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Gamma Degradation Process and Accelerated Model Combined Reliability Analysis Method for Rubber O-Rings

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ABSTRACT The sealing performance of rubber O-rings plays an irreplaceable role in ensuring the high reliability and safety of mechanical systems. However, traditional statistical analysis may not consider the aging process of rubber O-rings at different temperatures and the time-varying characteristics of rubber material parameters. In view of these problems, an improved reliability analysis method based on the accelerated aging test of rubber O-rings is proposed in this paper. This method is featured by a combination with the traditional accelerated model and the Gamma stochastic process. This paper first adopts the stationary Gamma process to describe the degradation of compression set which is the performance degradation indicator of rubber O-rings. Then, the reliability model is derived from the Gamma degradation model. Next, the Arrhenius model is used to represent the shape parameter of Gamma degradation model and the approach of maximum likelihood estimation is adopted to solve the parameters. Finally, the rubber O-rings of gas steering engine were selected and a series of accelerated aging tests were conducted to obtain the reliability and the storage lifetime of rubber O-rings at different temperature stresses through our improved method. Besides, the comparison with the traditional statistical method is also conducted to prove the advantage of this method.

INDEX TERMS Reliability analysis method, rubber O-rings, Gamma degradation model, accelerated aging test, compression set.

I. INTRODUCTION

Owing to the excellent temperature resistance and sealing performance, rubber O-rings have been widely used in a variety of mechanical systems in aerospace products. Once the O-rings fail, there will be a serious accident that would bring about fatal losses and casualties. However, it is inevitable that the sealing performance of rubber O-rings will degenerate over time because of the aging of rubber materials. Therefore, it is of great significance to study the performance degradation of the O-rings and to assess their reliability accurately to guarantee the safe operation of mechanical systems.

Considering the aging of rubber materials, the relevant researches have shown that rubber components are more prone to fail compared with other mechanical parts [1]–[3]. Based on the engineering experience, the change of compression set (CS) is a more representative indicator to describe the deterioration process of rubber O-rings on the sealing performance [4]–[6]. What's more, the CS of rubber O-rings is one of the most important performance indexes of rubber products, which relates to the elasticity and recovery of vulcanized rubber materials [7]. Thus, CS is chosen as the characteristic index to represent the degenerative character of rubber O-rings in this paper.

To describe the deterioration process of CS precisely, many researchers have conducted studies to resolve the problem by means of a great deal of experimental and theoretical research. Feyzullahoglu [8] compared the differences between the properties of several rubber materials under different working conditions and analyzed the degradation characteristics of CS. Patel and Skinner [9] put forward a time-temperature superposition model to analyze the experimental data and evaluate the reliability of rubbers utilizing

CS as the characteristic parameter. On the basis of superposition model, the relationship between the accelerated factor and the temperature was analyzed by the Arrhenius plot method. Morrell *et al.* [10] applied the superposition model to the study on the accelerated thermal aging of performance for nitrile-butadiene rubber (NBR) O-rings. Furthermore, an accelerated model depend upon the drift theory of Brownian movement was used by Wang *et al.* [11] to evaluate the results of step-stress accelerated aging test of NBR sealing rings. Coons *et al.* [12] studied the characteristics of CS for the silicon rubber materials adopting a first order kinetic model. In addition, Jung *et al.* [13] investigated the hyperplastic behavior of rubber seals through using axisymmetric finite-element analysis and applied the neo-Hookean constitutive equations to analyze nonlinear elastomeric materials. Moreover, Woo *et al.* [14] adopted a linear model to describe the relationship between elastic recovery and test time and predicted the lifetime of rubber components. What's more, the Bayesian network is an effective method to deal with various uncertainty problems on reliability modeling [15], [16].

In summary, these researchers adopted different methods to calculate the lifetime and reliability of rubber O-rings using CS as the degradation index. However, the data processing in accelerated aging test that these researchers have conducted is depend on the traditional statistical analysis method that is complex in calculation and depend on the prior information. In the traditional statistical analysis methods, the lifetime of all samples are first calculated respectively basing on the accelerated model. Then, the life distribution of products can be obtained by assuming the distribution of the results of lifetime and verifying the accuracy of results through the hypothesis test. But the performance index degradation of products usually shows some regularity with time. For example, the degradation of CS for rubber O-rings is non-negative and the degradation process is strictly monotone. In the previous research of our study group, it had been proved that the degradation process of CS for rubber O-rings monotonically increases with time under certain temperature range [17]–[19].

In order to better describe the performance degradation and simplify the calculation process, modeling methods basing on the stochastic process are widely used in the study of reliability assessment, especially Gamma process and Wiener process [20]–[23]. Besides, the stationary Gamma process is suitable for describing the regularity of monotonous increasing characteristics with time. Pan and Balakrishnan [24] established the accelerated degradation model based upon Wiener process and Gamma process in the step-up-stress accelerated test. Liu *et al.* [25] set up the performance degradation model depend on Gamma process to predict the lifetime of pulse capacitor.

