}{m}.$$

From the first inequality in (8), we have

$$\frac{L_m+1}{2} \geq \frac{\log(1/\gamma_1-1)}{\log(\beta/\alpha)} \simeq -\frac{\log(1/\gamma_1-1)}{k_1\rho}m. \tag{9}$$

Note that $-\log(1/\gamma_1-1) > 0$ since $1 \geq \gamma_1 > 1/2$ and $\rho > 0$. It is worthwhile to note that ρ depends on c^*, and is irrelevant to m.

If we replace k_1, γ_1 by $-k_2, \gamma_2$, then we can derive a similar result for the second inequality of (8) as follows:

$$\frac{L_m+1}{2} \geq \frac{\log(1/\gamma_2-1)}{\log(\alpha/\beta)} \simeq -\frac{\log(1/\gamma_2-1)}{k_2\rho}m. \tag{10}$$

Considering the adjustment speed of the ENN value, it is better to choose L_m as small as possible. When $\gamma = \gamma_1 = \gamma_2$, the smallest possible value of L_m occurs when $k_1 = k_2 = 1$ and is given by

$$L_m \simeq -2\log(1/\gamma-1)\cdot\frac{1-e^{-c^*}}{c^*}\cdot m.$$

With the above observation, for the A-RAP we propose to use

$$L_m = \lfloor \frac{1}{3}m \rfloor, \quad m \geq 2$$

In the conference version of this paper [32] we proposed to use $L_m = m$, but in order to make the adjustment speed faster we propose to use the above value for L_m. In this case, from $\rho = c^*/(1-e^{-c^*})$ we see

$$\gamma \simeq 1/(1+e^{-\rho/2}) \simeq 0.5516.$$

We will verify through simulation that the A-RAP with $L_m = \lfloor \frac{1}{3}m \rfloor$ provides a good performance.

C. OUTLIER PROBLEM IN THE A-RAP

In this subsection, we identify the outlier problem of the A-RAP and provide a solution for it. As aforementioned in the introduction, the A-RAP simplifies the estimation of the number of active nodes by only considering the transmission results and hence it is inevitable to have some error in the estimation. In most cases the error is shown to be not significant because the A-RAP provides similar short term fairness and throughput as the RAP as in Section IV-A. However, we recently find that outliers sometime appear who have much larger estimates of the number of active nodes than others in the A-RAP. The outlier problem becomes more significant when the number of active nodes are relatively large.

To explain the reason why the outlier problem arises in the A-RAP, we consider a saturated network in steady state where the ENNs of all nodes are close to the number of active nodes in the network. Assume that the number of active nodes is relatively large. We now add a new node with the ENN that is much larger than the number of active nodes, which is called the tagged node. Ideally, the tagged node must decrease its ENN in the A-RAP, but it rarely changes its ENN. This is because the large ENN of the tagged node makes it prevent packet transmissions too much and hence it has much less chances to decrease its ENN than the other nodes. Furthermore, since the number of active nodes is relatively large and the tagged node transmits its packets less frequently, the impact of the tagged node to the network is not significant. Therefore, it is difficult that the other nodes realize the existence of the tagged node. In a similar way, if a node happens to have a large ENN value, it has a high chance of being an outlier.

One way to solve the outlier problem is to decrease the outlier's ENN intentionally under some situations. To this end, we propose to *force a node whose ENN is maintained the same value over its Γ packet transmissions to decrease its ENN* until reaching the minimum value of 2 as follows:

$$\text{ENN} \leftarrow \max\left(\lceil \text{ENN} \times \delta \rceil, 2\right) \tag{11}$$

where δ is a constant and $\lceil x \rceil$ is the smallest integer greater than or equal to x.

The reason why we use δ in (11) is explained in the following. From the perspective of the convergence of ENN's, it is reasonable to decrease ENN in proportion to the current ENN value. Obviously, Γ and δ are important parameters because they affect the performance significantly, i.e., it has some beneficial effects as well as some side effects. When δ is too large, we can easily expect the A-RAP behaviors too aggressively. Our simulations confirm that $\delta = 7/8$ performs quite well in all scenarios and hence we propose to use $\delta = 7/8$ in this paper. For convenience, we name the improved A-RAP using Γ and δ as the A-RAP$^+$ to distinguish it from the A-RAP without using them.

In what follows we focus on the determination of a suitable value of Γ. Our main approach toward it is to use a random walk model. To start with the discussion on Γ, we first consider the beneficial effects and the side effects of the introduction of Γ. The beneficial effects are listed as follows:

(B1) Outliers do not appear by intentionally forcing their ENNs to decrease.

(B2) A node might decrease its ENN faster when it needs to.

(B3) Fairness performance is improved with the rearrangement of ENNs of nodes as well as the outlier. That is, all nodes maintain their ENN values almost invariant in steady state. So, by forcing some nodes with relatively high ENNs to decrease their ENNs, they transmit their packets a little bit aggressively, which results in the increase in the ENNs of some other nodes. So all ENNs of nodes have chances to be close to the actual number of active nodes.

