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A New S^2 Control Chart Using Multiple Dependent State Repetitive Sampling

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ABSTRACT The combined application of multiple dependent state sampling and the repetitive group sampling (RGS) is an efficient sampling scheme for industrial process monitoring as it combines the advantages of both the sampling schemes. In this paper, a new variance control chart has been proposed, when the interesting quality characteristic follows the normal distribution using the combination of these two efficient sampling schemes called multiple dependent state repetitive sampling. The control chart coefficients and parameters have been estimated through simulation for the in-control process by considering the target in-control average run lengths under different process settings. The efficiency of the proposed chart has been determined by computing the out-of-control ARL for different shift levels. The advantages of the proposed monitoring scheme have been discussed and compared with the existing RGS scheme and the single sampling scheme. A simulated example and a real industrial data have been included to demonstrate the application of the proposed monitoring scheme. It has been observed that the proposed chart is a valuable addition to the toolkit of the quality monitoring personnel.

INDEX TERMS Variance chart, multiple dependent state sampling, repetitive group sampling, chi-square distribution, average run length.

I. INTRODUCTION

Since the inception of the notion of a control chart by Shewhart A. Walter during 1920's for monitoring an industrial process, the tool of control charts is being applied extensively in many manufacturing processes under different sampling schemes. It detects the unusual changes in the process by signaling alarms when the process is beyond the threshold values commonly known as the control limits [1]. Several modifications have been introduced in these control charts but no robust technique has been developed yet. Generally, the most efficient control chart scheme is adopted by the administration of the manufacturing process in order to avoid losses incurred by the rework of the items or the scrape of the items. Once the out-of-control signal is observed with respect to some quality characteristic, the production process is clogged in some cases, and the quick remedial action is taken to bring the process back to the in-control condition.

Today, single sampling (SS), multiple dependent state sampling (MDSS) and repetitive group sampling (RGS) are the most widely used sampling schemes in the literature of the statistical process control. In SS a sample of n units is selected from the process and the declaration of in-control or out-of-control is determined based on the information contained in that sample [2]. The SS scheme is the simplest one and easy to understand by the quality control personnel for decision-making in the traditional univariate control charts but its efficiency is questionable [3].

An MDSS scheme is generally used one in acceptance sampling plans, which was initially introduced by [4] but it has been also equally implemented in the control chart techniques [5]. In MDSS the decision is made by considering the information from the previous decisions on the process [6]–[8]. Several researchers used the MDSS scheme in the control chart literature including [9]–[11].

The RGS scheme was initially introduced by [12] for proposing an attribute acceptance sampling plan, which is based upon two criteria for declaring the acceptance or the rejection and decision is postponed when it is not obvious [13]. In case of needing further information, we repeat the whole process by further selecting another sample and decide accordingly [14]. This sampling scheme has a lead over an SS scheme as it utilizes the smaller sample size for lot sentencing [13]. The RGS scheme has been explored by several researchers of the quality control including [3], and [14]–[21].

Two types of control charts are commonly considered for monitoring the process: charts for monitoring the process mean and charts for monitoring the process variance. During the last few years, the control charts for monitoring process variance have attracted the attention of many quality control researchers, for example, Riaz et al. [22]. In general, the within subgroup variation is attributable to the common cause of variation. So the overall process variation is targeted to explore the unusual variations in the process Tsai and Hsieh [23]. The variance control chart has been explored by many researchers including [24]–[29]. Among them, a S^2 control chart is most popularly used.

By exploring the literature, we did not find any work on S^2 control chart using the multiple dependent state repetitive sampling (MDSRS). In this paper, a new S^2 control chart using MDSS combined with RGS has been developed for the efficient monitoring of the shift in the variability of a production process. It is expected that the proposed control chart will be more efficient than the existing control charts in average run length. The rest of the paper is organized as follows: the designing of the new S^2 control chart is described in Section 2. In Section 3 the average run length properties of the proposed scheme have been derived. A simulation study of the proposed chart for the performance evaluation is given in Section 4. In Section 5 the comparative study with respect to MDSS, RGS and SS have been discussed. A simulated data and a real-world example have been discussed in Section 6. In the last section concluding remarks are given.

II. A NEW S^2 CONTROL CHART

In this section, we described the proposed S^2 control chart using multiple dependent state repetitive sampling (MDSRS). It is assumed that the quality characteristic follows a normal distribution with variance σ^2. The steps of the proposed control chart are as follows:

Step 1: A random sample of size n is selected at each subgroup from the production process and calculate the sample variance S^2.

Step 2: The process is declared as in-control if $LCL_2 \leq S^2 \leq UCL_2$ (these are called the interior control limits). Or the process is declared as out-of-control if $S^2 \geq UCL_1$ or $S^2 \leq LCL_1$ (these are called the exterior control limits). Otherwise, go to Step-3.

Step 3: Declare the process as in control if i preceding subgroups have been declared as in-control. Otherwise, go to **Step 1** and repeat the process.

For the case of known σ^2 the exterior control limits of the proposed control chart are given by

$$UCL_1 = \sigma^2 + k_1\sqrt{2\left(\sigma^2\right)^2/(n-1)}$$
$$LCL_1 = \sigma^2 - k_1\sqrt{2\left(\sigma^2\right)^2/(n-1)}$$

where k_1 is a constant to be determined. The interior control limits are:

$$UCL_2 = \sigma^2 + k_2\sqrt{2\left(\sigma^2\right)^2/(n-1)}$$
$$LCL_2 = \sigma^2 - k_2\sqrt{2\left(\sigma^2\right)^2/(n-1)}$$

where k_2 is a constant to be determined.