This paper proposed a reliability analysis method in accelerated degradation test for rubber O-rings. Firstly, CS was selected as the performance degradation index of rubber O-rings. Secondly, the reliability model was developed combined with accelerated model and Gamma stochastic process in Section II. Then, the parameters in the model were determined based on the approach of MLE. Finally, the case study demonstrated the availability and applicability of this improved method through the calculation results and analysis, as Section III presented.

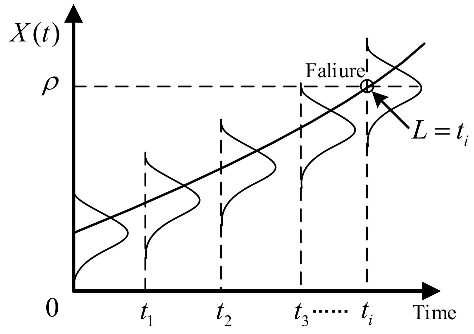


FIGURE 1. The degradation process of CS.

II. METHODOLOGY

A. RELIABILITY MODELING BASED ON GAMMA PROCESS

Considering the long-term loading and high-temperature work conditions, the main failure mode of rubber O-rings is aging caused by the destruction of vulcanized structure for rubber materials. The deterioration of CS can be regarded as a time-dependent stochastic process $\{X(t), t \geq 0\}$ that describes the degradation process of CS for rubber O-rings with time. When $X(t)$ reaches its failure threshold ρ which is a constant, rubber O-rings will fail. The random variable L which is the lifetime of rubber O-rings represents the first time of $X(t)$ to achieve the failure threshold ρ and can be described as $L = \inf\{t \,|X(t) \geq \rho, t \geq 0\}$. $F_L(t)$ is the cumulative distribution function (CDF) of L for rubber O-rings. According to the failure mode of rubber O-rings, $F_L(t)$ can be derived as

$$F_L(t) = P(L \leq t) = P(X(t) \geq \rho) \tag{1}$$

Thus the reliability function of rubber O-rings can be obtained as

$$R(t) = 1 - F_L(t) = 1 - P(X(t) \geq \rho) \tag{2}$$

Figure 1 schematically shows the degenerative process of CS for rubber O-rings. As shown in Figure 1, $X(t)$ that describes the degenerative process of CS increases with time and reaches the failure threshold ρ at the time t_i. So the rubber O-ring fails at t_i and its lifetime is $L = t_i$.

In practice, the degradation of CS presents a certain regularity in engineering application and accelerated tests. On the basis of the degradation characteristics of CS, $X(t)$ is monotonous gradual process with stable independent increments. Therefore, $X(t)$ is a stationary Gamma process that satisfies $X(t+\Delta t) - X(t) \sim Ga(\alpha \Delta t, \beta)$ for any $t \geq 0$ and $\Delta t \geq 0$, where $Ga(\alpha, \beta)$ stands for the Gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$.

The probability density function (PDF) of $Ga(\alpha, \beta)$ can be defined as

$$f_G(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} \exp(-x/\beta), \quad x > 0 \tag{3}$$

where $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$ is the Gamma function.

When utilizing the Gamma process to model the performance degradation index of products, the shape parameter α describes the effect of stress on the performance of products and the scale parameter β describes the influence of random factors such as environmental factors, human factors, material differences, etc. on the performance of products. Therefore, it is generally assumed that α is related to stress, while β is not related. Since $X(t)$ is a stationary Gamma stochastic process, it satisfies $X(t) \sim Ga(\alpha t, \beta)$. The PDF of $X(t)$ can be expressed as

$$f_X(x; \alpha(t), \beta) = \frac{1}{\Gamma(\alpha t) \beta^{\alpha t}} x^{\alpha t-1} \exp(-x/\beta), \quad x > 0 \tag{4}$$

According to the statistical properties of Gamma process, it is known that the expectation and the variance of $X(t)$ at the time t are respectively

$$E[X(t)] = \beta \cdot \alpha t, \quad Var[X(t)] = \beta^2 \cdot \alpha t \tag{5}$$

What's more, $F_L(t)$ can be derived based on (1).

$$\begin{aligned} F_L(t) &= P(L \le t) = P(X(t) \ge \rho) \\ &= \int_{\rho}^{\infty} f_X(x; \alpha(t), \beta)\, dx \\ &= \int_{\rho}^{\infty} \frac{1}{\Gamma(\alpha t) \beta^{\alpha t}} x^{\alpha t-1} e^{-\frac{x}{\beta}} dx \\ &= \frac{1}{\Gamma(\alpha t)} \int_{\frac{\rho}{\beta}}^{\infty} \xi^{\alpha t-1} e^{-\xi} d\xi \end{aligned} \tag{6}$$