From the above beneficial viewpoint, it is desirable to choose a small value of Γ because a small value of Γ speeds up the changes of the ENNs of nodes as mentioned in (B1) and (B2). Moreover, rearranging the ENNs of nodes in (B3) occurs more often and it has an advantage in terms of fairness. Note that there obviously exist differences in the ENNs of all nodes, but the ENNs remains almost invariant in steady state. This implies that, if a node happens to have a relatively large ENN in steady state, the node has less chances to transmit its packets and this is undesirable from the fairness perspective. Therefore, it is desirable to rearrange the ENNs of nodes to a certain extent to get better short term fairness.

On the other hand, there are also some side effects as follows:

(S1) A node might decrease its ENN when it has to increase because a failure to increase its ENN within its Γ packet transmissions forces the node to decrease its ENN.

(S2) A node might decrease its ENN when it does not need to, i.e., its ENN is close to the actual number of active nodes N.

From the viewpoint of the above side effects, it is necessary to guarantee the minimum time required to decrease the ENN intentionally, that is, Γ should not be too small.

To determine a suitable value of Γ to alleviate these side effects as much as possible, we first consider the following simple scenario where there are N nodes in the network and they are saturated. Suppose that every node has the same BSS $(N-d, 0)$ for a given positive integer d. We tag an arbitrary node and assume that the other nodes maintain their ENNs invariant. Let $S(N,d)$ denote the number of packet transmissions of the tagged node until it changes its ENN, i.e., it changes its ENN after $S(N,d)$ packet transmissions. The behavior of the BSS of the tagged node can be modeled as a lazy random walk as in Section III-B. Its transition diagram

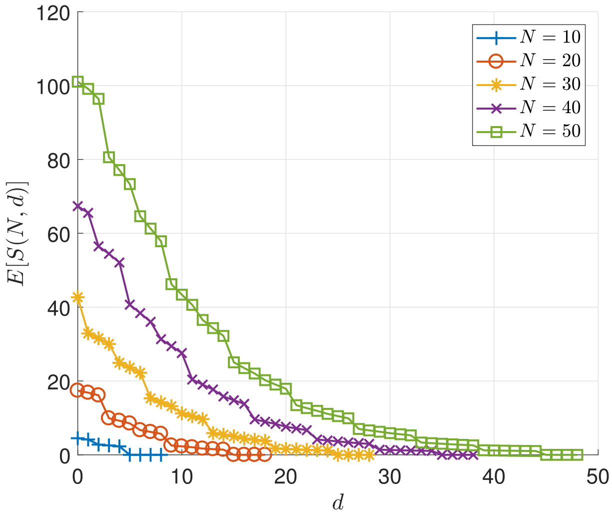


FIGURE 5. Expected number of transmissions required for a node to change its ENN when there are N active nodes whose ENNs are the same as $N - d$.

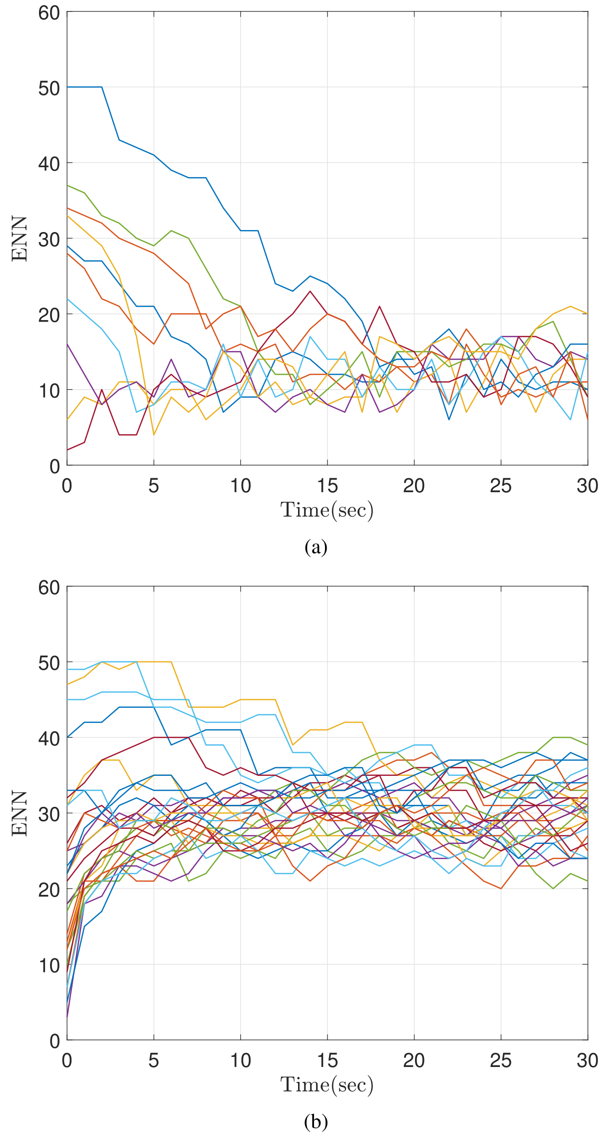


FIGURE 6. Trajectories of ENNs of nodes. (a) the number of active nodes $N = 10$, (b) the number of active nodes $N = 30$.