Example 1: A real-world example of Inside Diameter Measurements (mm) for Automobile Engine Piston Rings given on page 252; Table 6.3 from [2] has been used to construct the proposed chart in Figure 1. The variance is estimated from the sample data using $s^2 = \Sigma(x-\bar{x})^2/n-1$. Using the control chart coefficient, $k_1 = 12.37159455$ and $k_2 = 5.19449819$, the $UCL_1 = 0.0004212676$, $UCL_2 = 0.0001757226$ and likewise, the lower control limits are $LCL_1 = 0.000220014$ and $LCL_2 = 0.00002553$. Figure 1 has been constructed by posting the variances of each rational samples of size 5. It can be observed from the plotted statistic that the 14th and 25th samples are falling in the repetition region meaning that at this sample the process shows an indication of some deterioration. So in such a region, we neither declare out-of-control process nor the in-control process but we repeat the whole of the process.

Example 2: A real-world example of diameter casting which is an important quality characteristic is considered for the implementation of proposed control chart. The data is taken from [2] has been used to construct the proposed chart in Figure 2. The four control limits for the diameter of each casting when $n = 5$ are also shown in Figure 2. From Figure 2, it can be observed that although the process is in-control two subgroups fall in the in-decision area.

Example 3: Vane heights of casting is an important quality variable in the manufacturing of casting. We applied the proposed control chart to monitor the within casting vane height. This data of within casting vane height also selected from [2]. The four control limits for this data are shown in Figure 3. From Figure 3, it can be observed that although the process is in-control more than five subgroups fall in the in-decision area which need some attention.

III. ARLs OF IN-CONTROL AND OUT-OF-CONTROL PROCESSES

In this section, the in-control and the out-of-control ARLs of the proposed control chart have been obtained. Based on a single sample, the probability of declaring the process as out-of-control is

$$P_{out}^{(1)} = P\left(S^2 \geq UCL_1\right) + P\left(S^2 \leq LCL_1\right) \tag{1}$$

It is known that $(n-1)S^2/\sigma^2$ follows the chi-square distribution with $n-1$ degree of freedom for the in-control process. Thus, for the in-control process,

$$P\left(S^2 \geq UCL_1\right) = 1 - H\left(\frac{(n-1)\,UCL_1}{\sigma^2}\right)$$
$$= 1 - H\left((n-1)\left(1 + k_1\sqrt{2}/(n-1)\right)\right)$$

TABLE 1. ARLs and ASNs of the proposed chart for different shift levels when $r_0 = 200$.

Shift	n							
	4		5		6		7	
	k_1=4.1027;k_2=0.8976;i i=1		k_1=3.8421;k_2=2.2545;i i=3		k_1=3.7158;k_2=2.9948;i i=3		k_1=3.6234;k_2=2.5393;i i=5	
	ARL	ASN	ARL	ASN	ARL	ASN	ARL	ASN
1.00	200.00	4.41	200.00	5.01	200.00	6.00	200.00	7.01
1.10	113.89	4.46	110.26	5.03	105.93	6.00	101.68	7.03
1.20	71.15	4.53	67.37	5.06	62.88	6.01	58.39	7.07
1.30	47.72	4.59	44.50	5.09	40.71	6.02	36.75	7.12
1.40	33.85	4.67	31.22	5.13	28.18	6.03	24.83	7.19
1.50	25.11	4.75	22.98	5.19	20.58	6.05	17.73	7.28
1.60	19.32	4.83	17.59	5.25	15.67	6.08	13.24	7.38
1.70	15.32	4.91	13.89	5.31	12.36	6.11	10.26	7.49
1.80	12.46	4.99	11.27	5.38	10.03	6.14	8.20	7.60
1.90	10.36	5.07	9.35	5.45	8.34	6.17	6.73	7.71
2.00	8.77	5.14	7.90	5.52	7.07	6.21	5.65	7.81
3.00	3.14	5.64	2.86	5.99	2.66	6.47	2.12	8.21
4.00	1.98	5.71	1.86	6.04	1.76	6.51	1.48	8.01

TABLE 2. ARLs and ASNs of the proposed chart for different shift levels when $r_0 = 300$.

Shif t	n							
	4		5		6		7	
	k_1=5.0650;k_2=0.5336;i i=3		k_1=4.1868;k_2=1.3421;i i=3		k_1=4.0655;k_2=1.3942;i i=8		k_1=3.9168;k_2=1.6544;i i=2	
	ARL	ASN	ARL	ASN	ARL	ASN	ARL	ASN
1.00	300.00	8.89	300.00	5.15	300.00	6.39	300.00	7.06
1.10	153.35	9.00	156.00	5.24	146.36	6.60	144.87	7.13
1.20	87.09	9.19	90.29	5.36	80.24	6.87	79.45	7.22
1.30	53.71	9.42	56.70	5.50	48.14	7.17	47.95	7.35
1.40	35.38	9.66	37.95	5.67	31.06	7.49	31.15	7.52
1.50	24.61	9.91	26.74	5.85	21.26	7.81	21.45	7.71
1.60	17.90	10.16	19.65	6.04	15.29	8.12	15.49	7.92
1.70	13.53	10.39	14.96	6.23	11.47	8.41	11.63	8.15
1.80	10.56	10.59	11.74	6.43	8.92	8.67	9.03	8.39
1.90	8.48	10.77	9.46	6.61	7.15	8.90	7.21	8.62
2.00	6.98	10.91	7.79	6.79	5.89	9.10	5.91	8.84
3.00	2.31	10.66	2.51	7.68	2.03	9.51	1.93	9.87
4.00	1.56	9.24	1.63	7.45	1.41	8.75	1.34	9.34

where H is the distribution function of the chi-square distribution with degree of freedom of n-1.