Furthermore, $F_L(t)$ can be simplified as

$$F_L(t) = \frac{\Gamma(\alpha t, \rho/\beta)}{\Gamma(\alpha t)} \tag{7}$$

where $\Gamma(a, z)$ is the incomplete gamma function, $\Gamma(a, z) = \int_z^{\infty} \xi^{a-1} e^{-\xi} d\xi$. Besides, the PDF of L can also be obtained from (7).

$$\begin{aligned} f_L(t) &= \frac{dF_L(t)}{dt} \\ &= \frac{\alpha}{\Gamma(\alpha t)} \int_0^{\rho/\beta} \left[\ln(\xi) - \frac{\Gamma'(\alpha t)}{\Gamma(\alpha t)}\right] \xi^{\alpha t-1} e^{-\xi} d\xi \end{aligned} \tag{8}$$

However, the results include both the digamma function which is the derivative of the logarithmic gamma function and the generalized hypergeometric function, which will lead to many difficulties in subsequent calculations if differentiate directly the PDF of L. In order to avoid this problem, BS (Birnbaum-Saunders) distribution [26], [27] is used to approximate the distribution of L, which can be described as

$$F_L(t) \approx \Phi\left[\frac{1}{m}\left(\sqrt{\frac{t}{n}} - \sqrt{\frac{n}{t}}\right)\right], \quad t > 0 \tag{9}$$

where $\Phi(\cdot)$ is the standard normal distribution and $m = \sqrt{\beta/\rho}$, $n = \rho/\alpha\beta$. The corresponding PDF of L is written as

$$f_L(t; m, n) = \frac{\left(\frac{n}{t}\right)^{\frac{1}{2}} + \left(\frac{n}{t}\right)^{\frac{3}{2}}}{2\sqrt{2\pi}mn} e^{-\frac{1}{2m^2}\left(\frac{t}{n}-2+\frac{n}{t}\right)}, \quad t > 0 \tag{10}$$

According to (10), submitting that X is a random variable, the following changes are made like:

$$X = \frac{1}{2}\left[\left(\frac{L}{n}\right)^{\frac{1}{2}} - \left(\frac{L}{n}\right)^{-\frac{1}{2}}\right] \tag{11}$$

It can be transformed into:

$$L = m\left(1 + 2X^2 + 2X\sqrt{1 + X^2}\right) \tag{12}$$

It is obvious that $X \sim N\left(0, \frac{1}{4}m^2\right)$. Consequently, the expectation and the variance of L can be obtained by transformations.

$$E(L) = n\left(1 + \frac{m^2}{2}\right), \quad Var(L) = (mn)^2\left(1 + \frac{5m^2}{4}\right) \tag{13}$$

Generally, the reliability function of rubber O-rings can be derived based on (2) and (9).

$$R(t) = 1 - F_L(t) = 1 - \Phi\left[\frac{1}{m}\left(\sqrt{\frac{t}{n}} - \sqrt{\frac{n}{t}}\right)\right], \quad t > 0 \tag{14}$$

B. ACCELERATED DEGRADATION MODEL UNDER SINGLE STRESS

The accelerated performance degradation equation is generally derived from the traditional accelerated model in accelerated life test. The Arrhenius model is the most commonly used accelerated model of single stress when the stress applied to the accelerated degradation test of products is a single accelerated stress. Meanwhile, the Arrhenius model that is used to describe the effect of temperature on the chemical reaction rate of products is usually adopted in the design of accelerated degradation performance test and the analysis of test data with temperature as accelerated stress. So it can be utilized to construct the empirical accelerated degradation equation. The Arrhenius model [28] can be defined as:

$$A(T) = a \exp\left(-\frac{E_a}{KT}\right) \tag{15}$$

where $A(T)$ represents the reaction rate, the absolute temperature T(K), the undetermined parameter a, the activation energy E_a(eV), and K represents the Boltzmann constant.

In the data analysis of accelerated degradation test, $A(\cdot)$ can be indicated the parameters associated with degradation rate in the performance degradation model of a product. For instance, the shape parameter $\alpha(t)$ in the Gamma degradation model conform to this formula. Typically, the Arrhenius model is transformed into the following form

$$A(S) = a \exp\left(-\frac{b}{S}\right) \tag{16}$$

where S is accelerated stress, the thermodynamic temperature should be taken when the temperature stress is adopted and a, b are the undetermined parameters. Therefore, the shape parameter $\alpha(t)$ in the Gamma degradation model can be expressed as

$$\alpha\left(t, S\right) = a\exp\left(-\frac{b}{S}\right)t \tag{17}$$

C. PARAMETERS ESTIMATION

The parameters estimation of the proposed methodology is based on the accelerated test. In the trail, X_{ijk} denotes the j^{th} measurement of the i^{th} sample under the k^{th} accelerated stress level and t_{ijk} denotes the j^{th} measurement time of the i^{th} sample under the k^{th} accelerated stress level. Besides, $\Delta X_{ijk} = X_{ijk} - X_{i(j-1)k}$ stands for the increment of performance degradation and $\Delta t_{ijk} = t_{ijk} - t_{i(j-1)k}$ stands for the increment of time. According to the characteristic of the Gamma process, such can be known as $\Delta X_{ijk} \sim Ga(\alpha\Delta t_{ijk}, \beta)$.