is the same as given in Fig. 1 where $m = N - d$ and $p_c(t)$ is given by $p_c(t) = 1-(1 - c^*/m)^{N-1}$. Then $S(N, d)$ represents the sojourn time of the lazy random walk during which its ENN remains unchanged. The computation of $S(N, d)$ is important to determine the value of Γ because a node needs as many packet transmission as $S(N, d)$ to increase its ENN and hence Γ should be at least as large as $S(N, d)$. Since $S(N, d)$ is a random variable, for our purpose to determine the value of Γ, we simply focus on the expected value $E[S(N, d)]$.

To obtain a closed formula for $E[S(N, d)]$, we define $v(x)$ by the expected sojourn time of the tagged node with the initial BSS (m, x) while the other nodes maintain their ENNs as m. Then the expected value of $S(N, d)$ can be obtained by solving the following recursive equations:

$$v(-\lfloor m/2 \rfloor - 1) = v(\lfloor (m-1)/2 \rfloor + 1) = 0, \tag{12}$$

$$v(x) = \alpha v(x-1) + (1 - \alpha - \beta)v(x) + \beta v(x+1) + 1 \tag{13}$$

for $x = -\lfloor m/2 \rfloor, \cdots, \lfloor (m-1)/2 \rfloor$. By the definition of $v(x)$, we have $E[S(N, d)] = v(0)$ and obtain

$$E[S(N, d)] = \frac{m+1}{\beta - \alpha} \frac{(\alpha/\beta)^{\lfloor m/2 \rfloor + 1} - 1}{(\alpha/\beta)^{m+1} - 1} - \frac{\lfloor m/2 \rfloor + 1}{\beta - \alpha} \tag{14}$$

where $\alpha = (1 - p_c(t))a_m = (1 - c^*/m)^{d+1} - (1 - c^*/m)^{N-1}$, $\beta = p_c(t) = 1-(1-c^*/m)^{N-1}$. We plot $E[S(N, d)]$ in Fig. 5 for various values of N and d.

From the above discussion and results, it is reasonable to set $\Gamma = E[S(N, 0)]$ which can alleviate the side effects (S1) and (S2). Since nodes have their ENNs as $m = N - d$, they will change their ENNs after $E[S(N, d)]$ packet transmissions on average. If $\Gamma < E[S(N, d)]$, then the performance might be degraded because the introduction of Γ prevents nodes from increasing their ENNs and forces to decrease their ENNs. On the other hand, if $\Gamma > E[S(N, d)]$, the introduction of Γ does not prevent the increase of the ENN, i.e., the side effect of (S1) is much alleviated. Similarly, if we suppose that every node has the same ENN value of N, then Γ must be at least $E[S(N, 0)]$ to avoid the unnecessary decrease of the ENN, i.e. the side effect of (S2) is alleviated. Since $E[S(N, 0)] > E[S(N, d)]$ as seen in Fig. 5, it is reasonable to set $\Gamma = E[S(N, 0)]$. When we limit ourselves to scenarios where N is not greater than 50, from our extensive simulation we propose to use $\Gamma = 100 \simeq E[S(50, 0)]$. In the next numerical section, it will be shown that our careful choice of Γ guarantees that the performance of the A-RAP$^+$ is still good.

Before we go further, we verify that nodes using the A-RAP$^+$ are able to adaptively estimate the number of nodes in the network by showing the simulation results on their ENN trajectories in Fig. 6. In the simulation, to see if the

TABLE 1. Network parameters.

Payload	8184 bits
PHY Header	128 bits
MAC Header	272 bits
RTS Frame	PHYheader +160 bits
CTS Frame	PHYheader +112 bits
ACK Frame	PHYheader +112 bits
Time Slot	9 μs
SIFS	16 μs
DIFS	34 μs
Propagation Delay	1 μs
Data Rate	6.5 − 65 Mbps
CW_0	16
CW_{max}	1024

A-RAP$^+$ adaptively estimates the number of nodes well even in a bad situation, we consider a scenario where each node selects its initial ENN uniformly on [2, 50] and adjusts its ENN over time. As seen in the figure the ENNs of nodes gradually approach to the actual number of nodes and then move up and down near it. It can be also seen that even a node whose initial ENN is relatively large decreases its ENN to the actual number of nodes, which implies that the outlier problem is well resolved in the A-RAP$^+$.

IV. NUMERICAL VALIDATION

In order to evaluate the performance of the proposed A-RAP$^+$, we use MATLAB to conduct our experiments under a Rayleigh-lognormal fading channel with parameters given in Table 1. We consider variable physical data rates from 6.5 Mbps to 65 Mbps determined by the channel condition depending on the Rayleigh-lognormal fading. In the figures, otherwise mentioned, each result is an averaged value for 30 different seed values and the corresponding 99% confidence interval is also given. We investigate the performance of the A-RAP$^+$ from three perspectives.