Also,

$$P\left(S^2 \leq LCL_1\right) = H\left((n-1)\left(1 - k_1\sqrt{2/(n-1)}\right)\right)$$

Therefore, Eq. (1) becomes

$$P_{out}^{(1)} = 1 + H\left((n-1)\left(1 - k_1\sqrt{2/(n-1)}\right)\right) - H\left((n-1)\left(1 + k_1\sqrt{2/(n-1)}\right)\right) \quad (2)$$

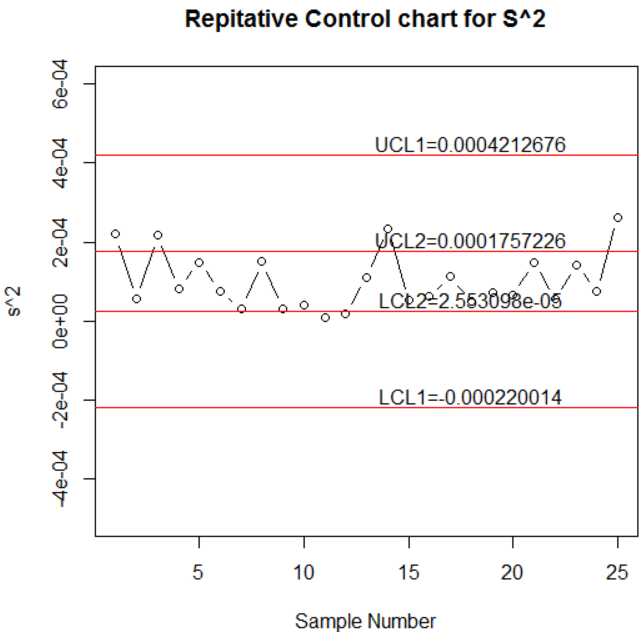

FIGURE 1. **The proposed control chart for the inside diameter measurements.**

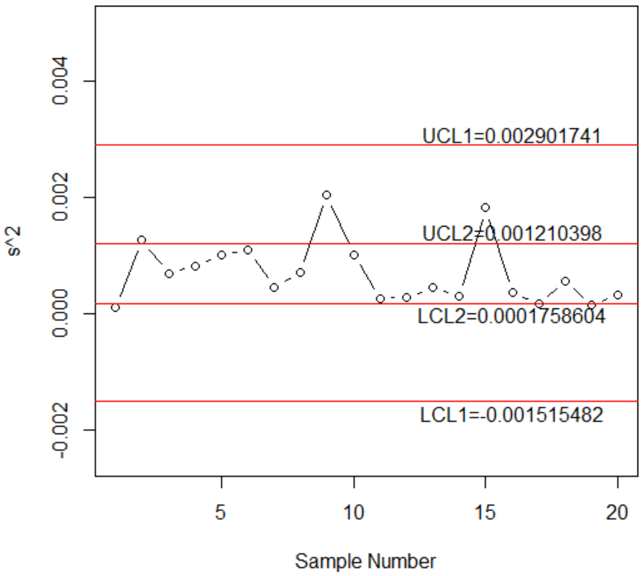

FIGURE 2. **The proposed control chart for the diameter casting data.**

Thus the probability of the single sample for declaring the process as in control becomes

$$\begin{aligned} P_{in}^{(1)} &= P\left(LCL_2 \le S^2 \le UCL_2\right) \\ &\quad + \left\{P\left(LCL_1 < S^2 < LCL_2\right) \right. \\ &\qquad \left. + P\left(UCL_2 < S^2 < UCL_1\right)\right\} \\ &\quad \times \left\{P\left(LCL_2 \le S^2 \le UCL_2\right)\right\}^i \\ P_{in}^{(1)} &= 1 + H\left((n-1)\left(1-k_1 r\right)\right) - H\left((n-1)\left(1+k_1 r\right)\right) \\ &\quad + \left\{1 + H\left((n-1)\left(1-k_1 r\right)\right) - H\left((n-1)\left(1-k_2 r\right)\right)\right. \\ &\quad \left. + 1 + H\left((n-1)\left(1+k_2 r\right)\right) - H\left((n-1)\left(1+k_1 r\right)\right)\right\} \\ &\quad \times \left\{1 + H\left((n-1)\left(1-k_2 r\right)\right) - H\left((n-1)\left(1+k_2 r\right)\right)\right\}^i \end{aligned} \tag{3}$$

where $r = \sqrt{2/(n-1)}$.

The probability of repetition is computed as:

$$\begin{aligned} P_{rep} &= \left\{P\left(LCL_1 < S^2 < LCL_2\right) + P\left(UCL_2 < S^2 < UCL_1\right)\right\} \\ &\qquad \times \left\{1 - P\left(LCL_2 \le S^2 \le UCL_2\right)\right\}^i \end{aligned}$$

So, finally, for the in-control process, it is

$$\begin{aligned} P_{rep} &= \left\{1 + H\left((n-1)\left(1-k_1 r\right)\right) - H\left((n-1)\left(1-k_2 r\right)\right)\right. \\ &\quad \left. + 1 + H\left((n-1)\left(1+k_2 r\right)\right) - H\left((n-1)\left(1+k_1 r\right)\right)\right\} \\ &\quad \times \left\{1 + H\left((n-1)\left(1-k_2 r\right)\right) - H\left((n-1)\left(1+k_2 r\right)\right)\right\}^i \end{aligned} \tag{4}$$

where $r = \sqrt{2}/(n-1)$

Therefore, the out-of-control process when actually it is in-control can be written in (5), as shown at the bottom of the next page.

The average sample number (ASN) for the in-control process can be stated in (6), as shown at the bottom of the next page.