The likelihood function is obtained from the performance degradation data

$$\begin{aligned} L(\alpha(S), \beta) &= \prod_{k=1}^{N_1}\prod_{i=1}^{N_2}\prod_{j=1}^{N_3} f_G\left(\Delta X_{ijk}; \alpha(t, S), \beta\right) \\ &= \prod_{k=1}^{N_1}\prod_{i=1}^{N_2}\prod_{j=1}^{N_3}\left\{\frac{\left(\Delta X_{ijk}\right)^{\alpha(S_k)\Delta t_{ijk}-1}}{\Gamma(\alpha(S_k)\Delta t_{ijk})\beta^{\alpha(S_k)\Delta t_{ijk}}}e^{-\frac{\Delta X_{ijk}}{\beta}}\right\} \end{aligned} \tag{18}$$

where $\alpha\left(S_k\right) = a\exp\left(-\frac{b}{S_k}\right)$, and N_1, N_2, N_3 represents respectively the number of the accelerated stress levels, the number of the trail samples under each accelerated stress level and the number of the measurements of each trial sample.

Then, the log-likelihood function also can be derived as

$$\begin{aligned} &\ln L(a(S_k), \beta) \\ &= \sum_{k=1}^{N_1}\sum_{i=1}^{N_2}\sum_{j=1}^{N_3}\Bigg[\left(\alpha(S_k)\Delta t_{ijk} - 1\right)\ln\Delta X_{ijk} - \frac{\Delta X_{ijk}}{\beta} \\ &\qquad - \ln\Gamma\left(\alpha(S_k)\Delta t_{ijk}\right) - \alpha(S_k)\Delta t_{ijk}\ln\beta\frac{\Delta X_{ijk}}{\beta}]\Bigg] \end{aligned} \tag{19}$$

By substituting the data of $\Delta X_{ijk}, \Delta t_{ijk}, S_k$ into (19) and adopting the approach of maximum likelihood estimation, the estimations of parameters a, b, β can be figured out. Then, the parameter α under different accelerated stress levels can also be calculated.

III. CASE STUDY

A. ACCELERATED AGING TEST AND PARAMETERS DETERMINATION

In the case study, the O-rings of a certain type of gas steering gear were selected as the trial objects. The material of O-rings is silicone rubber which possesses excellent performance of resistance to effect of heat. The major parameters of the O-rings are listed as Table 1.

TABLE 1. The major parameters of the O-rings.

Material	Silicone Rubber
Mounting Mode	Axial Mount
Sealing Mode	Static Seal
External Diameter	25mm
Wire Diameter	3.5mm
Compression Ratio	30%
Operating Temperature Range	0-150°C
Failure Threshold ρ	50%

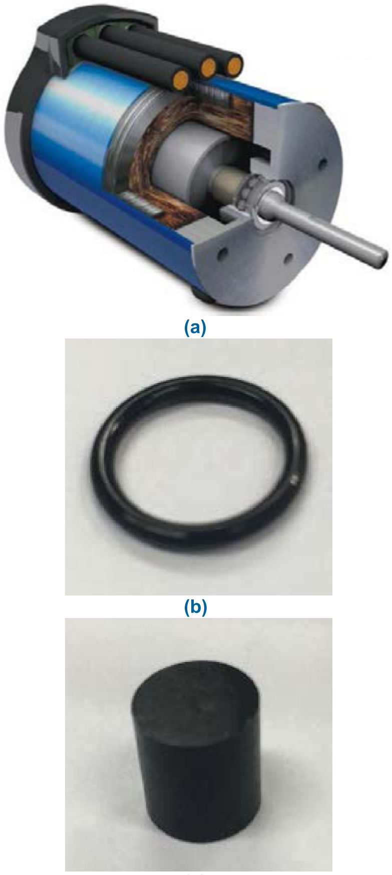

(a)
(b)
(c)

FIGURE 2. The schematic diagram of gas steering gear and rubber O-ring seals. (a) A type of gas steering gear. (b) Rubber O-ring seals in this gas steering gear. (c) Silicone rubber material sample.