Firstly, we examine the throughput and short-term fairness of the A-RAP$^+$. Secondly, we measure the transmission efficiency of a node that is defined by the ratio of the number of successfully transmitted packets over the total number of transmitted packets from the node. The transmission efficiency is a good metric to evaluate the performance of a protocol in terms of energy efficiency. For a comparison purpose, we also measure the performance of other backoff schemes proposed in the open literature. Among various schemes we choose EIED [16], and quadratic backoff (QB) [11] since they are *individual updating passive* backoff schemes as the A-RAP$^+$. We also consider the RAP [14] with the actual number of active nodes to confirm that the A-RAP$^+$ inherits the benefits of the RAP. Hence we compare the performances of the RAP, BEB, QB, EIED, A-RAP and A-RAP$^+$ in steady state. Detailed descriptions of EIED, and QB are given in Appendix E. Lastly, we investigate how adaptively and fast each node with the A-RAP$^+$ adjusts its ENN to the varying number of nodes in the network. In fact, we observe the trajectory of the ENN values of an arbitrarily tagged node while changing the number of nodes in the network.

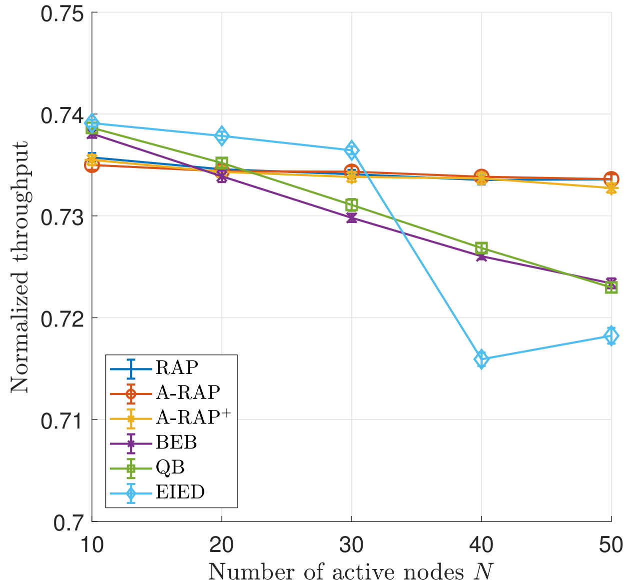


FIGURE 7. Throughput comparison of the RAP, BEB, QB, EIED, A-RAP, and A-RAP$^+$ versus the number of active nodes *N*.

A. THROUGHPUT AND SHORT-TERM FAIRNESS

We evaluate the throughput defined as the fraction of time the channel is used to successfully transmit payload bits as in [5]. Fig. 7 shows the throughput performances of the RAP, BEB, QB, EIED, A-RAP, and A-RAP$^+$. As shown in the figure, the throughputs of the A-RAP$^+$ and the RAP are almost the same and they have the highest throughput when $N > 30$. In the case of $N = 10, 20$, and 30, even though the throughput of the EIED is slightly higher than that of the A-RAP$^+$, the fairness of the EIED is very low as seen in Fig. 8. In addition, we see that the A-RAP$^+$ achieves the most stable throughput performance for various numbers of nodes. Note that the RAP, BEB, QB, and EIED are not adaptive,[2] so we use in the figure fixed parameter values proposed in the literature for each case. Even though only the A-RAP$^+$ is adaptively changing its parameters, i.e., BSS, the throughput of the A-RAP$^+$ is very close to that of the RAP[3] and higher than the throughputs of other backoff schemes. This shows the strong point of the A-RAP$^+$.

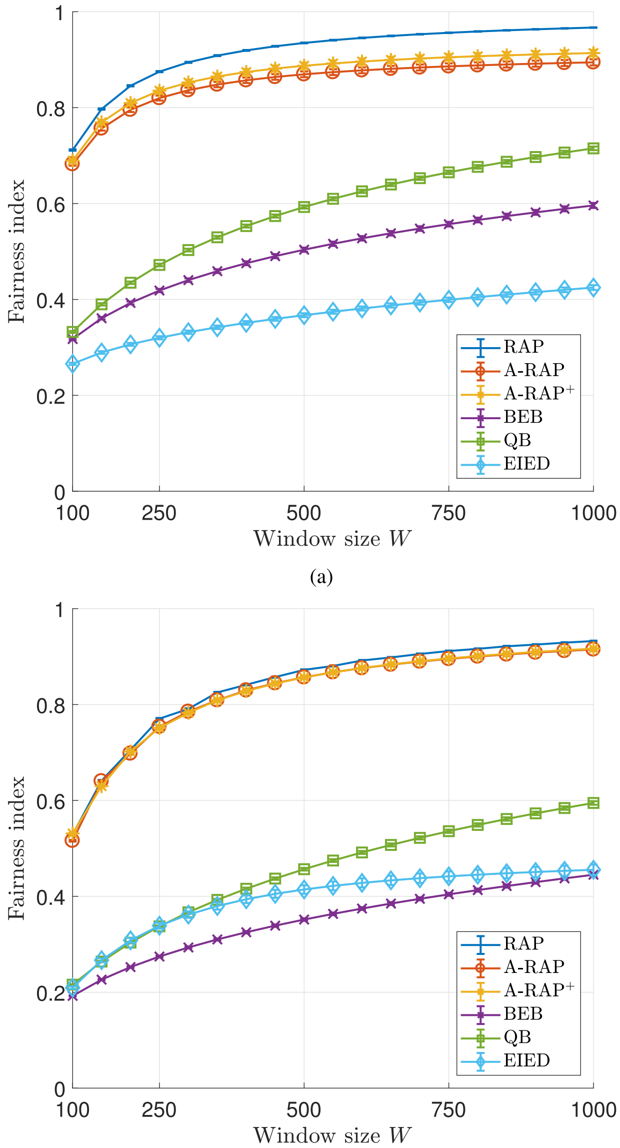
To evaluate the short-term fairness, the *Jain's Fairness Index* (JFI) is introduced in [33]. The JFI is defined by