For shifts in processes, the probability can be derived from the shifted process $\sigma_1^2 = c\sigma^2$ in which $(n-1)\,S^2/(c\sigma^2)$ also follow the Chi-Square distribution with the same $n-1$ degrees of freedom

In this situation, the probability of out-of-control process can be stated as

$$\begin{aligned} P_{out,shift}^{(1)} &= P\left(S^2 \ge UCL_1 | \sigma_1^2\right) + P\left(S^2 \le LCL_1 | \sigma_1^2\right) \\ &= 1 + H\left(\frac{n-1}{c}\left(1 - k_1\sqrt{2}/(n-1)\right)\right) \end{aligned}$$

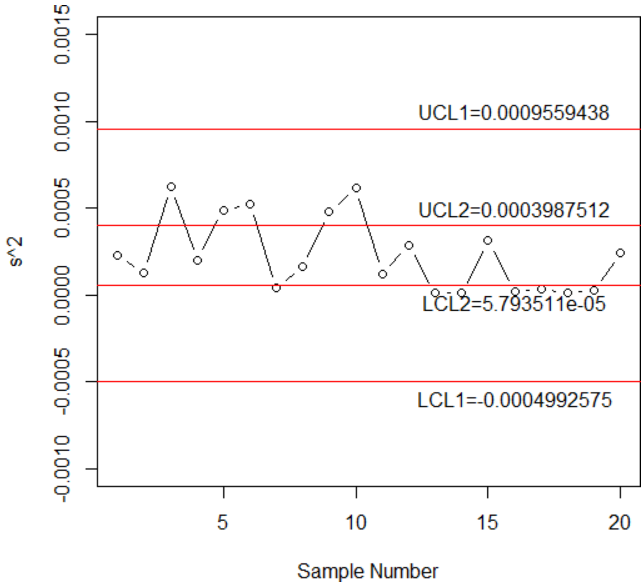

FIGURE 3. **The proposed control chart for the vane heights of casting.**

TABLE 3. ARLs and ASNs of the proposed chart for different shift levels when $r_0 = 370$.

Shift	n = 4 (k1=4.7272;k2=1.02918; ii=8)		n = 5 (k1=4.5063;k2=1.0554;i i=8)		n = 6 (k1=4.2816;k2=1.1085;i i=4)		n = 7 (k1=4.24005;k2=1.04306; ii=7)	
	ARL	ASN	ARL	ASN	ARL	ASN	ARL	ASN
1.00	370.01	4.88	370.02	6.24	370.04	6.93	370.03	9.06
1.10	192.59	5.04	181.98	6.45	175.59	7.14	165.69	9.38
1.20	111.21	5.22	100.30	6.70	93.98	7.42	84.51	9.81
1.30	69.63	5.41	60.41	6.97	55.17	7.75	47.72	10.31
1.40	46.50	5.60	39.06	7.26	34.86	8.12	29.24	10.85
1.50	32.72	5.79	26.76	7.54	23.38	8.50	19.17	11.39
1.60	24.05	5.97	19.23	7.81	16.49	8.89	13.30	11.92
1.70	18.33	6.15	14.40	8.07	12.14	9.28	9.68	12.40
1.80	14.41	6.31	11.15	8.31	9.27	9.63	7.35	12.82
1.90	11.63	6.46	8.90	8.52	7.31	9.96	5.78	13.18
2.00	9.60	6.59	7.28	8.71	5.93	10.24	4.69	13.46
3.00	3.07	7.14	2.34	9.19	1.93	10.75	1.63	12.96
4.00	1.92	6.88	1.55	8.45	1.36	9.57	1.22	10.95

$$-H\left(\frac{n-1}{c}\left(1+k_1\sqrt{2}/(n-1)\right)\right) \tag{7}$$

Therefore, the probability of out-of-control process when a shift has occurred can be stated in (8), as shown at the bottom of this page.

The average run length (ARL) when the process is in-control is given as

$$ARL_0 = \frac{1}{P_{out}} \tag{9}$$

The average run length (ARL) for the out-of-control process is stated as

$$ARL_1 = \frac{1}{P_{out,shift}} \tag{10}$$

IV. PERFORMANCE OF THE PROPOSED CHART THROUGH SIMULATION

Here, the performance evaluation of the offered chart has been discussed. The performance of any proposed chart

$$P_{out} = \frac{1+H((n-1)(1-k_1c))-H((n-1)(1+k_1c))}{1-\left[\{1+H((n-1)(1-k_1r))-H((n-1)(1-k_2r))+1+H((n-1)(1+k_2r))-H((n-1)(1+k_1r))\}\{1+H((n-1)(1-k_2r))-H((n-1)(1+k_2r))\}^i\right]} \tag{5}$$

$$ASN_0 = \frac{n}{1-\left[\{1+H((n-1)(1-k_1r))-H((n-1)(1-k_2r))+1+H((n-1)(1+k_2r))-H((n-1)(1+k_1r))\}\{1+H((n-1)(1-k_2r))-H((n-1)(1+k_2r))\}^i\right]} \tag{6}$$

$$P_{out,shift} = \frac{1+H\left(\frac{n-1}{c}(1-k_1c)\right)-H\left(\frac{n-1}{c}(1+k_1c)\right)}{1-\left[\left\{1+H\left(\frac{n-1}{c}(1-k_1r)\right)-H\left(\frac{n-1}{c}(1-k_2r)\right)+1+H\left(\frac{n-1}{c}(1+k_2r)\right)-H\left(\frac{n-1}{c}(1+k_1r)\right)\right\}\left\{1+H\left(\frac{n-1}{c}(1-k_2r)\right)-H\left(\frac{n-1}{c}(1+k_2r)\right)\right\}^i\right]} \tag{8}$$

TABLE 4. Average Run Lengths of S^2 chart using MDSS for different shift levels when $r_0 = 370$.

Shift	n							
	4		5		6		7	
	k_1=6.8561; k_2=1.9612i i=1		k_1=4.4746; k_2=2.6193i i=1		k_1=4.2345; k_2=2.8424i i=1		k_1=4.1134; k_2=2.8022i i=1	
	ARL	ASN	ARL	ASN	ARL	ASN	ARL	ASN
1.00	370.09	4.20	370.03	5.10	370.14	6.08	370.46	7.09
1.10	193.65	4.27	183.12	5.15	177.98	6.13	170.68	7.15
1.20	113.21	4.35	102.39	5.21	97.25	6.18	90.22	7.22
1.30	72.07	4.43	62.92	5.27	58.65	6.25	53.02	7.30
1.40	49.06	4.51	41.65	5.33	38.22	6.31	33.87	7.39
1.50	35.23	4.59	29.26	5.39	26.51	6.38	23.13	7.47
1.60	26.42	4.67	21.58	5.45	19.35	6.44	16.68	7.55
1.70	20.54	4.74	16.56	5.50	14.72	6.49	12.58	7.62
1.80	16.45	4.80	13.13	5.55	11.60	6.54	9.85	7.68
1.90	13.51	4.85	10.71	5.59	9.40	6.58	7.95	7.72
2.00	11.33	4.90	8.94	5.62	7.82	6.62	6.59	7.75
3.00	3.91	5.01	3.11	5.63	2.71	6.59	2.32	7.63
4.00	2.42	4.82	2.00	5.45	1.77	6.38	1.56	7.35

TABLE 5. Comparison of average run lengths of proposed versus MDSS charts when $r_0 = 370$.