Gas steering gear is a missile steering gear system which utilizes the solid powder to produce slow-combustion gas and drives the control surface movement. The essence of the gas steering gear is a kind of gas-type relay control pneumatic steering gear. Under the actual working condition, the working gas is a mixture of gas and air produced by the gas generator. While under the test room condition without electricity, the pressure of the gas rudder is only compressed air and the air pressure is applied from the inlet to the gas steering engine. The pictures of gas steering gear and the rubber O-rings ring are given in Figure 2.

According to the range of the operation temperature and the thermal analysis results of the rubber material using

TABLE 2. The testing time of CS for the rubber O-rings [17].

Number k	Test temperature S_k (°C)	Testing time t_j (day)									
		t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
1	110	1	6	12	18	25	37	49	65	95	126
2	120	1	6	12	18	25	37	49	65	95	126
3	130	1	6	12	18	25	37	49	65	95	126
4	140	1	3	5	7	11	15	21	27	35	44
5	150	1	3	5	7	11	15	21	27	35	44

synchronous thermal analyzer, five-group accelerated conditions at temperatures ranging from 110°C to 150°C were designed. Considering overall both the accuracy and the time costs, thirty O-rings from the same production batch were selected randomly as trail samples in the accelerated aging test.

FIGURE 3. The ageing oven.

First, all trail samples were fitted in the test fixture that consists of upper and lower two rectangle metal plates, ring limiters and binding bolts with 30% compression ratio. Secondly, all samples were removed after compression for one day and recovering the elastic deformation for one day at room temperature. Then, axial thickness of the four distribution points on the circumference was measured for each trail sample with rubber calibrator at room temperature. The mean value of the measurements as the initial value of the sample thickness. Next, the thirty trial samples were divided into five groups averagely and fitted each sample with the same steps as above. Then, the five-group samples that each group had six parallel samples were put in the ageing oven (as shown in Figure 3) at 110°C, 120°C, 130°C, 140°C and 150°C. Finally, each sample was removed at the time as listed in Table 2 [17], cooling and recovering at room temperature for one hour separately, and then was measured the axial thickness. The above steps were repeated until all the samples had been tested for ten times. The CS of O-rings can be calculated as

$$\mathrm{CS} = \frac{D_0 - D_t}{D_0 - D_x} \times 100\% \tag{20}$$

where D_0 stands for the initial value of the sample axial thickness, and D_x stands for the limiter thickness, and D_t stands for the axial thickness of trail samples after accelerated aging, cooling and recovering.

The calculations of CS for each sample are shown in Figure 4 (a)-(e) in which ΔX_{ijk} represents the degradation value of CS of thirty trial samples. From the Figure 4 (a)-(e), it can be obviously seen that the degradation trend of CS of each sample under the five accelerated temperature stresses is monotonically increasing over testing time. Therefore, it is appropriate and accurate to describe the degradation process of CS for the rubber seals with the Gamma degradation model.

By substituting the data of $\Delta X_{ijk}, \Delta t_j, S_k$ into (19) and adopting the method of MLE, the estimations of parameters a, b, β can be figured out