$$F_W = \frac{\left(\sum_{i=1}^{N} \gamma_W^i\right)^2}{N \sum_{i=1}^{N} (\gamma_W^i)^2} \tag{15}$$

where γ_W^i is the throughput of node i estimated during a time period of length W. We say that perfect fairness is

[2] To the best of the authors' knowledge, there are not adaptive and individually updating passive backoff schemes in the literature

[3] It is shown in [14] that the throughput that the RAP achieves is almost the same as the theoretical optimal throughput.



(a)

(b)

FIGURE 8. **Fairness index of the RAP, BEB, QB, EIED, A-RAP, and A-RAP$^+$ versus sliding window size W. (a) the number of active nodes $N = 20$, (b) the number of active nodes $N = 40$.**

achieved when the JFI is equal to one. In simulation, we use the sliding window method as in [12]. The results are plotted in Fig. 8. The figure shows that the A-RAP$^+$ achieves a similar fairness performance as the RAP and outperforms all other backoff schemes including the A-RAP. There are two main reasons for the improvement in fairness. The first reason is that the absence of the outliers obviously improves the short-term fairness, i.e., (B1), and the second reason is that the short term fairness is also improved by mixing the ENNs of nodes, i.e., (B2). Moreover, the little difference between the RAP and A-RAP$^+$ implies that the ENNs of all nodes are not significantly different. Thus, we conclude that the A-RAP$^+$ inherits the benefit of the RAP as well in the fairness perspective.

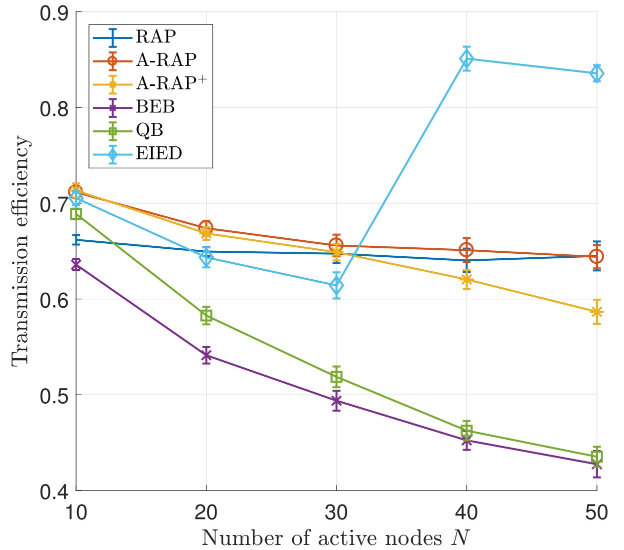


FIGURE 9. **Transmission efficiency of the RAP, BEB, QB,EIED, and A-RAP versus the number of active nodes N.**

B. TRANSMISSION EFFICIENCY

In this section, we measure the transmission efficiency and compare the transmission efficiency of the A-RAP$^+$ with those of other backoff schemes. From Fig. 9 we see that the A-RAP$^+$ achieves similar transmission efficiency as the RAP and surpasses the BEB and QB. This is because each node with the A-RAP$^+$ uses the near-optimal backoff counter selection distribution and thus it experiences less packet collisions compared to other backoff schemes. As discussed in [14], since the transmission efficiency implies the energy efficiency and both the A-RAP$^+$ and the RAP have similar transmission efficiency, we conclude that the A-RAP$^+$ is energy efficient in packet transmission.

Note that only the EIED shows the transmission efficiency higher than that of the A-RAP$^+$ when there are 40 or 50 nodes in the network, but this is because the EIED with $N = 40, 50$ transmits packets much less aggressively. This results in more packet transmission successes, but the channel utilization is quite low as seen in the throughput performance.