Shift	n							
	4		5		6		7	
	Proposed	MDSS	Proposed	MDSS	Proposed	MDSS	Proposed	MDSS
1.00	370.01	370.09	370.02	370.03	370.04	370.14	370.03	370.46
1.10	192.59	193.65	181.98	183.12	175.59	177.98	165.69	170.68
1.20	111.21	113.21	100.30	102.39	93.98	97.25	84.51	90.22
1.30	69.63	72.07	60.41	62.92	55.17	58.65	47.72	53.02
1.40	46.50	49.06	39.06	41.65	34.86	38.22	29.24	33.87
1.50	32.72	35.23	26.76	29.26	23.38	26.51	19.17	23.13
1.60	24.05	26.42	19.23	21.58	16.49	19.35	13.30	16.68
1.70	18.33	20.54	14.40	16.56	12.14	14.72	9.68	12.58
1.80	14.41	16.45	11.15	13.13	9.27	11.60	7.35	9.85
1.90	11.63	13.51	8.90	10.71	7.31	9.40	5.78	7.95
2.00	9.60	11.33	7.28	8.94	5.93	7.82	4.69	6.59
3.00	3.07	3.91	2.34	3.11	1.93	2.71	1.63	2.32
4.00	1.92	2.42	1.55	2.00	1.36	1.77	1.22	1.56

can be efficiently evaluated calculating the average run length (ARL), which is defined as the average number of samples falling in the in-control limits before the process indicates an out-of-control point [2]. The ARL of the in-control process is denoted by ARL_0 which should be an enough larger value for an efficient chart scheme and the out-of-control ARL denoted by ARL_1 should be a smaller value for the efficient detection ability of the proposed chart. However, the average sample number (ASN) is also equally important to consider the evaluation of the proposed chart which is the average number of units selected for posting on the control chart. Therefore, in comparison with various competitive charts the ARL_1 and the ASN is very important and commonly used criterion [30].

Simulation technique has been used for estimating the coefficients k_1 and k_2 for the proposed control chart using

TABLE 6. Average run lengths of S^2 chart using RGS for different shift levels when $r_0 = 370$.

shift	n							
	4		5		6		7	
	k1=4.64494;k2=1.30889		k1=4.34237;k2=2.83004		k1=4.19872;k2=2.23288		k1=4.067;k2=2.84737	
	ARL	ASN	ARL	ASN	ARL	ASN	ARL	ASN
1.00	370.00	4.44	370.00	5.07	370.00	6.19	370.00	7.08
1.10	194.84	4.58	190.55	5.11	180.23	6.29	174.78	7.14
1.20	113.85	4.72	109.95	5.16	99.34	6.41	94.36	7.21
1.30	72.09	4.86	69.21	5.22	60.17	6.54	56.39	7.30
1.40	48.64	5.01	46.63	5.28	39.25	6.69	36.47	7.39
1.50	34.55	5.16	33.16	5.34	27.16	6.83	25.12	7.49
1.60	25.60	5.30	24.65	5.40	19.72	6.98	18.20	7.59
1.70	19.65	5.44	19.00	5.46	14.91	7.12	13.76	7.69
1.80	15.53	5.57	15.10	5.52	11.65	7.25	10.76	7.78
1.90	12.60	5.69	12.31	5.58	9.37	7.38	8.67	7.87
2.00	10.44	5.80	10.26	5.63	7.72	7.50	7.16	7.96
3.00	3.35	6.36	3.41	5.95	2.53	7.99	2.41	8.31
4.00	2.07	6.27	2.12	5.96	1.64	7.76	1.59	8.14

TABLE 7. Comparison of average run lengths of proposed versus RGS charts when $r_0 = 370$.

Shift	n							
	4		5		6		7	
	Proposed	RGS	Proposed	RGS	Proposed	RGS	Proposed	RGS
1.00	370.01	370.00	370.02	370.00	370.04	370.00	370.03	370.00
1.10	192.59	194.84	181.98	190.55	175.59	180.23	165.69	174.78
1.20	111.21	113.85	100.30	109.95	93.98	99.34	84.51	94.36
1.30	69.63	72.09	60.41	69.21	55.17	60.17	47.72	56.39
1.40	46.50	48.64	39.06	46.63	34.86	39.25	29.24	36.47
1.50	32.72	34.55	26.76	33.16	23.38	27.16	19.17	25.12
1.60	24.05	25.60	19.23	24.65	16.49	19.72	13.30	18.20
1.70	18.33	19.65	14.40	19.00	12.14	14.91	9.68	13.76
1.80	14.41	15.53	11.15	15.10	9.27	11.65	7.35	10.76
1.90	11.63	12.60	8.90	12.31	7.31	9.37	5.78	8.67
2.00	9.60	10.44	7.28	10.26	5.93	7.72	4.69	7.16
3.00	3.07	3.35	2.34	3.41	1.93	2.53	1.63	2.41
4.00	1.92	2.07	1.55	2.12	1.36	1.64	1.22	1.59

the above- mentioned developed equations. When the exact form of the mean and other measures of the proposed chart is unavailable then the simulation approach is employed to study the numeric performance of the chart [31]. This simulation approach has been used by many authors including [32]–[35]. By considering the marked ARL_0, an R-language code program was written to determine the ARL_1 for the in-control process with the constraint of minimal