$$\begin{cases} a = 0.053 \\ b = 0.198 \\ \beta = 0.151 \end{cases} \tag{21}$$

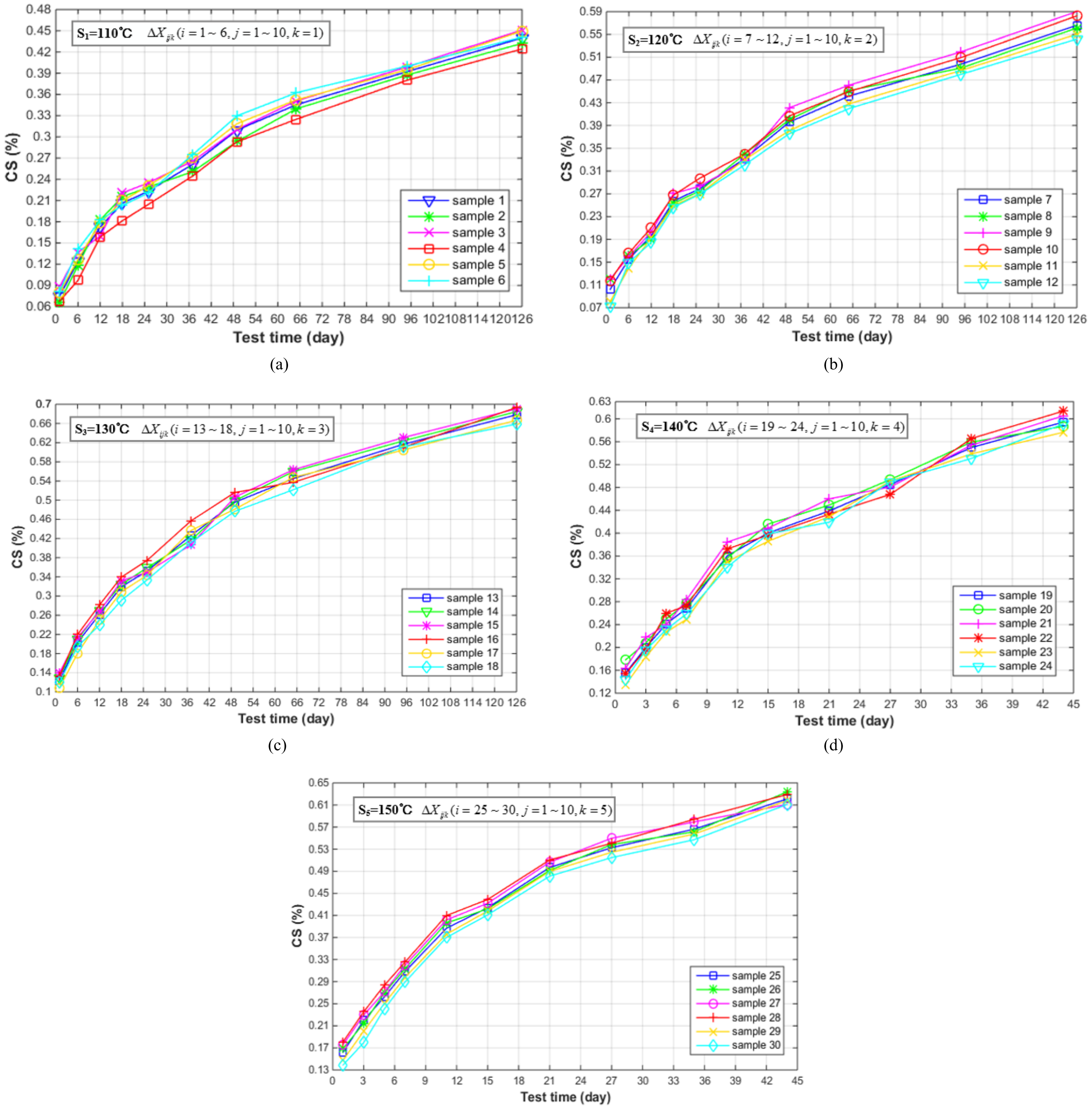
Then, the parameter $\alpha(t)$ can be obtained based on (14)

$$\alpha(t) = 0.053 \exp\left(-\frac{0.198}{S}\right) t \tag{22}$$

B. NUMERICAL RESULTS ANALYSIS

The time-variant reliability curve and numerical results of reliability for the rubber O-rings are elaborated in Figure 5. From the figure, we can see clearly the change of reliability over time at 30°C which is the normal working temperature. What's more, the results are illustrated apparently that the rubber O-rings maintain a high-level reliability between 1 and 0.9976 in 400 days. However, because the rubber O-rings are long-term under the aging and load-bearing conditions, CS of the rubber O-rings degenerates rapidly over time after the 400th day. Furthermore, the unrecoverable elastic deformation leads to a sharp drop of elasticity of the silicon rubber materials, which results in the deterioration of sealing performance for the rubber O-rings. It is obviously seen that, the reliability drops to 0.2994 at the 700th day.

For the sake of better comparing the impact of different temperature stresses on CS of the rubber O-rings in the accelerated aging test, the comparison results at different temperatures are also provided in this paper. The Figure 6 is well demonstrated the variation tendency of computational results at four temperatures. As can be seen from Figure 5, with the rise of the test temperature, the reliability of the rubber



(a) (b) (c) (d) (e)

FIGURE 4. **The CS degradation data of trial samples under five group accelerated stresses. (a) $S_1 = 110°C$. (b) $S_2 = 120°C$. (c) $S_3 = 130°C$. (d) $S_4 = 140°C$. (e) $S_5 = 150°C$.**

O-rings decreases gradually. In other words, the reliability of the rubber O-rings is inversely proportional to temperature. Besides, in the same test time, the higher the test temperature is, the faster the reliability decreases. The reason of this change is mostly that the oxidation rate and heat-degradation rate of silicone rubber materials are accelerated with the temperature rise.