C. ADAPTIVITY TO CHANGING NETWORK ENVIRONMENTS

In this section, we observe the trajectory of the ENN values of nodes when changing the number of nodes in the network. We assume that, when a node enters the network, it begins contending for the channel with the initial BSS $(2, 0)$ for convenience. The number of nodes in the network, N, varies as 10, 30, and 20, and the changes occur every 30 seconds. We evaluate the throughput, fairness, and collision probability during the time interval [0, 30], [30, 60], [60, 90] and the results are provided in Table 2. Table 2 shows they are close to the steady state results given in Section IV-A. In the table, the columns for '$N = 10$', '$N = 30$', and '$N = 20$' denote the steady state results when $N = 10$, 30, and 20, respectively.

TABLE 2. Adaptivity of the A-RAP.

	$[0,30]$	$[30,60]$	$[60,90]$	$N=10$	$N=30$	$N=20$
Throughput	0.7350	0.7330	0.7348	0.7355	0.7338	0.7343
JFI ($W=500$)	0.8903	0.8811	0.8942	0.8840	0.8831	0.8874
JFI ($W=1000$)	0.9028	0.9262	0.9204	0.8971	0.9271	0.9138
Transmission efficiency	0.7066	0.6448	0.6864	0.7133	0.6489	0.6685

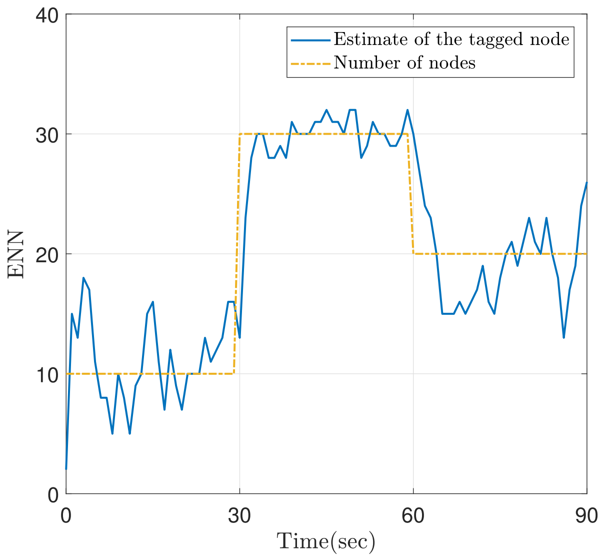


FIGURE 10. Trajectory of ENN's of the tagged node; the initial number of nodes is 10; the number of nodes increases to 30 at 30s and decreases to 20 at 60s.

To demonstrate the adaptivity of the A-RAP$^+$ explicitly, we tag an arbitrary node and observe the trajectory of the ENN value of the tagged node. Fig. 10 provides the trajectory of the ENN value of the tagged node. As seen in Table 2 and Fig. 10, the A-RAP$^+$ performs adaptively well enough to chase the varying number of nodes. Moreover, it achieves the steady state performance. Thus, we conclude that the A-RAP$^+$ is robust to the varying number of nodes in a practical network.

V. CONCLUSIONS

In this paper, we proposed an adaptive version of the Renewal Access Protocol, called the A-RAP. Two key parameters of the A-RAP, the adjustment probabilities and the numbers of phases, were carefully determined through mathematical analysis, so that the estimated numbers of nodes move toward the actual number of nodes in the network on average. The benefits of the A-RAP were verified through intensive numerical and simulation studies. In fact, the A-RAP achieves high throughput, short-term fairness, and adaptivity even in the time varying network. We also tackle the outlier problem of the A-RAP and propose an improved version of the A-RAP, called A-RAP$^+$. The A-RAP$^+$ resolves the outlier problem by introducing a method to intentionally decrease the ENN of a node where its ENN remains invariant for a certain time period. Simulation results are provided to validate our analysis and the improvement in the A-RAP$^+$.

APPENDIX A
PROOF OF THEOREM 2

Let $p_{c,1}$ and $p_{c,2}$ be the collision probabilities of node 1 and node 2, respectively, when they transmit packets, and $q := \prod_{k=1}^{N}(1-c^*/m_k)$. Since we assume that the packet transmissions in the network do not change m_1 and m_2, we drop the time index t in the proof for simplicity. If $m_1 = m_2$, both node 1 and node 2 have net drifts with the same sign because

$$p_{c,1} = 1 - \frac{q}{1-c^*/m_1} = 1 - \frac{q}{1-c^*/m_2} = p_{c,2}.$$

Next, consider the case of $m_1 < m_2$. If node 1 has a negative net drift, we then have

$$a_{m_1}\frac{q}{1-c^*/m_1} \geq 1 - \frac{q}{1-c^*/m_1}$$

which is equivalent to

$$q \geq \frac{1-c^*/m_1}{a_{m_1}+1}.$$

This leads to

$$q \geq \frac{1-c^*/m_2}{a_{m_2}+1}$$

because $\{(a_m+1)\,/\,(1-c^*/m)\}_{m\geq 2}$ is an increasing sequence. So node 2 also has a negative net drift. Similarly, it is easy to show that, if node 2 has a positive net drift, then node 1 also has a positive net drift.