TABLE 8. Average run lengths of S^2 chart using SS for different shift levels when $r_0 = 370$.

n	4	5	6	7
k_1	4.55366	4.33065	4.17509	4.05862
Shift	ARL			
1.00	370.00	370.00	370.00	370.00
1.10	202.74	192.35	183.76	176.40
1.20	123.17	112.18	103.48	96.30
1.30	80.98	71.42	64.12	58.27
1.40	56.65	48.71	42.81	38.20
1.50	41.63	35.07	30.32	26.68
1.60	31.84	26.39	22.52	19.61
1.70	25.17	20.59	17.39	15.02
1.80	20.44	16.55	13.87	11.90
1.90	16.99	13.64	11.36	9.70
2.00	14.39	11.48	9.52	8.10
3.00	5.16	4.05	3.34	2.85
4.00	3.17	2.51	2.11	1.85

ASN_0 given in Eq. (6). Table 1–3 has been generated for in-control ARL_0 (r_0 = 200, 300 and 370).

V. COMPARISON WITH CHARTS USING MDSS, RGS AND SS

In this section, a comparative study has been discussed for highlighting the benefits of the proposed chart. A control chart having the smaller ARL_1 value is considered as the better chart as compared to any other chart [2].

A. PROPOSED MDSRS CHART VERSUS MDSS CHART

The ARL_1 values of the MDSS chart are provided in Table 4. The proposed chart is compared with the MDSS chart for the fixed values of the r_0 = 370 and n=4, 5, 6 and 7.

A comparison table (Table 5) has been presented for the ready reference. We can observe that the proposed scheme has smaller values of the ARL_1 for all the shift levels. For example, the proposed MDSRS gives a better performance in detecting the out-of-control process as it required 69.63 samples to detect a shift of 1.30 when n = 4 as compared to existing chart of MDSS which utilizes 72.07 samples on the average to detect the same shift level. The same benefit of smaller ARL_1 of the proposed chart can be observed for all values of the process settings.

B. PROPOSED MDSRS CHART VERSUS RGS CHART

The ARL_1 values of the RGS chart are provided in Table 6. The proposed chart is compared with the RGS chart for the fixed vales of the r_0 = 370 and n=4, 5, 6 and 7.

A comparison table (Table 7) has been presented for the ready reference. We can observe that the proposed scheme has smaller values of the ARL_1 for all the shift levels. For example, the proposed MDSRS gives a better performance in detecting the out-of-control process as it required 47.72 samples to detect a shift of 1.30 when n = 7 as compared to existing chart of RGS which utilizes 56.39 samples on the average to detect the same shift level. The same benefit of smaller ARL_1 of the proposed chart can be observed for all values of the process settings.

C. PROPOSED MDSRS CHART VERSUS SS CHART

The ARL_1 values of the SS chart are provided in Table 8. Here, the proposed chart is compared with the SS chart for the fixed vales of the r_0 = 370 and n=4, 5, 6 and 7.

A comparison table (Table 9) has been presented for the ready reference. We can observe that the proposed scheme has smaller values of the ARL_1 for all the shift levels. For example, the proposed MDSRS gives a better performance in detecting the out-of-control process as it required 60.41 samples to detect a shift of 1.30 when n = 5 as compared to existing chart of RGS which utilizes 71.42 samples on the average to detect the same shift level. The same benefit of smaller ARL_1 of the proposed chart can be observed for all values of the process settings.

It can be very easily observed that the proposed chart is the most efficient chart while the SSS chart is the least efficient chart. For example, when n = 5 and ARL_0 = 370 a shift of 1.50 can be detected using the proposed MDSRS 26.76, using the MDSS 29.26, using the RGS 33.16 and using the SSS 35.07 sample on the average. The overall comparison of the proposed MDSRS, MDSS, RGS and the SS can be best seen in Table 10.

VI. A SIMULATED STUDY

In this section, a simulated data set has been generated and applied to the proposed control chart to demonstrate the efficiency. For this purpose Table 11 has been given in which the first 20 observations are generated from the normal distribution with mean 0 and $\sigma^2 = 4$ (considered as an in-control process) and the subsequent 20 observations are created from a shifted process with standard deviation $c^*\sigma$, where the shift constant c = 1.6.

Figure 4 is constructed for the proposed MDSRS using the simulated data given in Table 11. The control limits of the proposed control chart are obtained as $UCL_1 = 16.7457$, $LCL_1 = -8.7457$, $UCL_2 = 6.9851$ and $LCL_2 = 1.0149$. The proposed control chart shows that the process will be declared as out-of-control at the 36^{th} sample.

Figure 5 shows the control using RGS applied to the simulation data. The four control limits have been obtained as the $UCL_1 = 16.2821$, $LCL_1 = -8.2821$, $UCL_2 = 12.0046$ and $LCL_2 = 4.00456$. The statistics S^2 of the simulated data have been plotted on these four limits, which shows that process is declared as in-control. We can say that the RGS scheme is unable to detect an out-of-control process.

Figure 6 shows the control chart scheme of the MDS sampling using the simulation data. The four control limits have

TABLE 9. Comparison of average run lengths of proposed versus SS charts when $r_0 = 370$.

Shift	n							
	4		5		6		7	
	Proposed	SS	Proposed	SS	Proposed	SS	Proposed	SS
1.00	370.01	370.00	370.02	370.00	370.04	370.00	370.03	370.00
1.10	192.59	202.74	181.98	192.35	175.59	183.76	165.69	176.40
1.20	111.21	123.17	100.30	112.18	93.98	103.48	84.51	96.30
1.30	69.63	80.98	60.41	71.42	55.17	64.12	47.72	58.27
1.40	46.50	56.65	39.06	48.71	34.86	42.81	29.24	38.20
1.50	32.72	41.63	26.76	35.07	23.38	30.32	19.17	26.68
1.60	24.05	31.84	19.23	26.39	16.49	22.52	13.30	19.61
1.70	18.33	25.17	14.40	20.59	12.14	17.39	9.68	15.02
1.80	14.41	20.44	11.15	16.55	9.27	13.87	7.35	11.90
1.90	11.63	16.99	8.90	13.64	7.31	11.36	5.78	9.70
2.00	9.60	14.39	7.28	11.48	5.93	9.52	4.69	8.10
3.00	3.07	5.16	2.34	4.05	1.93	3.34	1.63	2.85
4.00	1.92	3.17	1.55	2.51	1.36	2.11	1.22	1.85

TABLE 10. Comparison of ARL_1 of Proposed MDSRS, MDSS, RGS and SS Charts when $r_0 = 370$ and n = 5.