Meanwhile, the calculations of lifetime for the rubber O-rings under different stress conditions are listed in Table 3 based on (11). The lifetime under normal working condition is about 747 days. Moreover, considering the storage temperature in storehouse is generally about 20° the lifetime of the rubber O-rings is approximately 840 days. From Table 3, the accelerating factor (AF) of each accelerated stress is also given by the following equation.

$$AF = \frac{L_{normal}}{L_{stress}} \tag{23}$$

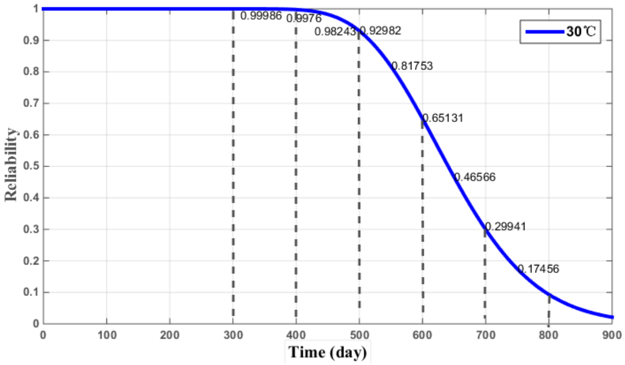


FIGURE 5. The time-variant reliability curve of O-rings at 30°C.

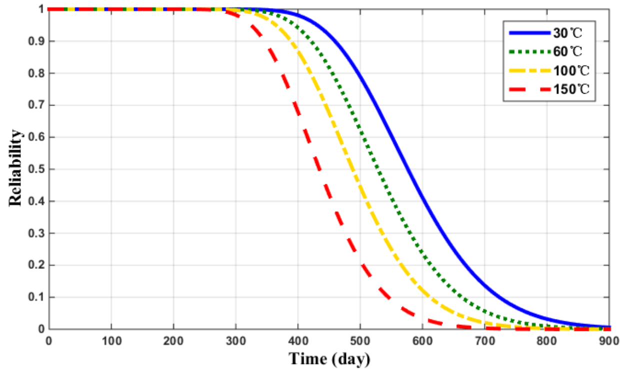


FIGURE 6. The time-variant reliability curve of O-rings at different temperature stresses.

TABLE 3. The lifetime of the rubber O-rings at different temperature.

Temperature (°C)	Lifetime of the O-rings (day)	Accelerating factor
20	940	/
30	747	/
60	672	1.11
100	611	1.22
110	579	1.29
120	560	1.33
130	525	1.42
140	487	1.53
150	473	1.58

where L_{normal} stands for the lifetime of products under normal stress, and L_{stress} represents the lifetime of products under accelerated stress. The AF in the accelerated test represents the effect of the accelerated test. The greater the AF, the more obvious the acceleration effect is and vice versa. In addition, the relationship between AF and temperature is displayed in Figure 7 and the data points are fitted by the binomial function curve. The fitting result can be expressed as

$$AF = 0.000049\mathrm{T}_{stress}^{2} - 0.0049\mathrm{T}_{stress} + 1.227 \tag{24}$$

and the fitting variance is 0.0185 and the correlation coefficient is 0.9959, which illustrates the fitting results are satisfactory.

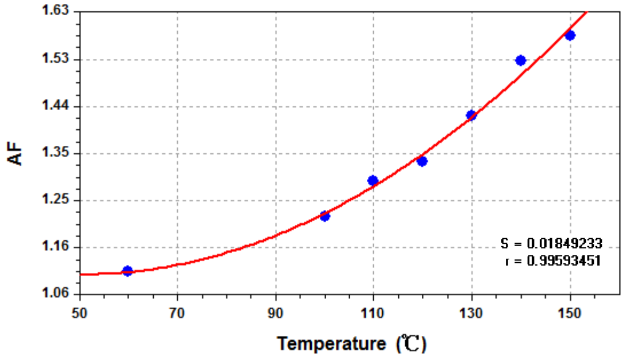


FIGURE 7. The fitting curve of AF under different temperatures.

C. COMPARISION ANALYSIS WITH TRADITIONAL MODEL

In order to compare the prediction results of the reliability for rubber seals based on the Gamma degradation process model, the traditional life prediction model is compared with this method in the paper.

The kinetic curve model is the most commonly used predictive model for predicting the lifetime of rubber materials in the traditional life prediction models. The kinetic curve method is a linear method based on the kinetic curve model through using the kinetic model to describe the performance parameter P changing with time t, and through some transformations, the original curve is transformed into a straight line. Furthermore, the rate constant k under each accelerated temperature stress can be obtained. What's more, the Arrhenius model formula is adopted to extrapolate the value of the rate constant k at normal temperature stress, thus the performance degradation model at normal temperature is established. In the practical application, the commonly used empirical formulas are as follows:

$$\begin{aligned} f(P) &= \exp(-kt) \\ f(P) &= A\exp(-kt) \\ f(P) &= B\exp(-kt^{\alpha}) \\ \xi &= kt^{n} \end{aligned} \tag{25}$$

where if $f(P)$ stands for the CS, then $P = 1 - \xi$; if $f(P)$ stands for other aging performance coefficients, then $f(P) = P/P_0$; k stands for the rate constant of $f(P)$; B, α stands for the constants independent of temperature.