APPENDIX B
PROOF OF THEOREM 3

For later use, we prove that $\{a_m\}_{m\geq 2}$ is an increasing sequence. Since $\{(a_m+1)/(1-c^*/m)\}_{m\geq 2}$ is assumed to be an increasing sequence,

$$a_m + 1 \leq \frac{1-c^*/(m+1)}{1-c^*/m}(a_m+1) \leq a_{m+1}+1. \quad (16)$$

(i) Suppose $N < m_k$ for $1 \leq k \leq N$. From (2) with $m = N+1$, we have

$$\frac{1}{(1-c^*/(N+1))^{N-1}} - 1 \leq a_{N+1}. \quad (17)$$

Let $q := \prod_{k=1}^{N}(1-c^*/m_k)$. The statement (i) is equivalent to

$$a_{m_k}\frac{q}{1-c^*/m_k} \geq 1 - \frac{q}{1-c^*/m_k}, \quad 1 \leq k \leq N$$

from which we obtain

$$\frac{1-c^*/m_k}{q}-1 \le a_{m_k}, \quad 1 \le k \le N. \tag{18}$$

Note that $(1-c^*/m_k)/q$ is maximized when $m_j = N+1$ for $j \in \{1,2,\cdots,N\}\backslash\{k\}$, and the maximum value of $(1-c^*/m_k)/q$ is given by

$$\frac{1}{(1-c^*/(N+1))^{N-1}}.$$

Then, for any set of $m_k > N$, $1 \le k \le N$, from (16) and (17), we have

$$\begin{aligned}\frac{1-c^*/m_k}{q}-1 &\le \frac{1}{(1-c^*/(N+1))^{N-1}}-1\\ &\le a_{N+1}\\ &\le a_{m_k}\end{aligned}$$

which shows that (18) is satisfied. This implies that every node has a negative net drift if (2) is satisfied.

(ii) Suppose that $m_k < N$ for $1 \le k \le N$. From (2) with $m = N-1$, we have

$$a_{N-1} < \frac{1}{(1-c^*/(N-1))^{N-1}}-1. \tag{19}$$

The statement (ii) is equivalent to

$$a_{m_k} < \frac{1-c^*/m_k}{q}-1, \quad 1 \le k \le N. \tag{20}$$

Note that $(1-c^*/m_k)/q$ is minimized when $m_i = N-1$ for $i \in \{1,2,\cdots,N\}\backslash\{k\}$, and the minimum value of $(1-c^*/m_k)/q$ is given by

$$\frac{1}{(1-c^*/(N-1))^{N-1}}.$$

Then, for any set of $m_k < N$, $1 \le k \le N$ from (16) and (19), we have

$$\begin{aligned}a_{m_k} &\le a_{N-1}\\ &< \frac{1}{(1-c^*/(N-1))^{N-1}}-1\\ &\le \frac{1-c^*/m_k}{q}-1\end{aligned}$$

which shows that (20) is satisfied. This implies that every node has a positive net drift if (2) is satisfied.

APPENDIX C
PROOF OF THEOREM 4

Suppose

$$a_m = \frac{1}{(1-c^*/m)^{m-r-1}}-1, \quad m \ge 2.$$

From Theorem 3, the values of $\{a_m\}_{m\ge2}$ satisfying the condition in (2) are represented as follows:

$$a_m = \frac{1}{(1-c^*/m)^{m-r-1}}-1, \quad \text{for some } r \in [-1,1].$$

Then $(a_m+1)/(1-c^*/m) = 1/(1-c^*/m)^{m-r}$.

Let $f(x) := (1-c^*/x)^{x-r}$ for $x \ge 1$. Then the sequence $\{(a_m+1)/(1-c^*/m)\}_{m\ge2}$ can be represented by $\{1/f(m), m \ge 2\}$. Since $f(x) > 0$ for $x \ge 2$, it suffices to show that $(\log f(x))' \le 0$. Note that

$$\begin{aligned}(\log f(x))' &= \left((x-r)\log(1-\frac{c^*}{x})\right)'\\ &= \log(1-\frac{c^*}{x}) + (x-r)\frac{c^*/x^2}{1-c^*/x}.\end{aligned} \tag{21}$$

Let $t := c^*/x$, then $0 < t < 1$. Then, in (21), the former term is given by

$$\log(1-\frac{c^*}{x}) = \log(1-t) = -t-\frac{t^2}{2}-\frac{t^3}{3}-\frac{t^4}{4}-\cdots,$$

and the latter term is given by

$$\begin{aligned}(x-r)\frac{c^*/x^2}{1-c^*/x} &= \left(\frac{c^*}{t}-r\right)\frac{t^2/c^*}{1-t}\\ &= \left(\frac{c^*}{t}-r\right)\frac{t^2}{c^*}(1+t+t^2+\cdots).\end{aligned}$$

Then we have

$$\begin{aligned}(\log f(x))' &= \log(1-t) + \left(\frac{c^*}{t}-r\right)\frac{t^2/c^*}{1-t}\\ &= \left(-t-\frac{t^2}{2}-\frac{t^3}{3}-\cdots\right)+(t+t^2+t^3+\cdots)\\ &\quad -\frac{r}{c^*}(t^2+t^3+\cdots)\\ &= \left(\frac{1}{2}t^2+\frac{2}{3}t^3+\cdots\right)-\frac{r}{c^*}(t^2+t^3+\cdots)\\ &= \left(\frac{1}{2}-\frac{r}{c^*}\right)t^2+\left(\frac{2}{3}-\frac{r}{c^*}\right)t^3+\cdots.\end{aligned}$$

Thus $r/c^* \ge 1$ implies that $f(x)$ is a decreasing function and $\{(a_m+1)/(1-c^*/m)\}$ is an increasing sequence.