Shift	Proposed MDSRS	MDSS	RGS	SS
1.00	370.02	370.03	370.00	370.00
1.10	181.98	183.12	190.55	192.35
1.20	100.30	102.39	109.95	112.18
1.30	60.41	62.92	69.21	71.42
1.40	39.06	41.65	46.63	48.71
1.50	26.76	29.26	33.16	35.07
1.60	19.23	21.58	24.65	26.39
1.70	14.40	16.56	19.00	20.59
1.80	11.15	13.13	15.10	16.55
1.90	8.90	10.71	12.31	13.64
2.00	7.28	8.94	10.26	11.48
3.00	2.34	3.11	3.41	4.05
4.00	1.55	2.00	2.12	2.51

been estimated as the $UCL_1 = 16.6562$, $LCL_1 = -8.6562$, $UCL_2 = 11.4085$ and $LCL_2 = 3.4085$. The statistics s^2 of the simulated data given in Table 5 have been plotted on these four limits which shows that process is in-control or we can say that the MDS sampling scheme is unable to detect an out-of-control process.

Figure 7 shows the control chart using SS applied to the simulation data. The two control limits have been estimated as the UCL = 16.24 and LCL = −8.24. The statistics S^2 of the simulated data have been plotted on these two limits, which shows that process is declared as in-control. Again, we can say that the SS scheme is unable to detect an out-of-control process.

VII. CONCLUDING REMARKS

This article has proposed a new S^2 chart by combining the features of the MDS sampling and the RGS scheme, called the S^2 control chart with MDSRS. The analysis of the proposed chart revealed the efficient detecting ability of the out-of-control processes for different shift levels as compared with

TABLE 11. Simulated data for the proposed control chart.

Sr#	Samples					S^2
	1	2	3	4	5	
1	1.63106	-2.72484	-1.35171	0.74399	-0.13008	2.95293
2	1.55391	-2.47700	-0.44054	-1.92541	-0.34623	2.48207
3	-2.77411	0.60127	2.39076	3.88763	-2.18262	8.22773
4	2.01237	1.27574	1.07938	-1.16430	-0.42822	1.71027
5	-1.68264	-5.94597	-2.32852	-2.83786	0.19329	4.98452
6	-0.01313	2.18607	-2.64597	1.50730	0.86858	3.52062
7	1.03516	-0.70303	-0.24265	1.04861	0.12907	0.60494
8	-0.86764	-0.05846	-0.86923	-4.11042	-0.09313	2.80468
9	-0.98278	0.28230	-1.35431	2.63292	-3.00470	4.41563
10	2.08466	-0.02800	0.18943	1.31041	2.77224	1.44359
11	2.92028	-0.93272	-2.21979	0.30998	1.27963	3.92262
12	-1.81145	0.08825	-0.98579	0.38062	-2.92653	1.86190
13	-1.80729	-2.48797	-1.95164	2.21667	2.53590	6.04076
14	0.58308	-0.97878	-0.32329	-3.64366	-0.05972	2.69271
15	0.11149	-1.80075	2.04242	1.28491	-0.04168	2.14241
16	0.05889	1.74951	-0.24133	-1.68638	-2.17695	2.41273
17	3.43103	0.70431	1.41900	2.46813	2.33023	1.09189
18	-0.84779	0.21316	-1.25673	-2.02000	2.19890	2.66815
19	0.67464	-0.38699	1.80344	-2.30608	4.86068	7.12125
20	0.54963	-0.39311	0.07366	0.41629	0.32989	0.13838
21	-0.46420	-1.91382	-1.07720	-0.51207	-6.97069	7.49085
22	-0.14014	-1.93384	-2.22979	2.99686	-2.51050	5.27515
23	-3.43280	3.57900	-4.05985	-0.31234	0.09210	9.44098
24	-4.18158	-1.73693	-4.79201	0.44755	3.15434	10.87738
25	-0.37308	-4.17973	-4.13064	2.50093	1.42036	9.60200
26	-2.98550	-1.95858	2.22064	0.24868	3.14868	6.89143
27	-3.69930	-4.32324	-6.93377	-2.59638	-1.42882	4.29389
28	1.46144	-4.96049	3.27466	-2.66248	-2.62579	11.34285
29	0.07832	-0.04321	-8.83739	-2.36672	-2.20680	13.19911
30	1.50473	3.09669	-3.43827	-0.70032	-2.94311	7.89934
31	-6.21001	-1.84666	-0.05954	-0.04125	3.75967	13.06166
32	-0.49872	-0.43926	5.14265	0.05575	0.36071	5.68770
33	4.30456	-3.27666	3.88102	-2.81879	-3.67086	16.31228
34	5.93879	-0.71003	1.37533	1.78048	0.25497	6.49292
35	3.01609	3.51745	-0.53523	2.38751	-0.92046	4.28921
36	5.92490	1.27528	-7.33237	-2.05000	3.17579	26.14626
37	-1.32994	-1.89781	-1.45543	-0.86030	-1.38426	0.13657
38	4.72893	-2.30266	2.85139	0.53082	-0.87936	7.99780
39	-0.54947	-0.52518	0.57600	3.71346	-5.94204	12.12990
40	3.44048	4.37454	-5.24580	-6.24346	1.78340	24.98453