Many researchers have studied the accuracy of the series equations in (25), and it proved that $f(P) = B\exp(-kt^{\alpha})$ is the most accurate in (25), which can be used to calculate the lifetime of products and can also be used to predict the performance change regulation of products. By adopting this equation to fit the degradation data of CS in Figure 4 (a)-(e), the equation of CS varies with time t under actual storage conditions is obtained:

$$P = 1.0945e^{-0.2094t^{0.57}} \tag{26}$$

When the failure threshold of CS is 50%, the storage lifetime of the rubber seals is about 980 days (2.68 years)., which is basically accord with the storage lifetime results of 940 days at 20°C calculated in Table 3. However, the method can only obtain the lifetime of a certain group of data under the temperature stress condition. It is not possible to extrapolate the value of the lifetime index of rubber seals under different temperature stress conditions, so the life prediction method adopted in this paper has more practical application value.

In addition, the reliability model can be extrapolated by using its storage life values calculated from (26), and the reliability function can be expressed as follows:

$$R(t) = \frac{1 - \Phi\left(\frac{t-\mu}{\sigma}\right)}{1 - \Phi\left(\frac{-\mu}{\sigma}\right)} \tag{27}$$

where μ is the expectation of products lifetime and σ the variance of products lifetime. The storage lifetime of rubber seals can be calculated through the kinetic curve model in the range of the specified failure threshold of CS. Then, the expectation and variance of the lifetime of rubber seals can be obtained by using the statistical analysis method. The reliability curve is shown in Figure 8.

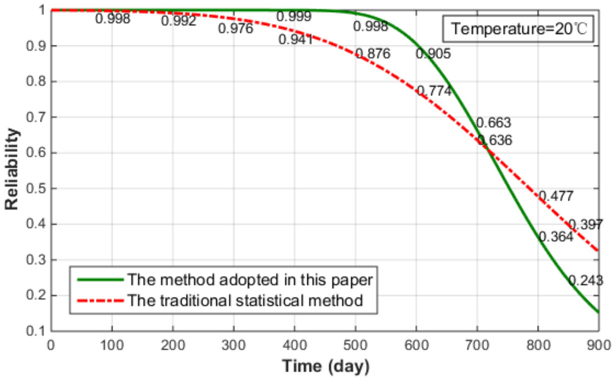


FIGURE 8. The reliability curves of the two method over time.

Compared with the results of the method adopted in this paper, the reliability curve obtained from the traditional statistical method decreases more slowly, and the reliability results under different temperature stress conditions cannot be extrapolated. As can be seen from Figure 7, the reliability curve of the traditional method does not have a rapidly decreasing turning point and the overall downward trend is relatively gentle with time increasing, which is suitable for estimating the aging tendency of the rubber seals in the low-temperature environment, but the traditional model method is not suitable for describing the aging mechanism and the degradation trend of the rubber seals under the low-temperature environment. So the prediction results obtained from the traditional statistical method are more conservative. What's more, the derivation of the reliability model (27) is based on the assumption that the product lifetime is subject to the normal distribution. However, the actual lifetime of rubber seals does not always have the statistical property of normal distribution. Therefore, the traditional statistical method has some limitations.

IV. CONCLUSION

Because of the lack of accuracy and simplicity of traditional statistical model, the research on reliability analysis based on Gamma stochastic process in accelerated degradation test is particularly important. This paper selected Gamma degradation model to describe the degenerative process of CS and the results of the case study proved the rationality of this model. In the case study, a type of rubber O-rings in the missile system were used to carry out the accelerated aging test at constant temperatures ranging from 110°C and 150°C. Meanwhile, the shape parameter in Gamma degradation model was determined by the Arrhenius model, which was well reflected the effect of temperature on the deterioration of CS for the rubber O-rings. Finally, the reliability curve of the rubber O-rings at different temperatures was obtained, which further showed that the temperature has a remarkable effect on the sealing performance of rubber O-rings. In addition, we can also conclude that:

(1) From the degradation trend of CS for rubber seals, it can be known that Gamma degradation model is more compatible to feature the monotone-increasing damage process which is accord with the degenerative process of CS for rubber O-rings.

(2) The selecting of accelerated models is depend on the property of accelerated degradation test, such as the Arrhenius model, the inverse power model and the generalized Aileen model. For instance, the generalized Aileen model is suitable to describe the influence of temperature and humidity on the performance of products in the temperature-humidity double stress accelerated test.

(3) From the fitting curve of AF, it can be known that the AF raises as the increase of accelerated stress. That's to say, the acceleration effect of this accelerated aging test is marked and effective.

(4) According to the comparison with the traditional statistical method, the method in this paper not only could extrapolate the value of the lifetime index of rubber seals under different temperature stress conditions, but also could calculated the value of reliability and lifetime at different times.

For the sake of improving the accuracy of this method, the follow-up work will take the randomness and variability of trial samples into account by combining Wiener degradation model and Gamma degradation model.

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