APPENDIX D
FIRST HITTING PROBABILITY $\widehat{P}_{N,m}$

Let $\{X_n\}_{n\ge1}$ be the lazy random walk with state space $\{LB, -\lfloor L_m/2\rfloor, \ldots, -1, 0, 1, \ldots, \lfloor (L_m-1)/2\rfloor, UB\}$. Then transition probabilities are given by

$$\begin{aligned}P(X_1 = l-1 \mid X_0 = l) &= \alpha,\\ P(X_1 = l \mid X_0 = l) &= 1-\alpha-\beta,\\ P(X_1 = l+1 \mid X_0 = l) &= \beta\end{aligned}$$

for $l \in \{-\lfloor L_m/2\rfloor, \ldots, -1, 0, 1, \ldots, \lfloor (L_m-1)/2\rfloor\}$ and 0 otherwise.

We consider the embedded time epochs where there occur changes in the value of $\{X_n\}_{n\ge1}$ and the embedded time epochs are indexed as $n_1, n_2, \cdots$. Then the embedded process $\{\widetilde{X}_j := X_{n_j}\}_{j\ge1}$ becomes an usual

TABLE 3. Optimal values of δ.

N	10	20	30	40	50
δ	1	1	1	1/8	1/8

random walk. Transition probabilities of $\widetilde{X}_j$ are given as follows:

$$P(\widetilde{X}_1 = l-1 \mid \widetilde{X}_0 = l) = \frac{P(X_1 = l-1 \mid X_0 = l)}{P(X_1 \in \{l+1, l-1\} \mid X_0 = l)} = \frac{\alpha}{\alpha+\beta},$$

$$P(\widetilde{X}_1 = l+1 \mid \widetilde{X}_0 = l) = \frac{P(X_1 = l+1 \mid X_0 = l)}{P(X_1 \in \{l+1, l-1\} \mid X_0 = l)} = \frac{\beta}{\alpha+\beta}$$

for $l \in \{-\lfloor L_m/2 \rfloor, \ldots, -1, 0, 1, \ldots, \lfloor (L_m-1)/2 \rfloor\}$ and 0 otherwise. Using the formula for the gambler's ruin probability in [34], we obtain

$$\widehat{P}_{N,m} = \frac{(\alpha/\beta)^{(L_m+1)/2} - (\alpha/\beta)^{L_m+1}}{1-(\alpha/\beta)^{L_m+1}} = \frac{(\alpha/\beta)^{(L_m+1)/2} - (\alpha/\beta)^{L_m+1}}{1-(\alpha/\beta)^{(L_m+1)/2} + (\alpha/\beta)^{(L_m+1)/2} - (\alpha/\beta)^{L_m+1}} = \frac{1}{1+(\beta/\alpha)^{(L_m+1)/2}}.$$

APPENDIX E
DESCRIPTION FOR EIED, AND QB

The EIED scheme is summarized by the following set of equations:

$$\begin{cases} x \leftarrow \max(x/2^{\delta}, CW_0) & \text{upon transmission success,} \\ x \leftarrow \min(2x, CW_{max}) & \text{upon collision.} \end{cases} \tag{22}$$

Parameter δ is chosen to be the optimal value in the fairness perspective. The values of δ used in the simulation are given in Table 3.

The QB scheme is summarized by the following set of equations:

$$\begin{cases} x \leftarrow CW_0 & \text{upon transmission success,} \\ x \leftarrow (1+\min(d, K))^2 CW_0 & \text{upon } d \text{ successive collisions.} \end{cases} \tag{23}$$

Smaller K refers to better fairness performance. In [11], it is shown that $K = 4$ is good enough to approach the limiting throughput with $K = \infty$. Hence, we use $K = 4$ in Section IV.

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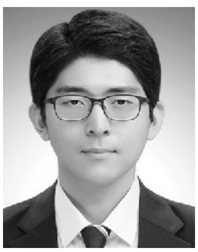

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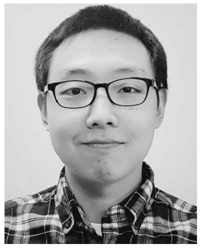

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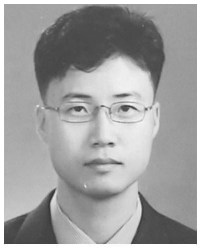

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