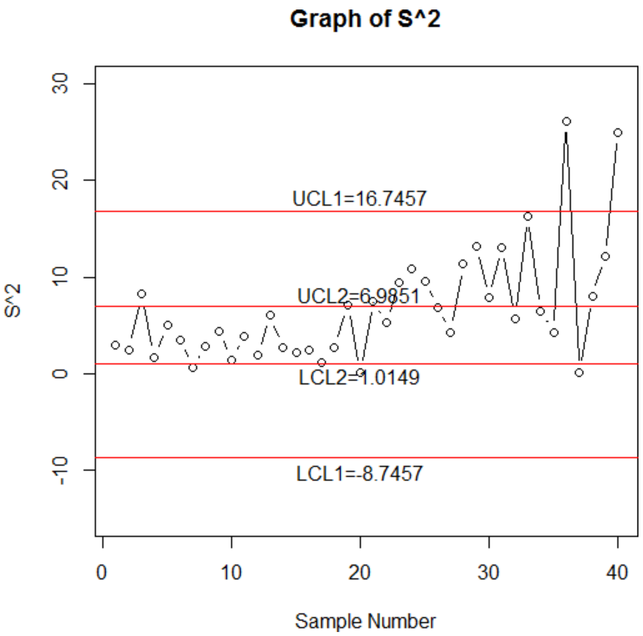

FIGURE 4. The proposed MDSRS control chart for the simulated data.

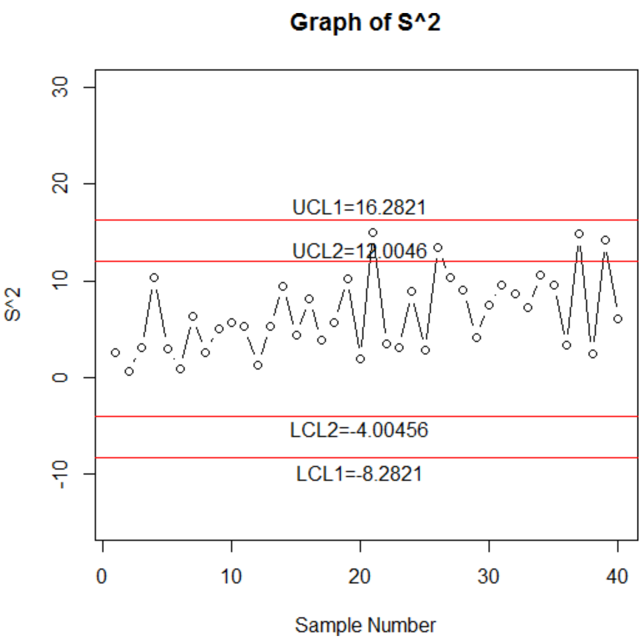

FIGURE 5. The control chart using RGS for the simulated data.

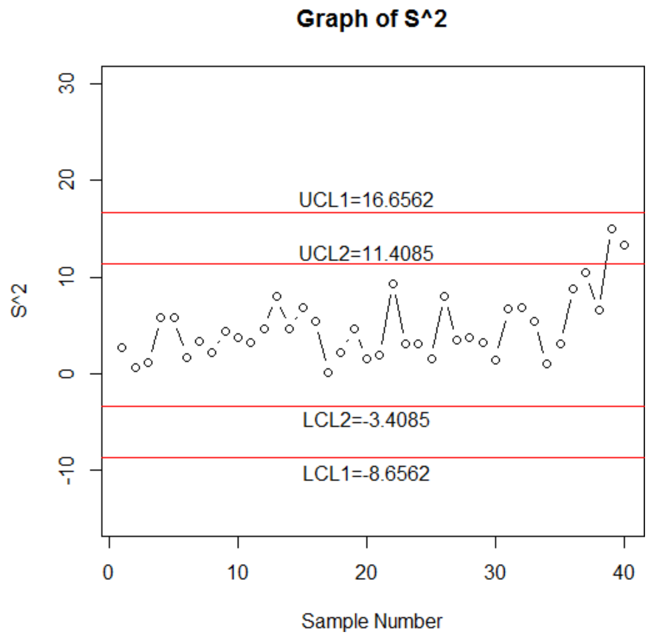

FIGURE 6. The control chart using MDSS for the simulated data.

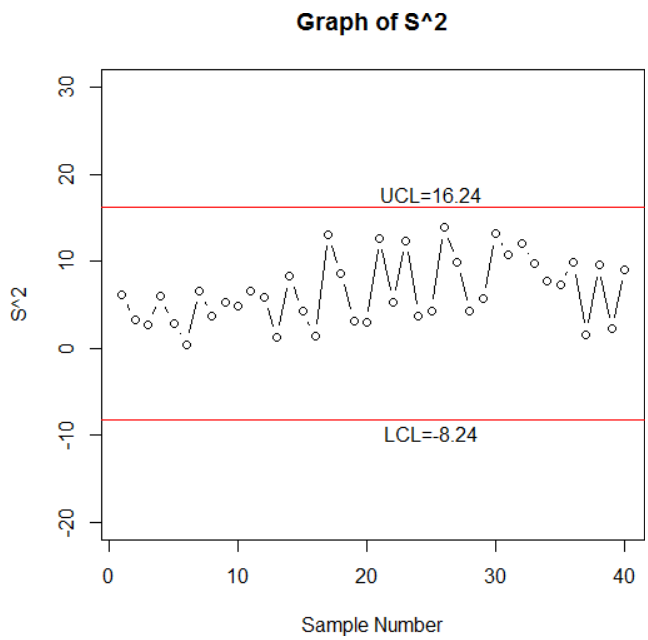

FIGURE 7. The control chart using SS for the simulated data.

the existing counterparts. The performance of the purposed chart has been evaluated using different process settings. The proposed chart is a valuable addition to the toolkit of the quality control personnel. The significance of the proposed chart to monitor the process variance has also been demonstrated using the simulated as well as the real-life example.

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